Actuarial, Financial and Statistical Aspects of Dependencies in Insurance and Financial Portfolios (GOA 2007/2011)

Deel 1: Wetenschappelijke vorderingen van het GOA-onderzoek

We developed part of the new theoretical framework. Actuarial risks are described mathematically by series of future cash flows, i.e. series of random future losses to be paid at random future times. The value of a series of cash flows is determined by analyzing and modeling the cash flows themselves, in addition to choosing a model that reflects the time value of money and adapting the cash flow payments for the cost of risk. We investigated concepts and developed probabilistic and statistical tools to analyze various cash flow streams that emerge in an actuarial and/or financial context. The research fields involved are actuarial science, finance, statistics and probability theory.

Modeling dependency structures.

In general, series of actuarial cash flows exhibit two main sources of randomness: the amounts of the cash flows and the timing when these amounts will be paid out. The randomness concerning the amounts can further be decomposed in the randomness concerning the frequency (incidence) and the randomness concerning the severity (claim size). The actuarial valuation of a series of cash flows involves the choice of time-dependent discount factors. These discount factors may be deterministic or random. In the latter case, a third source of randomness appears.

Multiplying the respective cash flow amounts by their appropriate discount factor and summing these discounted amounts leads to a univariate random variable, which is called the stochastic present value or also the actuarial loss random variable. Describing the randomness of this loss variable by determining its distribution function is in general a very complicated task, mainly because of the mutual dependency between the different terms in the sum involved. This dependency is caused by the dependency that may exist between the amounts and timing of payments, but also because the stochastic discount factors over different periods exhibit a dependency.

We investigated the dependence structure between the random variables described above, a topic that is crucial in valuating actuarial cash flows. We extended the results of the previous GOA, namely the class of multivariate distribution functions or stochastic processes for which one can use the comonotonic approximation technique, e.g. the class of elliptical distributions and the class of Lévy processes.

Risk measures and risk principles.

Modeling actuarial and financial risks as described in the previous subsections is only a first step in the valuation process. The next step consists of attaching one single real number to the underlying stream of cash flows. Nowadays, such a real number is commonly called a risk measure in the literature.

We have further investigated a distinction between “axiomatic risk measurement” and “optimal risk measurement”. The difference between these two classes of risk measures stems from the different levels on which they operate. An axiomatic risk measure is a functional that assigns a real number to a random variable (a risk), based on a given set of axioms. An optimal risk measure is a “derived” functional that assigns a real number to a random variable. It appears as a solution of an economic optimization problem (such as minimizing a cost function).

Any decision process will in general be based on both axiomatic and optimal risk measures, introduced in the process at different levels. In order to illustrate this procedure, consider the problem of determining the optimal solvency capital requirement from the regulator’s point of view. A first risk measure is introduced to describe the total cost (which is the sum of the insolvency cost and the cost of capital) for any possible level of solvency capital. The optimal solvency capital is then determined as the one that minimizes the cost function under consideration. In this example the axiomatic risk measure involved is the cost function. We obtained several results in the framework of solvency capital requirements.

Cost functions are introduced that are a compromise between the conflicting criteria of the parties involved (regulator, management, fiscal authority, consumer and/or rating agency). The regulator’s point of view is introduced in the model by considering the shortfall, whereas management’s point of view is introduced by taking into account the cost of capital. Introducing cost functions that take into account simultaneously the different points of view of the parties involved will allow us to set up a more realistic risk measurement framework for insurers than the mechanisms that are considered so far in the literature and that are mainly based on a single party point-of-view.

Analytical results versus simulation.

Asset Liability Management (ALM) techniques include simulation of cash-flows of liabilities (claims to be paid minus premiums to receive) and investments. From these simulations, in principle, one constructs a simulated distribution of the present value of the liability cash flow stream.

By simulating different paths, depending on the cash flows and the model that is used to discount the future payments, different trajectories are obtained. As a consequence, the distribution of the cash flow arises and the initial provision can be determined as a given percentile, e.g. the 99 - percentile.

A drawback of Monte Carlo simulation is its high computational cost, especially in a multi-dimensional setting. This will especially be the case in the above mentioned application where one is interested in a high percentile of the discounted cash flow (Value-at-Risk) or even in the whole far upper tail of the underlying distribution function (Tail Value-at-Risk).

To avoid simulation, we examined how to construct a convex order bound by replacing the copula by the comonotonic copula. This comonotonic upper bound can be computed very quickly, but it often gives only a rough approximation. Therefore, we have also developed more accurate bounds based on the idea of not only transforming the copula in the comonotonic copula but also changing the marginals involved.

In the next period we will start the implementation of our theoretical results in insurance and financial models.

Fore more details concerning the output we refer to the list of our publications.