Network for Studies on Pensions, Aging and Retirement

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Public Investment Funds and Value-Based Generational Accounting

Discussion Paper 01/2009 - 003

January 21, 2009

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This version: January 21, 2009

Abstract

We demonstrate how the methodology of value-based generational accounting reveals the position of various generations for any institutional arrangement sharing revenues and losses with current and future generations. The illustration in this section is based on a stand-alone pension fund with intergenerational risk sharing, but can also be applied to sovereign wealth funds and public finance. Changes in the stylized pension contract may easily lead to sizeable intergenerational value transfers as the allocation of risk amongst stakeholders changes substantially. We evaluate three types of policy reforms that are closely related to pension reforms in the past and current debate about pension reform: improvement of solvency risk management, the effect of a more conservative investment mix due to an ageing society, and challenges of life cycle theory

Keywords: Sovereign Wealth Fund, public investment funds, public pension funds, Value-based ALM, generational accounting, embedded options, intergenerational transfer, stochastic discount factor.

1 Introduction

Governments in various countries are holding large pools of resources managed for public goals. These pools are known under different names, like sovereign wealth funds, public investment funds, pension funds, saving funds, intergenerational funds, and the more, reflecting their different historical roots and orientation. Mitchell *et al.* (2008) make a distinction between three types of publicly held funds, or of public investment funds as they call these funds. First reserve funds held for currency stabilization and macroeconomic stabilization purposes. Secondly sovereign wealth funds (SWFs) accumulated from natural resource taxes or from fiscal surpluses aimed at sharing the revenues of the exploitation of natural resources with future generations (Norway, Kuwait, Abu Dhabi). Third, public pension funds built up either through an explicitly funded arrangement or the result of prefunding the foreseeable increase in social security benefits because of ageing (Japan, Canada).

*The views expressed here are those of the authors only and not necessarily of the institutions with which they are affiliated.

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SWFs and the public pension funds have in common an intergenerational dimension in their objectives (Mitchell *et al.* (2008) and Paulson (2009)). Questions faced by SWFs can be handled with insights derived in other fields. Universities and other private endowment funds provide guidance to public funds regarding the optimal spending rules. Lee *et al.* (2009) for example explore for a typical SWF how generational equity can be realized by coordination of savings, asset allocation strategies and spending rules. The challenge for a typical SWF is to define a strategy that can offer a relatively high level of spending while affording as much stability to both spending and ongoing fund value as possible. Furthermore Scherer (2009) provides a linkage between oil-funded SWFs and life cycle investing by interpreting the asset allocation problem of a SWF as the decision making problem of an investor with a non tradable endowed asset (oil reserves).

If generational equity is an important goal for SWFs and public pension funds, then one necessarily have to raise the question how to assess a certain strategy as fair or not. Which path of wealth accumulation, asset allocation and spending over time is desirable from the perspective of various generations. This is a difficult topic belonging to the field of normative economics. Moreover there are large uncertainties in revenues, financial market rates of return, domestic economic growth, and so on. These factors all have impact on the welfare position of the various generations. Which discount factor has to be used to compare the benefit position of the various generations?

In previous papers we have discussed for real-existing pension funds the usefulness of what we have called the tool of value-based ALM. This tool is a complement to the traditional ALM and it appears to be very useful to compare the positions of the various generations from the perspectives economic value (Ponds (2003) and Hoevenaars and Ponds (2008)). Main advantage of value-based ALM is its objective nature as the method is rooted in market prices and that risk can easily be handled by the use of stochastic discount factors.

We claim that the value-based ALM tool is a useful tool for SWFs and public pension funds aiming at realizing generational fairness when setting the path of wealth accumulation and spending over time. What we do in this chapter is to explain the tool of value-based ALM. Subsequently we apply the tool to a stylized pension fund considering policy changes. This pension fund is a stand-alone pension fund in which all risks now and in the future have to be borne by current and future generations. We reconstruct this fund into an aggregate of embedded generational options by making us of value-based generational accounting. Finally we discuss the helpfulness of the value-based tool for SWFs intending to establish generational equity.

2 Asset Liability Management

2.1 Classical ALM

In the pension industry, Asset Liability Management (ALM) is being used to come to optimal pension deals. A typical pension deal defines what is being promised, how these promises are funded (asset mix and contribution policy) and who of the stakeholders of the deal is bearing the risks in the funding process (risk allocation rules). Members of the Board of Trustees of the pension funds have to decide what the optimal combination is of funding strategy, indexation policy and investment strategy, as well as how risks best can be shared over the various stakeholders, i.e. the plan members and the employer. ALM often make use of Monte Carlo scenario simulations to project distributions regarding contributions, indexation and funding ratios to form an opinion on the attractiveness of the different strategy options being considered. Sensitivity analysis is usually carried out to explore specific policy variants in terms of asset mix, contribution policy and indexation rules. Policy variants are evaluated in terms of expected values and relevant risk measures for key variables—for example, the funding ratio, the contribution rate, the indexation rate, and so on. Moreover one can easily take care of specific constraints, like funding requirements of the supervisor (e.g., a minimum probability of underfunding) and a maximum level of contribution rate.

2.2 Value-based ALM

Despite its widespread popularity, one may feel uncomfortable with the classical ALM tool kit. Chapman *et al.* (2002) characterize ALM studies as producing merely "funnels of doubt", which serve only to demonstrate that taking more risk will imply more uncertainty about key outputs. It is difficult to rank policy variants using solely the classical ALM output. Is a risky strategy with, on average, a high but volatile funding ratio to be preferred above a less risky strategy that will end up with, on average, a low funding ratio with little risk? Younger members in a plan with intergenerational risk sharing may prefer a risky strategy that could yield a high pay-out per unit contribution, whereas older members will prefer a liability-hedged investment strategy to safeguard pension fund assets in order to reduce benefit risk. Practitioners solve the ranking problem by discovering the policy setting that is most acceptable given the interests of all participants, taking into account all constraints. However in seeking this 'most acceptable' policy variant, the ALM professionals and/or the board of trustees do not usually consider whether the policy variant is fair in economic value terms for all members. In financial markets, the no arbitrage principle guarantees that the market-based compensation for a taken risk is fair, so that risk taking is accompanied by an appropriate reward compensation. Within pension funds, the no arbitrage principle of financial markets is replaced by the rules of the pension contract defining the risk and reward allocation amongst the members.

Value-based ALM can deal with the fact that indexation cuts and contribution increases typically occur in bad times. Therefore these financial adjustments are very valuable. In good times funding ratios and return of financial assets will be high giving room for contribution cuts and additional (catch-up) indexation. However these adjustments probably are not that valuable in good times.

Contingent claim analysis is fruitful to test for possible value transfers. Restating the highly stylized framework of Sharpe (1976) into a realistic setting results in what is now called value-based ALM (Kortleve and Ponds (2006) and Hoevenaars and Ponds (2008)). Value-based ALM essentially uses the same output from scenario analysis as classical ALM, but the future outcomes are discounted back to the present with an appropriate risk adjusted discount rate. This is achieved by making use of a pricing kernel specification (see section 3.3 for technical details): low discount rates are assigned to bad scenarios, whereas high discount rates are assigned to good scenarios. This reflects the prevailing risk aversion in the market which implies that payoffs during bad times are more valuable than payoffs during good times. What we show in this paper is that value-based generational accounting enables us to rewrite any institutional generational arrangement, for example a pension fund or a sovereign wealth fund, can be rewritten as an aggregate of embedded generational options. This allows us to detect possible transfers of value resulting from policy changes by examining changes in the value of the various embedded generational options.

3 The pension fund as an aggregate of embedded generational options

3.1 Pension fund characteristics

Before deriving generational accounts in a pension fund setting, it is be useful to describe the specific institutional characteristics of the pension fund. All funding risks must be borne by current and future members of the pension plan. The content of the pension contract helps determine how surpluses and deficits in the funding process are allocated amongst participants. Essentially there are four ways to allocate the funding risks amongst the participants: (i) doing nothing by shifting forward in time a position of underfunding or overfunding, i.e. passing the gains/losses to future participants; (ii) adjusting the contribution rate; (iii) adjusting the indexation rate; or (iv) a reallocation of the asset mix.

The fund under study has the following features: The pension plan is an average-wage plan with indexed liabilities. Workers acquire for each year of service 2% of their pensionable wage as new accrued liabilities. The yearly indexation of benefits and accrued liabilities aims to follow the wage growth of the sector; however the actual indexation may be contingent on the financial position of the pension fund. Workers pay yearly contributions out of their wage income in order to fund new accrued liabilities in that year. Total contributions must be equal to the present value of new liabilities, where new liabilities are calculated with the expected rate of return on assets net of wage growth as the discount rate. All workers pay the same uniform contribution rate as a percentage of pensionable wages[1]. We assume a constant mix rebalancing policy in which the investment manager rebalances to fixed asset weights at the end of each year. The investment universe consists of stocks and bonds only. The duration of the nominal liabilities is 15 years (at a nominal rate of 4.6%) and the investment horizon is 40 years. The nominal funding ratio equals 150% at the start of the analysis.

3.2 Generational accounts as embedded options

The value of pension fund assets A_t is equal to the value of total pension fund nominal liabilities L_t plus the pension fund residue R_t

$$A_t = L_t + R_t \tag{1}$$

The balance sheet next period $(t+1)$ expressed in present value terms at t is

$$A_t + V_t\left[C_{t+1}\right] - V_t\left[PP_{t+1}\right] = V_t\left[L_{t+1}\right] + V_t\left[R_{t+1}\right] \tag{2}$$

Inherent to the stochastic discount factors approach (see e.g. Cochrane (2001)) is that the economic value of initial assets plus investments proceeds is equal to initial assets: $V\left[A_t(1+r_{t+1})\right] = A_t$ with

[1]This implies that young workers pay more contributions than the present value of their new accrued liabilities, whereas older workers contribute less than their new accrued liabilities. Younger workers grow older so that at the end of their careers there will be a balance between the value of paid contributions and the value of accrued liabilities.

r_{t+1} as rate of return in $t+1$. The term $V_t[C_{t+1}]$ is the economic value at t of contributions C_{t+1} paid in $t+1$ and $V_t[PP_{t+1}]$ is the economic value at t of pension payments PP_{t+1} in $t+1$. The term $V_t[L_{t+1}]$ stands for the economic value of accrued liabilities at the end of period $t+1$, being the sum of the accrued liabilities at the end of period t, including indexation *minus* the liabilities written off in $t+1$, as they have been reserved for pension payments in $t+1$ *plus* the new accrued liabilities in $t+1$ attributable to one year of additional service of working members. The term $V_t[R_{t+1}]$ is the economic value at t of the pension fund residue at the end of period $t+1$, R_{t+1}.

Using (1), we can rearrange (2) as

$$V_t[L_{t+1}] - L_t + V_t[PP_{t+1}] - V_t[C_{t+1}] + (V_t[R_{t+1}] - R_t) = 0 \tag{3}$$

This expression says that the one year change in the value of liabilities is backed by contributions and by either an increase or a decrease in the pension fund residue. This reflects the zero-sum nature of a pension fund. However, the zero-sum feature does not hold necessarily for the different age cohorts. We can split up expression (3) by age cohort. This results in

$$\Delta GA^x_{t+1} = V_t\left[L^x_{t+1}\right] - L^x_t + V_t\left[PP^x_{t+1}\right] - V_t\left[C^x_{t+1}\right] + \left(V_t\left[R^x_{t+1}\right] - R^x_t\right) \neq 0 \tag{4}$$

where x refers to cohort x. We call the term ΔGA^x_{t+1} the generational account option of cohort x, that is defined as the economic value at t of the change in the generational account of cohort x during $t+1$.

We assume that the pension fund residue can be allocated amongst the cohorts at all times proportionately to each cohorts's stake of nominal liabilities :

$$R^x_t = l^x_t R_t \tag{5}$$

with

$$l^x_t = \frac{L^x_t}{L_t} \tag{6}$$

The sum of all generational account options must be necessarily equal to 0, reflecting that the pension fund is a zero-sum game in value terms:

$$\sum_{x\in X} \Delta GA^x_{t+1} = 0 \tag{7}$$

We can split up ΔGA^x into two parts: the so-called net benefit option NB^x and the residue option ΔR^x:

$$\Delta GA^x_{t+1} = \underbrace{V_t\left[L^x_{t+1}\right] - L^x_t + V_t\left[PP^x_{t+1}\right] - V_t\left[C^x_{t+1}\right]}_{NB^x} + \underbrace{\left(V_t\left[R^x_{t+1}\right] - R^x_t\right)}_{\Delta R^x} \tag{8}$$

The net benefit option consists of the change in the value of liabilities $V_t\left[L^x_{t+1}\right] - L^x_t$ due to new nominal accruals and the writing off of planned nominal pension payments, plus the value of actual pension payments $V_t\left[PP^x_{t+1}\right]$ including indexation, and the value of paid contributions $V_t\left[C^x_{t+1}\right]$.

The net residue option says that a cohort gives away the certain claim on the current residue R^x_t by participating in the fund and it receives an uncertain claim on the residue at the end of the evaluation period R^x_{t+1}, with economic value equal to $V_t\left[R^x_{t+1}\right]$.

Below, we compare some alternative policy variants to study the impact on the generational accounts of cohorts. This comparison is based on the expression:

$$\Delta GA^{x}_{alternative} - \Delta GA^{x}_{basic} = (NB^{x}_{alternative} - NB^{x}_{basic}) + (\Delta R^{x}_{alternative} - \Delta R^{x}_{basic}) \quad (9)$$

Stepping over from the current pension contract to an alternative one may lead to a change of the generational account option of cohort x, and this can be split up into changes in the net benefit option and the residue option held by cohort x.

3.3 Pricing embedded options

The ALM framework is based on a simulation study which projects the development of the pension fund in many future scenarios. The investment universe consists of MSCI world stocks and (German) nominal 10 years zero coupon bonds. Furthermore we assume that wage inflation equals price inflation, so that real wage growth is zero[2].

In accordance with Campbell and Viceira (2002), we describe the return dynamics by a first-order vector autoregressive (VAR) model. The relevant economic factors $\mathbf{X}_t$ in the model include the 1-month interest rate ($y_t^{(1)}$), the 10-year zero coupon rate ($y_t^{(120)}$), price inflation (π_t), stock returns in excess of the 1-month interest rate (xs_t), and the corresponding dividend yield (dy_t).

$$\mathbf{X}_t = \left[y_t^{(1)}, y_t^{(120)}, \pi_t, xs_t, dy_t \right]^{\intercal}$$

Returns on a rolling 10-year constant maturity zero coupon bond portfolio are constructed from the nominal term structure. The VAR is written as

$$\mathbf{X}_{t+1} = \Phi_0 + \Phi_1 \mathbf{X}_t + \mathbf{\Sigma}\zeta_{t+1} \quad (10)$$

where $\zeta_{t+1} \sim N(0, I)$. In order to value the embedded options, we specify the pricing kernel as

$$-\log M_{t+1} = y_t^{(1)} + \frac{1}{2}\lambda_t^{\intercal}\lambda_t + \lambda_t^{\intercal}\zeta_{t+1} \quad (11)$$

where M_{t+1} is the stochastic discount factor (SDF) for valuation of embedded options in the pension deal and λ_t are the time-varying prices of risk. λ_t is a linear function of the state variables $\mathbf{X}_t$

$$\lambda_t = \lambda_0 + \mathbf{\Lambda}_1 \mathbf{X}_t \quad (12)$$

The pricing kernel defined in (10) to (12) is thus consistent with the VAR return dynamics and results in arbitrage free term structures (see the Appendix). This approach ensures that for each stochastic scenario besides the results for the $\mathbf{X}_t$ series also a series for the SDF is generated. For further technical details, we refer to Nijman and Koijen (2006) and Brennan and Xia (2002).

Summary statistics of the data and scenarios are provided in Table 1. Monthly data (1972:09-2008:08) are used to estimate the parameters in the VAR system. MSCI world stock returns[3] and

[2]The assumption of a real wage growth of zero avoids the problem of valuation in an incomplete market. As there are no wage-indexed assets, risk relating to real wage growth is not priced into the market. De Jong (2008) discusses several methods to value wage-indexed cash flows in an incomplete market.

[3]The returns are in Deutsch Mark / Euros, the USD exposure is fully hedged

dividend yield are from Factset, German interest rates are from the Deutsche Bundesbank and the German inflation is from Datastream. Stochastic scenarios are constructed by forward iterating the VAR. The views in Table 1 are used for the median of the scenarios.

The ALM model provides both classical and value-based outcomes. The classical results include probability distributions for the relevant ALM output variables as e.g. the distribution of the nominal funding ratios, indexation characteristics and probabilities of underfunding. We value embedded options in the pension contract using the pricing kernel specification (10) to (12). Low discount rates are assigned to bad scenarios, whereas high discount rates are assigned to good scenarios. This reflects the prevailing risk aversion in the market which implies that payoffs during bad times are more valuable than payoffs during good times. Multiplication of the future payoffs k periods ahead (P_{t+k}) by the corresponding SDF ($M^*_{t+k} = M_{t+1}M_{t+2}\cdots M_{t+k}$) and averaging over all scenarios gives the current economic value $V_t\left[P_{t+k}\right]$ (i.e., the option value) embedded in the pension contract:

$$V_t\left[P_{t+k}\right] = E_t[M^*_{t+k}P_{t+k}]$$

4 Empirical Results

Table 1 reports the summary statistics for the data. The Dickey-Fuller OLS F-statistic fails to reject the unit root at the 10% level for both interest rates and the dividend yield. We do not correct for the unit roots due to economic reasoning: it is consistent with the literature (e.g. Campbell and Viceira (2005) and Brennan *et al.* (1997)). Also the Dickey Fuller test statistic has a too low power.

Insert Table 1 here

Table 2 reports the parameters $\mathbf{\Phi}_1$ of the VAR model in (10) and the correlations and volatilities of the shocks ζ_t. The maximal eigenvalue of the coefficient matrix is 0.987. The system is stable, i.e. shocks to the system will dampen through time. The dividend yield has a significant predictive power for the stocks returns. The negative correlation between the shocks to the dividend yield and the stocks return indicates that a positive shock to the dividend yield implies a negative shock to the contemporaneous stocks return. The positive exposure of stocks to the lagged dividend yield implies that the future stocks returns will be affected positively. These effects imply mean reversion in stock returns. The high and significant coefficients of the 1-month rate, the 10-year rate and the dividend yield on their lagged values reflect the high persistence of these series (this is also reflected in the low values of the DF-test in Table 1).

Insert Table 2 here

Figure 1 shows the annualized conditional volatility of the cumulative holding period returns[4]. The results confirm the findings of Hoevenaars *et al.* (2008): stocks and bonds show mean reversion, while a strategy of monthly investing in the 1-month rate is more risky in the long run. Stocks are less risky in the long run due to the negative correlation between the shocks to dividend yield and stocks

[4] We refer to Campbell and Viceira (2005) for the technical details on the derivation of the term structures of risk

and the positive exposure of stocks to the lagged dividend yield: a positive shock to the dividend yield implies a negative shock to the contemporaneous stocks return but a positive effect on future stocks returns. Shocks in the 10-year yield are negatively correlated to contemporaneous bond returns, but positively to future bond returns, whereas the shocks to the 1-month rate are negatively correlated to both the current and the future returns. The mean reverting effect of the long yield dominates, as the 10-years yield is more persistent than the 1-month rate. The increasing risk of the 1-month rate reflects the reinvestment risk due to the persistent variation of both the inflation and short term real interest rate.

Insert Figure 1 here

Figure 2 shows the horizon dependent correlations with stocks. The diversification between stocks and bonds changes over time: in the short and long run, the correlation is low, however in the medium term it increases to 0.55. If the 10-years yield increases unexpectedly, the bond return will decline immediately, whereas the contemporaneous effect on the stock return is small (the correlation is -0.10). The subsequent effects on the bond return are positive as the bond profits from the higher yield, whereas the majority of the decrease of the stock return will occur in the next periods (the exposure is -4.08).

Insert Figure 2 here

Figure 3 shows the inflation hedging qualities of the assets: the 1-month rate is the best instrument to hedge unexpected inflation shocks. Bonds will suffer for several years from unexpected inflation increases, however they will profit ultimately from the higher yields. Increasing inflation will lead to lower real economic activity and this leads to lower stock returns. In particular, unexpected inflation is related to negative output shocks, which generally lead to falling stock prices through higher discount rates. The long run inflation hedge potential can be explained by a present-value calculation of real stock prices. The higher inflation will increase future dividends, which will boost stock prices in the long run (see e.g. Campbell and Shiller (1988)). The net effect on the long run is negative.

Insert Figure 3 here

5 Value transfers in an intergenerational fund

We demonstrate how the methodology of value-based generational accounting reveals the position of various generations for any institutional arrangement sharing revenues and losses with current and future generations. The illustration in this section is based on a stand-alone pension fund with intergenerational risk sharing, but can also be applied to sovereign wealth funds and public finance. Changes in the stylized pension contract may easily lead to sizeable intergenerational value transfers as the allocation of risk amongst stakeholders changes substantially. We evaluate three types of policy reforms that are closely related to pension reforms in the past and current debate about pension reform: improvement of solvency risk management (section 5.1), the effect of a more conservative investment mix due to an ageing society (section 5.2), and challenges of life cycle theory (section 5.3).

As our main purpose is about the intergenerational transfers we focus on the embedded options. The classical ALM output is in line with expectations: if we e.g. change the 50% stocks and 50% bonds portfolio to a 100% bonds portfolio (see section 5.2), the median nominal funding ratios will be lower and the volatility of the assets vs. the nominal liabilities will also be lower.

5.1 Improvement of solvency risk management after the 2001-2003 pension crisis

Falling stock returns and declining interest rates during 2001-2003 led to a revision of financial assessment frameworks for banks, insurance companies and pension funds around the world. Fair value accounting and solvency risk management became key aspects of the financial policy of pension funds. Pension funds in the Netherlands also responded by switching from a traditional defined benefit system to a hybrid system where the inflation compensation was no longer guaranteed. In general, the indexation policy became conditional on the funding ratio as shown in Figure 4.

Insert Figure 4 here

We explore how changes in pension plan design influence the embedded options held by the various age cohorts. Along the lines of the previous section, we decompose changes in the generational account options into changes in the underlying net benefit options and residue options as demonstrated in (8). The contribution rate remains fixed in all variants at 19%.

The replacement of the full indexation rule by the hybrid plan leads to value transfer from older generations to younger ones. Figure 5 displays the accompanying changes in embedded options for each cohort. The introduction of the hybrid plan implies that an unconditional indexation policy is replaced by a conditional indexation policy. All members lose value from this change in indexation policy. Typically, indexation cuts will occur in bad times and these cuts will then be very valuable. Catch-up indexation is provided in good times when the funding ratio is high; however, this additional indexation in good times is not as valuable in value terms. The flexible indexation has substantial effects on both the residue and net benefit option. The residue option improves, because the downside risks reduce, but at a cost of the net benefit option which deteriorates for all cohorts. The overall impact on the generational accounting option differs considerably between young and old plan members. This option improves strongly for the younger workers, whereas the older workers must accept a severe deterioration. The net changes turn negative from the current age of 32 onwards. Hence for workers older than 32, the loss in value due to a conversion to a conditional indexation policy more than outweighs the gain in value attributable to the lower downside risks due the conditional indexation. Workers younger than 32 have a longer planning horizon and thus a longer time for the indexation policy to catch up. The improved downside risks conditions are more valuable for them than the potential indexation cuts[5].

Insert Figure 5 here

[5] The contribution rate has no effect here because it remains fixed. Hoevenaars and Ponds (2008) show that all workers benefit from the replacement of the flexible contribution rate in a traditional DB plan by the fixed contribution rate in the hybrid plan. Workers no longer lose value due to contribution increases in bad times

5.2 Ageing society and a conservative investment mix

Most collective pension funds face problems over the coming years due to an ageing population and demographics. Mature funds may shift the focus of the investment policy to the interest of the elderly. Life cycle theory suggests a more conservative mix to safeguard payout benefits of the retirees. However, such a conservative mix might conflict with common financial planning advice for the younger plan members. An important question, arises as to who is bearing the additional risk? When a different asset mix will leads to a different financial position and indexation policy, then value transfers will inevitably occur from workers to retirees.

The analysis above has been evaluated using an asset mix of 50% bonds and 50% stocks. Figure 6 demonstrates the impact on the hybrid plan of moving the asset mix toward 100% bonds. As the contribution rate does not change in this setting, the return on the assets falls below the return on the liabilities in many scenarios. The significant deterioration of the residue option reveals the higher underfunding risks for all cohorts. On the other hand, the elderly benefit the most in terms of the net benefit option. Not only the lower volatility of the assets mix, but also the comfortable initial funding ratio contribute to this. Younger members still benefit from the lower volatility, however, they have less upside potential because the financial position deteriorates over the years. On balance the switch to a more conservative asset mix leads to a value distribution from young (younger than currently 35) to elderly.

Insert Figure 6 here

5.3 Challenges of life cycle theory

The theory of life cycle investing suggests that a uniform risk profile over the life cycle is not optimal from a theoretical perspective. When the human capital has a bond-like nature, the depletion of human capital over the life cycle introduces horizon effects in the optimal equity allocation. Below we propose an age-dependent indexation rule in combination with benefits of collectivity and risk-sharing(see Figure 4).

The proposed life cycle policy leads to substantial value transfers from young (younger than currently 40) to elderly (see Figure 7). Just as in the first setting, retirees benefit from the full wage inflation compensation. All members benefit from the higher upward potential of the indexation policy, but are depending on their age exposed to possibly severe downside risks. Low and even negative return indexation coincides with bad economic scenarios which have a high stochastic discount rate for the payoffs of the indexation and especially the residue option. Especially, for the younger participants the residue option and the net benefit option interact over the planning horizon. The residue option becomes worse as the fund matures, because retirees are guaranteed full indexation at the cost of the other members which leads to a deterioration of the financial position of the fund. The results suggest that the downside risks outweigh the upward potential the most for participants who are currently in their twenties.

Insert Figure 7 here

The proposed life cycle policy partly restores the value transfers from elderly to the young from the replacement of the full indexation system with the hybrid system after the previous pension crises. Interestingly, the value transfers from switching to a life cycle variant are in same direction as a switch to a more conservative asset mix in an ageing society. As the value transfers measured by the changes in the generational account options are much higher in magnitude in the life cycle variant than in the other two illustrations, the parameters of the life cycle elements in institutional arrangements should be twisted to find an acceptable set of value transfers between the current and future generations. For instance the change in the net benefit option will be positive for all generations and will resemble the change in Figure 6 if the life cycle policy is combined with a reduction of the contribution rate of 2 %-points in order to profit from the increase of the expected indexation over the horizon. Other policy settings like the addition of a collar construction to the life cycle variant seem worth investigation to reduce downside risks. Value based generational accounting can be used to accomplished this.

6 Conclusion

Sovereign wealth funds and other public funds often have aims at sharing revenues and losses with future generations. The background of these funds varies considerably, giving a broad spectrum of government-backed funds ranging from prefunding the ageing-driven increase in social security costs to a fair distribution of oil wealth over generations.

A challenge for the funds is to arrive at a generational fair policy. The decision-making by public fund management necessarily has to take place within a long horizon context with large uncertainties regarding economic growth and capital market returns. Therefore the welfare position of the various generations is difficult to evaluate. Setting a generational fair strategy regarding the funding path, the asset allocation and the spending path over time is also hard. The traditional ALM tool is of limited value as the ALM output of probability distributions provides no guidance in ranking the various strategies. We have proposed the method of value-based generational accounting to get an objective view on the consequences of different strategies for the position of the various generations. The method is rooted in market-based valuation wherein all cash flows from and to current and future generations can be compared by making use of stochastic discount factors. The positions of the various generations can be expressed in terms of the embedded generational options. These options can be derived as an institutional arrangement covering current and future generations has to be seen as a zero-sum game in economic value terms. A change in strategy can be evaluated on its generational effects by looking at the changes in the embedded options held by the various generations.

Appendix

The development of the state variables $\mathbf{X}_{t+1}$ is given by the VAR(1) model

$$\mathbf{X}_{t+1} = \Phi_0 + \Phi_1 \mathbf{X}_t + \mathbf{\Sigma} \zeta_{t+1} \tag{A-1}$$

where $\zeta_{t+1} \sim N(0, I)$. The pricing kernel is log normal

$$-\log M_{t+1} = y_t^{(1)} + \frac{1}{2} \lambda_t^{\intercal} \lambda_t + \lambda_t^{\intercal} \zeta_{t+1} \tag{A-2}$$

where M_{t+1} is the stochastic discount factor and λ_t are the time-varying prices of risk. λ_t is a linear function of the state variables $\mathbf{X}_t$

$$\lambda_t = \lambda_0 + \mathbf{\Lambda}_1 \mathbf{X}_t \tag{A-3}$$

The pricing equation for a n-period zero-coupon treasury bond is given by

$$P_t^{(n)} = E_t \left[M_{t+1} P_{t+1}^{(n-1)} \right]$$

or in log terms

$$p_t^{(n)} = E_t \left[m_{t+1} + p_{t+1}^{(n-1)} \right] + \frac{1}{2} Var_t \left[m_{t+1} + p_{t+1}^{(n-1)} \right] \tag{A-4}$$

where $p_t^{(n)} \equiv ln \left[P_t^{(n)} \right]$ and $m_t \equiv ln \left[M_t \right]$. We have

$$-p_t^{(n)} = A_n + \mathbf{B}_n^{\intercal} \mathbf{X}_t$$

As $p_t^{(0)} = 0$, $A_0 = 0$ and $\mathbf{B}_0 = \mathbf{0}$. For a *1*-period bond (A-4) can be rewritten as

$$-y_t^{(1)} = p_t^{(1)} = E_t \left[m_{t+1} \right] + \frac{1}{2} Var_t \left[m_{t+1} \right]$$

Let $y_t^{(1)} = \delta_1^{\intercal} \mathbf{X}_t$. The recursive relations for A_n and $\mathbf{B}_n$ are given by

$$\begin{aligned} A_n &= A_{n-1} + \mathbf{B}_{n-1}^{\intercal} \left(\mathbf{\Phi}_0 - \mathbf{\Sigma} \lambda_0 \right) - \frac{1}{2} \mathbf{B}_{n-1}^{\intercal} \mathbf{\Sigma} \mathbf{\Sigma}^{\intercal} \mathbf{B}_{n-1} \\ \mathbf{B}_n &= \delta_1 + \left(\mathbf{\Phi}_1 - \mathbf{\Sigma} \mathbf{\Lambda}_1 \right)^{\intercal} \mathbf{B}_{n-1} \end{aligned} \tag{A-5}$$

The n-period log zero coupon yields $y_t^{(n)}$ are an affine function of the state variables $\mathbf{X}_t$ as

$$y_t^{(n)} = -\frac{p_t^{(n)}}{n} = \frac{A_n}{n} + \frac{\mathbf{B}_n}{n}^{\intercal} \mathbf{X}_t = a_n + \mathbf{b}_n^{\intercal} \mathbf{X}_t \tag{A-6}$$

The real term structure can be derived along the lines in Ang *et al.* (2008).

Let the *1*-period return of a n-period zero coupon bond be given by $r_{t+1}^{(n)}$.

$$\begin{aligned} r_{t+1}^{(n)} &= p_{t+1}^{(n-1)} - p_t^{(n)} \\ &= -\mathbf{B}_{n-1}^{\intercal} \mathbf{\Sigma} \lambda_0 - \frac{1}{2} \mathbf{B}_{n-1}^{\intercal} \mathbf{\Sigma} \mathbf{\Sigma}^{\intercal} \mathbf{B}_{n-1} - (\mathbf{B}_{n-1}^{\intercal} \mathbf{\Sigma} \mathbf{\Lambda}_1 - \delta_1^{\intercal}) \mathbf{X}_t - \mathbf{B}_{n-1}^{\intercal} \mathbf{\Sigma} \zeta_{t+1} \end{aligned}$$

The parameters of the system (A-1) to (A-3) (i.e. $\mathbf{\Phi}_0$, $\mathbf{\Phi}_1$, $\mathbf{\Sigma}$, λ_0 and $\mathbf{\Lambda}_1$) will be estimated using a two-step procedure (see e.g. Ang *et al.* (2005)). In the first step we estimate the VAR parameters ($\mathbf{\Phi}_0$, $\mathbf{\Phi}_1$ and $\mathbf{\Sigma}$). In the second step we estimate conditional on the VAR estimates, the parameters of the prices of risk (λ_0 and $\mathbf{\Lambda}_1$). We assume that the yields are observed with a measurement error, i.e. $y_t^{(n)} = \widehat{y}_t^{(n)} + \epsilon_t^{(n)}$, where

$$\widehat{y}_t^{(n)} = a_n + \mathbf{b}_n^{\intercal} \mathbf{X}_t$$

and $\epsilon_t \sim N(0, \mathbf{\Omega})$. We minimize the sum of squared errors

$$\min_{\lambda_0, \mathbf{\Lambda}_1} \sum_{n=1}^{N} \sum_{t=1}^{T} \left(\widehat{y}_t^{(n)} - y_t^{(n)} \right)^2$$

As the 10-years yield is included in the set of state variables, the constraint is imposed that this yield is fitted exactly. The optimization is based on yields with maturities from 1 to 15 years[6]. Tables 3 and 4 report the results of the second step.

Insert Table 3 here
Insert Table 4 here

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Tables

Table 1: Summary statistics

The Table reports the summary statistics for the full period (1972:09-2008:08). The mean, stdev and Sharpe Ratio (SR) are annualized. The other statistics are on a monthly basis. XKurtosis is the excess kurtosis. The DF-test is the Dickey-Fuller statistic. We have adjusted the mean log returns by one-half of their variances so that they reflect mean gross returns. The statistics are based on the log returns and log yields. NOTE: the statistics for dy_t are based on the original series (DY_t=$exp(dy_t)$).
The last column shows the views for the (simple) yields and returns. The views are imposed by adjusting the constants Φ_0 in the VAR model and λ_0 in the pricing kernel

	Mean	Stdev	SR	Min	Max	Skewness	XKurtosis	DF-test	Views
$sy_t^{(1)}$	5.31	2.53		2.00	13.10	0.93	0.05	2.17	3.75
π_t	2.88	0.91		-0.47	1.40	0.94	1.77	104.27	2.00
$y_t^{(120)}$	6.54	1.74		3.16	10.71	0.01	-0.82	0.92	4.50
xs_t	3.54	14.41	0.25	-23.15	11.84	-0.81	2.61	165.82	2.75
DY_t	2.99	1.16		1.27	5.70	0.65	-0.85	0.75	2.25

Table 2: VAR estimation results.

The first panel reports the parameter estimates of Φ_1 in the VAR(1) model $\mathbf{X}_{t+1} = \mathbf{\Phi}_0 + \mathbf{\Phi}_1\mathbf{X}_t + \mathbf{\Sigma}\zeta_{t+1}$. The coefficient estimates and R^2 are in the 1^{st} row, the t-values and p-value of the F-statistic of joint significance in the 2^{nd}. The seond panel reports the cross-correlations of the shocks ζ_{t+1}. The volatility (%/month) is given on the diagonal.

	$y_t^{(1)}$	π_t	$y_t^{(120)}$	xs_t	dy_t	R^2/p
$y_t^{(1)}$	0.95	0.03	0.04	-0.00	-0.01	0.96
	61.10	3.89	1.48	-0.56	-1.04	0.00
π_t	0.39	0.17	0.05	0.00	0.08	0.26
	4.68	3.66	0.35	0.62	2.07	0.00
$y_t^{(120)}$	0.01	0.00	0.98	0.00	-0.00	0.98
	1.69	1.02	79.20	1.02	-0.16	0.00
xs_t	-1.11	-2.18	-4.08	0.10	2.48	0.06
	-0.74	-2.57	-1.57	2.10	3.38	0.00
dy_t	0.00	0.02	0.03	-0.00	0.98	0.99
	0.19	2.52	0.97	-1.39	129.46	0.00

	$y_t^{(1)}$	π_t	$y_t^{(120)}$	xs_t	dy_t
$y_t^{(1)}$	0.04				
π_t	0.03	0.22			
$y_t^{(120)}$	0.19	0.16	0.02		
xs_t	-0.04	-0.07	-0.10	4.03	
dy_t	0.06	0.05	0.12	-0.91	0.04

Table 3: Prices of Risk.

The Table reports the parameter estimates of $\lambda_t = \lambda_0 + \mathbf{\Lambda}_1\mathbf{X}_t$.

	λ_0	$y_t^{(1)}$	π_t	$y_t^{(120)}$	xs_t	dy_t
$y_t^{(1)}$	0.14	-140.00	303.61	-135.54	-5.48	-11.91
π_t	-0.00	4.79	-10.39	4.64	0.19	0.41
$y_t^{(120)}$	-0.14	143.55	-37.60	-126.17	2.31	-5.79
xs_t	3.09	-21.45	-46.13	-117.31	2.47	60.99
dy_t	6.83	-65.57	-133.00	-208.24	5.60	137.78

Table 4: Volatilities of measurement errors of yields.

The Table reports the volatilities of the measurement errors $\epsilon_t^{(n)}$ in $y_t^{(n)} = a_n + \mathbf{b}_n^\intercal \mathbf{X}_t + \epsilon_t^{(n)}$. The volatility of the 10-years yield ($y_t^{(120)}$) is by construction 0.00

$y_t^{(12)}$	0.67	$y_t^{(24)}$	0.71	$y_t^{(36)}$	0.68	$y_t^{(48)}$	0.60
$y_t^{(60)}$	0.50	$y_t^{(72)}$	0.39	$y_t^{(84)}$	0.28	$y_t^{(96)}$	0.18
$y_t^{(108)}$	0.09	$y_t^{(120)}$	0.00	$y_t^{(132)}$	0.07	$y_t^{(144)}$	0.14
$y_t^{(156)}$	0.19	$y_t^{(168)}$	0.24	$y_t^{(180)}$	0.28		

Figure 1: Term structure of risk: annualized volatility of cumulative holding period returns. The y-axis shows the annualized volatility, the x-axis shows the horizon in years.

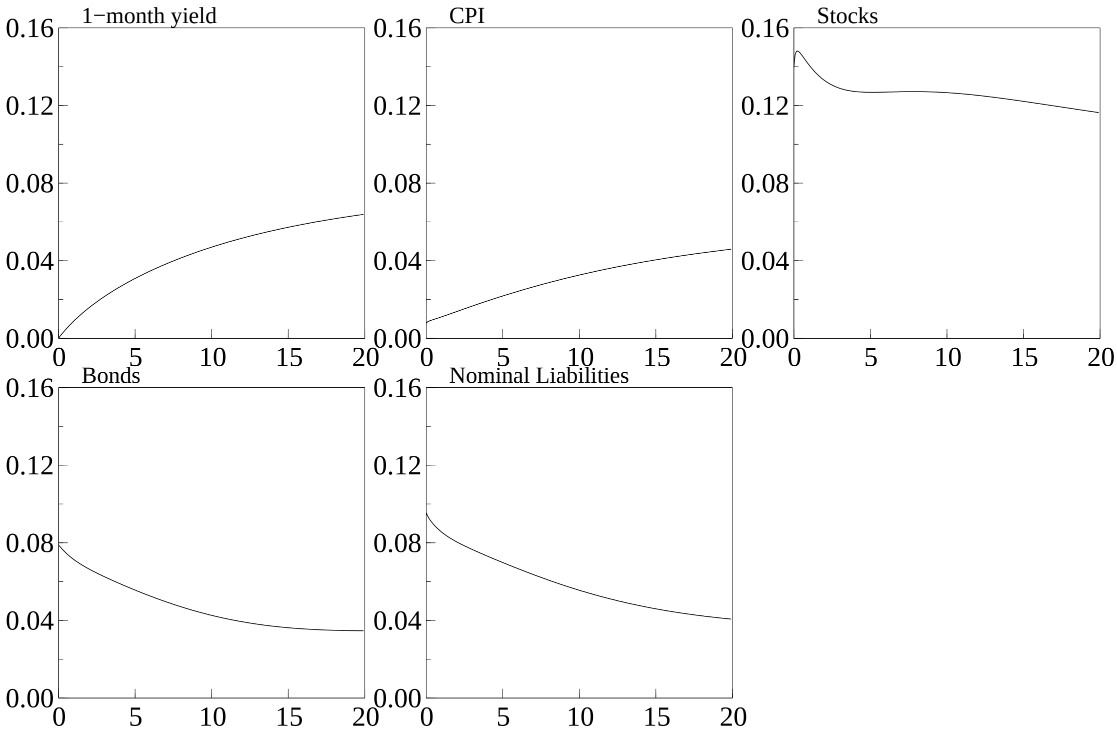

Figure 2: Term structure of risk (continued): correlation of cumulative holding period returns with stocks. The y-axis shows the correlation, the x-axis shows the horizon in years.

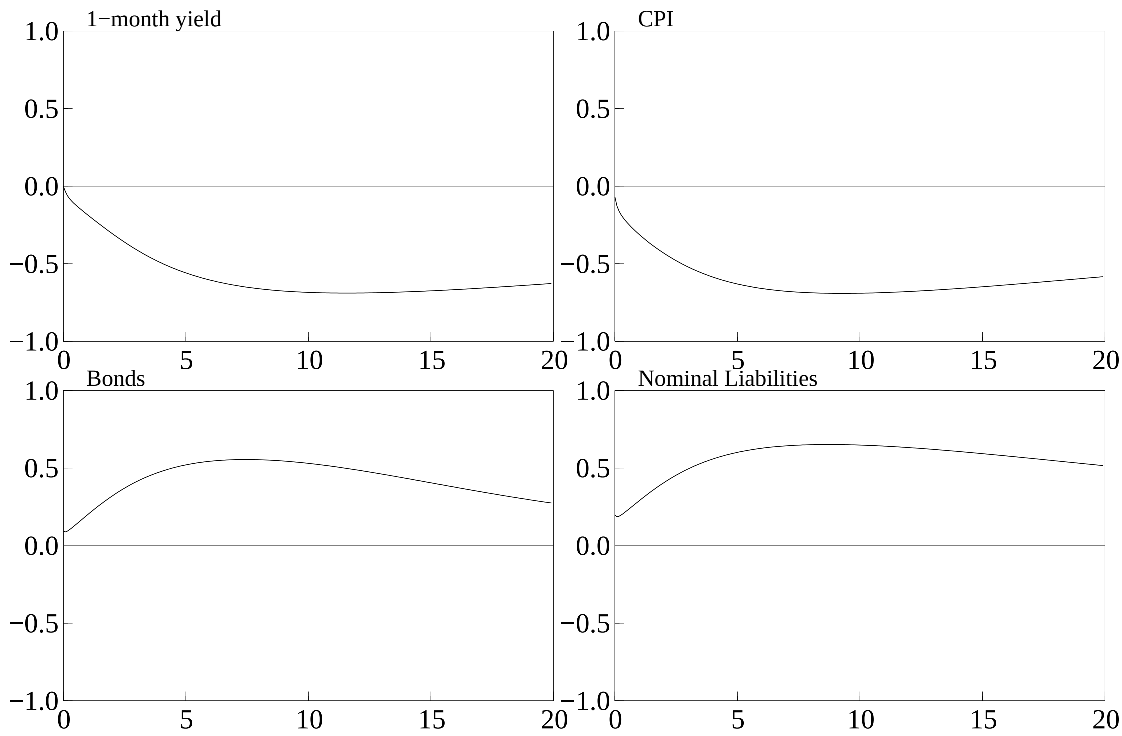

Figure 3: Term structure of risk (continued): correlation of cumulative holding period returns with CPI. The y-axis shows the correlation, the x-axis shows the horizon in years.

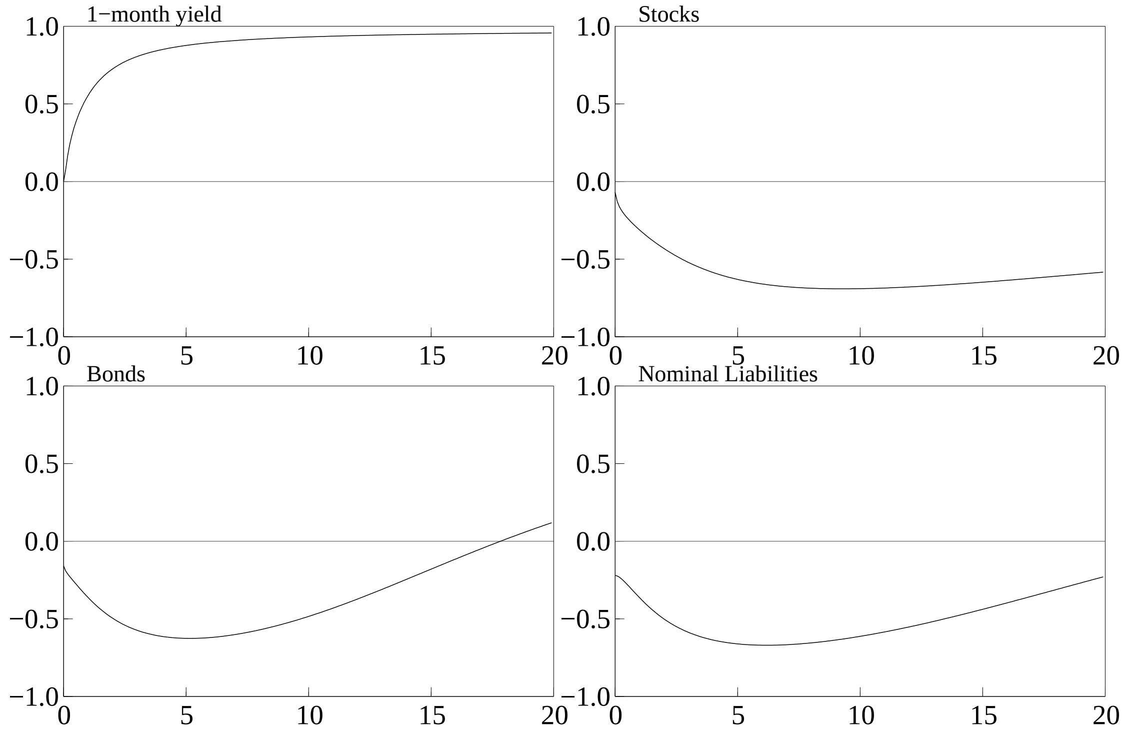

Figure 4: Different collective pension deals: pension deal 1 represents unconditional wage indexation; pension deal 2 is a hybrid plan with a dynamic wage indexation policy (with boundaries); pension deal 3 is a hybrid plan with dynamic indexation which depends on an age dependent combination of return indexation and unconditional wage indexation.

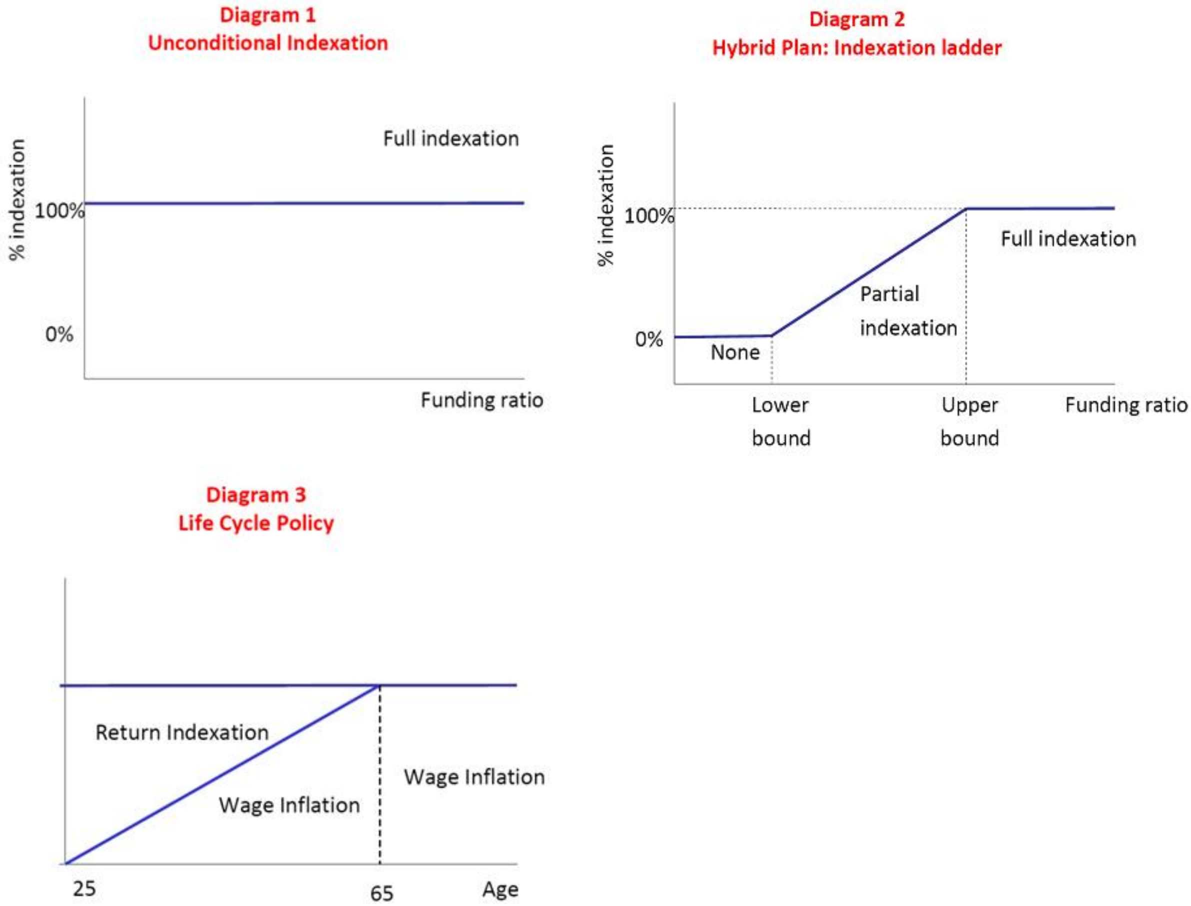

Figure 5: Generational effects when stepping over from the full indexation plan to the hybrid plan expressed as % of total nominal liabilities in 2008 (y-axis) for various age cohorts with age at 2008 on x-axis

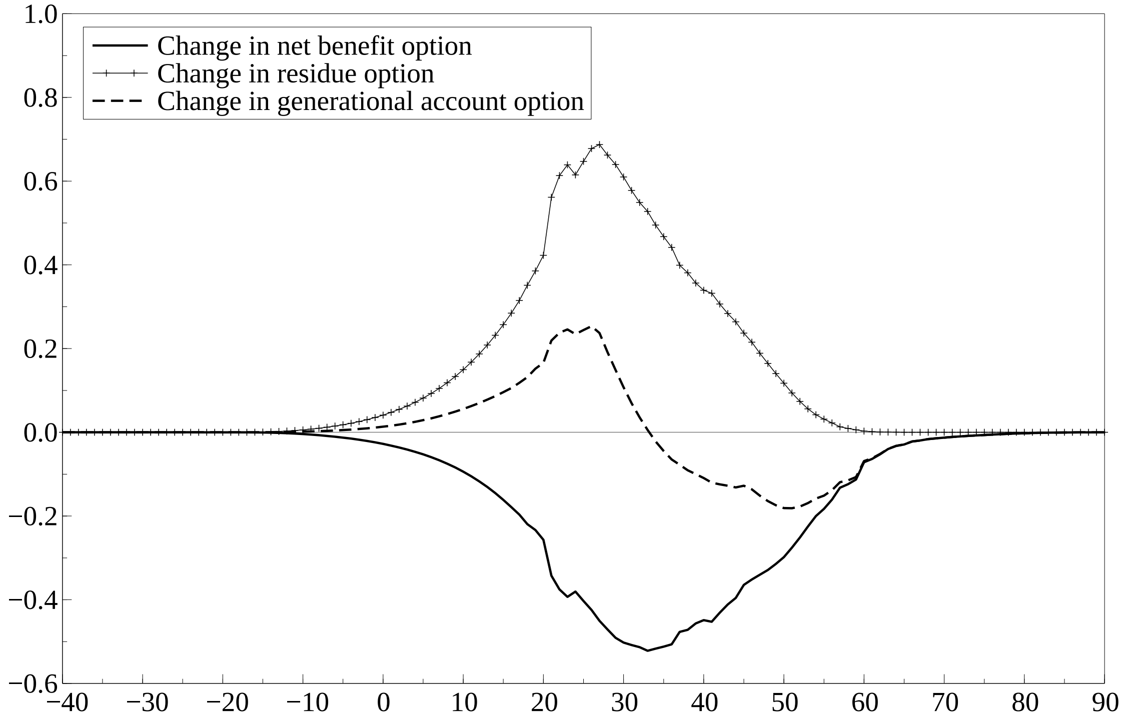

Figure 6: Generational effects when stepping over from the hybrid plan with 50% stocks and 50% bonds to the hybrid plan with the conservative mix (i.e. 100% bonds) expressed as % of total nominal liabilities in 2008 (y-axis) for various age cohorts with age at 2008 on x-axis

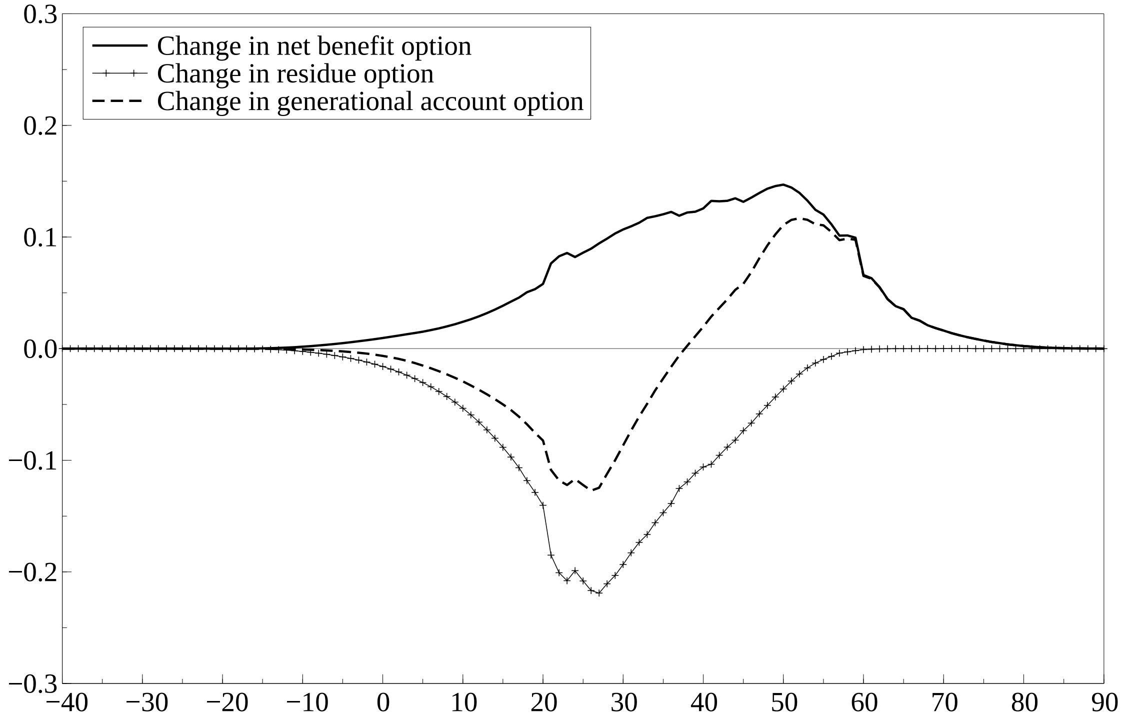

Figure 7: Generational effects when stepping over from the hybrid plan to the life cycle policy plan expressed as % of total nominal liabilities in 2008 (y-axis) for various age cohorts with age at 2008 on x-axis