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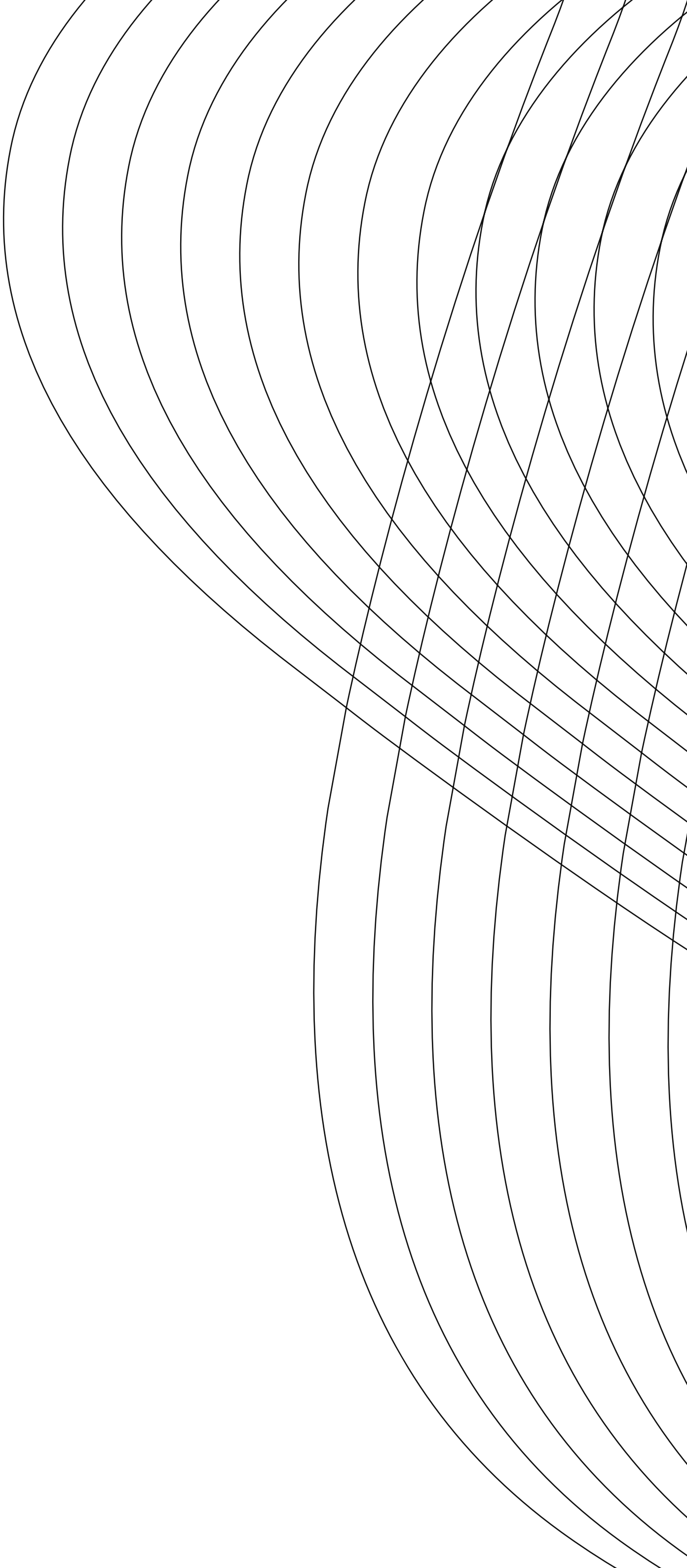
Netspar PANEL PAPERS

Frank de Jong and Frans de Roon

Illiquidity: implications for investors and pension funds



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PREFACE

Netspar stimulates debate and fundamental research in the field of pensions, aging and retirement. The aging of the population is front-page news, as many baby boomers are now moving into retirement. More generally, people live longer and in better health while at the same time families choose to have fewer children. Although the aging of the population often gets negative attention, with bleak pictures painted of the doubling of the ratio of the number of people aged 65 and older to the number of the working population during the next decades, it must, at the same time, be a boon to society that so many people are living longer and healthier lives. Can the falling number of working young afford to pay the pensions for a growing number of pensioners? Do people have to work a longer working week and postpone retirement? Or should the pensions be cut or the premiums paid by the working population be raised to afford social security for a growing group of pensioners? Should people be encouraged to take more responsibility for their own pension? What is the changing role of employers associations and trade unions in the organization of pensions? Can and are people prepared to undertake investment for their own pension, or are they happy to leave this to the pension funds? Who takes responsibility for the pension funds? How can a transparent and level playing field for pension funds and insurance companies be ensured? How should an acceptable trade-off be struck between social goals such as solidarity between young and old, or rich and poor, and individual freedom? But most important of all: how can the benefits of living longer and healthier be harnessed for a happier and more prosperous society?

The Netspar Panel Papers aim to meet the demand for understanding the ever-expanding academic literature on the consequences of aging populations. They also aim to help give a better scientific underpinning of policy advice. They attempt to provide a survey of the latest and most relevant research, try to explain this in a non-technical manner and outline the implications for policy questions faced by Netspar's partners. Let there be no mistake. In many ways, formulating such a position paper is a tougher task than writing an academic paper or an op-ed piece. The authors have benefitted from the comments of the Editorial Board on various drafts and also from the discussions during the presentation of their paper at a Netspar Panel Meeting.

I hope the result helps reaching Netspar's aim to stimulate social innovation in addressing the challenges and opportunities raised by aging in an efficient and equitable manner and in an international setting.

Henk Don
Chairman of the Netspar Editorial Board

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ILLIQUIDITY: IMPLICATIONS FOR INVESTORS AND PENSION FUNDS

Abstract

This paper discusses the implications of illiquidity, and in particular transaction costs and non-traded risks, for investors. First, transaction costs are derived from order-processing costs, inventory and search costs, and asymmetric information. In addition, trading volume is related to transaction costs as well. A final aspect of illiquidity is the temporary inability to trade.

Transaction costs affect optimal portfolio choices by lowering expected returns and by inducing a hedge demand for uncertain transaction costs. A similar hedge demand is derived for the presence of nontraded risks. In multiperiod models, transaction costs also have the effect of resulting in no-trade regions, and in lower trading frequencies.

The effect of transaction costs and non-traded risks on equilibrium prices for traded assets is to induce higher expected returns in order to compensate for transaction costs and to yield additional risk premiums beyond the CAPM, reflecting hedge demand for non-traded assets. Non-traded assets themselves can be valued using equivalent utility approaches.

Policy implications

This panel paper discusses the effects of illiquidity, and in particular transaction costs and non-traded risks, for investors. The implications for investors are as follows:

- Illiquidity is a multi-dimensional concept, reflecting at least a time, volume and price dimension. The price dimension is particularly reflected in transaction costs, whereas the time and volume dimension are mainly reflected in the (temporary) inability to trade certain assets. This affects the optimal portfolio composition in the following ways:

- Expected return inputs in portfolio models are reduced by transaction costs.

- Liquidity is a trade-off factor next to traditional portfolio trade-offs, resulting, for instance, in Mean-Variance-Liquidity efficient portfolios.

- Uncertain transaction costs and (temporarily) non-traded assets and risks induce a hedge demand for traded assets, similar to Asset-Liability Management portfolios.

- Due to transaction costs, it is optimal to lower trading frequency and to take into account no-trade regions.

- In addition, in equilibrium, transaction costs and non-traded assets and risks may change the expected returns on traded assets. In particular, to the extent that certain asset classes provide a hedge against non-traded risks, they may induce additional risk premiums, beyond the CAPM. Long-term investors that are not exposed to these risks may capture these risk premiums by deviating from the market portfolio and (over)investing in these asset classes. In a similar way, long-term investors (such as pension funds) may capture the liquidity premium by (over)investing in illiquid stocks.

- To the extent that investors are investing in the non-traded (illiquid) assets for which no market prices are available, they may value them using an equivalent-utility approach.

1. Introduction

Liquidity has become one of the most heavily discussed and researched aspects of financial markets. Although liquidity is a broad concept, this paper defines illiquidity as either the costs incurred while trading financial assets, or the (temporary) impossibility to trade an asset. Notice that such 'market' liquidity is different from 'funding' liquidity, which is about the ease with which banks or other financial intermediaries can attract capital or other financial funds. See Brunnermeier and Pedersen (2009) for more about this difference.

In general, liquidity has at least three dimensions:[1] a liquid asset is one that can be traded (i) quickly (the time dimension), (ii) with minimal price impact (the price dimension), and (iii) in sufficiently large volumes (the size dimension). Obviously, these dimensions are interlinked. For instance, the larger the required volume to be traded in a short period of time, the bigger the price impact will be. Although this paper focuses mainly on transaction costs (i.e., the price dimension,) liquidity may also be thought of as being a function of both size and time. Thus, the price impact of a trade may be thought of as a statistic that summarizes the size and time dimensions of the trade.

This paper focuses on the implications of (lack of) market liquidity for investors. Liquidity may important to investors for many reasons. First, there is a simple effect that high transaction or trading costs affect the returns on investments and hence the optimal portfolio composition. Second, the investor can adapt his trading strategies to minimize transaction costs. In efficient markets, investors are aware of the transaction costs they face,

[1]See, e.g., Lo, Petrov and Wierzbicki (2003).

and in equilibrium, the prices of the assets will reflect the expected trading costs. Third, a greater weight of illiquid assets in the portfolio makes it more difficult for the investor to adapt to unforseen shocks. For example, after a large drop in asset values the portfolio may become very unbalanced or too risky, and adjusting the portfolio composition is easier with more liquid assets than with less liquid assets. Such events may also trigger margin calls and regulatory requirements, which are easier to satisfy by adjusting (selling) liquid asset positions.

Although this paper focuses mainly on the price dimension, all three aspects of liquidity are discussed. First, by focusing on transaction costs, the paper explicitly deals with the price dimension. Next, when discussing the portfolio implications and the illiquidity discount, we also focus on the inability to trade an asset for a specific period of time, which therefore touches on the time dimension. Finally, also in relation to portfolio implications and asset pricing implications, we consider the case where investors are exposed to a non-tradable risk such as human capital or real estate, i.e., a risk or asset that they cannot trade at all (for a certain period) but that does affect their wealth. Since such an asset or risk cannot be traded at all, we may think of it as having zero trading volume.

Several papers have been written about the illiquidity discount, i.e. the difference between the price of a fully liquid asset and an, otherwise identical, illiquid asset. Two approaches can be distinguished. In the first approach, the illiquidity is caused by the inability to trade for a pre-specified period of time. In this situation, the investor can neither rebalance his portfolio during the holding periods nor sell off the assets and consume the proceeds. Both of these restrictions lower his utility from holding the asset. In the second approach, the investor can trade the asset, but only at a cost. These transaction costs, and the expectation of future transaction costs, lower the value of the asset.

The structure of this paper is as follows. Section 2 discusses the nature and causes of transaction costs and illiquidity. Section 3 shows the impact of transaction costs and other liquidity aspects on the optimal investment strategy, while section 4 shows the impact on the valuation of assets and liabilities. Section 5 summarizes the main results and draws lessons for pension funds.

Other useful reviews of the relation between liquidity and investments include Cochrane (2004), who focuses on trading, and Amihud and Pedersen (2005), who provide an extensive review of the relation between liquidity and asset prices.

2. Transaction costs and other liquidity measures

The standard way to think about illiquidity is as transaction costs that must be paid every time an asset is traded. An extensive academic literature addresses the reasons why financial markets are not perfectly liquid. The main theories can be classified in three groups: order-processing costs, inventory and search costs, and asymmetric information. Next to transaction costs, this section discusses other liquidity measures, related to size and time.

2.1. Order-processing costs

Order-processing costs refer to the costs that financial intermediaries such as market makers, dealers and exchanges make in processing orders. These could be costs like the back office, exchange fees and the like. With modern technology and increasing competition between exchanges, these costs are likely to be low for heavily traded products such as stocks, treasury bonds and large-issue corporate bonds. However, for structured products and in smaller markets such as the municipal bond market and real estate markets, these costs may be relatively high. For investors, order-processing costs also include any fees and taxes that are levied by the exchanges or the government.

2.2. Inventory and search costs

Consider a typical financial market that is centered around a relatively small number of dealers. Many financial markets have this structure, for example, the bond and foreign exchange market, the options and futures markets and the market for block trades in equities. These dealers typically trade on their own account and provide an important service to investors: the opportunity to trade immediately without the investors having to search for a counterparty to their trade. The dealers are thus liquidity providers. The cost of providing this immediacy is twofold. First, the dealers have to invest time and effort to find a counterparty. Second, the dealers often are the counterparty to the trade, until the lot is traded along to another investor, and in the meantime the dealer becomes the owner of the securities. These have price risk, and dealers are most likely quite risk averse, as they need to pledge their own capital as buffers against these risks. To compensate for the search cost and the inventory risk, the dealers charge a fee to the investors. Although this can be an explicit fee or commission, it is more usual for the dealer to charge different prices for buying the asset (a relatively low bid price) and selling the asset (a relatively high ask price). The difference between the bid and ask prices (called the bid-ask spread) is an implicit cost for the investors, as they buy at a high price and sell at a low price. Conversely, the bid-ask spread is a profit for the dealers. In competitive markets, the bid-ask spreads will be driven down to the level where the spread compensates exactly for the search costs and the inventory risk of the dealers. In non-dealer markets such as the modern electronic markets, the issuers of limit orders take the role of liquidity providers. They face the same type of risks as the dealers, in the sense that their limit orders have the risk of non-execution and do not immediately lead to a transaction, so there are waiting costs. These lead to very similar effects as the inventory and search costs. The specific market mechanism is therefore less important than the underlying economic mechanisms to explain transaction costs.

2.3. Asymmetric information

In many markets, the initiators of transactions know more about the quality of the goods than the potential counterparties do. A classical example is Akerlof's (1970) market for 'lemons', where the sellers of used cars are much more aware of the quality of the car than potential buyers are. To protect themselves from buying a 'lemon' (i.e. a low quality car), the buyers bid lower prices than in a situation with symmetric information.

In financial markets, the situation is not very different. Some traders may be better informed than others. However, these informed traders may be on both sides of the market (i.e. they may be buyers or sellers). The presence of such informed traders leads to a wedge between buying and selling prices. In the famous model of Kyle (1985), the prices are linear in the size of the order

$$p(x) = p_0 + \lambda x, \tag{1}$$

where x is the size of the order ($x > 0$ indicates a buy, and $x < 0$ a sell). The coefficient λ is the price impact of a trade, and indicates how much the transaction price is affected by the order. A high 'lambda' indicates a large price impact and an illiquid market in which small orders can move prices substantially. Interestingly, this price impact is permanent and not reversed in later trades. Kyle's lambda is an often-used measure of transaction costs in the empirical literature.

2.4. Estimating transaction costs

The measurement of transaction costs in markets is not always trivial. An often-used measure of transaction costs is the bid-ask spread: the difference between the buying price (ask price, a_t) and the selling price (bid price, b_t) of an asset (measured simultaneously)

$$S_{quoted} = E(a_t - b_t). \tag{2}$$

The average bid-ask spread over some period (say, a day or a month) provides an indication of the cost of trading. However, the bid-ask spread has some drawbacks. First, the average is usually taken over time, and not all times of the day or month are equally busy. Secondly, some trades are larger than the quantities of assets available and will trade at worse prices. Conversely, in some markets, the bid and ask prices are only indicative, and it is sometimes possible to negotiate a better price with the dealer. Therefore, a better measure of transaction costs is the effective spread, defined as the absolute difference between the transaction price and the mid-point of bid and ask quotes prevailing at the time of the trade. This can be expressed as follows:

$$S_{effective} = 2E|p_t - (a_t + b_t)/2|. \tag{3}$$

The calculation of both the quoted and the effective spread requires data on bid and ask quotes, and transaction prices. These are not always available. Roll (1984) introduced a simple way to measure the bid-ask spread from transaction prices only; for many markets, these are available at least on a daily frequency and often also on higher frequency, even if data on quotes are not available. Roll's model is

$$p_t = p_t^* + (S/2)Q_t, \tag{4}$$

where p_t is a transaction price and p_t^* is the 'efficient' price (i.e. the fair value of the asset). The variable Q_t indicates whether the asset is a buy ($Q_t = 1$) or a sell ($Q_t = -1$). Now Roll makes a few important assumptions: (i) the efficient price is a random walk; (ii) the buy/sell indicator Q is IID binomial with equal probability for buy and sell; (iii) the innovations in the efficient price and the buy-sell indicator are independent. The latter assumption means that the trading process does not have an impact on the efficient price (which is a strong assumption; in the Kyle model, for example, prices move in the direction of trading). Under these assumptions, one can show that the transaction price returns exhibit negative serial correlation induced by the bouncing of transaction prices between the bid and the ask price. The bid-ask spread can be estimated by

$$S = -2\sqrt{Cov(\Delta p_t, \Delta p_{t-1})}. \tag{5}$$

Hasbrouck (2009) suggests an improvement to Roll's method, based on Bayesian methods using a Gibbs sampler.

If observations on the buy/sell indicator Q are available, the effective bid-ask spread can be estimated directly from transaction data, using the regression

$$\Delta p_t = (S/2)\Delta Q_t + u_t. \tag{6}$$

Glosten and Harris (1988) extend this model to allow for price impact of transactions. This leads to the regression[2]

$$\Delta p_t = (S/2)\Delta Q_t + \lambda x_t + u_t. \tag{7}$$

The total transaction costs are then a sum of the fixed bid-ask half-spread $S/2$ and the price impact $\lambda|x|$, which is proportional to the transaction size.

[2]The assumption is that the asymmetric information component is proportional to the trade size, whereas the other costs are fixed.

A simple but approximate way to measure price impact is by dividing the daily price change by daily trading volume. This measure is useful if data are available on the gross trading volume but not on the net order flow (buy minus sell trades). This measure was introduced by Amihud (2002) and is commonly called *ILLIQ*:

$$ILLIQ_{it} = \frac{1}{D_{it}} \sum_{d=1}^{D_{it}} \frac{|\Delta p_{it,d}|}{v_{it,d}}, \tag{8}$$

where $|\Delta p_{it,d}|$ is the absolute price change for stock i on day d in month t, $v_{it,d}$ is the trading volume on that day, and D_{it} is the number of trading days for stock i in month t. Figure 1 illustrates this measure.

A final measure of liquidity was suggested by Pastor and Stambaugh (2003). The liquidity measure for stock i in month t is the estimate of γ_{it} in the regression

$$r_{it,d+1} - r_{mt,d+1} = \alpha_{it} + \beta_{it} r_{it,d} + \gamma_{it} \text{sign}(r_{it,d} - r_{mt,d}) v_{it,d} + \epsilon_{it,d+1}, \tag{9}$$

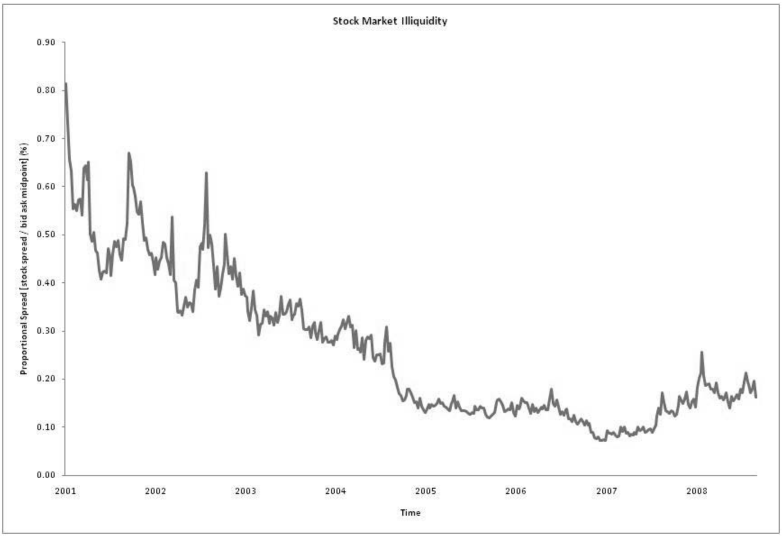
where $r_{it,d+1}$ is the return on stock i on day $d+1$ in month t, $r_{mt,d+1}$ is the return on the stock market index on day $d+1$ in month t, and $v_{it,d}$ is the trading volume for stock i on day d in month t. Effectively, γ_{it} measures the price reversal from day d to the next day $d+1$ after an upward or downward price move associated with a trading volume v.

2.5. Volume-based measures

Next to the transaction cost measures discussed above, other liquidity measures are often related to the size dimension of liquidity. A natural way to measure the ease with which a

Figure 1: Liquidity over time

The figure plots the average ILLIQ measure for all S&P500 stocks. Source: own calculations



security can be traded in sufficiently large quantities is to base the measure on trading volume. Although almost tautological, if a security is traded in large volumes at high frequencies it is also a liquid security. Thus, based on this dimension natural measures are (the log of) trading volume and turnover:

$$\log\left(\text{Trading Volume}_t\right) = \log\left(\#\text{ Shares Traded}_t * \text{Share Price}_t\right),$$

$$\text{Turnover} = \frac{\#\text{ Shares Traded}_t}{\#\text{ Shares Outstanding}_t}.$$

Obviously, there is also interaction between transaction costs and trading volume, as large trades are likely to be executed at higher bid-ask spreads, thereby increasing transaction costs. For instance, Loeb (1983) relates the effective trading costs to i) the total market capitalization of the stock and ii) the block size of the trade. His Table II provides some interesting relationships that, although perhaps obvious, illustrate an important point. For instance, if the block size is about $250,000 for a stock with a market capitalization of about $25 million (so the block size is 1% of the market capitalization), the effective transaction costs can be as high as 20% of the price. If the market capitalization is about $250 million (ten times as large), the same block trade would imply transaction costs of about 3%-4%. Although these numbers need to be updated for current market data, they clearly show the relevance of the volume-based measure as well as the importance of transaction costs for big trades.

3. Portfolio choice with transaction costs

This section discusses the optimal choice of an investment portfolio when the investor faces transaction costs or other forms of illiquidity. We discuss two approaches, one based on a simple one-period model, and one based on life-cycle asset allocation decisions.

3.1. Static models

3.1.1. Mean-variance optimal portfolio with transaction costs

This subsection presents a very simple model of portfolio choice when trading is subject to transaction costs. We follow the simple one-period mean-variance optimization of Markowitz (1959), adapted to transaction costs, as in Acharya and Pedersen (2005). Suppose the investor owns assets that yield a stochastic return (before transaction costs) r, and the stochastic transaction costs when selling the asset at the end of the period are c. Also assume that the investor has a mean-variance utility function with risk aversion parameter γ. It is then straightforward to show that the optimal portfolio for the investor is the standard mean-variance portfolio defined over net returns $r - c$:[3]

$$x^* = \frac{1}{\gamma} Var(r-c)^{-1} E(r-c). \tag{10}$$

This formula can be worked out a little more:

$$x^* = \frac{1}{\gamma} \left[Var(r) - C_{rc} - C'_{rc} + Var(c) \right]^{-1} \left[E(r) - E(c) \right], \tag{11}$$

[3]For an introduction to mean-variance portfolio models, see Campbell and Viceira (2002), Chapter 2.

with $C_{rc} = Cov(r, c)$. This formula shows that the demand for risky assets diminishes with high expected transaction costs $E(c)$, and also with high variance of the transaction costs $Var(c)$. Usually, transaction costs and returns are negatively correlated (i.e. when returns are negative, transaction costs increase). The covariance terms therefore decrease the asset demand even further.

3.1.2. Hedging pressure and limited access

In many markets (for example, emerging markets) there are restrictions on the holdings of assets by foreign investors. Thus, a large fraction of the assets has to be held by domestic investors. This implies that their portfolios are not well diversified and have more domestic country risk than is optimal. The domestic investors therefore require a higher return on holding domestic assets than in a situation without investment restrictions. Errunza and Losq (1985), Bekaert and Harvey (2000) and De Jong and De Roon (2005) formally model this situation. They draw the following conclusions:

- Without restrictions, a world CAPM holds, and
- With restrictions, idiosyncratic risk is priced and there are hedging pressure effects from cross-hedges of neighboring countries.

For futures markets, De Roon, Nijman and Veld (2000) show that untraded positions lead to hedging pressures, which affect the prices of traded assets if they are correlated with the untraded risk. If there is a positive correlation with the untraded risk, and this risk is positively priced, than the hedging pressures increase the risk premium. Examples are credit risk (corporate bonds have a credit risk premium above the CAPM market risk premium; see Elton et al. (2001) and De Jong and Driessen (2007)) and volatility risk.

The optimal portfolio with hedging pressure terms is

$$x^* = \frac{1}{\gamma} Var(r-c)^{-1} E(r-c) - Var(r-c)^{-1} Cov(r-c, R) q, \tag{12}$$

where R is some fixed, non-traded background risk, and the size of the agents' exposure to the non-traded risk is q. Examples of non-traded risks are the returns on human capital, an owner-occupied house or a private business. The important thing in the portfolio demand with the exogenous or non-traded risk R is that it can now be written in two parts: the first term is the standard Markowitz demand that looks at the risk-return characteristics of the traded assets only. This is also known as the *speculative* demand. The second term takes into account the fact that the agent is exposed to the non-traded risk and wants to hedge against this risk. The hedge positions in the traded assets that minimize the variance resulting from the non-traded risk are the regression coefficients from the non-traded risks on the traded asset returns. These regression coefficients are exactly equal to the second term in the above portfolio weights. Since these hedge positions are given per dollar exposure to the non-traded risk, they need to be scaled by the size of the exposure q in order to obtain the total hedge demand. This second part of the portfolio positions are therefore known as the *pure hedge* demand. The effects of transaction costs are pretty much the same as in the previous case, but now also the covariance between the transaction costs and the background risk $Cov(c, R)$ comes into play.

3.1.3. Optimal portfolio with short positions

Due to the hedge demand, the optimal position in the assets may be a short position. The analysis in the previous subsection assumed that the investor held long positions in all assets. With short positions, the effects of the transaction costs are somewhat different. The net return for an investor with a short position is $|x|(-r-c)$, which equals $x(r+c)$ when $x<0$. The optimal portfolio weight (excluding the hedging pressure terms) then is

$$x^* = \frac{1}{\gamma} Var(r+c)^{-1} E(r+c). \quad (13)$$

Then, the absolute portfolio weight for the short positions is

$$|x^*| = \frac{1}{\gamma} \left[Var(r) + C_{rc} + C'_{rc} + Var(c) \right]^{-1} \left[-E(r) - E(c) \right]. \quad (14)$$

Again, high transaction expected costs $E(c)$ and a high variance of transaction costs $Var(c)$ decrease the absolute position in the assets. There is, however, now a countervailing effect due to the covariance term $C_{rc} = Cov(r,c)$, which is usually negative and hence decreases the variance of the net returns $r+c$.

3.1.4. Mean-variance liquidity portfolios

Lo, Petrov and Wierzbicki (2003) propose including liquidity measures directly in the portfolio problem. They assume that each asset has a liquidity l_i which can be based on measures such as transaction costs, trading volume and the Loeb-measure, for instance. The portfolio liquidity is then equal to $x'l$, where l is the vector containing all of the asset-specific liquidities. Lo et al. then propose including the constraint $x'l = \delta$ into the Lagrangian formulation of the portfolio problem. If we rephrase their portfolio problem into excess-return space, then the solution to the optimal portfolio is

$$x^{*}=\frac{1}{\gamma}Var\left(r\right)^{-1}\left[E\left(r\right)-\varphi l\right], \tag{15}$$

with φ the Lagrange multiplier for the constraint $x'l=\delta$. Thus, given the level of risk aversion γ, φ an be interpreted as the fraction of wealth (or portfolio return) the investor is willing to give up in order to alleviate the liquidity constraint. It is straightforward to show that φ is a simple linear function of the required portfolio liquidity δ.[4] The solution in (15) implicitly assumes that x^{*} contains only long positions, or equivalently, that it applies to the subset of assets for which the short sales constraints are not binding.

Based on the modified efficient set constants[5]

$$\begin{aligned}\widetilde{A} &= l'Var\left(r\right)^{-1}l,\\ \widetilde{B} &= l'Var\left(r\right)^{-1}E\left(r\right),\\ \widetilde{C} &= E\left(r\right)'Var\left(r\right),^{-1}E\left(r\right).\end{aligned}$$

it is easy to show that expected portfolio return and portfolio variance are equal to

$$E\left(r_{p}\right)=\delta\frac{\widetilde{B}}{\widetilde{A}}+\lambda\left(\frac{\widetilde{A}\widetilde{C}-\widetilde{B}^{2}}{\widetilde{A}}\right), \tag{16a}$$

$$Var\left(r_{p}\right)=\delta^{2}\frac{1}{\widetilde{A}}+\lambda^{2}\left(\frac{\widetilde{A}\widetilde{C}-\widetilde{B}^{2}}{\widetilde{A}}\right), \tag{16b}$$

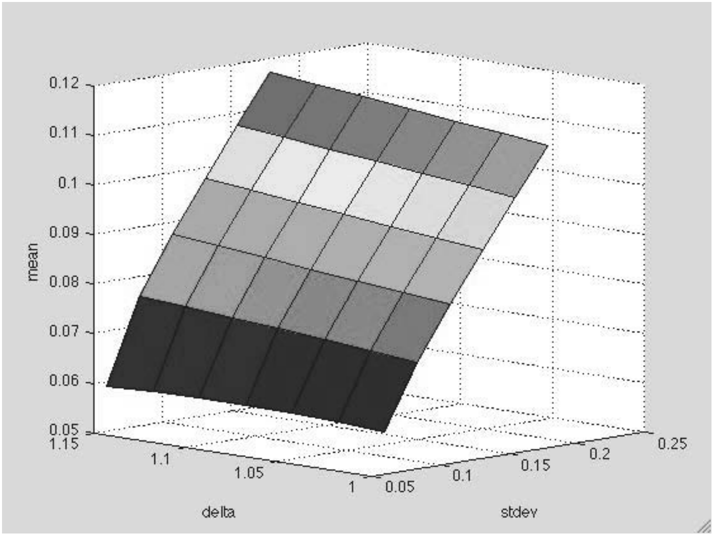
---

[4]In particular, using the modified efficient set constants defined below, we get $\varphi=\widetilde{B}/\widetilde{A}-\gamma\delta/\widetilde{A}$.

[5]In the standard case, the efficient set constants are defined in a similar way, replacing the vector l with a vector ι consisting of ones only, and based on total rather than excess returns.

Figure 2: Mean-Variance Liquidity Frontier

The figure plots the mean-variance liquidity frontier as a function of the liquidity constraint parameter δ.



with $\lambda = 1/\gamma$ the risk tolerance of the investor. These expressions show that expected portfolio return and variance are simple linear and quadratic functions of both risk tolerance λ and required liquidity δ. The solutions are very similar to those obtained in the standard case (see, e.g., Ingersoll (1987)), with modified definitions of the efficient set constants and an explicit role of portfolio liquidity. Figure 2 illustrates the mean-variance liquidity frontier as a function of δ.

3.1.5. Lock-up periods and optimal trading strategies

In the case of lock-up periods, the illiquidity is caused by the inability to trade for a pre-specified period of time. This happens, for example, after initial public offerings (IPOs), when the former owners of the company are forbidden to trade their stake in an initial period after the IPO (often, six months to one year). In the case of pensions and insurance, it is typically impossible or very difficult to trade the pension or insurance contract before the retirement date (and often thereafter as well). Also, investment vehicles such as private equity investments and hedge funds have lock-up and notification periods, making it difficult to withdraw money from such investments.

De Roon, Guo and Ter Horst (2009) show that lock-ups substantially reduce the performance of hedge fund investments. The model in the paper is fairly simple. They consider a multiple-month investment period. Stocks and bonds can be traded every month (i.e. the portfolio weight can be adjusted every month). The amount invested in hedge funds is fixed at the beginning of the period and cannot be changed for the entire investment period. The optimal dynamic investment strategy is found by applying the timing portfolio technology of Brandt and Santa-Clara (2006).

For instance, with an investment horizon of two months, the two-period return can be written as

$$\begin{aligned} r^p_{t\to t+2} &= \left(R_{f,t} + x_t' r_{t+1}\right)\left(R_{f,t+1} + x_{t+1}' r_{t+2}\right) - R_{f,t}R_{f,t+1} \\ &= x_t'\left(R_{f,t+1} r_{t+1}\right) + x_{t+1}'\left(R_{f,t} r_{t+2}\right) + \left(x_t' r_{t+1}\right)\left(x_{t+1}' r_{t+2}\right). \end{aligned}$$

By ignoring the last term, which is a second-order effect, the optimization is over a set of timing portfolios:

$$r^p_{t\to t+2} = x_t'\left(R_{f,t+1} r_{t+1}\right) + x_{t+1}'\left(R_{f,t} r_{t+2}\right).$$

The first timing portfolio with return $R_{f,t+1} r_{t+1}$ invests in the risky asset in the first period and in the risk-free asset in the second period, whereas the second timing portfolio with return $R_{f,t}r_{t+2}$ invests in the risk-free asset in the first period and in the risky asset in the second period. The portfolio problem then reduces to a single-period problem where we optimize the portfolio weights over the different timing portfolios. If we find that for the two-period holding period we cannot rebalance the illiquid asset, the portfolio returns become

$$r^{p}_{t\to t+2} = x'_{t}\left(R_{f,t+1}r_{t+1}\right) + x'_{t+1}\left(R_{f,t}r_{t+2}\right) + x_{q,t}r^{q}_{t\to t+2}, \quad (17)$$

where $r^{q}_{t\to t+2} = R^{q}_{t\to t+2} - R_{f,t}R_{f,t+1}$ is the two-period excess return on the illiquid asset. Thus, similar to Brandt and Santa-Clara, we end up optimizing over two timing portfolios in the liquid assets, where we invest in the risk-free asset in one period and in the risky assets in the other period. For the illiquid assets we have a similar structure, but here we always invest in the risky illiquid asset and not in the riskfree asset.

De Roon, Guo and Ter Horst (2009) then compare the expected utility of final wealth between this setting and a setting in which there are no restrictions on trading hedge funds (the portfolio weight in hedge funds can be adjusted every month). The paper finds that the lock-up period of three months costs the investor around 4% in certainty equivalent return per year. Of course, this is the premium that the investor would require if he only invests in one illiquid hedge fund. Investing in multiple funds with different starting dates (so called 'laddering') may mitigate the effects of illiquidity for the portfolio as a whole, thereby deminishing the utility loss.

3.2. Dynamic trading strategies

The model in the previous subsection considered only a one-period risk-return trade-off. Importantly, the investor was implicitly assumed to sell his full portfolio at the end of the investment period. In reality, investors hold assets for longer periods, but may want or need to rebalance their portfolio on a regular basis. However, there is no need for them (except in the final period) to trade the full portfolio, and also they can endogenously choose how much to rebalance. This subsection discusses several models in this class.

3.2.1. Constantinides' model

Constantinides (1986) considers a model like the consumption-saving model of Merton (1969) and extends it with proportional transaction costs. Thus, every time a value Y of the stock is traded, kY is lost in trading costs. In the Merton model, the investor optimally holds a fixed portfolio weight in the risky asset. To maintain this fixed weight, the investor has to trade continuously. With trading costs, this strategy is not feasible, as all wealth will be eaten up quickly by the continuous trading. Instead, the investor reacts to these trading costs by rebalancing his portfolio only infrequently. The results of Constantinides' model are quite neat:

- The investor has a no-trading range. This is characterized by $\underline{\lambda} \leq y(t)/x(t) \leq \overline{\lambda}$, where $y(t)/x(t)$ is the ratio of dollar wealth invested in the risky asset and the riskless asset. Only when $y(t)/x(t)$ is outside this range does the investor buy and sell the stock to get the ratio back within the range. The width of this range is increasing in the transaction costs k.

- The average allocation to risky assets is decreasing in the transaction costs, as is optimal consumption, but the effect on consumption is small.

- The amount of wealth needed to compensate the investor for transaction costs is small. Since the investor endogenously trades much less than in the Merton case, the compensation needed for a 1% transaction cost is only an extra 0.2% return on the risky asset for realistic parameters.

- The effect on the optimal consumption pattern is also small if one restricts the solutions to 'simple' policies. Constantinides defines a simple policy as a policy where the consumer consumes a constant fraction out of his riskless wealth. In the case of more complex consumption strategies, however, the effects are not fully clear from his analysis.

Without transaction costs, the results of Merton (1969) imply that the investor puts a fixed dollar amount in each asset; this amount is independent of the investment horizon if the returns are IID, as assumed. This strategy requires frequent (in fact, continuous) trading, as the number of shares has to be adjusted every time the stock price changes.

With transaction costs, the pictures changes. First, the investor no longer trades continuously, as this would imply infinite trading costs. The optimal strategy becomes an interval around the Merton value. The investor will trade only if the value of his position in the stock hits the upper or lower boundary of the interval. If the value of the position hits the upper bound, he will sell a fraction of his stock investment; if the value of the position hits the lower bound, he will buy additional shares. How much he trades depends on the nature of the transaction costs. With strictly proportional transaction costs (i.e. the costs are a fixed percentage of the value traded), the investor always trades the minimum quantity necessary to bring the value of his position back within the band. With fixed costs of trading (i.e. a fixed cost per transaction independent of the size of the trade) the investor brings back his position in the stock to the Merton level in one transaction. This pattern generates infrequent, lumpy trades. With both fixed and proportional trading costs, the pattern is different, although qualitatively the idea is the same: the investor has an optimal band of stock investment (a 'target zone').

Liu (2004) performs comparative statics on the expected trading frequency. Not surprisingly, the trading frequency is decreasing in the transaction costs. Interestingly, with positive expected returns, the average amount invested in the stock is higher than the amount under the Merton strategy.

Liu also extends the analysis in Constantinides (1986) to multiple assets. He considers a life-cycle asset allocation for an investor with CARA preferences and multiple assets. There are N assets, which all follow a geometric Brownian motion. On the investment objective side, the setting is an infinite-horizon CARA consumer without labor income. A limitation of Liu's paper is that the asset returns are assumed to be uncorrelated. Under that assumption, the optimal amount invested and the boundaries of the no-trade zones can be determined on a stock-by-stock basis. When returns are correlated, this no longer holds, and the solution becomes much more complex. The paper only shows some numerical solutions for a two-risky-assets case with low correlation. The more general model of many assets with possibly high correlation is not treated in the literature, as far as we know.

An important limitation of Constantinides' and Liu's models is that there is no predictability or time variation in investment opportunities. This is a serious limitation, as intertemporal hedge demands would induce more frequent trading and probably a bigger role for transaction costs. Jang, Koo, Liu and Loewenstein (2007) extend the analysis of Constantinides with intertemporal hedging demands. (Recall that Constantinides (1986) endogenizes trading frequency. He finds that the trading frequency declines quite rapidly with transaction costs. Therefore, the expected trading costs over the investment period of an asset are not increasing much with the bid-ask spread, and the equilibrium price effects are small.) They show that with intertemporal hedging demands, the presence of transaction costs can have first-order effects on the equilibrium price. The reason is that due to the hedging demands, the trading frequencies are not affected as much by transaction costs. The expected trading costs over the investment period could now be much larger than in the case without hedging demands. This would be reflected in much larger illiquidity discounts in the asset prices.

3.2.2. Other papers

Longstaff (2001) models the impact of illiquidity on optimal investment strategy and asset prices in a slightly different way. Instead of transaction costs, illiquidity is introduced as a bound α on the (absolute) fraction of shares that can be traded per unit of time. The strictest bound ($\alpha = 0$, so no trading at all) corresponds to a buy-and-hold strategy. The objective of the investor is to maximize log utility of terminal wealth. The asset price follows a geometric Brownian motion with possibly stochastic volatility.[6] As wealth has to remain positive at all times, the finite trading possibilities endogenously impose borrowing and short-sales constraints. The optimal portfolio weight is endogenously restricted to $0 \leq w(t) \leq 1$. This restriction is not very costly if the Merton weight w^* is below one, but for cases with $w^* > 1$ this restriction leads to a significant decline in the certainty equivalent of expected utility. This can be translated to a lower price that the investor is willing to pay for the asset (an illiquidity discount). For example, when $w^* = 2$ the discount is around 2.5%, and for $w^* = 5$ the discount is around 15%. If the volatility of the asset price is stochastic, the discounts can be twice as large.

Gârleanu and Pedersen (2009) present an asset allocation model with transaction costs that has explicit analytical solutions. They model the transaction costs as in the Kyle (1985) model (i.e. as price impact of trading, which is proportional to the trade size; hence total transaction costs are quadratic in trade size). The optimal investment portfolio in their model consists of a weighted average of (i) the mean-variance optimal portfolio and (ii) the portfolio in the previous period. The weight on the optimal portfolio is bigger the more liquid the market is, and the adjustment is smaller in illiquid markets. This result is quite nice because it is the only one (to the best of our knowledge) that uses price impact as a measure for illiquidity. This seems to be a natural choice, as institutional investors are fully aware of the impact that their large trades may have on prices. This structure also neatly avoids the no-trade range results of the older literature.

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[6]The stochastic volatility is not needed for the qualitative results, but quantitatively it matters for the illiquidity premium.

4. Liquidity and asset prices

This section discusses theoretical models that link asset prices to the liquidity of a particular asset. Parts of this section are based on Chapter 7 of De Jong and Rindi (2009).

4.1. Price effects of (known) transaction costs

From an investor's point of view, transaction costs reduce the return on investments. Rational investors will require compensation for anticipated transaction costs. This affects the price that an investor is willing to pay for an asset. As a result, in equilibrium, transaction costs lead to lower asset prices and therefore to higher expected returns gross of costs. This section discusses several formal models for the relation between transaction costs and asset prices.

4.1.1. The Amihud-Mendelson model

This sub-section discusses the model of Amihud and Mendelson (1986), which is a generalization of the Gordon dividend-growth model for asset valuation, where the value of a share with perpetual dividends d (and no growth of dividends) is $P = d/r$, and r is the risk-adjusted discount rate. The Amihud-Mendelson model has the following parameters: a perpetual per-period dividend d; the required risk-adjusted return r; the relative transaction costs c;[7] and the expected trading frequency ζ. The parameter ζ is the rate at which the investor turns over the asset. For example, a value of $\zeta = 0.50$ means that the investor trades 50% of his holdings of the asset per period.

Amihud and Mendelson (1986) show that under these

[7]These can be seen as the sum of the relative bid-ask spread S, the Kyle price impact λx and other costs such as exchange, clearing and settlement fees.

assumptions, the value of the asset is[8]

$$P = \frac{d}{r + \zeta c}. \tag{18}$$

The logic of this equation is simple: apart from the required net return r, the investor requires compensation for the expected per-period trading cost ζc. The sum of the two is the effective discount rate. The proof of equation (18) is quite simple. Consider the return on buying the asset and selling after one period. The asset is bought for P_0 and must generate an expected return of rP_0. The expected payoff at the end of the holding period equals its price P_1 less the expected transaction costs, $\zeta c P_1$, plus the dividend payment d. Equating expected payoff and the required return, we find the equilibrium pricing condition

$$P_1(1 - \zeta c) + d = (1 + r)P_0. \tag{19}$$

Now observe that the stream of cash flows generated by the asset at time 1 is identical to that generated by the asset at time 0 (i.e. the perpetual dividend d). Therefore, the end-of-period price P_1 must be equal to the beginning-of-period price P_0. This yields the equality

$$P(1 - \zeta c) + d = (1 + r)P. \tag{20}$$

Solving this gives the pricing equation (18).

The foregoing analysis does not allow for uncertainty in the next period's price. This is, however, easily captured. Let r now

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[8]We follow the convention in Amihud and Mendelson (1986) that P denotes the purchase (or ask) price of the asset. The investor then receives $P(1 - c)$ when he sells the asset. The gross returns on the asset are defined as ask-to-ask returns.

be the required, risk-adjusted expected return on the stock, P_1 the end-of-period price and P_0 the initial price; we can now generalize equation (19) to

$$E(P_1)(1-\zeta c)+d=(1+r)P_0. \tag{21}$$

Solving this expression for the expected return on the asset we find[9]

$$E(R)\equiv E\left(\frac{P_1-P_0+d}{P_0}\right)=r+\zeta c. \tag{22}$$

The interpretation of this result is highly intuitive: the expected gross return on the asset equals the risk-adjusted required return r plus the expected trading costs. Using the CAPM to determine risk-adjusted required returns we find

$$E(R)=r_f+\beta\left(E(R_m)-r_f\right)+\zeta c, \tag{23}$$

where r_f is the risk-free rate, $E(R_m)-r_f$ is the market risk premium and β is the asset's beta. Thus, expected returns are the sum of three components: (i) the risk-free rate of return; (ii) a risk premium, determined by the "beta" of the asset; and (iii) a liquidity premium, determined by the bid-ask spread and the trading frequency.

Amihud and Mendelson (1986) also discuss an extension of their model where the trading frequency ζ is different across investors. This generates a self-selection effect, where patient traders with low turnover frequency invest mainly in assets with high transaction costs, and impatient traders with high turnover rates invest mainly in assets with low transaction costs. Due to this clientele effect, in equilibrium the trading frequency of an

[9] In this solution, the small cross products $\zeta S\cdot r$ and $\zeta c\cdot d$ are omitted.

asset decreases with higher transaction costs. This effect will also be reflected in the equilibrium expected returns. The next section shows a simple model that yields the same result. The net result in the model of Amihud and Mendelson is that the equilibrium expected returns are increasing but concave in the transaction costs c. However, the empirical evidence for this concave relation between transaction costs and expected returns is weak. Nevertheless, this point is particularly relevant for pension funds. As they arguably have a longer investment horizon than most other investors, pension funds may be the likely clientele for less liquid investments. The next sub-section investigates how much pension funds then should invest in less liquid assets.

Lo, Mamansky and Wang (LMW, 2004) model general equilibrium in the style of Constantinides (1986) with fixed transaction costs per trade (thus, not per share). This gives rise to a no-trade region: only when the actual position in stocks deviates enough from the desired position does the investor trade. Because of the resulting suboptimal asset allocations, LMW show that in equilibrium this leads to a discount on the stock price. This discount is approximately proportional to the square root of the transaction costs. This is reminiscent of the clientele effect of Amihud and Mendelson: stocks with high transaction costs are traded less frequently, implying a price discount that is increasing but concave in the transaction costs.

4.1.2. A liquidity CAPM with heterogeneous trading frequencies

Consider a market with two types of agents. The first type is patient and trades with a low frequency ζ_1; the second type is impatient and trades with a higher frequency ζ_2, where obviously $\zeta_2 > \zeta_1$. For simplicity, we assume that both agents are equally wealthy and have the same risk aversion. We furthermore assume that there are N assets with gross returns r and transaction costs c. To keep the analysis as clear as possible, we assume that the transaction costs are not random (so that there is no liquidity risk). The demand for assets by agent i is then given by equation (10) as

$$x_i^* = \frac{1}{\gamma} Var(r)^{-1} E(r - \zeta_i c). \tag{24}$$

Again, we see that assets with high transaction costs have lower demand but this effect is stronger for the agent with a high trading frequency. Aggregating the demand of both agents, we find the equilibrium returns

$$E(r) = \zeta E(c) + \gamma Cov(r, r_m), \tag{25}$$

where ζ is the average trading frequency (i.e. the average of ζ_1 and ζ_2) and r_m is the return on the market portfolio. Now consider the net, after-cost expected return for the patient investor

$$E(r) - \zeta_1 E(c) = (\zeta - \zeta_1) E(c) + \gamma Cov(r, r_m) \tag{26}$$

This is higher than the risk-adjusted return because the patient investor's trading frequency ζ_1 is smaller than the market average trading frequency ζ. Plugging the equilibrium expected returns back into the optimal portfolio of equation (24), we find

$$\begin{aligned} x_i^* &= \frac{1}{\gamma} Var(r)^{-1} \left[(\zeta - \zeta_i) E(c) + \gamma Cov(r, r_m) \right] \\ &= x_m + \frac{\zeta - \zeta_i}{\gamma} Var(r)^{-1} E(c), \end{aligned} \tag{27}$$

where x_m is the vector of weights of the market portfolio. This formula reveals that the patient trader (with $\zeta - \zeta_1 > 0$) has a relatively high demand for illiquid assets with high expected transaction costs $E(c)$, whereas the impatient trader (with $\zeta - \zeta_2 < 0$) has a relatively low demand for such assets. This analysis therefore confirms a clientele effect, where impatient investors invest relatively more in high-cost assets, and impatient traders relatively more in liquid assets.

4.1.3. CAPM with liquidity constraints

Similar to the variation on the standard CAPM in (23), the portfolio set-up in Section 3.1.4 in which investors choose their mean-variance optimal portfolio subject to a liquidity constraint $w'c = \delta$ readily leads to a variation on the CAPM. Specificially, note that when aggregating the demand x^* over all agents (weighted by their wealth), we obtain the results that aggregate demand, or the market portfolio, x_m also satisfies Equation (15) for the market risk aversion γ_m and market-weighted value of the Lagrange multiplier φ_m. Introducing the Global Minimum-Variance-Liquidity (MVL) portfolio, $x_{MVL} = \left(c' Var\left(r\right)^{-1} c\right)^{-1} Var\left(r\right)^{-1} c$, and solving for γ_m and φ_m, then leads to a two-factor version of the CAPM:

$$E\left(R\right) = r_f + \beta_m E\left(r_m\right) + \beta_L E\left(r_{MVL}\right), \tag{28}$$

where β_m and β_L are the betas from regressing the asset returns on the market return r_m and the Minimum-Variance-Liquidity portfolio return r_{MVL}. Working out the last term, we can show that[10]

$$E\left(R\right) = r_f + \beta_m E\left(r_m\right) + \alpha c, \tag{29}$$

[10]It can be shown that $\beta_L = c$ and $\alpha = E\left(r_{MVL}\right)$.

which is observationally equivalent to the Amihud-Mendelson model, but with a different interpretation for the regression coefficient α of the transaction costs c.

The fact that agents care about portfolio liquidity implies that they no longer hold the market portfolio as in the standard CAPM, but that they want to adjust the market portfolio to get the desired level of liquidity. As agents may care about portfolio liquidity in different ways, they adjust the market portfolio accordingly but always proportional to the MVL portfolio. We therefore obtain the result that each agent holds a combination of the market portfolio and the MVL portfolio, which are then exactly the same portfolios that determine the expected returns on assets. Although this version of the CAPM is not derived in Lo et al. (2003), it follows readily from their analysis.

4.2. Liquidity risk and asset prices

The Amihud and Mendelson (1986) pricing model discussed in the previous sub-section, as well as the CAPM with liquidity constraints, show the sensitivity of asset prices to liquidity. In reality, liquidity is not a constant but fluctuates substantially over time. Figure 1, which graphs the quoted and effective bid-ask spread over time, illustrates this point. Recent studies on equity market liquidity, such as Chordia, Roll and Subramanyam (2000) and Hasbrouck and Seppi (2001), show that there is commonality in liquidity (i.e. shocks to the liquidity of individual stocks contain a common component). Moreover, returns on stocks tend to be correlated with changes in market-wide liquidity.

These results warrant investigating liquidity as a priced risk factor. In this set-up, it is not only the level of transaction costs that determines asset prices, but also the exposure of returns to fluctuations in market-wide liquidity. Indeed, the recent literature has shown that the (systematic) risk associated with common liquidity fluctuations is priced in the cross-section of expected equity returns. Pioneering work in this area was done by Amihud (2002). Important recent papers in this growing literature include Acharya and Pedersen (2005), Pastor and Stambaugh (2003) and Sadka (2006), who all document the significance of liquidity risk for the expected returns on equities.

Acharya and Pedersen (AP, 2005) extend the CAPM to include transaction costs. In their model expected returns are determined by expected transaction cost and the asset's beta using net (i.e. after transaction cost) returns. They start from the asset demand with transaction costs in equation (10), which is repeated here for convenience:

$$x^* = \frac{1}{\gamma} Var(r - c)^{-1} \left[E(r) - E(c) \right]. \tag{30}$$

Adding up all asset demands and putting the aggregate demand equal to aggregate supply yields an asset pricing relation similar to the usual CAPM

$$E(r_i - c_i) = \beta_i^{net} E(r_m - c_m), \tag{31}$$

where r_m is the value-weighted market return, and c_m a value-weighted average of an individual asset's transaction costs. Acharya and Pedersen refer to the coefficient as the "net" beta (i.e. the beta calculated using returns net of transaction costs):

$$\beta_i^{net} = \frac{Cov(r_i - c_i, r_m - c_m)}{Var(r_m - c_m)}. \tag{32}$$

For additional insight, notice that the net beta can be decomposed into four components, as follows:

$$\begin{aligned}\beta_i^{net} &= \beta_{1i} + \beta_{2i} - \beta_{3i} - \beta_{4i} \\ &= \frac{Cov(r_{it}, r_{mt}) + Cov(c_{it}, c_{mt}) - Cov(r_{it}, c_{mt}) - Cov(c_{it}, r_{mt})}{Var(r_{mt} - c_{mt})}.\end{aligned}$$

The first component, β_{1i} is the traditional CAPM beta, whereas the other betas measure different aspects of liquidity risk.

This theory can be tested by running regressions of the form

$$\bar{r}_i = \alpha + \zeta E(c_i) + \lambda \beta_i^{net} + u_i, \quad (33)$$

where $\bar{r}_i$ is the average excess return on asset i, and $E(c_i)$ is the expected transaction cost. The coefficient ζ measures the implicit trading frequency of the asset (like in Amihud and Mendelson's (1986) model). The coefficient λ is the risk premium for covariance with the market return. The intercept α should be zero. The model can also be extended by breaking up the net beta into its components, and running the regression

$$\bar{r}_i = \alpha + \zeta E(c_i) + \lambda_1 \beta_{1i} + \lambda_2 \beta_{2i} + \lambda_3 \beta_{3i} + \lambda_4 \beta_{4i} + u_i. \quad (34)$$

Bongaerts, De Jong and Driessen (BDD, 2009) present an equilibrium pricing model for assets that incorporates hedging demands (similar to the case discussed in section 3.1.2), short positions and positive or zero net supply. This model therefore extends the model of Acharya and Pedersen (AP, 2005) in several ways (AP only consider investors with long positions and no hedge demands). They show that illiquid assets can have lower expected returns if the short-sellers have more wealth, lower risk aversion or shorter horizon. The pricing of liquidity risk is different for derivatives than for positive-net-supply assets, and depends on the investors' net non-traded risk exposure. Formally, in case of a single benchmark asset the model is

$$E(\widehat{r}_i) = \zeta E(\widehat{c}_i) + \phi \frac{Cov(\widehat{r}_i - \widehat{c}_i, \widehat{r}_m - \widehat{c}_m)}{Var(\widehat{r}_m - \widehat{c}_m)}, \tag{35}$$

with

$$\widehat{r}_{it} = r_{it} - \beta_i^r r_{b,t}, \quad \beta_i^r = \frac{Cov(r_{it}, r_{b,t})}{Var(r_{b,t})}$$

and

$$\widehat{c}_{it} = c_{it} - E_{t-1}(c_{it}) - \beta_i^c r_{b,t}, \quad \beta_i^c = \frac{Cov(c_{it} - E_{t-1}(c_{it}), r_{b,t})}{Var(r_{b,t})},$$

and with r_b the return on the benchmark asset.[11] The 'market' return and cost factors $\widehat{r}_m$ and $\widehat{c}_m$ are value-weighted averages of the individual returns and costs. BDD estimate this model for the credit default swap market using GMM. They find strong evidence for an expected liquidity premium earned by the credit protection seller. The effect of liquidity risk is significant but economically small.

BDD assume exogenous transaction costs and exogenous trading frequency. But even with endogenous trading frequencies, Jang, Koo, Liu and Loewenstein (2007) show that transaction costs can have first-order effects on the equilibrium price. The reason is that due to the hedging demands, the trading frequencies are not affected much by transaction costs. Therefore, the expected trading costs over the investment period can be much larger than in the case without hedging demands. This will be reflected in much larger illiquidity discounts in the

[11] This model is simpler than the original BDD model: it assumes there are no non-traded risk factors with which the asset returns are correlated.

asset prices than is the situation in models without hedging demands such as Constantinides (1986)

4.3. Non-traded risks and asset returns

Whereas the Acharya and Pedersen (2005) model incorporates (stochastic) transaction costs, and therefore the price dimension of liquidity, a simple but insightful variation can also be obtained by focussin on the quantitiy dimension of liquidity. In particular, by assuming certain assets or risks cannot be traded at all, we can obtain a variation on the CAPM with non-traded risks or hedging pressure. The model by Bongaerts, De Jong and Driessen (2009) further extends this.

Based on the optimal portfolio choice in equation (12), at least two representations of an asset pricing model can be obtained. First, aggregating demand x^* over all agents, total demand equals the market portfolio x_m, which satisfies the same equation for the market risk aversion γ_m and exposure to the non-traded risk q_m. This means that expected returns on traded assets satisfy

$$E\left(r\right) = \gamma_m Cov\left(r, r_m\right) + \gamma_m Cov\left(r, R\right) q_m. \qquad (36)$$

It is then straightforward to show that this implies a familiar beta-representation:

$$E\left(r\right) = r_f + \beta_m E\left(r_m\right) + \beta_h E\left(r_h\right), \qquad (37)$$

where h is the hedge portfolio introduced in section 3.1.2 (i.e., the portfolio that has maximum correlation with the non-traded risk R). Importantly then, as with the liquidity constraint, the fact that the agents combine the market portfolio with the hedge portfolio implies that expected returns depend on the covariances or betas with exactly those two portfolios. If there is only one non-traded risk, this therefore leads to a two-factor model. If there are K non-traded risks, this leads to a $K+1$-factor model. This important insight underlying many linear factor models may help in understanding empirical factor models such as the Fama-French (1996) three-factor model. For instance, Heaton and Lucas (2000) motivate the (empirical) Fama-French factors based on Size and Book-to-Market as hedge portfolios for entrepreneurial risk (i.e., illiquid non-listed firm risk). Similarly, Kullmann (2001) interprets these same factors as hedges against the non-traded or illiquid real-estate risk to which many investors are exposed. Finally, Errunza and Losq (1985) and De Jong and De Roon (2005) derive a similar model in an international context, where the illiquid or non-traded assets are stocks in emerging markets that cannot be held by foreign investors and therefore create non-traded risk for domestic investors in those markets.

The factor-model in (37) illustrates how non-traded risks lead to additional risk premiums in traded assets via the hedge portfolio h. Intuitively, from (12), investors underweigh the market portfolio with respect to the hedge portfolio (i.e., they invest less in assets that are highly correlated with their non-traded risks). This means that in order to induce investors to invest in these assets, equilibrium expected returns must increase. Therefore, assets highly correlated with non-traded risks induced higher risk premiums.

An alternative representation based on (36), which provides further insight, is the following:

$$\begin{aligned} E\left(R_i\right) &= r_f+\beta_{im}E\left(r_m\right)+\theta_i q_m, \\ \theta_i &= \gamma_m Cov\left(\varepsilon_i, R\right), \end{aligned} \tag{38}$$

where ε_i is the residual from a CAPM-based regression:

$$R_i - r_f = \alpha_i + \beta_{im} r_m + \varepsilon_i,$$

and q_m is the market exposure to the non-traded risk factor, defined as the wealth-weighted sum of the individual traders' exposures. This representation of the same model further highlights the source of the additional risk premium for the traded asset i, which now equals $\theta_i q_m$. First, θ_i contains the market risk aversion γ_m, which reflects that (as for $E(r_m)$) the more risk-averse all investors in the market are, the higher the risk premium should be. Next, θ_i shows that the relevant element in the additional risk premium is the covariance between ε_i and the non-traded risk R, and not the covariance of the total return R_i with R. This reflects the fact that the part of R_i that is due to the market return is irrelevant for the hedge premium. Intuitively, to the extent that the market portfolio is highly correlated with the non-traded risk, investors can use the market portfolio itself for hedging and thus can still hold the market portfolio. However, if it is the residual return ε_i that is highly correlated with the non-traded risk, investors will want to invest less in that asset relative to the market, and will use the specific asset to hedge themselves against the non-traded or illiquid risk factor. Thus, in order to induce investors to include the asset in the market portfolio, the asset must generate an additional expected return or risk premium. This highlights the fact that due to correlation with illiquid or non-traded risks, idiosyncratic risk gets priced. Eiling (2009) uses this to show how human capital, perhaps the ultimate non-traded asset, is priced in financial markets. Finally, (38) shows that the risk premium is also proportional to q_m. This implies that a non-traded risk only gets priced in a traded asset if it is relevant on a market-level. Put differently, only if sufficiently many investors (weighted by their wealth) become exposed to non-traded risk, will this induce additional risk premiums in traded assets. This explains why illiquid assets such as non-traded firms and real estate may be the relevant risk factors, as these constitute important illiquid risks on a macro-economic level.

5. Valuation of illiquid assets

The previous section discussed optimal trading strategies and the valuation of traded assets in the presence of transaction costs. Some assets, however, cannot be traded, at least for some period of time. Examples are assets with lock-up periods such as private equity, major shareholdings in a business and real estate. Another important form of illiquidity are financial risks that cannot be traded at all because there are no financial instruments that are perfectly correlated with that risk. Examples here are macro risks such as inflation and productivity, and on an individual level labor income uncertainty. The literature has extensively discussed the implications of these frictions for the valuations of assets and liabilities. This section provides an overview.

The valuation of illiquid assets is relevant for accounting and when transferring (selling) assets or liabilities. This includes, for example, pension liabilities, the housing stock of a housing corporation, private business, and human capital. Also, pension funds increasingly invest in assets that generate steady cash flows but are difficult to trade, like direct real estate, infrastructure and private equity. Can we always use mark-to-market valuation, or are there alternatives, and what are the accounting rules around this?

There are several theoretical contributions in this area, including Grossman and Laroque (1990), Longstaff (2001) and Kahl, Liu and Longstaff (2003). These papers work from an equivalent utility approach, which is sometimes also called an indifference approach. They compare an investor who has access to a fully liquid asset to another investor, with the same preferences, who has a position in the illiquid asset. The models specify the optimal consumption-investment strategies of the two investors. The expected utility of the two investors is then compared. This approach can be used to determine how much of the liquid asset the investor should be endowed with in order to obtain the same expected utility as the investor with the illiquid asset. This value is then the value of the illiquid asset.

The model of Kahl, Liu and Longstaff (2003) is a good and simple example. There are three assets in the economy: a risk-free (cash) investment, a stock index fund with price M and a stock in the investor's firm with market price S. The investor can trade freely in the risk-free asset and the stock index fund, but his holdings in the firm are restricted until time $t = R$. After R, the stock can be traded freely. This situation is typical after an initial public offering (IPO) where the former owners of the company are forbidden to trade their stake in an initial period after the IPO. But we could also use the same idea to study a pension investor who is building up assets in a pension plan but is not allowed to take out the money (or borrow against the value of the pension plan) before retirement date R. The risk-free interest rate is fixed at r, whereas the values of M and S follow geometric Brownian motions with drift $r + \mu$ and $r + \lambda$, respectively (thus, μ and λ are the risk premiums on the stock index and firm's stock):

$$dM/M = (r + \mu)dt + \sigma dZ_1 \tag{39}$$

$$dS/S = (r + \lambda)dt + \nu dZ_2. \tag{40}$$

The correlation between the returns on the stock index and the firm's stock is ρ.

The investor maximizes a utility function over period $t = 0$ to

$t = T$

$$U = E\left(\int_0^T e^{-\beta t} U(c_t) + e^{-\beta T} U(W_T)\right), \qquad (41)$$

with a constant relative risk aversion (CRRA) utility function

$$U(c) = \frac{c^{1-\gamma}}{1-\gamma}, \qquad (42)$$

where γ is the coefficient of risk aversion. The policy instruments of the investor are his consumption and his asset allocation. Let x_M be the portfolio weight in the market and x_S the weight in the firm's stock. Then, utility is maximized under the budget constraint

$$dW/W = (r + x_M\mu + x_S\lambda)dt + x_M\sigma dZ_1 + x_S\nu dZ_2 \qquad (43)$$

and given initial wealth W_0.

We now have two settings. One investor faces a restriction on his portfolio weights before time R (i.e. $x_2(t) = \phi$ for $t < R$). After R, he can freely trade the firm's stock. The other investor is completely unrestricted and can freely trade all assets. Obviously, with the same initial capital W_0, the unrestricted investor will obtain a higher expected utility than the restricted investor. We can also turn the setting around and explore for which initial capitals do the restricted and unrestricted investors obtain the same expected utility? The difference between the required initial capitals then is the economic value of the liquidity of the asset (or the discount on the illiquid asset).

Obviously, the value of the restricted stock depends on the parameters of the model. Especially important are the length of the lock-up period; the asset's volatility (the higher the volatility, the higher the illiquidity discount); the correlation with the market (the higher the correlation, the lower the discount as the market can be used as a hedge against the illiquid asset's return fluctuations); and the fraction of initial wealth locked up in the illiquid asset (the higher this fraction, the higher the illiquidity discount). For example, a two-year lock-up for an asset with 30% volatility and no correlation with the market has a 10% discount for an investor with $\gamma = 2$ and half of his wealth locked up in the firm's stock. For a five-year lock-up period, the discount rises to 28%. More results can be found in the original paper of Kahl, Liu and Longstaff (2003). The reduced value of the illiquid asset also has a profound effect on the consumption level. For the same parameters, the investor reduces his consumption level to 93% of his consumption level without the lock-up period. For a five-year lock-up period, the consumption level is even lower, only 80% of the unrestricted level. So, the illiquidiy of assets has serious implications both for the valuation of the asset and for the consumption decisions of the investor.

De Jong, Driessen and Van Hemert (2007) study the investments of a homeowner. They model the situation of a homeowner similar to Kahl, Liu and Longstaff (2003). The house can be sold at retirement date R, and before retirement the house is a substantial fraction of total wealth. An additional complication is that the revenues of the house are partly 'in kind' (i.e. there is a convenience yield of living in the house), which makes the financial returns of the house investment low compared to other assets with similar risk characteristics. Figure 3 shows the implicit discount an investor applies to the value of his house in making investment decisions. The equation for the portfolio weight in stocks is

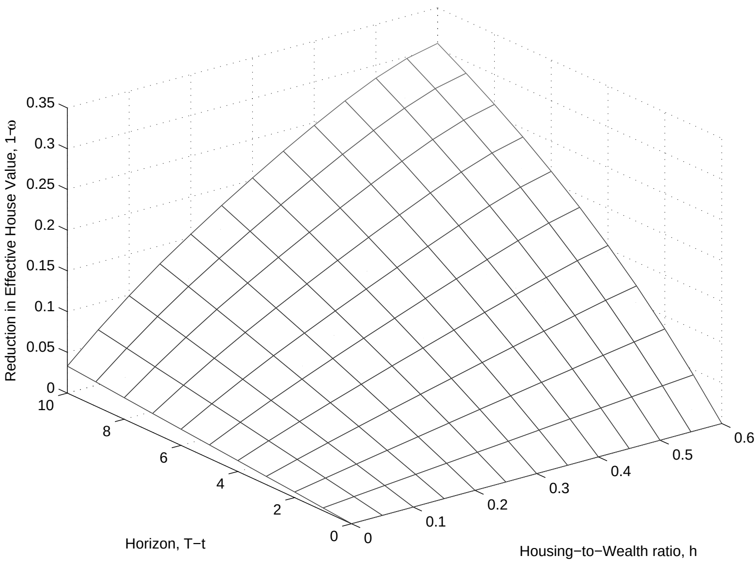
$$x_M = \frac{1-(1-\omega)h}{1-h}\frac{\mu}{\gamma\sigma} - \frac{\omega h}{1-h}\beta_h, \tag{44}$$

where h is the ratio of the value of the house to total wealth, and β_h is the beta of the house return.[12] The components of this expression are quite clear: the investor has the usual mean-variance optimal weight in stocks, $\mu/\gamma\sigma$, multiplied by a leverage factor $1/(1-h)$ and a factor that discounts the value of the house, $(1-\omega)h$. The final term is a hedge term that corrects for the correlation between the house value and the stock market. Again, in this term the value of the house is only a fraction ω of the market price of the house. This equation clearly shows the impact of illiquidity on investment decisions.

The optimal investment policy is also affected by the illiquidity of the holdings in the firm's stock. Figure 3 plots the portfolio weight of the market index fund as a fraction of the *liquid* wealth. There are two countervailing effects. First, the investor takes a higher weight in the stock market to get the same exposure to stock returns. This is a pure leverage effect: as the fraction of liquid assets to total assets becomes smaller, the investor leverages up his liquid investment portfolio. But there is a countervailing effect that diminishes the investments in risky assets. The presence of the illiquid asset leads to undiversified background risk, which makes the investor act in a more risk-averse manner. See also Grossman and Larqoue (1990) for an explanation of this effect.

[12]The beta is defined as the covariance between the return on the house and the return on the market index, divided by the variance of the market return.

Figure 3: Reduction in Effective Housing Wealth.
The figure plots the reduction in effective housing wealth, $1-\omega$, for a $\gamma = 5$ investor. Source: De Jong, Driessen and Van Hemert (2007)



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SUMMARY OF THE DISCUSSION

By Jacqueline van Leeuwen (Utrecht University and Netspar)

Illiquidity. Implications for investors and pension funds.
By Frank de Jong (Tilburg University and Netspar) and Frans de Roon (Tilburg University and Netspar)

Chairman: Peter Schotman (Maastricht University and Netspar)
Discussants: Berend Roorda (University of Twente and Netspar) and Rob van den Goorbergh (APG)

Berend Roorda opened the discussion by remarking that the paper's multidimensional definition showing the liquidity aspects of assets is quite clear. However, not enough attention was paid to the context of the investors. Roorda commented on the use of terminology: in his view liquidity risk relates not only to the costs *given a trade* – as is the topic of the paper – but also the risk of being confronted with these costs. Roorda focussed on the assumption that – in the one-period framework – the assets are sold again at the end of the period. Roorda maintained that we are used to thinking in terms of one-period returns, because this is entirely straightforward in ideal pricing: in that case it is irrelevant whether you actually sell, or not, at the end of the period. With transaction costs it does matter. Roorda observed that there are issues here, as it is tricky to perceive liquidity costs or to analyze liquidity effects in terms of one-period returns.

Roorda also mentioned that equilibrium models are often used as a building block in a backward recursive way, with the end state of a period serving as the initial state of the next period.

From this viewpoint, it would be more natural to assume in Section 4.1 that at each moment (both at the beginning and end of each period) a fraction z/2 of investors sells, and a same fraction buys, rather than assuming that everyone possesses assets at the beginning, and a fraction z sells. This is mainly a matter of exposition (see the discussion below).

He compared these points with CAPM. The weaker point of CAPM is that everyone has essentially the same portfolio. For modeling the price impact of illiquidity we are faced with a similar problem. Roorda noticed that in order to arrive at unambiguous answers we have to make very stylized assumptions on the trading behavior of all agents in the market. This raises the interesting question of what kind of trading behavior really determines the price effect in the market. During his discussion on the price impact of (un)known transaction costs, Roorda once again stressed the importance of the investor's context. Liquidity risk is not exclusively related to the asset side of the balance sheet: it may, in fact, arise from the liability side.

Roorda concluded his part of the discussion with the remark that he missed the L in the ALM. Anyone who really wants to examine liquidity risk has to pay attention to the broad setting.

Rob van den Goorbergh found this a very nice and rich review paper. Van den Goorbergh chose to focus on the models about portfolio choice and illiquidity premium. He noticed that one very relevant case not mentioned by the authors (about the one-period mean-variance investor) is the situation in which the transaction costs are relatively high, in which case it is optimal not to invest at all. Van den Goorbergh reasoned that in fact we have three cases, rather than two: the added case being the no-trade zone, in which it is optimal for the investor to do nothing, or 'far niente'.

Van den Goorbergh also commented on another dimension of illiquidity, namely the (temporary) inability to trade. The situation referred to in the paper is about hedge funds with a pre-specified three-month lock-up period. The paper gives a certainty equivalent return of about 4% per year for only a three-month lock-up, which is really high. If this were the case for hedge funds, he wondered, then what about private equity where the lock-ups can be ten years? How much of this premium depends on your assumption of timing ability?

In this context, Van den Goorbergh mentioned the possibility of 'laddering'. The investor might consider diversifying this type of investments across different vintages; he has lock-ups in each one of the investments but never in all of them at the same time. This type of diversification might mitigate the rebalancing problem somewhat and reduce the certainty equivalent return.

Another way of modeling the inability to trade is through stochastic trade times rather than pre-specified lock-up periods. Van den Goorbergh mentioned a recent paper by Ang, Papanikolaou and Westerfield (2010), which considers the case of random trade times. The paper is an elegant extension of the classic Merton model. In their case, a three-month average turnover (that is, an average trading frequency of four times per year) corresponds to a liquidity premium of only 0.63%. Therefore, Van den Goorbergh wondered if the authors could comment on the difference between this paper and the previous paper.

Bas Werker directed the following question at De Jong: The liquidity functions that we use are all state-independent. We expect discussion in these kind of situations. Certainly in situations where the liquidity is low, we have other problems as well. For instance, the regulator that forces you to do things you may or may not want to do. Wouldn't that be a useful extension then?

De Jong began by replying to Roorda's remarks on liquidity. **Roorda** mentioned that what De Jong calls liquidity does not reveal the presence of any variance of the transaction costs. Well, how could it? De Jong wondered. The degree to which you are exposed to fluctuations is transaction costs. De Jong agreed, however, that Roorda made a valid comment. De Jong thinks that the paper, at this point, does not feature a fully-developed framework capable of doing that. Regulatory constraints may force you to take less risk; therefore, selling may lead to additional trading and therefore to more exposure to liquidity risk. De Jong agreed with **Werker**'s suggestion that that will be a way to model state-dependent utility.

De Jong emphasized that the purpose of this paper was not to develop the ultimate model of pricing of illiquidity. The main idea was to summarize what is in the literature, what type of lessons can be learned and why these might be relevant for investors. Illustrating this in a one-period model gives you a lot of the key intuitions, but obviously not all of the key intuitions. **Roorda** pointed out that the issue is mainly a matter of exposition: without changing the results, the assumptions on trading and selling can be presented more symmetrically, in line with their use in a dynamic context. After all, De Jong's main suggestion is to make the different notions of one-period returns explicit in the paper, which may also help to unify the numerous approaches described in the paper.

In response to a question by **Van den Goorbergh**, **De Jong** reckoned that they could improve somewhat the way they describe the clientele effect in their paper. Investors will probably specialize in a particular asset. The investors with a long horizon will probably invest in high transaction cost assets, and they will determine the price of that asset. And the investors with a short investment horizon will probably specialize in the high liquidity assets, and they will determine the price of that asset. Of course many investors have many assets and it becomes quite complex. But this possibility was considered to be too complex for a panel paper to explain. Perhaps a simple example could be included. Regarding liabilities, **De Jong** mentioned that there is a model discussed in the paper that includes the background for this (like the type of risk exposure that you want to hedge). He thought of that as a liability, which may influence or impact the asset allocation. But, nothing was said about how to deal with the illiquidity of the liabilities themselves. In that set-up the liability or the background risk you want to hedge is not endogenous; you take it as given.

With regard to **Van den Goorbergh's** suggestion of layering investments with lock-up periods across different vintages, **De Jong** agreed that these types of strategies would lead to a reduced exposure to liquidity risks if you diversify in that dimension.

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23. *Illiquidity: implications for investors and pension funds* (2011)
Frank de Jong and Frans de Roon

Netspar

Illiquidity: implications for investors and pension funds

Liquidity has become one of the most heavily discussed and researched aspects of financial markets. This paper by Frank de Jong and Frans de Roon (both Tilburg University) discusses the implications of illiquidity, and in particular transaction costs and non-traded risks, for investors. Illiquidity is a multi-dimensional concept, reflecting at least a time, volume and price dimension. This affects the optimal portfolio composition.