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Optimal Retirement with Disability Pensions

Hans Fehr, Adrian Fröhlich

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Hans Fehr*
University of Wuerzburg, Netspar and CESifo

Adrian Fröhlich
University of Wuerzburg

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Abstract

This paper develops a general equilibrium life-cycle model with endogenous retirement and disability risk, in order to quantify the impact of recent pension reforms in Germany. At certain ages households may either apply for disability pensions (DP) or old-age pensions (OAP), depending on eligibility rules and the generosity of the two programs. Our policy analysis focus on the increase in the normal retirement age (NRA) from age 65 to 67 (Reform 2007) and the recent increase in the maximum assessment age (MAA) for DP benefits (Reform 2018). In contrast to the first reform, the second reform received hardly any attention in the public pension debate in Germany. Our simulation results indicate that with current eligibility and benefit rules, the second reform will almost neutralize the financial and economic benefits of the first reform. Consequently, securing the financial stability of the system will require a tightening of eligibility rules and/or a reduction of early retirment benefits in the future.

JEL Classifications: C68, D91, H55, J24

Keywords: overlapping generations, stochastic general equilibrium, endogenous retirement, disability pensions

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* *Corresponding author:*
Address: University of Wuerzburg, Sanderring 2, 97070 Wuerzburg, Germany
Email: Email: hans.fehr@uni-wuerzburg.de

1 Introduction

Almost all developed economies in the Western world are confronted with a rapidly ageing population which places considerable pressure on the future sustainability of pay-as-you-go financed pension systems. Since reducing the benefit levels is hardly possible without increasing old-age poverty, an obvious and seemingly simple reform strategy in countries such as the U.S., France, Italy or Germany is to delay retirement and encourage labor force participation and employment of older workers by increasing the normal retirement age (NRA) and reducing early retirement incentives. Although such policies may yield a double dividend in the form of financial and distributional benefits (Cremer and Pestieau, 2003), it is typically very unpopular among the affected population. Consequently, households seek for alternative routes of retirement and such behavioral reactions may seriously impair the effectiveness of the original reform.

The difficulty of raising effective retirement ages is at least partially due to the simultaneous provision of disability pensions (DP) and old-age pensions (OAP). DP allow individuals to retire earlier when they are in bad health with severe mental or physical work limitations. However, in practice one can often hardly observe the mental or physical condition of an agent who applies for DP. Consequently, fairly healthy agents who want to retire early may try to use DP as an alternative pathway into retirement.[1]

The present paper analyzes this interplay between OAP and DP retirement in Germany, which has enacted recent pension reforms that may be of particular interest also for other countries. Starting already in 1992 Germany has successively reduced OAP and DP benefits as well as incentives for early retirement. This reform process ended in 2007 with a phased-in increase of the NRA from age 65 to age 67 during the period 2012 until 2031. Börsch-Supan et al. (2020) document that these reforms steadily increased employment rates of elderly as well as pension claiming ages in Germany between 2000 and 2015. However, the increase of the NRA also turned out to be very unpopular among the German population due to the widespread fear that those with physically demanding and often low-paid jobs would be the main losers.[2] To mitigate this opposition, the pension reform of 2014 introduced an early retirement window at age 63 for employees with an extra long contribution record. In addition, various reforms until 2018 raised the generosity of DP benefits sequentially in order to reduce old-age poverty risk. As a consequence, DP benefits will become more and more attractive relative to OAP benefits, inducing a possible shift towards DP applications (and enrolments) in the future.

This paper aims to quantify such possible effects. Therefore, we capture the linkage between DP and OAP within the framework of a dynamic general equilibrium life-cycle model that features the institutional structure of the German tax and pension system. Individuals consume, save and decide whether to file a DP application and when to stop working, facing idiosyncratic health, earnings, disability claiming and mortality risk. We distinguish severe and less severe health shocks, which reduce work capacity. While the government can only partially identify severe and less severe work limitations, the DP application is associated with social stigma cost. Therefore, depending on the

[1] This problem is especially relevant during times of high unemployment and in countries with a generous DP program. A recent study by Maestas et al. (2021) found that the Great Recession during 2008-2012 increased the number of disability insurance beneficiaries in the US by almost 9 percent.

[2] For example, Scheubel et al. (2013) report of opinion polls which show that about 80 to 90 percent of the population opposed this reform.

prospects of recovery, the severity of the screening process, the benefit generosity and the social stigma cost associated with disability, households with work limitations will either apply for DP or continue to work on the labor market.

Our initial equilibrium reflects the situation, where the normal retirement age for OAP was still at age 65 and the generosity of DP benefits was quite low. Then we successively alter the benefit and eligibility rules for OAP and DP retirement and compute the resulting long-run financial and macroeconomic effects. Our simulations indicate four central results. First, the sole increase of the NRA from age 65 to age 67 will induce a shift towards DP retirment, so that the effective retirement age increases much less than two years. However, the Reform 2007 will significantly reduce the financial pressure on the pension system, while the induced increases in employment and savings improve improve long run welfare by 3.6 percent. Second, these positive effects are almost neutralized by the recent Reform 2018, which dramatically increases incentives for DP retirement, although DP pensions only play a minor role in the pension budget. Third, securing financial stability of the pension system in the long run requires a tightening of DP eligibility and/or a reduction of early retirement benefits. Finally, reforms of OAP eligibility rules may also suffer from spill-over effects towards DP retirment. Overall, disability pensions in Germany are quantitatively important in terms of their financial and macroeconomic consequences. It is therefore quite surprising how often they are neglected in the pension debate.

Our paper is related to the macroeconomic literature on social security reforms in overlapping-generations models pioneered by Auerbach and Kotlikoff (1987). While the early studies typically neglected the retirement decision, various recent contributions focus on retirement behavior and the interaction between health and retirement. İmrohoroğlu and Kitao (2012) apply a model with endogenous labor supply at the intensive and extensive margin in order to analyze the increase in the NRA from 66 to 68 in the U.S. Fehr et al. (2012) provide an analysis of the 2007 pension reform and alternative options to increase the effective retirement age in Germany. Börsch-Supan et al. (2018) model labor market participation cost which rise with age in order to quantify the impact of deductions combined with an earnings test for early retirement. They propose actuarially neutral deductions in combination with an elimination of the earnings test. All these studies abstract from disability risk and DP pensions. These features are considered by Diaz-Gimenez and Diaz-Saavedra (2009), Erosa et al. (2012), Fehr et al. (2013) or Kitao (2014), who study retirement in models with earnings uncertainty and disability risk. However, DP retirement is treated there as a pure exogenous process without an individual application decision and application process. Most closely related to our approach are therefore Laun and Wallenius (2015, 2016), Laun et al. (2019), Li (2018) and Galaasen (2021), who explore how the interaction between OAP and DP affects the labor supply of older workers in Sweden, the U.S. and Norway, respectively. These studies highlight that including DP is quantitatively important when analyzing pension reforms. For example, Li (2018) finds that an increase in DP receipt may offset about 40 percent of the fiscal gain from increasing the NRA in the U.S. In Norway, where DP recipients are much more important than in the U.S., the fiscal gain from OAP reforms could be even completely offset by increases in DP receipt, see Galaasen (2021). This is exactly where our paper sets in. However, we model the uncertainty structure of the health and productivity process as well as the disability application in more detail and focus on the institutional structure and the central elements of the most recent pension reforms in Germany.

The next section introduces the structure of the German pension system, the application process for DP pensions and a discussion of recent OAP and DP benefit reforms. Then the general equilibrium model which is applied for the quantitative analysis is presented. The forth section discusses the calibration of the baseline equilibrium, section five reports the results from the simulation exercises and section six concludes.

2 Old-age and disability pensions in Germany

The statutory German pension system (GRV) is a compulsory social insurance that offers benefits to elderly and disabled citizens as well as survivors. It is mainly financed by social security contributions by the working population and by public funding.[3] Table 1 provides some key indicators for 2019 with respect to old-age and disability pensions. The total budget of the system amounted to roughly 9.3 percent of GDP, mostly financed by payroll taxes of employers and employees which amount to 18.6 percent of the gross wage up to the contribution ceiling which is roughly the double of average income.

Table 1: Key pension indicators in Germany 2019

Total GRV budget 320 bn. € (9.3 % of GDP)				
Contribution ceiling 80.400 € (207 % of average income)				
Contribution rate 18.6 %				
	OAP	DP	Total	DP (in %)
Applications	868.373	369.499	1237.872	29.8
Inflow	816.129	161.534	977.663	16.5
with mental health problems (in %)		41.7		
with musculoskeletal problems (in %)		12.5		
Retirement age 2019	64.3	52.7	62.3	
(Retirement age 2011	63.5	50.5	60.8)	
Standard benefit (in €)	17.820 (45.8%)			
Av. annual benefit (in €)	14.244	9.924		69.0
DP (in % of pension budget)				6.7

*Source: Deutsche Rentenversicherung (2021); own calculations.

When employees retire from the workforce, they may either apply for an old-age pension (OAP) or - in case of partial or full disability - for a disability pension (DP). Both systems require a minimum contribution period of five working years in order to receive benefits.[4] Workers with a contribution record of at least 35 years are eligible for OAP benefits starting at the early retirement age (ERA), all others have to wait until they pass the NRA. DP pensions are therefore the only pathway to retirement before the ERA. The process of benefit application and approval is complicated, time consuming and uncertain. After all rehabilitation options have been exhausted, eligibility to DP requires a likely persistent inability to work for more than three (full disability) or more than six hours (partial disability) per day. Any DP application therefore requires to be out of the labor force, an extensive documentation of the disability and a final medical examination. In 2019 the average

[3] Public funding mostly finances not contribution-related benefits. Note also that civil servants are not covered by the statutory system, while self employed can voluntarily join.

[4] This so-called "waiting time" does not apply to persons who are disabled since birth. They may receive DP pensions starting at age 20.

processing time until final rejection and acceptance was ten months and six months, respectively. The acceptance rate of applications was only about 50 percent which explains the difference between the number of applications and the inflow of new pensioners in Table 1. About 10 percent of applicants did not fulfill the minimum waiting time and more than 30 percent were rejected because they did not pass the medical examination.[5] Those who are rejected have to go back to work and may apply again in a future period. As shown in Table 1 above, among those who were accepted, by far the most important diagnose were mental health and musculoskeletal problems (back injuries, etc.), which are both difficult to diagnose.[6] The average retirement age for OAP was 64.3 years in 2019, while the respective retirement age for DP was at 52.7 years.

The calculation of benefits is based on a point system, where an annual earnings point reflect the relative income position of a worker in that year. A year's contribution at the average earnings of contributors earns one earnings point, contributions based on lower or higher income (up to the contribution ceiling) earn proportionally less or more points. When the employee retires, the sum of the accumulated earning points is multiplied by the annual "pension value", which defines the benefit amount for each earning point. The latter is adjusted every year based on changes in wages and demographics. Since disability pensioners are usually forced to retire at an early age, the assessment period of their contributions is extended to the maximum assessment age (MAA), which in effect assumes that employees have continued to contribute up to the MAA. Finally, employees that retire with OAP before the NRA and with DP before the disability retirement age (DRA) face deductions which reduce the benefit permanently by 0.3 percent for each month (or 3.6 percent for each year) of early retirement. These deductions, however are capped at a maximum of 10.8 percent for disability pensioners. In 2019 the annual pension value of an earning point was 396 €, so that the "standard benefit" of a 65-year new retiree, who had contributed for 45 years at the average income amounted to ($45 \times 396 =$) 17.820 € or roughly 46 percent of average labor income, as shown in Table 1. However, the average annual OAP benefit was much lower than the standard benefit reflecting earlier retirement and deductions. Due to much earlier retirement, the average DP benefit in the same year amounted to roughly 70 percent of the OAP benefit. Overall, disability pensions amounted to 6.7 percent of total pension benefits. This number is fairly stable during the last 20 years after a major reform of disability pensions.

During the last decade Germany implemented a number of pension reforms which affected retirement incentives quite dramatically. Our starting point is the year around 2012 before the NRA increase (Reform 2007) was phased-in. As shown in Table 2 the normal retirement age was then set at 65 and employees who had contributed for at least 35 years could already retire earlier starting at age 63 with OAP benefits (i.e. ERA = 63). Of course, individuals who retired at age 63 had to accept a reduction of their benefits by 10.8 percent. Disability pensioners, on the other hand, could already retire without deductions at age 63 (i.e. DRA = 63). For those who retired before age 60, earning points were calculated as if they have worked until age 60 (i.e. MAA = 60). Then a maximum deduction of 10.8 percent was applied. Table 1 shows that due to these provisions, individuals retired significantly earlier in year 2011 than currently. In order to induce individuals to retire later and to stabilize the financial perspective of the system, the pension reform of 2007 introduced an announced and gradual increase of the NRA from 65 to 67. Starting in 2012 and the birth cohort 1947, the NRA

[5] We would like to thank Mathias Weber and Edgar Kruse from the German pension insurance (DRV-Bund) for providing this information.

[6] The rise of mental health problems especially in lower social classes and among women is discussed by Schuller and Weiss (2019).

increases by one month per year and birth cohort. After 2023 for birth cohorts born in and after 1959 the NRA increases by two months per year so that cohorts born in 1964 and later face a NRA of 67. In addition, those with a contribution record of at least 45 years could retire already at age 65 without deductions.[7]

Table 2: Changes of key retirement parameters in Germany

Reflecting year	Initial equilibrium 2011	Reform 2007	Reform 2014	Reform 2017	Reform 2018
		2031			
NRA	65	67			
NRA (contrib. years $\geq$ 45)		65	63		
ERA (contrib. years $\geq$ 35)	63				
DRA	63	65			
MAA	60		62	65	67

With respect to disability pensions, the reform of 2007 gradually increased the retirement age without deductions from 63 to 65. A maximum of 10.8 percent deductions applies in the future to all DP pensioners, who retire at age 62 or earlier. Surprisingly, the maximum assessment age for DP benefits was not altered initially. As a result, DP benefits remained fairly low compared to OAP benefits and were identified as a major source for the observed rise in old-age poverty. To dampen this development, various recent reforms successively increased the MAA significantly. First, the pension reform of 2014 increased the MAA for new disability pensioners by two years to 62. Then the reform of 2017 implemented a phased-in increase of the MAA up to age 65 until year 2024 for new DP entrants. Finally, the latest reform, the so-called 'Pension Pact' introduced a MAA of age 65 already for new retirees of year 2019 and phased-in a further increase of the MAA up to age 67 until year 2031. The fairly generous benefit level of current DP entrants in the year 2019 can be already observed from the relative DP/OAP benefit level in Table 1 above. The attractiveness of DP relative to OAP will further increase in the future. Simple back-of-the-envelop calculations show for a 63-year old in year 2031 that the considered reforms increase the DP/OAP benefit ratio by 20 percent and increase the DP benefit of a 60-year old in the same year by 17.5 percent.[8]

It should be quite clear that agents will increasingly try to retire with DP instead of OAP benefits, which counteracts the intended increase in retirement ages and all budget consolidation efforts. Duggan et al. (2007) forecast the rise in DI enrollment due to the NRA increase from 65 to 67 in the US. They project that after 2024 when the reform is fully phased in, the implied benefit reduction will raise the disability enrollment by 1 percentage point of male and 1.56 percentage points of female employees between the ages 45 and 64. Given a DP enrollment rate of 6.7 percentage points in this age group in 2005 this is a significant increase by about 15 percent. Mullen and Staubli (2016) have analyzed the elasticity of disability claiming with respect to disability generosity in Austria for the period 1987-2010. Their estimated elasticity of DP claiming with respect to DP generosity of 1.2 ap-

[7] In 2014, the NRA for employees with such a very long contribution record was even reduced to 63 years. But this reform only applies to a small fraction of employees.

[8] Jess et al. (2019, p.109) compute that retirees with a long contribution record could be up to 27.4 percent better off with a DP instead of a OAP in and after 2031.

plied to the German birth cohort 1971 implies a rise in DP applications of more than 20 percent.[9]

Of course, all these figures are very vague and may only provide a first intuition of the reform effects. A more comprehensive analysis requires a detailed modelling of the individual decision process with respect to retirment, which reflects health and productivity risk over the life cycle and takes into account the incentives and associated uncertainties provided by the German pension system. The following section presents a simulation model which is based on these criteria.

3 The model economy

In order to quantify the likely effects of the above described reform packages, we apply a general equilibrium simulation model with overlapping generations which is developed in this section. The theoretical structure is in the spirit of Li (2018), where households face survival risk, and face shocks to their labor productivity and health which reduce their work capacity at least temporary. During their life-cycle they have to decide about whether to be employed, whether to claim disability or old-age pensions and about how much to consume and save. Besides the individual health and productivity situation, these decisions depend on the incentives provided by the tax and pension system. The government taxes consumption and income from capital and labor in order to finance public goods and operates a pay-as-you-go financed pension system that reflects the institutional structure described above. The model considers a closed economy where wages and the interest rate are determined to balance the respective factor markets. Due to our steady state analysis we omit the time index in the following whenever possible.

3.1 Demographics

The economy is populated by J overlapping generations with a model period that equals a calendar year. At the beginning of each period, a new generation is born. The population grows with a constant rate n and the cohort size of newborns is normalized to unity. Individuals enter the economy at (model) age $j = 1$ (i.e. age 20) with perfect health and may live up to a maximum of J years after which they pass away with certainty. After working one period on the labor market they may be hit by health shocks which may induce them to apply for DP. Throughout their entire life, agents face idiosyncratic survival risk, which is determined by age and individual health status h. The conditional survival probability of an agent to survive from age $j-1$ to age j is denoted by $\psi_j(h)$ with $\psi_{J+1}(h) = 0$.

3.2 Preferences and endowments

Households have preferences over stochastic streams of consumption $c_j \geq 0$, labor supply $l_j \in \{0, 0.2, 0.5\}$ and the application for disability benefits $d_j \in \{0, 1\}$. They maximize discounted ex-

[9] Note that this figure is roughly in line with the US calculations. Assuming an application rate of 14 percent in 2005 (with a 50 percent acceptance), a 20 percent rise in applications increases the application rate by 2.8 and the enrollment rate by 1.4 percentage points.

pected utility

$$U = \mathbb{E}\left[\sum_{j=1}^{J} \beta^{j-1} u(c_j, l_j, d_j)\right] \quad \text{with} \quad u(c,l,d) = \frac{c^{1-\frac{1}{\gamma}}}{1-\frac{1}{\gamma}} - \chi I_{l>0} - \xi d.$$

Expectations are formed with respect to survival, productivity, health and employment risk and future utility is discounted with the constant time discount factor β. The intertemporal elasticity of substitution in consumption is denoted by γ. Agent face participation cost χ when employed that depend on age and education. The indicater variable $I_{l>0}$ is zero when retired and one when working. Finally, following Lau et al. (2019) we assume that applying for DP induces stigma cost ξ that are iid across households and again depend on age and education. In order to ensure that the latter are always positive, we assume a log-normal density function

$$\xi \sim LN(\mu, \sigma_{\xi}^2)$$

where $\Psi_{\xi}(\cdot)$ denotes the cumulative distribution function (at a specific age).

Labor productivity The modeling of productivity risk is taken from Kindermann and Püschel (2021) who have studied administrative data from the German pension insurance system to investigate the properties of individual labor earnings dynamics over the life cycle. They found that workers are exposed to significant earnings risk which could not be captured by the standard AR(1) process for log-earnings. On the one side, individual contribution records show periods where employees do not pay contributions because they are out of official employment. The latter includes households who work in the informal sector, stay at home because of illness or rely on social assistance benefits. On the other side, some employees typically spend a serious fraction of working years in low income episodes where they only make about 10 percent of average annual labor earnings. Due to these "mini-jobs", the distribution of labor earnings becomes bimodal with quite distinct dynamics across educational groups.

Kindermann and Püschel (2021) therefore distinguish between households with high school education ($s = 0$) and college education ($s = 1$), where the initial fraction of college educated households is denoted by ω_s. All workers of education level s share a common deterministic age-specific labor productivity profile $e_{j,s}$. Knowing their educational level, workers are again divided in two permanent subgroups m, which indicate whether they face a stable ($m = 0$) or an unstable career path ($m = 1$). The probability to draw the state $m = 1$ is independent of education and denoted by ω_m. Throughout their working life, individuals' labor productivity is due to idiosyncratic shocks η. Productivity of individuals with a stable career ($m = 0$) follows a standard AR(1) process

$$\eta^+ = \rho\eta + \epsilon^+ \qquad \text{with} \quad \epsilon^+ \sim N(0, \sigma_\epsilon^2), \tag{1}$$

where innovations ϵ^+ are iid across households with education level s. In order to capture low income episodes for individuals with unstable careers ($m = 1$), this standard shock process (1) is augmented by a persistent (but not permanent) low productivity shock η_0. Productivity in this low productivity state is independent of age and education. The transition into and out of low earnings is modeled as a first-order discrete Markov process, where households with unstable careers ($m = 1$) face the education-specific probability $\pi^s_{low,0}$ of a normal earner to transition into the low earnings state in the next period, while $\pi^s_{low,1}$ is the probability to remain in the low earnings state. In the initial year a fraction ω^s_{low} of individuals with education s in the unstable career subgroup starts as a low earnings individual.

Health and employment risk In our model households are also exposed to health risks which affect their work capacity. We distinguish a good health state ($h = 0$), where households can work without impairment, a medium state ($h = 1$), where they face some minor injuries and suffer from mental problems which reduce work capacity slightly and a bad health state ($h = 2$), where work capacity is reduced severely.[10] These shocks again follow a first-order discrete Markov process where households face probabilities of the future health state $\pi_h(h^+ \mid j, s, h)$ that depend on age, education and the current health state:

$$h^+ = f(j, s, h). \tag{2}$$

In order to model the impact of these health shocks on productivity, we further augment the productivity process of Kindermann and Püschel (2021) by assuming that bad health conditions reduce the productivity of workers by θ_h percent and force a fraction π_h^u to stay at home (i.e. where we set $l = 0$). Details of the exact parametrization of the health and employment dynamics are presented in the calibration section.

Consequently, labor productivity $z(j, s, m, \eta, h)$ depends on age, education, career stability, the idiosyncratic productivity shock and the health state. Individual labor income y is then given by

$$y = w \times z(j, s, m, \eta, h) \times l,$$

where the wage per efficiency unit w is multiplied by labor productivity and the working hours.

Individuals enter the labor market without assets ($a_1 = 0$), during their working years they accumulate assets to finance retirement and to self insure against uncertainty. Since our model abstracts from annuity markets, individuals that die before the maximum age of J may leave accidental bequest b that will be distributed equally in a lump-sum fashion among all individuals beyond age 60.

3.3 Accumulation of pension wealth and benefit calculation

As already explained above, the statutory German pension system is based on so-called "earning points", which reflect the relative income position (and therefore contribution level) during employment years. Agents who receive an average income $\bar{y}$ in a specific year accumulate one earning point in their retirement account. In case of higher income, earning points are increased proportionally (up to the contribution ceiling which is roughly $2\bar{y}$) and vice versa in case of lower income. Consequently, earning points ep in the model are accumulated as

$$ep^+ = ep + \min[y/\bar{y}; 2.0] \tag{3}$$

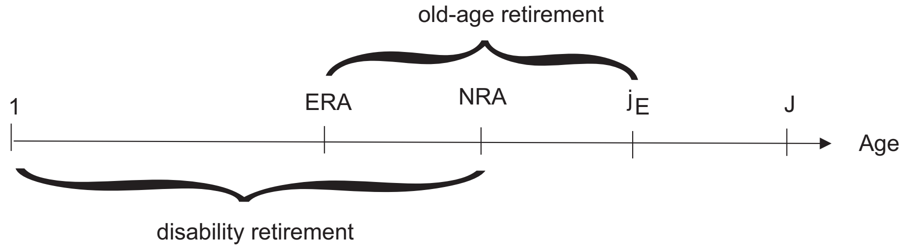
until agents decide to stop working at age j_R and receive retirement benefits. We abstract from temporary retirement and from separating the decision of labor supply and benefit receipt.[11] No earning points are therefore accumulated after retirement.

Figure 1 shows the retirement windows for OAP and DP in the model. Retirement with OAP is

[10] Note that we do not model any additional cost (hospital, etc.) associated with theses health shocks, since in Germany full coverage of health insurance is mandatory.

[11] In Germany an earning test reduces incentives to work during early retirement with OAP and there is hardly any work after the NRA. In the case of DP receipt, medical examinations are sometimes required even after retirement which might induce a forced return to the labor market, but such cases of temporary retirement are very rare.

Figure 1: Overlapping retirement windows for OAP and DP



possible either in and after the ERA with a contribution record of more than 35 (and later 45) years, or in and after the NRA for all other employees up to the maximum retirement age j_E in which all remaining employees in the model are forced to retire.[12] Not all employees qualify for early retirement, since those with health problems might end up without labor income in some periods. In order to check eligibility for early retirement, we keep track of the number of non-contributory years before retirement:

$$nc^+ = \begin{cases} nc & \text{if } y > 0 \\ nc + 1 & \text{otherwise.} \end{cases} \tag{4}$$

In contrast to OAP, retirement with DP depends on the individual health status. Agents enter the labor market in full health ($h = 0$), but after one year their health may deteriorate which reduces or even temporarily eliminates their work capacity. Employees may then either stay in the labor market and hope for future health improvements or they restrict labor market participation from $l = 0.5$ to $l = 0.2$ and submit an application for DP.[13] After one "waiting period", the DP board accepts an application with an (exogenous) probability $1 - q(h)$ and the agent starts receiving DP benefits independent of the actual health condition. If the application is rejected with probability $q(h)$, the agent either works again full time or files another application depending on the actual health condition or decide to retire with OAP. As shown in Figure 1, DP retirement is possible before reaching the NRA, so that the two retirement windows may overlap in some periods for specific individuals.

If agents are eligible for OAP benefits they may retire without any delay. At retirement, the accumulated earning points are multiplied with an adjustment factor $\nu(j_R)$ which takes into account early or late retirement, and the annual point value (APV), which reflects the worth of one earning point in pension income. For simplicity, we derive the APV from the replacement rate of a so-called "standard pensioner", who has worked for 45 years and always received an average income:

$$p = \nu(j_R) \times ep \times \underbrace{\kappa \times \bar{y}/45}_{\text{APV}}. \tag{5}$$

Since all agents in the model enter the labor market at age 20 and the normal retirement age for OAP in the initial equilibrium is 65 years, the standard pensioner has accumulated $ep = 45$ earning points at retirement. The adjustment factor at the NRA is unity, so that OAP benefits of the standard pensioner are simply computed by $p = \kappa \times \bar{y}$ where κ denotes the replacement rate shown in the last

[12] The maximum retirement age j_E, as well as the number of non-contributory years (instead of contributory years) are specified for purely technical reasons to reduce the state space.

[13] As already explained above, in Germany full DP benefits are only provided to applicants who can only work less than forty percent of full working time.

line of Table 1. For each year of early retirement before the NRA, OAP benefits are reduced by 3.6 percent and for each year of delayed retirement they are increased by 6 percentage points, i.e.

$$\nu(j_R) \quad = \quad \begin{cases} 1 - (\text{NRA} - j_R) \times 0.036, & j_R < \text{NRA} \\ 1 + (j_R - \text{NRA}) \times 0.060, & j_R \geq \text{NRA}. \end{cases}$$

In principle, the calculation of DP benefits is based on the same formula (5). However, earning points and the adjustment factor are computed slightly different. Since DP retirement is typically much earlier than OAP retirement (see Table 1 above), accumulated earnings points are upgraded to the maximum assessment age (MAA):

$$ep = ep \times \max\left[1.0; \frac{\text{MAA}}{j_R}\right].$$

In the initial equilibrium the maximum assessment age is set at 60. Someone who receives DP benefits at age 50 would therefore receive a 20 percent increase in accumulated earning points. In addition, the DP adjustment factor $\nu(j_R)$ is defined relative to the disability retirement age (DRA), reductions of DP benefits are limited to 10.8 percent and no increase in DP benefits is granted for delayed retirement, i.e.

$$\nu(j_R) \quad = \quad \begin{cases} 1.0 - \min[0.108; (\text{DRA} - j_R) \times 0.036], & j_R < \text{DRA} \\ 1.0 & j_R \geq \text{DRA}. \end{cases}$$

3.4 The dynamic optimization problem

In order to describe the individual decision process we need to distinguish three retirement states rs for each household, which specify the labor market situation in the previous year. If the agent has worked before ($rs = 0$) he/she needs to decide about retirement (with DP or OAP), consumption and savings. If the agent has either filled a successful DP application or has already received a DP benefit before ($rs = 1$), he/she only need to decide about consumption and savings. Of course, the same applies when the agent was already retired with OAP benefits before ($rs = 2$), but the benefit calculation is different. Therefore, the current state of a household is described by a vector

$$x = (j, s, m, rs, a, ep, nc, \eta, h)$$

where $a \in [0, \infty]$ define the financial assets at the beginning of the period and the remaining variables summarize the household's current age, education, career stability, retirement state, earning points, non-contributory periods, labor productivity and health. In each period the age-j cohort is fragmented in subgroups, according to the initial distribution at age $j = 1$ as well as idiosyncratic shocks to productivity and health and optimal household decisions. Let $\Phi(x)$ be the corresponding cumulated measure so that[14]

$$\int d\Phi(x) = 1 \qquad \text{with} \quad x = (1, s, m, 0, 0, 0, 0, \eta_1, 0)$$

must hold since we have normalized the initial cohort size to be unity and endowed initial agents with productivity η_1. In the following we will omit the state index x for every variable whenever possible.

[14] Actually, set the initial number of non-contributory years for high-skilled to 3 in order to account for their longer education time.

Optimal household decisions regarding consumption, savings and retirement with DP or OAP will be formulated recursively starting in the last phase of life (see Figure 1). Since we abstract from bequest motives, households who have survived until the final age J simply consume their resources:

$$V(x) = u(c, 0, 0) \quad \text{with} \quad c = [(1 + r)a + p - T(p, ra)]/(1 + \tau^c),$$

where p either denotes DP ($rs = 1$) or OAP ($rs = 2$) benefits, $T(p, ra)$ defines the (progressive) income tax levied on pension income and asset income and τ^c is the consumption tax rate (see below).

Younger households who are beyond the maximum retirement age j_E must be also retired (i.e. $rs \in \{1, 2\}$), but they may still face health shocks which affect their life expectancy. Optimal consumption $c(x)$ and savings $a^+(x)$ are then derived according to

$$V(x) = V_2(x) = \max_{c,a^+} \quad u(c, 0, 0) + \beta\psi_{j+1}(h)\mathbb{E}\left[V(x^+)\middle|h\right] \tag{6}$$

subject to (2) and the budget constraint

$$a^+ = (1 + r)a + p - T(p, ra) - (1 + \tau^c)c.$$

Of course, the same also applies to all retired households who are below the maximum retirement age j_E and in one of the absorbing states $rs \in \{1, 2\}$.

Those who are in or beyond the NRA and were still working in the previous year (i.e. $rs = 0$) have to decide whether to continue working or to exit the labor market and receive OAP benefits. In case of working, their optimal consumption and saving is derived from[15]

$$V_0(x) = \max_{c,a^+} \quad u(c, 0.5, 0) + \beta\psi_{j+1}(h)\mathbb{E}\left[V(x^+)\middle|\eta, h\right] \tag{7}$$

subject to (1), (2), (3), (4) and the budget constraint

$$a^+ = (1 + r)a + y - T_p(y) - T(y - T_p(y), ra) - (1 + \tau^c)c, \tag{8}$$

where $T_p(y)$ defines the contribution to the pension system.[16] In case of retirement, the optimization problem (6) applies. The participation decision $l(x)$ is then derived from maximizing

$$V(x) = \max_l [V_0(x), V_2(x)]. \tag{9}$$

If the household decides to retire, the future retirement status changes to the absorbing state $rs = 2$. Otherwise he/she remains in $rs = 0$. The same decision applies to working agents in good health who are in the early retirement window for OAP.

Agents younger than the NRA, who are in the medium or bad health state (i.e. $h \in \{1, 2\}$) and who have worked in the previous year may also consider to apply for DP. In this case they need to take into account that on the one side they can work only part-time and face stigma cost during the application period and on the other side their application may be rejected with probability $q(h)$. Optimal consumption and savings in case of a DP application are derived from

$$\tilde{V}_1(x) = \max_{c,a^+} \frac{c^{1-\frac{1}{\gamma}}}{1 - \frac{1}{\gamma}} - \chi + \beta\psi_{j+1}(h)\left\{q(h)\mathbb{E}\left[V(x^+)\middle|\eta, h\right] + (1 - q(h))\mathbb{E}\left[V_2(x^+)\middle|h\right]\right\} \tag{10}$$

[15] Note that the expectation operator here also includes the risk of unemployment in medium or bad health states.

[16] For simplicity we assume that taxation of pensions is completely back-loaded, although this will be fully implemented in Germany only after 2040.

subject to (1), (2), (3), (4) and the budget constraint (8), which now may also include unintended bequest b which are received at ages younger than 60. Note that $\tilde{V}_1(x)$ denotes the value function in case of an application net of stigma cost. If the application is accepted with probability $1 - q(h)$, agents receive DP benefits in the next period, they are assigned to the retirement state $rs = 1$ and the productivity uncertainty disappears.[17] In case of denial, expected future utility depends on the future productivity and health condition. In case of good health, they may still decide whether to participate in the labor market or not (if they are eligible for early OAP retirement), or they have to return to the labor market. If they end up again in the medium or bad health state, they may apply again for DP benefits.

The application decision $d(x)$ depends on individual stigma cost $\xi(x)$. Since the distribution of these cost is given by the age- and education-dependent cumulative function Ψ_ξ, the probability that an individual in state x decides to apply is

$$P(d = 1|x) = \Psi_\xi(\tilde{V}_1(x) - \hat{V}(x)).$$

Ignoring stigma cost, the difference $\tilde{V}_1(x) - \hat{V}(x)$ reflects the expected utility gain from a DP application. The term $\Psi_\xi(\cdot)$ on the right side then denotes the fraction of households with state x with stigma cost below this utility gain, who will apply for DP. The utility in case of not applying for DP

$$\hat{V}(x) = \begin{cases} \max[V_0(x); V_2(x)] & \text{ERA} \le j < \text{NRA} \\ V_0(x) & j < \text{ERA}. \end{cases}$$

reflects the options for those in and above the early retirment age who may either continue to work or receive OAP and the younger ones who can only continue to work. The final value function at this age is therefore a weighted average of the value functions with and without disability application:

$$V(x) = P(d = 1|x)V_1(x) + (1 - P(d = 1|x))\hat{V}(x),$$

where now $V_1(x)$ also includes the expected stigma cost of applicants.

Finally, the decision problem of healthy young employees at age $j < ERA$ is given by (7) except that the budget constraint also includes unintended bequest b. Table 3 summarizes the final decision problem of an agent in retirement state $rs = 0$ depending on age and health status:

Table 3: Decision problem for working individuals

	$h = 0$	$h = 1$	$h = 2$
$1 \le j <$ ERA	V_0	$\max(V_0, V_1)$	
ERA $\le j <$ NRA	$\max(V_0, V_2)$	$\max(V_0, V_1, V_2)$	
NRA $\le j < j_E$	$\max(V_0, V_2)$		

When entering the labor market everybody is healthy and working, so that there is only the decision problem how much to consume and save. Those who are younger than ERA and not healthy have to decide whether to file a DP application or work. Of course, healthy agents will always work. The period between the early and the normal retirement age is most interesting. Healthy agents (i.e. where $h = 0$) with a long contribution record have to chose whether to work or to retire early with

[17] Note that the value function $V_2(x)$ applies to both, old-age pensioners and disability pensioners.

OAP. Those with work limitations (i.e. $h \in \{1,2\}$) may in principle file a DP application, work or retire (if qualified) with OAP. These are exactly the groups where the government may induce even more instead of less retirement. Finally, at ages between the normal retirement age and the end of the retirement window j_E, all households still working have to decide whether to continue working or receive an OAP.

3.5 The government sector

The government in our model splits into a general budget and a pension system. Both budgets are closed separately. We abstract from public debt and corporate taxes so that general government expenditure G, which is fixed in absolute terms, is financed by income and consumption taxes, i.e.

$$T_y + \tau^c C = G, \tag{11}$$

where C defines aggregate consumption and T_y the revenues of income taxation. The latter are computed from

$$T_y = \int T(y, p, ra) \mathrm{d}\Phi(x) \qquad \text{with} \quad T(y, p, ra) = 2T16(\tilde{y}/2) + \tau_r ra.$$

Due to deferred taxation of pensions taxable labor income $\tilde{y}$ is computed from gross labor income net of pension contributions $T_p(y) = \tau^p \min[y, 2\bar{y}]$ and – after retirement – public pensions:

$$\tilde{y} = y - T_p(y) + p.$$

Given taxable income, we apply the German progressive tax code of 2016 $T16(\cdot)$ to labor income and assume that all households are married couples (i.e. full income splitting). With respect to taxable interest income we apply a constant rate τ^r which reflects the flat capital income tax in Germany.

The pension system pays in every period disability and old-age benefits $p(x)$ to retired households and collects payroll contributions from labor income below the contribution ceiling. The budget of the pension system must be balanced in every period by adjusting the contribution rate τ^p:

$$\int p(x) \mathrm{d}\Phi(x) = \tau^p \int \min[y(x); 2\bar{y}] \mathrm{d}\Phi(x). \tag{12}$$

3.6 The production sector

The production sector is populated by large firms which hire capital K and effective labor L on perfectly competitive factor markets to transform it into a single good according to the Cobb-Douglas production technology

$$Y = AK^{\alpha} L^{1-\alpha},$$

with α the capital share in production and A as technology parameter. Capital is rented from households through an intermediary at the riskless rate r and depreciates over time with rate δ. Labor inputs are paid the competitive wage w. Factor prices are then determined by marginal productivity conditions, i.e.

$$w = (1-\alpha) A \left(\frac{K}{L}\right)^{\alpha} \tag{13}$$

$$r = \alpha A \left(\frac{K}{L}\right)^{\alpha-1} - \delta. \tag{14}$$

3.7 Equilibrium conditions

Given a specific fiscal policy, an equilibrium path of the economy has to solve the households decision problems (6)-(10), reflect competitive factor prices (13) and (14) and balance aggregate inheritances with unintended bequests

$$\int b(x)\mathrm{d}\Phi(x) = \int \frac{1-\psi_{j+1}(h)}{1+n}(1+r)a^{+}(x)\mathrm{d}\Phi(x). \tag{15}$$

Furthermore, in the closed economy aggregation holds,

$$L = \int l(x)z(x)\,\mathrm{d}\Phi(x), \tag{16}$$

$$C = \int c(x)\mathrm{d}\Phi(x), \tag{17}$$

$$K = \int a^{+}(x)\mathrm{d}\Phi(x), \tag{18}$$

the budgets of the general government (11) and the pension system (12) are balanced and the goods market clears in every period, i.e.

$$Y = C + G + (n+\delta)K.$$

The computational method to solve the model numerically follows the Gauss-Seidel procedure of Auerbach and Kotlikoff (1987). We start with a guess for aggregate variables, bequest distribution and policy parameters. Then we compute factor prices, individual decision rules and value functions which involves discretization of the state space and interpolation, see Fehr and Kindermann (2018). Next we obtain the distribution of households and aggregate assets, labor supply and consumption as well as payroll and consumption taxes in order to update the initial guesses. The procedure is repeated until the initial guesses and the resulting values of macro variables and policy parameters have sufficiently converged.

4 Calibration of the initial equilibrium

In what follows, we describe the parameterisation of the model. This is a two-stage process. We first specify parameter values which are estimated outside the model. Some of these parameters are taken directly from the literature. Then we calibrate further parameters by matching moments of our model to the data.

4.1 Demographic structure

In our model, one period covers one year. Agents therefore start their economic life at age 20 ($j = 1$) and may receive health shocks after the initial work year. They may work until age 70 ($j_E = 51$) and face a maximum possible life span of 99 years ($J = 80$). The specification of the retirement window reduces the state space significantly, but this restriction is not very binding. Appendix 1A shows that in 2019 only 0.5 percent of retirees retired after age 69. The growth rate of the population is set at $n = 0.0065$ which reflects the average population growth during 2012 until 2017 (mainly due to refugee migration) but also generates a fairly realistic old-age dependency ratio. The ratio of cohorts age 65+ relative to cohorts at ages 20-64 is 31.6 percent in the model while it was at roughly 34 percent around 2012, see DRV (2021, 288). Finally, the share of college educated workers $\omega_s = 0.2373$ as well as the share of workers with a stable career $\omega_m = 0.5$ is taken from Kindermann and Püschel (2021, 24) who base their estimates on administrative data from the German pension insurance (Versichertenkontenstichprobe 2017).

4.2 Health transitions

Since labor productivity is affected by the health state h, we need to describe the health process over the life cycle first. The model distinguishes three life cycle phases for health transitions. Appendix 1A shows that disability application only increase slightly between ages 20 and 44. Here we assume that agents may receive only a bad health shock with probability $\pi^h_{j,s}$ from which they will not recover at all. Consequently

$$\pi_h(h^+ \mid j, s, h) = \begin{cases} \pi^h_{j,s} & h = 0, h^+ = 2 \\ 1 - \pi^h_{j,s} & h = 0, h^+ = 0 \\ 1.0 & h = 2, h^+ = 2 \\ 0.0 & \text{otherwise,} \end{cases}$$

where the probabilities $\pi^h_{j,s}$ are derived as in Fehr et al. (2013) from an exponential function

$$\pi^h_{j,s} = \varrho_s \times \exp(v_s \times j)$$

with $\varrho_s = (0.003, 0.0015)$ and $v_s = (0.027, 0.033)$. This procedure generates bad health probabilities between 0.2 percent and 0.8 percent which in turn lead to realistic disability applications and disability rates close to those reported in Hagen et al. (2010) for these two groups.

Appendix 1A documents that in 2019 more than 80 percent of disability applications were submitted by individuals older than age 44. Following Jürges et al. (2015), we applied a principal component analysis to estimate two health transition matrices for each age group from the German sub-sample of the Survey of Health, Ageing and Retirement in Europe (SHARE). The data selection and more details on our estimation approach as well as the resulting health transition probabilities for the two age and education groups are explained in Appendix 1B. We also compare there the distribution of health states in three life cycle phases for the two educational groups and then the aggregate model data with German health data for the years 2008-2019 from Eurostat (2021). College graduates are significantly healthier than high school graduates in all considered age cohorts which explains the differences in life expectancy.

4.3 Survival probabilities

Given the health transition matrices, we can determine the age- and health-dependent survival probabilities $\psi_j(h)$. The calibration starts with the average male survival probabilities $\bar{\psi}_j$ taken from the Human Mortality Database (2020). We follow the approach of Kindermann and Püchel (2021) and compute the health-dependent probabilities according to

$$\psi_j(h) = \frac{1}{1 + \exp(-\iota_h \times \bar{x}_j)} \quad \text{with} \quad \bar{x}_j = -\log\left(\frac{1}{\bar{\psi}_j} - 1\right).$$

The three parameters $\iota_h = [1.35, 1.345, 0.518]$ are calibrated to match exactly the average life expectancy of newborn and 65-year old men in Germany as well as the difference in life expectancy at age 65 for the two education groups. Table 4 reports these targeted data moments and the resulting education-specific life expectancies.

Table 4: Education-specific and targeted life expectancies

	High school $s = 0$	College $s = 1$	Targeted average	Data source
Life expectancy at birth	78.4	82.9	79.5	HMD (2020)
Life expectancy at age 65	82.5	85.0	83.0	HMD (2020)
Difference at age 65			2.5	Luy et al. (2015)

The HMD (2020) reports an average life expectancy of newborn men in Germany of 79.5 years and of 83 years for those who have reached age 65. In addition, Luy et al. (2015) report that 65-year old college graduates in Germany face a 2.5 year higher life expectancy than high school graduates. We determine the three parameters ι_h by solving a non-linear equation system in life expectancies. As Table 4 shows, the resulting education-specific difference in life expectancy even amounts to 4.5 years at birth, but then decreases to 2.5 years for those who have reached retirement age.

4.4 Productivity process

Given health transitions and survival probabilities, the productivity process over the life cycle can be calibrated with our simulation model in order to match empirical targets taken from Kindermann and Püschel (2021).[18] As explained before, individual labor income is defined by $y = w \times z(j, s, m, \eta, h) \times l$. We normalize the wage rate to unity in the initial equilibrium and specify the productivity term

$$z(j, s, m, \eta, h) = \begin{cases} \exp(e_{j,s} + \eta) \times \theta_{h,s} & \text{employed normal earners } (1 - \pi_h^u) \\ \exp(\eta_0) \times \theta_{h,s} & \text{employed low earners } (1 - \pi_h^u) \\ 0 & \text{unemployed } (\pi_h^u), \end{cases}$$

which is similar to the one in Kindermann and Püschel (2021), but allows for health-related productivity shocks $\theta_{h,s}$ and temporary unemployment π_h^u. Consequently, we need to calibrate the following fifteen parameters:

1. The education-specific innovation variances $\sigma^2_{\epsilon,s}$ of the normal labor productivity processes.
2. The five coefficients of a third-order polynomial for the deterministic productivity profile over the life cycle for each education group.
3. The three unemployment probabilities π_h^u for each health state.

The calibration target for the innovation variances of normal labor productivity risks is the unconditional variance of labor incomes computed by Kindermann and Püschel (2021) which is reported in

[18] Appendix 1C describes in more detail their approach using administrative data from the German pension insurance to study the dynamics of labor earnings over the life cycle.

Appendix 1C. We combine their processes for normal and low earnings with health-related productivity drops from Capatina (2015) to simulate the earnings process with an endogenous innovations variance until we generate the targeted data moments. Table 5 reports the exogenous parameter values and the resulting calibrated parameter values. As shown, a fairly high fraction of college

Table 5: Exogenous and calibrated parameters of productivity process

	High school ($s=0$)	College ($s=1$)	Source or targeted data moments
Exogenous parameter values			
Autocorrelation normal earnings $\hat{\rho}_s$	0.9869	0.9900	Kindermann and Püschel (2021)
Health-related productivity drop $\theta_{0,s}$	1.000	1.000	Capatina (2015)
$\theta_{1,s}$	0.810	0.860	
$\theta_{2,s}$	0.640	0.720	
Low earnings: - productivity level $\exp(\eta_0)$	0.1000		Kindermann and Püschel (2021)
- initial share ω^s_{low}	0.2040	0.8136	
- inflow probability $\pi^s_{low,0}$	0.0063	0.0051	
- probability to stay $\pi^s_{low,1}$	0.8399	0.7324	
Calibrated parameters			
Innovation variance $\sigma^2_{\epsilon,s}$	0.00445	0.00397	unconditional variance (KP,2021)

graduates starts in the low earner group, but afterwards the chance to transition into a low earnings episode is very small (less than 1 percent for both education groups). Being in the low income state, however, has quite some persistence. The average duration of a low earnings episode is 6.24 and 3.7 years for high school and for the college educated, respectively.

Based on the correlation parameters $\hat{\rho}_s$ and the calibrated innovation variances $\sigma^2_{\epsilon,s}$, the AR(1) processes for the two education levels are discretized using a Rouwenhorst method as described in Kopecky and Suen (2010) with 3 approximation points.[19].

Next, the age-productivity profiles are estimated as in Kindermann and Püschel (2021) by assuming

$$e_{j,s} = b_{0,s} + b_{1,s}\frac{\min(j, j_{M,s})}{10} + b_{2,s}\left[\frac{\min(j, j_{M,s})}{10}\right]^2 + +b_{3,s}\left[\frac{\min(j, j_{M,s})}{10}\right]^3. \quad (19)$$

This functional form captures both a hump-shaped ($j_{M,s} = \infty$) and a stagnating ($j_{M,s} < 60$) life-cycle labor productivity profile where productivity is constant from age $j_{M,s}$ onward. The parameters $b_{i,s}$ and $j_{M,s}$ of the polynomials are selected so that the model matches the age-fixed effects of the income process estimated by Kindermann and Püschel (2021). Table 6 summarizes the resulting parameter values and Appendix 1C shows the match of the age-productivity profiles.

Finally, the unemployment probabilities $\pi^u_h = [0.04, 0.59, 0.80]$ are specified in order to match realistic fractions of long-term and very long-term employed in the data, see below.

4.5 Preference and technology parameter

In order to calibrate the parameters of the utility function we first set the intertemporal elasticity of substitution (IES) γ at 0.5, which is in the range of commonly used parameters in these types of

[19] More nodes would only change the results slightly but increase computational time dramatically.

Table 6: Calibrated parameters of age-productivity profile

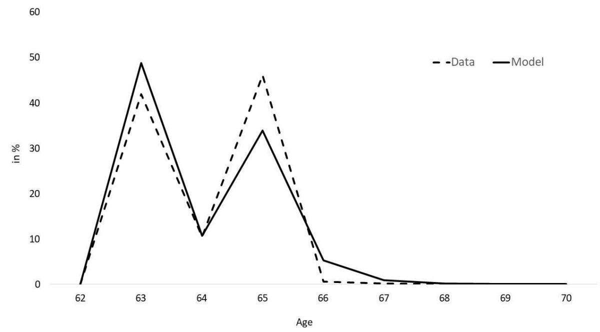
	Variable	High school $s = 0$	College $s = 1$
Intercept	$b_{0,s}$	-1.3006	-5.7498
Linear age term	$b_{1,s}$	0.8411	3.9905
Quadratic age term	$b_{2,s}$	-0.0767	-0.7503
Cubic age term	$b_{3,s}$	0.0000	0.0481
Stagnation threshold	$j_{M,s}$	∞	52

models, see Conesa et al. (2009, p. 33). The time preference rate β is set to 0.98 in order to calibrate a realistic capital-output share of 330 percent. Labor disutility

$$\chi_{j,s} = \zeta_{1,s} j + \zeta_{2,s} \max[0, j - (NRA - 1)]$$

increases with age and the respective growth parameters $\zeta_{1,s}$ and $\zeta_{2,s}$ are selected to match the average retirement ages and the retirement pattern of the two education types for OAPs. The parameters $\zeta_{2,s}$ can be interpreted as reference points as in Seibold (2021) and help to generate the two peaks in old-age retirement inflows. Figure 2 compares the data (when we abstract from retirement before

Figure 2: Old-age retirement inflow pattern in model and data



Source: Computed from https://statistik-rente.de for the year 2014.

ERA) and the model results. Similarly, the education-specific stigma cost $\xi_{j,s} \sim LN(\mu_{j,s}, \sigma_{\xi}^2)$ are calibrated to match the respective retirement ages for DPs. More specifically, we assume a polynomial form

$$\mu_{j,s} = \varphi_{0,s}(NRA - j) + \varphi_{1,s}(NRA - j)^2 \quad \text{with} \quad \varphi_{0,s}, \varphi_{1,s} \geq 0,$$

so that stigma cost decrease with age. Finally, the variance σ_{ξ}^2 of the stigma cost is independent of age and education and specified to match the shares of OAP and DP retirees in the data.

The capital share in production is set at $\alpha = 0.30$ to match the capital income share. In addition, we choose a value for the technology parameter A in order to normalize the wage rate for effective labor to unity. The depreciation rate δ on capital is set at 5 percent which guarantees together with the population growth rate a realistic investment to GDP ratio of 21.8 percent for Germany.

4.6 Government policy parameters

Government tax policy in our model reflects the main features of the German tax system. We neglect public debt and corporate taxes and fix the public goods expenditures to 20 percent of output. The German income tax code of 2016 is applied to labor and pension income, i.e. the marginal tax rate schedule rises after a basic allowance from 15.8 to 44.3 percent. We assume that the transition towards deferred pension taxation is already finished. Consequently, pension contributions are fully deductible from tax base while pension benefits are fully taxable.[20] With respect to the tax base we apply the German income splitting method in order to compute individual labor tax revenue. Returns from savings are taxed linearly at the rate 15 percent. This reflects the quasi dual income taxation in Germany. The resulting income tax revenue in the initial equilibrium is 10.5 percent of GDP. The budget is balanced by the consumption tax, where the equilibrium tax rate of 16 percent generates a revenue of 9.5 percent of GDP.

With respect to the pension system we consider an initial equilibrium which reflects the situation in Germany before the implementation of the retirement reform 2007, see Table 2. Consequently, NRA and ERA for OAP benefits are set to 46 and 44 (i.e. ages 65 and 63), respectively. With respect to DP, DRA for receiving benefits without deductions is set at 44 (i.e. age 63) and the MAA is set at 41 (i.e. age 60).

We assume that all DP applicants in good health are all rejected (i.e. $q(0) = 1.0$) if they file a DP application. For individuals with a negative health shock, we set the probability of rejection to 49%, regardless if the health shock is severe or less severe (see Deutsche Rentenversicherung, 2021). Finally, replacement rates are identical for both pension types and are set at $\kappa = 0.56$ which yields a realistic contribution rate. Table 7 summarizes the parameter values and the respective calibration targets or data sources.

4.7 Initial equilibrium

Given the dynamics of health and productivity over the life cycle, our chosen preference and policy parameters generate an initial equilibrium which reflects some key indicators of the German pension system in the years when the NRA increase was phased-in. Of course, the decision to file a DP application depends on many different parameters, such as the current health condition , alternative employment and retirement options and also stigma cost. Nevertheless, our model replicates DP applications and DP retirement ages quite well. Figure 3 compares the distribution of DP inflows over the life cycle in the model and in year 2014. As one can see, the shape is very similar but in the data the peak of inflows is at age 60 while it is (roughly) age 57 in the initial equilibrium of the model. As shown in Table 1, the effective retirement age has increased significantly since 2011. Consequently, the shift in the profile can be easily explained and even improves the calibration. Besides that the two curves are very similar especially with the steep increase after age 45.

Table 8 compares some model results with respective micro data in 2016 provided by the research center of the German pension insurance, see FDZ-RV(2019).[21] More than 75 percent of cohorts aged 60-65 were long-term insured (i.e. have more than 35 insurance years) while only 1.4 percent even

[20] In reality the transition period until all pension benefits are fully taxable lasts until 2040.

[21] This was the earliest micro data set available, but there is no reason to expect too dramatic differences in earlier years.

Table 7: Parameter values and calibration targets

Parameter	Values $s=0$	$s=1$	Source/Target
Demographics and labor productivity			
Population growth (n)	0.0065		Dependency ratio (65+/20-64) 31.6 percent
Survival probabilities (ι_h)	1.775, 1.405, 0.554		Luy et al. (2015), HMD(2020)
Education shares (ω_s)	0.7627	0.2373	Kindermann and Püschel (2021)
Stable career share (ω_m)	0.50		
Unemployment probabilities (π_h^u)	0.04, 0.59, 0.8		Fractions of long-term insured
Preference and technology parameters			
Inter-temporal elast. of subst. (γ)	0.50		Conesa et al. (2009)
Time discount factor (β)	0.98		Capital-output ratio 330 percent
Work disutility ($\zeta_{1,s}$)	0.041	0.018	average OAP retirement ages
($\zeta_{2,s}$)	2.800	2.800	
Stigma costs ($\varphi_{0,s}$)	0.75	0.77	average DP retirement ages
($\varphi_{1,s}$)	0.02	0.00	
(σ_ξ^2)	3.0		DP pensioners/ OAP pensioners
Capital share in production (α)	0.30		Capital income share
Technology parameter (A)			Wage rate normalized to unity
Depreciation of capital (δ)	0.05		Investment to GDP ratio 21.8 percent
Government policy			
Public good to GDP ratio (G/Y)	0.20		Ratio tax revenue to GDP
Income tax code ($T16(\cdot)$)			German tax law of 2016
Pension system (κ)	0.56		Contribution rate 18.9 percent
($q(h)$)	1.00, 0.49, 0.49		DRV(2021)

qualify for very long-term insured (i.e. have at least 45 insurance years). Table 8 also shows that our

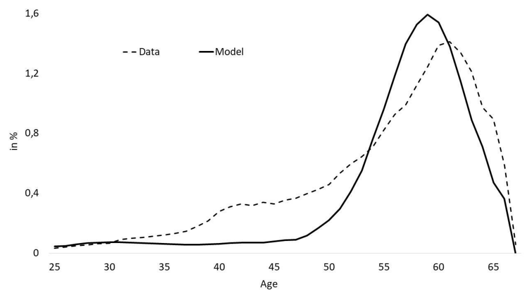
Table 8: Long-term insured (in %) and average retirement ages in model and data

		High school ($s=0$)	College ($s=1$)	Average
Long-term insured (>35)	Model	78.7	68.2	76.0
	Data*	77.8	71.6	76.4
(>45)	Model	1.8	0.0	1.4
	Data*	3.0	1.5	2.7
Retirement ages OAP	Model	63.9	64.3	64.0
	Data*	63.8	64.3	63.9
Retirement ages DP	Model	52.7	54.6	52.9
	Data*	52.6	54.4	52.8
Retirement ages average	Model	61.4	63.2	61.8
	Data*	61.3	63.4	61.7

*Source: FDZ-RV(2019): SUFVVL2016

model captures important empirical facts with respect to retirement behavior. On the one side and

Figure 3: DP inflows in the model and the data



Source: Computed from https://statistik-rente.de for the year 2014.

quite surprisingly, the average retirement age for OAPs is (more or less) independent of skill level in our data. On the other side, the retirement age for DP benefits rises significantly with the skill class. As in the data, the average retirement age in the model rises significantly with the educational background, since high-skilled retire later with DP and high-skilled have a lower share of DPs.

The first column in Table 10 reports the resulting average retirement ages as well as some key aspects of the initial equilibrium. Total outlays of the pension system amount to 12.6 percent of GDP and about 7 percent of total pension expenditure is spent for DP benefits.[22] DP pensioners receive on average about 70 percent of average OAP benefits and the contribution rate is at 18.9 percent. These numbers describe quite well the situation of the German pension system in the period 2012-2013 when the 2007 reform was implemented. Note that only 1.9 and 7.8 percent of 50-65 year old individuals with a medium and bad health state respectively apply for DP benefits. These rates seem quite low but they reflect economic and psychological factors in the model such as expectations about future recovery, benefits, the existing wealth and stigma cost of applications.[23] In combination with the exogenously set rejection rates they determine the pension structure.

With respect to some key macroeconomic statistics, the capital-output ratio of 330 percent is close to the target value of 320 percent which we derived from official data. Of course, the consumption and investment fractions of GDP are somewhat higher than in the official data, reflecting the fact that Germany is a net exporter and we model a closed economy. The endogenously determined interest rate is at 3.1 percent and the unintended bequest which are distributed lump sum to working cohorts amount to 3.2 percent of GDP. Finally, the budget of the general government is financed by income and consumption taxes and closed by the consumption tax rate which is at 16.2 percent. This rate is not unrealistic, given the very stylized modelling of the income tax system.

Before we analyze the consequences of the reforms described above, it is useful to discuss the welfare effects of DP pensions in the present model. However, quantifying the welfare of the whole system is not so easy. Of course, we could simply set the rejection rates $q(h)$ for all DP applications to unity so that nobody applies for DP and compute the resulting long run welfare effects. This exercise would yield very similar long run welfare gains as the elimination (or privatization) of unfunded pensions. Due to lower contributions and higher (precautionary) savings, future cohorts would benefit from such a reform. In order to avoid such intergenerational redistribution effects, we follow another

[22] Note that the budget figure reported in Table 1 does not include civil servants pensions which are included here.

[23] Further disaggregation shows that the fraction of applicants rises with education background since college graduates have accumulated more wealth.

route and quantify the benefits and costs of DP in the initial equilibrium. The budgetary cost of DP (B^{DP}) are already reported (in % of the pension budget) in Table 10 below.[24] In order to compute the welfare benefits of the system, we first consider those individuals which would in principle apply for a DP, i.e. those younger than the NRA and where $\tilde{V}_1(x) > \hat{V}(x)$. Next, we compute the required compensation in assets $v(x)$, which makes them indifferent between applying and not applying, i.e.

$$\tilde{V}_1(x) = \hat{V}(\dots, a + v(x), \dots).$$

The welfare gain W is then computed as the aggragated compensation payments of those who finally apply and are accepted:

$$W = \int (1 - q(h)) P(d = 1|x) v(x) \mathrm{d}\Phi(x).$$

The difference betwen the welfare gain and the budgetary cost is then our net welfare measure which is expressed in percent of aggregate consumption $\Delta W = (W - B^{DP})/C$. The net welfare gain reflects the trade off between insurance benefits and the labor supply distortions of DP. Table 9 reports the net welfare gain for various initial equilibria with different rejection rates. By definition, the net welfare gain is zero without a DP system. But even when all applications are accepted we find a positive net welfare effect of DPs. As it seems, the current rejection rate of 49 percent is fairly optimal in terms of net welfare.

Table 9: Net welfare gains of DP*

q	0.00	0.30	0.49	0.60	0.80	1.00
ΔW (in %)	2.55	3.26	3.55	3.27	0.90	0.00

Of course, we could further analyze optimal rejetion rates by differentiating between health states and/or ages. However, the main motivation of the paper is the analysis of pension reforms to which we turn next.

5 Simulation of pension reforms

This section presents the long-run macroeconomic and welfare consequences of alternative pension arrangements for workers in Germany. The simulations always start from the initial equilibrium described in the previous section and the first column of 10, then various policy parameters are adjusted and a new long run equilibrium is computed. The reported welfare effects are computed as a Hick'sian equivalent variation (HEV), i.e. the required relative change in consumption in the initial equilibrium that would yield the after-reform welfare level.

5.1 Impact of pension reforms 2007 and 2018

In this section we implement successively the reform packages of 2007 and 2018. The second column of Table 10 reports the consequences of an increase in the normal retirement ages for OAP and DP

[24] However, there is a slight adjustemnt. In Table 10 we follow the official calculations and only aggregate DP benefits payed to pensioners younger than the NRA. Now we add to this number the benefits of those pensioners beyond the NRA which are due to upgrading.

benefits by two years. In our model the reform of 2007 increases the effective retirement for OAP benefits only by 0.7 years up to 64.7 years while the DP retirment age increases by 1.7 years up to age 54.6.[25] Later retirement of DP pensioners (relative to OAP pensioners who retire now with higher deductions) also increases the relative DP benefit level and the DP budget share slightly. Mainly due to later OAP retirement and higher deductions, the total pension budget decreases significantly to 11.5 percent of GDP and the contribution rate can be lowered by 1.7 percentage points. At the aggregate level, the delay of retirement (and the fall in contributions) increases labor supply significantly, but savings and the capital stock rise even more by 3.5 percent. This is somewhat surprising given the shorter retirement phase. However, facing higher stigma cost, individuals may save more to better self-insure against bad health conditions in the future. The higher capital stock increases the real wage and reduces the interest rate slightly. Higher wages and employment in turn increase income tax revenues and allow to reduce the consumption tax rate by 1.1 percentage points. Not surprisingly, the reform increases long run welfare of all households by about 3.6 percent of initial consumption.

Table 10: Economic effects of recent pension reforms*

	Initial	Reform	Reform 2018		
	equilibrium	2007	Fiscal effect	GE effect	higher rejections
Retirement ages (av.)	61.8	62.8	62.8	61.8	62.9
OAP	64.0	64.7	64.7	64.6	64.7
DP	52.9	54.6	54.6	53.6	55.2
Pension budget					
Total budget (in % of GDP)	12.6	11.5	11.8	12.5	11.7
DP (in % of pension budget)	7.1	7.4	8.5	11.6	7.9
DP/OAP benefit	69.6	71.1	82.6	82.6	82.2
DP applications (50-65)	1.9/7.8	1.7/8.3	1.7/8.3	2.8/9.5	2.5/10.4
Rejection rate (average)	49	49	49	49	60
Labor supply	–	1.5	1.6	0.5	1.5
Capital	–	3.5	3.0	0.1	2.9
Wage rate	–	0.6	0.4	-0.1	0.4
Interest rate (in pp)	3.1	-0.1	-0.1	-0.1	0.0
Contribution rate (in pp)	18.9	-1.7	-1.2	-0.2	-1.3
Consumption tax rate (in pp)	16.2	-1.1	-1.0	-0.2	-0.3
Welfare	–	3.6	2.1	0.3	1.2

*Changes are reported in % of initial equilibrium if not stated otherwise.

The remaining columns of Table 10 consider the 2018 reform which adds to the previous one the increase in the MAA from age 60 to 67. In order to better understand the economic effects, we first simulate the resulting increase in benefits while fixing the retirement behavior of DP pensioners at the 2007 reform level. In this so-called "fiscal effect" simulation, DP and OAP retirement ages are the same as in the previous simulation, but the DP benefit ratio as well as the DP share in the pension

25 If we would alter only the NRA for OAP, especially individuals in bad health will switch towards DP, so that DP applications in the age group 50-65 increase by 16 percent. This figure is strikingly close to the relative rise in the US DP enrollment rate projected by Duggan et al. (2007) for the same reform.

budget now increase significantly.[26] Due to expected higher DP benefits the capital accumulation is dampened and the contribution rate increases relative to the previous simulation. Factor prices now remain roughly (due to demand effects on factor markets) constant and long run welfare decreases compared to the 2007 reform.

When we allow individuals to adjust their retirement behavior in the column "GE effect", the OAP retirement age falls slightly and the DP retirment age falls by one year. Not surprisingly, the DP share of the pension budget rises strongly, which in turn raises the total budget and the contribution rate significantly. At the aggregate level, labor supply now decreases as well as capital accumulation, reducing the wage by 0.5 percent. The higher maximum assessment age therefore induces a massive increase in DP applications especially at the intermediate health state and (with constant health-specific rejection rates) an inflow of DP pensioners at all ages. The average retirement age falls significantly and the pension budget and the whole economy suffer strongly from the policy reform. As it turns out, higher contribution and tax rates as well as the lower long-run wage reduce now long-run welfare by 3.3 percent of initial consumption (relative to the Reform 2007). Of course, this reaction of DP applications in the model is very strong. On first impact, the reform increases DP benefits by roughly 15 percent (i.e. (8.5-7.4)/7.4 in the "fiscal effect" simulation). When we allow for adjustments in retirement behavior, DP applications increase for individuals with bad health by 23 percent (i.e. 2.3/10.0), so that the resulting elasticity of 1.5 is close to the figure computed by Mullen and Staubli (2016) for Austria.

Our computed effects are somewhat unrealistic, since despite the huge increase in applications we have kept the eligibility rules (and the resulting rejection rates) unaltered. If in the future such a massive DP inflow would arise, one could be almost sure that also rejection rates would increase significantly either due to applicants in better health or due to a stricter screening process. The last column therefore shows the consequences when the rejection rate increases from 49 to 60 percent for all applicants. As one can see, households now apply later for and retire later with DP benefits when their health has deteriorated. This has significant positive effects on the pension budget and the overall economy. Consequently, the contribution rate now declines again and long-run welfare increases compared to the previous simulation.

The central message from the results of Table 10 is therefore, that without further adjustments the Reform 2018 will most likely induce a dramatic shift towards disability retirement which reduces the average retirement age, raises the pension budget and hurts the whole economy significantly. In order to mitigate these effects, the tightening of eligibility rules which leads to higher rejection rates for DP applicatnts seems to be a realistic policy option. The next subsection explores alternative options for future reforms.

5.2 Options for future pension reforms

Since the demographic pressure on the pension budget will continuously rise, reforms that increase the retirement age will be required in the future.[27] Starting from the Reform 2018, this subsection an-

[26] Note that we report here the increase in average DP/OAP benefit ratio, while specific ages might experience much stronger benefit increases.

[27] There is alredy an ongoing discussion to increase the normal retirement age beyond age 67 after year 2030, see Deutsche Bundesbank (2022).

alyzes two modest and two more radical reforms of the OAP system intended to induce households to retire later in the future. The two modest reforms adjust the eligibility criteria for early OAP retirment and are shown in the first and second column of Table **??**. The two remaining policy reforms in Table **??** implement a phased in increase of the adjustment factors for early retirement from 3.6 to the US level of 6.3 percent.[28]

Quite surprisingly, the reform of 2007 has not adjusted the early retirement age (ERA) and the required number of contribution years for OAP benefits. In principle, households who have contributed 35 years could retire currently at age 63 when they accept a benefit reduction of 14.4 percent. Not surprisingly, early retirement is widespread in Germany and this explains the very modest increase in the effective OAP retirment age shown in the second column of Table 10. In the first column of Table **??** we increase the early retiremt age from 63 to 65, while in the second column we keep the EAR at 63, but increase the required number of contribution years for early retirment from 35 to 37.

Table 11: Economic effects of alternative OAP reforms*

	Eligibility rules		Benefit calculation	
ERA	65	63	63	
Contrib. years	≥ 35	≥ 37	≥ 35	
Adjust. factor	both 3.6 %		OAP 6.3 %	both 6.3%
Retirement ages	62.4	61.8	62.6	63.3
OAP	65.7	64.8	65.6	65.6
DP	54.0	54.1	54.4	55.0
Pension budget				
Total budget (in % of GDP)	12.5	12.6	11.8	11.2
DP (in % of pension budget)	12.6	13.1	12.2	9.1
DP/OAP benefit	79.2	82.6	82.6	75.6
DP applications (50-65)	3.3/10.2	3.4/10.4	3.0/10.0	2.3/9.1
Rejection rate (average)	49	49	49	49
Labor supply	1.8	0.5	1.7	2.3
Capital	-0.6	-0.5	1.5	3.7
Wage rate	-0.7	-0.3	-0.1	0.4
Interest rate (in pp)	0.2	0.1	0.0	-0.1
Contribution rate (in pp)	-0.1	0.1	-1.2	-2.2
Consumption tax rate (in pp)	-0.8	-0.1	-0.9	-1.5
Welfare	0.1	-0.4	2.0	4.1

*Changes are reported in % of initial equilibrium if not stated otherwise.

The increase in the ERA has a significant effect on retirement behavior. On the one side there is a shift towards DP applications and DP retirement. At the same time, households retire later with OAP benefits. As a consequence, both, the OAP and the DP retirement age increases significantly by 1.1 and by 0.4 years, respectively (compared to the forth column in Table 10). The total pension budget (and the contribution rate) is hardly affected, but the share of DP benefits in the pension budget rises. Later retirment increases employment and the revenues from income taxes, so that the consumption tax rate could be reduced. The slight reduction in long-run welfare reflects the dampened intergenerational redistribution implied by the reform.

[28] According to Börsch-Supan et al. (2018) this is about the average actuarial value in Europe and the US.

Keeping the current ERA, but increasing the rerquired number of contribution years for early retirment even induces a stronger shift towards DP retirment. As the second column shows, the OAP retirment age rises in this case only slightly and employment is hardly affected. As a consequence, the pension budget increases slightly as well as the contribution rate. Slightly lower wages and higher consumption taxes now result in a reduction of long-run welfare by 0.4 percent.

Not surprisingly, the phased increase of the adjustment factor from currently 3.6 up to 6.3 percent in the two remaining columns of Table **??** have a very strong effect. When the increase only applies to OAP benefits, it induces again a strong shift towards DP applications and benefits and later OAP retirment. Consequently, the effective retirment age increases for both groups significantly, while at the same time the pension budget and contribution rate is reduced. People now work longer and save more, which in turn also increases income tax revenues and lowers the consumption tax rate. Overall there is a strong redistribution towards future cohorts so that long-run welfare increases now by 2 percent.

Increasesing the adjustment rate for both pension types has the strongest effects on long-run retirement, the pension budget and the aggregate economy. The effective retirement ages for DP and OAP benefits increase by 1.4 and 1 year, respectively. Lower benefits reduce the contribution rate by 2 percentage points. Higher employment and savings boost income tax revenues, so that the consumption tax rate falls by 1.5 percentage points. Consequently, long-run welfare now increases by even 4.1 percent.

Overall, the two reforms considered at the end indicate the power of higher adjustment factors relative to modest reforms of eligibility rules. Even more modest increases would induce a significant fiscal improvement and dampen the long-run pressure on the whole economy. Furthermore, these subsection also documents the strong substituition effects when OAP rules are adjusted in isolation. Often pension reform discussions and studies neglect the interaction between OAP and DP pension completly. Our study highlights that this neglect may result in stongly misleading conclusions on the impact of the reform.

6 Conclusion

This paper provides a first step to model retirement behavior in Germany in a more realistic way by considering the interlinkages between OAP and DP benefits. Since it is not possible to monitor perfectly health-related work limitations (i.e. burn out etc.), households substitute twoards early DP retirment even if they are still able to work. Already the pension reform of 2007 will induce such a behavior, since later retirement is very unpopular. Applications with mental work limitations are steadily rising currently and our simulation results indicate a further increase when the Reform 2007 is fully implemented after yerar 2030.

Compared to the Reform 2007, the various DP reforms afterwards have gained surprisingly little attention in the public debate in Germany. Our simulations indicate that this reform package induces an even more dramatic increase of DP applications, when it is fully implemented after year 2034. With current eligibility rules it may even completly offset or neutralize the fiscal savings and economic benefits of the Reform 2007. Consequently, politicians maybe forced in the future to tighten eligibility rules resulting in higher DP rejection rates.

Finally, our simulations also clearly demonstrate that the interlinkage between OAP and DP benefits also strongly affects reforms of OAP pensions. Since many studies neglect this interaction complety, conclusions drawn there suffer from a severe shortcomming.

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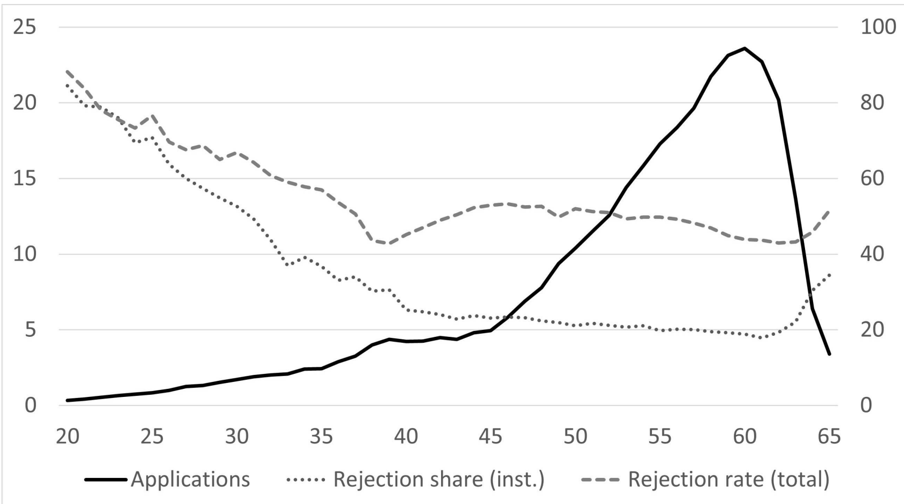
Appendix 1: Detailed description of data processing

In this appendix we describe the parameterisation of the model. After discussing the recent data an disability applications, we focus on the income process and its interaction with health shocks during the working phase of households. Finally we discuss the data which is used to calibrate some of the model parameters internally.

1A: Applications and rejections for disability pensions in 2019

The most recent available data on disability applications is from year 2019. We focus on the most recent applications, since older data (i.e. around year 2010) is only available in much less detail. In 2019 about 348.200 applications for disability pensions were submitted to the pension insurance, of which 170.100 were rejected. Figure **??** shows on the left vertical axis the absolute application numbers (in 1000) per age group and on the right axis the percentage rates of rejections. Less than 3 percent of all applications were submitted by households younger than 30. This group has a high rejection rate of more than 70 percent. In addition, most of the rejections were due to institutional reasons (which are not captured in our model). The latter refers to applicants with a total contribution record of less than five years or no contributions during the last three years or applications were sent to another insurance agency. After age 30 applications for disability pensions increase steadily until about age 45. About 14 percent of all applications were submitted by cohorts in the age group 31-44. The rejection rates falls in this group with rising age and also institutional rejections become less and less important. Overall about 24 percent of rejections were due to institutional reasons.

Figure 4: DP applications and rejections in 2019



Starting with age 45 applications increase steadily until they peak roughly at age 60, before they fall again sharply to zero by age 65. Of course, after age 60 there are alternative routes into retirement (such as severely handicapped retirement), which is not captured by the model. Especially when they were formerly unemployed, households may also retire with reduced old-age pensions before, and can only retire with full old-age pensions in and after the normal retirement age. Note that the rejection rate is roughly constant after age 45 and that institutional rejections are very low.

Besides the above described institutional reasons, most rejections were declared because applicants did not pass the medical examination (about 56 percent). In the remaining cases rejections were mostly due to either non-cooperation of or withdrawal by applicants. This behavior might indicate that the applicants were afraid not to pass the medical test. The average processing time of an application was about 160 days, where the average acceptance took roughly three weeks longer. However there is a huge spread in the processing time ranging from 56 days (when sent to other agency) up to 200 days (rejection because of non-cooperation).

1B: Health transitions for age and education groups

The transition matrices of the health state for individuals older than age 45 are obtained from the German sub-sample of the Survey of Health, Ageing and Retirement in Europe (SHARE). A detailed description of the data set can be found in Börsch-Supan et al. (2013). The SHARE is a longitudinal data set that includes a wide range of micro-data on socio-economic status, social and family networks, as well as health across European countries. The latest SHARE Release 7.0.0 includes about 140,000 individuals aged 50 or older in 28 countries. The German sub-sample currently includes seven waves between 2004 until 2018 with a total of 8.788 persons. However, the original sample had to be reduced considerably for our purposes. As shown in Table **??**, we need to subtract first 337 persons because of missing information of some health variables, age or education background. The remaining sample of 8.451 persons with 17.444 observations was used to construct the health

Table 12: Selection of sample from German SHARE data 2004-2018

	High school ($s = 0$)	College ($s = 1$)	$\sum$
Original sample			8.788
- missing identification variables			-337
Sample for health index weights			8.451
- only one observation			-3.221
Final sample for transition matrix	3.666	1.564	5.230
Transitions	5.946	2.768	8.714

index weights which are discussed below. In order to construct the transition matrices for the two education groups and age brackets we needed to exclude further all persons who only provided full information in one wave. This reduced our sample for the transition matrix further to 5.230 persons with 8.714 health transitions. Table **??** shows that our final sample roughly contains the double number of people and observations for high school graduates compared to college graduates. In both educational groups the transitions are roughly the same in the two age brackets 45-64 and 65+.

Following Jürges et al. (2015) or Poterba et al. (2017) health is measured by constructing a health index after performing a principal component analysis using data from six waves of SHARE.[29] Using all observations at once a continuous health index is computed from the first principal component of twenty-one health indicators listed in Table **??**. The weights (or loadings) of the first principal component reported in Table **??** are chosen to maximize the variance of the projected health data. As one can see, the weights are fairly stable across all waves. Highest weights are given daily life health

[29] The third wave contained only retrospective data on early lives.

Table 13: Health index weights (principal component loadings)

Health measure	Loads
Difficulty walking 100m	.300
Difficulty lift/carry	.294
Difficulty push/pull	.294
Difficulties with an ADL	.297
Difficulty climbing stairs	.276
Difficulty stoop/kneel/crouch	.288
Difficulty getting up from chair	.279
Difficulty reach/extend arms up	.257
Difficulty sitting two hours	.224
Difficulty picking up a coin	.179
Ever experience heart problems	.123
Ever experience stroke	.130
Ever experience high blood pressure	.119
Ever experience lung disease	.103
Ever experience diabetes	.119
Ever experience cancer	.074
Nursing home stay during last year	-.089
Hospital stay during last year	-.149
BMI at beginning of period	.114
Self-reported health	.299
Psychological problems (Euro-D-scale)	.233
N	17.444

measures (i.e. difficulties in walking, lifting, climbing, etc.), while much less weight is given to questions about whether the respondent ever experienced specific health problems. This is very much in line with results in Poterba et al. (2017), who argue that this probably reflects a high correlation between many self-reported measures.

Using the weights from table **??** the individual data is converted into a "raw health index" in each wave for each person with more than one observation. In the next step, the raw health indices of each wave are put together and normalized on the $[0, 1]$-interval, where a higher value indicates a worsening of health. The interval is split into three areas $[0 - k_0, k_0 - k_1, k_1 - 1]$ and all normalized indices are shifted back to their original wave. Now we can observe transitions and allocate them back into the cells of the respective (h, h^+) matrix. Finally the resulting transitions matrix has to be recalculated on a yearly basis. Tables **??** and **??** report the resulting health transition matrices for the two age and education groups.

Given the transition probabilities we can compute the distribution and dynamics of the health status over the life cycle and compare it with the data. The calibrated interval borders $k_0 = 0.43$ and $k_1 = 0.80$ then minimize the difference between the model generated and the observed dynamics of the health status. Table **??** compares in the upper part the model results for the two educational groups in three phases of their life cycle. High school graduates are clearly less healthy than college graduates. The lower part of Table **??** compares the aggregate health data from the model with the respective German health data.

Table 14: Health transition matrix for age group 45-64 ($j = 26, \dots, 45$)

	High school ($s=0$)			College ($s=1$)		
	$h=0$	$h=1$	$h=2$	$h=0$	$h=1$	$h=2$
$h=0$	0.8738	0.1195	0.0066	0.9140	0.0813	0.0047
$h=1$	0.1346	0.7849	0.0806	0.1349	0.8117	0.0534
$h=2$	0.0050	0.1649	0.8302	0.0090	0.1829	0.8081

Table 15: Health transition matrix for age group 65+ ($j = 45, \dots, 80$)

	High school ($s=0$)			College ($s=1$)		
	$h=0$	$h=1$	$h=2$	$h=0$	$h=1$	$h=2$
$h=0$	0.8340	0.1557	0.0103	0.8691	0.1309	0.0000
$h=1$	0.0934	0.8117	0.0948	0.1131	0.7960	0.0910
$h=2$	0.0005	0.1028	0.8968	0.0000	0.1287	0.8713

Table 16: Distribution of health status in the data and in the model (in %)

	Age cohorts					
	45-54		55-64		65+	
	s=0	s=1	s=0	s=1	s=0	s=1
Good health	64.0	75.7	47.2	59.8	35.7	47.2
Medium health	25.9	20.1	37.9	32.4	43.5	39.1
Bad health	10.1	4.2	14.9	7.8	20.8	13.7
	Data*	Model	Data*	Model	Data*	Model
Good health	65.1	66.8	52.0	50.3	40.0	38.8
Medium health	26.4	24.5	35.3	36.5	45.4	42.3
Bad health	8.5	8.7	12.7	13.2	14.6	18.9

*Source: Eurostat (2021), average between 2008-2019.

1C: Productivity profiles and productivity risk

The parameters to calibrate the earnings process and earnings risk over the life cycle are taken from Kindermann and Püschel (2021), who use administrative data from the German public pension insurance system. The scientific use file of the Versichertenkontenstichprobe 2017, contains information from the insurance accounts of 69,520 individuals, with information on age, gender, education and (most importantly) a monthly history of accumulated pension claims which could be used to compute individual earnings. The estimations are based on observations between year 2000 and 2016 of males aged between 25 and 60 with available information on educational background. Labor earnings comprises income from regular work, marginal employment and short-term unemployment (up to one year), all other sources of pension claims (like times of care for children or sickness) are counted as zero earnings. Individuals with a full year of zero earnings were excluded from the sample. This selection procedure left a total of 15,242 individuals and 189,184 annual earnings observations. The share of individuals with a college degree in this group was 23,73 percent, all remaining persons were assumed to have a high school degree. Due to the contribution ceiling the raw data was top-coded and due to part-time jobs or so-called mini-jobs a substantial fraction of the sample showed an earnings level below 25 percent of average income. Consequently, a simple log-normal AR(1) process is not rich enough to describe the earnings dynamics of households in Germany. While there exists a "'normal"' earnings process that follows some regular AR(1) dynamics, individuals can also experience very low earnings episodes. In order to develop a statistical model that can fit the data on low earners by age and education, Kindermann and Püschel (2021) split the data set into two parts where the first group contains all labor incomes below an earnings threshold of 23 percent of average labor earnings. These low earnings individuals can be thought of as having some months of temporary unemployment or non-employment throughout a year or as being marginally employed. All individuals with labor earnings above the threshold have normal labor earnings. In order to specify a statistical model with low earnings episodes, Kindermann and Püschel (2021) assume that individuals either follow a stable or an unstable career path, where both groups have the same probability (i.e. $\omega_m = 0.5$) independent of the educational background. Those who are on a stable career path are always in the normal earner group, while those on the unstable career path may experience earnings shocks, which put them temporarily in the low earner group.

The normal earner group

The normal earner sample is then split between high school and college graduates and the earnings dynamics of both groups are described by a standard AR(1) process in logs. The estimated statistical model

$$\log(y_{isjt}) = \hat{\kappa}_{t,s} + \hat{\theta}_{j,s} + \hat{\eta}_{isjt} \quad \text{with} \quad \hat{\eta}_{isjt} = \hat{\rho}_s \hat{\eta}_{is,j-1,t-1} + \hat{\epsilon}_{isjt} \quad \text{and} \quad \hat{\epsilon}_{isjt} \sim N(0, \hat{\sigma}^2_{\epsilon,s})$$

explains individual labor earnings y_{isjt} of an individual i with education s, age j and year t with a year fixed effect $\hat{\kappa}_{t,s}$ that controls for earnings changes along the business cycle, an age fixed effect $\hat{\theta}_{j,s}$ that informs us about the age-earnings relationship and a noise term $\hat{\epsilon}_{isjt}$ that is assumed to follow a normal distribution with mean zero. The method of moment estimation controls for the top-coding and the truncation at the lower earnings threshold, see Kindermann and Püschel (2021) for more details. The point estimates of the age fixed effects show a steep increase for both education groups up to age 45 and a stagnation or slight decline afterwards for both education groups. The college wage premium implied by the profiles is about 60 percent, which seems fairly realistic, see OECD (2016). The profiles of the age fixed effects are used to calibrate the age-productivity profiles for the two education groups in the model.

Table **??** reports the parameter estimates for the residual earnings process. Both earnings groups exhibit a high persistence in labor earnings with an unconditional variance of roughly 15-20 percent. Following Kindermann and Püschel (2021), we directly apply the estimated autocorrelation terms in our model but calibrate the innovation variance in order to replicate the unconditional variances with our model.

The low earner group

The fraction of individuals in an age cohort that are in the low earnings group declines over the life cycle for both educational groups. However, college educated mostly experience low labor earnings

Table 17: Parameters of residual log-earnings process

	High school ($s = 0$)	College ($s = 1$)
Autocorrelation $\hat{\rho}_s$	0.9869	0.9900
Innovation variance $\hat{\sigma}^2_{\epsilon,s}$	0.0046	0.0039
Unconditional variance $\frac{\hat{\sigma}^2_{\epsilon,s}}{1-\hat{\rho}_s}$	0.1780	0.1982

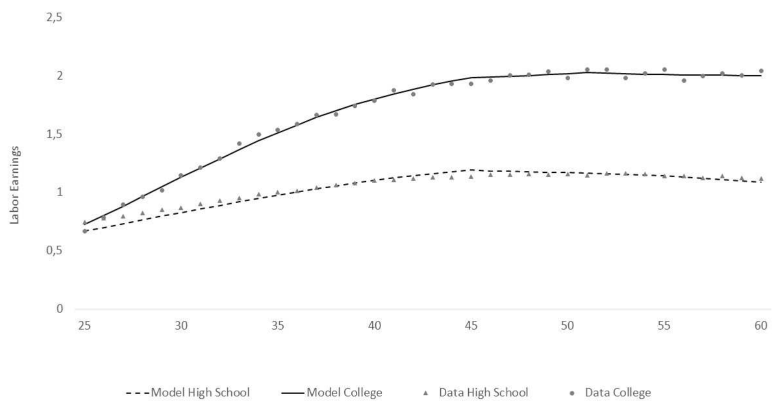
early in their career and after age 35 their share converges to almost zero. High school workers experience low earnings episodes much less at younger ages but this sharer declines also much less later in life. The data also shows that individual labor earnings in the low earnings group are by and large independent of age and education and amount on average to roughly 10 percent of average labor earnings (i.e. $\exp(\eta_0) = 0.1$). Kindermann and Püschel (2021) therefore distinguish within each educational level two groups that exhibit different degrees of career stability. Some workers follow a stable career paths, while others frequently transition into and out of employment. Individuals therefore draw at the beginning of working life a discrete shock whether they face a stable or unstable career path during their working life. They assume the same probabilities $\omega_m = 0.5$ to belong to one of the two groups. The transition into and out of low earnings for those who face unstable careers is then modeled as a first-order discrete Markov process for both educational groups. Households with unstable careers $(m = 1)$ face an education specific transition matrix

$$\Pi^s_{low} = \begin{bmatrix} 1 - \pi^s_{low,0} & \pi^s_{low,0} \\ 1 - \pi^s_{low,1} & \pi^s_{low,1} \end{bmatrix}$$

where the probability $\pi^s_{low,0}$ indicates the likelihood of a normal earner to transition into the low earnings state in the next period, while $\pi^s_{low,1}$ is the probability to remain in the low earnings state. Furthermore they assume that, at age 25, a fraction ω^s_{low} of individuals of education s starts as a low earnings individual. The point estimates which are reported in the calibration section provide the best fit to the data in a least squares sense.

Age-earnings profile

Figure 5: Education-specific age-productivity profiles



The original data is taken from Kindermann and Püschel (2021), while the straight line is simulated with the parameters from the model.

1D: Scientific use file of fully insured lives (Vollendete Versicherungsleben) SUFVVL2016

In order to calibrate parameters and the initial equilibrium of the model we use the administrative data set of fully insured lives in 2016 which is available as a scientific use file (SUF) from the research center of the German pension insurance. The representative sample contains 25 percent of all pensioners who received in this year the first time a pension benefit in Germany. The data set contains biographical information on sex, age, address, citizenship, family status, number of children and education, but also all information necessary to compute the benefit level (sum of earning points, etc.). In year 2016 832.664 people entered retirement, so that the full sample contains 208.166 insured persons. As Table **??** documents, we have excluded all persons where we assume that their employment history differs significantly from a regular work history (women, foreigners, no residence, no contribution record, long absence). In addition, we only wanted to consider retirement with old age

Table 18: Selection from German VVL 2016 sample

	Initial full sample (persons)	208.166
-	women	-110.679
-	pensions from unemployment, severely handicapped, etc.	-7.827
-	retirement before 2015	-1.745
-	pensions without contribution record	-2.533
-	no German citizenship/residence	-3.981
-	no information on education	-25.931
-	long absence from pension insurance	-403
=	final data base	55.067

or with disability pensions who had information on their educational background. This procedure left us with roughly more than 55.000 persons of which were 44.483 high school graduates and the remaining 10.584 had a college degree.

Literature

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