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Regret and Asset Pricing

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Abstract

I investigate the consequences of regret aversion for asset prices in an otherwise standard model of financial markets. This paper shows that accounting for investors' regret aversion can help explain the risk-free rate puzzle, excess volatility, the downward sloping term structure of equity risk premiums, and the predictability of stock returns both in the time series and in the cross section. The model also evaluates bond behavior and predicts a downward sloping real yield curve. I provide an empirical measure of regret which confirms empirically the main model's testable predictions. This paper is the first to document the linkage between regret aversion and many stylized facts concerning asset prices.

Keywords: equity, bonds, asset pricing puzzles, stylized facts, regret aversion

JEL Codes: G12, G41

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I. Introduction

It appears to be somewhat of a challenge to provide a unifying explanation for stylized facts that have been uncovered in the asset pricing literature. In the time series, typical stock returns are excessively volatile and predictable using lagged prices scaled by fundamentals such as dividends (Shiller, 1981; Campbell and Shiller, 1988). The risk-free rate is low and stable (Weil, 1989), with unpredictable consumption and dividend growth. The unconditional term structure of equity risk premiums is downward sloping (van Binsbergen, Brandt, et al., 2012; van Binsbergen and Koijen, 2017).[1] In the cross section, there is a value premium (Basu, 1983; Fama and French, 1992) and "long-term reversal" (De Bondt and Thaler, 1985). Bond yields produce an unconditional downward sloping real yield curve (Piazzesi and Schneider, 2006).[2]

I present a model that helps explaining these stylized facts in a unifying way. The central and only ingredient is regret and the aversion to it, added to an otherwise standard power utility function and standard financial market. Regret-averse investors are concerned not only about the returns they receive, but also about the foregone returns they could have received, had they invested differently. Investors anticipate disutility from a state of the world where they could have had higher consumption, weighted by a regret-aversion parameter. In case the foregone return is large, regret and marginal utility in that state of the world are high, such that risky asset prices are high and expected returns fall.

Why is regret aversion relevant for asset prices? Extensive psychological, experimental, and neuroscientific research provides abundant evidence for the role of regret in investment

[1]Bansal, Miller, et al. (2021) argue that the unconditional equity term structure is upward sloping.

[2]This is consistent with the evidence in the U.K., but it is more ambiguous in U.S. data. For the latter, we only have a small sample (Piazzesi and Schneider, 2006).

decisions and in trading behaviour.[3] Psychological evidence comes from Lin, Huang, et al. (2006), who study actual stock investors' behavior and document that regret influences investors' investment decisions through counterfactual thinking. Lohrenz et al. (2007) and Frydman and Camerer (2016) use neural scientific data, gathered simultaneously with actual investment behavior, to show that investors exhibit and experience regret while trading. More generally, using a neuro-psychological experiment, Camille et al. (2004) and Bourgeois-Gironde (2010) find that their respondents think counterfactually, anticipate regret and consider regret while making risky decisions.[4] Using detailed trading data, Strahilevitz et al. (2011) and Magron and Merli (2015) emphasize the important role of regret in financial decisions and they relate regret to the disposition and repurchase effects.[5]

I model regret, and the aversion to it, in line with three observations from the literature. First, at the moment of investment decision making, investors anticipate that they experience the feeling of regret in the future. Neuroscientific evidence (Camille et al., 2004) and the review of Zeelenberg (2018) show that anticipating future regret influences current decision making and, thus, current investors' holdings. In the model, regret enters in an otherwise standard power utility function as a multiplicative component that yields disutility, weighted by a regret-aversion parameter (Quiggin, 1994). Regret follows from counterfactual thinking

[3]Evidence for regret in decision making has been extensively investigated by Larrick (1993), Gilovich and Medvec (1995), Larrick and Bowles (1995), Zeelenberg (1999), Connolly and Zeelenberg (2002), Connolly and Butler (2006), and Zeelenberg and Pieters (2007). Zeelenberg and Pieters (2004) document regret in a real-life non-student sample. According to Connolly and Zeelenberg (2002), regret is the emotion that has received the most attention from decision theorists. Saffrey et al. (2008) investigate the intensity and frequency of twelve emotions, and their participants rate regret as being the most intense negative emotion. Zeelenberg (2020) argues that regret fulfills all conditions for being classified as a basic emotion.

[4]Bourgeois-Gironde (2010) state that regret helps to optimize decision behaviour. They define regret as a rational emotion.

[5]When an investor sells a stock, she is less likely to repurchase this same stock if the price has increased since the sale, compared with when the price has decreased since the sale. Strahilevitz et al. (2011) call this the repurchase effect. Fioretti et al. (2021) use a stock market experiment to study the influence of regret aversion on the decision to sell an asset if prices change over time.

about foregone returns. The representative investor invests in a portfolio and anticipates regret by a comparison (ex-post) of her realized consumption with the best unchosen alternative (i.e., "if only I made another investment decision") and the inaction alternative (i.e., "if only I did not invest").

Second, emotions, and thereby regret, follow laws (Frijda, 1988; Frijda, 2007). Regret is time varying and reverts over time around a mean (Wilson and Gilbert, 2005), but only slowly and gradually. Regret is persistent, extending up to years. Especially negative feelings and emotions, to which regret belongs, are persistent phenomena (Coricelli et al., 2005; Wilson and Gilbert, 2005; Hajcak and Olvet, 2008). This feature produces mean-reversion in prices and return predictability in the time series and cross section. Consumption growth and dividend growth follow simple white noise lognormal processes (Campbell and Cochrane, 1999), with means and standard deviations consistent with the empirical asset pricing literature. Because of the law of emotional control (Frijda, 1988; Frijda, 2007), regret is not too volatile such that the risk-free rate in the economy remains stable.

Third, the main premise of regret theory (Bell, 1982; Loomes and Sugden, 1982), being an alternative to expected utility theory (Neumann and Morgenstern, 1947), is that we are averse to regret. Bleichrodt et al. (2010) introduce the first quantitative measurement of a regret aversion parameter. They estimate a regret-aversion parameter which implies more disutility when foregone consumption is high. Their evidence confirms regret aversion at the individual and aggregate level.

My results show that regret aversion is a helpful ingredient to understand behavior of assets in the time series and in the cross section, not only in terms of sign, but also in terms of magnitude consistent with the empirical asset pricing literature. I find a low stable risk-free rate with unpredictable, low, and stable consumption growth and dividend growth. Stocks are more volatile than the underlying dividends, and returns are predictable by the lagged price-dividend ratio and lagged returns. Regret produces an unconditional term structure of risk premiums that is downward sloping. In the cross section, I document long-term reversal and a value premium: stocks with low price-dividend ratios (i.e., value stocks) yield higher subsequent returns than stocks with high price-dividend ratios (i.e., growth stocks). The analysis on bonds shows that regret produces an unconditional downward sloping real yield curve, and bond returns are predictable by regret.

To understand the mechanisms, consider first the case of predictability in the time series and the cross section. If foregone returns on the risky asset are high, then regret is high. Investors regret having invested too little and demand more of the risky asset, which pushes up prices today. Prices relative to dividends become overvalued such that future returns fall. In the cross section, regret is asset specific. Stocks with high regret are typically growth stocks, or winner stocks, that yield subsequent lower returns. The mispricing of these stocks and the regret on growth stocks are highly persistent, consistent with (Arisoy et al., 2021; van Binsbergen, Boons, et al., 2021). Regret volatility, amplified by regret aversion, makes returns more volatile than the underlying cash flows. Because regret-averse investors are concerned with regret in the short run, as they confront their performance annually, investors require a premium to hold short-term assets such that the term structures of equity risk premiums and interest rates is downward sloping.

The risk-free rate in the economy is low and stable, since regret is not too volatile. When regret is high, the representative investor feels poor such that she starts to save more which drives down the equilibrium risk-free rate. Finally, regret-averse investors theoretically require a regret risk premium to hold risky assets through negative correlations between consumption and regret, and dividend and regret. If regret is high, then the foregone return on risky assets is high, such that marginal utility is also high. Regret-averse investors do not like such states of the world, as the could have been better off. Therefore, investors require a premium to hold risky assets. Theoretically this holds true, but in my calibration the regret premium is small and, thus, regret does not resolve the equity premium puzzle. Though, I study the equity premium on unlevered claims, which is typically lower than the equity premium on firms with leverage included (Abel, 1999).

An empirical measure of regret confirms the main model's predictions. In line with my regret model and the measure of Arisoy et al. (2021), I empirically measure annual regret by the highest return in the cross section per year. This regret measure is highly persistent and behaves historically similar to the price-dividend ratio, as predicted by the model that prices relative to dividends are a function of investors' regret. Moreover, the regret measure predicts future returns with a negative sign in the time series, specifically when the forecasting horizon is long. Arisoy et al. (2021) empirically study the implications of regret-sorted portfolios in the cross section and they find that growth stocks are stocks with high regret, as predicted by my theoretical model.

Regret-averse investors are concerned with the positive skewness of returns. A simple exercise shows that regret, or foregone returns, are on average high when the skewness of the underlying returns is positive and high. The more dispersion cross-sectionally in returns, the higher the probability of a foregone missed opportunity. Drerup et al. (2022) provide direct evidence for my main mechanism of predictability of returns, as the authors find a positive correlation between skewness expectations and investment decisions. That is, investors indeed increase portfolio allocations when skewness expectations are high, i.e., high expected skewness possibly yields high foregone returns which implies high regret. Eeckhoudt et al. (2007) and Gollier (2018) find that regret-averse agents have a preference for positively skewed risks and longshots. These upside risk concerns of regret-averse investors contrast with the downside risk capital asset pricing model of Lettau, Maggiori, et al. (2014), in which investors are concerned with downside risks.

My paper is the first to study regret in a consumption-based model and the first to show that regret also explains stylized facts in terms of magnitude. I contribute to the asset pricing literature on regret, which contains a few studies with explicit regret-theoretic models. Dodonova and Khoroshilov (2005) study regret aversion in a one-period terminal wealth asset pricing model with two firms and two types of agents. They theoretically find excess volatility and long-run negative autocorrelations of stock prices, but their regret-utility specification deviates from the original regret theory of Bell (1982), Loomes and Sugden (1982), and Quiggin (1994). Muermann et al. (2006) study optimal portfolio choice between a risky and risk-free asset in DC schemes when investors are regret averse, by following additive regret. Qin (2020) presents a regret-CAPM model, and indicates that a regret-related beta can help explain cross-sectional returns and possibly the high equity premium. Arisoy et al. (2021) empirically document a regret premium in the cross section.

Compared to leading asset pricing models, regret is able to match the downward sloping term structure of equity risk premiums and the cross-sectional stylized facts. Habit formation (Campbell and Cochrane, 1999) needs high risk aversion to explain the equity-premium risk-free rate puzzle, it produces a growth premium rather than a value premium (Santos and Veronesi, 2010) and the term structure of equity risk premiums is upward sloping (van Binsbergen, Brandt, et al., 2012). The long-run risk model has difficulties with the absence of predictability in consumption and dividend growth (Beeler and Campbell, 2012), needs to rely on time-varying consumption volatility for predictability, and produces an unconditionally upward sloping term structure of equity risk premiums. Disaster risk (Barro, 2006) requires time variation in disasters to match predictability, and disaster risk produces a flat term structure for equity and bonds (Gabaix, 2012). Prospect theory and loss aversion models (Barberis, Huang, and Santos, 2001; Pagel, 2016) have difficulties solving the risk-free rate puzzle and produce upward sloping equity term structures, but Barberis and Huang (2001) can math the value premium and long-term reversal.

Regret relates to long-term experience effects caused by the persistence and strength of emotions (Malmendier, 2021). Malmendier (2021) states that we as economists typically pay little attention to emotions, while we might want to reconsider choice behavior and beliefs as a function of emotional inputs rather than (only) informational inputs. The implications of regret on asset prices relate to return extrapolation (Atmaz, 2021) and optimism (Brunnermeier et al., 2007) as regret-averse investors chase good foregone returns. Moreover, regret relates to ex-ante rational expectations-based reference-dependent models (Köszegi and Rabin, 2006; Pagel, 2016) in which references points are (fixed) forward looking beliefs over all possible outcomes (i.e., good and bad news) rather than backward looking as in habit formation or prospect theory. Finally, regret aversion relates to disappointment aversion (Gul, 1991), but regret is neurologically different and significantly more intensely felt than disappointment (Camille et al., 2004).

Regret appears implicitly in the early behavioral finance literature (Shefrin and Statman, 1984; Shefrin and Statman, 1985). Taking regret more explicitly into account, Muermann et al. (2006) and Baule et al. (2019) study optimal portfolio choice for regret-averse investors with additive regret, whereas Gollier and Salanié (2012) study risk-sharing and portfolio allocation in a complete market for a general bivariate regret-utility function. Qin (2015) studies bubbles, herding, and market turbulence by regret over action and inaction, while Fogel and Berry (2006) find that regret aversion explains the disposition effect. Solnik and

Zuo (2012) present a global equilibrium asset pricing model to explain the home bias. Regret also appears in the literature of insurance and pensions (Braun and Muermann, 2004; Frehen et al., 2008), as well as currency hedging (Michenaud and Solnik, 2008).

A. Related concepts

In this section, I discuss how the predictions of the regret model compare to leading asset pricing models. The section ends with economic and psychological related concepts to regret.

1. Asset pricing models

This section provides a comparison between the regret model and other leading asset pricing models such as habit formation, long-run risks, disaster risk and prospect theory. To the extent of my knowledge, current leading asset pricing models have difficulty explaining the behavior of equity and bonds in the time series and the cross section. Table 1 presents an overview of the models, and I explain it below.

Table 1: **Overview of asset pricing models.** This table presents an overview of the stylized facts that asset pricing models can explain, together with the number of parameters. It compares the regret model to four leading asset pricing models: habit formation, long-run risks, disaster risk and prospect theory. [a] Required to add economic uncertainty (i.e., time-varying consumption volatility). [b] Required to add that disaster risk changes over time. [c] Narrowing framing required.

	Regret aversion	Habit formation	Long-run risk	Disaster risk	Prospect theory
Risk-free rate puzzle	Yes	Partial	Yes	Yes	No
Equity premium puzzle	Partial	Partial	Partial	Yes	Yes
Excess volatility	Yes	Yes	Yes	Yes	Yes
St. Dev. Price-dividend ratio	Yes	Yes	Partial	Yes[b]	No
Time-varying risk premium	No	Yes	Yes[a]	Yes[b]	
AC(1) returns	Yes	Yes	Yes[a]		Yes
AC(1) price-dividend ratio	Yes	Yes	Yes[a]		Yes
Stock predictability	Yes	Yes	Partial[a]	Yes[b]	Partial
Unpredictable Δc	Yes	Yes	No[a]	Yes[b]	Yes
Unpredictable Δd	Yes	Yes	No[a]	Yes[b]	Yes
Value premium	Yes	No	Yes		Partial[c]
Long-term reversal	Yes	No	No		Yes[c]
Equity term structure	Downward	Upward	Upward	Flat	Upward
Real yield curve	Downward	Upward	Downward	Flat	
St. dev. yields	Decreasing	Increasing			
Total parameters	12	11	13	16	16

Habit formation - Campbell and Cochrane (1999) present an asset pricing model that explains many asset pricing phenomena by including one simple ingredient in an otherwise standard model: external habit formation. The habit is slow moving and yields time-varying risk aversion, whereas the regret model implies time-independent risk aversion. The external habit model delivers a high equity premium, excess volatility, with low mean consumption growth, and volatility, unpredictable consumption and dividend growth, and a low and slowly varying risk-free rate. But, the habit model does not always have low risk aversion and, as such, does not resolve the equity-premium risk-free rate puzzle.[6]

Also, the habit model delivers the observed return predictability, the countercyclical variation of stock market volatility, and time-varying risk premia. The regret model has all of the above, including low stable risk aversion, and as such has the potential to explain the equity premium and risk-free rate puzzles. However, in the current setup, regret lacks the time-varying stock market volatility (i.e., volatility is higher after a price drop) and time-varying risk premia.

The external habit formation model yields an upward sloping yield curve (Campbell and Cochrane, 1995), which contradicts the evidence in the U.K. (Piazzesi and Schneider, 2006). Yields on long-term bonds vary more than yields on short bonds, which contradicts the data. The habit model implies an unconditional upward sloping term structure of risk premiums and volatility (Campbell and Cochrane, 1999), which slope upward indefinitely through the non-stationary state variable, inconsistent with the empirical findings of van Binsbergen, Brandt, et al. (2012).

In the cross section, the habit model produces a growth premium rather than a value premium (Santos and Veronesi, 2010), which is at odds with the data (Basu, 1983; Fama

[6] The risk aversion of investors varies over time to more than 100 when the probability of a recession becomes larger (Mehra, 2012).

and French, 1992). Overall, the habit model requires eleven parameters in total.[7] Note that the authors set mean consumption growth and mean dividend growth equal.

Long-run risks - Bansal and Yaron (2004) present an asset pricing model that explains many stylized facts by using two ingredients: (i) recursive Epstein-Zin preferences, and (ii) small persistent shocks (i.e., news) to consumption and dividend growth. News about long-run future consumption growth is the state variable. However, the data seems to suggest that consumption growth is closer to a random walk than the assumed persistence (Beeler and Campbell, 2012). The model produces a low and stable risk-free rate, and excess stock return volatility. It can produce the high equity premium with high risk aversion and low consumption volatility, or low risk risk aversion and high consumption volatility.

When adding economic uncertainty (i.e., time-varying volatility of consumption growth) to the long-run risk model, it produces time-varying risk premia and return predictability, but the latter is less than observed in the data (Beeler and Campbell, 2012). Also, the model-implied volatility of the log price-dividend ratio appears low compared to the data (Beeler and Campbell, 2012). The long-run risk model produces a downward sloping real yield curve, with real yields below zero for a maturity of ten years or higher. Risks for the long-run imply an unconditional upward sloping term structure of risk premiums and volatilities (van Binsbergen, Brandt, et al., 2012). The model produces a value and size premium in the cross section, as well as momentum (Bansal, Dittmar, et al., 2005). Overall, the long-run risks model (including time-varying volatility of consumption growth) needs thirteen parameters in total.[8]

Rare disasters - Barro (2006) presents an asset pricing model that explains the basic asset pricing moments by including disaster risk in an otherwise standard model. Disasters

[7]Based on Table I in Campbell and Cochrane (1999).

[8]Based on the long-run risks parameters in Table I in Beeler and Campbell (2012).

reflect huge market crashes; one objection to the disaster model is that we might have seen too few disasters (Cochrane, 2017). The model resolves the equity premium and risk-free rate puzzles with low risk aversion, and explains excess stock return volatility.

To get rare disasters to account for return predictability, one needs to specify that the risk of a rare event changes over time (Gabaix, 2012; Cochrane, 2017). Gabaix (2012) shows that time-varying disaster risk produces the observed volatility of the log price-dividend ratio, the time-varying risk premia, and the return predictability as observed in the data. As shown by van Binsbergen, Brandt, et al. (2012), the unconditional term structure of equity risk premiums is flat, which appears to be at odds with the data (Bansal, Miller, et al., 2021).

The yield curve on real bonds is flat: all yields are equal to the risk-free rate (Gabaix, 2012). Disasters with recoveries produce a downward sloping yield curve. To the extent of my knowledge, it is unknown whether disaster risks produce a value premium or long-term reversal in the cross section. Overall, the disaster model requires sixteen parameters.[9] Similar to Campbell and Cochrane (1999), the authors set the growth rates of consumption and dividends equal.

Prospect theory - Barberis and Huang (2001) and Barberis, Huang, and Santos (2001) present an asset pricing model that explains several stylized facts by using four ingredients of prospect theory. First, investors derive direct utility not only from consumption, but also from gains and losses itself. Second, investors are loss averse such that agents are more sensitive to losses than to gains. Third, loss-averse investors are risk-seeking over (large) losses. Finally, investors use narrowing framing and mental accounting (Barberis and Huang, 2001), or they distort probabilities (Barberis, Huang, and Santos, 2001).

Barberis, Huang, and Santos (2001) explain the equity premium puzzle and excess volatil-

[9]Based on Table I in Gabaix (2012).

ity, with low and stable consumption growth that is not predictable like their dividend growth. However, the model produces a too high risk-free rate and a too low log price-dividend ratio volatility. Their model yields return predictability, with an R^2 increasing with the return horizon, but the R^2 are lower than found in the data. It is unclear whether the model produces time-varying risk premiums. de Vries (2021) argues that the model of Barberis, Huang, and Santos (2001) implies an upward-sloping term structure of equity premiums and risks, but quantitative predictions are absent. I am unaware of a model with these ingredients that studies (real) bond yields.

Barberis and Huang (2001) use similar ingredients as Barberis, Huang, and Santos (2001), but the authors include narrow framing and mental accounting. This model has some success in the cross section as well, as it creates a value premium and De Bondt-Thaler premium. Though, the value premium appears too high. On the other hand, Barberis, Jin, et al. (2021) also apply prospect theory to the cross section, but they find that prospect theory works especially poor in explaining the value premium. Overall, the model of Barberis and Huang (2001) needs sixteen parameters.[10]

In a similar vein, Pagel (2016) presents an asset pricing model by using the ex-ante rational expectations-based reference-dependent model of Köszegi and Rabin (2006) together with loss aversion. The reference point is forward looking, rather than backward looking as in habit formation (Campbell and Cochrane, 1999) or prospect theory (Barberis, Huang, and Santos, 2001). Specifically, the reference point is based on fully probabilistic rational beliefs about current and future consumption that the agent formed in the previous period. Investors receive gain-loss utility from unexpected changes in present consumption and from revisions in expectations over future consumption, such that gain-loss utility can be inter-

[10] Based on Table I in Barberis and Huang (2001).

preted as utility over good and bad news. In contrast, the "reference point" in regret theory is simply based on the true best ex-post realization.

The model of Pagel (2016) explains the equity premium puzzle, the excess volatility and predictability of returns. The model fails to explain the risk-free rate puzzle and autocorrelation of returns. To match these empirical findings, the author needs to introduce long-run risks with time-varying consumption volatility (Bansal and Yaron, 2004) as well as time-varying disaster risk (Barro, 2006) and sluggish belief updating.

2. Economics and psychology

I end this section by discussing how regret relates to other economic and psychological concepts. Barberis and Huang (2001) and Barberis, Huang, and Thaler (2006) study implicitly the notion of regret by arguing that consumption is not the only carrier of utility and that regret is a possible interpretation for narrow framing and loss aversion. However, anticipated regret already occurs before any losses actually materialize (Janis and Mann, 1997), such that the basic emotion of regret (Zeelenberg, 2020) could be more primary than the aversion of realizing losses (Frydman and Camerer, 2016). The concept that losses loom larger than gains in prospect theory bears similarities with the finding that regret (after a negative experience) is more strongly felt than rejoicing (after a positive experience).

Proceeding on prospect theory, regret runs over final wealth levels as in expected utility theory, rather than gains or losses in cumulative prospect theory (Tversky and Kahneman, 1992). Loss aversion requires the specification of a reference point, similar to the counterfactual specification of regret aversion, but the psychological literature has found well-defined counterfactuals for investors (Lin, Huang, et al., 2006). Furthermore, regret enters convex in the utility function (i.e., the larger the foregone alternative, the higher regret, the more disutility) and is symmetric if one would consider rejoicing, rather than the asymmetric S-shaped concave-convex utility function of gains and losses in prospect theory.

Regret relates to long-term experience effects caused by the persistence and strength of emotions (Malmendier, 2021). The implications of regret on asset prices relate to return extrapolation (Atmaz, 2021) and optimism (Brunnermeier et al., 2007) as regret-averse investors have a preference for positively skewed risks (Gollier, 2018) and chase good returns. Frydman and Camerer (2016) argue that regret itself can provide a microfoundation for realization utility (Barberis and Xiong, 2012). Other interpretations of regret relate to cognitive dissonance (Chang et al., 2015) and belief-based explanations (Frydman and Camerer, 2016).

Regret is not a unique emotion when making decisions, because other emotions, such as disappointment, relief, anger, envy, satisfaction, and pride, are also often felt in a decision making context. However, all these other emotions can also be felt without one having made a decision (e.g., one can be disappointed in the weather, and proud of one's children), but regret is always linked to a decision (Zeelenberg, 2020). Thus, regret theory does not require an ex-ante fixed (probabilistic) reference point like the rational expectations-based model of Köszegi and Rabin (2006) and Pagel (2016), but regret uses the ex-post realized alternatives (see Lin, Huang, et al. (2006)).

Camille et al. (2004) provide neurological evidence that regret, based on ex-post alternative realizations, is different from disappointment, based on ex-ante (rational) expectations. Contrary to disappointment, which is experienced when a negative outcome happens relative to prior ex-ante expectations, regret is strongly associated with a feeling of responsibility for the ex-post outcome of the decision that has been made. The authors also report that disappointment is insignificant in the decision making process, while regret is significant and more intensely felt neurologically.

II. The Model

This section describes the preferences of the investors and how they set prices for equity and bonds in the market.

A. Preferences

Identical agents maximize expected discounted utility today t, with subjective discount factor δ, over the fraction ξ_t invested

$$\max_{\{\xi_t\}} u\left(C_t(\xi_t), X_t\right) + \delta \mathbb{E}_t \left[u\left(C_{t+1}(\xi_t), X_{t+1}\right)\right]. \tag{1}$$

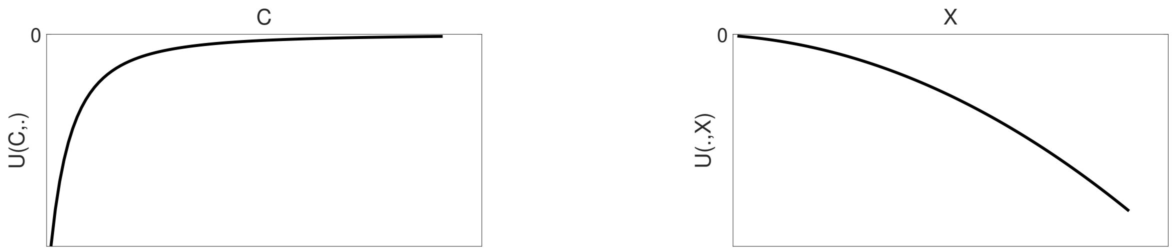
Here, C_t is the realized consumption level, based on realized returns, while $X_t \geq C_t$ is the foregone consumption level, based on foregone returns. I follow the multiplicative regret theory of Quiggin (1994), which is based on the additive regret theory as originally formalized by Bell (1982) and Loomes and Sugden (1982). Regret theory is an alternative to expected utility theory (Neumann and Morgenstern, 1947).

The multiplicative regret-utility function is defined as

$$u(C_t, X_t) = \frac{C_t^{1-\gamma}}{1-\gamma} X_t^{\kappa}, \quad \gamma > 1 \tag{2}$$

with γ the risk-aversion parameter and κ the regret-aversion parameter.[11] The first term in the regret-utility function is standard CRRA utility. The second multiplicative term reflects the disutility of regret. Because $\gamma > 1$, the regret-utility function has a negative range and a positive domain. When the foregone consumption level X_t is larger than the realized

[11] γ retains the classical interpretation of the Arrow-Pratt definition of risk aversion (Gollier, 2018).



(a) $u(C, X)$ for varying consumption levels C with fixed foregone consumption level X

(b) $u(C, X)$ for varying foregone consumption levels X with fixed consumption level C

Figure 1: **Regret-utility function.**

consumption level C_t, investors feel regret as they could have been better off. If there is no regret aversion, i.e., $\kappa = 0$, then the regret-utility function is a standard CRRA utility function.

Figure 1 provides intuition for the behavior of the regret-utility function. For varying consumption levels C and a fixed foregone consumption level X, Figure 1a graphs the standard well-known behavior of CRRA utility: increasing and concave. A low consumption level corresponds to a bad state of the world. For varying foregone consumption levels X and a fixed consumption level C, Figure 1b shows that the regret-utility function is decreasing in foregone consumption. A large foregone consumption level yields high disutility, which corresponds to a bad state of the world for investors, since marginal utility is high. The strength of disutility depends on the regret-aversion parameter κ. To ensure that (i) marginal utility of consumption increases as foregone consumption increases and (ii) utility exhibits aversion to the foregone alternative, we need the condition $\gamma - 1 \geq \kappa \geq 1$.[12] The conditions are essential properties for modelling regret (Gollier, 2018).

Instead of multiplicative regret (Quiggin, 1994), one could also model additive regret

[12]In line with Gabillon (2011), see properties P3 and P4b in Table 10 in the Appendix. The theorem of Diecidue and Somasundaram (2017) shows that regret theory holds in line with their behavioral foundation if the inequalities are strict, which I can easily assume as well (implying $\gamma - 1 > \kappa > 1, C > 0, X > 0$.

as originally and independently developed in the regret theory of Bell (1982) and Loomes and Sugden (1982). However, the multiplicative specification simplifies our calculations and leads to closed-form tractable results, as opposed to the additive specification which yields a nested utility function in a convex regret function. Conceptually, multiplicative and additive regret should lead to the same results. But, multiplicative regret, compared to additive regret, excludes rejoicing.

Rejoicing is an additional emotion, which is the opposite feeling of regret and results from downward counterfactual thinking. Rejoicing is felt when realized consumption turns out to be the more desirable result than the foregone consumption X_t, i.e., $C_t > X_t$: "the extra pleasure associated with knowing that, as matters have turned out, the agent has taken the best decision" (Loomes and Sugden, 1982). However, the emotional impact of regret is greater than rejoicing (Larrick and Bowles, 1995; Zeelenberg, Beattie, et al., 1996; Humphrey, 2004; Zeelenberg, 2020) since counterfactual thinking is primarily triggered after upward counterfactual thinking and negative experiences rather than downward counterfactual thinking and positive experiences (Kahneman and Tversky, 1979; Roese and Olson, 1995; Roese and Olson, 1997), as generally negative information exerts a greater influence on choices than positive information (Beattie et al., 1994) and pains persist longer than joys (Frijda, 1988; Frijda, 2007). For this reason, I only model regret, which is in line with the multiplicative regret theory of Quiggin (1994) which excludes the feeling of rejoicing compared to additive regret.

To ease interpretation, think of the unit of time as a year, so that consumption and foregone consumption are measured annually. Investors might check their portfolio more often than that, but I assume that it is only once a year that investors confront their performance in a serious way.[13] Without loss of generality of results, decision problem (1) can be extended to a multi-period model with a finite or an infinite horizon.

Consumption and foregone consumption

Having described regret utility, we now turn towards realized consumption and foregone consumption. Investors can buy or sell a risky asset with gross return R_{t+1}. We can think of the risky asset as the market portfolio. The representative investor invests a fraction ξ_t of her wealth w_t. e_t denotes the consumption level if the agent decides to invest all of her wealth, which ensures that consumption remains non negative. I exclude short selling such that $0 \leq \xi_t \leq 1$. Then, realized consumption today C_t and realized consumption next year C_{t+1} equal

$$\begin{aligned} C_t &= e_t + w_t \left(1 - \xi_t\right), \\ C_{t+1} &= e_{t+1} + w_t \xi_t R_{t+1}. \end{aligned} \tag{3}$$

Foregone consumption is defined as the largest level of consumption that would have been attainable if another decision would have been made (Bell, 1982; Loomes and Sugden, 1982; Quiggin, 1994). If there is a foregone alternative that yields higher consumption than realized consumption, then regret is felt. In case the foregone alternative equals realized consumption, the investor feels no regret as her decision is the best she could have made ex post.

Thus, regret follows from upward counterfactual thinking using foregone alternatives. Upward counterfactuals follow after a negative experience and take the form of "if only..."

[13] This annual review period is in line with the yearly review periods of Benartzi and Thaler (1995) and Barberis and Huang (2001).

statements (Lin, Huang, et al., 2006). Investors consider a state of the world where they would have been better off in terms of consumption levels, i.e., $X_t \geq C_t$ for all t: "If only I had made another decision, I would have had a higher consumption level". Lin, Huang, et al. (2006) study real investors and they find that investors base their foregone alternatives on two counterfactuals: the inaction alternative and the best unchosen alternative.[14] In my model, investors use these two counterfactuals to determine their foregone consumption levels today X_t and next year X_{t+1}

$$\begin{aligned} X_t &= e_t + w_t, \\ X_{t+1} &= e_{t+1} + w_t \tilde{R}_{t+1}. \end{aligned} \tag{4}$$

First, the foregone consumption level today X_t originates from the inaction alternative. This is the counterfactual thought of not having invested anything in the risky asset, i.e., $\xi_t = 0$. The representative investor considers a state of the world where she would have been better off by just holding her wealth: "if only I did not invest." Second, the foregone consumption level next year X_{t+1} originates from the best unchosen alternative, which is the counterfactual thought of having invested all wealth, $\xi_t = 1$, in the risky asset (i.e., portfolio of assets) with a higher return than realized returns, i.e., $\tilde{R}_{t+1} \geq R$. We can think of $\tilde{R}$ as the foregone return on the risky asset. The representative investor anticipates the experience of a state of the world where she would have been better off by any alternative investment: "if only I made another investment decision." Hence, the foregone consumption level today X_t

[14] The authors also report the expected outcome as a potential reference point. However, as also stated by Lin, Huang, et al. (2006), if an outcome does not match the investor's expectation, then disappointment is felt. Camille et al. (2004) provide neurological evidence that regret is distinct from disappointment, such that disappointment falls outside the scope of the current analysis. Moreover, disappointment differs from the foregone and inaction alternative in the sense that the latter two are evoked after a negative experience only.

follows from the foregone inaction alternative, while the foregone consumption level next year X_{t+1} follows from the foregone best unchosen alternative. So, investors experience utility from realized consumption, but also experience disutility regarding the consumption they could have received, had they made a different decision. The foregone consumption levels are the main mechanism that drive investors' regret.

It is convenient to think of regret as the ratio of the foregone consumption levels, i.e., X_{t+1}/X_t, such that we can interpret regret in terms of foregone returns. Namely, the market-wide foregone return that ex-post higher alternative risky returns would have yielded over not investing. Thus, regret is high when the market-wide ex-post unrealized risky returns could have been high.

Regret

I model regret in line with three observations from the literature. First, at the moment of investment decision making investors anticipate the feeling of regret, as shown in the representative investor's problem (1). Neuroscientific evidence (Camille et al., 2004) and the review of Zeelenberg (2018) show that anticipating future regret influences current decision making under uncertainty. Although regret is only felt when consumption is realized and foregone consumption is known, the investor anticipates and takes into account this emotion when making her investment decision today such that counterfactual thinking influences the investor's holdings today. If we think of the risky return R as the market return, then the anticipation of the market-wide foregone return $\tilde{R}$ directly influences the investors' decisions today and, thereby, the composition and prices of the market today and consequently future to be realized market returns. Anticipation and the feeling of regret are possible, because asset prices are available after any investment decision such that feedback is always received.

Second, emotions, and thereby regret, follow laws. Regret varies over time around a mean, and is persistent, and is not too volatile. The laws of change and habituation (Frijda, 1988; Frijda, 2007) state that regret is time varying and reverts over time to a steady-state mean (Wilson and Gilbert, 2005), but only slowly and gradually. Additionally, the laws of hedonic asymmetry and conservation of emotional momentum (Frijda, 1988; Frijda, 2007) state that regret is persistent, which extends up to years. Especially negative feelings and emotions, to which regret belongs, are persistent phenomena (Coricelli et al., 2005; Hajcak and Olvet, 2008). People have the tendency to overestimate the anticipated intensity and duration of their emotional feelings (Wilson and Gilbert, 2005). Arisoy et al. (2021) find that regret extends up to years by empirically showing that regret-sorted portfolios are highly persistent.[15] The law of care for consequence manifests the presence of emotional control (Frijda, 1988; Frijda, 2007), such that the emotion of regret is not too volatile.

Third, investors are averse to regret. The main premise of regret theory (Bell, 1982; Loomes and Sugden, 1982) is that we are averse to regret. Bleichrodt et al. (2010) introduce the first quantitative measurement of regret aversion, and their evidence confirms regret aversion at the individual and aggregate level. To the best of my knowledge, they are the first and only ones estimating a utility curvature parameter and a regret-aversion parameter based on power utility forms of regret. They estimate a regret-aversion parameter $\kappa = 2$, which implies increasing disutility in the foregone alternative because $\kappa > 1$. $\kappa < 1$ implies decreasing disutility in the foregone alternative, and $\kappa = 1$ corresponds to linear regret. Regret aversion (Bell, 1982; Loomes and Sugden, 1982; Quiggin, 1994), which generates the distinctive predictions of regret theory, implies that regret should enter convex, i.e., $\kappa > 1$.

[15] Using financial market data, Arisoy et al. (2021) show that regret is a highly persistent phenomenon, extending up to years. The authors form portfolios based on a regret measure (i.e., the maximum return in the same industry), and they show that a regret stock remains in the highest regret quintile for five months up to years with a probability up to 60%.

Dynamics

To complete the description of preferences, I specify how they develop over time. First, given the laws of emotions, log regret $x_t \equiv \log\left(\frac{X_{t+1}}{X_t}\right)$ evolves as an AR(1) process

$$x_{t+1} \equiv \log\left(\frac{X_{t+1}}{X_t}\right) = \phi x_t + \mu_x + \varepsilon_{x,t+1}, \tag{5}$$

in which μ_x, σ_x, and $0 < \phi < 1$ are parameters such that regret is stationary and positively autocorrelated. In line with the aforementioned psychological, neuroscientific and experimental evidence, regret evolves as a time-varying and persistent process with coefficient ϕ, gradually and slowly mean reverting around μ_x with a low emotional volatility of regret σ_x. The persistence coefficient ϕ ensures that regret slowly varies over time. Since regret equals the foregone return, we can think of long-run mean of regret, $\mu_x/(1-\phi)$, as the steady premium that the foregone risky return would have offered. Regret is subject to shocks, but emotions behave in a controlled and stable manner such that volatility of regret σ_x is low. Later, I specify the calibration of all these parameters.

Second, we close the model by describing the processes for consumption and dividends. It is convenient to introduce dividend growth already here, since we need dividend growth for computing returns. Following Campbell and Cochrane (1999), I model realized consumption growth and dividend growth as independently and identically distributed (i.i.d.) lognormal processes. Thus,

$$\Delta c_{t+1} \equiv \log\left(\frac{C_{t+1}}{C_t}\right) = \mu_c + \varepsilon_{c,t+1}, \tag{6}$$

in which the parameter μ_c is mean consumption growth and the parameter σ_c denotes the volatility of consumption growth. And dividend growth is white noise

$$\Delta d_{t+1} \equiv \log\left(\frac{D_{t+1}}{D_t}\right) = \mu_d + \varepsilon_{d,t+1}, \tag{7}$$

in which the parameter μ_d is mean dividend growth and the parameter σ_d denotes the volatility of dividend growth.

All three processes have the following correlation structure

$$\begin{bmatrix} \varepsilon_{c,t} \\ \varepsilon_{x,t} \\ \varepsilon_{d,t} \end{bmatrix} \sim N\left(0, \begin{bmatrix} \sigma_c^2 & \sigma_{c,x} & \sigma_{c,d} \\ \sigma_{c,x} & \sigma_x^2 & \sigma_{d,x} \\ \sigma_{c,d} & \sigma_{d,x} & \sigma_d^2 \end{bmatrix}\right) \tag{8}$$

and i.i.d. over time, with covariances $\sigma_{c,d} = \sigma_c \sigma_d \rho_{c,d}$, $\sigma_{c,x} = \sigma_c \sigma_x \rho_{c,x}$ and $\sigma_{d,x} = \sigma_d \sigma_x \rho_{d,x}$. $\rho_{c,d}$, $\rho_{c,x}$ and $\rho_{d,x}$ respectively denote the correlation between consumption and dividends, the correlation between consumption and regret, and the correlation between dividends and regret. The growth rates of consumption and dividends are weakly correlated (Campbell and Cochrane, 1999). Consumption growth and regret (i.e., foregone returns) correlate negatively, and dividend growth and regret correlate negatively as well. Intuitively, when consumption or dividends are low, regret is high as the investor missed out on a good opportunity.[16]

[16]One may wonder if the specified dynamics for regret (5) and consumption growth (6) fulfil the condition $X_t \geq C_t$. A simulation exercise, with 7000 simulations and 500 years of data, shows that $X_t \geq C_t$ holds true for all dates t. The simulation for regret starts in the steady state. Initial values for both consumption and foregone consumption are normalized to one.

B. Equity

I now compute equilibrium equity asset prices and returns to show how regret influences these. As evident from the representative agent's problem (1), regret is independent of the invested fraction ξ since regret follows from the counterfactuals of having invested all wealth differently. Therefore, marginal utility with respect to consumption, its first argument, is

$$u_1(C_t, X_t) = C_t^{-\gamma} X_t^{\kappa}, \tag{9}$$

Intuitively, when realized consumption C_t is low, regret X_t is high due to their negative correlation such that marginal utility of consumption is relatively high as investors find themselves in a bad state of the world.

Taking the first-order conditions in this economy yields the stochastic discount factor

$$M_{t,t+1} = \delta \frac{u_1\left(C_{t+1}, X_{t+1}\right)}{u_1\left(C_t, X_t\right)} = \delta \left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} \left(\frac{X_{t+1}}{X_t}\right)^{\kappa}. \tag{10}$$

It is related to the time-discount factor, innovations in consumption and regret, and the aversion to risk and regret. Since consumption growth and regret are negatively correlated, the stochastic discount factor is more volatile than under standard CRRA utility (i.e., $\kappa = 0$). We can now compute moments of the stochastic discount factor and find equity asset prices. To do so, I follow the approach of Lettau and Wachter (2007) who price zero-coupon equity as long-lived assets in the economy. With this approach, we can also easily price zero-coupon bonds in the economy, as I do later below.

Let $P_{n,t}$ be the value of a dividend paid n periods from now. Absence of arbitrage implies

$$P_{n,t} = \mathbb{E}_t[M_{t,t+n} D_{t+n}], \tag{11}$$

with boundary condition $P_{0,t} = D_t$, because equity maturing today must be worth aggregate dividend. The law of iterated expectations, as in a standard Lucas-tree model, yields the recursion

$$P_{n,t} = \mathbb{E}_t[M_{t,t+1} P_{n-1,t+1}], \quad n \geq 1. \tag{12}$$

Following standard practice, I guess that a solution to the recursion depends on the state variable of regret x_t

$$\frac{P_{n,t}}{D_t} = F_n(x_t), \tag{13}$$

such that, after dividing both sides of (12) by D_t,

$$F_n(x_t) = \mathbb{E}_t \left[M_{t,t+1} F_{n-1}(x_{t+1}) \frac{D_{t+1}}{D_t} \right]. \tag{14}$$

Assuming joint lognormality of consumption growth, dividend growth and regret, we find a closed-form solution for the price-dividend ratio

$$\frac{P_{n,t}}{D_t} = F_n(x_t) = \delta^n e^{a_n + b_n x_t}, \tag{15}$$

with the coefficients recursively defined as

$$
\begin{aligned}
a_n &= a_{n-1} - \gamma\mu_c + \kappa\mu_x + \mu_d + b_{n-1}\mu_x + \frac{1}{2}\left(\gamma^2\sigma_c^2 + (\kappa + b_{n-1})^2\sigma_x^2 + \sigma_d^2\right) \\
&\quad - \gamma(\kappa + b_{n-1})\sigma_c\sigma_x\rho_{c,x} - \gamma\sigma_c\sigma_d\rho_{c,d} + (\kappa + b_{n-1})\sigma_d\sigma_x\rho_{d,x}, \\
b_n &= \kappa\phi + b_{n-1}\phi,
\end{aligned} \tag{16}
$$

and initial conditions $a_0 = b_0 = 0$. The Online Appendix gives a derivation with intermediate steps. Hence, prices relative to dividends depend on the time-discount factor, the time-invariant constants a_n and b_n, and the time-varying, but persistent, state variable of regret. Please notice that the recursive parameter b_n is an increasing function of the regret-aversion parameter κ and the persistence coefficient of regret ϕ.

Returns, predictability and volatility

To find returns, let $R_{n,t+1}$ denote the one-period return on zero-coupon equity that matures in n periods

$$
R_{n,t+1} = \frac{P_{n-1,t+1}}{P_{n,t}} = \frac{F_{n-1}(x_{t+1})}{F_n(x_t)}\frac{D_{t+1}}{D_t}. \tag{17}
$$

The price-dividend ratio on the aggregate stock market is a claim to all future dividends such that the aggregate price-dividend ratio is the sum of the price to aggregate dividend ratios for all n-period claims

$$
\frac{P_t^m}{D_t} = \sum_{n=1}^{\infty}\frac{P_{n,t}}{D_t} = \sum_{n=1}^{\infty}F_n(x_t). \tag{18}
$$

Then, the return on the aggregate stock market equals

$$R_{t+1}^m = \frac{P_{t+1}^m + D_{t+1}}{P_t^m} = \frac{P_{t+1}^m / D_{t+1} + 1}{P_t^m / D_t} \frac{D_{t+1}}{D_t}. \tag{19}$$

For the simulated results later below, I use the aggregate price-dividend ratio and I calculate aggregated results as expected returns, volatility, predictability and other interesting quantities. However, I present intuition for my results based on analyzing zero-coupon equity returns $R_{n,t+1}$. The intuition derived from zero-coupon equity extends to the aggregate market.

Starting with Shiller (1981) and Campbell and Shiller (1988), the literature shows that returns are excessively volatile and predictable using lagged prices scaled by fundamentals, such as dividends. Fama and French (1988) and Poterba and Summers (1988) find that returns have small negative autocorrelations due to slow mean reversion. To illustrate the intuition for predictability, we start by computing the returns $R_{n,t+1}$. Substituting the price-dividend ratio and the processes for dividend growth and regret, we find

$$\begin{aligned} \log(1 + R_{n,t+1}) &= \log\left(\frac{F_{n-1}(x_{t+1})}{F_n(x_t)}\right) + \log\left(\frac{D_{t+1}}{D_t}\right), \\ &= -\log(\delta) + a_{n-1} - a_n + b_{n-1}x_{t+1} - b_n x_t + \mu_d + \varepsilon_{d,t+1}, \\ &= -\log(\delta) + a_{n-1} - a_n + (b_{n-1}\phi - b_n)x_t \\ &+ b_{n-1}\mu_x + b_{n-1}\varepsilon_{x,t+1} + \mu_d + \varepsilon_{d,t+1}, \end{aligned} \tag{20}$$

in which the final equality follows from the process for regret x_{t+1}. Substitution of the recursion for b_n from (16) implies the log one-period holding return on the n-period dividend strip

$$\log(1 + R_{n,t+1}) = -\log(\delta) + a_{n-1} - a_n - \kappa\phi x_t + b_{n-1}\mu_x + b_{n-1}\varepsilon_{x,t+1} + \mu_d + \varepsilon_{d,t+1}, \quad (21)$$

such that log conditional expected returns are given by

$$\begin{aligned} \log \mathbb{E}_t \left[1 + R_{n,t+1}\right] = &-\log(\delta) + a_{n-1} - a_n - \kappa\phi x_t + b_{n-1}\mu_x + \mu_d \\ &+ \frac{1}{2}\left(b_{n-1}^2\sigma_x^2 + \sigma_d^2\right) + b_{n-1}\sigma_d\sigma_x\rho_{d,x}. \end{aligned} \quad (22)$$

Thus, returns are predictable and affected by the investor's regret through counterfactual thinking. If regret x_t is high today, prices relative to dividends are high, see (15), and future returns are low. Intuitively, if regret today is high, then the foregone return on the risky asset is high. The representative investor regrets having invested too little and demands more of the risky asset, which pushes up risky prices today. Consequently, prices relative to fundamentals are high today such that expected future returns must fall. For this reason, the lagged price-dividend ratio predicts future returns.

The emotion of regret also delivers the small negative autocorrelation of returns as observed in the data. High returns today forecast low returns in the future. Note that standard CRRA utility (i.e., $\kappa = 0$) or white noise regret (i.e., $\phi = 0$) yield no predictability or autocorrelation of returns, as returns are not time varying.

Regret also creates excessive volatility of prices relative to dividends and, consequently, high volatility of returns. The unconditional variance of returns (21) equals

$$V\left[\log(1 + R_{n,t+1})\right] = \kappa^2\phi^2\frac{\sigma_x^2}{1-\phi^2} + b_{n-1}^2\sigma_x^2 + 2b_{n-1}\sigma_d\sigma_x\rho_{d,x} + \sigma_d^2. \quad (23)$$

Returns are more volatile than the volatility of the underlying dividends σ_d alone, as the stochastic discount factor is volatile. The volatility in returns arises from the emotional volatility of regret σ_x, which by itself is not too large, but the regret-aversion parameter κ and the persistence coefficient ϕ enlarge the impact of regret volatility through b_{n-1}. Similar to Guo and Wachter (2021), most excess volatility on the market comes from the second term, $b_{n-1}^2\sigma_x^2$, as it is an order of magnitude larger than the first and third terms. Intuitively, the return on the market is best represented for long-maturity equity strips, (i.e., large n) which implies that the recursion coefficient b_{n-1} is large. Returns in the economy are more volatile than standard CRRA utility or when the emotion of regret would be white noise, as in both cases return volatility equals dividend volatility.

Risk-free interest rate and equity premium

The equity-premium risk-free rate puzzle (Mehra and Prescott, 1985; Weil, 1989) is the stylized fact that stock returns are high and the risk-free rate is low, compared to the implications of a standard CRRA model. Empirically, the model needs low mean consumption growth and volatility, unpredictable consumption and dividend growth, matching market volatility, a slowly varying risk-free rate, with low risk aversion and a low subjective discount rate. To provide intuition for these stylized facts, we compute the risk-free rate and equity premium.

A risk-free asset exists in the economy as well, and is assumed to be in zero-net supply. Then, the real one-period risk-free rate is given by the reciprocal of the conditionally expected stochastic discount factor

$$R_{f,t} = 1/\mathbb{E}_t\left[M_{t,t+1}\right]. \tag{24}$$

The risk-free asset is priced with the investor's Euler equation as

$$1 = \mathbb{E}_t \left[\delta \left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} \left(\frac{X_{t+1}}{X_t} \right)^{\kappa} R_{f,t} \right], \tag{25}$$

implying the log risk-free rate

$$\log(1 + R_{f,t}) = -\log\delta + \gamma\mu_c - \kappa(\mu_x + \phi x_t) - \frac{1}{2}\gamma^2\sigma_c^2 - \frac{1}{2}\kappa^2\sigma_x^2 + \gamma\kappa\sigma_c\sigma_x\rho_{c,x}. \tag{26}$$

The risk-free rate is low and stable. The term $-\kappa(\mu_x + \phi x_t)$ reflects an intertemporal substitution effect. Intuitively, when regret x_t is high, the investor feels poor as she missed out on a high alternative return. So, the agent is willing to save more, which drives down the equilibrium interest rate. The risk-free rate is stable, because the volatility of consumption σ_c and regret σ_x are not too large. If regret aversion is absent, then we fall back in the standard class of CRRA utility models and find the classical risk-free rate puzzle.[17]

Theoretically, regret could explain the high mean excess return that we observe empirically in the stock market. The equity premium equals the difference between the log conditional expected one-period return on the n-period dividend strip (22) and the log one-period risk-free rate (26) such that

$$\begin{aligned} \log\mathbb{E}_t\left[(1 + R_{n,t+1})/(1 + R_{f,t})\right] = {} & a_{n-1} - a_n + b_{n-1}\mu_x + \mu_d + \frac{1}{2}\left(b_{n-1}^2\sigma_x^2 + \sigma_d^2\right) \\ & + b_{n-1}\sigma_d\sigma_x\rho_{d,x} - \gamma\mu_c + \kappa\mu_x + \frac{1}{2}\left(\gamma^2\sigma_c^2 + \kappa^2\sigma_x^2\right) \\ & - \gamma\kappa\sigma_c\sigma_x\rho_{c,x}. \end{aligned} \tag{27}$$

[17] Stronger regret aversion implies a lower and more volatile risk-free rate. The intertemporal substitution effect becomes stronger but also the emotional volatility.

Substituting the recursion for a_n from (16) yields

$$\begin{aligned}\log \mathbb{E}_t\left[(1+R_{n,t+1})/(1+R_{f,t})\right] &= \log \mathbb{E}\left[(1+R_{n,t+1})/(1+R_{f,t})\right] \\ &= \gamma\sigma_c\sigma_d\rho_{c,d} - \kappa\sigma_d\sigma_x\rho_{d,x} - \kappa b_{n-1}\sigma_x^2 + \gamma b_{n-1}\sigma_c\sigma_x\rho_{c,x},\end{aligned} \tag{28}$$

which could produce a higher equity premium compared to standard CRRA utility.

The first term represents the standard consumption risk premium as in CRRA utility models, i.e., the risk aversion multiplied by the covariance between consumption and dividends. The second term represents a regret risk premium for regret-averse investors, in line with the finding of Qin (2020). Intuitively, if dividends are low, regret is high such that marginal utility is high as investors do not like such a state of the world. However, as we will see below, the regret risk premium is small in my calibration and, thus, the equity risk premium remains similar to the standard consumption risk premium. The reason is that $\kappa\sigma_d\sigma_x\rho_{d,x}$ influences the volatility of returns (23) and the risk premium with different signs, such that there exists a trade-off. If $\kappa\sigma_d\sigma_x\rho_{d,x}$ is strongly negative, then the volatility of returns is smaller, while the risk premium will be larger. Vice versa, a small negative $\kappa\sigma_d\sigma_x\rho_{d,x}$ produces higher return volatility, but a lower risk premium.

The last two terms reflect a bond risk premium, which is decreasing in the maturity of the claim n, since $\rho_{c,x}$ is negative. Therefore, regret produces a downward sloping term structure of equity risk premiums. Regret-averse investors demand a higher premium on short-term assets than long-term assets. Intuitively, investors are concerned with regret in the short run as they confront their performance annually, such that investors require a higher premium for holding risky assets in order to compensate for potential regret in the short term. If regret aversion is absent $\kappa = 0$, then we find a flat equity term structure.

Observe that the equity risk premium is not time varying as the conditional and unconditional equity premium are equal to each other. The reason is that the state variable of regret x_t enters the risk-free rate and the risky return, such that it cancels in the equity risk premium. Though, introducing time-varying volatility of regret would produce time-varying risk premiums. However, I am unaware of behavioral studies regarding time-varying volatility of emotions, such that I cannot support or oppose this claim.[18]

C. Bonds

Analogous to zero-coupon equity, we can price zero-coupon bonds. Let $P^B_{n,t}$ denote the real price of a real bond maturing in n periods. I highlight the differences with equity by adding an additional B to the expressions for bonds. Bond prices are determined recursively by the investor's Euler equation

$$P^B_{n,t} = \mathbb{E}_t \left[M_{t,t+1} P^B_{n-1,t+1} \right], \quad n \geq 1, \tag{29}$$

in which the stochastic discount factor $M_{t,t+1}$ is the same as before and given by (10). We work with a real stochastic discount factor such that bond prices are in real terms. When $n = 0$, the bond is worth one unit of consumption good, implying the boundary condition $P^B_{0,t} = 1$. Similar to zero-coupon equity, regret x_{t+1} is a state variable and bond prices are an affine function of time-varying regret x_t up to some time-invariant constants. The solution to the recursion of the bond prices takes the form

$$F_n(x_t) = P^B_{n,t}, \tag{30}$$

[18] Likewise, the conditional variance of returns is not time-varying, but an extension of the model with time-varying regret volatility would do so.

such that we can write (29) as

$$F_n(x_t) = \mathbb{E}_t \left[M_{t,t+1} F_{n-1}(x_{t+1}) \right]. \tag{31}$$

Since the processes for consumption growth and regret are jointly lognormal, we find a closed-form analytical solution for the price of an n-period real bond at time t

$$P_{n,t} = F_n(x_t) = \delta^n e^{a_{B,n} + b_{B,n} x_t}, \tag{32}$$

with the coefficients recursively defined as

$$\begin{aligned} a_{B,n} &= a_{B,n-1} - \gamma\mu_c + \kappa\mu_x + b_{B,n-1}\mu_x \\ &\quad + \frac{1}{2}\left(\gamma^2\sigma_c^2 + (\kappa + b_{B,n-1})^2\sigma_x^2\right) - \gamma(\kappa + b_{B,n-1})\sigma_c\sigma_x\rho_{c,x}, \\ b_{B,n} &= \kappa\phi + b_{B,n-1}\phi, \end{aligned} \tag{33}$$

and initial values $a_{B,0} = b_{B,0} = 0$. The Online Appendix gives a derivation with intermediate steps. Similar to zero-coupon equity, notice that the recursive parameter $b_{B,n}$ is an increasing function of the regret-aversion parameter κ and the persistence coefficient of regret ϕ.

Returns and yields

Piazzesi and Schneider (2006) find that real bond yield curves are unconditionally downward sloping in U.K. data, while U.S. data suggests an unconditional upward sloping curve. To illustrate intuition for the real yield curve in the regret model, we want to find bond yields and bond returns. In line with Wachter (2006), I define the real return on an n-period bond as

$$R_{n,t}^{B} = \frac{P_{n-1,t+1}^{B}}{P_{n,t}^{B}}, \tag{34}$$

and the (continuously compounded) yield on the n-period real bond as

$$y_{n,t} = -\frac{1}{n} \log P_{n,t}^{B}. \tag{35}$$

Substituting the bond prices from (32), we find the one-period holding return on the n-period bond

$$\log(1 + R_{n,t+1}^{B}) = -\log \delta + a_{B,n-1} - a_{B,n} - \kappa\phi x_t + b_{B,n-1}\mu_x + b_{B,n-1}\varepsilon_{x,t+1} \tag{36}$$

and the bond yield

$$y_{n,t} = -\log \delta - \frac{a_{B,n}}{n} - \frac{b_{B,n}}{n} x_t, \tag{37}$$

with the yield on an one-period bond as

$$\begin{aligned} y_{1,t} &= -\log \delta - a_{B,1} - b_{B,1} x_t, \\ &= \log(1 + R_{f,t}). \end{aligned} \tag{38}$$

Thus, bond returns and bond yields are functions of regret x_t and the time-invariant recursion coefficients, which depend on the maturity n. As such, bond returns and yields would theoretically be predictable by regret. The one-period yield is identical to the risk-free rate in equation (26).

The unconditional mean yield curve follows from

$$\mathbb{E}\left[y_{n,t}\right] = -\log\delta - \frac{a_{B,n}}{n} - \frac{b_{B,n}}{n}\frac{\mu_x}{1-\phi} \tag{39}$$

and the unconditional mean yield spread equals

$$\mathbb{E}\left[y_{n,t} - y_{1,t}\right] = \left(a_{B,1} - \frac{a_{B,n}}{n}\right) + \left(b_{B,1} - \frac{b_{B,n}}{n}\right)\frac{\mu_x}{1-\phi}. \tag{40}$$

Bond yields and the yield spread are decreasing functions of maturity n, producing an downward sloping real term structure of interest rates. Intuitively, the model's bond risk premium predicts that long-term bonds have a lower risk premium than short-term bonds and, thus, the bond yield curve slopes downward. If the long-run mean of regret $\frac{\mu_x}{1-\phi}$ is large, or regret aversion κ (implicit in $b_{B,n}$) is strong, then the yield curve becomes steeper. In case of CRRA utility (i.e., no regret), the real yield curve is exactly flat as there is no bond risk premium.

D. Cross section

We have seen that regret can explain stylized facts in the time series. In this part, I show that regret also explains two cross sectional features of equity: the value premium (Basu, 1983; Fama and French, 1992) and the De Bondt-Thaler premium (De Bondt and Thaler, 1985), also known as long-term reversal.

Basu (1983) and Fama and French (1992) show that stocks with low price-to-fundamentals ratios (value stocks), such as price-dividend ratios, exhibit significantly higher subsequent returns than stocks with high price-to-fundamentals ratios (growth stocks). A stock's price-dividend ratio predicts the stock's subsequent return with a negative sign. The difference in returns earned by "value" stocks with low price-dividend ratios and "growth" stocks with high price-dividend ratios is known as the value premium.

De Bondt and Thaler (1985) show that a stock's return over the past three to five years (i.e., portfolio formation period) predicts the stock's subsequent return (i.e., portfolio evaluation period) with a negative sign in the cross section. The difference in the future returns earned by losing stocks and winning stocks is known as the De Bondt-Thaler premium. A slight change in interpretation of the model presented above accounts naturally for both stylized facts. The results above rely on the interpretation of regret by counterfactually considering the investor's realized *portfolio* returns with the inaction alternative and the foregone best unchosen alternative (Lin, Huang, et al., 2006). For the cross section, the two counterfactuals remain the same, but we are interested in the behavior of an individual risky asset itself. As such, we study the asset pricing implications if the investor counterfactually compares the return on a chosen individual risky *asset* with the two foregone alternatives of Lin, Huang, et al. (2006).

Consider a market with two risky assets k and l. The representative agent invests in both risky assets. However, rather than being concerned with the foregone return on her portfolio, the representative investor is concerned with the foregone return on each individual asset. That is, the investor holds two separate regret processes for each asset. An interpretation is that the agent holds an individual mental account for each asset. Thus, the investor considers the performance and feedback on stock k and stock l separately.

Consistent with the finding of investors' mental individual stock accounting (Thaler, 1985; Barberis, Huang, and Santos, 2001), agents maximize utility over allocations $\xi_{j,t}$ to individual assets $j = l, k$

$$u(C_t, X_t) \equiv \sum_j u(C_{j,t}, X_{j,t}), \quad \forall t \tag{41}$$

with realized and foregone consumption such that $\xi_{j,t} = [0, 1]$

$$C_{j,t} = e_{j,t} + w_{j,t}(1 - \xi_{j,t}) \quad C_{j,t+1} = e_{j,t+1} + w_{j,t}\xi_{j,t}R_{j,t+1} \tag{42}$$

$$X_{j,t} = e_{j,t} + w_{j,t} \qquad X_{j,t+1} = e_{j,t+1} + w_{j,t}\tilde{R}_{j,t+1} \tag{43}$$

The representative investor's foregone consumption today on asset j, $X_{j,t}$, reflects the inaction alternative (i.e.,"if only I did not invest in asset j"). Foregone consumption next year on asset j, $X_{j,t+1}$, follows from the (anticipated) foregone return $\tilde{R}_{j,t+1} \geq R_{j,t+1}$ on risky asset j.[19]

Thus, rather than a single process for regret, the investor holds two separate processes for regret based on each risky asset j such that log regret evolves as

$$x_{j,t+1} \equiv \log\left(\frac{X_{j,t+1}}{X_t}\right) = \phi x_{j,t} + \mu_x + \varepsilon_{x,j,t+1}, \text{ for } j = \{l, k\}, \tag{44}$$

in which μ_x, σ_x, and $0 < \phi < 1$ are the same parameters as before in equation (5). Intuitively, investors' regret is subject to same laws of emotion (Frijda, 1988; Frijda, 2007) such that regret inhibits the same persistence, mean and volatility, but the innovations to regret are different for each asset j. I assume that regret shocks and dividend shocks are uncorrelated across assets, such that the correlation structure per asset j in equation (8) remains.

Intuition for the value premium and long-term reversal now follows easily. Since asset j

[19] In case the realized return $R_{j,t+1}$ is negative, the representative investor imagines a situation where she would have invested in a risk-free asset.

is subject to the asset-specific process of regret $x_{j,t+1}$, prices relative to dividends for asset j depend on asset-specific regret

$$\frac{P_{j,t}}{D_{j,t}} = \sum_{n=1}^{\infty} F_n(x_{j,t}). \tag{45}$$

Consequently, the log conditional expected one-period return on the n-period dividend strip on stock j equals

$$\begin{aligned} \log \mathbb{E}_t \left[1 + R_{j,n,t+1}\right] = & -\log(\delta) + a_{n-1} - a_n - \kappa\phi x_{j,t} + b_{n-1}\mu_x + \mu_d \\ & + \frac{1}{2}\left(b_{n-1}^2\sigma_x^2 + \sigma_d^2\right) + b_{n-1}\sigma_d\sigma_x\rho_{d,x}. \end{aligned} \tag{46}$$

The return on stock j is identical to the market return in equation (22), however returns now are asset specific through the asset specific regret $x_{j,t}$. Namely, the investor experiences regret on each asset separately rather than on her portfolio. The asset-specific regret drives the cross-sectional findings.

To fix ideas, suppose that regret of holding stock k is higher than regret of holding stock l, i.e., $x_{k,t} > x_{l,t}$. It follows from the price-dividend ratio that stock k has a high price-to-fundamentals ratio, whereas stock l has a low one. Thus, stock k is overpriced relative to stock l. So, stock k identifies as a growth stock and stock l as a value stock. The value premium predicts that stock k has lower subsequent returns, while stock l has higher subsequent returns.

Actually, we can compute the differential expected excess return on the value-minus-growth stock. It equals

$$\log \mathbb{E}_t \left[1 + R_{l,n,t+1}\right] - \log \mathbb{E}_t \left[1 + R_{k,n,t+1}\right] = \kappa\phi(x_{k,t} - x_{l,t}), \tag{47}$$

such that, value stocks, identified as stock l, indeed offer a premium over growth stocks, identified as stock k. The value premium equals the difference in regret multiplied by the regret-aversion parameter κ and the persistence parameter ϕ. Clearly, a value premium is absent in standard CRRA utility models (i.e., $\kappa = 0$) or when regret is white noise (i.e., $\phi = 0$).

Intuitively, if regret $x_{j,t}$ on stock j is high today, then this stock typically generated good returns in the past (a winner stock). The investor regrets having invested too little in stock j and demands more of stock j, pushing up the price-dividend ratio today. As a result, prices compared to fundamentals today are overvalued (a growth stock) such that expected future returns are worse. Overall, this mechanism explains the value premium, i.e., growth stocks earn lower future returns than value stocks. Also, it explains the De Bondt-Thaler premium: winner stocks, formed on past performance, earn lower future returns than loser stocks. One mechanism, namely regret, explains both stylized facts in the cross section, which is plausible since long-term reversal is an alternative proxy for value (Fama and French, 1996; Gerakos and Linnainmaa, 2018).

III. Model evaluation

In this section, I evaluate the implications of the model in magnitudes. I simulate data by drawing the normally distributed shocks for consumption growth, dividend growth and regret. The simulated data for equity and bonds replicates many interesting statistics consistent with the empirical asset pricing literature.

A. Parameter values

Table 2 summarizes the parameter choices for my simulations, which are done at an annual frequency. I simulate 7000 samples, each consisting of 500 years of data from the model, to calculate population values for a variety of statistics. The amount of samples and the length of each sample is chosen such that the population moments are close to the theoretical moments.

I choose the parameters in the model in three ways. First, I choose several parameters to match certain moments as found in the empirical asset pricing literature. The parameters for dividend growth and consumption growth are chosen in line with Barberis and Huang (2001) and Lettau and Wachter (2007). Dividend growth is a white noise process with mean zero and a standard deviation σ_d of 14%, while consumption grows annually with a mean μ_c of 1.84% and a standard deviation of σ_c 3.82%. Correlation between dividend and consumption growth is imperfect (Campbell and Cochrane, 1999) and set to 0.30. The persistence coefficient of regret is set as $\phi = 0.81$ to closely match the serial correlation of the log price-dividend ratio as found in the empirical asset pricing literature (Campbell and Cochrane, 1999).

Second, I choose the preference parameters in line with the experimental literature. Using the estimated value of Bleichrodt et al. (2010), I set the regret-aversion parameter to $\kappa = 2$. This value implies that marginal utility of consumption is increasing in the foregone consumption level. The subjective annual time-discount factor equals $\delta = 0.97$ (Frederick et al., 2002), which yields a very plausible annual discount rate of 3%. I set the risk-aversion parameter γ to 10, which is considered to be plausible by Mehra and Prescott (1985), Bansal and Yaron (2004), and Beeler and Campbell (2012).

Third, I need to calibrate the four parameters in the regret process to match moments as found in the empirical asset pricing literature. The mean μ_x and standard deviation σ_x of regret are chosen such that the risk-free rate is low and stable in the economy, and such that we can interpret the long-run mean of log regret as the foregone premium that risky assets offer over risk-free assets. I set $\mu_x = 0.0125$, such that the long-run mean of regret $\mu_x/(1-\phi)$ equals 6.51%, which yields a plausible interpretation of the premium that foregone risky assets offer. I set $\sigma_x = 0.015$, so emotional volatility of regret is not too large. We have seen that the correlations between consumption and regret must be negative, as well as between dividends and regret. I set $\rho_{c,x} = -0.1$ and $\rho_{d,x} = -0.1$ such that regret produces excess volatility and a downward sloping term structure of equity risk premiums.

Table 2: **Parameter choices and calibration.** This table shows the parameters used in the simulations. The values are based on the asset pricing literature, experimental literature and a calibration. All values are annual.

Parameter	Variable	Value
Asset pricing literature:		
Mean consumption growth	μ_c	1.84%
Standard deviation consumption growth	σ_c	3.82%
Standard deviation dividend growth	σ_d	14%
Correlation consumption growth and dividend growth	$\rho_{c,d}$	0.30
Persistence coefficient	ϕ	0.81
Experimental literature:		
Regret aversion	κ	2
Time-discount factor	δ	0.97
Risk aversion	γ	10
Calibration:		
Mean regret	μ_x	1.25%
Standard deviation regret	σ_x	1.50%
Correlation consumption growth and regret	$\rho_{c,x}$	-0.1
Correlation dividend growth and regret	$\rho_{d,x}$	-0.1

B. Results aggregate market

Table 3 shows the basic moments of the aggregate market from the simulations, namely: the risk-free rate, the equity premium, the market and risk-free rate volatilities, the volatility of the log price-dividend ratio, and the mean and volatility of regret.

Table 3: **Simulated moments for the regret model.** This table reports the means and standard deviations of simulated moments on the aggregate market. R^m denotes the return on the market, R_f the real risk-free rate, $R^m - R_f$ the equity premium, $p^m - d$ the log price-dividend ratio on the market, x regret and $\sigma(.)$ the standard deviation. The model is simulated at an annual frequency. All values are annualized.

Statistic	Model	Statistic	Model	Statistic	Model
$\mathbb{E}[R_f]$	1.02%	$\sigma[R_f]$	4.18%	$\sigma[p^m - d]$	0.22
$\mathbb{E}[R^m]$	2.05%	$\sigma[R^m]$	18.90%	$\mathbb{E}[x]$	6.51%
$\mathbb{E}[R^m - R_f]$	1.03%	$\sigma[R^m - R_f]$	20.04%	$\sigma[x]$	2.57%

The results confirm the intuition from the earlier theoretical analysis. The model produces a low risk-free rate of 1.02%, which is also stable with a volatility of 4.18%. As such, regret solves the risk-free rate puzzle (Weil, 1989) since consumption growth is unpredictable with a realistic mean and low volatility, and the model has low risk aversion with positive time discounting. Notwithstanding these resolved stylized facts, the regret model has difficulty with explaining the equity premium puzzle (Mehra and Prescott, 1985). The equity premium is small, 1.03%, and mostly driven by the standard consumption CAPM. One might argue that the equity premium is low, because I compute the equity premium on unlevered claims. Empirically observed equity returns include firms with leverage and Abel (1999) shows that a correction for levered equity produces a higher equity premium in general equilibrium models.

Regret solves the excess volatility puzzle (Campbell and Shiller, 1988). Risky assets have a return volatility of nearly 19% and the volaility of the equity premium is about 20%, which is larger than the underlying dividend volatility. The volatility of the log price-dividend ratio is 0.22. All reported values of these moments are similar to the values reported in Bansal and Yaron (2004). The simulated mean and volatility of log regret equal their theoretical counterparts given by its AR(1) property, respectively $\mu_x/(1-\phi)$ and $\sqrt{(\sigma_x^2/(1-\phi^2))}$. Regret is not too volatile and the steady-state mean of regret can be interpreted as the foregone return.

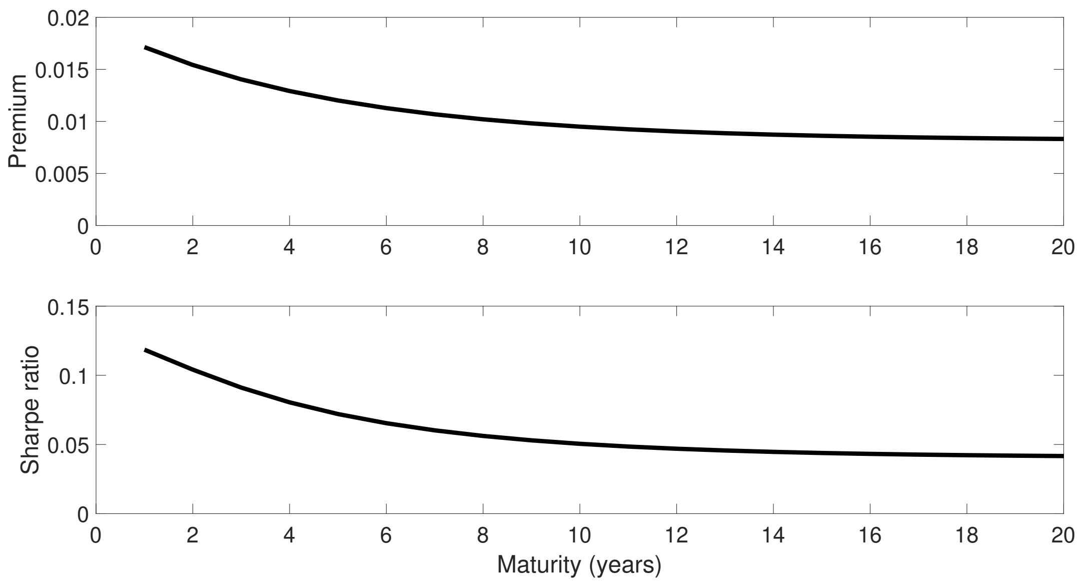


Figure 2: **Term structures of equity risk premiums and Sharpe ratios.** The graph shows the term structures of equity risk premiums and Sharpe ratios implied by the regret model. The graph plots the first 20 years of dividend strips.

Figure 2 shows the term structures of equity risk premiums and Sharpe ratios. The graphs show that the term structures of excess returns and Sharpe ratios are unconditionally downward sloping, consistent with the empirical evidence of (van Binsbergen, Brandt, et al., 2012; van Binsbergen and Koijen, 2017). In terms of shape, the regret model matches this new stylized fact in the asset pricing literature. The earliest dividend strips have an annual average excess return equal to 1.8 percent, while the latest dividend strips earn a annual average excess return of somewhat less than one percent. Note however that Bansal, Miller, et al. (2021) find an unconditionally upward sloping term structure of equity risk premiums.

C. Predictability

Here, I document autocorrelations, long-horizon predictability and cross-sectional predictability, consistent with the empirical asset pricing literature.

Autocorrelations

Table 4 presents autocorrelations for the excess returns, the price-dividend ratio and regret from the simulated data. The model fits the slight negative autocorrelation of returns (Fama and French, 1988; Poterba and Summers, 1988; Guo and Wachter, 2021). This produces mean reversion and time-series predictability: high returns today, predict low returns tomorrow. The price-dividend ratio and regret have identical autocorrelations as the price-dividend ratio is a function of regret and the persistence coefficient ϕ is chosen to generate the first-order annual autocorrelation of the log price-dividend ratio, which is close to the value of 0.78 as found in the data by Campbell and Cochrane (1999).

Long-horizon regressions and the cross section

Table 5 documents time-series and cross-sectional predictability. Panel A reports regressions of lagged price-dividend ratios on future returns, consumption and dividends. The column with 'Returns' replicates our earlier intuition that the price-dividend ratio predicts subsequent returns with a negative sign. R^2 statistics are increasing in the horizon and range between 5% to 16%. The coefficients and R^2 are very similar to the reported values

Table 4: **Autocorrelations of simulated data.** This table reports for several yearly lags the autocorrelations of the return on the market R^m, the equity premium $R^m - R_f$, the log price-dividend ratio on the market $p^m - d$, and regret x. The model values are based on time-aggregated annual values with a yearly simulation interval.

	Lag (years)				
Variable	1	2	3	5	7
R^m	-0.04	-0.03	-0.03	-0.02	-0.01
$R^m - R_f$	-0.02	-0.01	-0.01	-0.01	-0.01
$p^m - d$	0.80	0.64	0.51	0.33	0.20
x	0.80	0.64	0.51	0.33	0.20

by Campbell and Cochrane (1999) and Guo and Wachter (2021). Columns 'Consumption' and 'Dividend' show, in contrast, that consumption growth and dividend growth are unpredictable, because they are simply white noise in the economy. Cochrane (2008), Beeler and Campbell (2012), and Cochrane (2017) argue that dividend growth and consumption growth are in fact unpredictable. The unpredictability is a necessary ingredient for a successful explanation of the equity-premium risk-free rate puzzle (Cochrane, 2017).

Panel B confirms our earlier analysis of the cross section. Each year, the two stocks in the economy are sorted based on their price-dividend ratio, and the returns of both stocks over the next year are measured. The value premium is the time-series mean of the difference between the returns. Regret produces a value premium of 4.73% with a standard deviation of the value-minus-growth portfolio of 23.66 percentage points. The magnitude of the value premium and its standard deviation are similar to the values of 5.42% and 20.39 percentage points as observed in the data by Guo and Wachter (2021). Additionally, the value premium is highly persistent in the model. On average, a growth (value) stock remains overpriced (underpriced) up to five years after portfolio formation, consistent with the finding of van Binsbergen, Boons, et al. (2021). As such, the 'mispricing' resolves gradually. Stated

Table 5: **Time-series and cross-sectional predictability.** This table reports predictability results for the time series and the cross section. Panel A reports predictive coefficients and R^2-statistics from annual long-horizon regressions of cumulative log risky returns, consumption growth and dividend growth on the log price-dividend ratio: $\sum_{j=1}^{H} \log(1 + R^m_{t+j}) = \beta_0 + \beta_1(p^m_t - d_t) + \varepsilon_{t+H}$, $\sum_{j=1}^{H} \Delta c_{t+j} = \beta_0 + \beta_1(p^m_t - d_t) + \varepsilon_{t+H}$, $\sum_{j=1}^{H} \Delta d_{t+j} = \beta_0 + \beta_1(p^m_t - d_t) + \varepsilon_{t+H}$. Panel B reports the mean annual value premium and its standard deviation (in percentage points). Panel C reports the mean annual De Bondt-Thaler premium for winner-loser formation periods n and for winner-loser evaluation periods N.

Panel A: Time series						
	Returns		Consumption		Dividend	
Horizon (years)	Coefficient	R^2	Coefficient	R^2	Coefficient	R^2
1	-0.20	0.05	0.00	0.00	0.00	0.00
2	-0.36	0.09	0.00	0.00	0.00	0.00
3	-0.49	0.12	0.00	0.01	0.00	0.01
5	-0.67	0.15	0.00	0.01	0.00	0.01
7	-0.80	0.16	0.00	0.01	0.00	0.01
Panel B: Price-to-fundamentals ratio						
Value premium	4.73%					
Standard deviation	23.66 pp					
Panel C: Long-term reversal						
Formation period n	3 years	5 years	5 years			
Evaluation period N	3 years	3 years	5 years			
De Bondt-Thaler premium	4.48%	5.01%	7.25%			

differently, a stock on average remains a low (i.e., value) or high (i.e., growth) regret stock for five years, in line with the observation from Arisoy et al. (2021) that regret on stocks is persistent.

In Panel C we see that regret produces a De Bondt-Thaler premium between 4.48% to 7.25%, depending on the portfolio formation and evaluation periods. The portfolio formation period sorts stocks in winners and losers, and the evaluation period tracks their subsequent returns. Every n years (formation period), the two stocks in the economy are sorted based on their n-year cumulative prior return, and the returns of both the winner and loser stocks over the next N years (evaluation period) are measured. The De Bondt-Thaler premium is the time-series mean of the difference between the returns of the loser and winner stocks over all non-overlapping periods. The premiums are smaller than observed by De Bondt and Thaler (1985), but do indicate that regret creates long-term reversal in asset prices.[20]

D. Bonds

Table 6 shows the implications of the model for means and standard deviations of real bond yields and returns. The maturities demonstrate that the average unconditional yield curve on real bonds is downward sloping, which is consistent with the empirical findings of Piazzesi and Schneider (2006) on U.K. indexed bonds. However, U.S. data on indexed bonds suggests that the yield curve is unconditionally upward sloping, but the data series is very small. The magnitude of the mean yields on the one-year to five-years real bonds is similar to the model's average real yields of Piazzesi and Schneider (2006). Volatilities on real yields are decreasing in maturity. Short-term yields are more volatile than long-term yields, a finding which Piazzesi and Schneider (2006) empirically support with data.

The regret model implies that bond returns decrease in maturity, which contradicts the empirical asset pricing literature, while their volatilities rise with maturity, consistent with the empirical findings. The one-year bond return in logs behaves identical to the one-year bond yield, i.e., the risk-free rate in the economy, by construction. For this reason, the one-year bond return volatility appears high. The other maturities show that long-term bonds have higher volatility than short-term bonds.

[20]In the current exposition, regret cannot explain momentum. As regret is concerned with long-term persistence, it does not mean revert quickly enough in the short run to create momentum. Capturing both long-term reversal and momentum is a long-standing challenge (Barberis, 2018).

Table 6: **Moments for real bond yields and bond returns.** This table reports the means and standard deviations for yields and returns on real zero-coupon bonds (i.e., bonds that pay off in units of aggregate consumption) in the simulated model for an annual holding period. Yields and returns are in annual percentages. Maturity is in years.

Bond yields	Mean	St. dev.	Bond returns	Mean	St. dev.
1 Year	0.96	4.12	1 Year	1.05	4.17
2 "	0.85	3.73	2 "	0.82	3.36
3 "	0.74	3.39	3 "	0.64	3.90
4 "	0.63	3.09	4 "	0.49	4.91
5 "	0.53	2.83	5 "	0.37	5.92
6 "	0.44	2.59	6 "	0.27	6.81
7 "	0.35	2.39	7 "	0.19	7.56
8 "	0.27	2.21	8 "	0.13	8.17
9 "	0.19	2.05	9 "	0.08	8.68
10 "	0.13	1.91	10 "	0.03	9.10
15 "	-0.13	1.39	15 "	-0.08	10.27
20 "	-0.29	1.07	20 "	-0.12	10.68

IV. Empirical findings

In this section, I propose an empirical measure of regret based on stock returns. Consequently, I empirically show that the behavior of regret is in line with the model's predictions. The section concludes with the statistical properties of regret, which shows that regret-averse investors are concerned with positively skewed returns.

Testable predictions

Empirically, I focus on the following three main testable predictions regarding aggregate stock returns. First, regret is a persistent phenomenon. Second, the price-dividend ratio is a function of regret. High (anticipated) regret today implies high prices relative to dividends today. Third, (anticipated) regret today predicts future returns with a negative sign in the time series. High (anticipated) regret today predicts lower future returns.

My model also predicts a value premium in the cross section when considering regret. Stocks with high (low) regret are growth (value) stocks. Arisoy et al. (2021) extensively study empirically the behavior of regret sorted portfolios in the cross section. Besides their finding that regret is a persistent phenomenon, they also find that stocks with low (high) regret have low (high) price-dividend ratios, statistically significant at any reasonable level.[21] Thus, low regret stocks are value stocks and high regret stocks are growth stocks. This finding precisely matches my regret model's prediction in the cross section.

A. Regret measure

The basic premise of regret utility is that high regret corresponds to a high foregone alternative, which equals the maximum return that an investor could have achieved by an alternative foregone investment. I propose a regret measure based on counterfactual thoughts that approximate investors' regret as

$$REG_t^a = \max_j \left(R_{j,t} \right). \tag{48}$$

Intuitively, in line with the definition of regret in the theoretical model, REG_t^a is the annual foregone return during year t. The measure resembles the market-wide regret investors experience during year t. When REG_t is high, the foregone return is high such that regret is high, which decreases investor's utility in (2).

Specifically, $R_{j,t}$ is the return during year t for stock j trading on the market. The representative agent considers all stocks j on the market as counterfactuals and compares her realized market portfolio return to the maximum return that could have been attained

[21] See Table 2, which displays the average stock characteristics of regret portfolios.

across these counterfactuals over the same period, which creates the measure REG_t. My proposed market-wide regret measure is in line with the individual-stock regret measure of Arisoy et al. (2021), who posit a regret measure for individual stocks by taking the maximum return over all stocks in the same industry. As shown later, regret relates to skewness, so my empirical regret measure also relates to the cross-sectional measures of Bali et al. (2011) and Lin and Liu (2018) who proxy expected skewness by the maximum return of a stock within a month.

As a robustness check, I also consider two other specifications of the regret measure. First, regret defined as the maximum foregone monthly return within a year, which I measure as

$$REG_t^m = \max_j \left(R_{j,t,m}\right) \text{ for each } t = 1, ..., T. \tag{49}$$

Here, $R_{j,t,m}$ equals the return of stock j during month m within year t. Second, regret defined as annual regret by averaging all monthly maximum foregone returns within year t, which I measure as

$$REG_t^{\bar{a}} = \frac{1}{m} \sum_{m=1}^{12} REG_{t,m} \quad \text{with} \quad REG_{t,m} = \max_j \left(R_{j,t,m}\right). \tag{50}$$

Here, the representative agent forms annual regret during year t by averaging all maximum foregone monthly returns within a year. To ensure comparability with model's regret x_{t+1}, I consider the log of the regret measures. Intuitively, x_{t+1} measures the foregone return in logs and $\log(REG_t)$ does likewise.

B. Data

To construct the regret measures, I download stock prices. Individual monthly stock prices come from the Center for Research in Security Prices (CRSP). I take all individual stocks that trade on the New York Stock Exchange (NYSE), American Stock Exchange (AMEX), and NASDAQ with share codes 10 and 11. I exclude stocks with share prices less than $1 and more than $1,000, such that the results are not driven by small and illiquid stocks. All these individual stocks arguably form the market.

To test the model's predictions, I download annual variables from Professor Shiller's website. The main interest is in the real price-dividend ratio and the market return, represented by the S&P500. I consider a postwar sample from 1947 to 2012, and a long sample from 1926 to 2012.[22]

C. Results

1. Persistence

First, I show that regret is a persistent phenomenon. Figure 3 presents the history of the log real price-dividend ratio in the postwar sample with the annual log regret measure $\log(REG_t^a)$. The graphs indicate that regret is a time-varying phenomenon, but gradually and slowly moving. In fact, regret is highly persistent.

Table 7 presents the first-order autocorrelation coefficients for the three regret measures, in the postwar and long samples. In the postwar sample the serial correlation coefficients of regret are somewhat higher than in the long sample. Overall, the persistence coefficients range from 0.68 to 0.92. So, this is in line with the dynamics of regret as prescribed by the

[22] The sample stops at 2012 as the annual dataset of Professor Shiller's website ends in 2012.

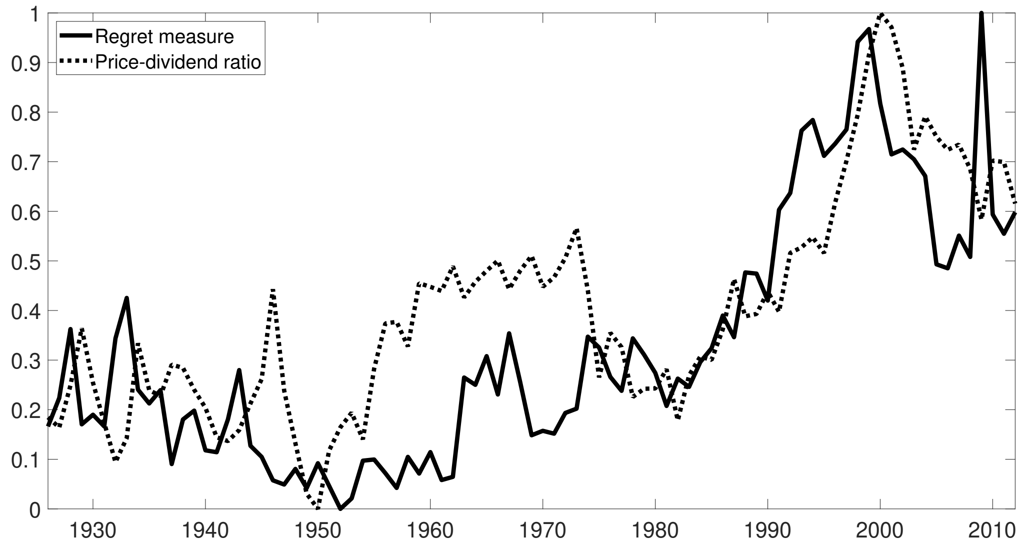


Figure 3: **Historical price-dividend ratio and regret.** The graph shows the normalized time-series of the real log price-dividend ratio and log regret measure $REG_t^{\bar{a}}$, which measures annual regret by averaging all monthly maximum foregone returns within a year. The values are based on annual data in the long sample (1926-2012).

laws of emotions (Frijda, 1988; Frijda, 2007) as found in the psychological, experimental and neuroscientific literature.

Table 7: **Autocorrelations of regret measures.** This table reports first-order autocorrelation, AR(1), coefficients of the regret measures in the postwar sample (1947-2012) and long sample (1926-2012). The values are based on annual data.

	$\log(REG_t^a)$	$\log(REG_t^m)$	$\log(REG_t^{\bar{a}})$
Postwar sample	0.73	0.81	0.92
Long sample	0.68	0.75	0.91

2. Price-dividend ratio

Second, the price-dividend ratio is a function of regret. The behavior of regret is similar to the behavior of the price-dividend ratio. Especially from the 1970s onward. The regret model predicts that high regret today implies high prices relative to dividends today. Table 8 confirms our eyeballing from Figure **??** and shows the regression results of regressing the price-dividend ratio on regret. In both samples and for all regret measures, the coefficients on regret are positive and highly statistically significant.[23] The results imply that high regret today yields high prices relative to dividends today. The R^2 in the postwar sample range from 42% to 56%. The relation between regret and prices relative to dividends is somewhat stronger in the postwar sample than in the long sample.

Table 8: **Price-dividend ratio and regret.** This table reports OLS regression of the real price-dividend ratio on the regret measure i: $\log(P_t/D_t) = a + b\log(REG_t^i) + \varepsilon_t$. Newey-West standard errors $\sigma(b)$ to correct for heteroskedasticity and serial correlation. The values are based on annual data.

	Postwar sample			Long sample		
Measure	Coefficient	$\sigma(b)$	R^2	Coefficient	$\sigma(b)$	R^2
$\log(REG_t^a)$	0.41	0.09	0.42	0.37	0.09	0.31
$\log(REG_t^m)$	0.65	0.13	0.48	0.61	0.13	0.38
$\log(REG_t^{\bar{a}})$	1.13	0.19	0.56	1.18	0.18	0.54

3. Return predictability

The third prediction to test is that high regret today implies lower future aggregate stock returns. Table 9, in a similar spirit to the earlier long-horizon regressions, regresses future returns on lagged regret. The main takeaway of the regression results is that the point estimates for the coefficients on regret are negative: high regret today predicts lower future returns. The relation between regret and future returns becomes less statistically insignificant when the horizon increases. At a seven year horizon, the R^2 ranges from 5% to 8% in the postwar sample.

[23]Throughout this section, we calculate standard errors using the Newey-West procedure with a lag of $T^{1/4}$, where T equals the number of observations.

Table 9: **Future returns and regret.** This table reports predictive coefficients, standard errors and R^2-statistics from long-horizon regressions of cumulative log real risky returns on regret measure i: $\sum_{j=1}^{H}\log(R^m_{t+j}) = a + b\log(REG^i_t) + \varepsilon_{t+H}$. Newey-West standard errors $\sigma(b)$ to correct for heteroskedasticity and serial correlation. The values are based on annual data.

		Postwar sample			Long sample		
Measure	Horizon (years)	Coefficient	$\sigma(b)$	R^2	Coefficient	$\sigma(b)$	R^2
$\log(REG^a_t)$	1	-0.02	0.03	0.01	0.02	0.03	0.01
"	2	-0.05	0.04	0.02	0.02	0.05	0.00
"	3	-0.09	0.06	0.05	0.00	0.07	0.00
"	5	-0.14	0.09	0.06	-0.06	0.09	0.01
"	7	-0.16	0.10	0.06	-0.11	0.09	0.03
$\log(REG^m_t)$	1	-0.03	0.06	0.01	-0.01	0.04	0.00
"	2	-0.07	0.09	0.02	-0.04	0.08	0.00
"	3	-0.08	0.12	0.02	-0.05	0.12	0.00
"	5	-0.13	0.18	0.03	-0.11	0.17	0.02
"	7	-0.22	0.19	0.05	-0.21	0.17	0.05
$\log(REG^{\bar{a}}_t)$	1	-0.04	0.07	0.01	-0.02	0.07	0.00
"	2	-0.12	0.14	0.02	-0.06	0.14	0.00
"	3	-0.17	0.20	0.03	-0.09	0.19	0.00
"	5	-0.30	0.28	0.05	-0.22	0.27	0.02
"	7	-0.43	0.29	0.08	-0.35	0.27	0.05

Overall, this section confirms empirically the three main regret model's predictions: (i) regret is persistent, (ii) prices relative to dividends are a function of regret, and (iii) regret predicts future returns in the time series with a negative with an economically meaningful R^2.

Properties of regret

One may wonder which properties of returns drive regret-averse investors. To provide intuition, this section presents a simple theoretical exercise.

Assume that stock returns are i.i.d. lognormally distributed in the cross section, such that each individual stock return is distributed as $R_i \sim \log N(\mu, \sigma^2)$ in which R_i is the gross return on stock $i = 1, ..., n$. Using the standard properties of a lognormal distribution, the mean and variance of the lognormally distributed returns depend on the parameters μ and σ, while the skewness of the returns depends on the parameter σ only.

Define cross-sectional regret, in line with the earlier measures, as

$$REG = \max_i (R_i) . \tag{51}$$

Then, the CDF $F_{REG}(x)$ of the random variable REG equals

$$F_{REG}(x) = \Phi\left(\frac{\log(x) - \mu}{\sigma}\right)^n , \tag{52}$$

where Φ denotes the CDF of the standard normal distribution. The PDF $f_{REG}(x)$ is

$$f_{REG}(x) = \frac{dF_{REG}}{dx}(x). \tag{53}$$

Consequently, we can compute the first moment of the function $f_{REG}(x)$ by

$$m_1 = \int_{-\infty}^{\infty} x f_{REG}(x) dx, \tag{54}$$

which is the mean, or expected value, of regret. We can find the variance and skewness of regret by computing the *centralized* moments $n = 2, 3$ respectively:

$$m_n = \int_{-\infty}^{\infty} (x - m_1)^n f_{REG}(x) dx. \tag{55}$$

We find that mean regret m_1 is high when the skewness of the underlying returns is positive and high, or when the volatility of the underlying returns is high.[24] Namely, the behavior of regret is dominated by the parameter σ, i.e., the skewness of the returns, while the parameter μ has a small effect on regret. Intuitively, a high cross-sectional dispersion in returns leads to a higher probability of a foregone missed return. Regret itself is substantially less positively skewed than the underlying returns, i.e., the third centralized moment m_3 is small. So, regret-averse investors are mainly concerned about the positive skewness of the returns, which is in line with the findings of Eeckhoudt et al. (2007) and Gollier (2018) who report that regret-averse agents have a preference for positively skewed risks and longshots.

VI. Conclusion

I explore the idea whether regret can provide a unified explanation for the behavior of asset prices. The central ingredient is regret and the aversion to it, added to an otherwise standard asset pricing model. Regret is based on the intuition that an investor is concerned not only about the outcome she receives, but also about the outcome she could have received, had she invested differently. The central finding is that regret aversion has the potential to explain asset pricing stylized facts in the time series and in the cross section. I provide evidence for the main model's predictions by using an empirical measure of regret, especially that prices relative to dividends are a function of persistent regret.

Three features of the regret model are unique, compared to leading asset pricing models. First, one simple ingredient has the ability to explain several stylized facts in a unifying way. In the current setup, regret cannot yet explain the equity premium puzzle, due to the calibration trade-off between the regret risk premium and excess volatility. Since I am

[24] I used simulations with $n = 1000$ individual stocks and parameter choices for individual stock behaviour with mean $\mu = \{0.01, 0.1\}$ and standard deviation $\sigma = \{0.04, 0.4\}$.

unaware of behavioral studies regarding time-varying volatility of emotions, the regret model does not make any predictions about time-varying risk premiums. Second, regret produces a downward sloping term structure of equity risk premiums, whereas most other models predict the opposite. Third, regret-averse investors produce a value premium and long-term reversal in the cross section. Regret-sorted stocks are persistent and the value premium is persistent, consistent with the empirical findings of Arisoy et al. (2021) and van Binsbergen, Boons, et al. (2021).

A potential avenue for further research is the link between institutional investors, regret, benchmarks and asset prices. Regret, and their counterpart of rejoicing, could potentially be linked to the investment industry as institutional investors typically try to follow an investment benchmark. If they do not achieve their benchmark, then investors could experience regret as they could have made an alternative investment decision. Finally, psychologists interested in finance might be inspired to study the time-varying volatility of regret.

Appendix

Table 10: **Properties multiplicative regret-utility function** $u(c, x)$**.** u_1 (u_2) denotes the partial derivative with respect to its first (second) argument. The same rule applies to u_{11}, u_{12} and so on.

Property	
P1a Utility is increasing (states without regret)	$u_1(c, c) + u_2(c, c) \geq 0$
P1b Utility is concave (states without regret)	$u_{11}(c, c) + u_{12}(c, c) + u_{21}(c, c) + u_{22}(c, c) \leq 0$
P2a Utility increases with consumption	$u_1(c, x) \geq 0$
P2b Utility decreases with regret	$u_2(c, x) \leq 0$
P2c Utility is globally increasing	$u_1(c, x) + u_2(c, x) \geq 0$
P3 Utility is sensitive to regret	$u_{12}(c, x) = u_{21}(c, x) \geq 0$
P4a Utility exhibits diminishing marginal consumption utility	$u_{11}(c, x) \leq 0$
P4b Utility exhibits aversion to the foregone alternative	$u_{22}(c, x) \leq 0$

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Online Appendix

Zero-coupon equity

This section derives the prices for zero-coupon equity in the economy along the lines of Lettau and Wachter (2007).

Time t, $n = 1$

Assuming joint log-normality of consumption growth, regret and dividend growth it follows that

$$\begin{aligned} \frac{P_{1,t}}{D_t} &= \mathbb{E}_t \left[M_{t,t+1} \frac{D_{t+1}}{D_t} \right] \\ &= \mathbb{E}_t \left[\delta \left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} X_{t+1}^{\kappa} \frac{D_{t+1}}{D_t} \right] \\ &= \mathbb{E}_t \left[\delta e^{-\gamma(\mu_c + \varepsilon_{c,t+1}) + \kappa(\phi x_t + \mu_x + \varepsilon_{x,t+1}) + \mu_d + \varepsilon_{d,t+1}} \right] \end{aligned} \tag{56}$$

We can write this as

$$e^{w_{t,1} + x\varepsilon_{c,t+1} + y_1 \varepsilon_{x,t+1} + z\varepsilon_{d,t+1}} \tag{57}$$

with (please note the difference between the time-independent parameter x with time-varying regret x_t)

$$\begin{aligned} w_{t,1} &= -\gamma\mu_c + \kappa(\phi x_t + \mu_x) + \mu_d \\ &\equiv w_{1,1} + w_{1,2}x_t \\ w_{1,1} &= -\gamma\mu_c + \kappa\mu_x + \mu_d \\ w_{1,2} &= \kappa\phi \end{aligned} \tag{58}$$

$$x = -\gamma\sigma_c \tag{59}$$

$$y_1 = \kappa\sigma_x + y_0, \quad y_0 = 0 \tag{60}$$

$$z = \sigma_d \tag{61}$$

Compute conditional expectation

$$\begin{aligned} \frac{P_{1,t}}{D_t} &= \delta e^{-\gamma\mu_c + \kappa(\phi x_t + \mu_x) + \mu_d + \frac{1}{2}\gamma^2\sigma_c^2 + \frac{1}{2}\kappa^2\sigma_x^2 + \frac{1}{2}\sigma_d^2} \\ &\cdot e^{-\gamma\kappa\sigma_c\sigma_x\rho_{c,x} - \gamma\sigma_c\sigma_d\rho_{c,d} + \kappa\sigma_x\sigma_d\rho_{x,d}} \end{aligned} \tag{62}$$

We can write this as

$$\frac{P_{1,t}}{D_t} = F_1(x_t) = \delta e^{a_1 + b_1 x_t} \tag{63}$$

with

$$\begin{aligned} a_1 &= -\gamma\mu_c + \kappa\mu_x + \mu_d + \frac{1}{2}\gamma^2\sigma_c^2 + \frac{1}{2}\kappa^2\sigma_x^2 + \frac{1}{2}\sigma_d^2 \\ &- \gamma\kappa\sigma_c\sigma_x\rho_{c,x} - \gamma\sigma_c\sigma_d\rho_{c,d} + \kappa\sigma_x\sigma_d\rho_{x,d} \\ &\equiv w_{1,1} + \frac{1}{2}(x^2 + y_1^2 + z^2) + xy_1\rho_{c,x} + xz\rho_{c,d} + y_1 z\rho_{x,d} \end{aligned} \tag{64}$$

$$\begin{aligned} b_1 &= \kappa\phi \\ &\equiv w_{1,2} \end{aligned} \tag{65}$$

Time $t+1$, $n = 1$

$$\frac{P_{1,t+1}}{D_{t+1}} = F_1(x_{t+1}) = \delta e^{a_1 + b_1 x_{t+1}} = \delta e^{a_1 + b_1(\phi x_t + \mu_x + \varepsilon_{x,t+1})} \tag{66}$$

Time t, $n = 2$

$$\begin{aligned} \frac{P_{2,t}}{D_t} &= F_2(x_t) = \mathbb{E}_t\left[M_{t,t+1}\frac{P_{1,t+1}}{D_{t+1}}\frac{D_{t+1}}{D_t}\right] \\ &= \mathbb{E}_t\left[\delta^2 e^{-\gamma(\mu_c+\varepsilon_{c,t+1})+\kappa(\phi x_t+\mu_x+\varepsilon_{x,t+1})+\mu_d+\varepsilon_{d,t+1}+a_1+b_1(\phi x_t+\mu_x+\varepsilon_{x,t+1})}\right] \end{aligned} \tag{67}$$

We can write this as

$$e^{w_{t,2}+x\varepsilon_{c,t+1}+y_2\varepsilon_{x,t+1}+z\varepsilon_{d,t+1}} \tag{68}$$

with

$$
\begin{aligned}
w_{t,2} &= -\gamma\mu_c + \kappa(\phi x_t + \mu_x) + \mu_d + a_1 + b_1(\phi x_t + \mu_x) \\
&\equiv w_{2,1} + w_{2,2} x_t \\
w_{2,1} &= -\gamma\mu_c + \kappa\mu_x + \mu_d + a_1 + b_1\mu_x \\
w_{2,2} &= \kappa\phi + b_1\phi = (\kappa + b_1)\phi
\end{aligned} \tag{69}
$$

$$
x = -\gamma\sigma_c \tag{70}
$$

$$
y_2 = \kappa\sigma_x + b_1\sigma_x = (\kappa + b_1)\sigma_x \tag{71}
$$

$$
z = \sigma_d \tag{72}
$$

Compute conditional expectation

$$
\begin{aligned}
\frac{P_{2,t}}{D_t} &= \delta^2 e^{-\gamma\mu_c + \kappa(\phi x_t + \mu_x) + \mu_d + a_1 + b_1(\phi x_t + \mu_x) + \frac{1}{2}\gamma^2\sigma_c^2 + \frac{1}{2}(\kappa + b_1)^2\sigma_x^2 + \frac{1}{2}\sigma_d^2} \\
&\cdot e^{-\gamma(\kappa + b_1)\sigma_c\sigma_x\rho_{c,x} - \gamma\sigma_c\sigma_d\rho_{c,d} + (\kappa + b_1)\sigma_x\sigma_d\rho_{x,d}}
\end{aligned} \tag{73}
$$

We can write this as

$$
\frac{P_{2,t}}{D_t} = F_2(x_t) = \delta^2 e^{a_2 + b_2 x_t} \tag{74}
$$

with

$$
\begin{aligned}
a_2 &= -\gamma\mu_c + \kappa\mu_x + \mu_d + a_1 + b_1\mu_x + \frac{1}{2}\gamma^2\sigma_c^2 + \frac{1}{2}(\kappa + b_1)^2\sigma_x^2 + \frac{1}{2}\sigma_d^2 \\
&\quad - \gamma(\kappa + b_1)\sigma_c\sigma_x\rho_{c,x} - \gamma\sigma_c\sigma_d\rho_{c,d} + (\kappa + b_1)\sigma_x\sigma_d\rho_{x,d} \\
&\equiv w_{2,1} + \frac{1}{2}(x^2 + y_2^2 + z^2) + xy_2\rho_{c,x} + xz\rho_{c,d} + y_2z\rho_{x,d}
\end{aligned} \tag{75}
$$

$$
\begin{aligned}
b_2 &= \kappa\phi + b_1\phi \\
&\equiv w_{2,2}
\end{aligned} \tag{76}
$$

General t, n

$$
\frac{P_{n,t}}{D_t} = F_n(x_t) = \delta^n e^{a_n + b_n x_t}, \quad a_0 = b_0 = 0 \tag{77}
$$

with

$$
\begin{aligned}
w_{t,n} &= -\gamma\mu_c + \kappa(\phi x_t + \mu_x) + \mu_d + a_{n-1} + b_{n-1}(\phi x_t + \mu_x) \\
&\equiv w_{n,1} + w_{n,2}x_t \\
w_{n,1} &= -\gamma\mu_c + \kappa\mu_x + \mu_d + a_{n-1} + b_{n-1}\mu_x \\
w_{n,2} &= \kappa\phi + b_{n-1}\phi = (\kappa + b_{n-1})\phi
\end{aligned} \tag{78}
$$

$$
x = -\gamma\sigma_c \tag{79}
$$

$$
y_n = \kappa\sigma_x + b_{n-1}\sigma_x = (\kappa + b_{n-1})\sigma_x \tag{80}
$$

$$
z = \sigma_d \tag{81}
$$

$$
a_n = w_{n,1} + \frac{1}{2}(x^2 + y_n^2 + z^2) + xy_n\rho_{c,x} + xz\rho_{c,d} + y_nz\rho_{x,d} \tag{82}
$$

$$
b_n = w_{n,2} \tag{83}
$$

Thus, recursively

$$
\begin{aligned}
a_n &= -\gamma\mu_c + \kappa\mu_x + \mu_d + a_{n-1} + b_{n-1}\mu_x + \frac{1}{2}(x^2 + y_n^2 + z^2) + xy_n\rho_{c,x} + xz\rho_{c,d} + y_n z\rho_{x,d} \\
&= -\gamma\mu_c + \kappa\mu_x + \mu_d + a_{n-1} + b_{n-1}\mu_x + \frac{1}{2}(\gamma^2\sigma_c^2 + (\kappa + b_{n-1})^2\sigma_x^2 + \sigma_d^2) \\
&- \gamma(\kappa + b_{n-1})\sigma_c\sigma_x\rho_{c,x} - \gamma\sigma_c\sigma_d\rho_{c,d} + (\kappa + b_{n-1})\sigma_x\sigma_d\rho_{x,d}
\end{aligned}
\tag{84}
$$

$$
\begin{aligned}
a_{n-1} - a_n &= \gamma\mu_c - \kappa\mu_x - \mu_d - b_{n-1}\mu_x - \frac{1}{2}(x^2 + y_n^2 + z^2) - xy_n\rho_{c,x} - xz\rho_{c,d} - y_n z\rho_{x,d} \\
&= \gamma\mu_c - \kappa\mu_x - \mu_d - b_{n-1}\mu_x - \frac{1}{2}\gamma^2\sigma_c^2 - \frac{1}{2}(\kappa + b_{n-1})^2\sigma_x^2 - \frac{1}{2}\sigma_d^2 \\
&+ \gamma(\kappa + b_{n-1})\sigma_c\sigma_x\rho_{c,x} + \gamma\sigma_c\sigma_d\rho_{c,d} - (\kappa + b_{n-1})\sigma_x\sigma_d\rho_{x,d} \\
&= \gamma\mu_c - \kappa\mu_x - \mu_d - b_{n-1}\mu_x - \frac{1}{2}\gamma^2\sigma_c^2 - \frac{1}{2}\kappa^2\sigma_x^2 - \frac{1}{2}b_{n-1}^2\sigma_x^2 - \kappa b_{n-1}\sigma_x^2 - \frac{1}{2}\sigma_d^2 \\
&+ \gamma\kappa\sigma_c\sigma_x\rho_{c,x} + \gamma b_{n-1}\sigma_c\sigma_x\rho_{c,x} + \gamma\sigma_c\sigma_d\rho_{c,d} - \kappa\sigma_x\sigma_d\rho_{x,d} - b_{n-1}\sigma_x\sigma_d\rho_{x,d}
\end{aligned}
\tag{85}
$$

and

$$b_n = \kappa\phi + b_{n-1}\phi \tag{86}$$

$$b_{n-1}\phi - b_n = -\kappa\phi \tag{87}$$

□

Zero-coupon bonds

The derivation for zero-coupon bond prices is completely analogous to the derivation for zero-coupon equity, but the starting equation is (29) rather than (11). □