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Ed Westerhout, Lex Meijdam, Eduard Ponds and
Jan Bonenkamp

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Should we revive PAYG? On the optimal pension system in view of current economic trends[*]

Ed Westerhout,[†] Lex Meijdam,[‡] Eduard Ponds,[§] and Jan Bonenkamp[¶]

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Abstract

In many countries, both pay-as-you-go (PAYG) and funding are used to finance pensions, although the balance between the two principles differs a lot between countries. Over the last decades, many countries made a gradual transition to more funding. In this paper, we develop an analytical framework that includes three models of pension design, allowing us to study the role of efficiency aspects, redistributional aspects and political-economy aspects. We subsequently analyze the impact of several trends (a permanent decline in the rate of return on financial markets, a decline in the average rate of economic growth, decreased output volatility and increased capital market volatility) on the optimal balance between PAYG and funding. We argue that it may be optimal to revise the gradual transition to more funding and to revive PAYG.

Keywords:
PAYG, Funding, Pensions, Aaron Rule

JEL classification numbers:
H21, H55

1 Introduction

Many countries use both the pay-as-you-go (PAYG) and the funding principle to finance their pensions. The balance between the two principles differs a lot however, not only between countries, but also over time (OECD, 2016). In the past decades, we have generally seen a shift towards more funding.

The shift towards more funding can be regarded as a consequence of population ageing, which directly deteriorates the relation between contributions and benefits in PAYG schemes. Moreover, Rachel and Summers (2019) estimate that the rate of productivity growth and that of potential output have declined, which also drives down the rate of return in PAYG schemes. On the other hand, there is also increasing evidence of a downward trend in the return to funding, probably (partly) due to ageing as well (Caballero *et al.*, 2017; Blanchard, 2019; Rachel and Summers, 2019). Additionally, there is evidence that capital market volatility has increased in the past, whereas two recent global crises (the 2008 financial crisis and the 2020 Corona crisis) may have made people more aware of the risks involved in investing pension contributions in financial markets. At the same time, we observe a decrease of output volatility (Jordà *et al.*, 2017).

In the future, these trends may be reversed, but they may also persist for some time. Interest rates may continue to hover around their current low levels, economic growth may remain low in the decades to come (Gordon, 2016) and capital market volatility and output volatility may continue to develop in different directions. These future perspectives raise a number of questions. What do we know about the optimal balance between PAYG and funding? How will this balance shift if the developments we currently observe persist? Should we revive PAYG?

More than 50 years ago, Aaron (1966) devised a simple rule to decide what is the best principle to finance pensions: PAYG is to be preferred if the implicit rate of return of this system (*i.e.*, the growth rate of the economy) exceeds the rate of return on the capital market. This so-called Aaron rule provides a nice framework to start thinking about the optimal balance between the two principles. The rule as such is much too simple however for a number of reasons. In particular, in contrast to what Aaron assumes, the capital market rate of return and the rate of economic growth are not constants, but vary stochastically over time. Further, any transformation from a PAYG scheme to a funded scheme (or vice versa) exerts not only effects upon economic efficiency, but also upon the distribution between generations. In addition, in reality decisions are taken not by a social planner, but by governments that serve the interests of different groups of voters,

thereby creating room for all kinds of politico-economic factors and for the possibility that policies deviate from earlier expectations due to unforeseen changes in economic circumstances.

In this paper, we develop an analytical framework that takes into account all these three aspects: efficiency aspects, redistributional aspects and politico-economic aspects. Using this framework, we explore how structural changes in the economic environment change the optimal balance between PAYG and funding. The structural changes we study match the trends we described above: declining interest rates, a lower average rate of economic growth, increasing capital market volatility and decreasing output volatility. This then allows us to answer the question whether the balance between PAYG and funding should be adapted if the trends observed continue into the future.

Our contribution to the literature is twofold. Firstly, we generalize existing models for optimal pension design. In particular, we extend the portfolio approach to pensions (specifically, Matsen and Thøgersen, 2004) by explicitly incorporating intergenerational redistribution. That is, we assume a social planner that weighs the efficiency effects of the pension system (as measured by the consequences for the utility of currently young and future generations) against the equity effects (as measured by the effects for currently old generations). As the extended approach is still time-inconsistent, we modify it further along the lines of the time-consistent approach in d'Amato and Galasso (2010) by extending their approach with stochastic productivity growth. The modified model allows us to study optimal pension design that is flexible and that accounts for the sharing of financial and economic risks between generations when the income of the old generation is lower than expected. Secondly, we contribute to the literature by analyzing the effect of the trends described above on the optimal balance between PAYG and funding. In particular, we explore whether the different trends change the optimal balance between the two financing principles in the same direction and whether the three models that we distinguish draw similar conclusions.

The structure of this paper is as follows. The next section takes a look at pension schemes around the world and focuses on the variety in schemes, both across countries and over time. In addition, it presents an overview of movements over time in the average and volatility of interest rates, of the equity rate of return and of the rate of economic growth. Section 3 explores the determinants of the share of PAYG from different perspectives (economic efficiency, redistribution and political economy). Section 4 develops an analytical framework to capture the perspectives in different models. We use these models to, first, explore the determinants of the balance between

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PAYG and funding and, second, to assess how the mix between PAYG and funding might develop in the future. Finally, section 5 concludes.

2 The dynamics of pension systems and capital markets

There is widespread diversity in pension systems across the world. Despite this diversity, there is a common shift away from PAYG towards funding. At the same time, financial and economic conditions are changing. Interest rates have been declining as have economic growth rates, whereas output volatility has decreased and capital market volatility has increased. This section looks at these issues in somewhat more detail.

2.1 Variety of pension schemes in the world

There is a large variety in size and design of PAYG retirement systems. Their early roots date back to begin 1900s but most of their expansion occurred in the middle part of the 20th century, more specifically in the 30s and 50s (Perotti and Schwienbacher, 2009). These systems have in common that they aim at reducing poverty in old age. Furthermore, consumption smoothing and risk sharing are important objectives for PAYG plans (Hinz, 2012). The size and design of the plans differ greatly across countries, reflecting cultural and historical differences (Aggarwal and Goodell, 2013; Rivera-Rozo *et al.*, 2018).

A clear distinction can be made between the so-called Beveridge-style PAYG plans, primarily oriented at redistribution to the lower incomes, and the Bismarck-style plans, more oriented at insurance and featuring a stronger link between individual contributions and benefits. Countries with Bismarck-style plans have integrated employment-based pension entitlements in the public plan structure, whereas countries with a Beveridge-type of public plans have separated funded plans to cover private sector employment-based pension accrual.

Figure 1 reports OECD projections of gross replacement rates from the public and private sector plans, assuming a full career till formal retirement age. The reported countries are listed to Bismarck-style and Beveridge-style plans according to the classification of Disney (2004). Overall, the average gross replacement rate is equal to 65.2%. The weights of the public and private sector plans differ fundamentally, however. Countries with Bismarckian plans derive on average 59.7% replacement rates from the public plans.

Most of these countries have no private sector plan, but in case they have, these plans provide on average only 13.0% replacement rates. The shares of public and private plans are very different in countries with Beveridgean plans. These countries have on average only 28.5% replacement rates from the public plans and 38.2% from the private plans.

2.2 Changes in pension schemes over time

Disney (2004) reports for the OECD countries a steady increase in the effective contribution rate for PAYG-financed public pension plans from 17.4% in 1955 to 28.0% in 1995. This increase is primarily driven by higher benefit generosity. The replacement rate from the public plan for the median income increased from 47.5% in 1955 to 65.4% in 1995, whereas the dependency ratio remained more or less stable. For many countries it can be foreseen that the ongoing process of ageing will drive up the contribution rate even more, primarily by an increase in the dependency ratio, unless the generosity is scaled back by parametric reform measures like lower benefit levels or a higher retirement age, or more fundamental plan reforms, like a reset to a notional DC plan (Bonenkamp *et al.*, 2017).

Table 1 is very instructive to reveal the immense challenge for policy makers to tame the rise of the PAYG burden due to ageing. EU-wide ageing will lead to an increase in the dependency ratio (measured as the ratio of population aged 65+ over the population aged $25-64$) over the period 2016-2070 of 7.0%pt. Without reform or more labor participation, the burden of the PAYG pension on EU GDP will increase with 7.0%pt, implying an increase of the effective contribution rate of at least more than 10%pts. The table also reports EU-wide intended reform plans and labor market efforts to absorb the impact of the rise in the dependency ratio. The combined effort of the planned increase in the formal retirement age (-2.2%pt), the lowering of benefit generosity (-3.1%pt) and the assumed higher participation of the labor force (-1.0%pt) will outweigh more or less the impact of relatively more elderly on public pension expenditure as measured by the increase in the dependency ratio.

Despite these trends in PAYG systems, funding has become more important relative to PAYG over the last few decades. Chile is probably the most well-known example of a country that transformed its formerly PAYG scheme into one based on funding. A number of other Latin American countries followed the Chilean example, implying a shift away from PAYG towards funding. But these reforms are not the only reason for this shift.

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Figure 1: Projections gross pension replacement rates from public sector and private sector plans (as percentage of mean individual earnings)

Sources: Classification Bismarck and Beveridge: Disney (2004); Projected figures: OECD (2017)
Note: Projections based on the assumption of full labour time career till formal retirement age
*: Grey-marked voluntarily participation, otherwise mandatory participation private sector plans

BISMARCK-like				BEVERIDGE-like			
	Public	Private *	Total		Public	Private *	Total
Austria	78,4		78,4	Australia	13,5	32,1	45,6
Belgium	46,7	14,2	60,8	Canada	41,0	34,2	75,2
Finland	56,6		56,6	Denmark	14,8	71,6	86,4
France	60,5		60,5	Ireland	34,1	38,0	72,1
Germany	38,2	12,7	50,9	Netherlands	28,7	68,2	96,9
Greece	53,7		53,7	New Zealand	40,0	18,8	58,8
Italy	83,1		83,1	Switzerland	24,2	17,9	42,1
Luxembourg	76,7		76,7	United Kingdom	22,1	30,0	52,2
Norway	39,2	5,9	45,1	United States	38,3	33,0	71,3
Portugal	74,0		74,0				
Spain	72,3		72,3				
Sweden	36,6	19,2	55,8				
Average	**59,7**	**13,0**	**64,0**	**Average**	**28,5**	**38,2**	**66,7**

Table 1: Decomposition of gross public pension expenditure change over 2016-2070 – in pts of GDP for EU27

	2016 level	2070 level	Depend. ratio	Coverage ratio	Benefit ratio	Labour market	Cross effects
EU27	10.4	10.5	+7.0	-2.2	-3.1	-1.0	-0.5

Source: European Commission (2018), Table II.1.11

This can be seen by looking at figures for the OECD area.

OECD (2019) compares 2015 with 1990. In 1990, the share of private pensions in total pension spending was 13 percent. In 2015, this had increased to 20 percent. The same picture arises when one looks at the increases in public and private spending over the same period. Public spending increased from 6.3 to 8.0 percent of GDP, an increase of about 25 percent. The share of private spending in GDP however more than doubled, from 0.7 to 1.5 percent. For individual OECD countries like the US and Switzerland, a similar picture arises. The UK fits also in this picture. Whereas the share of public pension spending in GDP increased about 40 percent, the share of private spending increased more than fivefold. The pattern is not universal, however; for France and Italy, the picture is different.

One explanation of the shift towards funding is that many countries reform their public schemes, anticipating an ageing population, for example through reducing benefits or raising the pension eligibility age. The effect of this is a reduction of public pension spending. Individuals will probably save more via private plans to compensate for this reduction. Given that many countries are involved into this kind of parametric reforms, one may expect that the trend of shifting away from PAYG towards funding has continued after 2015 and will further continue in the future.

2.3 Trends in capital market rates of return and rates of economic growth

The evidence that the decline in interest rates across the world is a structural phenomenon is strong. Caballero *et al.* (2017) show that the US interest rate has been declining for 35 years from a level of more than 10 percent in 1980

to a low in 2015 of less than one percent. Del Negro *et al.* (2019) show that the same has occurred in other countries, notably Canada, France, Germany, Italy, Japan and the UK. Holston *et al.* (2017) focus on the natural rate of interest, a measure which is free from transitory disturbances. They show that in four regions in the world - the US, Canada, the euro area, and the UK - the natural rate of interest has declined sizably to a historically low level in 2016.[1]

Holston *et al.* (2017) also report substantial comovement between the natural rates of interest in different regions. The latter is confirmed by Del Negro *et al.* (2019). They find that the world interest rate - composed of interest rates in seven advanced economies in the world - almost coincides with the US interest rate since the late 1970s, which they attribute to an increased level of integration in international asset markets.

The global nature of the trend in interest rates suggest that the factors that drive these trends are also global. One candidate factor is population ageing. Indeed, several studies (Summers, 2015; Favero *et al.*, 2016; Lunsford and West, 2019) find evidence that the trend to lower rates of interest is correlated with an ageing demographic structure and the associated changes in saving and investment propensities. Carvalho *et al.* (2016) calculate that demograhic changes may have reduced the equilibrium interest rate with more than one and a half percentage point between 1990 and 2014. Lunsford and West (2019) further find a long-run correlation between safe interest rates and growth in hours of labor worked. They do not find a positive correlation with labor productivity, however, which is what one would expect based on economic theory. Del Negro *et al.* (2019) also find a role for demographics. However, they also find that since the beginning of this millennium demographic factors have changed and have been putting upward pressure on interest rates.

The latter suggests that there have been also other factors at work. Del Negro *et al.* (2019) argue that one such factor is the convenience yield, the premium that investors are willing to pay to hold safe and liquid assets. According to their paper, the role of the convenience yield is large, but also quite recent. Until 1990, the role of the convenience yield was negligible. After 1990 (1997), changes in the convenience yield accounted for a decline of the world safe interest rate of 120 (80) basis points. Qualitatively, these results are in line with Caballero *et al.* (2017), who argue that the price of

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[1]Schmelzing (2020) shows that the decline in interest rates goes back to much earlier periods. He constructs global real interest rates going back to the 14th century and shows that, since the late middle ages, real interest rates have displayed a trend decline between 0.6–1.6 basis points per year.

safe assets has increased since the 1980s, reflecting a shortage of especially safe assets. This then implies that the trends in safe interest rates may be disconnected to movements in the rate of return on capital. Indeed, Caballero *et al.* (2017) find for the US stable returns on productive capital. According to their study, the equity risk premium has thus increased. This is in line with Jordà *et al.* (2019) who find that over more than a century the equity risk premium is volatile and the risky return often smoother than the safe real interest rate.

On this issue there does not seem to be full agreement, however. Blanchard (2019) argues that not only the interest rate, but also the marginal product of capital may have decreased. Rachel and Summers (2019) present evidence that during the last five decennia, variations in the equity risk premium have been fairly modest.

Like the safe interest rate, economic growth also exhits variation over time. Holston *et al.* (2017) report a declining trend in the four regions in the world they study. The decline in the growth rate is comparable with the decline in the safe interest rate or of smaller magnitude. Rachel and Summers (2019) present evidence on trend growth rates of labor productivity and total factor productivity in advanced economies. Both trend growth rates have been declining since 1970. Combined with declining population growth rates and declining numbers of years working, it is easy to infer that trend economic growth rates have been decreasing as well. Indeed, Rachel and Summers show that the trend rate of economic growth has decreased in the 1980-2016 period, but far less than the natural rate of interest.

On volatilities, Jordà *et al.* (2017) demonstrate that output growth has become less volatile over time, reflecting the well-known *Great Moderation*. After WW II, the volatility of output was only half as large as before WWII. On the other hand, stock prices have become more volatile in the second half of the twentieth century relative to the period before WW II. In addition, Jordà *et al.* (2019) show that during the last decades capital market returns have become more correlated across countries. This holds true not only for safe returns, but also and even more for risky returns. This suggest that over time diversification possibilities have reduced, contributing to increasing capital market volatility.

The combination of facts raises a question. Despite large differences between countries, many of them have shown a shift towards more funding. Population ageing and the productivity slowdown have lowered the rate of return on PAYG schemes, whereas the structural decline in interest rates has contributed to a lower return on funded schemes. So, should pension plans be redesigned if these trends persist? The answer is not that simple

and depends on which perspective one takes.

3 The role of PAYG: different perspectives

In order to understand the role of PAYG pensions, it is useful to distinguish between various perspectives. As almost any form of government intervention, PAYG pensions cause both efficiency and redistribution or equity effects. Often, these effects impact on welfare in opposite ways, so that a welfare-optimizing government has to balance efficiency versus equity. In addition, in reality decisions on PAYG pensions are made in the political arena, where various arguments may play a role. Therefore, in this section, we will not only distinguish between the efficiency and the redistribution perspective on PAYG pensions, but also discuss the political-economy perspective.

3.1 Efficiency perspective

Aaron (1966) was one of the first to point out that the attractiveness of funding and PAYG relates to their rates of return: the capital market rate of return (R) in case of funded schemes and the rate of economic growth (Y) in case of PAYG schemes. Aaron's analysis is based on two important assumptions. Firstly, it assumes that the factors that determine the efficiency of PAYG relative to funding, *i.e.* R and Y, are given and constant. Secondly, the decision on the level of the PAYG contribution rate is assumed to be a once-and-for-all decision. That is, Aaron basically analyses the effect of PAYG pensions for a generation in a steady state. Consequently, he ignores that actual decision making on PAYG pensions may lead to contribution rates that vary over time. As a consequence, the redistribution effects of changes in the size of a PAYG scheme play no role in his analysis. He thus focusses on efficiency only and concludes that countries should rely fully on either PAYG or funding, depending on which of the two has the highest rate of return. Obviously, in reality it is more complex to compare the efficiency of PAYG versus funding as both the growth rate of the economy and the rate of return on the financial market fluctuate over time. Moreover, "the" return on financial capital is difficult to observe as financial markets redistribute the gross reward to capital into the security market with a straight line between expected return and systematic risk.

The so-called portfolio approach in pension plan design (Dutta *et al.*, 2000; Wagener, 2003; Matsen and Thøgersen, 2004; De Menil *et al.*, 2016) explicitly takes into account that rates of return are stochastic. As a result,

this approach advocates a mix of PAYG and funding as a better alternative above either full funding or full PAYG. The best mix will not only depend on the expected rates of return of both finance methods, but also on their risk aspects and the risk tolerance in society. This approach can thus be viewed as an extension of the analysis of Aaron. However, just like Aaron, most papers within this approach assume a once-and-for-all decision on the size of the PAYG scheme and neglect the effects of changes in the size of the scheme.

3.2 Redistribution perspective

The efficiency perspective focusses on the effects of PAYG in steady state and neglects the fact that a PAYG pension scheme always implies inter- and often also intragenerational redistribution. It is well known that introducing or extending PAYG pensions implies a windfall gain for the first generation of elderly: they receive a pension benefit without having paid for it. In fact, this redistribution towards the elderly was an important reason to introduce PAYG pensions in many counries after WWII. Perotti and Schwienbacher (2009) explain the international diversity in pension systems by the impact of severe economic shocks in the interwar period on the then prevailing political preferences. Large inflationary shocks devastated middle class savings in a number of countries, among them typically the countries in Continental Europe with nowadays a large call on PAYG financing and government-based insurance (Germany, France, Italy, Spain). The political majority in these countries shifted support away from pension savings and free markets towards social insurance and a strong role for state intervention.[2] The mirror image of this windfall gain appears when a pension system is reformed and PAYG pensions are (partly) replaced by funding: without additional policy measures, this leaves the elderly at the time of the reform with no (or lower) pensions.

Taking these intergenerational redistribution effects of PAYG pensions into account reveals that the efficiency perspective focussing on the steady state is too narrow and provides a false image. For example, it is well-documented that for the textbook case of a dynamically efficient (*i.e.* $R > Y$) overlapping generations economy without risk, the efficiency gain of moving from a low-returning PAYG scheme to high-returning funded pensions

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[2]However, causality may also run the other way, as argued by Scharfstein (2018). Pension system choices drives financial system performance and institutional structure. Overall, the size and depth of a country's financial system has been stronger, the larger the call on funding instead of PAYG.

is fully absorbed by the burden to compensate all those with accrued rights in the old setting (Breyer, 1989; Verbon, 1989; Feldstein, 2005; Sinn, 2000). Hence, a Pareto-improving transformation is not possible, *i.e.*, the existing PAYG scheme is Pareto efficient. Likewise, it can easily be shown that in this textbook case the loss for current and future young generations resulting from the introduction of a PAYG scheme is the annuity of the windfall gain for the first generation of elderly. Therefore, one cannot say that the PAYG scheme is inefficient. More generally, the conclusion is that, abstracting from risk and taking redistribution into account, there are no real efficiency effects of adjusting the size of a PAYG scheme. It is primarily intergenerational redistribution, and these redistribution effects should not be ignored when modelling the optimal mix of PAYG and funding.

Changes in the size of the PAYG scheme may have additional effects, however. It is, for example, well-documented both for closed economies and small open economies (Krueger and Kubler, 2006) that the introduction of more PAYG may be accompanied with a lower fertility rate and lower labor supply. It is sometimes argued that, due to these effects, a Pareto-improving reform to more funding in a world without risk may still be possible. For example, Homburg (1990) shows that a Pareto-improving reform may be possible when the PAYG contribution rate is perceived as a tax on labor income. In that case, the move to funding may imply higher labor supply so part of the reform burden can be borne from the increase in higher (taxable) labor income. However, this reasoning ignores the reason why the PAYG scheme is financed by a tax on labor income, namely in order to affect the intragenerational distribution. If the government wants to retain this intra-generational redistribution effect, it will have to use another distortionary tax instrument which offsets the welfare increase resulting from abolishing the PAYG scheme (Fenge, 1995). Likewise, Van Groezen *et al.* (2003) show that even if a PAYG scheme leads to a suboptimally low level of fertility, the scheme is still Pareto efficient as replacing the implicit debt of the PAYG scheme by explicit government debt in order to compensate the elderly at the time of the change will not improve fertility.

The conclusion, assuming dynamic efficiency and taking redistribution into account, that there are no real efficiency effects of adjusting the size of a PAYG scheme, is not necessarily true if one allows for risk, however. As noted by Matsen and Thøgersen (2004), the portfolio-diversification model allows for a Pareto-improving extension of the PAYG scheme as long as the size of the scheme is below the level that is optimal for young and future generations.

3.3 Political-economy perspective

Taking into account all redistribution effects above, a benevolent social planner may decide to introduce a PAYG scheme. A number of seminal contributions have defined a role for the government in setting up a PAYG scheme as a kind of insurance which provides a stable income in good and bad economic times in the form of public benefit for the retirees (Enders and Lapan, 1982; Merton, 1983; Verbon, 1989). The role for a benevolent government is then to bind current and future generations to a risk-sharing contract, including yet to be born generations. In these models, the initialization of a public pension plan typically will occur in bad performing financial markets, like the interbellum period, or a period of widespread poverty as in the 30s and 50s.

In a representative democracy however, election-oriented politicians may provide too much social security compared to the welfare-maximizing benevolent government. They gain political support from the elderly by providing them a generous pension. The costs of this for the young working generation (and thus the loss of their political support) will be small as these generations can lower savings for retirement, anticipating the continuation of a generous PAYG scheme by the next generation of politicians (d'Amato and Galasso, 2010). Moreover, the elderly form an important interest group with a lot of time to defend their interests.

In a direct democracy, the reduced time horizon of the median voter implies that, once granted, a generous benefit structure has a strong tendency to preserve (Browning, 1975). Downscaling a too generous public pension plan is increasingly difficult with an ageing electorate. Reform will be blocked by the median voter, who will become older and older, so surpassing the age of indifference at which the benefits and costs of reform exactly match (Sinn, 2005).

Generally speaking, the intergenerational redistribution effects form an important element in explaining political decision making on PAYG pensions. However, also intragenerational redistribution may play an important role in understanding the politics of PAYG pensions (Hansson and Stuart, 1989; Tabellini, 2000). If a PAYG scheme redistributes from high to low incomes, as is often the case, low-income young people may support a PAYG scheme, even if it hurts their generation.[3]

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[3]For an overview of the literature on political decision making with respect to PAYG pensions, see Galasso and Profeta (2002).

4 Modelling the optimal mix between PAYG and funding

In the previous section, we presented three perspectives that are relevant for determining the optimal balance between PAYG and funding: the efficiency perspective, the redistribution perspective, and the politico-economic perspective.

In this section, we focus on the welfare-maximizing design of the pension system and develop an analytical framework that takes the three perspectives into account. Using this framework, we sketch the effects of the trends that we described above upon the optimal balance between PAYG and funding.

In order to keep the framework as simple as possible, we make some assumptions. Our analysis distinguishes two overlapping generations, *i.e.* we build on the well-known Samuelson-Diamond framework. It abstracts from intragenerational heterogeneity and, in line with papers like Matsen and Thøgersen (2004), assumes that members of each generation consume only in the second period of life. In order to allow for time-consistent decision making by a social planner as in d'Amato and Galasso (2010), our analysis assumes a quadratic utility function.[4]

Another assumption is that there is only one type of financial asset. This can be thought of as a market portfolio consisting of riskless bonds and risky equity. We argue that there is little need to distinguish explicitly between bonds and equity in case one wants to study the effects of a downward trend in the interest rate and in the rate of return on equity, as suggested in Blanchard (2019) and Rachel and Summers (2019). If only the interest rate decreases over time and the average rate of return on equity is more or less a constant, as suggested in Caballero *et al.* (2017), our assumption deserves more attention. Theoretically, it is possible that a decline of the interest rate induces such a big portfolio shift towards equity that on net the portfolio rate of return increases. We think that such a large portfolio shift is unlikely, however. This is even more true if at the same time capital market volatility is increasing. In this analysis, we will abstract from this possibility and assume that a downward trend in interest rates is reflected in a decline of the rate of return on the only financial asset in our model.

Finally, as we will show below, the PAYG contribution rate that is opti-

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[4]Apart from the part on time-consistent decison making, our framework could also be based on a CRRA utility function as in Matsen and Thøgersen (2004). This would not significantly change most of our results. Derivations available upon request.

mal in the three models that we study is not restricted to the $(0, 1)$ domain. A contribution rate higher than one is meaningless in our framework however as it would imply negative income for the young generation. Similarly, a contribution rate below zero is also meaningless as it would imply transfers from the old towards the young. Hence, we assume that the set of exogenous variables and parameters that determine the level of the optimal contribution rate in the three models is such that this contribution rate is non-negative and not higher than one.

4.1 The joint return-growth distribution

All the models we study include moments of the joint return-growth distribution. At some points, we need to have an idea about the signs of these moments and of some statistics that are based on these moments. Statistics of the means and variances based on annual data are not that meaningful, given that our unit period covers half the economic life of a generation. Therefore, we adopt an approach based on variation across countries.[5] Using data of 15 countries from the database in Jordà *et al.* (2017, 2019),[6] we calculate for each country the average rate of economic growth and the average real capital market rate of return in the 1991-2015 period.[7] Taking the calculated return and growth realizations of the 15 countries as draws from the same joint return-growth distribution, we can use these realizations to estimate the moments of the return-growth distribution.

Table 2 below summarizes the information. Here, we use μ_Y to denote the average rate of economic growth and σ_Y to denote the corrsponding standard deviation. Similarly, we use μ_R and σ_R to refer to the average and the standard deviation of the capital market rate of return respectively. Further, we use ρ to denote the correlation between the two variables. The rest of the variables will be defined when we discuss the models.

A few points are worth mentioning. First, the world economy is dynamically efficient (in the sense that $\mu_R > \mu_Y$, see below), the capital market rate of return is far more volatile than the rate of economic growth ($\sigma_R \gg \sigma_Y$) and the two variables are positively correlated ($\rho > 0$), although far from perfect. In line with these results, the unhedged risk of

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[5]This approach differs from the one in Matsen and Thøgersen (2004) and De Menil *et al.* (2016) who allow the return-growth distribution to be different for different countries.

[6]We did not use data for Belgium and Canada, as data on the rate of return on wealth (see footnote 7) were missing for these countries.

[7] We use Jordà *et al.* (2017)'s rate of return on wealth, defined as a weighted average of the rates of return on housing, equity, bonds and bills.

funding $(X_R = \sigma_R^2 - \sigma_{RY}^2)$ is much higher than the unhedged risk of PAYG $(X_Y = \sigma_Y^2 - \sigma_{RY}^2)$.

Table 2: Data (1991-2015)

μ_Y	0.6
μ_R	4.5
σ_Y	0.2
σ_R	2.8
ρ	0.2
n	0.6
X_Y	-0.1
X_R	7.6
S_Y	2.6
S_R	38.3
S_{RY}	8.9
S_{R-Y}	23.1
S_G	1.0

4.2 Efficiency: a portfolio diversification model

As discussed above, the Aaron rule is simple: finance pensions fully on the basis of PAYG (funding) if the rate of economic growth exceeds (falls below) the capital market rate of return. But, as we argue, it is too simple as it does not recognize that both rates vary over time. The so-called portfolio approach in pension plan design explicitly takes this into account. Therefore, we start building our framework by setting up a portfolio diversification model that fits with this approach.

We assume an agent who lives two periods, earns labour income W in the first period of his life and consumes c in the second period of his life. Hence, the utility function for the agent born in period t reads as

$$u_t = -\frac{1}{2}E_t(c_{t+1} - \tilde{c}_{t+1})^2 \tag{1}$$

where $\tilde{c}_{t+1}$ refers to the bliss point of consumption. We assume that this bliss point is proportional to wealth in the previous period: $\tilde{c}_{t+1} = \gamma W_t$. This formulation has some association with the concept of habit formation. A given amount of consumption is valued less the higher one's income in

earlier times. In order to ensure that the marginal utility of consumption is always positive, we choose γ sufficiently high. To see what this means, we formulate the consumption equation.

Consumption when old is made up of two components: income that equals savings plus the returns from investing these savings in the financial market (*i.e.*, funding) and income that comes from transfers by the young at that time (*i.e.*, PAYG):

$$c_{t+1} = (1 - \tau_t)W_t(1 + R_{t+1}) + \tau_{t+1}W_t(1 + Y_{t+1}) \tag{2}$$

Here, R_{t+1} denotes the rate of return on the capital market which we assume has maximum value $\overline{R}$. Y_{t+1} denotes the rate of economic growth with maximum value $\overline{Y}$. From this consumption equation it follows that the maximum of c_{t+1}/W_t is $1 + \overline{R}$ or $1 + \overline{Y}$, whichever is highest. Hence, $\gamma > max(1+\overline{R}, 1+\overline{Y})$ is a sufficient condition to ensure that the marginal utility of consumption is always positive.[8]

As regards the rate of return on the capital market and the rate of economic growth, we introduced in the previous subsection μ_R, σ_R, μ_Y and σ_Y, the mean and standard deviation of the rate of return on the capital market and the rate of economic growth respectively. We also introduced ρ as the coefficient of correlation between the two variables, so that we can define σ_{RY}^2 as their covariance. Taken together, these variables define the time-invariant joint distribution of the rate of return on the capital market and the rate of economic growth, to which we will refer below as the return-growth distribution.

We make two further assumptions as regards the moments of this distribution. The first is that output is positive: $1 + \mu_Y > 0$. The second is that of dynamic efficiency. We take the economy to be dynamically efficient in the sense that on average the rate of return on saving exceeds the rate of economic growth: $\mu_R > \mu_Y$. Given that the processes for R and Y are imperfectly correlated, this leaves open the possibility that the rate of return is lower than the rate of economic growth in a particular state. To explain this assumption more precisely, it is important to recall our interpretation of the financial asset in our model as a basket of bonds and equity. Assuming dynamic efficiency thus does not mean that we assume the interest rate to be higher than the rate of economic growth (which would be an assumption

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[8]The requirement that the marginal utility of consumption is always positive is the only reason that we make the assumption that Y and R are bounded from above. Note that this is entirely consistent with the stochastic nature of the distributions of the two variables, as long as the right tails of these two distributions are bounded.

hard to defend). It does mean that we assume that the rate of return on the portfolio of bonds and equity exceeds the rate of economic growth on average.[9] The results in Jordà *et al.* (2019) suggest this is a very mild assumption. That paper finds that the rate of return on wealth has exceeded the rate of economic growth across most countries over the past 150 years.

Throughout the paper, we will also use the expected value of the square of the rate of return on the capital market (S_R) and of the rate of economic growth (S_Y). Hence, $S_R \equiv (1+\mu_R)^2+\sigma_R^2$ and $S_Y \equiv (1+\mu_Y)^2+\sigma_Y^2$. Similarly, we define S_{RY} as $(1+\mu_R)(1+\mu_Y)+\sigma_{RY}^2$ and S_{R-Y} as $(\mu_R-\mu_Y)^2+\sigma_{R-Y}^2$ (where $\sigma_{R-Y}^2 \equiv \sigma_R^2+\sigma_Y^2-2\sigma_{RY}^2$).

Finally, we decompose economic growth into non-stochastic population growth (n) and stochastic productivity growth (G). Hence, the distribution of $1+G_t \equiv (1+Y_t)/(1+n)$ is proportional with that of economic growth: $1+\mu_G = (1+\mu_Y)/(1+n)$, $\sigma_G^2 = \sigma_Y^2/(1+n)^2$ and $S_G = S_Y/(1+n)^2$.

In line with Aaron (1960) and Matsen and Thøgersen (2004), we optimize the size of the PAYG scheme for a generation in the steady state. That is, we assume that the contribution rate is constant (i.e. $\tau_t = \tau_{t+1} = \tau$) and maximize utility in equation (1) with respect to this contribution rate.[10] This yields the following expression,

$$\tau_{pd}^* = \frac{-(\gamma-(1+\mu_R))(\mu_R-\mu_Y)}{(\mu_R-\mu_Y)^2+X_R+X_Y} + \frac{X_R}{(\mu_R-\mu_Y)^2+X_R+X_Y} \qquad (3)$$

where subscript τ_{pd}^* refers to the optimal contribution rate in the portfolio diversification model and $X_R \equiv \sigma_R^2 - \sigma_{RY}^2$ and $X_Y \equiv \sigma_Y^2 - \sigma_{RY}^2$ stand for the unhedged risk of funding and PAYG respectively.[11]

As in Dutta *et al.* (2000) and Matsen and Thøgersen (2004), the expression in equation (3) has two components, an *excess return* component

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[9]As we include bonds in the portfolio, our concept of dynamic efficiency seems stronger than the assumption tested in the literature on dynamic efficiency that the average marginal product of capital exceeds the rate of economic growth (Abel *et al.* 1989). It seems less strong as we impose the inequality between the rate of return and the rate of growth to hold only on average. It is not possible to draw a strong conclusion on this, however, given the difficulties that are inherent to measuring the marginal product of capital (Blanchard, 2019).

[10]We assume that the wage rate when young is known and focus on traditional risk sharing, in contrast to Rawlsian risk sharing where the stochastic wage rate when young is assumed not to be realized yet. Given that we assume that the trend in income growth is stochastic, the difference between traditional and Rawlsian risk sharing is small and and only stems from the correlation of wage growth and returns on the financial markets (see Matsen and Thøgersen (2004), section 4).

[11]One can easily derive that the second-order condition of the optimization problem holds true: $\partial^2 u_t/(\partial\tau)^2 < 0$ for all values of τ.

and a *hedging* component. The excess return component echoes the Aaron condition. Indeed, if we abstract from risk (*i.e.* $\sigma_R^2 = \sigma_Y^2 = 0$),[12] we find that $\tau_{pd}^* = 0$ (funding dominates PAYG) if $\mu_R > \mu_Y$ and $\tau_{pd}^* = 1$ (PAYG dominates funding) if $\mu_R < \mu_Y$.[13] However, in a dynamically efficient environment with risk as we assume here, the optimal share of PAYG is a continuous and decreasing function of $\mu_R - \mu_Y$. In particular, a higher expected growth rate μ_Y increases the optimal PAYG contribution rate, whereas a higher expected rate of return on the capital market μ_R decreases the optimal PAYG contribution rate (See appendix A for the elaboration of the relevant derivatives).

The hedging component indicates which portfolio, *i.e.* which shares of PAYG and funding, minimizes the variance of the portfolio rate of return. The share of PAYG is increasing in the unhedged risk of funding X_R, *i.e.* in the variance of return on savings as far as this return is not correlated to the return on the PAYG scheme. Similarly, an increase in the unhedged risk of PAYG X_Y decreases the optimal size of the PAYG scheme. Related, the optimal contribution rate is increasing in capital market volatility. The impact of a change in output volatility cannot be unambiguously signed, however, when $X_Y < 0$, as borne out by the data.

Increasing the correlation between the rate of return on the capital market and the rate of economic growth (for given variances σ_Y^2 and σ_R^2) lowers both the unhedged risk of funding and that of PAYG. This raises or reduces the share of PAYG in case the initial contribution rate τ_{pd}^* is higher respectively lower than 50 percent.[14] The latter corresponds with the numerical results in Matsen and Thøgersen (2004).

Equation (3) makes clear that it is crucial to account for the time-varying nature of both the rate of return on the capital market and the rate of economic growth. In contrast to what the Aaron rule prescribes, in general it will be optimal to rely on a mix of PAYG and funding, with the composition of the mix depending on the expected returns and the unhedged risks of both ways of financing.

This conclusion remains valid if we change our utility function and assume that the bliss point $\tilde{c}_{t+1}$ is proportional to wealth in the current period instead of the previous period, *i.e.* $\tilde{c}_{t+1} = \gamma_{rel} W_{t+1}$, where we define $\gamma_{rel} \equiv \gamma/(1 + \mu_G)$ so that the expected value of the bliss point remains the

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[12]For obvious reasons, for this exercise we do not impose dynamic efficiency and do not require that the interior solution of the model lies between zero and one.

[13]Note that $\gamma > max(1 + \bar{R}, 1 + \bar{Y})$ implies $\gamma > 1 + \mu_R$.

[14]Note that τ_{pd}^* refers to the share of contributions to the PAYG scheme in total collective and individual pension contributions.

same. This changes the association with habit formation into an association with relative utility. A given amount of consumption when old is valued less the higher the wealth of the current young generation. The optimal contribution in that case equals:

$$\tau^*_{pd,rel} = \frac{(\gamma - (1 + \mu_R))(\mu_Y - \mu_R)}{(\mu_Y - \mu_R)^2 + X_R + X_Y} + \frac{X_R + \gamma X_Y}{(\mu_Y - \mu_R)^2 + X_R + X_Y} \qquad (4)$$

Comparing equations (3) and (4) shows that $\tau^*_{pd,rel} > \tau^*_{pd}$ if $X_Y > 0$. That is, if unhedged income risk is positive, relative utility makes PAYG more attractive as it provides a hedge against changes in the value of the benchmark income (*i.e.*, the income of the young). In other words, it insures pensioners against the risk of being outperformed. This conclusion is in line with Knell (2010).[15]

The portfolio diversification model is an important step forward compared to the Aaron setup. However, it suffers from a serious drawback: it analyses the optimal size of the PAYG scheme for steady-state generation and does not pay attention to redistribution between generations, which may be as relevant as the efficiency considerations that are accounted for by the portfolio diversification model. Indeed, the portfolio diversification model cannot explain cross-country differences in the share of PAYG financing that relate to the (lack of) income of retired generations at the time the PAYG scheme was set up. Therefore, we will now add redistribution considerations to the portfolio diversification model and thus construct a model that combines equity and efficiency perspectives.

4.3 A combined efficiency-redistribution model

In order to account for the interests of successive generations, we assume that the contribution rate for the PAYG scheme is set once-and-for-all by a benevolent social planner that takes into acount the effects on all current and future generations. That is, the social planner maximizes a social welfare function that consists of the (discounted) utilities of the generation who is old when the size of the PAYG scheme is chosen, of the generation who is young at that time and of all successive generations:

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[15]Bilancini and D'Antoni (2012) point to another potential effect: relative consumption may make the PAYG system less effective in hedging financial market risk. Therefore, they argue, in general the net effect is ambigous. As is evident from equations (3) and (4), in our model the net effect of changing from habit formation to relative utility is unambiguously positive, as long as $X_Y > 0$.

$$SWF_t = -\frac{1}{2}(c_t - \tilde{c}_t)^2 - \frac{1}{2}(1+n)\tilde{\Delta}E_t\Big((c_{t+1} - \tilde{c}_{t+1})^2\Big) \qquad (5)$$
$$- \frac{1}{2}(1+n)^2\tilde{\Delta}^2 E_t\Big((c_{t+2} - \tilde{c}_{t+2})^2\Big) - ...$$

Note that $1+n$ measures the size of generation i relative to that of generation $i-1$. $\tilde{\Delta}$ indicates the social discount rate.[16] We define $\tilde{\Delta}$ as Δ/S_G. We correct the pure discount rate Δ for expected productivity growth in order to avoid the counterintuitive outcome that the policymaker chooses a low contribution rate such as to compensate the young and future generations for the utility losses that are due to productivity growth (productivity growth produces an efficiency loss as the bliss level of consumption is dynamic in our framework, *i.e.* increases if the economy grows).

The time-invariant nature of the return-growth distribution allows us to derive a more compact expression for the social welfare function. As always, we need the assumption that Δ is not too large in order for the infinite sum of terms in equation (5) to converge. Here, this implies that $\Delta < 1/(1+n)$.

We adopt the same specification for the bliss level of consumption as before, *i.e.* $\tilde{c}_{t+1} = \gamma W_t$. Hence, $c_t - \tilde{c}_t$, which refers to the old generation, reads as $c_t - \gamma W_{t-1}$. However, we prefer to write this expression in terms of a PAYG part and the rest, *i.e.* $c_t - \gamma W_{t-1} = W_t(\tau(1+n) + \omega_t)$, where ω_t is the relative wealth level of the old, defined as $(1 - \tau_{t-1})(1 + R_t)/(1 + G_t) - \gamma/(1 + G_t)$, *i.e.* the financial wealth of the old (minus the bliss point of consumption), compared to the human capital of the young. In order to ensure that the marginal utility of consumption is always positive, we need to assume now that $\gamma > 2 + \bar{R} + \bar{Y}$.[17] Note that this assumption implies that $\omega_t < 0$.

Using this expression for $c_t - \tilde{c}_t$ and the assumptions regarding the joint return-growth distribution, the social welfare function can be written as

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[16]The Benthamite utility function in equation (5) implies that the effective discount rates applied to successive generations differ a factor $(1+n)\tilde{\Delta}$. The Millian utility function does not attach a higher weight to future generations just because these generations are bigger, thus replacing the factor $(1 + n)\tilde{\Delta}$ with $\tilde{\Delta}$ (Canton and Meijdam, 1997). Such a change of utility function would not change our results fundamentally. Effectively, moving from a Benthamite to a Millian utility function would increase the weight the social planner attaches to the interests of the generation that is old at the time of introduction of the scheme relative to the weight of all other generations if $n > 0$. If $n < 0$, the opposite holds true.

[17]This is different from the condition in the previous section $(\gamma > max(1 + \bar{R}, 1 + \bar{Y}))$, as the model in this section features two PAYG contrbution rates (the optimal rate and the historical rate), which can be different.

follows:

$$
\begin{aligned}
SWF_t &= -\frac{1}{2}W_t^2(\tau(1+n)+\omega_t)^2 \tag{6}\\
&\quad - \frac{1}{2}W_t^2\left((1+n)\hat{\Delta} + S_G(1+n)^2\hat{\Delta}^2 + S_G^2(1+n)^3\hat{\Delta}^3 + ...\right) \times\\
&\quad \times \left[\left((1-\tau)(1+\mu_R) + \tau(1+\mu_Y) - \gamma\right)^2 + \right.\\
&\quad \left. \left((1-\tau)^2\sigma_R^2 + \tau^2\sigma_Y^2 + 2\tau(1-\tau)\sigma_{RY}^2\right)\right]
\end{aligned}
$$

where we have used $\hat{\Delta}$ as a shortcut for $\tilde{\Delta}/(1 - \tilde{\Delta}S_Y/(1+n))$.

Note that the fact that the social welfare function can be rewritten as in equation (6) implies that the contribution rate that would be optimal for future generations is exactly the same as the rate τ_{pd}^* that is optimal for the current young generation. This is remarkable as the wage rate for the current young is known, while the wage rate for future generations is still uncertain. The fact that future generations face a wage risk affects their weight in the social discount factor (as can be seen from the S_Y in $\hat{\Delta}$) but not their preferred contribution rate. In other words, in contrast to, for example, Matsen and Thøgersen (2004), Rawlsian risk sharing plays no role in our model. It can easily be checked that this relates to the quadratic specification of the utility function we use. Adding an additional term for risk aversion[18], for example, would imply that the social welfare function cannot be rewritten as in equation (6). Consequently, in that case there would be a role for Rawlsian risk sharing and the contribution rate that is optimal for future generations would be different from the optimum for the current young.

It follows from equation (6) that the contribution rate τ^* that optimizes the social welfare function can be written as a weighted average of the contribution rate that maximizes utility of the current old τ_o^* and the rate that maximizes the utility of the current young and future generations τ_{pd}^*:

$$
\tau_{er}^* = \beta\tau_o^* + (1-\beta)\tau_{pd}^* \tag{7}
$$

where the subscript er refers to the combined efficiency-redistribution model, β defines the weight given to the current old generation, *i.e.* $\beta = (1 + n)/((1+n) + \hat{\Delta}S_{R-Y})$, and $\tau_o^* = -\omega_t/(1+n)$.[19]

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[18]In that case the utility function would look like $u_t = -\frac{1}{2}E_t(c_{t+1}-\tilde{c}_{t+1})^2 - \beta Var_t(c_{t+1}-\tilde{c}_{t+1})$ where $Var_t(c_{t+1} - \tilde{c}_{t+1})$ stands for the variance of consumption.

[19]Strictly speaking, τ_o^* and τ_{er}^* should carry a time index t. We omit these time indices

This result shows that, although a change from funding towards PAYG is a zero-sum game in financial terms, *i.e.* it is merely redistribution between generations as stressed by Sinn (2000), there is a unique level of the contribution rate that maximizes social welfare. One reason for this is that our model not only looks at the returns of PAYG and funding, but also includes risk factors. But even if we abstract from these risk factors, there is a well-defined optimal contribution rate. The reason for this is that, even if it is merely redistribution, a social planner can decide to introduce a PAYG scheme, depending on the relative weights of the various generations in the social welfare function and their marginal utility of income. For example, in case of a relatively poor generation of elderly, for example due to a low wage income in the past and/or a low return on their savings (and a correspondingly high marginal utility of income), the social planner may decide to offer some wealth of young and future generations (with relatively low marginal utility of income) and introduce a PAYG scheme in order to give the old a decent level of consumption. Actually, this may have been the primary reason that countries introduced PAYG schemes after the great depression and WWII. Indeed, our result shows that the expected rate of return on savings and the expected rate of economic growth, their volatilities and their correlation, as well as the relative wage income and the realized rate of return on saving of the old are all relevant.

In order to explore how the contribution rate that is optimal according to the combined efficiency-redistribution model relates to the moments of the return-growth distribution, we again employ differentiation. We use the expression in equation (7) to write $d\tau_{er}^*/dx$ as $(1-\beta)\partial\tau_{pd}^*/\partial x + (\tau_o^* - \tau_{pd}^*)\partial\beta/\partial x$ for $x = \mu_R, \mu_Y, \sigma_R, \sigma_Y$. Given that $0 < \beta < 1$ and making the plausible assumption that $\tau_o^* > \tau_{pd}^*$, it then follows that $d\tau_{er}^*/dx$ and $\partial\tau_{pd}^*/\partial x$ will have the same sign if the signs of $\partial\beta/\partial x$ and $\partial\tau_{pd}^*/\partial x$ are equal.

In appendix B, we formally derive expressions for $\partial\beta/\partial\mu_R$, $\partial\beta/\partial\mu_Y$, $\partial\beta/\partial\sigma_R$ and $\partial\beta/\partial\sigma_Y$. There we find that $\partial\beta/\partial\mu_R < 0$. A higher expected capital market rate of return raises the opportunity costs of a marginal increase in the PAYG contribution rate for the young and future generations, calling for a lower PAYG contribution rate. Hence, a permanent decline in the expected rate of return on the capital market raises the optimal PAYG contribution rate in the combined efficiency-redistribution model, just as in the portfolio diversification model.

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to stress that the PAYG contribution rate calculated in equation (7) is set at a value that is supposed to last forever. Obviously, this is time-inconsistent, an issue we will return to below.

The effect of a change in μ_Y upon β is opposite: $\partial\beta/\partial\mu_Y > 0$. A higher expected growth rate reduces the weight attached to the young and future generations, calling for a higher PAYG contribution rate. Again, the two models have qualitatively the same result: lower expected economic growth calls for a lower PAYG contribution rate.

The same cannot be said about a change in capital market volatility. Appendix B derives that $\partial\beta/\partial\sigma_R < 0$. Higher capital market volatility increases the expected square capital market rate of return and thus increases the opportunity costs of an increase in the PAYG contribution rate for young and future generations. This calls for a lower PAYG contribution rate. Hence, the effect of higher capital market volatility in the combined efficiency-redistribution model is ambiguous. Efficiency calls for a higher PAYG contribution rate, redistribution for a lower one.

Furthermore, $\partial\beta/\partial\sigma_Y > 0$. Given that we derived that the effect of a change in output market volatility on the PAYG contribution rate is ambiguous in the portfolio diversification model, the corresponding effect is ambiguous in the combined efficiency-redistribution model as well.

The combined efficiency-redistribution model developed in this section assumes that the social planner sets the contribution rate once-and-for-all. This is a strong assumption, however. Reoptimizing the same welfare function at a future moment in time will only lead to the same outcome if the relative wealth position of the old generation is identical to that of the old at the moment the initial decision was made. This will in general not be the case, as the PAYG-scheme affects savings and both wage growth and the return on savings are stochastic. In other words, the solution of the efficiency-redistribution model is time-inconsistent. To resolve this problem, we will in the next section adopt a time-consistent approach as in d'Amato and Galasso (2010).

4.4   A time-consistent efficiency-redistribution model

The model in this section generalizes the benevolent government model in d'Amato and Galasso (2010) by accounting for stochastic productivity growth. We use the same utility function and social welfare function as in the previous sections (see equations (1) and (5)). But instead of assuming that the social planner sets the contribution rate to the PAYG scheme once-and-for-all, we now assume that the planner in period t only decides about the contribution rate in that period. The contribution rate in period $t+1$ will only be chosen after period $t+1$ has arrived. Consequently, for the decision by the benevolent government about the period-t contribution rate

only the utilities of the two generations alive in period t are relevant. This approach involves finding a policy function that relates the optimal PAYG contribution rate in a period to the state at the beginning of that period, *i.e.* the relative wealth position of the elderly as indicated by ω_t.

In the previous section, we assumed that $\gamma > 2 + \bar{R} + \bar{Y}$. In this section, we follow d'Amato and Galasso (2010) and also assume that $\gamma > S_R/(1+\mu_R)$ to ensure that a higher contribution rate for the young without any higher benefit in the future lowers the expected utility of the young. Combining the two assumptions, we thus impose that $\gamma > max(2+\bar{R}+\bar{Y}, S_R/(1+\mu_R))$.

The policy function can then be shown to read as follows (see appendix C for the derivation),

$$\tau_{tc,t}^* = \frac{(\gamma(1+\mu_R) - S_R)((1+n) - \tilde{\Delta}S_R)}{\tilde{\Delta}S_R(S_R - S_{RY})} - \frac{1}{\tilde{\Delta}S_R}\omega_t \tag{8}$$

where we use subscript tc to refer to the time-consistent model.[20]

As in the combined redistribution-efficiency model, the expression for the optimal PAYG contribution rate consists of a time-invariant part and a part that is responsive to the state of the economy. The time-invariant part is strictly negative if we impose that $\Delta > S_Y/(S_R(1+n))$. We impose this condition for two reasons. Without it, first, the marginal utility of consumption of the old generation would become non-positive. Second, intergenerational risk sharing would be more than 100%, *i.e.* $dc_t/d\omega_t|_{\tau_t=\tau_{tc,t}^*} < 0$, whereas $dc_t/d\omega_t|_{\tau_t=0} > 0$.

Starting with the state-dependent part (the last term at the RHS of equation (8)), the impact of the state of the economy upon the optimal PAYG contribution rate is negative. The state of the economy relates to the relative wealth position of the old generation. The lower its relative wealth, the higher will be the PAYG contribution rate. Both a higher value for the expected square of the rate of return on the capital market and a higher social discount rate increase the weight of future generations in the social welfare function, making the optimal PAYG contribution rate less responsive to a change of the state.

It is interesting to compare the slope of the policy function, $d\tau_{tc,t}^*/d\omega_t$, with the corresponding slope in the combined efficiency-redistribution model, $d\tau_{er}^*/d\omega_t$, in equation (7). We derive that the former slope is larger than the

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[20]The expression in equation (8) generalizes the one in d'Amato and Galasso (2010). One can easily derive that the result obtained by d'Amato and Galasso (2010) emerges if productivity risk and productivity growth in equation (8) are put to zero.

latter (in an absolute sense).[21] Our interpretation is as follows. When the social planner sets the contribution rate once-and-for all, any change in the contribution rate is permanent. As the interests of the old generation on the one hand and the young and future generations on the other hand diverge, it is generally suboptimal to set a higher contribution rate in period t and all other periods in order to compensate for a relatively low wealth position of the elderly in period t. In the time-consistent model, the perspective for the social planner is different as he does not decide on the contribution rate in future periods, but only indirectly affects decision making in these periods via the effect of PAYG contributions on savings. Hence, the social planner has more freedom to compensate a relatively low wealth position of the elderly in a particular period by setting a higher contribution rate for that period.

The intercept in the equation for the optimal contribution rate (the first term on the RHS of equation (8)) is also a function of the expected rate of growth, the expected capital market rate of return and capital market volatility. In appendix D, we elaborate the derivatives $\partial \hat{\tau}_{tc}^*/\partial x$ for $x = \mu_R, \mu_Y, \sigma_R, \sigma_Y$, where $\hat{\tau}_{tc}^*$ denotes the intercept. We find that $\partial \hat{\tau}_{tc}^*/\partial \mu_R < 0$. A higher expected capital market rate of return increases the opportunity cost for the young generation of an increase in the PAYG contribution rate, calling for a lower PAYG contribution rate. In this respect, the time-consistent model is similar to the combined redistribution-efficiency model. The same holds true with respect to the impact of a higher expected rate of economic growth. Again, higher expected economic growth reduces the weight of the young generation and brings the social planner to choose for a higher PAYG contribution rate.

The impact of higher capital market volatility is ambiguous in both models. In the time-consistent model, an additional mechanism plays a role. Higher capital market volatility makes the future policymaker less responsive to a change in the state variable ω_{t+1}. This implies less insurance for the currently young when they become old, for which the current policymaker wants to compensate them in the form of a lower contribution rate. This mechanism also explains that higher output volatility increases the

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[21]In particular, we derive that $|d\tau_{tc,t}^*/d\omega_t| > |d\tau_{er}^*/d\omega_t|$ as long as the following condition holds true: $(S_{R-Y}/S_R) > (1-S_Y/((1+n)\Delta S_R))(1-(1+n)\Delta)$. This will be the case if $\Delta - S_Y/(S_R(1+n))$ is not too large (see appendix D, where we use the same argument to sign derivatives of the intercept of the policy function). Suppose that we would impose for the sake of argument that $\Delta - S_Y/(S_R(1+n)) = 0$. Then the RHS of this inequality condition would be zero and the condition would hold true. Invoking a continuity argument, the same result applies as long as $\Delta - S_Y/(S_R(1+n))$ is not too large.

optimal contribution rate. Higher output volatility makes the future policymaker more responsive to shocks in the state variable, thereby increasing the insurance for the currently young in the following period.

5 Concluding remarks

We started this paper with a question: do the worldwide trends that we have observed in the past imply that we should consider reviving PAYG? More precisely, should we consider to increase the share of PAYG in the financing of nationwide pension schemes? Obviously, the future is unknown and we cannot be sure that the trends observed will continue. So, let us rephrase our question in a conditional sense: should we consider reviving PAYG if the low average rate of economic growth, the low average capital market rate of return, the low volatility of economic growth and the high volatility of the capital market rate of return persist in the coming decennia?

Table 3: Results

	Portfolio diversification model	Combined efficiency-redistribution model	Time-consistent model
$\partial\tau^*/\partial\mu_R$	$-$	$-$	$-$
$\partial\tau^*/\partial\mu_Y$	$+$	$+$	$+$
$\partial\tau^*/\partial\sigma_R$	$+$	$?$	$?$
$\partial\tau^*/\partial\sigma_Y$	$?$	$?$	$+$

Table 3 summarizes the results of the three models studied in this paper. The results may be a bit puzzling at first sight, as the three models give different answers on the impact of trends on the optimal size of PAYG. Looking closely, the results are more informative. The three models differ in their assessment of the impact of changes in volatility. The trends in capital market volatility and output volatility look less dramatic than the other trends however, particularly that of a declining interest rate. On the impact of the other trends, those of declining average growth rates and capital market rates of return, the three models give the same assessment, despite their differences in focus.

One may approach the results in table 3 also from a different angle. Given that of the three models studied in this paper, only the time-consistent

efficiency-redistribution model incorporates efficiency, redistributional and political-economic aspects, one might give this model a larger weight in evaluating the impact of the four trends. What we then notice is that the overall impact of the four trends is unclear. The decline in the average capital market rate of return points in the direction of a larger PAYG scheme, whereas the trends of lower average growth and lower output volatility point in the other direction.

Given these ambiguities in results, it is difficult to draw a sharp conclusion. The fact that the trend of a decline in interest rates is more dominant than the other trends (occurring over a longer period of time and stronger in relative terms) suggests that this trend could be given a larger weight, however. And on this trend, the three models draw a similar conclusion. Hence, we conclude that, despite the differences between models and trends, there is a case for reconsidering the role of PAYG in pension schemes.

References

[1] Aaron, H. (1966). The Social Insurance Paradox, *The Canadian Journal of Economics and Political Science* 32, 371-374.

[2] d'Amato, M. and V. Galasso (2010). Political intergenerational risk sharing, *Journal of Public Economics* 94, 628-637.

[3] Abel, A.B., N.G. Mankiw, L.H. Summers and R.J. Zeckhauser (1989). Assessing Dynamic Efficiency: Theory and Evidence, *Review of Economic Studies* 56, 1-20.

[4] Aggarwal, R. and J.W. Goodell (2013). Political-economy of pension plans: Impact of institutions, gender, and culture, *Journal of Banking and Finance* 37, 1860-1879, http://dx.doi.org/10.1016/j.jbankfin.2012.05.008.

[5] Bilancini, E. and M. D'Antoni (2012). The desirability of pay-as-you-go pensions when relative consumption matters and returns are stochastic, *Economics Letters* 117, 418-422.

[6] Blanchard, Olivier (2019). Public Debt and Low Interest Rates, *American Economic Review* 109, 1197-1229.

[7] Bonenkamp J., L. Meijdam, E. Ponds and E. Westerhout (2017). Ageing-driven pension reforms, *Journal of Population Economics* 30, 953-976.

[8] Breyer, F. (1989). On the Intergenerational Pareto Efficiency of Pay-as-you-go Financed Pension Schemes, *Journal of Institutional and Theoretical Economics* 145, 643-658.

[9] Browning, E.K. (1975). Why the social insurance budget is too large in a democracy, *Economic Inquiry* 13, 373-388.

[10] Caballero, R.J., E. Farhi and P.-O. Gourinchas (2017). The Safe Assets Shortage Conundrum, *Journal of Economic Perspectives* 31, 29-46.

[11] Canton, E. and L. Meijdam (1997). Altruism and the Macroeconomic Effects of Demographic Changes, *Journal of Population Economics* 10, 317-334.

[12] Carvalho, C., A. Ferrero and F. Nechio (2016). Demographics and real interest rates: Inspecting the mechanism, *European Economic Review* 88, 208-226.

[13] Disney, R. (2004). Are contributions to public pension programmes a tax on employment? *Economic Policy* 19, 267-311.

[14] Dutta, J., S. Kapur and J.M. Orszag (2000). A portfolio approach to the optimal funding of pensions, *Economics Letters* 69, 201-206.

[15] Enders, W. and H.E. Lapan (1982). Social Security Taxation and Intergenerational Risk Sharing, *International Economic Review* 23, 647-658.

[16] European Commission (2018). *The 2018 Ageing Report: Economic and Budgetary Projections for the EU Member States (2016-2070)*, Brussels.

[17] Favero, C.A., A.E. Gozluklu and H. Yang (2016). Demographics and the behavior of interest rates, *IMF Economic Review* 64, 732-776.

[18] Feldstein, M. (2005). Structural reform of Social Security, *Journal of Economic Perspectives* 19, 33-55.

[19] Fenge, R. (1995). Pareto-efficiency of the pay-as-you-go pension system with intragenerational fairness, *Finanzarchiv* 52, 357-363.

[20] Galasso, V. and P. Profeta (2002). The Political Economy of Social Security: A Survey, *European Journal of Political Economy* 18, 1-29.

[21] Gordon, R.J. (2016). Perspectives on the rise and fall of American growth, *American Economic Review* 106, 72-76.

[22] Groezen, B.J.A.M. van, T. Leers and A.C. Meijdam (2003). Social security and endogenous fertility: Pensions and child allowances as Siamese twins, *Journal of Public Economics* 87, 233-251.

[23] Hansson, I. and C. Stuart (1989). Social security as trade among living generations, *American Economic Review* 79, 549-559.

[24] Hinz, R. (2012). The World Bank's pension policy framework and the Dutch pension system: a paradigm for the multi-pillar design?, in L. Bovenberg, C. van Ewijk and E. Westerhout (eds.), *The Future of Multi-Pillar Pensions*, CUP, 46-89.

[25] Holston, K., T. Laubach and J.C. Williams (2017). Measuring the natural rate of interest: International trends and determinants, *Journal of International Economics* 108, 559-575.

[26] Homburg, S. (1990). The Efficiency of Unfunded Pension Schemes, *Journal of Institutional and Theoretical Economics* 146, 640-647.

[27] Jordà, Ò., M. Schularick and A.M. Taylor (2017). Macrofinancial History and the New Business Cycle Facts, in M. Eichenbaum and J.A. Parker (eds.), *NBER Macroeconomics Annual 2016* 31, Chicago, Chicago University Press, consulted 19-04-2020.

[28] Jordà, Ò., K. Knoll, D. Kuvshinov, M. Schularick and A.M. Taylor (2019). The rate of return on everything, 1870-2015, *Quarterly Journal of Economics* 134, 1225-1298.

[29] Knell, M. (2010). The Optimal Mix Between Funded and Unfunded Pension Systems When People Care About Relative Consumption, *Economica* 77, 710-733.

[30] Krueger, D. and F. Kubler (2006). Pareto-Improving Social Security Reform when Financial Markets are Incomplete!?, *American Economic Review* 96, 737-755.

[31] Lunsford, K.G. and K.D. West (2019). Some Evidence on Secular Drivers of US Safe Real Rates, *American Economic Journal: Macroeconomics* 11, 113-139.

[32] Matsen, E. and Ø. Thøgersen (2004). Designing social security - a portfolio choice approach, *European Economic Review* 48, 883-904.

[33] Menil, G. de, F. Murtin, E. Sheshinski and T. Yokossi (2016). A rational, economic model of paygo tax rates, *European Economic Review* 89, 55-72.

[34] Merton, R.C. (1983). On Consumption Indexed Public Pension Plans, in Z. Bodie and J.B. Shoven (eds.), *Financial Aspects of the United States Pension System*, Chicago: UCP, 259-290.

[35] Negro, M. Del, D. Giannone, M.P. Giannoni and A. Tambalotti (2019). Global trends in interest rates, *Journal of International Economics* 118, 248-262.

[36] OECD (2016). *OECD Pensions Outlook 2016.*

[37] OECD (2017). *Pensions at a Glance 2017; OECD and G20 Indicators.*

[38] OECD (2019). *Pensions at a Glance 2019; OECD and G20 Indicators*, OECD Publishing, Paris, https://doi.org/10.1787/b6d3dcfc-en.

[39] Perotti, E. and A. Schwienbacher (2009). The political origin pension funding, *Journal of Financial Intermedation* 18, 384-404.

[40] Rachel, L. and L.H. Summers (2019). On Falling Neutral Real Rates, Fiscal Policy, and the Risk of Secular Stagnation, *Brookings Papers on Economic Activity*, Spring.

[41] Rivera-Rozo, J.A., M.E. García-Huitrón, O.W. Steenbeek and S.G. van der Lecq (2018). National culture and the configuration of public pensions, *Journal of Comparative Economics* 46, 457-479.

[42] Scharfstein, D.S. (2018). Presidential Address: Pension Policy and the Financial System, *Journal of Finance* 73, 1463-1512.

[43] Schmelzing, P. (2020). Eight centuries of global real interest rates, R-G, and the 'suprasecular' decline, 1311–2018, Bank of England Staff Working Paper 845.

[44] Sinn, H.-W. (2000). Why a Funded Pension System is Useful and Why It is Not Useful, *International Tax and Public Finance* 7, 389-410.

[45] Sinn, H.-W. (2005). Europe's Demographic Deficit, a Plea for a Child Pension system, *De Economist* 153, 1-45.

[46] Summers, L.H. (2015). Demand Side Secular Stagnation, *American Economic Review* 105, 60-65.

[47] Tabellini, G. (2000). A Positive Theory of Social Security, *Scandinavian Journal of Economics* 102, 523-545.

[48] Verbon, H.A.A. (1989). Conversion Policies for Public Pension Plans in a Small Open Economy, in B.A. Gustafsson and N.A. Klevmarken (eds.), *The Political Economy of Social Security*, Contributions to Economic Analysis 179, Elsevier, 83-95.

[49] Wagener, A. (2003). Pensions as a portfolio problem: fixed contribution rates vs. fixed replacement rates reconsidered, *Journal of Population Economics* 16, 111–134.

Appendices

A Determinants of the optimal PAYG rate in the portfolio diversification model

Recall the expression of the optimal contribution rate in the main text, equation (3):

$$\tau_{pd}^* = \frac{(\gamma - (1 + \mu_R))(\mu_Y - \mu_R)}{(\mu_Y - \mu_R)^2 + X_R + X_Y} + \frac{X_R}{(\mu_Y - \mu_R)^2 + X_R + X_Y}$$

Further, recall our assumptions that $\gamma > max(1 + \bar{R}, 1 + \bar{Y})$, $\mu_R > \mu_Y$ and $0 < \tau_{pd}^* < 1$. We can then derive the following about the signs of the derivatives of this optimal contribution rate with respect to μ_R and μ_Y:

$$\partial \tau_{pd}^* / \partial \mu_R = \frac{-(\gamma - (1 + \mu_R) - (\mu_R - \mu_Y)) - 2\tau_{pd}^*(\mu_R - \mu_Y)}{S_{R-Y}}$$

If we further assume that $\gamma > (1 + \mu_R) + (\mu_R - \mu_Y)$, $\partial \tau_{pd}^* / \partial \mu_R < 0$.

$$\partial \tau_{pd}^* / \partial \mu_Y = \frac{(\gamma - (1 + \mu_R)) + 2\tau_{pd}^*(\mu_R - \mu_Y)}{S_{R-Y}}$$

Without any further assumptions, we note that $\partial \tau_{pd}^* / \partial \mu_Y > 0$.

$$\partial \tau_{pd}^* / \partial \sigma_R = \frac{2(1 - \tau_{pd}^*)(X_R / \sigma_R) + \rho \sigma_Y}{S_{R-Y}}$$

If we further assume that $X_R > 0$ and $\rho > 0$ (see Table 2), $\partial \tau_{pd}^* / \partial \sigma_R > 0$.

$$\partial \tau_{pd}^* / \partial \sigma_Y = \frac{-2\tau_{pd}^*(X_Y / \sigma_Y)}{S_{R-Y}} - \frac{\rho \sigma_R}{S_{R-Y}}$$

If we further assume that $\rho > 0$ $X_Y < 0$ (see Table 2), $\partial \tau_{pd}^* / \partial \sigma_Y$ cannot be signed.

$$\partial \tau_{pd}^* / \partial \rho = \frac{-(1 - 2\tau_{pd}^*)\sigma_R \sigma_Y}{S_{R-Y}}$$

If we further assume that $\tau_{pd}^* > 0.5$ ($\tau_{pd}^* < 0.5$), $\partial \tau_{pd}^* / \partial \rho > 0$ ($\partial \tau_{pd}^* / \partial \rho < 0$).

B Determinants of the weight of the old generation in the combined efficiency-redistribution model

Recall the expression of the weight of the old generation in the main text,

$$\beta = \frac{(1+n)}{(1+n) + \hat{\Delta} S_{R-Y}}$$

which, given the definition of $\hat{\Delta}$, can be written as

$$\beta = \frac{(1+n) S_G (1 - \Delta(1+n))}{(1+n) S_G (1 - \Delta(1+n)) + \Delta S_{R-Y}}$$

Further, recall our assumption that $\mu_R > \mu_Y$. We can then derive the following about the signs of the derivatives of this weight with respect to μ_R and μ_Y:

$$\partial \beta / \partial \mu_R = \frac{-2\beta\Delta(\mu_R - \mu_Y)}{(1+n) S_G (1 - \Delta(1+n)) + \Delta S_{R-Y}}$$

Without any further assumptions, we note that $\partial \beta / \partial \mu_R < 0$.

$$\partial \beta / \partial \mu_Y = \frac{2(1-\beta)((1+\mu_Y)/(1+n))(1 - \Delta(1+n)) + 2\beta\Delta(\mu_R - \mu_Y)}{(1+n) S_G (1 - \Delta(1+n)) + \Delta S_{R-Y}}$$

Without any further assumptions, we note that $\partial \beta / \partial \mu_Y > 0$.

$$\partial \beta / \partial \sigma_R = \frac{-2\beta\Delta(X_R/\sigma_R)}{(1+n) S_G (1 - \Delta(1+n)) + \Delta S_{R-Y}}$$

If we further assume that $X_R > 0$ (see Table 2), $\partial \beta / \partial \sigma_R < 0$.

$$\partial \beta / \partial \sigma_Y = \frac{2(1-\beta)\sigma_Y(1 - \Delta(1+n))/(1+n) - 2\beta\Delta(X_Y/\sigma_Y)}{(1+n) S_G (1 - \Delta(1+n)) + \Delta S_{R-Y}}$$

If we further assume that $X_Y < 0$ (see Table 2), $\partial \beta / \partial \sigma_Y > 0$.

$$\partial \beta / \partial \rho = \frac{2\beta\Delta\sigma_R\sigma_Y}{(1+n) S_G (1 - \Delta(1+n)) + \Delta S_{R-Y}}$$

Without any further assumptions, $\partial \beta / \partial \rho > 0$.

C   Derivation of the policy function of the benevolent social planner model

In order to find the policy function that maximizes the social welfare function in equation (6), we have to take three steps. First, we derive the first-order condition for optimal policies, $\partial SWF_t/\partial \tau_t = 0$, where the values of the policy instrument in other periods, $\tau_{t+i}\ i \neq 0$, are all taken as given:

$$
\begin{aligned}
&\tau_t(1+n) + \omega_t \\
&-\ \tilde{\Delta}\Big(E\big[\tau_{t+1}(1+Y_{t+1})(1+R_{t+1})\big] + (1-\tau_t)S_R - (1+\mu_R)\gamma\Big) \\
&=\ 0
\end{aligned}
\tag{C.1}
$$

The second step is to make a guess for the functional form of the policy function $\tau_t(\omega_t)$. Given the quadratic nature of the utility function, we assume a linear form:

$$
\tau_t = \xi_0 + \xi_1\omega_t
\tag{C.2}
$$

This implies that the expectations term on the second line of equation (C.1) can be elaborated as follows:

$$
\begin{aligned}
E\big[\tau_{t+1}(1+Y_{t+1})(1+R_{t+1})\big] &= \\
\xi_0 S_{RY} + \xi_1(1+n)(1-\tau_t)S_R &- \xi_1\gamma(1+n)(1+\mu_R)
\end{aligned}
\tag{C.3}
$$

Substituting this result in equation (C.1) and rearranging terms gives the following expression for τ_t:

$$
\begin{aligned}
\big((1+n) + \tilde{\Delta}S_R + (1+n)\xi_1\tilde{\Delta}S_R\big)\tau_t &= \\
-\ \omega_t + \tilde{\Delta}S_R - \tilde{\Delta}(1+\mu_R)\gamma &+ \tilde{\Delta}\xi_0 S_{RY} \\
+\ \tilde{\Delta}\xi_1(1+n)S_R - \tilde{\Delta}\xi_1\gamma(1+n)(1+\mu_R)
\end{aligned}
\tag{C.4}
$$

The third step is to match this expression with our earlier guess (equation (C.2)), which yields the following expressions for the two coefficients of the policy function:

$$
\begin{aligned}
\xi_0 &= \frac{\tilde{\Delta}S_R - \tilde{\Delta}(1+\mu_R)\gamma + \tilde{\Delta}\xi_0 S_{RY}}{(1+n) + \tilde{\Delta}S_R + (1+n)\xi_1\tilde{\Delta}aS_R} \\
&+ \frac{\tilde{\Delta}\xi_1(1+n)\,(S_R - \gamma(1+\mu_R))}{(1+n) + \tilde{\Delta}S_R + (1+n)\xi_1\tilde{\Delta}S_R}
\end{aligned}
\tag{C.5}
$$

$$
\xi_1 = \frac{-1}{(1+n) + \tilde{\Delta}S_R + (1+n)\xi_1\tilde{\Delta}S_R}
\tag{C.6}
$$

Equation (C.6) is a quadratic equation for ξ_1 with two candidate solutions of which only one yields that $\xi_0 \neq 0$: $\xi_1 = -1/(\tilde{\Delta} S_R)$.

The corresponding expression for ξ_0 reads as follows:

$$\xi_0 = \frac{\tilde{\Delta} S_R - (1+n) + \gamma(1+\mu_R)\left(\frac{(1+n)}{S_R} - \tilde{\Delta}\right)}{\tilde{\Delta}(S_R - S_{RY})} \tag{C.7}$$

Substituting the expressions for ξ_0 and ξ_1 into equation (C.2) and rewriting gives us the expression of the policy function in the main text (equation (8)):

$$\tau_{tc,t}^* = \frac{(\gamma(1+\mu_R) - S_R)((1+n) - \tilde{\Delta} S_R)}{\tilde{\Delta} S_R(S_R - S_{RY})} - \frac{1}{\tilde{\Delta} S_R}\omega_t \tag{C.8}$$

D Determinants of the optimal PAYG rate in the time-consistent model

We rewrite the expression for the optimal PAYG rate in the benevolent social planner model (equation (8)) by substituting $\Delta(1+n)^2/S_Y$ for $\tilde{\Delta}$,

$$
\begin{aligned}
\tau_{tc,t}^* &= \frac{(1+n)S_Y(\gamma(1+\mu_R) - S_R)(1 - \Delta(1+n)S_R/S_Y)}{\Delta(1+n)^2 S_R(S_R - S_{RY})} \\
&\quad - \frac{S_Y}{\Delta(1+n)^2 S_R}\omega_t \\
&= \hat{\tau}_{tc}^* - \tilde{\omega}_t
\end{aligned} \tag{D.1}
$$

where the last line defines implicitly $\hat{\tau}_{tc}^*$ and $\tilde{\omega}_t$.

In order to sign the intercept, $\hat{\tau}_{tc}^*$, note that we assumed in the main text that $\gamma > max(2 + \bar{R} + \bar{Y}, S_R/(1+\mu_R))$ and that $\Delta > S_Y/(S_R(1+n))$. If we further assume that $S_R > S_{RY}$ and $X_R > 0$ (see Table 2), the intercept is negative: $\hat{\tau}_{tc}^* < 0$.

In order to find out how the moments of the return-growth distribution affect the optimal contribution rate, we again employ differentiation. Note that equation (D.1) implies that $\partial \tau_{tc,t}^*/\partial x = \partial \hat{\tau}_{tc}^*/\partial x - \partial \tilde{\omega}_t/\partial x$ for $x = \mu_R, \mu_Y, \sigma_R, \sigma_Y, \rho$. Let us focus first on the case $x = \mu_R$. Note that, since $\omega_t < 0$, $\partial \tilde{\omega}_t/\partial \mu_R > 0$. Hence, a *sufficient* condition for $\partial \tau_{tc,t}^*/\partial \mu_R$ to be negative is that $\partial \hat{\tau}_{tc}^*/\partial \mu_R < 0$.

Upon differentiation of the intercept with respect to μ_R, we find the following expression:

$$\partial \hat{\tau}_{tc}^*/\partial \mu_R = -\frac{(\gamma - 2(1 + \mu_R))}{(S_R - S_{RY})} - \frac{\gamma(1 + n)((1 + \mu_R)^2 - \sigma_R^2)}{\Delta S_R^2 (S_R - S_{RY})/S_G}$$
$$- \hat{\tau}_{tc}^* \frac{(2(1 + \mu_R) - (1 + \mu_Y))}{(S_R - S_{RY})}$$

In order to sign this derivative, we make three additional assumptions. First, we assume that $\overline{Y} > \mu_R$, which, given that we already assumed $\gamma > 2 + \overline{R} + \overline{Y}$, implies that $\gamma > 2(1 + \mu_R)$. Second, we assume $(1 + \mu_R) > \sigma_R$ (see Table 2). Thirdly, we assume $\Delta - S_Y/(S_R(1 + n))$ not too large. Then, $\partial \hat{\tau}_{tc}^*/\partial \mu_R$ is strictly negative.

The expression for $\partial \hat{\tau}_{tc}^*/\partial \mu_R$ shows clearly the role of the assumption that $\Delta - S_Y/(S_R(1 + n))$ is not too large. If we would impose for the sake of argument that $\Delta - S_Y/(S_R(1 + n)) = 0$, $\hat{\tau}_{tc}^*$ would be zero and $\partial \hat{\tau}_{tc}^*/\partial \mu_R$ would be strictly negative. Invoking a continuity argument, the same result applies as long as $\Delta - S_Y/(S_R(1 + n))$ is not too large.

For the other derivatives, the reasoning is similar. The derivative with respect to the average rate of growth is as follows:

$$\partial \hat{\tau}_{tc}^*/\partial \mu_Y = \frac{2(1 + \mu_Y)}{(1 + n)} \frac{(\gamma(1 + \mu_R)/S_R - 1)}{\Delta(S_R - S_{RY})} + \hat{\tau}_{tc}^* \frac{(1 + \mu_R)}{(S_R - S_{RY})}$$

The first term on the RHS is positive, the second one negative. As long as $\Delta - S_Y/(S_R(1 + n))$ is not too large, $\partial \hat{\tau}_{tc}^*/\partial \mu_Y$ is thus positive. Since $\partial \tilde{\omega}_t/\partial \mu_Y < 0$, this ensures that $\partial \tau_{tc,t}^*/\partial \mu_Y$ is positive.

The expression for the derivative with respect to the standard deviation of the capital market rate of return is more complicated:

$$\partial \hat{\tau}_{tc}^*/\partial \sigma_R = -2\sigma_R \frac{(S_Y/(1 + n) - \Delta S_R)}{\Delta S_R (S_R - S_{RY})} - 2\sigma_R \frac{(\gamma(1 + \mu_R) - S_R)}{S_R (S_R - S_{RY})}$$
$$- \hat{\tau}_{tc}^* \frac{(2\sigma_R(S_R - S_{RY}) + \sigma_R S_R + S_R(X_R/\sigma_R))}{S_R(S_R - S_{RY})}$$

We make the further assumption that $X_R > 0$ (see Table 2). We then have that the first and third term on the RHS are positive and the second one negative. Even if Δ is not too large, we cannot conclude anything about the sign of $\partial \hat{\tau}_{tc}^*/\partial \sigma_R$.

The derivative with respect to the standard deviation of the growth is simpler:

$$\partial \hat{\tau}_{tc}^*/\partial \sigma_Y = \frac{(2\sigma_Y/(1 + n))(\gamma(1 + \mu_R) - S_R)}{\Delta S_R (S_R - S_{RY})} + \hat{\tau}_{tc}^* \frac{\rho \sigma_R}{(S_R - S_{RY})}$$

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We assume that $\rho > 0$ (see Table 2). Again, if Δ is not too large, $\partial \hat{\tau}_{tc}^* / \partial \sigma_Y$ is positive. Since $\partial \tilde{\omega}_t / \partial \sigma_Y < 0$, this ensures that $\partial \tau_{tc,t}^* / \partial \sigma_Y$ is positive.

Finally, the impact of the degree of correlation on the contributon rate is negative, even without assuming that Δ is not too large:

$$\partial \hat{\tau}_{tc}^* / \partial \rho \quad = \quad \hat{\tau}_{tc}^* \frac{\sigma_R \sigma_Y}{(S_R - S_{RY})}$$

Since $\partial \tilde{\omega}_t / \partial \rho = 0$, this ensures that $\partial \tau_{tc,t}^* / \partial \rho$ is negative as well.

Summing up, we can thus in general say something about the signs of the various derivatives if we make some additional assumptions. The above reasoning leaves open the question however whether the derivatives change sign if $\Delta - S_Y / (S_R (1+n))$ turns relatively large since we analyzed a sufficient condition, not a necessary condition. To find this out, we adopt a numerical approach. We evaluate the various derivatives of the optimal contribution rate, $i.e.$ $\partial \tau_{tc,t}^* / \partial x$ for $x = \mu_R, \mu_Y, \sigma_R, \sigma_Y, \rho$, using values for the moments of the return-growth distribution as displayed in table 2, using $\bar{R} = 6.75$ and $\bar{Y} = 2.85$, using values for γ over a wide range that obey the inequality condition $\gamma > max(2 + \bar{R} + \bar{Y}, S_R / (1 + \mu_R))$ (specifically, γ should be higher than 11.6 and the values used are in between 16.6 and 156.6), using values for Δ over a wide range that obey the minimum and maximum condition, $\Delta > S_Y / (S_R (1+n))$ and $\Delta < 1/(1+n)$ respectively (specifically, Δ should be higher than 0.04 and lower than 0.63 and the values used are in between 0.05 and 0.52), and using three different values for the state variable: $1 + \bar{R} - \gamma$, $0.5(1 + \mu_R) - \gamma$, and $-\gamma$. In general, we find that the results found under the condition that Δ is not too large, extend to higher values of Δ, except for the case of the derivative of $\tau_{tc,t}^*$ with respect to σ_R: the sign of this derivative remains ambiguous.