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Optimal pension contract for heterogeneous agents accommodating for life events

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Optimal pension contract for heterogeneous agents accommodating for life events

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Abstract

The current Dutch cabinet has announced reforms into the direction of further individualizing the pension contract. This could lead to personalized pensions for heterogeneous agents accommodating for life events with collective risk sharing. This thesis derives the optimal dynamic contribution scheme and investment strategy over the life cycle for pension plan participants. A suboptimal pension contract, in terms of suboptimal contribution scheme and suboptimal investment strategy, will lead to welfare losses. These welfare losses are crucial in the policy discussion of the attractiveness of a customized pension contract. Tailoring the pension contract, in terms of contribution scheme and asset allocation, on heterogeneity characteristics such as risk aversion and assumed wage profile provides substantial welfare gain under the assumptions made in this thesis. Adjusting the contribution scheme in the pension contract based on life events such as receiving an unexpected bequest, housing wealth and temporary unemployment is also significantly welfare enhancing. Adjusting the investment strategy optimally in the pension contract based on life events is marginally attractive. Including the relative importance of the state pension in the analysis (performed for a limited number of cases) yields similar policy implications, though the quantitative results change. For example, the welfare gain of adjusting the contribution rate in the pension contract on the accumulation of housing wealth crucially depends on the ratio of housing wealth to labor income. Adjusting the contribution scheme in the pension contract to realized asset returns might be attractive as well.

Keywords: Individual Defined Contribution (IDC), flexible pension contract, accumulation phase, heterogeneity and life events

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Beleidssamenvatting

Aanleiding

De laatste jaren verschuift de pensioensector langzaam maar zeker van Defined Benefit (DB) naar Defined Contribution (DC) pensioencontracten. In Bovenberg en Nijman (2017) [13] wordt voor deze transitie een duidelijke achtergrond gegeven. Deze ontwikkeling roept de vraag op of iedere participant een flexibel pensioencontract op maat moet krijgen. Een eerste implicatie is dat het pensioencontract afgestemd kan worden op de heterogeniteit die in de samenleving aanwezig is, al bij het betreden van de arbeismarkt (bijvoorbeeld risicopreferenties en verwacht carrièrepatroon). Een andere implicatie is dat het pensioencontract flexibiliteit (openbreken van het huidige pensioencontract) kan bieden bij veranderende karakteristieken (life events), zoals de aankoop van een eigen woning, scheiding of huwelijk en promotie.

Opzet modelanalyse

Deze scriptie is gericht op het bepalen van het optimale pensioencontract voor heterogene deelnemers waarvan de karakteristieken kunnen veranderen. Dit komt overeen met de bepaling van de flexibele, optimale premie-inleg en pensioenuitkering enerzijds. Anderzijds, wordt het flexibel, optimaal beleggingsbeleid afgeleid. Het kwantificeren van welvaarstverliezen van een uniform en inflexibel pensioencontract heeft als doel te identificeren waar maatwerk een toegevoegde waarde heeft voor de deelnemer. Deze scriptie kan gezien worden als aanvulling op Van Ewijk et al. (2017) [24], met overlap in en uitbreiding op de casussen. Belangrijk verschil in de modelanalyse is dat deze scriptie premiesturing toestaat.

Nadat het standaard life cycle model is besproken, dat gekalibreerd is op vlakke verwachte consumptie, zal een achtergrond worden gegeven van de wetgeving op fiscaal gefaciliteerde pensioenopbouw in de Nederlandse setting. Hierna zullen een aantal illustratieve casussen omtrent heterogeniteit worden geconstrueerd. In deze scriptie zullen de implicaties van heterogeniteit in de risicopreferenties en het verondersteld carrièrepatroon voor het gewenste pensioencontract geanalyseerd worden.

Naast verschillen aan de start van de life cycle, zijn er veel omstandigheden die wijzigingen in de gewenste inleg, uitkering of eerder voorgenomen beleggingsbeleid rechtvaardigen. Deze wijzingen vatten we samen onder de term 'wijzigingen in het pensioencontract' alhoewel daar juridisch geen sprake van hoeft te zijn. Wederom is gekozen voor een aantal illustratieve casussen in de vorm van het ontvangen van een erfenis (schok in financieel kapitaal), het aankopen van een huis (schok in additioneel kapitaal) en het tijdelijk werkloos zijn (schok in menselijk kapitaal). De casussen zijn gekozen op een manier zodat de afleiding van het optimale pensioencontract van een specifiek veranderde karakteristiek, die niet besproken wordt in deze scriptie, te herleiden valt tot een van deze casussen. Bijvoorbeeld het aanpassen van de pensioenleeftijd (schok in menselijk kapitaal).

Er dient gerealiseerd te worden dat het optimale pensioencontract en welvaartseffecten, gegeven het standaard life cycle model als uitgangspunt, doorgaans afgeleid zijn voor inkomenscategorieën waarvoor het relatieve belang van de AOW gering is. Door het meenemen van de AOW in de analyse kan in hoofdstuk 5 het optimale pensioencontract en welvaartswinsten worden afgeleid voor alle inkomensgroepen. Merk ook op dat de inkomensdistributie, door het relatieve belang van de AOW aan het model toe te voegen, een vorm van heterogeniteit wordt. Ook is zorgvuldigheid genoodzaakt bij de interpretatie van het optimale pensioencontract voor de belanghebbenden in de verschillende pijlers. Pensioenfondsen, sociale partners en deelnemers dienen zich te realiseren dat de resultaten in termen van spaar- en ontspaarbeslissing en beleggingsbeleid gecorrigeerd moeten worden voor het relatieve belang van de AOW. Voor een gelimiteerd aantal casussen is het optimale pensioencontract in de tweede pijler/derde pijler gegeven de eerste pijler afgeleid.

Ook zal de aantrekkelijkheid van aanpassingen in het pensioencontract op basis van reeds opgebouwd pensioenvermogen geanalyseerd worden. Het opgebouwd pensioenvermogen wordt bepaald door premie-inleg, beleggingsbeleid en gerealiseerde rendementen. Merk op dat gerealiseerde rendementen gezien kunnen worden als een speciale vorm van een veranderende karakteristiek. Bij tegenvallende gerealiseerde rendementen zal geanalyseerd worden in hoeverre afwijkingen in het beleggingsbeleid (garanties/risicoprofiel verhogen) wenselijk zijn, of dat additionele premie-inleg meer geschikt is om een eventueel target te kunnen behalen.

De strategieën zullen in de analyse beperkt blijven tot de opbouwfase en geëvalueerd worden op basis van de distributie van beschikbaar pensioenkapitaal op de pensioenleeftijd.

Beleidsrelevante conclusies
In de volgende alinea's zullen de belangrijkste conclusies van deze scriptie besproken worden. Als uitgangspunt is er gekozen voor het gangbare life cycle model (Merton model). De welvaartsanalyses zijn uitgevoerd op basis van het gestandardiseerde verschil in zekerheidsequivalent tussen de suboptimale strategie en de optimale strategie en worden gebruikt bij het in kaart brengen waar het aanbieden van maatwerk in het pensioencontract aantrekkelijk is voor zowel deelnemer als pensioenuitvoerder. Deze welvaartswinst duidt aan hoeveel verandering van het vermogen in een suboptimaal contract dat contract even aantrekkelijk maakt als het optimale contract. Dit betreffen welvaartsverliezen van suboptimaal beleggingsbeleid en een suboptimale spaar- ontspaarbeslissing.

Heterogeniteit

Risicopreferenties
Onderscheid maken in de risicopreferenties leidt tot hoge welvaartswinsten. Deze zijn hoger dan die worden gevonden in Van Ewijk et al. (2017) [24]. Een oorzaak kan zijn dat in deze scriptie de pensioendeelnemer kan lenen tegen menselijk kapitaal, wat in de beginjaren leidt tot een enorme hoeveelheid aandelenrisico. Echter, de aanname van premiesturing in deze scriptie (anders dan in Van Ewijk et al. (2017) [24] gaat men meer premie betalen na tegenvallende rendementen) lijkt numeriek belangrijker. Ondanks lagere welvaartswinsten, al gecorrigeerd voor AOW, geven de resultaten uit Van Ewijk et al. (2017) [24] ook aan dat het aanbieden van verschillende risicoprofielen aantrekkelijk is. Welvaartsverliezen van suboptimale premie-inleg zijn in de setting van deze scriptie substantieel lager dan de waarde van maatwerk in beleggingsbeleid.

Verondersteld carrièrepatroon
Bij een sterk verwacht groeiend carrièrepatroon zal het onderscheid maken naar risicoprofielen in beleggingsbeleid leiden tot welvaartswinsten van 0.5 %. Pensioenuitvoerders bieden tegenwoordig verschillende risicoprofielen aan, gedifferentieerd op risicopreferenties. In de onderliggende bepaling van de verschillende beleggingsprofielen zit slechts één verondersteld carrièrepatroon. Het meenemen van verschillende carrièrepatronen of het op een slimme manier kiezen van een onderliggend verondersteld carrièrepatroon lijkt wenselijk onder de hier gemaakte veronderstellingen. Het aanpassen van de premie-inleg op het verwachte carrièrepatroon kan binnen deze analyse leiden tot een welvaartswinst van 10 % bij een sterk verwacht carrièrepatroon. In het bijzonder is het voor deelnemers met een hoog menselijk kapitaal (hoog opleidingsniveau) aantrekkelijk om in de eerste jaren van de levenscyclus weinig bij te dragen aan pensioen.

Veranderende karakteristieken
In de veranderende karakteristieken die hier geanalyseerd zijn kan worden geconcludeerd dat het aanpassen van het beleggingsbeleid leidt tot marginale welvaartswinsten. Dit in tegenstelling tot de besproken vormen van heterogeniteit, waar beleggingsbeleid een dominantere factor is. Het optimaal spreiden van consumptie (dat wil zeggen het aanpassen van de premie) over de life cycle is bij veranderende karakteristieken veelal de dominante factor.

Ontvangst van een erfenis
Bij het ontvangen van een erfenis van 3.3 jaarsalarissen liggen de welvaartsverliezen voor gangbare consumptiestrategieën tussen de 5-10 %. In deze situatie is het voor de deelnemer, met als target vlakke verwachte consumptie, interessant om extra bij te sparen. Binnen de grenzen van het Witteveenkader, kan dit fiscaal gefaciliteerd altijd in de tweede of derde pijler. Een benadering van het optimum zou kunnen zijn dat er in de volgende jaren een deel van de erfenis (ad hoc bepaald) binnen de grenzen van het Witteveenkader wordt bijgespaard om ook het langlevenrisico af te kunnen dekken. Er is geen uitgebreide analyse gedaan in hoeverre bijsparen, niet fiscaal gefaciliteerd, optimaal is. Dit omdat wordt afgezien van belastingen.

Huis
In Van Ewijk et al. (2017) [24] leidt afstemming van de premie-inleg, bij opbouw van pensioenvermogen via

de eigen woning (gedefinieerd als percentage oversparen) tot welvaartswinsten van 0.5 % - 6.8 % afhankelijk van de waarde van de woning. Deze scriptie kwantificeert welvaartswinsten van suboptimale premie-inleg, als op de pensioenleeftijd voor een omkeerhypotheek wordt gekozen, op 0.7 % - 10.7 % en 0.8 % - 8.4 % onder de aanname dat de waardeontwikkeling van de eigen woning risicovrij respectievelijk onzeker wordt beschouwd. Kortom, het optimale pensioencontract moet de deelnemer de optie geven tot minder premie-inleg ter voorkoming van oversparen. Het is belangrijk de premie-inleg in het contract te conditioneren op de keuze rondom de eigen woning. Als de participant in deze setting in de eigen woning blijft wonen en deze nalaat als erfenis treden hoge welvaartsverliezen op (bijvoorbeeld 17.0 % bij een risicovrije middencategorie woning). Dit welvaartsverlies zal in de praktijk een stuk kleiner zijn, aangezien dit model veronderstelt dat het nalaten van de eigen woning via een erfenis geen nut oplevert. Ook zijn de resultaten hierbij nog niet gecorrigeerd voor AOW dat kan worden geinterpreteerd als een minimaal consumptielevel.

Werkloosheid
In de ongelukkige situatie waarin de deelnemer onvrijwillig werkloos wordt, is de huidige situatie dat consumptie beperkt wordt tot de werkloosheidsuitkering (geen verdere inleg, maar ook geen opname van pensioenvermogen). Geld onttrekken aan de pensioenpot, wat in sommige landen (Singapore en Maleisië) mogelijk is, lijkt onder de hier gemaakte veronderstellingen verstandig in termen van welvaartswinsten (+1.1 %). Aansluitend hierop kan geconcludeerd worden dat in een tijd van werkloosheid het niet aangaan/aflossen van een dure lening met pensioenvermogen aantrekkelijk kan zijn (Van Ewijk et al. (2017) [24]). Bijsparen voor pensioen blijkt onder de hier gemaakte veronderstellingen een slecht devies. Hierbij moet zelfs geconcludeerd worden dat doorsparen mogelijk is, maar dat een werkgeversbijdrage vaak vervalt. Dit leidt ertoe dat het welvaartsverlies in deze situatie een te optimistisch beeld weergeeft.

Relatief belang AOW op de inkomensheterogeneit: toegevoegde waarde maatwerk pensioencontract in de tweede en derde pijler
Zoals vermeld, zijn de resultaten van alle casussen tot nu toe alleen interpreteerbaar voor inkomenscategorieën waarvoor het relatieve belang van de AOW gering is. Interpretatie voor andere inkomenscategorieën is mogelijk als de analyse gecorrigeerd wordt voor de relatieve impact van de AOW. In feite kunnen we concluderen dat het inkomensniveau ook een belangrijke dimensie van heterogeniteit is in een model waarin AOW wordt meegenomen. Kortom, belangrijk is dat bij bepaling van het optimale pensioencontract (in de tweede pijler en derde pijler) en welvaartswinsten beide gecorrigeerd moeten worden voor het relatieve belang van de AOW.

Algemeen kan gesteld worden dat het optimale beleggingsbeleid in de tweede pijler offensiever wordt als het voor de AOW gecorrigeerd wordt. Verder kan er als hypothese gesteld worden dat de welvaartswinsten halveren voor de gemiddelde deelnemer.

Gezien de praktische relevantie is er gekozen om de casussen omtrent risicopreferenties en de aankoop van een huis uitgebreider te analyseren in een model waarin gecorrigeerd wordt voor het relatieve belang van de AOW. Het meenemen van het relatieve belang van het veronderstelde risicovrije staatspensioen in termen van beleggingsbeleid leidt voor participanten tot marginale welvaartswinsten. Zoals verwacht nemen de welvaartsverliezen van een suboptimaal contract, gedifferentieerd op de risicopreferenties af. De beleidsimplicaties blijven desondanks intact. In deze meer realistischere setting waar gekeken wordt naar welvaartswinsten voor mensen met een eigen woning die op de pensioenleeftijd kiezen voor een omkeerhypotheek blijven de welvaartswinsten significant. Net als in de setting waar het relatieve belang van de AOW gering is, hangen deze sterk af van de ratio waarde eigen woning ten opzichte van het salaris.

Gerealiseerde rendementen
Na constatering van het achterlopen op schema, in termen van opgebouwd pensioenvermogen, zal het aanpassen van het beleggingsbeleid (risicoprofiel verhogen binnen redelijke grenzen) maar gedeeltelijk helpen om tot een beoogde vervangingsratio te komen. Extra premie-inleg is in dat geval het devies. In het bijzonder is het aantrekkelijk om een dynamische premie-inleg te hebben ten opzichte van opbouwen via een a priori (constante) vastgestelde staffel.
Als de garantiestrategie in termen van beleggingsbeleid wordt toegepast, zal er sprake zijn van een minimaal

consumptielevel tijdens pensioen (een garantieniveau).

Vervolgonderzoek
In deze scriptie is een versimpelde versie van de werkelijkheid geschetst. Dit betekent dat renterisico, langlevenrisico, belastingeffecten, onzekerheid over het veronderstelde carrièrepatroon en leenrestricties genegeerd zijn. Het meenemen van deze risico's en beperkingen zal als gevolg hebben dat het optimale pensioencontract afwijkend kan zijn. Het analyseren van deze setting, waarin deze extra factoren meegenomen worden, zal voor vervolgonderzoek zijn. Echter, naar verwachting heeft dit geen gevolgen voor de richting van de beleidsimplicaties. Toch is dit vervolgonderzoek erg van belang voor de nauwkeurigere bepaling van het optimale pensioencontract (bijvoorbeeld ten behoeve van productontwikkeling).

Hoofdpunten
Hieronder de belangrijkste beleidsimplicaties nogmaals puntsgewijs geformuleerd.

- Het feit dat sociale partners veelal tegen de maximale grens van de fiscaal gefaciliteerde pensioensopbouw uit onderhandelen heeft als nadeel weinig flexibiliteit[1] in de premie-inleg voor deelnemers.
- Maatwerk in het pensioencontract, gedifferentieerd op risicopreferenties, is in een setting met premiestuur nog relevanter dan gevonden in Van Ewijk et al. (2017) [24].
- Pensioencontributies afstemmen op het veronderstelde (deterministische) carrièrepatroon lijkt wenselijk. In het bijzonder kan dit de eerste jaren na het betreden van de arbeidsmarkt leiden tot weinig/geen opbouw voor hoog opgeleiden die een sterk verondersteld carrièrepatroon hebben.
- Na het ontvangen van een significante erfenis, is extra premie-inleg bij een pensioenfonds gewenst om de consumptiestandaard tijdens pensioen te kunnen behouden, evenals het kunnen afdekken van het langlevenrisico.
- Lagere premie-inleg is gerechtvaardigd voor eigenwoningbezitters ten opzichte van huurders. Het pensioencontract zal de lagere premie-inleg moeten conditioneren op het al dan niet inzetten van de waarde van de woning tijdens pensioen.
- Geld opnemen uit de pensioenpot kan in tijden van werkloosheid een aantrekkelijke optie zijn.
- In de absentie van heterogeniteit en veranderende karakteristieken, zijn er in termen van beleggingsbeleid geringe welvaartswinsten te behalen als het relatieve belang van het veronderstelde risicovrije staatspensioen wordt meegenomen.
- In een meer realistischere setting, waar AOW wordt toegevoegd aan het model, blijven voor de geanalyseerde casussen (risicopreferenties en aankoop eigen woning) de beleidsimplicaties intact.
- Premiesturing is een krachtiger middel dan beleggingsbeleid indien een vooraf vastgestelde vervangingsratio noodzakelijk is om te behalen.

[1] In het bijzonder wordt hier gerefereerd aan het feit dat minder premie-inleg dan is afgesproken door de sociale partners niet mogelijk is.

Contents

1 Introduction

In the Netherlands, the pension system is characterized by three pillars. The first pillar is a state pension, for all inhabitants with the aim to alleviate poverty. The current working generation is responsible for financing this basic income of the current retirees, which is a Pay As You GO (PAYGO) system. The second pillar is mandatory for all employees in the Netherlands, with the aim to maintain a standard of living during retirement. The second pillar pension income basically depends on four aspects: pre-retirement income, pension contribution rates, asset allocation and the investment returns. The second pillar is financed via a funded system. Voluntary additional savings can be made in the third pillar.

In the second pillar, the pension industry is slowly, but surely moving from Defined Benefit (DB) pension schemes towards Defined Contribution (DC) pension schemes. A detailed background on this transition can be found in Bovenberg and Nijman (2017) [13]. Individual Defined Contribution (IDC) schemes are becoming more and more popular. Currently there is a discussion taking place on whether or not to move the whole Dutch pension system to IDC. Together with the Socio Economic Council (SER) the current cabinet soon hopes to reach an agreement to develop personal pensions with collective risk sharing[2]. An advantage of IDC schemes is that the investment policy is tailored to the investment horizon of the participant, whereas in a Defined Benefit (DB) or Collective Defined Contribution (CDC) scheme there is just one investment policy for all participants. The difference between the IDC and CDC pension scheme is whether the pension contributions and investment returns will be on a private or collective account. The focus of this thesis will be on personalized pensions. This individual component raises the question whether the pension industry should move in the direction of further individualizing the life cycles of the participants. This individualization could be in terms of asset allocation, pension contributions and dissaving strategies, motivated by a heterogeneous population accommodating for life events. It is well known that age only explains one dimension of the heterogeneity in the population. However, other important heterogeneous aspects are still ignored at the moment. One could think of heterogeneous aspects (risk aversion, assumed wage profile, targeted replacement rates and differences in survival probabilities) which are known at the start of the life cycle. In addition to this, changes in the characteristics of the participant (life events) will arise during the life cycle of a participant (bequest, housing wealth and temporary unemployment). A special case of a life event will be the realized asset returns.

The second pillar is the most relevant pillar to incorporate the implications of the heterogeneous population (tailored pension contract) and the implications of life events (flexibility in the pension contract) in the pension system. The first pillar has little flexibility, whereas the third pillar is relatively small for most Dutch inhabitants.

One commonly used approach to determine the optimal asset allocation in IDC is based on the investment horizon of the participant (i.e. age) and the risk preferences of the average pension plan participant. Note that this level of risk aversion cannot be explained by observable characteristics (Alserda, Dellaer, Swinkels & van der Lecq, 2016) [3]. As a consequence, pension funds use of elicitation method to estimate the risk preferences of the participant. As a final step, the pension fund has to motivate the risk profile in the life cycle. This risk profile will typically reflect the average risk preference of the participants. Some pension funds develop several life cycles based on different risk profiles. The participants are then divided into one of the life cycles based on a test. For example, NN distinguishes between a defensive, neutral and offensive life cycle profile. Being able to calculate the risk preferences in itself does not define a life cycle, additional assumptions are needed for this, for more details see Subsection 2.1. For the pension contributions, a limited number of contribution rates (premiestaffels) are available. The pension contributions during the accumulation phase can then be derived from the contribution rates. There can be substantial heterogeneity in the pension contributions among different pension funds. Since the introduction of the Witteveenkader (2015), contribution rates are more restricted. In September 1 2016 the 'Wet Verbeterde Premieregelingen' was introduced. This law ensured additional choices on how to annuitize the pension wealth at the (flexible) retirement date. The retiree could either choose a variable annuity which implies some investment risk during

[2] In the coalition agreement (Vertrouwen in de toekomst, 2017) between VVD, CDA, D66 and ChristenUnie, it is stated that pension reforms will be an important challenge for the current cabinet.

retirement or a fixed annuity which is offered as the default by pension funds.

The actual replacement rate[3] refers to the percentage of labor income that could be used for consumption during retirement. The first and second pillar pension income are the main components of the retirement income. Private wealth (among others pillar 3 and bequest) and housing wealth (link with life events) can increase the actual replacement rate significantly. Knoef et al. (2014) [37] and Rhuggenaath, Been, Goudswaard, Caminada, and Knoef (2017) [42] show the existence of significant variation in the actual replacement rates of different subgroups. The heterogeneity characteristics and life events could justify deviations from the prescribed asset allocation and (dis)saving decision at a specific point in time. The purpose is to smooth consumption optimally over the life cycle (i.e. have an optimal pension contract). These heterogeneity characteristics and life events could lead to significant variation in the actual replacement rates.

De Bresser and Knoef (2015) [15] analyze whether the accumulated pension wealth of the pension plan participants meets the preferred and minimum retirement income. The substantial heterogeneity in these (preferred) self-indicated goals, justifies a tailored (dis)saving and/or investment strategy. According to De Bresser, Kools, and Knoef (2017) [16] the income level will be an important factor explaining the variation in targeted replacement rates. Higher income levels need lower targeted replacement rates, whereas lower income levels need higher targeted replacement rates. In the UK, the PC (Pension Comission, 2014) advised integrating income dependent targeted replacement rates in the pension industry. De Bresser et al. (2015) [16] also concluded that targeted replacement rates have changed as a reaction to the financial crisis. This literature is relevant to mention, although this thesis will not go into the details of heterogeneity in the targeted replacement rates based on the income level[4].

To conclude, a suboptimal pension contract implies that consumption is not smoothened optimally over the life cycle (either a suboptimal (dis)saving decision or investment decision took place). The goal of this thesis is to identify heterogeneity characteristics and life events, resulting in different desirable pension contract in the second pillar.

According to the optimal pension contract both suboptimal contribution schemes (limited choice on how much to contribute) and suboptimal asset allocations (typically several life cycle profiles available) could lead to welfare losses. These welfare effects will be calculated by the method of consumption certainty equivalents if a functional form of the utility is imposed. This quantifies the importance to have a tailored pension contract for the participant and can guide pension funds in determining the attractiveness of incorporating the individualization in their product innovation. Otherwise, the suboptimality of the contract can be characterized by an overview of the distribution of the available pension wealth at retirement age.

Increased customization could have disadvantages as well. On the one hand, it could give the participant an incentive to make a strategic choice that harms the solidarity of the pension system. On the other hand individual mistakes could lead to welfare losses as well. In Van Ewijk, Hoet, Mehlkopf, and Van den Bleeken (2017) [24], it was concluded that there are limited concerns for strategic decision making, whereas the institutional setting should be designed in a way that limits the welfare losses of a wrong decision. Moreover, more customization might impose higher administrative costs.

Both components (contribution scheme and asset allocation) are relevant in practice. Nowadays, the asset allocation should reflect the investment horizon of the participant and the risk preferences of the pool. Pension funds are able to influence the asset allocation. It was typically assumed that many Dutch participants are characterized by oversaving, although several vulnerable groups have inadequate pension income. Currently, the participant has little control over the amount of pension contributions. Still, the pension provider could advise these participants to save more in the third pillar pension in case of undersaving. At this point in

[3]This thesis ignores taxes.

[4]In Section 5, this thesis will take into account the state pension to calculate the optimal pension contract for heterogeneous (in labor income) agents accommodating for life events. This could lead to heterogeneity in the optimal replacement rates for the income distribution. Note also that this thesis will not take into account 'complicated' utility functions other than CRRA utility that could lead to heterogeneity in the optimal replacement rates.

time we could think of housing wealth or private savings that could justify this undersaving. Although it is not allowed yet, less accumulation of pension wealth can be desirable in case housing wealth is accumulated.

In this thesis, several illustrated cases regarding heterogeneity and life events will be constructed. This was done in Van Ewijk, Hoet, Mehlkopf, and van den Bleeken (2017) [24] as well. This thesis will introduce different forms of heterogeneity and life events and will replicate and extend the cases of Van Ewijk, Hoet, Mehlkopf, and Van den Bleeken (2017) [24] in different ways. One of the major differences with respect to the assumptions in this thesis is that participants can accumulate pension wealth in line with a stochastic contribution scheme.

1.1 Research questions

This introduction leads to the following research questions:

1. What would desirable individualized life cycles for heterogeneous agents accommodating for life events look like?
2. What are the welfare effects of further tailoring the life cycle?
3. How would this backfire when the assumptions about preferences are incorrect?

The first research question basically refers to how the individualized optimal pension contract in terms of contribution strategy, asset allocation strategy and dissaving strategy looks like. Deviation from the conventional life cycle should be motivated by heterogeneity characteristics and/or life events. The optimal pension contract will be derived for heterogeneous (in Section 3) agents accommodating for life events (in Section 4) where the relative importance of the state pension is small. In Section 5, the state pension is taken into account for a limited number of heterogeneity characteristics and life events to be able to derive the optimal pension contract for all income categories. In particular, this thesis will replicate and extend on the heterogeneity characteristics and life events introduced in Van Ewijk et al. (2017) [24]. In Section 6, the pension contract will be adjusted to realized asset returns.

The second research question can be interpreted as the welfare effect for the pension plan participant arising from deviations from the standard pension contract. In Section 3 and Section 4 welfare effects were calculated for heterogeneous agents accommodating for life events. Tailoring the life cycle could be welfare enhancing on the individual level (adjust the contribution rate in case housing wealth is accumulated). Still one should take into account that financial illiteracy and irrational behavior could lead to suboptimal choices (i.e. shortsightedness in the consumption of a bequest). Therefore, welfare losses could arise as well. It is important to carefully define the setting of the decision architecture. Free choice could lead to these welfare losses, whereas in a setting of customization one could not employ all the information to attain these welfare gains. For example, the pension fund does not know that a bequest is received. Nudging is a combination of both concepts that guide the participant to choose for a specific default, whereas the participant is free to deviate from this advise.

An additional concern will be (in case of free choice) the actuarial fairness of the choice architecture. This could lead to strategic choices and intergenerational transfers. One could think of strategic decision making regarding a lump sum at retirement, or so called 'hoog laag constructies'. In Van Ewijk et al. (2017) [24] it is concluded that welfare losses arising from financial illiteracy is a main concern for policymakers (few retirees annuitize pension wealth if annuitization is not mandatory), whereas strategic decision making is unobserved. It is beyond the scope of this thesis to quantify this second effect.

The third research question has a double interpretation. In this thesis a basic initial setting is chosen, however we want to analyze the effect on the optimal pension contract and welfare effects in case different parameter values (i.e. economic, preference and life cycle) are assumed. Therefore this question could be interpreted as a robustness check against the economic parameters in Section 5 and the preference and life cycle parameters in Section 7. It could also be interpreted to obtain an idea about the welfare losses in case the institutional setting is defined where the pension fund creates defaults for the participant (an estimation error will lead to a suboptimal pension contract). In this case, a welfare loss could arise from incorrect assumptions about the participants individual situation.

2 Life cycle model

The basic model in the literature regarding life cycle investing is still the one proposed by (Merton, 1969 [40], 1971 [39]; Samuelson, 1969 [44]) who have derived a theory about an optimal consumption pattern and an optimal asset allocation strategy over the life cycle. Labor income was neglected in their research, implying a constant optimal asset allocation strategy. The important contribution of Bodie, Merton, and Samuelson (1992) [11] is that a decreasing exposure to the risky asset is optimal, as the investment horizon decreases, under the assumption of a risk free labor income. In their setting optimal pension contributions can be derived as well. Life cycles in a DC pension system are developed based on these findings.

The remaining structure of Section 2 will be to guide the reader through the important formulas of the Merton model, which is the basic model used to explain life cycle optimization in Subsection 2.1. The technical derivations of the formulas regarding the Merton model can be found in Appendix $A.1$. To continue, in Subsection 2.2 a graphical illustration of the life cycle model will be presented, including the implications for the optimal pension contract after a life event. In Subsection 2.3, the implications of heterogeneity characteristics, life events and realized asset returns for the optimal pension contract in the life cycle model will be discussed. In Subsection 2.4, an overview of technical restrictions in the Merton model will be presented. Finally, in Subsection 2.5 some background information regarding the institutional setting of the pension system will presented.

2.1 Merton model

In this section, we will start by explaining the assumptions with respect to financial markets (Subsection 2.1.1), preferences (Subsection 2.1.2) and the labor market (Subsection 2.1.3). In Subsection 2.1.4, we formulate the life cycle optimization problem that solves for the optimal consumption pattern, hence prescribes an optimal saving decision and solves for the optimal asset allocation over the life cycle. In Subsection 2.1.5, expressions for optimal consumption and optimal asset allocation in this model will be presented. Additional formulas presented in Subsection 2.1.5 are needed for the purpose of this thesis and were derived in the context of the Merton model. The notation in this section will be (to a large extent) in line with the notation used in Bovenberg, Koijen, Nijman, and Teulings (2007) [12].

2.1.1 Financial markets

In the Merton model there exist two assets in the financial market, a risky asset (stock) denoted by S_t and a risk free asset (bond) denoted by B_t. This implies that an investor can allocate his or her saving amounts (pension contributions) to these assets respectively. The price dynamics for S_t and B_t are given as follows:

$$dS_t = \mu S_t dt + \sigma S_t dZ_t \tag{1}$$

$$dB_t = rB_t dt \tag{2}$$

The stochastic process for the stock is a geometric Brownian motion (GBM), with drift parameter (μ), volatility term (σ) and Z_t is i.i.d standard normal and can be referred to as a standard Brownian motion (BM). The bond price is not characterized by any uncertainty, since it is not exposed to any stochastic process. The drift parameter for the bond dynamics will be denoted by a constant interest rate r. For the sake of completeness the analytical solutions of these asset pricing dynamics can be found in Appendix $A.1.2$. Also, we abstract from inflation in this model. Additionally, it is assumed that the investor could trade on a continuous basis without transaction costs. There are no borrowing constraints, which implies that money could be borrowed (short in the bond) in the financial system to buy the stock. This implication is also valid for stocks.

2.1.2 Preferences

In the standard life cycle model, it is assumed that the investor's utility in each time period, derived from consumption (C_t) only, is described by a time separable constant relative risk aversion (CRRA) utility function ($U(.)$) presented below:

$$U(C_t) = \begin{cases} \frac{C_t^{1-\gamma}}{1-\gamma} & \gamma \neq 1 \\ \log(C_t) & \gamma = 1 \end{cases} \tag{3}$$

In this setting the parameter γ has a double role. On the one hand, it characterizes the (constant) relative risk aversion of the investor. A derivation of this can be found in Appendix $A.1.3$. The assumption of a constant relative risk aversion parameter implies that the optimal asset allocation is independent of the initial wealth level of an investor. The parameter γ represents the willingness to smooth consumption across economic scenarios. On the other hand, the parameter γ defines the Elasticity of Intertemporal Substitution (EIS) by $\frac{1}{\gamma}$ derived in Appendix $A.1.4$. The EIS refers to our willingness to smooth the consumption across time.

The economic interpretation of γ is that a high value for γ implies relatively high risk aversion. A high risk aversion implies reluctance to smooth consumption across economic scenarios yielding a relatively high allocation to the risk free asset. A high value of γ implies a low value of intertemporal substitution preferring a stable consumption across time. This again yields a relatively high allocation to the risk free asset.

In this utility function, leaving a bequest yields no utility at all, since only consumption is valued by the participant.

2.1.3 Labor market

In the Merton model it is assumed that from a specific start age, a labor income L_t will be earned each year t until a preset retirement age (T). Since the labor income will be constant over the life cycle, the subscript t could be omitted. This implies that we abstract from a career pattern and we have a guaranteed job and risk free salary. This setting of the labor market gives rise to the idea of human capital (H_t). The main motivation behind the idea of human capital will be that individuals will work during their active phase (from time 0 to time T) and therefore will receive labor income (L) in the future. After time T, one is not able to work anymore and therefore human capital is fully depleted. Human capital is by definition at a specific point in time the value of all future labor income.

$$H_t = \begin{cases} \frac{L}{r}\left(1 - e^{-r(T-t)}\right) & t \in [0, T] \\ 0 & t \in [T, D] \end{cases} \tag{4}$$

During the accumulation phase every employee remains alive. The retirement phase is assumed to have a fixed time span. We can define survival probabilities (p_t). This is demonstrated by equation (5) implying a deterministic date of death (D).

$$p_t = \begin{cases} 1 & t \in [0, D] \\ 0 & t > D \end{cases} \tag{5}$$

2.1.4 Life cycle optimization problem

Recall that the idea of the pension system is to enable the investor to smooth his or her consumption over his or her life cycle. Contributions to a pension scheme need to be made in order to keep the consumption during retirement (no labor income anymore) at acceptable levels. The investor will maximize his or her indirect utility of consumption over the life cycle $J(W_t, t)$ as follows:

$$J(W_t, t) = \max_{f_t, C_t} E_{W_t, t}\left(\int_t^D \frac{e^{-\delta(s-t)}}{1-\gamma} C_s^{1-\gamma} ds\right) \tag{6}$$

Observe that δ is the time preference parameter. A higher value of δ implies a preference for consumption today instead of postponing consumption. The allocated fraction of total wealth (W_t) to the risky asset will be denoted by f_t. We have to optimize the function $J(W_t, t)$ subject to the wealth dynamics, defined as:

$$dW_t = \Big(W_t(r + f_t\sigma\lambda) - C_t\Big)dt + W_t f_t \sigma dZ_t \tag{7}$$

The first term of expression (7) represents the deterministic component of the investment return (λ is the risk premium) and the consumption in time period dt. The second term of expression (7) is the stochastic component of the investment return. Equation (6) and equation (7) define the life cycle optimization problem.

As a side note, in the Merton model a distinction was made between human capital (H_t) and financial wealth (F_t) that sum up to total wealth (W_t).

$$W_t = H_t + F_t \tag{8}$$

At the beginning of the life cycle total wealth will be equal to human capital and there is no financial wealth. This naturally brings us to the description how financial wealth accumulates over time, which can be found in Appendix A.1.5. Still, the dynamics of financial wealth will be presented. It should be noted that f_t^* is the fraction of financial wealth allocated to stocks.

$$dF_t = \Big(F_t(r + f_t^*\sigma\lambda)dt + F_t f_t^* \sigma dZ_t\Big) + (L - C_t)dt \tag{9}$$

At least two methods exist to solve the Merton model such that the optimal consumption pattern and the optimal asset allocation can be determined. The formulation of the problem through formulas (6) and (7) implies the Hamilton-Jacobi-Bellman (HJB) method as the appropriate solution approach. A detailed derivation of the analytical results via this method can be found in Appendix A.1.1. In Appendix A.1.6, the life cycle problem is formulated in order to solve it via the martingale method.

2.1.5 Analytical results

From the life cycle optimization problem, the optimal exposure of total wealth to the risky asset is

$$f_t = \frac{\lambda}{\sigma\gamma} \tag{10}$$

The optimal allocated fraction to stocks in terms of total wealth is constant over the life cycle. Under the assumption that human capital is risk free, it can be easily seen that stochastic fluctuations in total wealth (7) and financial wealth (9) must coincide.

$$f_t \sigma W_t dZ_t = f_t^* \sigma F_t dZ_t \tag{11}$$

From equation (11) and substitution of equation (8) we can derive the optimal fraction of financial wealth allocated to the risky asset.

$$f_t^* = f_t(1 + \frac{H_t}{F_t}) \tag{12}$$

From formula (12) it can be verified that a decreasing asset allocation over the life cycle (i.e. accumulation phase) to the risky asset will be optimal. This is because human capital decreases over the life cycle and financial wealth typically increases until retirement. In the retirement phase, this model prescribes a constant optimal asset allocation ($f_t^* = f_t = f$) since human capital is completely depleted at the retirement age.

Under the assumptions of the Merton model, we are also able to provide an explicit expression for the optimal consumption pattern (C_t).

$$C_t = \frac{W_t}{g(t)} \tag{13}$$

$$g(t) = \frac{1}{A}\left(e^{A(D-t)} - 1\right) \tag{14}$$

$$A = \frac{(1-\gamma)r - \delta}{\gamma} + \frac{(1-\gamma)\lambda^2}{2\gamma^2} \tag{15}$$

It might be important to express that the optimal consumption pattern indirectly defines the optimal pension contributions or saving decision (PC_t) as follows:

$$PC_t = L - C_t \tag{16}$$

Recall that the labor income during the accumulation phase is equal to the constant L, whereas the consumption over time is stochastic. This leads to the fact that the replacement rate (RR_t) is a random variable. In expectation, RR_t for t $\in$ [T,D] is expressed as follows:

$$E(RR_t) = \frac{E(C_t)}{L} \tag{17}$$

Still, it might be appropriate to present a more elaborate background on the consumption pattern (in particular in retirement). The optimal expected dynamics of the consumption pattern are derived in Bovenberg et al. (2007) [12] and is as follows:

$$E\left(\frac{dC_t/dt}{C_t}\right) = \left(\frac{r - \delta + \frac{1+\gamma}{2}\frac{\lambda^2}{\gamma}}{\gamma}\right) \tag{18}$$

In the situation in which a person is infinitely risk averse ($\gamma = \infty$) a constant expected consumption pattern is optimal, hence a constant expected replacement rate in retirement is optimal. If the interest rate exceeds the time preference parameter, an increasing expected consumption pattern is optimal. This implies an increasing expected replacement rate in retirement. It is also possible to construct the case where the decreasing expected consumption is optimal over the life cycle. The time preference parameter should exceed the interest rate and the term in equation (18) that depends on the risk premium and the risk aversion.

The certainty equivalent consumption (ce_t), is the deterministic amount of consumption that yields the same utility as the utility from the stochastic consumption stream. Mathematically this is presented as follows:

$$E_t\left(\int_t^D e^{-\delta(s-t)} \cdot u(C_s)ds\right) = \int_t^D e^{-\delta(s-t)} \cdot u(ce_t)ds \tag{19}$$

The left-hand side of equation (19) is equal to a deterministic constant (since it is an expectation). This term will be referred to as Utility Stochastic Consumption Stream (USCS). Solving for the certainty equivalent the following expression is found.

$$ce_t = \left(\frac{USCS \cdot (1-\gamma)}{\int_t^D e^{-\delta(s-t)}ds}\right)^{\frac{1}{1-\gamma}} \tag{20}$$

Detailed derivations of equation (20) can be found in Appendix $A.1.7$. It is difficult to derive an analytical expression for $USCS$, therefore it might be appropriate to approximate it by a Monte Carlo estimate.

In Appendix $A.1.8$ it is shown that the certainty equivalent can be related directly to the certainty equivalent found by Bovenberg et al. (2007) [12]:

$$ce = \left(W_0^{1-\gamma} g(D)^{\gamma} \frac{\delta}{(1-e^{-\delta D})}\right)^{\frac{1}{1-\gamma}} \tag{21}$$

It is important to emphasize that the certainty equivalent in this thesis is more general compared to the certainty equivalent derived in Bovenberg et al. (2007) [12]. This implies that in this thesis the subscript t

could be found, whereas in Bovenberg et al. (2007) [12] t is set to zero. The general case is important, to be able to quantify the certainty equivalents that arise from life events. However, the certainty equivalent of Bovenberg et al. (2007) [12] could be used for the calculation of certainty equivalents that arise from heterogeneity characteristics.

To calculate the welfare losses from a suboptimal life cycle strategy, the difference in certainty equivalent consumption between the optimal and suboptimal strategy ($ce_t^* - ce_t$) standardized by the certainty equivalent consumption under the optimal life cycle strategy (ce_t^*) is used.

$$\text{Welfare Loss}_t = \left(\frac{ce_t^* - ce_t}{ce_t^*} \right) \cdot 100\% \qquad (22)$$

2.2 Graphical illustration of impact of a life event

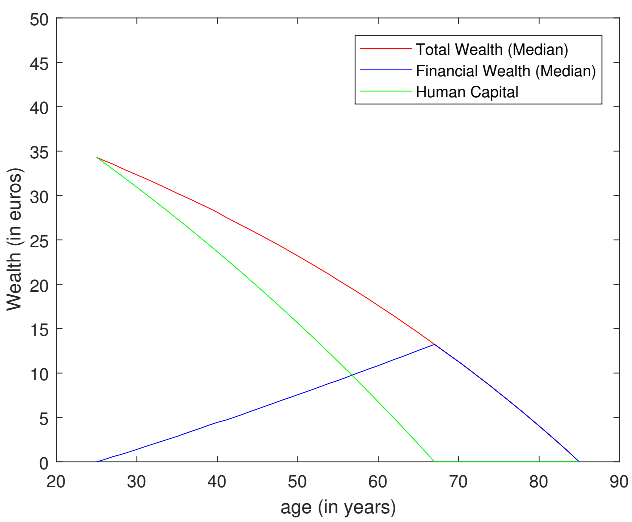
In Subsection 2.1, a description of the Merton model was presented. In particular, analytical formulas of the main variables (W_t, F_t, H_t, C_t, f_t^* $E(RR_t)$ and PC_t) were presented. This section will focus on presenting a graphical illustration of the important variables in the standard life cycle model. In addition to this, the situation in which the labor income of a 45 year old person is unexpectedly doubled will be analyzed. The purpose is to illustrate the implications of a life event that arises during the life cycle for the optimal pension contract. An economic motivation for doubling the wage, will be that the accumulated knowledge and experience gained, will be extremely valuable in a specific sector. By switching jobs, the increased labor income will be realized.

We will start to define the parameter values in Table 1 that were used to obtain the results.

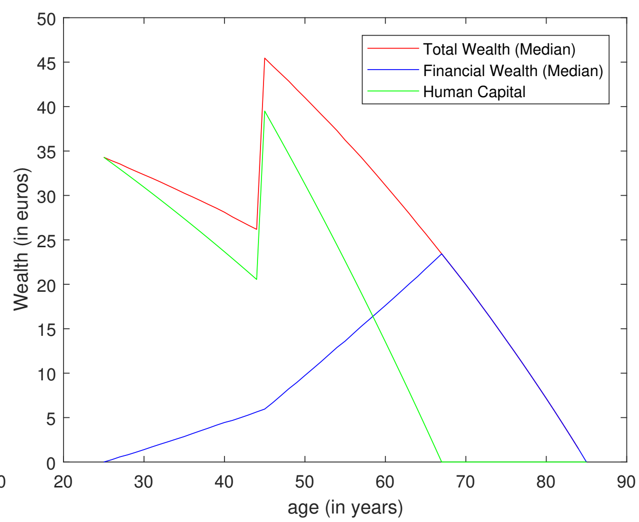
Table 1: Parameter values

Parameter names	Parameter values
Start age	25
Retirement age	67
Deterministic date of death	85
Normalized salary	1
Interest rate	1%
Volatility of stock returns	20%
Risk premium	4%
Risk aversion level	10
Time preference	3.2%
Scenarios	5000

Observe that the value of the time preference parameter (δ) is chosen in a way, to obtain a constant expected consumption pattern (18). This simplifies the problem, since constant expected pension contributions will be optimal.



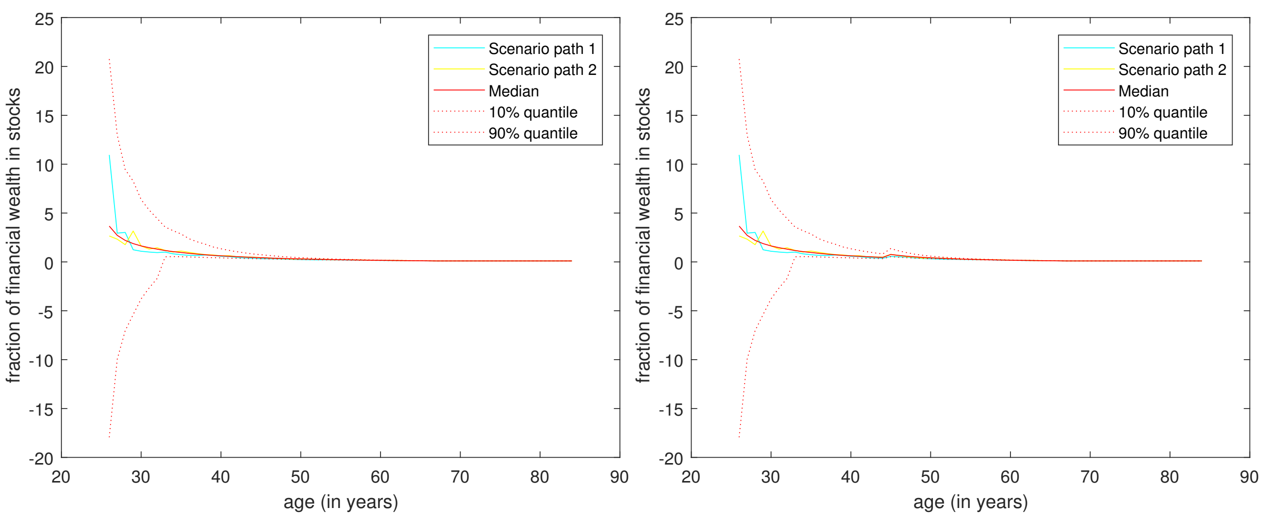
(a) Optimal wealth evolutions (standard)



(b) Optimal wealth evolutions (shock human capital)

Figure 2: Wealth dynamics

In Figure 2, we observe the median for the evolutions of total wealth, financial wealth and human capital for the standard case and the case of an exogenous shock to human capital respectively. Observe that in general, human capital is highest at the start age and gradually decreases and it is completely depleted at the retirement age. In case of doubling the labor income at the age of 45, an exogenous increase in human capital is observed. From this age onwards, the human capital will depreciate at a higher rate (since labor income will be higher). Human capital is still completely depleted at the retirement age. At the start age, human capital exactly equals total wealth. Observe, that total wealth gradually decreases since the depreciation of labor income is dominating the pension contributions and the investment returns. In the case of an exogenous shock to human capital, total wealth increases with the value of the human capital increase. Observe that total wealth decreases at a higher pace, since the labor income is dominating the pension contributions and investment returns even more. Financial wealth is equal to zero at the start age and gradually increases through pension contributions and investment returns. From the age of 45, in case of an exogenous shock to human capital, more contributions to the pension account are made, hence more investment return could be realized. This results in a sharper increase of financial wealth until the retirement age. Financial wealth reaches the maximum value at the retirement age. Since we consume out of our financial wealth (in retirement), in case of an exogenous shock to human capital we consume at a higher rate, yielding a sharper decrease of financial wealth during retirement. At the deterministic date of death there is no financial wealth remaining.



(a) Optimal portfolio mix (standard) (b) Optimal portfolio mix (shock human capital)

Figure 3: Optimal portfolio

From Figure 3, we observe two sample paths, the 10%, 50% and 90% quantile of the asset allocation in terms of financial wealth for both cases. The asset allocation is extremely volatile at younger ages, resulting in unrealistic asset allocations due to a relatively small financial wealth level. In practice, such allocations are not allowed. Figure 3a and 3b coincide until the age of 45.

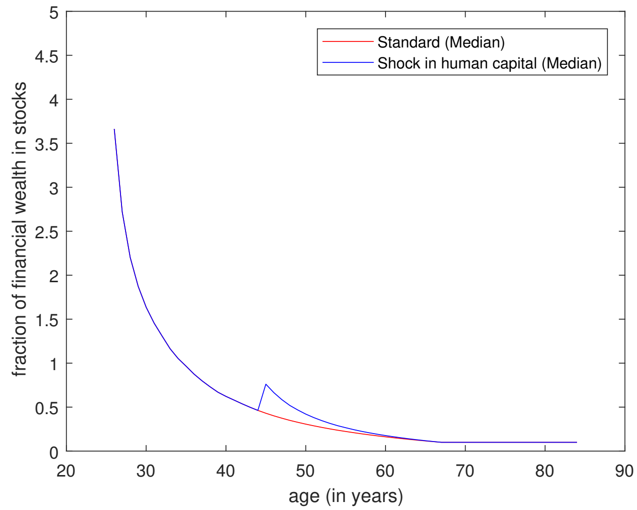
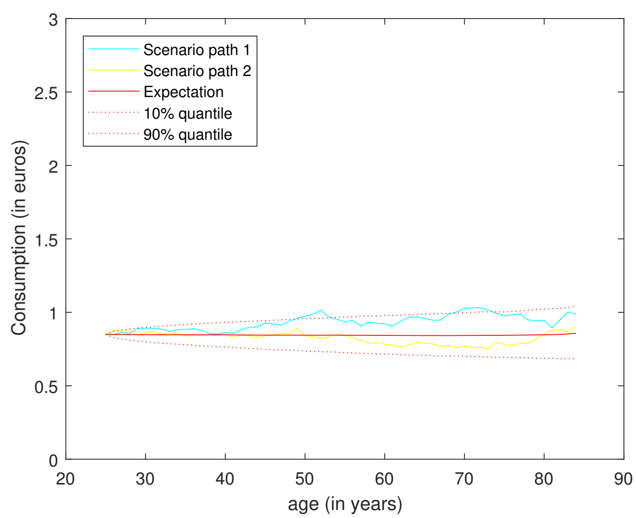
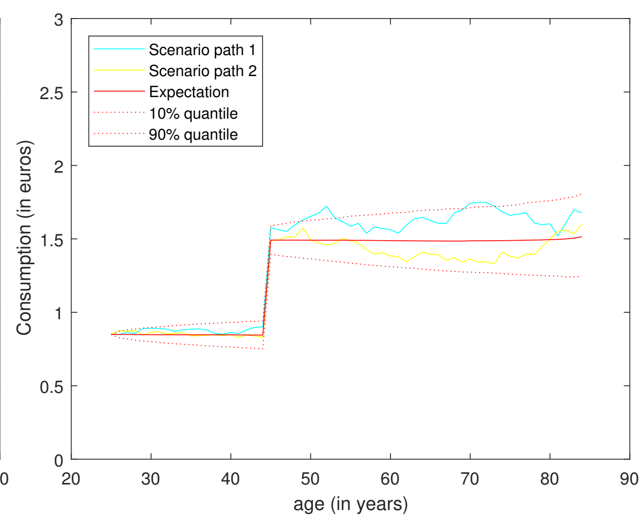


Figure 4: Asset allocation paths (zoom in)

In Figure 4, the median asset allocation strategy for both cases will be presented seperately to visualize the implications of a shock in human capital for the optimal asset allocation strategy. Note that the scale of the vertical axis will be different compared to Figure 3a and 3b. The interpretation of an exogenous increase to human capital will be the same as an additional claim to a portfolio of bonds (risk free asset). Therefore, the optimal allocated fraction to stocks in terms of financial wealth should be increased.



(a) Optimal consumption (standard)



(b) Optimal consumption (shock human capital)

Figure 5: Consumption

From Figure 5, we observe two sample paths, the expectation, the 10% and 90% quantile of the consumption pattern for both cases. First observe that a constant expected consumption pattern is optimal due to the parameter values defined in Table 1. In case of an exogenous shock to human capital, the expected consumption will be changed to a higher constant level. Observe that by standardizing the labor income to one, from Figure 5*a*, we can interpret the replacement rates and pension contributions as well. Implicitly the pension contributions can be interpreted from these figures by reducing the labor income with the consumption. Observe that the pension contributions will be adjusted dynamically, implying more pension contributions after underperformance of the financial market. For the replacement rates, it should be noticed that we have to divide consumption by the final labor income. So, looking at the retirement years, a decent replacement rate (84 %) is observed.

Pension contributions and replacement rates can be interpreted from Figure 5*b* as well. Note that from the age of 45, in the case of an exogenous shock to human capital, both labor income and consumption pattern have changed. Still, the difference will be defined as pension contributions (i.e. higher contributions after the shock). In case of an exogenous shock to human capital, lower replacement rates (in terms of the final salary) were obtained (74 %). Observe that Figure 5*b* would be the main input to calculate the certainty equivalent (20) for the optimal pension contract (i.e. a suboptimal pension contract was not considered here).

2.3 Implications of heterogeneity and life events in life cycle model

In this section we will present an overview of the implications of heterogeneity and life events in the conventional Merton model. In Subsection 2.3.1 the implications of heterogeneity will be discussed. In Subsection 2.3.2 the implications of life events will be discussed. In Subsection 2.3.3, the implications of deviating from the contribution strategy and investment strategy in the conventional Merton model based on realized asset returns will be discussed. Recall that the constant expected consumption pattern is the starting point (i.e. target) in this thesis. These heterogeneity characteristics and life events will justify changes in the (dis)saving decision and the asset allocation decision in the current pension contract. This flexibility will be necessary to offer the participant an optimal pension contract.

2.3.1 Heterogeneity

In the standard setting, it can be easily observed that several forms of heterogeneity known at the beginning of the life cycle (deterministic) can be naturally incorporated in the pension contract. These heterogeneity characteristics could raise the question whether a different life cycle should be used a priori. The heterogeneity characteristics will be identified and summarized in Table 2.

Table 2: An overview of the heterogeneity characteristics

Heterogeneity number	Heterogeneity characteristic
1	Risk preferences
2	Starting salary and assumed wage profile
3	Targeted replacement rates
4	Pension contributions
5	Survival probabilities

In Alserda et al. (2016) [3] it is concluded that pension plan participants have a significant variation in their risk preferences. The Merton model takes care of this by allowing for infinitely many life cycle profiles. In practice, only a few asset allocation strategies are offered.

An important heterogeneous aspect is the wage difference at the start of the life cycle. Observe in this context the difference in starting salary for high educated versus low educated persons. Wage differences could also arise from gender discrimination, although it is undesirable to implement this in the life cycle. Also differences in the assumed wage profile based on education level will be present. This is easy to incorporate since the model could be adapted by imposing a different labor income, implying a different structure of the human capital (4).

From Knoef et al. (2014) [37] it was concluded that significant differences exist between the actual replacement rates. The conclusion of de Bresser and Knoef (2015) [15] might be even more interesting since heterogeneity exists across the targeted replacement rate and minimal replacement rate. It is assumed that this is known at the beginning of the life cycle. It might not be so trivial to adapt the CRRA utility function (3) to a utility function taking into account utility arising from approaching this targeted replacement rate.

Depending on the pension scheme, the amount of pension contributions could be quite different among the participants. The pension contributions are one of the important components that determine the pension wealth at retirement. In the standard life cycle model, pension contributions will be dynamically adjusted based on the realized asset returns (i.e. accumulated financial wealth).

One important side note to make at this point in time is to discuss that in some aspects it could be undesirable to implement differentiation or it could be prohibited by law. Brown (2002) [17] concludes that mortality rates differ markedly over gender, ethnic groups and socio economic groups. A uniform and mandatory annuitization pricing principle yields value transfers. Still, in Brown (2002) [17] it is concluded that the insurance value of annuitization is larger than these value transfers under mild administrative costs. Davidoff, Brown and Diamond (2005) [23] and Poterba (2006)[34] found welfare enhancing effects in unfair priced annuities as well, in the absence of a bequest motive. This is completely neglected in the Merton model, since we assume a deterministic date of death, see Subsection 2.1.3.

2.3.2 Life events

In the standard setting, it can be easily observed that several life events that could arise during the life cycle can be naturally incorporated in the pension contract. This could raise the question whether it will be optimal to deviate from the (dis)saving and asset allocation decisions made unconditional on these life events. Therefore, flexibility in the pension contract will be desirable. Life events will be identified and summarized in Table 3.

Table 3: An overview of the life events

Life event number	Life event
1	Shock in financial wealth (bequest, rich partner, divorce, private wealth, existing pension wealth)
2	Shock in additional asset (housing wealth)
3	Shock in human capital (different job, unemployment, flexibility retirement age)
4	Birth of a child

In addition to the existing pension wealth (at the current pension fund or at another pension fund), households could have private savings as well. According to Knoef et al. (2014) [37], these private savings have a substantial positive effect on the pension wealth and significant heterogeneity within the population exists. Still, some concern might be appropriate since a bequest motive (neglected in the description of the Merton model) could also be a motivation for these private savings. In line with this, receiving a bequest could significantly increase financial wealth. Most of this information is unknown by the current pension provider. Hence, if this information will become available, it could be interpreted more or less as an exogenous shock in the financial wealth of the participant from the point of view of the pension provider.

Many households possess/accumulate housing wealth (i.e. additional asset), which is a rather illiquid asset, whereas other households rent a house. Still this housing wealth can be valuable, since rent expenses are avoided (imputed rental income). In addition to this, there are several ways to deplete housing wealth during retirement. In Knoef et al. (2014) [37] it is explained that housing wealth can significantly increase the pension annuity. In the paper of van Ewijk et al. (2017) [24] it was concluded that adjusting the contribution rate optimally in this setting yields welfare gains between 0.5 % - 6.8%, depending on the value of the housing wealth. The welfare gains in terms of adjusting the asset allocation in the pension contract were unambiguous according to Olear, de Jong, and Minderhoud (2017) [31]. In the coalition agreement (Vertrouwen in de toekomst, 2017) concrete steps are considered to reduced pension contributions in case of the accumulation of housing wealth. This is in line with the results of van Ewijk et al. (2017) [24].

In this setting, it will be assumed that everyone has a guaranteed job and risk free, constant salary. In a more realistic setting, it could be possible to switch jobs. This could lead to a different labor income. In addition to this, the pension plan participant might loose his job, leading to a period of unemployment. Periods of voluntary unemployment will be possible as well. This all can be interpreted as a shock in human capital.

Each life event challenges the optimality of the standard pension contract, in terms of optimal asset allocation and optimal contribution rate, in a different way. The implications will be discussed in detail in Section 4.

2.3.3 Realized asset returns

In Chiappinelly and Thirukkonda (2015) [20], the concept of sequence risk is illustrated by introducing two hypothetical persons. These persons are identical in almost all aspects of the pension scheme, in particular in realizing identical average returns over the life cycle. The only difference will be the sequence of the realized asset returns. In stylized cases it can be shown that this 'wrong order' of realized asset returns can significantly reduce the available pension wealth at the retirement age. Introducing a dynamic life cycle is one method to mitigate the effect of this risk. In DC pension schemes, this will be a justified concern since individual retirement accounts are considered. If a targeted replacement rate is of crucial importance, it could be argued that the order of realized asset returns (the sequence risk) should be incorporated in the life cycle. In principle three scenarios could arise. Either the stock market underperformed, performed as expected or outperformed. In the second scenario there is no need to deviate from the standard setting. In the other cases, we could either deviate from the contribution rate and/or asset allocation strategy to increase the probability that the targeted replacement rate is attained. The current situation is that a participant enters a DC pension fund and the complete life cycle is more or less deterministic.

An example, of this life event, could be that a 45 year old participant has faced disappointing results at the stock market. Three asset allocation strategies will be discussed that could be incorporated in the pension contract of the participant:

- Guarantee asset allocation strategy: In this strategy no rebalancing (based on underperformance of the stock market) takes place and we keep the number of stocks. This implies that the fraction of financial wealth which is allocated to stocks is decreased due to low returns (compared to the optimal Merton rebalancing strategy, where it does depend on the performance of the stock market). This strategy implies some guaranteed pension wealth (i.e. guarantees) since no rebalancing (based on underperformance of the stock market) takes place. In this strategy the participant will gradually reduce (i.e. rebalance independent of the shock) the allocated fraction to stocks as the remaining investment horizon becomes shorter.

- Rebalancing asset allocation strategy: The second strategy is what the Merton model prescribes as optimal. Rebalancing (based on underperformance of the stock market) takes place to retain the optimal exposure in terms of financial wealth. This implies that additional stocks need to be bought. This strategy prescribes a life cycle as well, in which the optimal allocated fraction will be decreased (i.e. rebalance independent of the shock) as the remaining investment horizon becomes shorter.

- Gambling for resurrection asset allocation strategy: In this strategy additional exposure to stocks is chosen. This implies that additional stocks should be bought (rebalance based on underperformance of the stock market) to end up with the optimal rebalancing asset allocation strategy. On top of this, additional stocks have to be bought in order to increase the allocation to stocks in terms of financial wealth. Again, we will incorporate a decreased asset allocation (i.e. rebalance independent of the shock) as the remaining investment horizon becomes shorter.

For the pension contributions the participant could be restricted to a preset deterministic contribution scheme. It might be relevant to consider the optimal constant pension contributions at the start of the life cycle. This implies that this person will retain to these optimized contributions after the shock. In case of a deterministic contribution scheme, the participant could be allowed to reoptimize the constant contributions after the shock. Still, we could allow for a dynamic contribution scheme.

Analogue to the situation in which the stock market underperformed, a situation in which the stock market outperformed the expectations could be constructed. It seems natural to pose the question whether it could be attractive for this person to deviate from the pension contributions and/or change the equity exposure for some time in this case. The risk preferences and the importance of the target will definitely influence this decision. More importantly, a more sophisticated utility function will be crucial in determining to which extend it will be optimal for this participant to deviate from the conventional life cycle. For this thesis it is more appropriate to focus on the distribution of the available pension wealth at retirement age. In a setting in which the participant is completely rational, incorporating this option in the pension system could only enhance the welfare (of the participant). Pension participants are typically characterized by financial illiteracy. Moreover, suboptimal deviations could lead to welfare losses as well. Chiappinelly and Thirukkonda (2015) [20] focused on the sequence risk (in the last 10 years) when the pension wealth is enormous and contributions are the highest. In addition to a dynamic asset allocation strategy, they propose an investment strategy for stocks based on the P/E ratios as a reliable strategy to mitigate the effect of nasty returns in the final 10 years. Basu, Byrne, and Drew (2009) [6] determine an average investment return (one could interpret this as different a replacement rate) and if the realized returns are behind schedule one will not reduce the allocation to stocks (not rebalance independent of the shock). If the realized returns are above average, the allocation to stocks will be reduced. No additional conservative strategy will be followed in the case when the stock market outperformed. In their paper they conclude that the dynamic strategy outperforms the conventional life cycle strategy.

To conclude this section, we will state which heterogeneity characteristics and life events will be considered in this thesis:

- Heterogeneity in risk preferences, see Subsection 3.1
- Heterogeneity in the assumed wage profile, see Subsection 3.2
- Life event shock in financial wealth (i.e. bequest), see Subsection 4.1
- Life event shock in additional asset (i.e. housing wealth), see Subsection 4.2
- Life event shock in human capital (i.e. temporary unemployment), see Subsection 4.3
- Life event adjusting the contract to realized asset returns, see Section 6

There could be heterogeneity characteristics and life events that are not mentioned in this section. At least, the events are related to some extent to the heterogeneity characteristics and life events discussed in Subsection 2.3.1 and 2.3.2 in terms of adaptations of the life cycle (i.e. optimal pension contract).

2.4 Limitations of the Merton model

In this section, the relevant limitations of the Merton model will be discussed. Later on, (in Section 3, 4 and 6) it could be decided which limitations will be implemented in order to derive the optimal pension contract and investigate the welfare effects (or distribution of available pension wealth) arising from the constructed cases in a more realistic setting.

Table 4: Technical limitations Merton model

Limitation number	Restrictions Merton model
1	Interest rate plus inflation rate risk
2	Loss aversion and endogenous updated reference level
3	Discrete trading and transaction costs
4	Exogenous borrowing plus going short constraint
5	No wage profile
6	State pension
7	Risk free human capital
8	Deterministic death
9	Consumption path in retirement
10	No dynamic pension contributions

2.4.1 Interest rate plus inflation risk

In the Merton setting, the interest rate is defined as a given constant. It might be appropriate to introduce a stochastic process for the interest rate. A common choice to model interest rates is to impose a Vasicek model, which is defined as:

$$dr_t = a(b - r_t)dt + \sigma dZ_t \tag{23}$$

$$r_0 = r(0) \tag{24}$$

The economic interpretation of the parameter b is the mean reversion level and the parameter a is the speed of mean reversion. (Z_t) is the stochastic term (BM) similar similar to the term in the stock price process, combined with a volatility term (σ). In Munk (2015) [41], derivations for the optimal asset allocation can be found under the assumption of stochastic interest rates. The implications are considered for a one factor and a two factor Vasicek model.

Inflation is ignored in the Merton model as well (i.e. set to zero). Typically it is assumed that the stock portfolio is an appropriate hedge against inflation risk. In Brennan and Xia (2002) [14], the optimal asset allocation was derived under stochastic processes (Ornstein-Uhlenbeck) for the interest rate and the inflation rate.

2.4.2 Loss aversion and endogenous updated reference level

Tversky and Kahneman (1992) [33] provide empirical evidence that people exhibit loss aversion. This economic concept refers to the fact that people will value gains and losses differently (i.e. more sensitive to losses).

In van Bilsen, Laeven, and Nijman (2014) [8] the optimal consumption pattern and the optimal portfolio choice are derived for a loss averse individual having a consumption reference level updated endogenously. This is in contrast with the conventional Merton setting, in which losses and gains are valued equally and the reference level is assumed to be zero. The setting of Van Bilsen et al. (2014) [8], in order to incorporate these characteristics, is changed at three points with respect to the original setting. For the sake of completeness this will be presented and the intuition will be provided.

In their setting, the utility of the investor depends on whether the consumption amount is above or below the reference level (θ_t). The parameter κ, if it is chosen larger than 1 indicates loss aversion. The choice of γ_G and γ_L will determine the curvature of the gains and losses.

$$u(C_t, \theta_t) = \begin{cases} (C_t - \theta_t)^{\gamma_G} & \text{if } C_t \geq \theta_t \\ -\kappa(\theta_t - C_t)^{\gamma_L} & \text{if } C_t < \theta_t \end{cases} \tag{25}$$

The parameter values for α and β, in equation (26) specify the dynamics of the endogenous reference level. Abstracting from this process will result in a constant exogenous reference level.

$$d\theta_t = (\beta C_t - \alpha \theta_t)dt \tag{26}$$

The third change restricts the yearly difference between the consumption level and the reference level. This difference is then defined by the parameter L_t^{max}.

$$\theta_t - C_t \leq L_t^{max} \tag{27}$$

Analogue to the traditional Merton setting, an optimization problem arises and analytical solutions for the consumption and portfolio choice can be found. It is beyond the scope of this thesis to explain the exact details. One of the main contributions of Van Bilsen et al. (2014) [8] (focusing on financial market risk), is to differentiate the life cycles based on the realized asset returns, in a different way as the traditional Merton model. More concrete, their model implies soft guarantees for the consumption pattern. This is because the participant values protection after a negative shock in financial wealth.

2.4.3 Discrete trading and transaction costs

In the standard Merton model trading occurs on a continuous basis to retain the optimal asset allocation. In reality, we are restricted to trading in a discrete time setting in which we could approach a continuous trading strategy. A concern of such a trading strategy could be the costs associated with it. These costs are nonnegligible and therefore a trade-off should be made between transaction costs and approaching the continuous trading strategy prescribed. Bovenberg et al. (2007) [12] concluded that a discrete trading strategy leads to a welfare loss of 0.3%.

2.4.4 Exogenous borrowing plus going short constraint

Optimizing the life cycle prescribes an optimal allocation to the risky asset in terms of total wealth. Rewriting this optimal allocated fraction in terms of total wealth, to an optimal allocated fraction in terms of financial wealth yields optimal median allocations of more than 100 % in the risky asset during the first years of the accumulation phase. This means that the investor borrows money in the financial system (is short in the bond) and invests additional money in the risky asset. This is restricted by law and therefore the maximum allocation to the risky asset is 100 %. In some scenarios, an optimal allocation results in a fraction of the risky asset below 0 %. This is observed in simulations that are performed. By the same reason this is not allowed. Bovenberg et al. (2007) [12] concluded that this suboptimal asset allocation strategy leads to a welfare loss of approximately 3 % in certainty equivalent consumption.

2.4.5 No wage profile

By assuming a constant salary the Merton model does not consider a wage profile which is typical in real life. In reality a wage profile will exist because of increasing working experience of the participant. This will be translated into a salary increase each year. This could be easily extended in the model, whereas Olear et al. (2017) [31] conclude that this would lead to minimal welfare gains in terms of adjusting the asset allocation strategy in the pension contract.

2.4.6 State pension

The model ignores the state pension (AOW). The results regarding the optimal pension contract can directly be interpreted as applicable for income categories in which the relative importance of the state pension will be small. Since the purpose of this thesis is to derive an optimal pension contract for all income categories, it might be appropriate to incorporate the state pension into the model. Section 5 explains in detail how to do this.

2.4.7 Risk free human capital

Risk free human capital implies that a risk free labor income will be obtained during the accumulation phase. This implies that human capital has the dynamics of a bond. In Benzoni, Collin-Dufresne, and Goldstein (2007) [7] it is argued that human capital, discounted future labor income, is highly correlated to the stock market. Continuing their line of reasoning, this implies that young participants already have a high exposure to the stock market via their human capital. Deriving an optimal exposure in terms of financial wealth, the optimal asset allocation is completely reversed. Young investors should short the stock market and gradually increase their allocation to the risky asset as their human capital start to deplete.

2.4.8 Deterministic death

In the standard life cycle model, a deterministic date of death is assumed. This is equivalent to abstracting from micro longevity risk, which is the risk that the participant is still alive after this deterministic date or passes away earlier. As a consequence of the life cycle optimization, in the first case insufficient financial assets will be in place. This is not such an extreme shortcoming of the model in case the design of the pension scheme includes a mechanism that shares this risk. In the Dutch setting the pension participant has to annuitize (recently several options are available) the pension wealth at the flexible retirement date that will guarantee a life long pension income stream. To conclude, if the deterministic date of death is the average life expectancy of the population the shortcoming mitigates. If this estimate for the population is incorrect, problems could arise and the risk is borne by the pension participant. The latter risk component is typically referred to as macro longevity risk. To create a more realistic setting, one could incorporate uncertainty in future survival probabilities (and heterogeneity of survival probabilities) into the model for each year.

2.4.9 Dynamic pension contributions

In DC schemes different contribution rates will be used, from which we can derive the pension contributions to the individual account. For the participants there are limited choices available to decide how high their contribution will be in the second pillar (i.e. agreements by social partners). Additional savings on a voluntary, tax incentive basis are always possible within the Witteveenkader. The contribution rates in the DC schemes will be independent of the realized asset returns (i.e. accumulated pension wealth).

In the Merton setting something remarkably arises if the dynamic pension contributions are studied in more detail. Pension contributions will be defined as the difference between the labor income and the consumption. For the expected consumption (if the interest rate is increased by 1 percent point) we see that the consumption exceeds the labor income (approximately 10 years) before retirement, motivated by the parameter value choice of equation (18). This implies that no pension contributions are made to the individual pension account from this ages onwards. In addition to this, it implies that dissaving already takes place before retirement.

To conclude, these results indicate the importance and the power of dynamic pension contributions as a tool to reach a targeted available pension wealth at retirement age.

To conclude this section, it will be explained which limitations this thesis will incorporate.

- Analyzing the cases in Matlab, a discrete setting will be chosen to avoid the unrealistic assumption that agents trade frictionless in continuous time. Annual time points (Δt will be set to one) are introduced. An illustration of how this works will be given for the dynamics of the stock market (since this is the starting point of the model) as follows:

$$S_{t+\Delta t} = S_t + \mu S_t \Delta t + \sigma \sqrt{\Delta t} S_t X \tag{28}$$

  where X is a standard normal random variable (i.e. X $\sim$ N(0,Δt)).

- As a starting point, the heterogeneity characteristics (Section 3) and life events (Section 4), will be analyzed in the standard life cycle model excluding the state pension. This implies that the results in Section 3 and 4 will be applicable for income categories with a low relative importance of the state pension. In Section 5, this thesis will take into account the state pension and derive the optimal pension contract and welfare gains for a limited number of cases discussed in Section 3 and Section 4. This is done in order to be able to generalize the results regarding the optimal pension contract for all income categories.

- In deriving the optimal pension contract in Section 5, a borrowing constraint will be imposed.

- In Subsection 3.2, the optimal pension contract in case of different assumed wage profiles will be derived.

In the illustrated cases, hypothetical persons are assumed to form a single household. Also, observe that an overview of the short hand notation to refer to the hypothetical persons, the strategies (contribution, asset allocation, etc..) will be presented in Subsection A.3.

2.5 Intermezzo: Taxes and current legislation

The purpose of this thesis is to derive the optimal pension contract for heterogeneous agents accommodating for life events. Some institutional background of the (Dutch) pension system will be required to advise pension participants on how to choose or adjust the pension contract optimally. Moreover, the institutional background should be taken into account in order to propose policy changes if this will be welfare enhancing for the pension participants. The institutional background is presented below.

In the Netherlands, the accumulation of pension wealth is fiscally facilitated in line with the so called 'Witteveenkader'. This framework concerns the accumulation of pension wealth in addition to the Pay As You GO state pension. The Salary tax Act ('Wet op de loonbelasting') article 18a clause 1 and clause 2 state that the maximum contribution rates of a DB pension contract are 1.657 % in case of a final salary pension scheme and 1.875 % in case of a career-average pension scheme. Article 18a clause 3 then defines the maximum contribution rates for a DC pension scheme. In DC pension schemes, the annual contribution rates are based on an accumulation period of 40 years. The accumulated pension rights cannot exceed 75 % of the average pensionable income. A DC pension scheme is aiming for the same benefit level as a career-average pension scheme.

Article 18 of the Pension Act is a general law which is applicable to the accumulation of pension wealth in the second and third pillar[5]. In the Dutch setting, the social partners make specific agreements on the pension contract in the second pillar on behalf of the employees in a specific sector or company. In practice this can lead to significant variations in the pension contributions of the participants and in the exact details of the pension contract. However, there are no possibilities for the participant to pay lower pension contributions in the second pillar than agreed on by the social partners.

Chapter 4 of The regulation on the execution of Pension Act and The law on mandatory occupational pension schemes ('Het Besluit uitvoering Pensioenwet en Wet verplichte beroepspensioenregeling') is concerned with investment policies of pension funds. If there is only one risk profile, the investment policy of the pension fund has to represent the risk preferences of the average participant. A minority of pension funds offers more than one risk profile to their participants. In that case, the participant has to make a test measuring the risk attitude in order to determine a suitable investment policy.

Concluding, the current institutional setting of the second pillar pension system offers limited options to take into account the heterogeneity of the participant. Moreover, there is limited flexibility to adapt the pension contract to life events. Note that this limited flexibility regarding the downward adjustment of pension contributions is especially due to the agreements which are made by the social partners and that the law is not that restrictive.

If the agreements made by the social partners do not allow for additional fiscally attractive pension contributions, the third pillar offers possibilities for this. Under the assumptions that the upper bound of Article 18 is not reached (in Dutch: indien jaarruimte). In the third pillar, there could be more flexibility regarding the annual contributions and the investment policy. To what extent flexibility is available depends on the exact product specifications offered by pension funds.

At retirement age, the participant always has the opportunity to buy the optimal pension income stream. If for example, a variable annuity or the insurance of the longevity risk is not offered by the the current pension fund, the participant can buy a pension income stream at a different pension fund (in Dutch: shoprecht). This can be found in Article 80 and 81 of the Pension Act and is applicable for the second and third pillar.

In the current Dutch situation, contribution rates are typically close to the maximum. Therefore, few flexibility regarding the downward adjustment of pension contributions is possible. This will lead to the hypothesis that it could be attractive that social partners would agree on lower contribution rates. This enables the participant to choose the optimal contribution scheme that will take into account the heterogeneity and life

[5] Article 18 defines an upper bound for the accumulation of pension wealth. A lower bound does not exist.

events. Even the participant can adjust the contribution scheme on realized asset returns.

2.5.1 NN products

For the sake of completeness, an overview of the products offered by NN will be provided:

- Second pillar: NN offers three different products during the accumulation phase[6], namely: Comfort Pensioen, Bewust Pensioen and Essentie Pensioen. The employer will decide on this. This thesis will focus on Essentie Pensioen. In this product, the participant is completely free in additional pension contributions than agreed on by the social partners. The participant has the right to change these additional contributions at any point in time. Note that the upper bound of the 'Witteveenkader' is always applicable. These additional pension contributions in the second pillar enables the participant to insure the longevity risk and choose for the variable annuity for the complete accumulated pension wealth at retirement age.
- Third pillar: In the third pillar, NN offers 'Aanvullend PensioenSparen'[7] only. Here the participant is also completely free in adjusting the pension contributions at any point in time. The participant can choose between a variable or a fixed interest rate. At retirement age, the participant can deplete the accumulated pension wealth via a fixed annuity not insuring the micro longevity risk (i.e. Aanvullende PensioenUitkering). This can be attractive in case of a bequest motive.

[6]In Comfort Pensioen, the pension outcome is guaranteed. In Bewust Pensioen, the pension outcome could be guaranteed with a maximum of 50 % of the pension wealth. The remaining part will be invested via a life cycle, where the participant has the right to exercise the click feature. In Essentie Pensioen the complete pension wealth is invested via a life cycle. The life cycles are motivated by three risk profiles.

[7]This product can be referred to as 'banksparen'.

3 Heterogeneity

In this section some illustrative heterogeneity characteristics that are present at the start of the life cycle will be analyzed. These heterogeneity characteristics will challenge the optimality of a (too a large extent) uniform pension contract. In particular, the participant may want to have a tailored pension contract, in terms of contribution and asset allocation strategy. Therefore, welfare effects of having a suboptimal pension contract will be quantified. A clear distinction will be made between welfare losses a suboptimal contribution strategy and a suboptimal asset allocation strategy. To ensure the practical relevance, it will be concluded to what extent flexibility in the pension contract is possible. In addition to this, this thesis will contribute to the discussion where a tailored pension contract (if it is currently not possible) will be welfare enhancing. This tailored optimal pension contract will be derived as well.

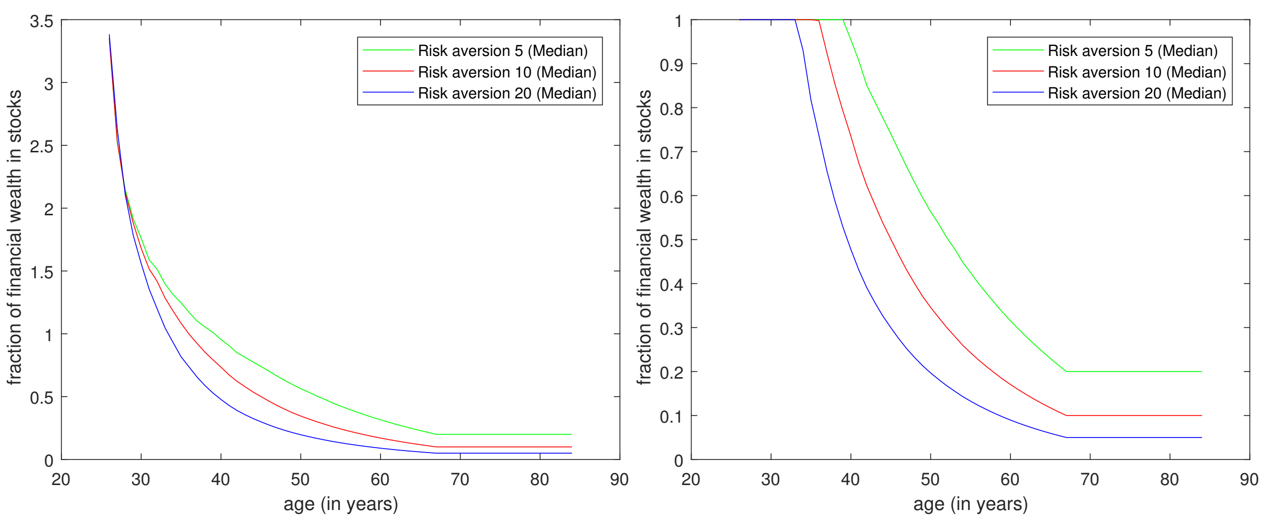
In subsection 3.1, the impact of the individual risk preferences on the optimal pension contract will be investigated. In the current setting, the asset allocation strategy for most pension funds reflects the risk preferences of the average participant, yielding one risk profile. Some pension funds, among others NN, develop several risk profiles and the participant will be assigned to a risk category based on a test regarding risk attitude. In line with the NN life cycles, we will define a defensive, neutral and offensive asset allocation strategy and calculate the welfare effects, of having a suboptimal asset allocation strategy implying that the participant was assigned incorrectly. The individual risk preferences will raise the question whether it will be optimal to have a tailored contribution scheme. The welfare effects will be investigated within the standard life cycle model.

In Subsection 3.2, the presence of an assumed wage profile could motivate a different life cycle (i.e. tailored pension contract). It was concluded that the typical pension fund has one asset allocation strategy, motivated by the risk preferences of the average participant. Still, assumptions regarding the assumed wage profile has to be made. Substantial heterogeneity in these assumed wage profiles could motivate a different pension contract in terms of asset allocation and pension contributions. Again, the welfare implications of having a suboptimal contribution strategy and having a suboptimal asset allocation strategy will be analyzed.

3.1 Mismatch risk preferences

In the life cycles provided by NN, the pension plan participant will be assigned to an asset allocation strategy based on the risk preferences. In particular, NN offers a defensive, neutral and offensive life cycle strategy. Relatively few participants make an active choice (NN PPI), hence they will be assigned to the default. Based on financial market conditions, in particular low interest rates, NN decided to offer the neutral risk profile as the default. It might be relevant to quantify welfare losses that arise from assuming an incorrect risk profile. Still, deriving the optimal asset allocation based on the risk preferences in the pension contract will be even more relevant. The life cycles of NN could be related to the life cycles prescribed by the Merton model. The life cycles of NN will be calibrated for the appropriate level of risk aversion (γ), see equation (12). Still, the risk aversion in the Merton setting that estimates the appropriate NN life cycle will not exactly match. This we will neglect and match as good as possible. In addition to this, by choosing the level of risk aversion, we will take into account that realistic expected pension contributions are made to the individual pension account (i.e. calibrate on pension contributions as well).

Calibrating the life cycles for the appropriate risk aversion level yields risk aversion parameters of 20, 10 and 5 for the defensive, neutral and offensive asset allocation strategy respectively. In Binswanger and Schunk (2008) [10], it was shown that in the Dutch setting the 25 %, 50 % and 75 % quantile of the risk preference distribution corresponds with risk preference parameters of 4, 7 and 12 respectively. In Van Rooij, Kool and Prast (2007) [43], it was concluded that the risk preferences were highest in the pension domain. Therefore, it will be argued that the life cycles will represent the distribution of risk preferences accurately. In Figure 6, the median unbounded (i.e. no exogenous borrowing constraint) and the median bounded (i.e. truncated) asset allocation strategies for these values of the risk preferences are presented. These were derived in the standard Merton setting (see Subsection 2.1).



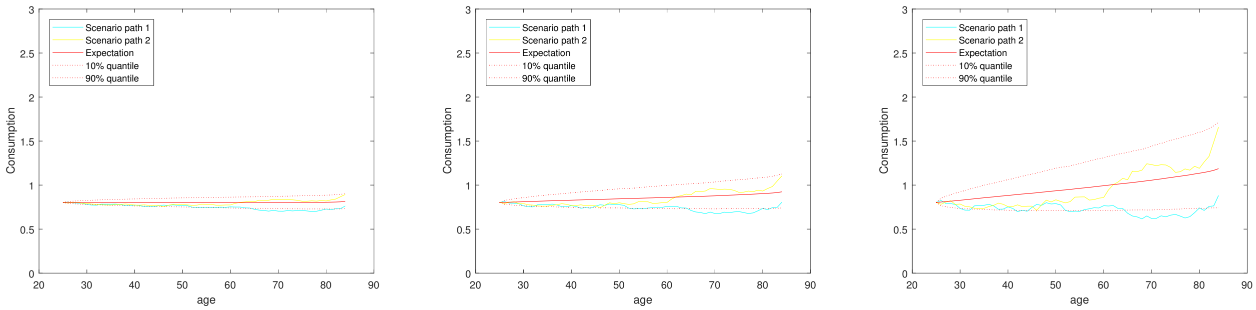
(a) Risk profiles (no exogenous borrowing constraint)

(b) Risk profiles (truncated)

Figure 6: Risk profiles

Welfare losses arising from assuming incorrect risk preferences were investigated in the Merton setting, in line with the setting of Bovenberg et al. (2007) [12][8]. In this thesis, an explicit distinction will be made between welfare losses arising from a suboptimal asset allocation strategy and a suboptimal contribution strategy.

[8] In Bovenberg et al. (2007) [12] an explicit distinction of welfare losses (Table 4 on page 52) from a suboptimal asset allocation strategy and a suboptimal contribution was not made. As we could read welfare losses were calculated with respect to the first-best solution in the case in which consumption and investment decision are based upon the wrong risk aversion parameter (and time preference parameter).

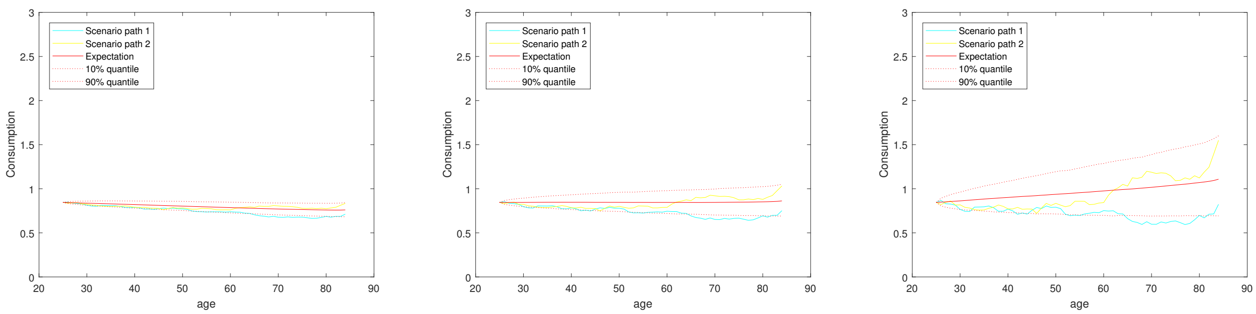


(a) Consumption path (Opt. C + Opt. (Defensive) A)

(b) Consumption path (Opt. C + Subopt. (Neutral) A)

(c) Consumption path (Opt. C + Subopt. (Offensive) A)

Figure 7: Consumption paths (Risk profile Defensive)

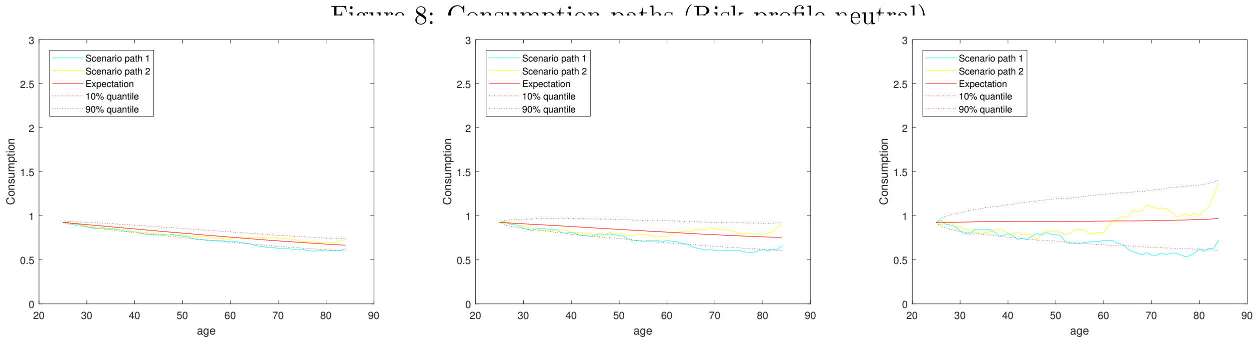


(a) Consumption path (Opt. C + Subopt. (Defensive) A)

(b) Consumption path (Opt. C + Opt. (Neutral) A))

(c) Consumption path (Opt. C + Subopt. (Offensive) A)

Figure 8: Consumption paths (Risk profile neutral)



(a) Consumption path (Opt. C + Subopt. (Defensive) A)

(b) Consumption path (Opt. C + Subopt. (Neutral) A)

(c) Consumption path (Opt. C + Opt. (Offensive) A)

Figure 9: Consumption path (Risk profile offensive)

In Figure 7a, 7b and 7c the optimal consumption paths (using the true risk preference parameter) combined with a defensive, neutral and offensive asset allocation strategy (under the assumption that the defensive strategy is optimal) are presented. These figures will be the main input to quantify welfare losses if a person will be assigned to the neutral or offensive asset allocation strategy, whereas the defensive asset allocation strategy will match the risk preferences. From Figure 8a, 8b, 8c welfare losses can be calculated if the person will be assigned to the defensive or offensive asset allocation profile, while the neutral asset allocation profile will match the risk preferences. From Figure 9a, 9b, 9c welfare losses can be calculated if the person will be assigned to the defensive or neutral asset allocation profile, while the offensive asset allocation profile will match the risk preferences. To conclude, these figures will be the main input to calculate welfare losses arising from a suboptimal asset allocation strategy based on the risk preferences.

The results of welfare losses arising from a suboptimal asset allocation strategy can be found in Table 5. Observe that t will be equal to zero (start age is normalized to zero) in equation (20), to be able to calculate the certainty equivalents. The table should be interpreted as follows: On the rows, a risk profile is defined and the elements in this row define welfare losses if a different risk profile was optimal (defined in the column). To be concrete, Figure 7a, 7b and 7c will be used to calculate welfare losses in column 1 of Table 5, Figure 8a, 8b, 8c will be used to calculate welfare losses in column 2 and Figure 9a, 9b, 9c will be used to calculate welfare losses in column 3. In particular, we are interested in row 2, since this is defined to be the default of NN, hence many participants will end up in this category.

Table 5: Welfare effects (asset allocation), incorrect risk preferences

	Defensive (Optimal)	Neutral (Optimal)	Offensive (Optimal)
Defensive	0 %	-1.21 %	-5.44 %
Neutral	-3.14 %	0 %	-2.27 %
Offensive	-25.86 %	-6.05 %	0 %

Table 5 present the results found in this thesis that should be interpreted based on the rows. Significant welfare losses are present if a person will be assigned to an incorrect risk profile. From these results, it is definitely not recommendable to take the offensive risk category as the default. Significant welfare losses (-6.05 %) will arise for this person if the neutral profile will be optimal, whereas enormous welfare losses (-25.86 %) will be present if this person has defensive risk preferences.

Based on these results, defining the neutral profile as the default risk category might be a good choice. The pension fund could have an idea about the distribution of the risk preferences of the participants and therefore chooses a different default. In particular, the pension fund could estimate that many of the pool belong to one specific risk category. Then the pension fund should take into account the (expected) summation of the welfare losses.

Before the results obtained in this thesis will be further analyzed, an intermezzo will be made where welfare gains obtained in Van Ewijk et al. (2017) [24] will be replicated and interpreted in the setting of this thesis. This is done in order to compare the results under different assumptions and therefore obtain some robustness in the results.

3.1.1 Intermezzo: Risk preferences as in Van Ewijk et al. (2017)

This thesis replicates the results found in Van Ewijk et al. (2017) [24] and will extend the findings. To start, an overview of the different assumptions in Van Ewijk et al. [24] will be provided, where a division will be made between major differences (**A**) and minor differences (**B**):

- (A, DC, ⇃): In their setting a deterministic consumption during the accumulation phase was used, implying constant pension contributions to the individual pension account. This instead of a dynamic contribution scheme which is used in this thesis.
- (A, BC, ⇃): In their setting it will be an easy task to have an optimal asset allocation in case of an exogenous borrowing constraint (i.e. truncated asset allocation strategy will be optimal). If stochastic consumption was used, numerical techniques should be used to find the optimal asset allocation in case of a borrowing constraint. In this thesis no exogenous borrowing constraint will be imposed.
- (A, AOW, ⇃): Their setting takes into account the state pension. The implications for the analysis of excluding or including the state pension will be extensively discussed in Subsection 5.1.2 for this case.
- (B): Their setting incorporates survival probabilities in the model with a maximum age. This implies that the model incorporates micro longevity risk, still abstracts from macro longevity risk. Here, a deterministic date of death will be assumed.
- (B): In Van Ewijk et al. (2017) [24] some different benchmark values will be chosen for the economic, preference and life cycle parameters. In particular, an interest rate of 2 %, risk preferences of 5, time preference parameter of 2 %, labor income of 40.000 will be chosen. The optimal constant contribution rate will be set at 9.2 % [9], that is already corrected for the accumulation of pension wealth via the state pension.

Based on these major differences (A) in the setting, this thesis is likely to find larger welfare gains compared to the setting of Van Ewijk et al. (2017) [24].

In the setting of Van Ewijk et al. (2017) [24], the asset allocation strategy is defined as a weighted average equity exposure over the life cycle. In particular, this weighted average equity exposure corresponds with a risk aversion parameter in the standard life cycle model. In their paper welfare gains were calculated with respect to a 40 % optimal weighted average equity exposure corresponding with a risk aversion parameter of 5[10]. Their setting considered deviations from one optimal asset allocation strategy. This thesis extended the results by considering deviations from the defensive, neutral and optimal risk profile. In Figure 10, the results are shown after replication (i.e. small differences could arise since the risk aversion parameter that corresponds with the weighted average equity exposure was not presented and this is matched as good as possible). For example, a weighted average equity exposure of 20 % was matched with a risk aversion parameter (y) of 12 (see also Figure 10).

[9] On page 56 in Van Ewijk et al. (2017) [24], a typo was made. The AOW level should be 16.000 instead of 13.000. The results were replicated with an AOW level of 13.000...

[10] The risk aversion parameter that corresponds with an average equity exposure will be 5 (i.e. rounded). In the true calculations, the decimal risk aversion parameter corresponding the optimal equity exposure over the life cycle will be used.

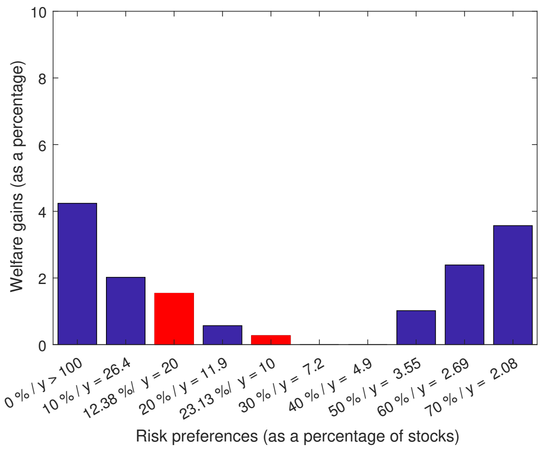
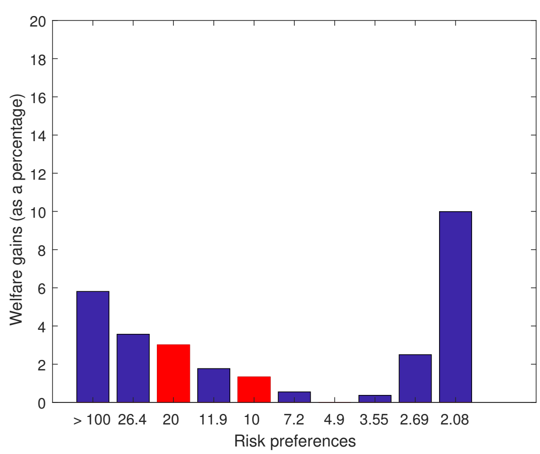


Figure 10: Replicated welfare gains of Van Ewijk et al. (2017) [24]

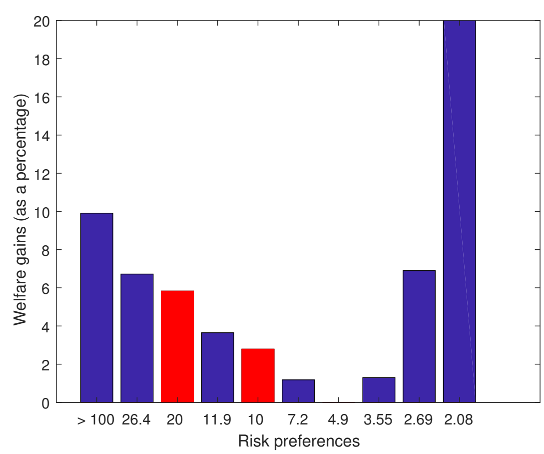
Based on the replicated results of Van Ewijk et al. (2017) [24] in Figure 10, the direct comparison with the results obtained in this thesis can only be made with respect to the third column of Table 5 (i.e. optimal risk aversion is 5). The red bars presented in Figure 10 represent the results of Table 5, column 3 in the setting of Van Ewijk et al. (2017) [24].

Two extensions of Van Ewijk et al. (2017) [24] are considered here:

- In Figure 11a (adjusted scale vertical axis) the results that Van Ewijk et al. (2017) [24] would obtain in a setting that excludes a borrowing constraint (A, BC) is presented. In Figure 11b (adjusted scale vertical axis) the results that Van Ewijk et al. (2017) [24] would obtain in a setting that incorporates a stochastic contribution scheme (A, DC) and excludes a borrowing constraint (A, BC) is presented.



(a) Replicated welfare gains of Van Ewijk et al. (2017) [24] Extended by A, BC



(b) Replicated welfare gains of Van Ewijk et al. (2017) [24] Extended by A, DC and A, BC

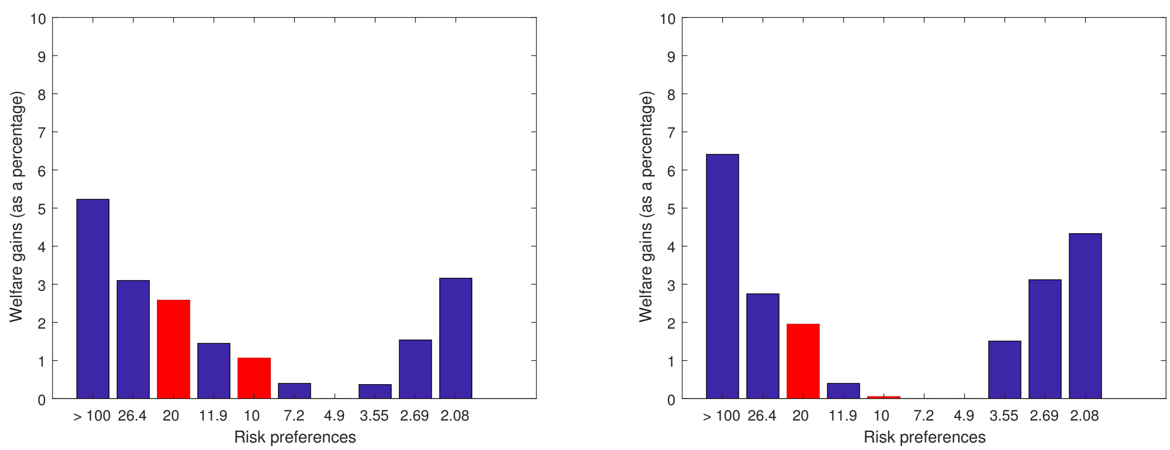
Figure 11: Replicated welfare gains of Van Ewijk et al. (2017) [24] (Extended, major changes)

The results from Figure 11a and Figure 11b could indicate that the stochastic component in the contribution scheme will be the dominant factor for the higher welfare effects found in this thesis compared to Van Ewijk et al. (2017) [24]. A formal proof will be skipped, based on a time constraint, since numerical techniques will be needed to analyze the effect on the welfare gains in the absence of a borrowing constraint and allowing for a stochastic contribution scheme.

- It might be relevant to calculate the results obtained in Table 5 under the assumptions of Van Ewijk et al. (2017) [24]. Observe that this thesis will take care of the fact that the optimal contribution rate have changed (17.8 % and 16.8 % instead of 15.3 % for column 1 and 2 respectively). Welfare losses of being divided to the defensive risk profile and offensive profile, while the neutral risk profile is optimal will be -0.23 % and -5.25 % respectively. If the defensive profile correspond to the optimal risk preferences and the participant will belong to the neutral category, a welfare loss of -2.55 % arises. In line with the results of this thesis, enormous welfare losses will be realized if the person will be divided to the offensive risk profile, while the person has an optimal defensive risk profile.
- For an elaborate analysis, where the relative importance of the state pension is taken into account, will be referred to Section 5.

Here, it will be shown that the minor differences (B) in the setting of Van Ewijk et al. (2017) [24] compared to this thesis will change the welfare gains insignificantly.

- The results of Van Ewijk et al. (2017) [24] can be naturally extended to a setting that will abstract from micro longevity risk (see Figure 12a).
- The results of Van Ewijk et al. (2017) [24] can be naturally extended to a setting that will have different economic and life cycle parameters (see Figure 12b for the interest rate equal to the setting of this thesis).



(a) Replicated welfare gains of Van Ewijk et al. (2017) [24] (Micro longevity risk)

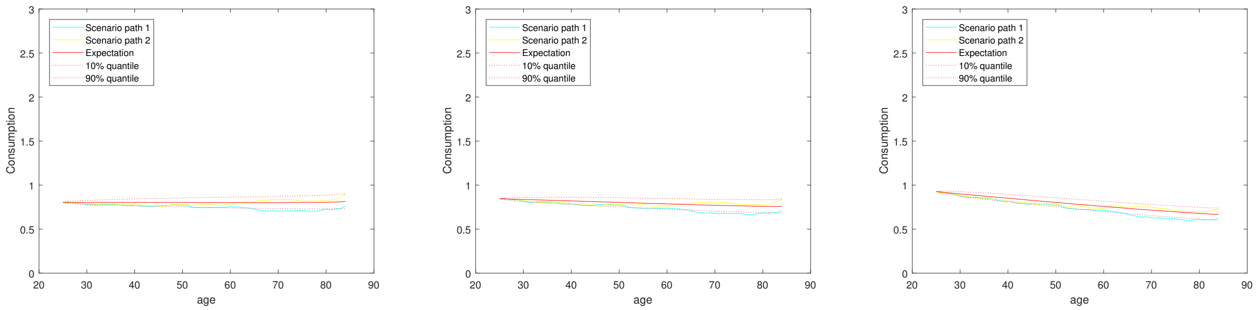
(b) Replicated welfare gains of Van Ewijk et al. (2017) [24] (Interest rates)

Figure 12: Replicated welfare gains of Van Ewijk et al. (2017) [24] (Extended, minor changes)

It could be observed that excluding the micro longevity risk, will change the welfare gains marginally. In addition to this, increasing the interest rate will yield for all risk aversion levels insignificant higher welfare gains. This confirms the hypothesis that these differences in the setting will be higher order effects.

3.1.2 Incorrect risk preferences in contribution scheme

In the first part of this subsection, welfare losses of having a suboptimal asset allocation strategy were calculated. In the remaining part of this subsection, welfare losses of having a suboptimal contribution strategy motivated by an incorrect estimation of the risk preferences will be quantified. Observe that the risk aversion parameter is present in the formula for the optimal consumption directly. Recall that this is motivated by the fact that the parameter γ will play a double role. It defines the risk aversion (γ) directly and indirectly the elasticity of intertemporal substitution ($\frac{1}{\gamma}$). As a consequence, a different risk aversion level will have a different optimal consumption smoothing over the life cycle independent of the asset allocation strategy. This will be mathematically derived in Appendix A.1.9 and some intuition will be presented. Analogue to a suboptimal asset allocation strategy, welfare losses could be quantified if suboptimal consumption smoothing takes place. More importantly, the optimal contribution scheme for different levels of risk aversion could be derived.

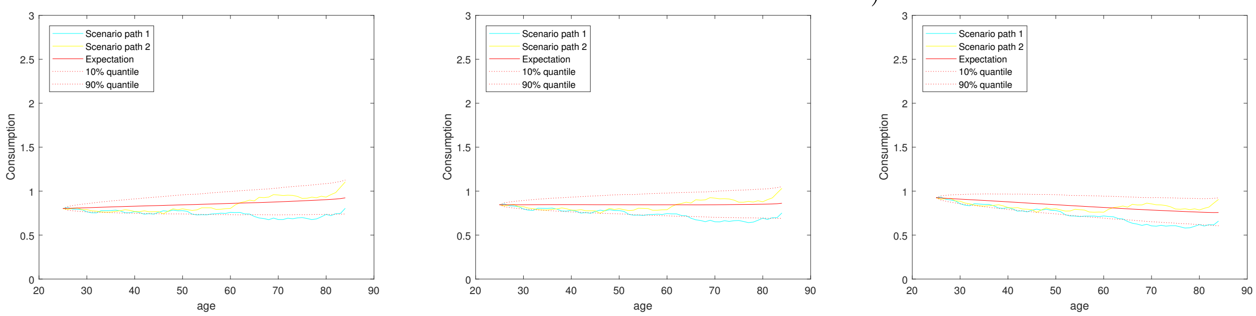


(a) Consumption path (Opt. C (Defensive) + Opt. A)

(b) Consumption path (Subopt. C (Neutral) + Opt. A)

(c) Consumption path (Subopt. C (Offensive) + Opt. A)

Figure 13: Consumption path (Risk profile Defensive) + Opt. A

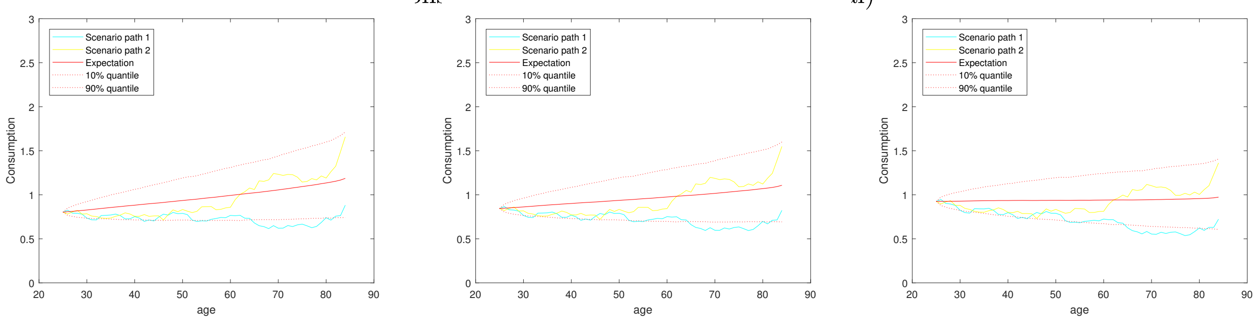


(a) Consumption path (Subopt. C (Defensive) + Opt. A)

(b) Consumption path (Opt. C (Neutral) + Opt. A)

(c) Consumption path (Subopt. C (Offensive) + Opt. A)

Figure 14: Consumption paths (Risk profile Neutral) + Opt. A



(a) Consumption path (Subopt. C (Defensive) + Opt. A)

(b) Consumption path (Subopt. C (Neutral) + Opt. A)

(c) Consumption path (Opt. C (Offensive) + Opt. A)

Figure 15: Consumption paths (Risk profile Offensive) + Opt. A

In figure 13a, 13b and 13c the optimal defensive, suboptimal neutral and suboptimal offensive contribution strategy (under the assumption that this person has defensive risk preferences) combined with the optimal asset allocation paths (using the true risk preference parameter) are presented. These figures will be the main input to quantify welfare losses if a person will be assigned to the neutral or offensive contribution strategy, whereas the defensive contribution strategy will match his risk preferences. In Figure 14a, 14b and 14c the optimal neutral, suboptimal defensive and suboptimal offensive contribution strategy (under the assumption that this person has neutral risk preferences) combined with the optimal asset allocation paths (using the true risk preference parameter) are presented. These figures will be the main input to quantify welfare losses if a person will be assigned to the defensive or offensive contribution strategy, whereas the neutral contribution strategy will match his risk preferences. In Figure 15a, 15b and 15c the optimal offensive, suboptimal defensive and suboptimal neutral contribution strategy (under the assumption that this person has offensive risk preferences) combined with the optimal asset allocation paths (using the true risk preference parameter) are presented. These figures will be the main input to quantify welfare losses if a person will be assigned to the defensive or neutral contribution strategy, whereas the offensive contribution strategy will match his risk preferences.

The results of welfare losses arising from a suboptimal contribution strategy based on the risk attitude can be found in Table 6. To be concrete, Figure 13*a*, 13*b* and 13*c* will be used to calculate column 1 of Table 6, Figure 14*a*, 14*b*, 14*c* will be used to calculate column 2 and Figure 15*a*, 15*b*, 15*c* will be used to calculate column 3.

Table 6: Welfare effects (contribution strategy), incorrect risk preferences

	Defensive (Optimal)	Neutral (Optimal)	Offensive (Optimal)
Defensive	0 %	-0.5 %	-2.03 %
Neutral	-1.22 %	0 %	-0.86 %
Offensive	-8.03 %	-2.1 %	0 %
Deterministic, constant	-4.82 %	-5.17 %	-8.7 %

From table 6, it could be concluded that welfare losses from a suboptimal contribution strategy (based on the risk preferences) yields lower welfare losses than a suboptimal asset allocation strategy differentiated on the risk preferences in the setting of this thesis.[11]

To be able to calculate the welfare losses of a deterministic contribution strategy[12], the deterministic contribution staffel should be defined in this setting first. In the analyses, the optimal constant contribution rate (i.e. calculated in a similar way as in Bovenberg et al. (2007) [12] and Van Ewijk et al. (2017) [24]) will be used, whereas a deterministic contribution scheme in line with the 3 % DC staffel as defined in Table 7 was possible as well.

Table 7: DC staffel

Age	Contribution rate
25-29	7.6 %
30-34	8.8 %
35-39	10.3 %
40-44	11.9 %
45-49	13.9 %
50-54	16.2 %
55-59	19.1 %
60-64	22.6 %
65-66	25.6 %

The introduction of the deterministic contribution scheme will make it interesting to analyze the unattractiveness of mandatory (constant) contributions in line with a preset contribution scheme compared to a stochastic contribution setting, where the optimal stochastic contribution scheme will be differentiated on the risk aversion level. The unattractiveness will be summarized in welfare losses (see fourth horizontal line in Table 6). It could be interesting to take into account this constant contribution scheme for all the heterogeneity characteristics (in the remaining of this section) and life events in Section 4, however this will be omitted based on a time constraint.

[11]In Bovenberg et al. (2007) [12] it was stated that a stochastic contribution scheme will be more beneficial compared to an optimal asset allocation. This contrasts the results (i.e. smaller welfare effects) found in Table 6 compared to Table 5.

[12]As in Bovenberg et al. (2007) [12] (Figure 11, page 36), the asset allocation is optimally adjusted given the constant saving rates (i.e. incorporate a borrowing constraint).

3.1.3 Implications risk preferences for optimal pension contract

Based on the results of this thesis it will be welfare enhancing to divide pension participants in the appropriate risk profile in terms of the optimal asset allocation. The results in this thesis were obtained in a setting that neglects a borrowing constraint, incorporates a stochastic contribution scheme and abstracts from a state pension. These additional constraints were taken into account in Van Ewijk et al. (2017) [24] where one optimal risk profile was defined. Their benchmark assumes a DC setting. In this thesis the results of Van Ewijk et al. (2017) [24] were extended for all the risk profiles defined in the setting of this thesis. Based on a different set of assumptions it could be more or less concluded that the setting of this thesis defines an upper bound for the welfare effect, whereas the setting of Van Ewijk et al. (2017) [24] defines a lower bound for the welfare effect. Observe that the welfare gains that could be realized in the current Dutch institutional pension setting will be close to the results obtained in Van Ewijk et al. (2017) [24]. Still, it is interesting to observe that the welfare gains in a setting that allows for a stochastic contribution scheme instead of a DC staffel will significantly increase. Having a dynamic contribution scheme could be a desirable feature of the pension system. Still from the setting of Van Ewijk et al. (2017) [24] it is concluded that dividing participants in the appropriate risk category will be important.

By taking into account a stochastic contribution scheme, this thesis could extend the results of Van Ewijk et al. (2017) [24] by quantifying to what extent a suboptimal risk aversion parameter in the contribution scheme will be costly to the participant. Somewhat unexpected, smaller welfare losses were found compared to having a suboptimal asset allocation. This contrasts the fact that Bovenberg et al. (2007) [12] stated that it will be more important to have the correct stochastic contribution scheme instead of reflecting the asset allocation strategy accurately.

As a side note, it should be noticed that the NN risk profiles were calibrated based on the risk aversion level by taking into account realistic pension contributions. In particular, the risk aversion parameter of 20 might fail to reflect the defensive risk preferences.

3.2 Assumed wage profile

In this subsection, the optimal pension contract will be derived for different assumed wage profiles. This subsection, will assume simple deterministic wage profiles and therefore also no correlation between the stock market and the assumed wage profile as is argued by Benzoni et al. (2007) [7]. Abstracting from a correlation between the stock market and the assumed wage profile seems justified by the results of Cocco, Gomes and Maenhout (2005) [21]. Here, we will restrict to the riskless view of human capital having bond like characteristics. This section will neglect that the volatility of the human capital could depend on individual characteristics such as age, gender, sector of employment as was shown in Cocco et al. (2005) and Campbell, Cocco, Gomes and Maenhout (1999) [19]. These individual risk factors were taken into account in determining the optimal asset allocation in Olear et al. (2017) [31].

In this simplified setting, an assumed (increasing) wage profile implies that the value of the human capital will be higher compared to the initial human capital (no assumed wage profile). Since we can consume out of total wealth, as presented in equation (33), a higher consumption standard will be optimal. This implies that in the presence of an assumed wage profile (in the beginning years) less contributions are made to the pension account. It is observed that a higher claim to a risk free asset will be present in case of an assumed wage profile. In terms of the asset allocation, defined in equation (12) as a fraction of financial wealth, it is optimal to have a higher equity exposure at the start of the accumulation phase at first sight. Observe that the higher consumption standard in case of an assumed wage profile leads to lower accumulation of financial wealth (even dissaving for many sample paths). Paths were the accumulated financial wealth becomes negative will reduce the optimal equity exposure as can be verified by equation (12). The exact evolution of the optimal asset allocation pattern will (in addition to this) depend on the exact assumption of the wage profile. In Table 8 several assumed wage profiles are defined.

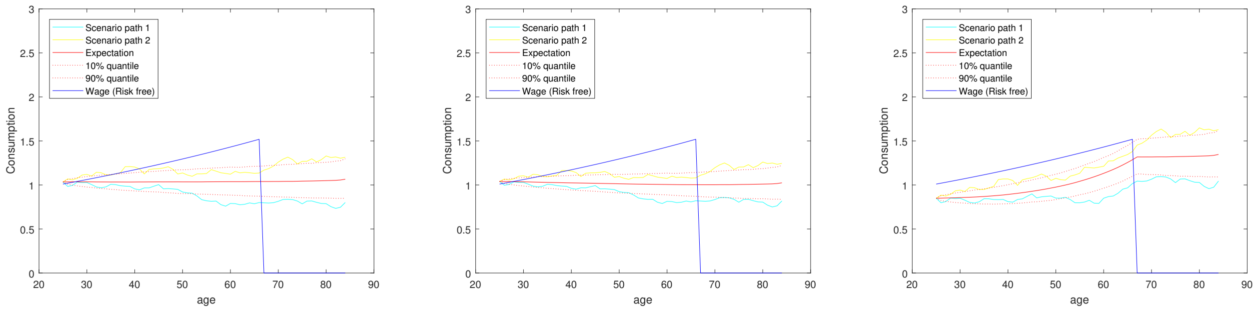
Table 8: Assumed wage profiles

	age 25-35	age 36-45	age 46-55	age 56-67
No wage profile	0 %	0 %	0 %	0 %
Wage profile 1 %	1 %	1 %	1 %	1 %
Wage profile 1.5 %	1.5 %	1.5 %	1.5 %	1.5 %
Wage profile 3-2-1-0 %	3 %	2 %	1 %	0 %

The initial wage will be normalized to 1 for all assumed wage profiles. For wage profile 3-2-1-0 %, during the first ten years of the accumulation phase, an annual wage increase of 3 % will take place. From the age of 36, the employee will have an annual wage increase of 2 % each year for the upcoming ten years. This elaboration shows how to interpret the assumed wage profiles defined in Table 8.

The main purpose of this section will be to derive the optimal pension contract, in terms of dynamic contribution and asset allocation strategy, for the different assumed wage profiles. To be able to quantify the importance of tailoring the pension contract based on the assumed wage profile, welfare losses will be calculated to quantify the importance for a tailored pension contract. In particular, in a DC setting it will be extremely relevant what is assumed with respect to the assumed wage profile (i.e. value of the claim on the risk free asset) in the optimal asset allocation strategy. In the current DC setting, contribution rates will be in line with a DC staffel. However, in a stochastic contribution scheme the pension contributions crucially depends on the assumed wage profile.

In Figure 16a, the consumption pattern will be presented, where an optimal dynamic contribution strategy and optimal asset allocation strategy (i.e. optimal pension contract) will be used under the assumption of wage profile 1 %. In figure 16b, the consumption pattern will be presented where an optimal contribution strategy and suboptimal asset allocation strategy (assuming no wage profile) will be used. In figure 16c the consumption pattern will be presented, where the suboptimal contribution strategy (assuming no wage profile) and optimal asset allocation strategy will be used. In Figure 17 and 18, an identical structure will be used where we have assumed wage profile 1.5 % and 3-2-1-0 % respectively. In all figures[13], at each point in time, the wage is plotted. Recall that the contribution strategy in the optimal pension contract in the absence of an assumed wage profile was graphically illustrated in Subsection 2.2.

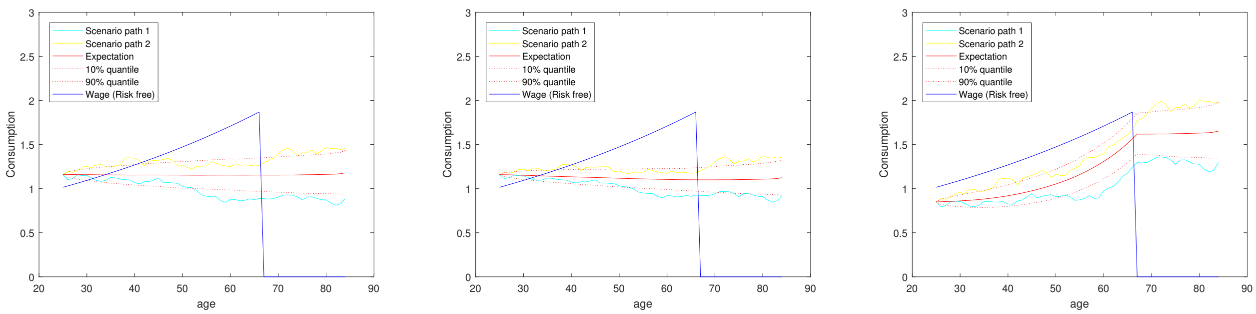


(a) Consumption path (Opt. C (profile 1 %) + Opt. A)

(b) Consumption path (Opt. C (profile 1 %) + Subopt. A)

(c) Consumption path (Opt. C (no profile) + Opt. A)

Figure 16: Consumption paths (wage profile 1 %)

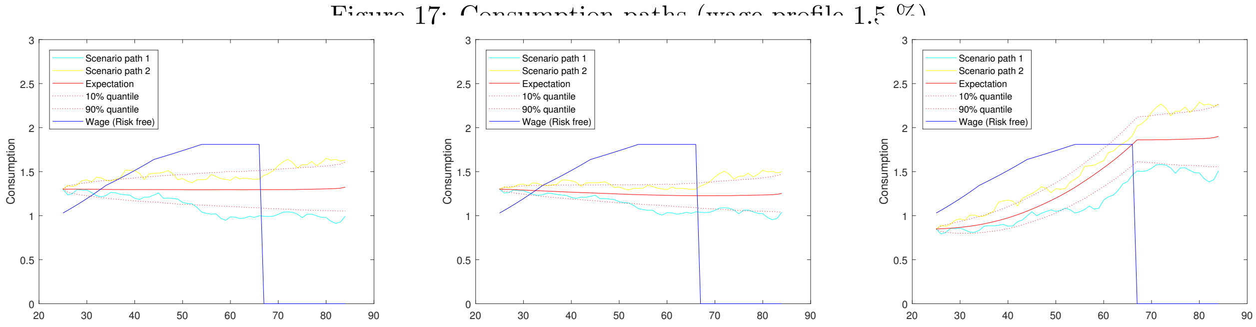


(a) Consumption path (Opt. C (profile 1.5 %) + Opt. A)

(b) Consumption path (Opt. C (profile 1.5 %) + Subopt. A)

(c) Consumption path (Opt. C (no profile) + Opt. A)

Figure 17: Consumption paths (wage profile 1.5 %)



(a) Consumption path (Opt. C (profile 3-2-1-0 %) + Opt. A)

(b) Consumption path (Opt. C (profile 3-2-1-0 %) + Subopt. A)

(c) Consumption path (Opt. C (no profile) + Opt. A)

Figure 18: Consumption path (wage profile 3-2-1-0 %)

In Figure 16*a*, a flat expected consumption pattern will be optimal prescribed by the parameter values defined in Table 1. Observe that assumed wage profile 1 % in this case implies (small) dissaving at the start of the life cycle. In Figure 16*b*, the dissaving at the start of the life cycle will be present as well. Still, a slight decreasing expected consumption will be observed, since a suboptimal asset allocation strategy will be used. In Figure 16*c*, an increasing consumption standard will be observed in expectation. Observe that during the

[13] It is important to present the consumption pattern that represents the optimal contribution strategy under assumed wage profile 1 %, 1.5 % and 3-2-1-0 %. A limited number of consumption patterns in case a suboptimal pension contract will be used are presented. Still, these welfare losses will be calculated in Table 9 and Table 10.

complete accumulation phase, contributions to the pension account will be made (i.e consumption assumes no wage profile, enormous oversaving)[14]. In retirement, an increased consumption standard (i.e. variable annuity) will be implied by the accumulated financial wealth, that will be constant in expectation by the assumptions. A similar consumption structure in terms of consumption pattern can be observed if we have assumed wage profile 1.5 % and assumed wage profile 3-2-1-0 %. Still, it should be noted that in case of wage profile 1.5 % and 3-2-1-0 %, an even higher consumption standard will be optimal, implying more dissaving at the start of the life cycle. Observe that for wage profile 3-2-1-0 %, the highest consumption standard can be attained (i.e. highest value of total wealth at the start of the life cycle). It is interesting to mention that in case of an assumed wage profile, increasing contributions over time will be apparently optimal, coinciding with the actual DC contribution rates. Still, the stochastic component of the contribution strategy will be incorporated in this setting.

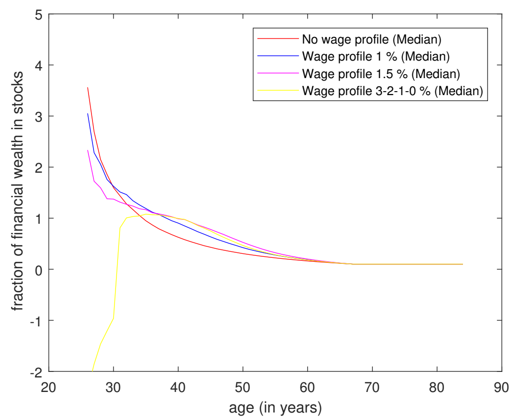


Figure 19: Median asset allocation paths (wage profiles)

In Figure 19, the median optimal asset allocation paths are presented for the different assumed wage profiles. From the optimal asset allocation strategy defined in equation (12), it is expected that in case of an assumed wage profile (higher claim on a risk free asset), a higher equity exposure will be optimal in terms of financial wealth. Still, one should not overlook the fact that a different accumulation of financial wealth will be present as well. In fact, this dissaving at the start of the life cycle will lead to the fact that it will be optimal to have a short position (i.e. negative financial will dominate the increased value of the human capital) or decreased equity exposure compared to the standard setting in the first ten years of the life cycle. After 10 years, it is noticed that the higher value of human capital will dominate the different accumulation of financial wealth and a higher equity exposure will be observed in case of an assumed wage profile compared to the standard setting.

In Table 9, the welfare effects of having a suboptimal asset allocation strategy are presented, in case of an assumed wage profile. In the rows, the assumed wage profile is defined and in the column the asset allocation strategies belonging to the assumed wage profiles are defined. On the diagonal of Table 9 the optimum is defined for each assumed wage profile respectively. In the first column of Table 9, one can find the welfare losses of having a suboptimal asset allocation assuming no wage profile. The welfare losses can be calculated by comparing the certainty equivalent consumption of Figure 16*a* and Figure 16*b* for wage profile 1 %, Figure 17*a* and Figure 17*b* for wage profile 1.5 % and Figure 18*a* and Figure 18*b* for wage profile 3-2-1-0 %. In the remaining columns, the welfare losses are calculated for different suboptimal asset allocation strategies in case of an assumed wage profile.

[14]One will consume out of total wealth (financial wealth and human capital (no wage profile)). By construction the initial consumption level will be identical to the standard setting. Since a higher labor income will be present, more contributions are made to the individual account, hence a different accumulation of financial wealth takes place. As a consequence, a sharper increase in financial wealth (compared to the standard setting) will result in an increasing consumption pattern during the accumulation phase. Recall that in the standard setting, the optimal consumption will take place out of the accumulated financial wealth

Table 9: Welfare effects asset allocation, including wage profiles

	Allocation (no profile)	Allocation (profile 1 %)	Allocation (profile 1.5 %)	Allocation (profile
Wage no profile	0 %	-0.57 %	-1.48 %	-2.64 %
Wage profile 1 %	-0.13 %	0 %	-0.18 %	-0.50 %
Wage profile 1.5 %	-0.35 %	-0.02 %	0 %	-0.11 %
Wage profile 3-2-1-0 %	-0.53 %	-0.12 %	-0.01 %	0 %

From Table 9, it is observed that a suboptimal asset allocation strategy in the absence of a wage profile could lead to significant welfare losses. In practice, pension funds offer the participant one or a few asset allocation strategies differentiated on the risk preferences. This leads to the fact that a misspecification of the underlying assumed wage profile can lead to significant welfare losses. Observe the asymmetry in the welfare losses found in Table 9. In particular, an overestimation of the assumed wage profile in the asset allocation leads to an overallocation to stocks.

In this setting, the welfare effects with respect to a mismatch in the dynamic contribution strategy based on the assumed wage profile will be investigated. Observe that a misspecification in the assumed wage profile will be directly related by oversaving in case the wage profile is underestimated (see Figure 16*c*, 17*c* and 18*c*) and undersaving in case the assumed wage profile is overestimated. In Table 10 the welfare effects will be presented. In the rows the assumed wage profile is defined and in the columns the contribution strategy is defined.

Table 10: Welfare effects contribution strategy, including wage profiles

	Contr. (no profile)	Contr. (profile 1 %)	Contr. (profile 1.5 %)	Contr. (profile 3-2-1-0 %)
Wage profile 1 %	-10.6 %	0 %	-8.78 %	-27.22 %
Wage profile 1.5 %	-18.17 %	-4.10 %	0 %	-4.22 %
Wage profile 3-2-1-0 %	-24.88 %	-10.42 %	-3.03 %	0 %

Under the assumptions in this thesis, welfare losses of a misspecification in the underlying assumed wage profile in the contribution scheme can be relatively high. Note that having assumed wage profile 3-2-1-0 %, and being restricted to a contribution scheme that abstracts from an assumed wage profile yields enormous welfare losses. Still, this case will be extremely stylized. From Table 10, it will be noticed that avoiding these extreme cases, significant welfare losses will be present in case of a suboptimal contribution scheme. For example, having assumed wage profile 1.5 % and being restricted to the contribution scheme of assumed wage profile 3-2-1-0 % yields a welfare loss of 4.2 %.

In particular, the welfare losses of an overspecification of the assumed wage profile will be significantly higher compared to an underspecification of the assumed wage profile (see Table 10). This asymmetric effect will become smaller in the presence of the state pension.

3.2.1 Implications assumed wage profile for optimal pension contract

In the setting of this thesis, it can be concluded that it will be more beneficial to have a tailored dynamic contribution scheme in terms of the assumed wage profile compared to an accurate reflection of the assumed wage profile in the asset allocation strategy.

Regarding the results of Table 10 (i.e. welfare losses of a suboptimal contribution scheme), it will be beneficial to present the participant a tailored dynamic contribution scheme differentiated on the assumed wage profile. Some stylized cases will generate enormous welfare losses and will have a difficult interpretation in reality. Having access to a dynamic contribution scheme compared to a constant contribution will be welfare enhancing. For this specific setting, the effect was not quantified.

Given that the typical pension fund has one underlying assumed wage profile in the risk profiles offered and the participant has a different assumed wage profile, under the assumptions of this thesis, nonnegliable welfare losses will arise. Still, choosing the underlying risk profile in a clever way (i.e. choose a column in Table 9 with minimal welfare losses) can mitigate the welfare losses and potentially reduces the attractiveness of offering the risk profiles with different underlying assumed wage profiles.

4 Life events

In this section some illustrative life events that could arise during the life cycle will be analyzed. These life events will challenge the optimality of the current pension contract in a second way in addition to the initial heterogeneity discussed in Section 3. In particular, the participant may want to reconsider the (dis)saving and the asset allocation strategy. Therefore, welfare effects of having a suboptimal pension contract will be quantified. A clear distinction will be made between welfare losses arising from a suboptimal contribution strategy and a suboptimal asset allocation strategy. To ensure the practical relevance, it will be concluded to what extent, conditional on the current pension contract, flexibility in the pension contract is possible. In addition to this, this thesis will contribute to the discussion where flexibility in the pension contract (if it is currently not possible) will be welfare enhancing. Observe that for some life events, the participant can perfectly anticipate on the life event to end up with the optimal pension contract (in the setting of this thesis).

Recall that in Subsection 2.2, the impact of an exogenous positive shock to human capital (hence total wealth) was illustrated. In particular, this implies that we have an additional claim to a risk free asset. As a consequence, the pension contract should allow for a higher allocated fraction to the risky asset in terms of financial wealth that can be verified from equation (12). The exogenous shock leads to an increased constant consumption pattern which is prescribed by formula (33). The optimal pension contributions in the pension contract will be increased to maintain the consumption standard (in retirement).

In Subsection 4.1, the case of receiving a cash bequest, increasing financial wealth, will be discussed. Compared to the case of an exogenous shock in human capital, one is able to allocate this bequest optimally over the two assets. In the optimal situation the complete available financial wealth (accumulated financial wealth and bequest), will be allocated in accordance with equation (12). A higher consumption standard, defined in formula (33), can be realized due to this higher available financial wealth. Pension contributions are difficult to analyze in this situation, since the consumption of the bequest will be spread over the accumulation phase and the retirement phase. The proportion of the bequest consumed during the accumulation phase and the retirement phase will be different for each sample path. Hence, part of the bequest will be interpreted as a one time (stochastic) contribution to the pension scheme. This requires flexibility in the (higher) pension contributions and as a consequence the asset allocation should be adjusted accordingly.

In Subsection 4.2, the consequences of owning (i.e. buying) a house will be analyzed. This implies that an additional asset will be available in the financial market that could have some correlation to the risky asset. It is assumed that the fraction of total wealth invested in the house will be given and the pension contract should offer flexibility to reoptimize the allocation of accumulated pension wealth over the stock and the bond. Observe that accumulating housing wealth is an implicit form of saving for retirement. Therefore, lower pension contributions could be desirable. In retirement, either one can deplete housing wealth (reverse mortgage, sell the house) or one has a decreased consumption pattern. As a starting point, in Subsection 4.2.2 it is assumed that the housing wealth is a risk free asset. In Subsection 4.2.3, the setting will be analyzed where the dynamics of housing wealth will be geometric Brownian motion.

In Subsection 4.3, the implications of involuntary temporary unemployment were investigated. This implies that part of the human capital will be unanticipated depleted. In terms of the asset allocation decision, a claim on a risk free asset will be lost. In the optimal situation the asset allocation in the pension contract should be adjusted in line with formula (12). The trade off between consumption now and making pension contributions (under the assumption of an unemployment benefit level) will be interesting to analyze. In this case, the pension contract should offer flexibility in already decumulation of accumulated pension wealth or still contributing to the pension account.

4.1 Life event (cash) bequest received

In this life event, a hypothetical pension plan participant aged 45 years will receive a bequest of $\frac{1}{3}$ and $\frac{1}{6}$ times the value of the human capital at that point in time. In the optimal scenario in the standard life cycle model, discussed in Subsection 2.1, receiving a bequest will not be possible by the assumption of a deterministic date of death. Here the bequest should be interpreted as an unexpected exogenous shock in financial wealth. This yields that a simplified setting of a bequest received will be analyzed. In this subsection, the consequences of a shock in financial wealth will be quantified for the optimal pension contract implying that different interpretations (instead of a bequest received) will be possible as well. For example, Love (2009) [38] quantifies the optimal life cycle portfolio in case of changes in marital status (i.e. divorce, widowhood) and family composition.

For this life event, the optimal pension contract in terms of optimal consumption strategy and optimal asset allocation strategy will be derived. We will differentiate between four consumption strategies and three asset allocation strategies to continue the life cycle. The main idea is to see the impact on optimal consumption smoothing and optimal investment summarized in welfare losses. These welfare losses could arise from inflexibility of the pension contract (i.e. asset allocation cannot be changed), shortsightedness of the pension plan participant (consume within 10 years). From these welfare losses, it could be determined where flexibilities are most important and / or attractive for the individual. For the pension fund it will interesting to know which product designs are attractive to implement and how to tune its products to the specific aspects that the participant requires.

The following consumption strategies can be distinguished, where it should be noted that the agent is assumed to pass away in 40 years:

- C_{10}: Consume the bequest within 10 years
- C_{20}: Consume the bequest within 20 years
- C_{30}: Consume the bequest within 30 years
- C_{40}: Consume the bequest within 40 years (i.e. until the deterministic date of death)

In all consumption strategies, the expected consumption pattern for the bequest is assumed to be constant implied by the parameter values defined in Table 1.

An overview of the considered asset allocation strategies:

- Opt. A: The asset allocation strategy in which we will rebalance after the bequest amount is received will be analyzed. The bequest amount will be added to the financial wealth, hence the allocation towards the risky asset (in terms of financial wealth) will be reduced as can be verified from equation (12). The reduction in equity exposure depends on the size of the shock in financial wealth, which equals the bequest amount. An other representation of the optimal asset allocation strategy will be to act in line with the initial optimal asset allocation for the accumulated pension wealth and use the constant optimal asset allocation strategy from formula (10) for the bequest amount. One can mathematically show that these asset allocation strategies are equivalent.
- Opt. A unchanged: The asset allocation strategy in which we do not take into account that the bequest was received will be analyzed. This implies that the asset allocation strategy will coincide with the optimal asset allocation strategy in the standard setting (see Subsection 2.1) since we will not increase the financial wealth level in formula (12) with the bequest amount.
- 100-age: The asset allocation strategy that has an equity exposure in line with the 100-age asset allocation rule will be analyzed as well.

This implies that we have to distinguish between 6 cases. In the first case it is allowed to optimize consumption over the remaining life cycle and optimization over the asset allocation is allowed (implying rebalancing after the shock in financial wealth because of the bequest). This case reflects the optimal pension contract and will be referred to as (C_{40} + Opt. A).

Three cases can be constructed in which a suboptimal consumption strategy will be chosen.

- Scenario 2 (C_{10} + Opt. A): It is assumed that the bequest is completely consumed within 10 years, whereas optimization over the asset allocation is allowed.
- Scenario 3 (C_{20} + Opt. A): It is assumed that the bequest is completely consumed within 20 years, whereas optimization over the asset allocation is allowed.
- Scenario 4 (C_{30} + Opt. A): It is assumed that the bequest is completely consumed within 30 years, whereas optimization over the asset allocation is allowed.

Two cases can be constructed in which a suboptimal asset allocation strategy will be chosen.

- Scenario 5 (C_{40} + Opt. A unchanged): In this scenario, it is allowed to optimize consumption over the remaining life cycle. The asset allocation strategy that is optimal if no bequest is received will be used.
- Scenario 6 (C_{40} + 100-age): In this scenario, it is allowed to optimize consumption over the remaining life cycle. The asset allocation strategy will be determined by the 100-age rule.

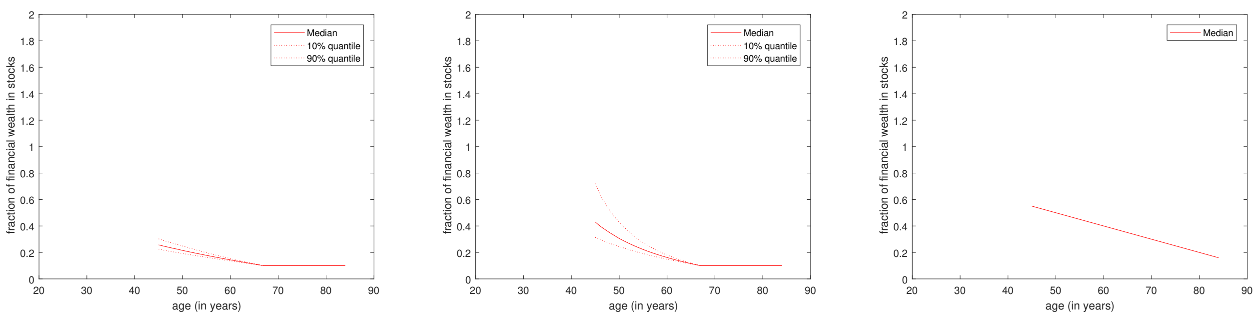
As a starting point, in all the cases we need to optimize the life cycle following the standard setting discussed in Subsection 2.2, until the age at which the bequest is received. After this age, the cases will be different. We will restrict to highlight the consumption pattern and the asset allocation (in terms of financial wealth) in the cases only.

The optimal pension contract and welfare effects of predetermined consumption strategies and asset allocation strategies will be investigated for different bequest amounts in Subsection 4.1.1 and Subsection 4.1.2. Observe that a bequest amount of $\frac{1}{3}$ and $\frac{1}{6}$ of the human capital at the age of 45 will be analyzed. In particular, this implies a numerical bequest amount of 6.6 and 3.3 times the annual salary. In Subsection 4.1.3, the main conclusions and recommendations for the optimal pension contract concerning this life event will be discussed.

4.1.1 Bequest $\frac{1}{3}$ Human Capital

In Figure 20, the median, 10 % and 90 % quantile of the asset allocation strategies are presented. Observe that the asset allocation will be presented after the bequest is received (from the age of 45 onwards). In Figure 20b, it is not allowed to optimize over the asset allocation, hence the asset allocation in Figure 20b does not take into account the exogenous increase in financial wealth. Recall that if one is able to optimize over the asset allocation, the asset allocation of Figure 20b will be used for the accumulated financial wealth, whereas the constant optimal equity exposure in terms of total wealth will be chosen for the bequest amount. If we combine, the asset allocation strategy in terms of financial wealth, for a smoothing period of 40 years is represented in Figure 20a. Optimally adjusting the asset allocation strategy implies a decreased exposure to the risky asset for all the asset allocation quantiles respectively. In the same way, the combined (optimal) asset allocation strategies in terms of financial wealth can be represented for a smoothening period of 10, 20 and 30 years as well.

In Subsection 4.1.2, the implications of a smaller bequest amount will be analyzed. Observe that again it is optimal to decrease the exposure to the risky asset compared to the asset allocation in Figure 20b. However, the asset allocation will not be reduced as much as in Figure 20a. This is explained by a smaller bequest amount, hence a smaller increase in financial wealth. In Figure 20c, the asset allocation that at each age has an equity exposure of 100-age is presented. In this strategy, there will be no uncertainty in the asset allocation, therefore no quantiles will be observed.



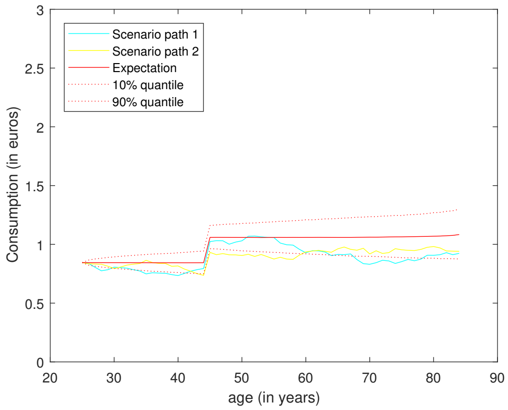
(a) Asset allocation (scenario 1, C_{40} + Opt. A)

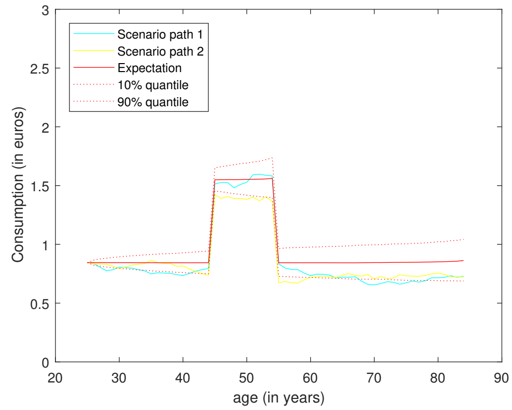
(b) Asset allocation (scenario 5, C_{40} + Opt. A unchanged)

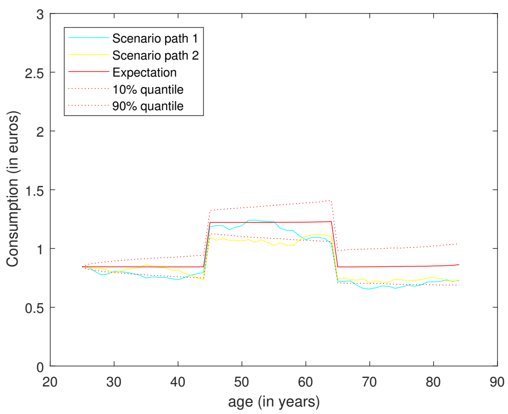
(c) Asset allocation (scenario 6, C_{40} + 100-age)

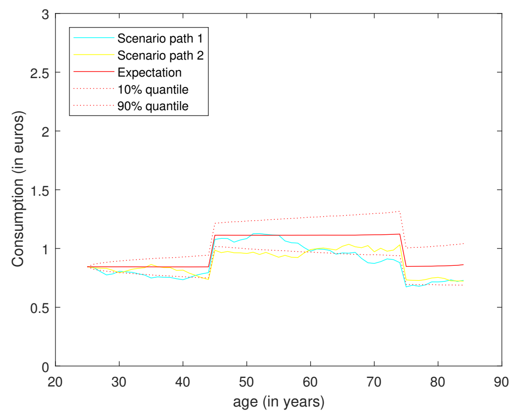
Figure 20: Asset allocation paths (Bequest $\frac{1}{3}$ Human Capital)

In Figure 21, the consumption paths for each case is presented.

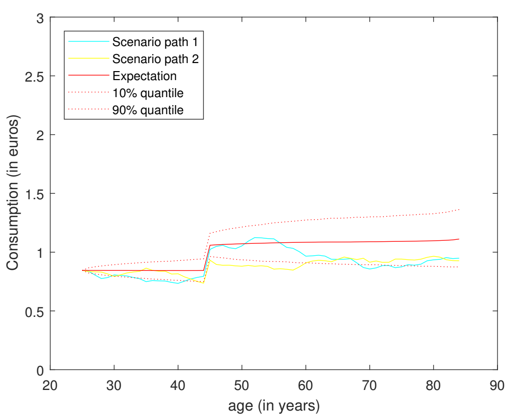




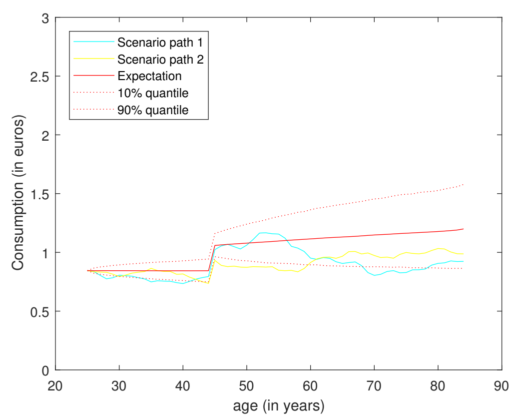




(d) Consumption (scenario 4, C_{30} + Opt. A)



(e) Consumption (scenario 5, C_{40} + Opt. A unchanged)



(f) Consumption (scenario 6, C_{40} + 100-age)

Figure 21: Consumption paths (Bequest $\frac{1}{3}$ Human Capital)

Observe that in Figure 21a, 21b, 21c and 21d the optimal asset allocation strategy will be used and different smoothing periods for the bequest will be analyzed. In Figure 21a, the bequest can be smoothened over the remaining life, which is the most optimal to do. Figure 21b, 21c and 21d represent suboptimal smoothing periods and an increased expected consumption standard will persist for the smoothing period. Observe that the shorter the smoothing period, the higher the consumption standard will be for this time. After the smoothing period, the consumption will fall to the same level as in the case in which no bequest was received. As a consequence of the optimal asset allocation strategy used in these figures, a constant expected consumption pattern will be observed. These figures will be the main input to calculate welfare losses arising from suboptimal smoothing of consumption.

In Figure 21a, 21e and 21f, an optimal smoothing period will be chosen, whereas in Figure 21e and 21f a suboptimal asset allocation strategy will be considered. In Figure 21e and 21f, a slight increasing consumption pattern will be observed (in expectation), which is explained by a higher exposure to the risky asset in the suboptimal asset allocation strategies compared to the optimal setting. These figures will be the main input to calculate welfare losses arising from suboptimal asset allocation strategies.

Based on the consumption patterns in Figure 21, it is expected that the dominant effect (in terms of welfare losses) will be explained by the consumption smoothing decision, whereas the asset allocation strategy will be a higher order effect. The welfare losses are summarized in Table 11.

Table 11: Welfare losses, bequest ($\frac{1}{3}$) Human capital at age 45

	Opt. A	Opt. A unchanged	100-age
Consume in 10 years	-18.37 %	X	X
Consume in 20 years	-14.22 %	X	X
Consume in 30 years	-8.58 %	X	X
Consume in 40 years	0 %	-0.63 %	-2.21 %

If we take a closer look at the welfare losses presented in Table 11, it is observed that a suboptimal asset allocation strategy leads to welfare losses that will be in the order of less than 2.5 %. Suboptimal consumption smoothing lead to welfare losses in the order of 8%-20%. Recall that Figure 21a and Figure 20a, define the optimal pension contract for an agent receiving a bequest.

4.1.2 Bequest $\frac{1}{6}$ Human Capital

In this part, the same analysis will be performed, however a smaller received bequest amount will be assumed. In particular, a bequest amount of $\frac{1}{6}$ times the value of the human capital will be received at the age of 45. It is decided to restrict in this part to the discussion of the results in Table 12 only. Compared to the optimal asset allocation strategy in case of a higher bequest (Subsection 4.1.1) a higher exposure to the risky asset will be maintained motivated by a smaller increase in financial wealth. The increase in expected consumption standard will be smaller compared to the setting of a higher bequest as in Subsection 4.1.1.

Table 12: Welfare losses, bequest ($\frac{1}{6}$) Human capital at age 45

	Opt. A	Opt. A unchanged	100-age
Consume in 10 years	-8.76 %	X	X
Consume in 20 years	-5.42 %	X	X
Consume in 30 years	-2.41 %	X	X
Consume in 40 years	0 %	-0.24 %	-0.78 %

If we take a closer look at the welfare losses presented in Table 12, it is observed that a suboptimal asset allocation strategy leads to welfare losses that are in the order of less than 1 %. Suboptimal consumption smoothing leads to welfare losses in the order of 2%-10%.

4.1.3 Implications (cash) bequest for optimal pension contract

Overall, it was concluded that the higher the bequest received will be, the more important the decision to reconsider the consumption smoothing strategy and asset allocation strategy will be.

In case of a bequest received, it is initially important to focus on optimal consumption smoothing. Even when the person uses a smoothing period of 30 years, significant welfare losses will be present (see Table 11 and 12). Even for a lower bequest amount, significant welfare losses will arise if a suboptimal smoothing period is chosen.

In case of a bequest received there will be relatively few gains if the asset allocation strategy will be reconsidered (see Table 11 and 12). Still if the bequest will be enormous, it might be attractive to reconsider the asset allocation strategy.

Recall that in this simplified Merton setting part of the bequest will be used to increase the consumption standard during the accumulation phase and the other part will be used for consumption in the retirement phase and the proportions will be different for each sample path and will be unknown ex ante. Therefore, part of the bequest has the interpretation of a stochastic one time contribution. This is because the consumption during the accumulation phase and the retirement phase are completely integrated. In particular, in the optimal situation the consumption of the bequest is extremely flexible.

In the setting of this thesis, the pension plan participant can replicate the optimal pension contract without the need of changes in the current pension contract. The optimal asset allocation can be replicated by a constant equity exposure of the bequest amount in line with formula (10) and no changes in the asset allocation of the accumulated financial wealth. In addition to this, the participant can spread the consumption of the bequest over a 40 year horizon.

- In practice, the typical pension participant will not be able to act in line with the optimal scenario, due to financial illiteracy.
- Shortsightedness, implying to choose for a suboptimal smoothing period, will lead to significant welfare losses.
- In a more realistic setting, the pension plan participant will be unable to deal with the micro longevity risk neglected in the setting of this thesis.

Based on these arguments, it might be attractive to contribute part of the bequest as a one time (stochastic) contribution to the pension account. However, the institutional rules might be a barrier to contribute this in the second pillar or third pillar. Recall that some institutional background regarding fiscally facilitated saving for retirement was presented in Subsection 2.5. Also note that taxes were neglected in the analysis.

- In principle the pension contributions in the second pillar will be fixed. Under some circumstances (depending on the exact agreements of the social partners), additional pension contributions could be made in the second pillar. Still, the participant will be restricted in making pension contributions that are fiscally attractive in line with the 'Witteveenkader'. In the last years, these contribution rates were significantly reduced.
- In addition to the second pillar, pension wealth can be accumulated in the third pillar. This could be interesting if the agreements in the second pillar do not allow for additional pension contributions and one can still contribute fiscally attractive ('vrije jaarruimte'). In a typical third pillar contract, there is complete flexibility regarding the contributions. Still, fiscally attractive saving will be restricted here as well. In a third pillar contract there will be more flexibility regarding the asset allocation.
- Still, additional contributions could be made in the third pillar, not fiscally facilitated.

In general, the additional pension contributions that could be made in the second and third pillar will be significantly lower than the optimal one time (stochastic) pension contribution in case of a bequest amount in the order as discussed in Subsection 4.1.1 and Subsection 4.1.2. In principle there will be a trade off for the participant to contribute each year maximally fiscally facilitated to his pension account, till the point that the maximum optimal part of the bequest is contributed (corrected for realized asset returns in the meanwhile) or contribute in the third pillar part of the bequest without a tax incentive. This could be beneficial to insure to some extent the longevity risk, although this is not considered in the model here.

As a result, changes in the asset allocation strategy will be optimal. Observe that in the optimization problem, accumulated financial wealth could be divided over three accounts (second pillar, third pillar and a remaining part privately held without longevity insurance). This requires an integrated asset allocation approach over the different financial wealth accounts.

In the current setting, the typical pension fund offers one life cycle (based on the average risk preferences of the population) in the second pillar. In principle, the remaining financial wealth (in third pillar and privately held) can be invested to end up with the optimal asset allocation. This might be too much to expect from the participant. However, this might not be too big of a problem, since a suboptimal asset allocation (i.e. Opt. A unchanged) yields minimal welfare losses. To contribute the optimal amount will be the dominating effect, however this might be a challenging task as well. It might be appropriate to come up with a rule of thumb. Contribute $\frac{1}{3}$ of the bequest (nominal amount) to the pension account and use $\frac{2}{3}$ for consumption in the accumulation phase (the fractions are motivated based on expected asset returns).

A side note could be that by contributing part of the bequest to the pension system, the flexibility regarding these contributions will be gone. This is because we are restricted to consume this from the retirement age onwards in the pension setting.

4.2 Life event housing wealth

In this case, a hypothetical pension plan participant will be confronted with an additional asset in the financial market, namely housing wealth $(HW_t)^{15}$. This situation challenges the optimality of the current pension contract in a different way compared to a bequest received. The availability of this additional asset challenges the optimal asset allocation (12) and optimal consumption pattern (33) over the life cycle. If it is assumed that the housing wealth will be used to generate a pension income stream, less accumulation of financial wealth (i.e. less pension contributions) could be desirable in the pension contract. Even a reduced consumption standard after the attainment of housing wealth will justify lower pension contributions. In particular, the size of the housing wealth motivates less (or no) pension contributions and even dissaving before retirement might be optimal. In addition to this, one might reconsider the optimal asset allocation strategy in the pension contract. The implications for the optimal asset allocation strategy depends on the assumed dynamics for housing wealth.

In case of an additional available asset in the financial market, the adjusted financial market can be represented as follows:

$$dB_t = rB_t dt \tag{29}$$

$$dS_t = \mu_S S_t dt + \sigma_S S_t dW_{S,t} \tag{30}$$

$$dHW_t = \mu_{HW} HW_t dt + \sigma_{HW} HW_t \left(\rho_{HW,S} dW_{S,t} + \sqrt{1 - \rho_{HW,S}^2} dW_{HW,t} \right) \tag{31}$$

The dynamics for housing wealth (HW_t) were presented in a general form. Under this specification, the assets S_t and HW_t, are two correlated geometric Brownian motions with correlation coefficient $\rho_{HW,S}$. By setting this correlation coefficient $(\rho_{HW,S})$ to zero, S_t and HW_t will be denoted as two independent geometric Brownian motions. Observe that μ_{HW} and σ_{HW} define the drift and volatility parameter for the housing wealth process. If these are assumed to be equal to the risk free rate and zero, the dynamics for housing wealth will be equal to the bond dynamics (i.e. risk free asset).

For the optimal asset allocation strategy, it might be convenient to maximize the fraction of financial wealth in stocks conditional on the housing wealth. This implies that the allocated fraction to housing wealth is assumed to be given. It might be too hopeful that individuals will buy a specific house and keep in mind the prescribed optimal allocation in terms of total wealth. Conditional on the housing wealth, the approximated optimal allocation to the risky asset in terms of financial wealth, found in Olear et al. (2017) [31], can be given as follows:

$$f_t^* = \frac{\lambda_S}{\gamma \sigma_S} \left(1 + \frac{H_t + HW_t}{F_t} \right) - \frac{HW_t}{F_t} \frac{\sigma_{S,H}}{\sigma_S^2} \tag{32}$$

In addition to incorporating the housing wealth in the optimal financial portfolio, De Jong, Driessen and Van Hemert (2007) [28] analyze the economic benefits of including housing futures in the portfolio.

The first term of expression (32) coincides more or less with the initial optimal allocation without the housing asset. The additional term in equation (32) represents a hedging demand term, where a higher correlation between the stock and the house yields a decreased exposure to the stock as well. If it is assumed that housing wealth is a risk free asset ($\rho_{HW,S}$=0, σ_{HW}=0 and μ_{HW}=r), the hedging demand term will be equal to zero. Then, the fraction of financial wealth allocated to the risky asset should be increased at all points in time, since the housing wealth will have a similar effect as human capital. The exact implications on the optimal asset allocation depends on the assumptions regarding the process for housing wealth.

If a closer look is taken at the derivation of the optimal consumption in Appendix $A.1.1$, optimal consumption can be defined in a similar way.

[15] HW_t refers to the value of the housing wealth asset. Later on, NHW_t (net housing wealth) is defined that reduces the value of the housing wealth with the outstanding mortgage.

$$C_t = \frac{W_t}{g_1(t)} \tag{33}$$

Observe that this implies that this person also consumes out of his housing wealth (reducing financial wealth). This might be realistic under the assumption that housing wealth will be used to generate a pension income stream.

In the Dutch setting, there will be a strong cross sectional correlation between the start salary and the housing wealth. In particular, banks will restrict individuals to invest in housing wealth to approximately 4 until 6 times their annual salary (before taxes). If we observe formula (32) in detail, for large amounts of housing wealth, optimal financial wealth can become negative, resulting in a short position in the stock. This is because, the complete pension wealth (and even more than needed) will be invested in housing wealth. By restricting the housing wealth in line with banks, these unrealistic cases will be avoided.

At this point in time, the house will be bought and the individual will go to the bank to borrow the necessary amount. In the above derivations of the optimal consumption smoothing and optimal asset allocation, it was assumed that there is no debt (i.e. housing wealth was received 'as a bequest'). Now we consider the case where the individual has to repay the mortgage and will need to make interest payments. In this standard life cycle model, this means that we short the bond, to be able to take a long position in housing wealth. To implement the attainment of housing wealth, there will be two necessary conditions:

- Condition 1: The mortgage should be repaid. It is assumed that full repayment will be done in 30 years.
- Condition 2: Annual interest payments should be made, which is in line with the payments of shorting a bond.

In Van Hemert [32] (2010) a detailed analyses of the optimal financial planning taking into account several possibilities regarding the mortgage contract (adjustable mortgage rate versus a fixed mortgage rate) can be found. This optimal mortgage choice was also considered in De Jong et al. (2007) [28]. In this thesis, the setting will be restricted to a fixed mortgage rate which equals the risk free rate.

Net housing wealth (NHW_t) will be increased by repayments (RP_t) to the bank in the following way (i.e. net housing wealth will be increased by the dynamics of housing wealth as well), where $t \geq 30$ (representing that NHW_t is defined from the age of 30):

$$NHW_t = HW_t - \sum_{j=30}^{t} RP_j \tag{34}$$

Definition (34) implies that we should adjust the optimal asset allocation in (32) and the optimal consumption in (33) derived for the case where housing wealth is received 'as a bequest'.

For the optimal consumption pattern, one should add NHW_t to equation (33) instead of HW_t. Otherwise, one would not optimally smooth consumption over the life cycle. The adjusted (approximately) optimal consumption pattern can be represented as follows:

$$C_t = \frac{F_t + H_t + NHW_t}{g_1(t)} \tag{35}$$

It is more challenging how to adjust the optimal asset allocation for the presence of housing wealth. In the first term of expression (32) we should add NHW_t since this concerns the total wealth level. In the second term of expression (32), we should not change. This is motivated since we are fully exposed to the geometric Brownian motion of housing wealth. The adjusted optimal asset allocation can be represented as follows:

$$f_t^* = \frac{\lambda_S}{\gamma \sigma_S}\left(1 + \frac{H_t + NHW_t}{F_t}\right) - \frac{HW_t}{F_t}\frac{\sigma_{S,H}}{\sigma_S^2} \tag{36}$$

In order to show the impact of an adequate contribution scheme for houseowners, the welfare losses reported in Van Ewijk et al. (2017) [24] will be replicated in Subsection 4.2.1. In Subsection 4.2.2, the optimal pension contract will be derived, where it is assumed that a mortgage will be repaid to obtain a final amount of housing wealth, where the dynamics of housing wealth will coincide with the risk free asset. In Subsection 4.2.3, the (approximated) optimal pension contract will be derived, where it is assumed that a mortgage will be repaid to obtain a final amount of housing wealth, where the dynamics of housing wealth will be a geometric Brownian motion with correlation to the risky asset. In both cases, welfare gains for realistic (suboptimal) contribution schemes and reasonable (suboptimal) strategies with respect to housing wealth at the retirement age will be calculated. Additional parameter values should be defined before we can discuss these cases. The parameter values of the housing wealth asset are defined in Table 13[16] in line with Olear et al. (2017) [31].

Table 13: Parameter values

Parameter names	Parameter values
μ_{HW}	1.13 %
σ_{HW}	13.72 %
$\rho_{HW,S}$	0.069
HW_0	2.89
Rental costs	0.10
Age at which house is bought	30

In our setting housing wealth will be obtained at the age of 30, whereas in Olear et al. (2017) [31] the house was bought at the age of 37 (i.e. house will be repaid at retirement age). Normalizing the price of the housing wealth with respect to the annual labor income in the setting of Olear et al. (2017) [31], one would get 5.23. In the setting of Olear et al. (2017) [31], one could calculate the certainty equivalent over a period in which the participant only owned housing wealth. This implies that this setting should not worry about rental costs. This thesis though had to come up with an estimate of the rental costs. This will be necessary to incorporate in the model, if the setting should be calibrated to obtain equivalence between renting a house and owning a house.

Before the results obtained in this thesis will be presented, an intermezzo will be made where welfare gains obtained in Van Ewijk et al. (2017) [24] will be replicated. In Van Ewijk et al. (2017) [24] all numbers can be directly interpreted in euros. In the setting of this thesis, all numbers will be normalized with respect to the annual labor income (ALI) which is set to 1[17].

For the sake of completeness, the major differences in the setting of Van Ewijk et al. (2017) [24] with respect to the setting of this thesis will be repeated.

- (A, DC, ↓): In their DC setting a constant contribution scheme was used. This instead of a dynamic contribution scheme (i.e. adjust the contributions to realized asset returns) which is used in this thesis.
- (A, BC, ↓): In their setting it will be an easy task to have an optimal asset allocation in case of an exogenous borrowing constraint (i.e. truncated asset allocation strategy will be optimal). If stochastic consumption was used, numerical techniques should be used to find the optimal asset allocation in case of a borrowing constraint. In this thesis no exogenous borrowing constraint will be imposed.
- (A, AOW, ↓): Their setting takes into account the state pension. The implications for the analysis of excluding or including the state pension will be extensively discussed in Subsection 5.1.2 for this case.

[16] The first three parameter values defined in Table 13 define the dynamics of the housing wealth asset in case of a geometric Brownian motion. A start value of housing wealth of 2.89 is used in the figures. In the tables, also smaller start values of housing wealth will be considered.

[17] Shorthand notation will be used for normalized with respect to the annual labor income (i.e. normalized w.r.t. ALI)

4.2.1 Intermezzo: Impact of housing wealth as in suboptimal contract (as in Van Ewijk et al. (2017))

Firstly, an elaboration on the welfare gains found in Van Ewijk et al. (2017) [24][18] will take place. After this, the results of this thesis regarding the optimal pension contract in the situation where housing wealth is accumulated will be discussed. To start, we replicate the results found in Figure 2 of Van Ewijk et al. (2017) [24]. In their Figure 2 welfare losses were calculated in case of undersaving. Minimal differences were found in the replication (i.e. welfare losses of 0.15 %, 0.64 %, 1.55 %, 2.95 % and 5.02 % for undersaving percentage of 10 %, 20 %, 30 %, 40 % and 50 % respectively). The results of oversaving via housing wealth were calculated in an analogous way. Observe that the oversaving amount will be optimally allocated between the risky asset and the risk free asset. Therefore, no concrete dynamics for the housing wealth asset is imposed. It was assumed that during the life cycle, housing wealth will be accumulated as an annuity with terminal values at retirement of 50k, 100k, 150k, 200k and 250k respectively. Recall that we should define the percentage oversaving (i.e. with respect to the optimal contribution rate, 15.3 %). Therefore, the annual annuity (i.e. how much do we contribute nominally each year in addition to the optimal contributions, 3.67 k) will be calculated first. From this we can define the additional contributions as a percentage of the labor income minus social security (i.e. franchise). This enables us to define the percentage oversaving for the different values of housing wealth. These numbers are presented in Table 14.

Table 14: Calculating the percentage oversaving as in van Ewijk et al. (2017) [24]

Housing wealth at retirement age	Annuity (each year)	Contribution (%)	Oversaving (%)
50 k	0.76 k	3.15 %	20.59 %
100 k	1.51 k	6.30 %	41.18 %
150 k	2.27 k	9.45 %	61.76 %
200 k	3.02 k	12.60 %	82.35 %
250 k	3.78 k	15.74 %	102.88 %

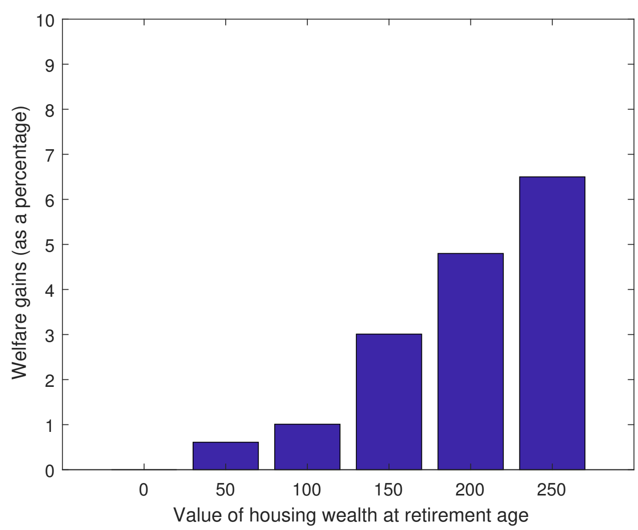


Figure 22: Replicated welfare gains of Van Ewijk et al. (2017) [24] of oversaving because mortgage repayments do not affect the contribution rate

Based on the oversaving percentage in Table 14, the welfare gains are replicated. This is presented in Figure 22. This can also be found in Figure 1 in Van Ewijk et al. (2017) [24]. Observe that the numerical welfare

[18]Here I was already aware of the typo on page 56 in Van Ewijk et al. (2017) [24] and therefore replicated the results with an AOW level of 16k.

gains can be found in their Table 7.

In this thesis the results of Van Ewijk et al. (2017) [24] will be extended[19], where it will either be assumed that housing wealth will have the dynamics of a risk free asset (Subsection 4.2.2) or that housing wealth will have the dynamics of a geometric Brownian motion (Subsection 4.2.3). Note that this thesis calibrated the initial value of housing wealth (in the risk free case) in such a way that the (expected) terminal value at retirement coincides with the value of housing wealth at retirement age in Van Ewijk et al. (2017) [24]. In the setting of this thesis, all numbers are normalized with respect to ALI which is set to 1.

[19]This thesis will not consider the results that Van Ewijk et al. (2017) [24] will obtain under the assumptions in this thesis. A different methodology is used in this thesis.

4.2.2 Housing wealth in optimal pension contract with housing wealth as a risk free asset

In this subsection, the implications for the optimal pension contract will be analyzed in the case in which housing wealth will be attained and repaid as a mortgage. This will be under the assumption that housing wealth will be a risk free asset. We abstract from equity exposure (which is optimal if risk premium $\lambda = 0$). For simplicity, the time preference parameter will be adjusted to the parameter value of the risk free rate (see Table 1). This is done to obtain a flat consumption pattern which is prescribed by equation (18). Note that in this setting, the consumption pattern, financial wealth dynamics and housing wealth dynamics are all deterministic. This is because we abstract from equity exposure.

One can distinguish between two consumption strategies, during the accumulation phase, in the setting in which the housing wealth is assumed to be risk free. There is only one asset allocation strategy available that is by definition optimal in the setting in which there is no equity exposure.

- Scenario (Acc, Opt. C): In this contribution strategy, optimization over the contribution rate (i.e. mortgage repayments reduce pension contributions) will take place after the housing wealth is obtained. The welfare gain and to what extent reduced pension contributions[20] will be optimal is highly dependent on the decision regarding housing wealth at retirement age. Housing wealth can be used to generate a pension income stream or rental costs will be avoided.
- Scenario (Acc, Subopt. C): In this contribution strategy, a fixed contribution rate (i.e. mortgage repayments at time t reduce consumption at time t) will be used. This coincides with the current Dutch institutional setting.

One can distinguish between two cases, during the retirement phase[21], in the setting in which the housing wealth is assumed to be risk free.[22]

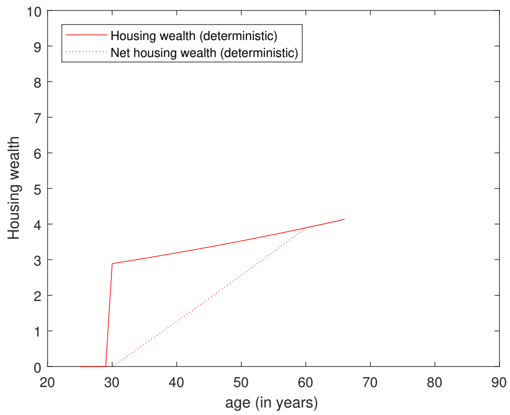
- Retirement scenario (RM): In line with Olear et al. (2017) [31], the financial wealth will be converted to a fixed annuity, whereas housing wealth will be depleted by a reverse mortgage. For simplicity, it is assumed that the housing wealth will generate a constant consumption stream[23].
- Retirement scenario (not use HW): In this situation housing wealth will not be used to generate a pension income stream. This implies that the housing wealth will be left as a bequest. At the age of 30 (after the housing wealth will be bought) an optimization problem arises, that in the solution implicitly assumes that we will use the housing wealth to generate a pension income stream. If we would not do this, the participant consumes too much during the accumulation phase and the standard of living should be decreased during retirement. Still, decreased pension contributions will be justified in this scenario compared to a participant renting a house. This is because this person will have a different consumption pattern (no rental costs).

[20] In the coalition agreement (Vertrouwen in de toekomst, 2017) between VVD, CDA, D66 and ChristenUnie, it is explicitly stated that the new individualized pension contract should integrate the traditional pension wealth and housing wealth. In particular, this will allow houseowners to accumulate less traditional pension wealth by having lower pension contributions compared to renters.

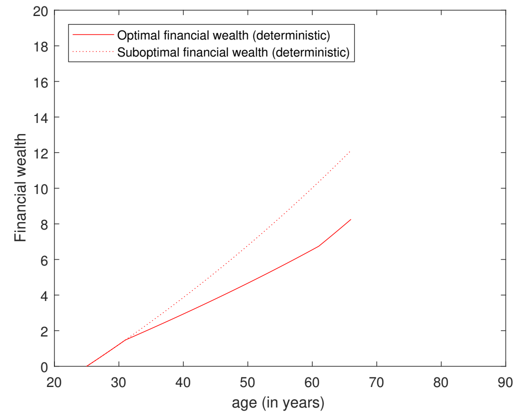
[21] In the coalition agreement (Vertrouwen in de toekomst, 2017) no particular section focused on the integration of traditional pension wealth and housing wealth in the retirement phase.

[22] In a setting where we exclude equity exposure, a variable annuity will be a fixed annuity. In the retirement phase, the consumption pattern should be reduced with the rental costs. For simplicity the variable annuity case will not be considered.

[23] Merton predicts, at a conference in St. Louis 'that home equity conversion, whether it is called a home pension, an equity release or a reverse mortgage, is going to be a crucial part of solving the retirement income problem'.



(a) Housing wealth



(b) Financial wealth (Acc, Opt. C) and Financial wealth (Acc, Subopt. C)

Figure 23: Accumulated deterministic pension wealth (housing wealth risk free)

In Figure 23*a*, the (deterministic) evolution of housing wealth and net housing wealth is presented. In Figure 23*b* the evolutions of financial wealth are presented for contribution strategy (Acc, Opt. C) and (Acc, Subopt. C). All numbers are normalized with respect to ALI which is set to 1. Observe that in Figure 23*b*, reduced pension contributions yield a smaller accumulation of financial wealth (Acc, Opt. C) in the age period 30-60 years old (i.e. mortgage repayments reduce pension contributions). In this setting, pension contributions will be in the order of 0.25, whereas mortgage repayments will be 0.10 (i.e. start value of housing wealth is 3[24]). In this deterministic setting, dissaving before retirement will not be required in the optimal solution. In Figure 23*b*, scenario (Acc, Subopt. C), the pension plan participant is not allowed to have decreased pension contributions (i.e. mortgage repayments reduce consumption immediately). As a consequence, financial wealth will be accumulated in line with the standard setting (i.e. oversaving).

[24]To be precise, the start value of housing wealth will be 2.89. Observe that housing wealth is considered at the individual level. A natural extension of the results can be made if the household consists of two persons.

Intermezzo: Equivalence houseowners versus renters

In this intermezzo we discuss the equivalence between renting a house and owning a house in the setting of this thesis. In the remainder of this subsection, the optimal pension contract is derived and welfare gains are calculated under the assumption that the housing wealth is risk free.

For simplicity, hypothetical persons are introduced in the following way:

- Hypothetical person (Rent): This person will rent housing wealth during the complete life cycle.
- Hypothetical person (Acc, Opt. C + RM): This person will buy housing wealth at the age of 30 and will repay the mortgage in 30 years. This person will have reduced pension contributions in the mortgage repayment period and will use his housing wealth to generate a pension income stream (i.e. reverse mortgage, owner without the bequest) during retirement [25].
- Hypothetical person (Acc, Opt. C + not use HW): This person will buy housing wealth at the age of 30, will repay the mortgage in 30 years. This person will have reduced pension contributions in the mortgage repayment period, will use housing wealth to have a lower consumption standard during retirement and leave the housing wealth as a bequest. Leaving the housing wealth as a bequest will have zero utility in this model.

In Table 15 an overview of the expenses for the hypothetical persons is presented. Hypothetical person (Rent) will have rental costs during the complete life cycle. Before hypothetical person (Acc, Opt. C + RM or not use HW) buys housing wealth, rental costs will be present. In the upcoming 30 years, interest payments and mortgage repayments will be present, whereas maintenance costs should be paid until the deterministic date of death. These expenses will be independent of the decision regarding housing wealth at retirement age.

Table 15: Overview of the characteristics of the hypothetical persons (risk free)

Age	Hypothetical person (Rent)	Hypothetical person (Acc, Opt. C + RM or not use HW)
25-30	Assumed rental costs (ARC_t)	Assumed rental costs
30-60	Assumed rental costs	Interest payments (IP_t)
		mortgage repayments (MR_t)
		and maintenance ($Maint_t$)
60-67	Assumed rental costs	Maintenance
67-85	Assumed rental costs	Maintenance

In this model, the equivalence between renting a house and owning a house, will be dependent on the decision regarding housing wealth at retirement age (i.e. reverse mortgage, or leave housing wealth as a bequest). Some intuition will be provided and it will be argued that leaving housing wealth as a bequest in retirement is a suboptimal retirement strategy.

Indifference between the consumption pattern of hypothetical person (Rent) and hypothetical person (Acc, Opt. C + RM) will be discussed.

- Mortgage repayments are optimally invested and contributed to the pension account via the risk free housing wealth. By a reverse mortgage in the setting of this thesis, the entire housing wealth can be used to generate a pension income stream. Therefore mortgage repayments and lower pension contributions do not have a negative influence on the optimal consumption smoothing of the agent over the life cycle.

[25] The reverse mortgage will be defined in a similar way as in Olear, de Jong, and Minderhoud (2017) [31]. This implies capitalizing the entire housing wealth at retirement age as a fixed annuity using the risk free rate.

- Interest payments will restrict at first sight the optimal smoothing of consumption. In the setting where housing wealth is assumed to be risk free, interest payments are paid whereas the housing wealth asset will accumulate with the same amount [26]. Therefore, we could allow to reduce financial wealth with these interest payments, since the interest payments will be directly added to the housing wealth asset. In retirement, the entire housing wealth can be converted into a cash flow.

- Assumed rental costs will reduce for hypothetical person (Rent) consumption equally over the life cycle. In case the house is owned, maintenance costs need to be made.

To conclude, for indifference between the consumption pattern of hypothetical person (Rent) and hypothetical person (Acc, Opt. C + RM), the rental costs should equal the maintenance costs.

Mathematically, this stylized equivalence can be presented in the following way (i.e. discrete setting):

$$\sum_{t=0}^{D} \frac{ARC_t}{r^t} = \sum_{t=0}^{D} \frac{Maint_t}{r^t} \tag{37}$$

$$ARC_t = Maint_t \tag{38}$$

Indifference between the consumption pattern of hypothetical person (Rent) and hypothetical person (Acc, Opt. C + not use HW) will be discussed.

- It will no longer hold that interest payments will cancel against the accumulation of housing wealth and mortgage repayments will cancel against the lower pension contributions. This is because housing wealth will not be used as a pension income stream. The accumulation of net housing wealth (mortgage repayments and increased value by the dynamics of housing wealth) will be beneficial for survivors who will receive the bequest only.

For indifference between hypothetical persons (Acc, Opt. C + not use HW) and (Rent) the present value of the assumed rental costs should be in the same order as the present value of the maintenance costs, the interest payments and the mortgage repayments.

Mathematically, this stylized equivalence can be presented in the following way (i.e. discrete setting):

$$\sum_{t=0}^{D} \frac{ARC_t}{r^t} = \sum_{t=0}^{D} \frac{Maint_t}{r^t} + \sum_{t=5}^{35} \frac{IP_t}{r^t} + \sum_{t=5}^{35} \frac{MR_t}{r^t} \tag{39}$$

From these results, it can be argued that in the setting of this thesis (i.e. leaving a bequest yields zero utility), we can refer to the reverse mortgage and not use housing wealth in retirement as the optimal and suboptimal consumption retirement strategy respectively.

An overview will be presented to obtain indifference between hypothetical person (Rent) and hypothetical person (Acc, Opt. C + RM). In addition the stylized indifference will be presented for hypothetical person (Rent) and hypothetical person (Acc, Opt. C + not use HW). In Table 16 and Table 17, the indifference is presented for initial values of housing wealth 1.74 and 2.89 respectively (normalized with respect to ALI).

Observe that lower maintenance costs will be present (Acc, Opt. C + not use HW) to obtain indifference. This is explained since mortgage repayments and interest payments (i.e. accumulation of net housing wealth) will be left as a bequest.

[26] This will hold if and only if the interest rate on the mortgage will be the same as the drift term of the risk free housing wealth asset and the reverse mortgage could be used defined in the setting of this thesis.

Table 16: Indifference hypothetical person (Rent), hypothetical person (Acc, Opt. C + RM) and hypothetical person (Acc, Opt. C + not use HW) for start value housing wealth normalized w.r.t. ALI of 1.74 at age 30

Assumed rental costs	Maintenance costs (Acc, Opt. C + RM)	Maintenance costs (Acc, Opt. C + not use HW)
0.10	0.10	0.058
0.15	0.15	0.108
0.20	0.20	0.158

Table 17: Indifference hypothetical person (Rent), hypothetical person (Acc, Opt. C + RM) and hypothetical person (Acc, Opt. C + not use HW) for start value housing wealth normalized w.r.t. ALI of 2.89 at age 30

Assumed rental costs	Maintenance costs (Acc, Opt. C + RM)	Maintenance costs (Acc, Opt. C + not use HW)
0.10	0.10	0.0324
0.15	0.15	0.0824
0.20	0.20	0.1324

This ended the intermezzo regarding equivalence of houseowners and renters.

Five hypothetical persons[27] can be distinguished in this setting. The two contribution strategies during the accumulation phase (Acc, Opt. C and Acc, Subopt. C) will be combined with the two consumption strategies in retirement (RM and not use HW) respectively. This will lead to the introduction of hypothetical person (Acc, Subopt. C + RM) and hypothetical person (Acc, Subopt. C + not use HW).

To start, welfare effects will be calculated by setting rental costs and maintenance costs in accordance with equation (38) (i.e. $ARC_t = Maint_t$). The following welfare effects will be calculated:

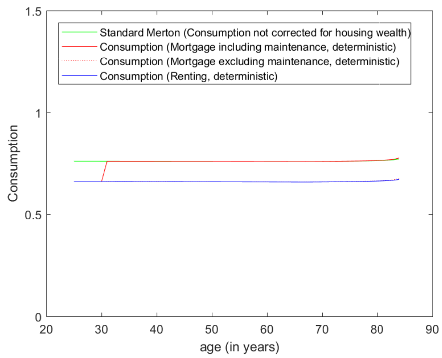
- Welfare gain of being able to adjust the contribution strategy in case housing wealth will be accumulated and a reverse mortgage will be used in retirement.
- Welfare gain of using a reverse mortgage at retirement age compared to leave the housing wealth as a bequest.
- Welfare loss of a 'suboptimal' contribution scheme in case it is decided (unexpected) not to deplete housing wealth at retirement age. Housing wealth will be left as a bequest.

In the upcoming figures, the results were plotted for an initial value of housing wealth of 3, whereas in the tables the welfare effects for smaller values of housing wealth will be presented as well. In the figures, a rental cost of 0.10 is imposed. Maintenance costs will be set to 0.10 as well[28]. To end this subsection, assumed rental costs and maintenance costs will be set in such a way that the participant will have an incentive to accumulate housing wealth[29]. Welfare effects with respect to hypothetical person (Rent) as the benchmark will be calculated here. Recall that all numbers are normalized with respect to ALI which is set to 1.

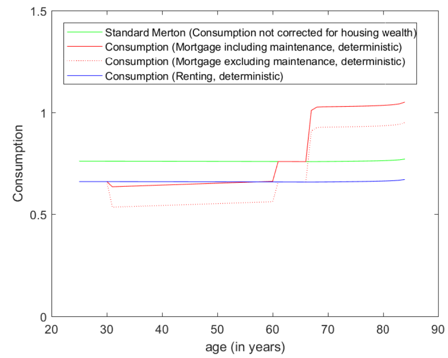
[27] Hypothetical person 1: (Rent), Hypothetical person 2: (Acc, Opt. C + RM), Hypothetical person 3: (Acc, Opt. C + not use HW), Hypothetical person 4: (Acc, Subopt. C + RM) and Hypothetical person 5: (Acc, Subopt. C + not use HW).

[28] For smaller start values of housing wealth, a similar percentage (of the start value of housing wealth) for rental costs and maintenance costs is chosen as for the case where the start value of housing wealth is 3.

[29] Assumed rental costs will be 0.10. The maintenance costs will be set to 0.05 and 0.025 respectively.



(a) Consumption path (Acc, Opt. C + RM)



(b) Consumption path (Acc, Subopt. C + RM)

Figure 24: Consumption path (Reverse mortgage)

In figure 24a, the optimal consumption path (i.e. hypothetical person Acc, Opt. C + RM) is presented, where mortgage repayments and interest payments will reduce the accumulation of financial wealth[30]. From the age of 25 until 30, the optimal consumption (i.e. hypothetical person Acc, Opt. C + RM) coincides with the consumption of hypothetical person Rent (blue line). This is because the participant buys the house at the age of 30. In the optimal situation, an increased constant consumption standard (compared to hypothetical person (Rent)) will be observed. A higher consumption standard is present because the rental costs will be avoided and the mortgage repayments and interest payments at each point in time will not harm the target of having a constant consumption pattern over the remaining life cycle. This increased consumption standard includes the maintenance costs. If the maintenance costs are set in line with equation (38), equivalence between hypothetical person (Acc, Opt. C + RM) and hypothetical person (Rent) is achieved. Therefore, the dotted red line (red line reduced by maintenance costs) coincides with the blue line.

In Figure 24b, the suboptimal consumption path (i.e. hypothetical person Acc, Subopt. C + RM) is presented, where mortgage repayments and interest payments at time t will reduce consumption at time t. The main difference with respect to Figure 24a is that during the mortgage repayment period a lower consumption standard will be obtained. In the age period 60-67 years, the consumption of hypothetical person (Acc, Opt. C) and (Acc, Subopt. C) coincides more or less[31]. In retirement a higher consumption standard (i.e. compared to the optimal consumption path) can be realized. The consumption pattern for hypothetical person (Acc, Subopt. C + RM) includes the maintenance costs. If the maintenance costs for hypothetical person (Acc, Subopt. C + RM) will be set to obtain equivalence in line with statement (38), the consumption standard for this person will become slightly lower presented by the red dotted line.

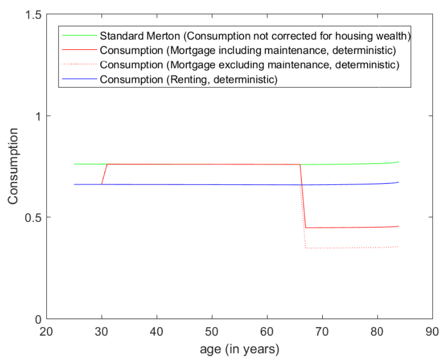
Table 18: Welfare gains (Acc, **Opt. C** + RM versus Acc, **Subopt. C** + RM)

Start value housing wealth (normalized w.r.t ALI), at age 30	Welfare gain (ARC=Maint=0)	Welfare gain (ARC=Maint=0.1[32])
0.58	+0.74 %	+0.78 %
1.16	+2.62 %	+2.86 %
1.74	+5.07 %	+5.76 %
2.31	+7.78 %	+9.14 %
2.89	+10.69 %	+12.96 %

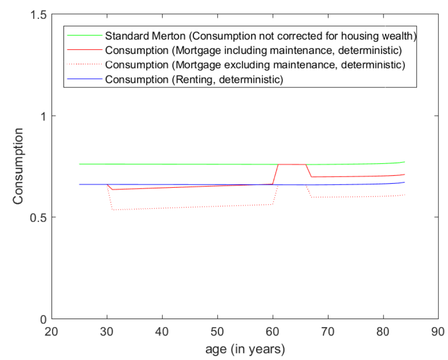
[30] Recall that in this situation the mortgage costs will not harm the target of having a flat consumption over the life cycle.
[31] The mortgage is completely repaid. Therefore, mortgage repayments will no longer reduce consumption.
[32] For lower values of housing wealth (than housing wealth is 3), an identical fraction compared to the start value of housing wealth is assumed. Still numerically this value is lower. For example, for start value of housing wealth 2.31, rental and maintenance costs of 0.08 will be assumed.

Adjusting the contribution rate in the pension contract optimally with respect to the accumulation of housing wealth, that will be used to generate a pension income stream (i.e. reverse mortgage), yields welfare gains as presented in Table 18[33]. Observe that the different initial values of housing wealth were chosen in such a way that it will match the normalized value of housing wealth with respect to ALI at retirement age as in van Ewijk et al. (2017) [24].

Welfare gains from adjusting the optimal contribution rate in the pension contract will increase with the start value of housing wealth. This will be explained by the fact that for higher values of housing wealth, the participant will oversave more. A more realistic reverse mortgage contract will be neglected in the setting of this thesis.



(a) Consumption path (Acc, 'Opt. C' + not use HW)



(b) Consumption path (Acc, 'Subopt. C' + not use HW)

Figure 25: Consumption path (not use HW)

In Figure 25a (Acc, Opt. C + not use HW) and Figure 25b (Acc, Subopt. C + not use HW), the 'optimal' and 'suboptimal' contribution strategy over the life cycle are presented. In retirement it is (unexpected) decided not to deplete housing wealth. During the accumulation phase, the optimal contribution strategy will take implicitly into account that housing wealth will be depleted in retirement (i.e. used for consumption). In this model, the retirement strategy not use HW implies that the housing wealth will be left as a bequest with an assumed zero utility. In this figure, the consumption corrected for the maintenance costs (red dotted line) is set in line with equivalence statement (38).

Now, we are able to calculate welfare gains from having access to a reverse mortgage market at retirement (i.e. choose a reverse mortgage, instead of leaving housing wealth as a bequest). In table 19, the welfare gains will be presented for different start values of housing wealth.

From Table 19, it can be observed that enormous welfare gains (32.08 % and 39.15 % for housing wealth 2.89 times the normalized ALI at retirement) can be realized by having access to the reverse mortgage market. This enormous effect can be explained by the assumption of a zero utility of leaving a bequest. In addition to this, the reverse mortgage contract will be too beneficial for the participant with respect to a more realistic setting.

If we allow for decreased pension contributions (i.e. mortgage repayments reduce accumulated pension wealth) and we will not deplete housing wealth in retirement, a decrease in consumption standard will be realized (Figure 25a). This yields relatively high welfare losses (see Table 20). In fact, the optimal contribution strategy during the accumulation phase will be extremely suboptimal if it is unexpectedly decided not

[33]In case $ARC = Maint = 0$, the red lines (from Figure 24a and Figure 24b) will be the input to calculate certainty equivalents. In case $ARC = Maint = 0.1$, the red dotted lines (from Figure 24a and Figure 24b) will be the input to calculate certainty equivalents. Therefore welfare gains will be marginally different for $Maint = 0$ versus $Maint = 0.10$.

Table 19: Welfare gains (Acc, Opt. C + **RM** versus Acc, Opt. C + **not use HW**)

Start value housing wealth (normalized w.r.t ALI), at age 30	Welfare gain (ARC=Maint=0)	Welfare gain (ARC=Maint=0.1)
0.58	+6.66 %	+6.92 %
1.16	+11.73 %	+12.74 %
1.74	+17.90 %	+20.33 %
2.31	+24.73 %	+29.17 %
2.89	+32.08 %	+39.15 %

to deplete housing wealth in retirement. In the end, the suboptimal consumption path in Figure 25*b* might be helpful to maintain a consumption standard in retirement when it is unexpectedly decided not to deplete housing wealth. This indicates that the pension contract should be conditioned on the purpose of housing wealth at retirement age.

Table 20: Welfare losses (Acc, **Opt. C** + not use HW versus Acc, **Subopt. C** + not use HW)

Start value housing wealth (normalized w.r.t ALI), at age 30	Welfare gain (ARC=Maint=0)	Welfare gain (ARC=Maint=0.1)
0.58	-2.37 %	-2.51 %
1.16	-5.98 %	-6.71 %
1.74	-10.75 %	-12.7 %
2.31	-16.22 %	-20.06 %
2.89	-22.39 %	-28.79 %

Still, in retirement scenario (not use HW), it will be interesting to analyze to what extent reduced contributions will be desirable. This will remain a question for future research[34]. For example, the utility function should reflect the utility of leaving a bequest.

In the remaining part, rental costs will be set to 0.10 and maintenance costs will be set to 0.05 and 0.025 respectively. Welfare effects for the four hypothetical persons will be calculated with respect to hypothetical person (Rent). The results will be presented in Table 21 and Table 22.

It can be concluded that hypothetical person (Acc, Opt. C + RM) will achieve the highest welfare. For maintenance costs, that are significantly lower than the rental costs, hypothetical person (Acc, Subopt. C +RM) will have a higher certainty equivalent compared to hypothetical person (Rent) for housing wealth in the order 0.58-2.31 normalized with respect to ALI. In the setting of this thesis, hypothetical person (Rent) will always be better off than a hypothetical person leaving the house as a bequest. This is mainly explained by the fact that leaving the house as a bequest yields zero utility. In addition to this, the optimal contribution scheme in case housing wealth is not used to generate a pension income stream was not derived.

[34]In Figure 25*b* a suboptimal contribution scheme was used. By setting the maintenance costs in line with equivalence statement (39) still yields a higher certainty equivalent for hypothetical person (Rent) compared to hypothetical person (Acc, Subopt. C + not use HW). Both persons will have an identical cost structure during the life cycle. The higher certainty equivalent for hypothetical person (Rent) can be explained by a suboptimal contribution scheme for hypothetical person (Acc, Subopt. C + not use HW). In addition to this a higher certainty equivalent is obtained for hypothetical person (Rent) compared to hypothetical person (Acc, Opt. C + not use HW). This proofs that in the situation where housing wealth will not be used as a pension income stream, reduced pension contributions compared to hypothetical person (Rent) will be optimal. Still, higher contributions need to be made compared to a person depleting housing wealth via a reverse mortgage. The optimal contribution scheme was not quantified in this thesis.

Table 21: Welfare effects (Rent vs. Others) with Rent=0.10 and Maint=0.05

Start value housing wealth (normalized w.r.t ALI), at age 30	Welfare effect (Acc, Opt. C + RM)	Welfare effect (Acc, Subopt. C + RM)	Welfare effect (Acc, Opt. C + not use HW)	Welfare effect (Acc, Subopt. C + not use HW)
0.58	+1.35 %	+0.59 %	-5.52 %	-3.15 %
1.16	+2.72 %	-0.02 %	-11.07 %	-3.65 %
1.74	+4.12 %	-1.33 %	-15.58 %	-4.43 %
2.31	+5.57 %	-3.02 %	-22.07 %	-5.47 %
2.89	+7.16 %	-4.93 %	-30.36 %	-6.77 %

Table 22: Welfare effects (Rent vs. Others) with Rent=0.10 and Maint=0.025

Start value housing wealth (normalized w.r.t ALI), at age 30	Welfare effect (Acc, Opt. C + RM)	Welfare effect (Acc, Subopt. C + RM)	Welfare effect (Acc, Opt. C + not use HW)	Welfare effect (Acc, Subopt. C + not use HW)
0.58	+2 %	+1.25 %	-4.81 %	-2.47 %
1.16	+4.18 %	+1.37 %	-8.28 %	-2.26 %
1.74	+6.06 %	+0.86 %	-13.21 %	-2.29 %
2.31	+8.12 %	+0.04 %	-19.17 %	-2.52 %
2.89	+10.22 %	-1.09 %	-26.10 %	-3.05 %

In terms of the optimal pension contract, in this simplified setting, one can only conclude about the optimal contribution rate in the pension contract and the decision regarding housing wealth at retirement age. We cannot conclude about the asset allocation since equity exposure was excluded. In the optimal pension contract (see coalition agreement, Vertrouwen in de toekomst, 2017), it could be incorporated that if a mortgage will be repaid lower pension contributions will be possible. In this simplified setting, it was shown that it will be important to condition these lower contributions in the pension contract on the purpose of housing wealth in retirement. An elaborate discussion (and an international perspective) of the implications for the desired pension contract will be discussed at the end of Subsection 4.2.3, after the case where the dynamics of housing wealth follow a geometric Brownian motion. In this setting, we can also conclude about the optimal asset allocation in the pension contract.

4.2.3 Housing wealth in optimal pension contract with housing wealth as geometric Brownian motion

In this subsection the implications for the (approximated) optimal pension contract will be analyzed in the case in which housing wealth will be a geometric Brownian motion with correlation to the risky asset.

One can differentiate between three strategies, during the accumulation phase, in the setting where housing wealth is assumed to have the dynamics of a geometric Brownian motion. Since equity exposure will be included, there will be an optimal asset allocation strategy given by formula (36) and a suboptimal asset allocation strategy presented in equation (12). Recall that there was only one asset allocation strategy available in the case where housing wealth was assumed to be risk free, since equity exposure was excluded.

- Scenario (Acc, Opt. C, Opt. A): In this contribution strategy, optimization over pension contributions (i.e. mortgage repayments will reduce the pension contributions) and optimization over the asset allocation will be considered.
- Scenario (Acc, Subopt. C, Opt. A): In this case, optimization over the asset allocation will take place, whereas a suboptimal contribution scheme (mortgage related payments will reduce the consumption level) is used. This contribution strategy implies that we do not take into account that we implicitly save for retirement via the housing wealth.
- Scenario (Acc, Opt. C, Subopt. A): In this strategy, the pension contributions will be adjusted optimally (i.e. mortgage repayments will reduce the pension contributions), whereas the asset allocation used will not take into account the exposure to the housing wealth asset.

At the retirement age, financial wealth will be available to generate a pension income stream. In addition to this, the accumulated housing wealth might be used to generate a pension income stream. Several possibilities how to deal with the accumulated pension wealth (housing wealth and financial wealth) in retirement will be discussed.

- Retirement scenario (RM): In line with Olear et al. (2017) [31], the financial wealth will be converted to a fixed annuity and housing wealth will be depleted by a reverse mortgage. For simplicity, it is assumed that the housing wealth will generate a constant consumption stream.
- Retirement scenario (VA): The situation in which we will sell the house at the retirement age will be considered as well. This implies that we could act in line with the standard optimal Merton setting (variable annuity). A subtle change with respect to the other possibilities (RM and not use HW) will be the fact that we should rent a new house. Therefore this strategy requires a higher consumption path (including rental costs).
- Retirement scenario (not use HW): The situation in which we will not use the housing wealth to generate a pension income stream will be considered as well. This implies that the housing wealth will be left as a bequest. Note that at the age of 30 (after buying the house) an optimization problem arises, that implicitly assumes that we will use the housing wealth to generate a pension income stream. If we will not do this, the participant consumed too much during the accumulation phase and the standard of living should be decreased during retirement. Still, decreased pension contributions will be justified in this strategy since this person will have a different consumption pattern (i.e. no rental costs).

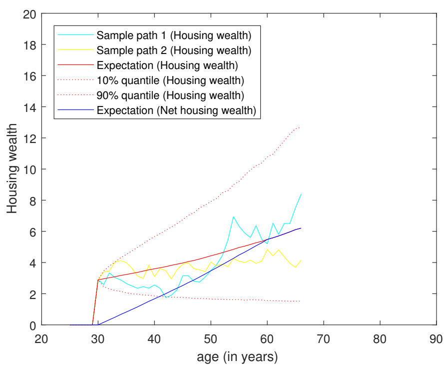
The remaining structure will be to present the accumulated pension wealth and the asset allocation strategy for the three scenarios (during the accumulation phase). Observe that the asset allocation strategy in retirement is implicitly defined by the chosen consumption method. After this, the consumption and asset allocation strategies during the accumulation phase will be combined with the chosen consumption method in retirement[35]. These consumption patterns will be the main input to calculate the certainty equivalents

[35] In the figures a start value of housing wealth of 3 is assumed. In the tables welfare effects will be calculated for smaller starting values of housing wealth as well. The consumption pattern of house owners will include the maintenance costs. If also the consumption pattern excluding the maintenance costs was present in the figures, an accurate interpretation of the lines was difficult since quantiles are plotted here as well.

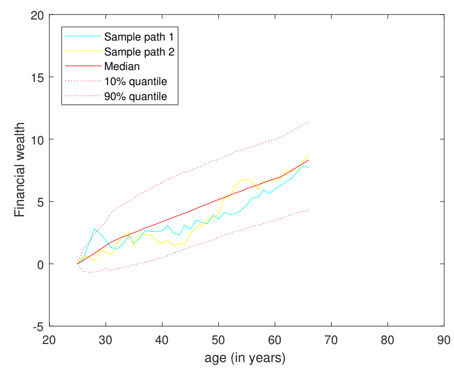
and hence welfare effects. Recall that all numbers will be normalized with respect to ALI which is set to 1.

The line of reasoning regarding the indifference of houseowners and renters will no longer hold in case housing wealth will have correlation with the risky asset. A detailed analyses is therefore not included. Still, welfare effects will be calculated at the end of this subsection for some ad hoc chosen values of the rental costs and maintenance costs.

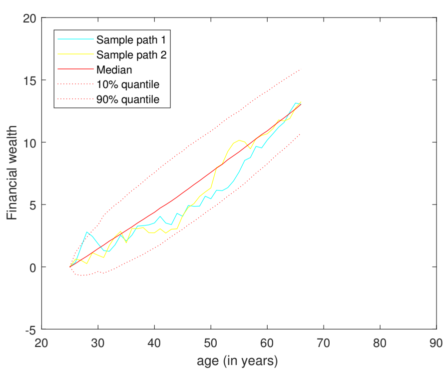
In Figure 26*a*, the dynamics of the housing wealth (i.e. start value housing wealth is 3) is presented. Observe that we are exposed to the housing wealth asset from the age of 30. Before the retirement age (at the age of 60), the mortgage is completely repaid. In Figure 26*b*, 26*c* and 26*d* the evolutions of financial wealth will be presented. In (Acc, Opt. C, Opt. A) and (Acc, Opt. C, Subopt. A), it is observed that the accumulation of financial wealth is 'relatively' low in the period where the participant is between 30 and 60 years old. This is explained by reduced pension contributions because the mortgage should be repaid. As a direct consequence, a lower level of financial wealth will be accumulated. In (Acc, Subopt. C, Opt. A), financial wealth is accumulated in line with the idea of the standard setting. This is because the mortgage related payments reduce the consumption. These two components (housing wealth and financial wealth) will sum up to the available pension wealth at the retirement age.



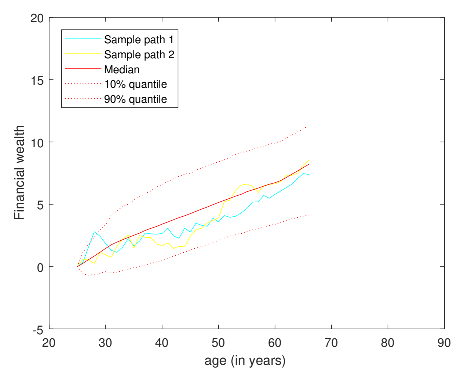
(a) Housing wealth



(b) Financial wealth (Acc, Opt. C, Opt. A)



(c) Financial wealth (Acc, Subopt. C, Opt. A)



(d) Financial wealth (Acc, Opt. C, Subopt. A)

Figure 26: Accumulated stochastic pension wealth (housing wealth geometric Brownian motion)

In Figure 27, the (median) asset allocation strategies will be presented. The optimal asset allocation path (Acc, Opt. C, Opt. A) has a higher exposure to the risky asset compared to the standard optimal asset allocation path (Acc, Opt. C, Subopt. A) except for the first years. This is explained since after a few years the first term is dominating the second term of equation (32). Note that the optimal asset allocation path (Acc, Subopt. C, Opt. A), will be slightly different from the optimal asset allocation path (Acc, Opt. C, Opt. A) since a different accumulation of financial wealth will be used (i.e. not presented in the figure). Differences between (Acc, Opt. C, Opt. A) and (Acc, Opt. C, Subopt. A) will become even more insignificant for smaller values of housing wealth.

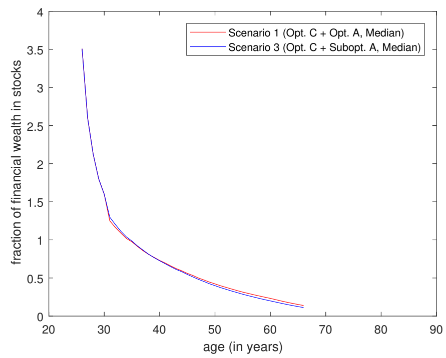
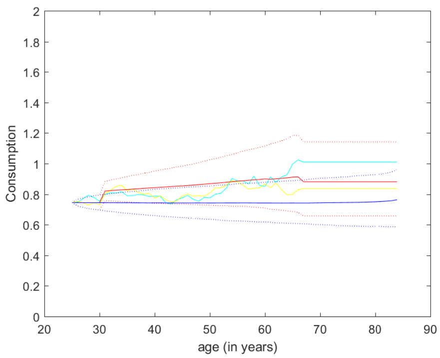


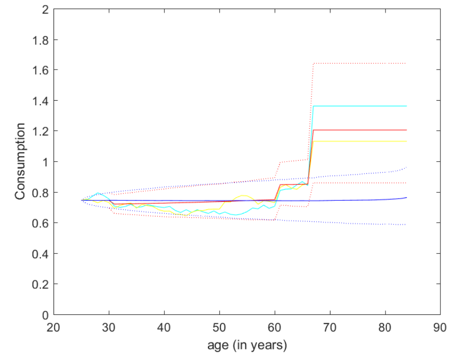
Figure 27: Asset allocation paths (housing wealth geometric Brownian motion)

In Figure 28*a*, 28*b* and 28*c* the consumption paths are presented belonging to consumption method in retirement (RM). The expected consumption for a participant who is renting a house over the complete life cycle will be presented as well for rental costs 0.10. It is observed that for all the consumption graphs, the consumption level will be identical for strategy (Acc, Opt. C, Opt. A), (Acc, Subopt. C, Opt. A) and (Acc, Opt. C, Subopt. A) for the ages 25-30. This consumption level will be equal to the consumption level implied by the life cycle model minus a reduction for the assumed rental costs. From the age of 30 onwards (until 60), the consumption standard will be changed based on three effects. Consumption will not be reduced by rental costs. Interest payments will generate a lower decreased consumption standard. These two are applicable for all the scenarios. The difference will be explained by the mortgage repayments. In strategy (Acc, Opt. C, Opt. A) and (Acc, Opt. C, Subopt. A), mortgage repayments reduce financial wealth, whereas in strategy (Acc, Subopt. C, Opt. A) mortgage repayments will completely reduce consumption. Combining all the effects, this yields an increasing consumption pattern[36] over time (in the mortgage repayment period). The decreased consumption standard (in the repayment period) in scenario (Acc, Subopt. C, Opt. A) will be more compared to scenario (Acc, Opt. C, Opt. A) and (Acc, Opt. C, Subopt. A) (i.e. motivated by oversaving). Implied by the chosen consumption strategy presented by equation (33), a relatively high increase in consumption standard will be realized after the mortgage is repaid until retirement (age 60 until age 67) for strategy (Acc, Subopt. C, Opt. A). This is because the mortgage is completely repaid (i.e. no mortgage repayments). In retirement, a consumption standard will be available that will be deterministic and flat by assumption (reverse mortgage). The highest consumption standard can be obtained for strategy (Acc, Subopt. C, Opt. A) (i.e. oversaving).

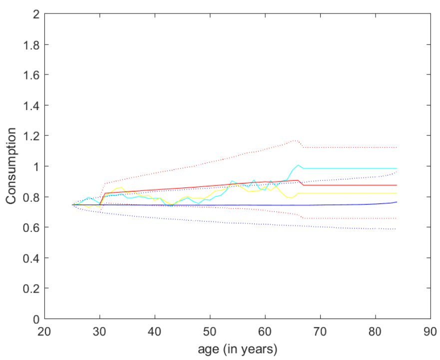
[36]Recall that the consumption pattern is not corrected for maintenance costs.



(a) Consumption (Acc, Opt. C, Opt. A + RM)



(b) Consumption (Acc, Subopt. C, Opt. A + RM)



(c) Consumption (Acc, Opt. C, Subopt. A + RM)

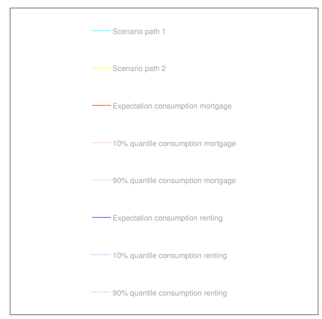


Figure 28: Consumption path (reverse mortgage)

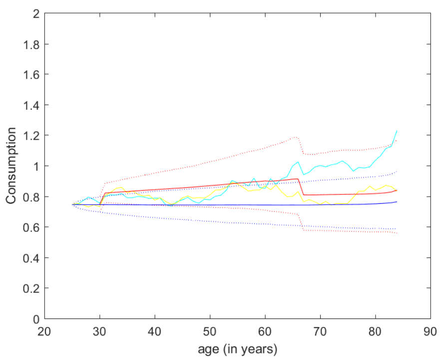
From Table 23 it can be concluded that welfare gains from adjusting the optimal contribution rate in the pension contract, in case housing wealth will be accumulated and used to generate a pension income stream (i.e. reverse mortgage), will be in the order of 0.79%-8.37%[37]. The size of the welfare gains increase with the ratio of house value to ALI. Observe that the welfare gains will be slightly lower as compared to the setting where housing wealth was assumed to be risk free (see Table 18). Somewhat unexpected, the (at first sight) optimal asset allocation strategy is found to be suboptimal, since we are -0.02% worse of by using the 'optimal' asset allocation strategy. Observe that a similar conclusion was made in Olear et al. (2017) [31]. This can probably explained since the optimal asset allocation strategy in formula (36) was an approximation.

[37] In Table 18, the welfare gains are calculated under the assumption $ARC_t = Maint_t = 0$. If $ARC_t = Maint_t = 0.10$ welfare gains of a suboptimal contribution scheme will be changed (marginally) to +0.82 %, +2.65 %, +4.71 %, +7.07 % and + 9.26 %.

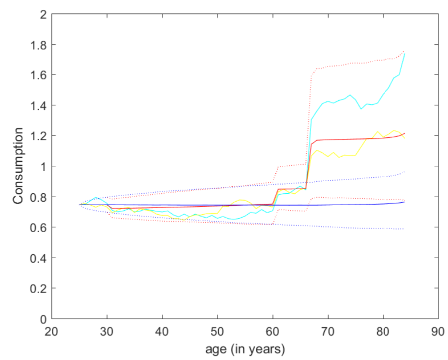
Table 23: Welfare gains (Optimal contribution and Optimal asset allocation with reverse mortgage)

Start value housing wealth normalized w.r.t ALI, at age 30	Acc, **Opt. C**, Opt. A + RM versus Acc, **Subopt. C**, Opt. A + RM	Acc, Opt. C, **Opt. A** + RM versus Acc, Opt. C, **Subopt. A** + RM
0.58	+0.79 %	-0.02 %
1.16	+2.52 %	-0.02 %
1.74	+4.47 %	-0.02 %
2.31	+6.43 %	-0.02 %
2.89	+8.37 %	-0.02 %

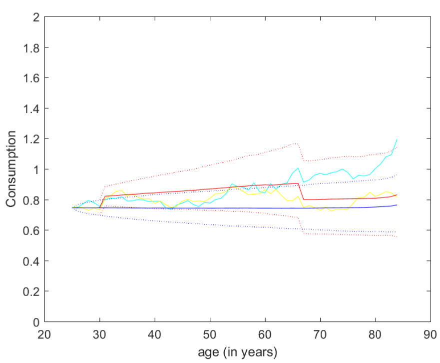
In Figure 29a, 29b and 29c the consumption paths are presented belonging to consumption method in retirement (VA). Again, the expected consumption for a participant who is renting a house over the complete life cycle will be presented. During the accumulation phase, the consumption pattern will be identical to the consumption pattern presented in Figure 28a, 28b and 28c. In retirement, the obtained consumption standard crucially depends on the rental costs. Observe that a variable annuity implies to retain the optimal investment risk in retirement.



(a) Consumption (Acc, Opt. C, Opt. A + VA)



(b) Consumption (Acc, Subopt. C, Opt. A + VA)



(c) Consumption (Acc, Opt. C, Subopt. A + VA)

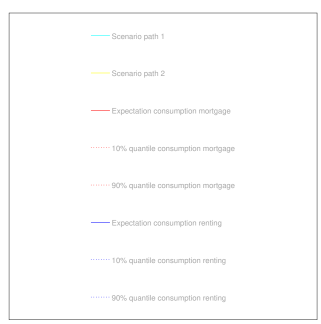


Figure 29: Consumption path (variable annuity)

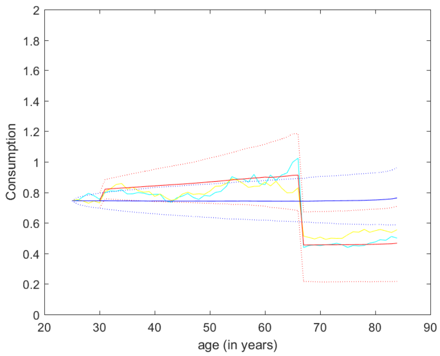
From Table 24 it is noted that welfare gains from adjusting the optimal contribution rate in the pension contract, in case housing wealth will be accumulated and used to generate a pension income stream (i.e. variable annuity), will be in the order 0.89%-8.79%[38]. The size of the welfare gains increase with the ratio of housing wealth to ALI. The welfare gains will be slightly above the welfare gains in case the reverse mortgage was chosen. This can be explained by the fact that in case of the variable annuity, we act in line with the optimal Merton setting. Observe that if the rental costs exceed the maintenance costs (in retirement) marginally, the reverse mortgage will outperform the variable annuity in terms of optimal decision how to deplete housing wealth at retirement. Again unambiguous results were found with respect to the asset allocation strategy in the pension contract.

Table 24: Welfare gains (Optimal contribution and Optimal asset allocation with variable annuity)

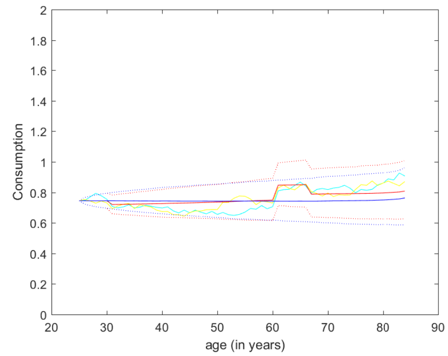
Start value housing wealth normalized w.r.t ALI, at age 30	Acc, **Opt. C**, Opt. A + VA versus Acc, **Subopt. C**, Opt. A + VA	Acc, Opt. C, **Opt. A** + VA versus Acc, Opt. C, **Subopt. A** + VA
0.58	+0.89 %	-0.00 %
1.16	+2.78 %	-0.00 %
1.74	+4.81 %	-0.00 %
2.31	+6.82 %	-0.01 %
2.89	+8.79 %	-0.02 %

[38]In Table 24, the welfare gains are calculated under the assumption $ARC_t = Maint_t = 0$. If $ARC_t = Maint_t = 0.10$ welfare gains of a suboptimal contribution scheme will be changed (marginally) to +0.94 %, +2.85 %, +5.12 %, +7.44 % and + 9.75 %.

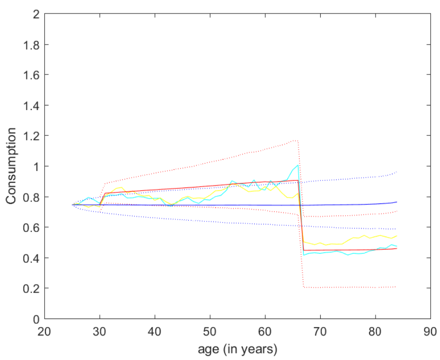
In Figure 30a, 30b and 30c the consumption paths are presented belonging to consumption method in retirement (not use HW). Again, during the accumulation phase the consumption pattern will be identical to the consumption pattern presented in Figure 28a, 28b and 28c respectively. By 'coincidence', the consumption standard in 30b can be (to some extent) maintained at 'acceptable levels'.



(a) Consumption (Acc, Opt. C, Opt. A + not use HW)



(b) Consumption (Acc, Subopt. C, Opt. A + not use HW)



(c) Consumption (Acc, Opt. C, Subopt. A + not use HW)

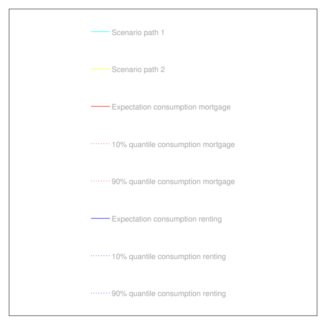


Figure 30: Consumption path (not use HW)

In strategy (Acc, Opt. C, Opt. A) and (Acc, Opt. C, Subopt. A), an enormous reduction in consumption standard will be present. This implies enormous welfare losses compared to having access to the reverse mortgage market (Figure 28a) or annuity market (Figure 29a). In addition to this, the suboptimal contract (Figure 30b) will be ex post beneficial if housing wealth will be left as a bequest. The welfare losses will not be formally calculated, since certainty equivalents will be extremely low (i.e. negative consumption in several sample paths[39]). This setting will overestimate the results, since the state pension will be neglected, guaranteeing a lower bound for the consumption. In addition to this, leaving the house as a bequest yields zero utility in this model.

Although a detailed analysis was not performed, some ad hoc chosen values for the rental costs and maintenance costs can gain some insight into the robustness of the results against the different values of the rental costs and maintenance costs. The analysis will be restricted to rental costs of 0.10 and maintenance costs of

[39]In sample paths were housing wealth will be generous at retirement, yielded a high consumption standard during the accumulation phase since we consume out of total wealth. Overlooking the housing wealth in retirement, when more or less the complete pension wealth will be present in the house, yields an extremely low consumption standard. This is because financial wealth will be extremely low. An example, for housing wealth (2.89) sample path 123 yields a terminal housing wealth of 15, yielding a negative consumption in retirement

0.05. The following (limited number of) welfare effects will be calculated:

- In column 1 of Table 25 it will be shown to what extent a reverse mortgage will be preferred over the variable annuity.
- In column 2 of Table 25 it will be shown whether hypothetical person (Acc, Opt. C + RM) will be better off compared to a person renting a house.

Table 25: Welfare effects (Acc, Opt. C + RM) versus (Acc, Opt. C + VA) and (Acc, Opt. C + RM) versus Rent

Start value housing wealth normalized w.r.t ALI, at age 30	(Acc, Opt. C + RM) versus (Acc, Opt. C + VA)	(Acc, Opt. C + RM) versus Rent
0.58	+0.08 %	+0.95 %
1.16	+0.59 %	+2.18 %
1.74	+1.29 %	+3.19 %
2.31	+2.23 %	+3.81 %
2.89	+3.65 %	+4.17 %

In the setting of this thesis, it is expected that for higher rental costs compared to maintenance costs, the reverse mortgage is preferred over the variable annuity. Indeed, this can be concluded from the results in Table 25. Not surprisingly, hypothetical person (Acc, Opt. C + RM) will be better of compared to hypothetical person (Rent). The effect is somewhat smaller compared to the case where housing wealth is assumed to be risk free (i.e. first column of Table 21).

4.2.4 Implications housing wealth in optimal pension contract

In the current Dutch setting, many individuals will not deplete their housing wealth in retirement and will act in line with retirement scenario (not use HW). It was concluded that relatively high welfare losses will arise if we will not use housing wealth to generate a pension income stream and allowed to reduce financial wealth (i.e. pension contributions) with the complete mortgage repayments[40]. These relatively high welfare losses were calculated in a setting where leaving a bequest yields zero utility. Still, reduced pension contributions might be desirable compared to a participant renting a house, since rental costs are avoided in retirement. Therefore, it will be justified that the pension system should allow for flexibility in the pension contract, in terms of decreased pension contributions if housing wealth is accumulated. If and only if the accumulated housing wealth will be used to generate a pension income stream in retirement, even less contributions might be desirable in the pension contract. Welfare gains were quantified in the setting of this thesis in the order of 0.74 % - 12.96 % depending on the ratio of house value to ALI and dynamics of the housing wealth asset. Van Ewijk et al. (2017) [24] found welfare gains in the order of 0.5 % - 6.8 % yielding similar policy implications, although the exact quantitative results are different. This will guarantee some robustness in the obtained welfare gains. Recall that in their DC setting, the participant cannot adjust the pension contributions to realized asset returns. In principle, the pension fund can condition the contributions in the pension contract on the purpose (either to use as a consumption stream or to use as a reduced consumption standard) of the housing wealth during retirement. This to guarantee the optimal contribution rate in the pension contract for the participant.

It is not completely clear until now, why the optimal asset allocation strategy that will take into account the accumulation of housing wealth will be suboptimal ex post. It is noted that Olear et al. (2017) [31] found similar results, when a reverse mortgage was chosen. This probably has to do with the fact that the optimal asset allocation in formula (36) will be an approximation.

Important lessons regarding the integration of housing wealth and 'traditional' pension wealth might be learned from abroad as described in Adriaansen, Bovenberg and Kortleve (2014) [1]. An overview of this integrated pension contract will be discussed for several countries in terms of optimal contribution scheme (Switzerland en United States) and asset allocation (in a different way as discussed in this thesis) (Switzerland en Canada).

- In **Switzerland**, to purchase residential property, it is allowed to deplete accumulated pension wealth (it is taxed, however the amount will not be maximized) for mortgage repayments. If the house is sold, the participant is obliged to return these amounts to the pension account. If the participant will not deplete housing wealth, the participant will have a lower consumption standard.
- In the **United States**, it is allowed to use 10.000 $ of the accumulated pension wealth (it will be taxed, no additional fine) for the attainment of housing wealth. There is no restriction to return the money to the pension account when the house will be sold.
- In **Switzerland**, the accumulated pension wealth can be used as a collateral for the attainment of housing wealth. This will lead to lower interest payments since the bank will impose a lower interest rate, hence a higher consumption standard.
- In **Canada**, the Home Buyer's Plan (HBP) allows to borrow money from the third pillar pension account maximized at 25.000 $. As a consequence, a lower nominal amount should be borrowed from the bank which reduces the interest payments, hence increases the consumption standard.

In Bart, Boon, Bovenberg, van Ewijk, Kortleve, Rebers, and Visser (2016) [5], it was shown that in Singapore and Malaysia where an integrated approach between pension, care and housing wealth is in place, two times a nominal amount may be used for the repayment of the housing wealth retrieved from the ordinary account.

[40] For the case where housing wealth is risk free this refers to hypothetical person (Acc, Opt. C). For the case where housing wealth has the dynamics of a geometric Brownian motion this refers to hypothetical persons (Acc, Opt. C, Opt. A) and (Acc, Opt. C, Subopt. A).

Also, Bart et al. (2016), [5], show that countries like the United states, Australia, Norway and France have an established market for reverse mortgages (consumption in retirement method) to use the accumulated housing wealth for consumption. In particular, in the United States, the government will guarantee the risk of having a negative housing wealth balance via 'no negative equity'. This even provides the participant the possibility to insure the longevity risk. In the Netherlands, a few commercial parties will offer a reverse mortgage contract, with less favorable contract agreements compared to abroad.

In these countries, that have incorporated the attainment of housing wealth in the pension design, the institutional pension setting will be characterized by DC schemes. In the expected new pension contract in the Netherlands (shift from DB to DC), incorporating the accumulation of housing wealth in the pension contract will be perfectly possible without harming the solidarity mechanisms that will be present in the current Dutch DB pension schemes.

4.3 Life event Temporary unemployment

In this case, the implications of temporary unemployment will be investigated. It is assumed that until the age of 45 no life event took place. The parameter values defined in Table 1 (for example labor income) will be used here as well. Unfortunately the hypothetical person will become unemployed at the beginning of age 45. In this setting, this implies that this person will be at least (unanticipated) unemployed for one year. Therefore, the human capital should be adjusted by depletion of the labor income for this year. After one year, the situation of this person should be evaluated in terms of having a job or not. If this person has found a job again, the life cycle optimization will continue as usual (small difference could be that less financial wealth is accumulated). If this person has not found a job, it implies that human capital should be adjusted by depletion of the labor income at the age of 46. This evaluation procedure should be repeated iteratively until this person has found a job.

This person does not have a labor income in the unemployment period. If this person can consume in this setting, this person will consume out of his pension wealth. It is even impossible for this person to contribute any amount to his pension account. Therefore, it is decided to introduce an unemployment benefit to make this case more realistic. It is assumed that this will be significantly lower as the salary (70 % of the labor income). This will increase human capital again. To conclude, in a year of unemployment, human capital should be depleted by the labor income minus the unemployment benefit. Introducing this unemployment benefit makes the situation interesting to analyze. In terms of contribution and asset allocation, several strategies could be constructed.

For the contribution strategy, the following strategies will be constructed.

- Contribution (Opt. Mert.): This will be in line with the optimal Merton setting. Under the chosen parameters, this will imply that the complete unemployment benefit will be used for consumption. In addition to this, part of the pension wealth will be used to maintain the slightly decreased consumption standard. This is because of a lower value of human capital.
- Contribution (Unempl. Ben.): In this situation, the consumption pattern will be restricted by the fact that we are not allowed to use the accumulated pension wealth for consumption during the unemployment period. This implies that we will approximate the optimal setting and will consume the complete unemployment benefit.
- Contribution (Unempl. Ben. min PC): In this situation, part of the unemployment benefit will be used for the accumulation of pension wealth (pension contributions). Therefore, a decreased consumption standard (unemployment benefit minus pension contributions) during the unemployment period will be present.

For the asset allocation strategies, the following strategies will be constructed.

- Asset allocation (Opt. A): The asset allocation strategy that will take into account the depletion of part of the human capital. A lower equity exposure will be optimal in terms of financial wealth because of the unanticipated depletion of part of the human capital, which is a risk free asset.
- Asset allocation (Old A): The asset allocation strategy that neglects the depletion of labor income minus unemployment benefit in the unemployment period. This coincides with the standard optimal Merton asset allocation strategy.

To summarize, contribution strategy (Opt. Mert.), (Unempl. Ben.) and (Unempl. Ben. min PC) in combination with asset allocation strategy (Opt. A) will be denoted by (Opt. Mert., Opt. A), (Unempl. Ben., Opt. A) and (Unempl. Ben. min PC, Opt. A) respectively. Contribution strategy (Opt. Mert.), (Unempl. Ben.) and (Unempl. Ben. min PC) in combination with asset allocation strategy (Old A) will be denoted by (Opt. Mert., Old A), (Unempl. Ben., Old A) and (Unempl. Ben. min PC, Old A) respectively.

The remaining structure will be to present some intuition about the methodology that will be used. The intuition of a simplified case, where it is ex ante known that the participant is unemployed for five years, will be provided. To continue, the optimal pension contract is derived for an agent, where at each year it will be evaluated whether the agent has found a job or not. It will be assumed that after the third evaluation moment this person has found a job. The corresponding median asset allocation strategies will be presented. After this, the consumption paths for all the scenarios will be presented. These will be the main input to calculate the certainty equivalents and hence quantify welfare effects.

In Figure 31, the evolution of human capital from the age of 45 is depicted for several depletion periods (of the labor income minus the unemployment benefit).

- Methodology unemployment 5 years, ex ante known: In the case where we will assume that the person will be unemployed for five years, the human capital for five depletion periods will be taken into account. This would imply that 'human capital (5 years depletion)' from Figure 31 should be taken into account in the optimal contribution strategy[41] and optimal asset allocation strategy from the age of 45 onwards. Observe that by construction the 'human capital (initial)' coincides with the 'human capital (5 years depletion)' after the 5 unemployment years.
- Methodology unemployment 3 years, ex post known: In the case where at each year it will be evaluated whether the person has found a job will be more challenging. The first year the person will be unemployed, the human capital depleted by the 1 year labor income minus unemployment benefit at age 45 should be taken into account in the optimal pension contract. In the second year of unemployment, the human capital depleted by 2 years of labor income minus unemployment benefit at the age of 45 and 46 should be taken into account. This coincides with the 'human capital (1 year depleted)' and 'human capital (2 years depleted)' from Figure 31 respectively. To conclude, the human capital with x depletion periods will be taken into account for the xth unemployment year. This method will be iteratively repeated until this person find a job again.

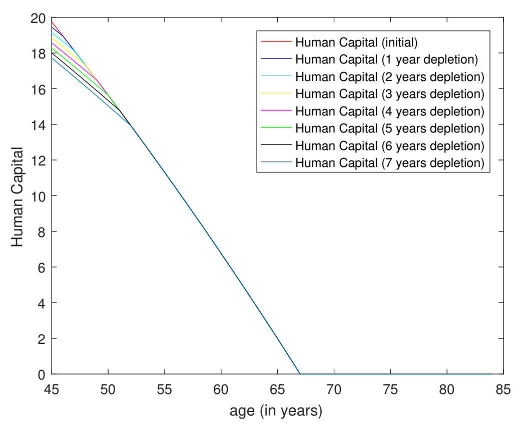


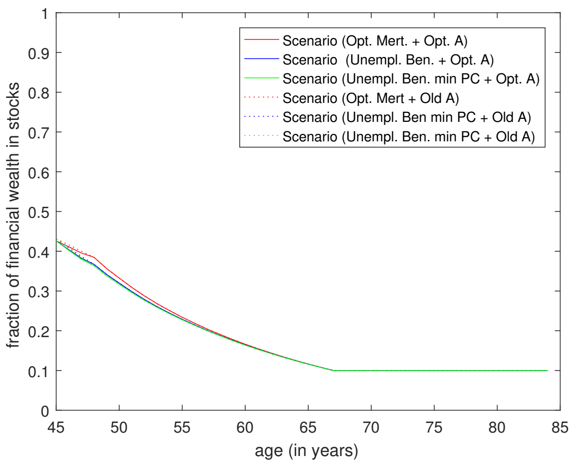
Figure 31: Depletion of human capital (several periods considered)

General remark: At this point in time it cannot be expected from the reader that in terms of deriving the optimal pension contract and welfare effects, it is not relevant to incorporate the more challenging method (i.e. methodology unemployment x years, ex post known). It is sufficient to approximate the unemployment period and directly take into account the human capital depleted by the approximated unemployment period. Still, the more complicated case, where it will be evaluated each year whether the person has found a job, will be analyzed in the remainder of this subsection.

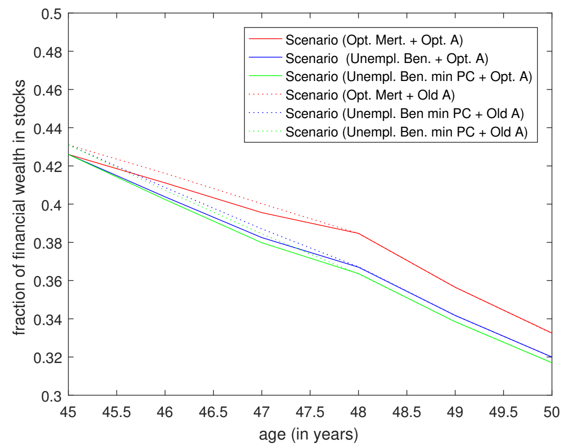
It will be assumed that the participant will become unemployed and each year we will evaluate whether this person has found a job. This evaluation procedure will be repeated iteratively and the situation where the person has found a job after three years of unemployment will be considered.

[41] Observe that this is not applicable for contribution strategy (Unempl. Ben.) since contributions are deterministic.

To start, the optimal and suboptimal (median) asset allocation paths will be presented in Figure 32a. Figure 32b will zoom in to observe the differences in the optimal fraction of financial wealth in the unemployment years. This figure also shows that the optimal asset allocation will not be identical for all the contribution strategies (this is observed for the suboptimal asset allocation strategy as well). This can be motivated by a different accumulation of financial wealth. Observe that the latest effect yield larger deviations in the asset allocation strategy as whether or not one corrects for the depletion of part of the human capital. Therefore, it can be concluded that the reduced equity exposure to correct for the depletion of part of the human capital is insignificantly small.



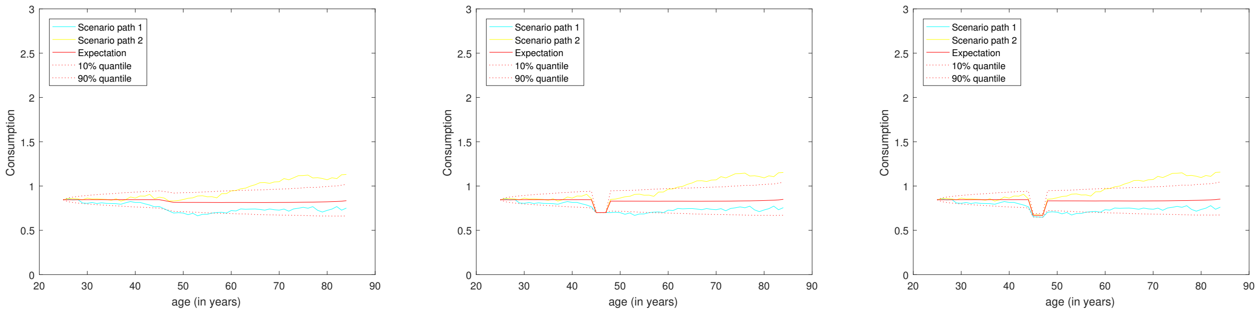
(a) Asset allocation paths (temporary unemployment)



(b) Asset allocation paths (temporary unemployment, zoom in)

Figure 32: Asset allocation paths (temporary unemployment)

In Figure 33 the consumption pattern for scenarios (Opt. Mert., Opt. A), (Unempl. Ben., Opt. A) and (Unempl. Ben. min PC, Opt. A) are presented. In scenario (Opt. Mert., Opt. A), the consumption standard will be more or less maintained in the unemployment period. This will come at a cost that less financial wealth will be accumulated (and even dissaving). As a consequence, this participant has to keep this decreased consumption standard after the unemployment period. In scenario (Unempl. Ben., Opt. A), for the three unemployment years a decreased (deterministic) consumption standard will be available. After this, the initial consumption standard can be remained more or less. This will be slightly lower (difficult to see), since no pension contributions were made in the unemployment years. In scenario (Unempl. Ben. min PC, Opt. A), part of the unemployment benefit is contributed to the pension account. This yields a significant decrease in the consumption standard for the unemployment years. After these years, the consumption standard can be retained. In Figure 34 the contribution strategies combined with the suboptimal asset allocation strategy are presented. It is observed that the chosen asset allocation strategy influences the consumption pattern marginally.

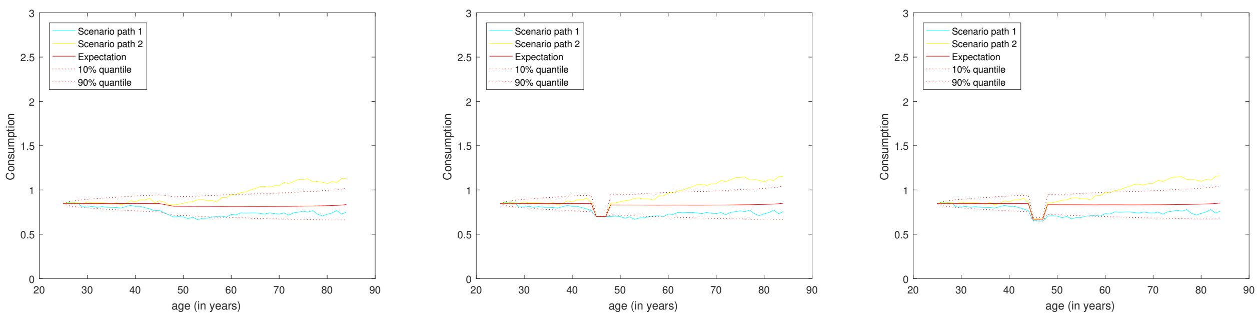


(a) Consumption (scenario, Opt. Mert. + Opt. A)

(b) Consumption (scenario, Unempl. Ben. + Opt. A)

(c) Consumption (scenario, Unempl. Ben. min PC + Opt. A)

Figure 33: Consumption paths (Temporary unemployment)



(a) Consumption (scenario, Opt. Mert. + Old A)

(b) Consumption (scenario, Unempl. Bem. + Old A)

(c) Consumption (scenario, Unempl. Ben. min PC + Old A)

Figure 34: Consumption paths (Temporary unemployment)

It might be reasonable that welfare effects will be calculated based on the current institutional setting. This implies that the starting point will be contribution strategy (Unempl. Ben.) and asset allocation strategy (Old A). As a consequence, we will investigate whether deviations to a different contribution strategy or switch to the asset allocation strategy that takes into account the depletion of human wealth will be welfare enhancing.

Table 26: Welfare effects, temporary unemployment (3 years, method ex post known)

	Asset allocation (Opt. A)	Asset allocation (Old A)
Consumption (Opt. Mert.)	X	+1.1 %
Consumption (Unempl. Ben.)	+0.00 %	0 %
Consumption (Unempl. Ben. min PC)	X	-0.8 %

Overall, it can be concluded that the welfare gains of an adjusted asset allocation strategy will be insignificant small (0.00 %). Therefore, flexibility in the pension contract regarding a different asset allocation will be unnecessary.

In terms of contribution strategy, it will not be attractive to consider to save part of the unemployment benefit for retirement (-0.8 %), whereas consumption out of the pension wealth can have significant welfare gains (+1.1 %) as well.
In the Dutch setting it is possible to accumulate pension wealth in the second pillar for three years after the person lost his job (article 54, Pension Act). Still, the pension contributions should be paid typically by the unemployed person. The former employer will no longer pay part of the pension contributions. Therefore,

the setting of this thesis will underestimate the welfare losses quantified. In addition to this, it is difficult to judge whether it is doable for the participant still making contributions to the pension account (minimal consumption level).
Under the assumptions made in this thesis, the pension contract should allow to consume part of the pension wealth. In van Ewijk et al. (2017) [24], it was concluded that in some scenarios (to repay a loan etc.) it can be attractive to consume part of the pension wealth. Such a situation might be applicable in case of temporary unemployment.[42]

4.3.1 Implications temporary unemployment for optimal pension contract

As already concluded in van Ewijk et al. (2017) [24], it can be attractive to deplete part of the pension wealth to maintain a consumption standard in the unemployment period. Under the assumptions of this thesis it can be concluded that consumption out of pension wealth can be beneficial. Being able to deplete pension wealth before retirement and the implications for society (in terms of harming the solidarity mechanisms) should be taken into account in the discussion.

[42] Observe that in the setting of this thesis, where the participant targets a flat expected consumption over the life cycle, it will be intuitive that contribution strategy (Opt. Mert.) will be optimal. This is because contribution strategy (Opt. Mert.) will adjust (by construction) the expected consumption standard flat over the remaining life cycle.

Intermezzo: Pension engagement of heterogeneous agents accommodating for life events

In Section 3 and 4 the optimal pension contract was derived for heterogeneous agents accommodating for life events. In Brüggen, Blakstad, and Post (2017) [18], it was investigated to what extent pension engagement was observed after a life event. In the setting of Brüggen et al. (2017) [18], a life event was defined as follows: 'Life events are time discrete transitions that mark the beginning or end of a specific status'. The heterogeneity characteristics, in the setting of Brüggen et al. (2017) [18] will be captured by the life event start of working life. To conclude about increased pension engagement after a life event, it should be shown that a life event will be characterized as a teachable moment. Teachable moments are supposed to make the participant more open to adjust the pension contract optimally.

Still, it might be a challenging task for a pension fund to anticipate on a life event (the implications for the optimal contract discussed in Section 3 an 4 in time)[43]. The information regarding a life event arrives typically at a delayed basis and the awareness typically lasts for a short period of time (Williams, Brown, Patton, Crawford, and Touquet (2005) [46]. It was considered how big data and predictive modelling could help in identifying a life event in time. Based on qualitative interviews, it was concluded that using predictive modelling and big data was identified as a threat in terms of privacy concerns. Being able to detect a life event in time, still has additional concerns because of substantial heterogeneity in terms of the economic and non-economic impact (i.e. moving to another house may be motivated by pleasant or unpleasant reasons). In the absence of pension communication, the impact of behavior (i.e. change the pension contract) may be small as a reaction to a life event.

This thesis will not go any further in the details of how to give heterogeneous agents accommodating for life events an incentive in actively changing the pension contract. Stating that this will be a challenging task might be useful.

[43]For example, the attainment of housing wealth will be known after a change of address. This will typically not coincide with the purchase-agreement. A similar line of reasoning can be used for signing a new labor contract.

5 Relative importance of the state pension on the income heterogeneity: quantifying the importance of tailoring the pension contract in the second pillar

In the Merton model, optimal contribution rate and asset allocation do not depend on the income level. This will be illustrated for person high who earns a higher labor income than person middle[44]. In particular, the labor income will be four times higher. This life cycle model then implies that the human capital (4) of person high will be four times higher (at each point in time). Since there is no financial wealth at the start age, the total wealth (8) will be four times higher initially. The consumption for person high will be four times higher than the consumption of person middle, because the participant consumes (33) part of total wealth. As a logical implication of this setting, person high contributes (16) four times as much to his pension account as person middle. There will be no difference in terms of the optimal asset allocation (12). This is because the four times higher human capital cancels against the four times higher accumulated financial wealth.

In reality, as indicated by Alserda et al. (2016) [3], a higher equity exposure (i.e. second and third pillar[45]) is reasonable for person middle compared to person high if the state pension (AOW, assumed to be risk free) is taken into account. Observe that for both persons the optimal equity exposure in the second pillar will increase compared to the setting where the relative importance of the state pension is small (discussed in Subsection 2.1). This is due to the implicit claim on a risk free asset (AOW). For person middle the claim on the state pension will be relatively higher, hence a higher equity exposure in the second pillar will be chosen. Overall, both person high and person middle will target a flat consumption (under the assumed parameter values in Table 1) over the life cycle.

Some observations regarding the optimal pension contract taking into account the relative importance of the state pension, that will be analyzed in more detail, will be made:

- In terms of the asset allocation, the optimal exposure in terms of total financial wealth will be known (i.e. shown for each heterogeneity characteristic (see Section 3) and life event (see Section 4) seperately under the assumption that AOW is small). For each income level, the optimal asset allocation strategy will be different in the second pillar. This is explained by the decreasing relative importance of the state pension for higher income levels. For high income categories the optimal asset allocation will converge to the Merton solution because AOW is relatively small.
- In terms of pension contributions, the normalized labor income should be interpreted as pensionable income ('pensioengevend salaris'). By taking into account the relative importance of the state pension, the pension base ('pensioengrondslag') will be defined as pensionable income minus the AOW franchise. The optimal contribution scheme in the second pillar, should be corrected for mandatory premiums to the state pension.

To conclude, for pension funds it might be relevant that one could differentiate the pension contract based on the income level of the participant. Taking into account the relative importance of the state pension will have implications for the optimal pension contract in the second pillar, in terms of optimal asset allocation and optimal contribution scheme. By extending the model with the relative importance of the state pension, the income level can be interpreted as a heterogeneity characteristic justifying a different pension contract.

To show the implications of including the relative importance of the state pension in terms of the optimal pension contract for different labor income categories, 4 hypothetical persons are defined that differ in their

[44]Later on, the hypothetical persons will be introduced formally in Table 27

[45]For simplicity, when a reference is made to the second pillar in this section, this can be interpreted as second and third pillar. For the average participant, the third pillar accumulated pension wealth will be relatively small since accumulation will typically take place in the second pillar. Still, if the participant will only accumulate in the third pillar, the results will have an equivalent interpretation.

annual labor income (ceteris paribus)[46]. The hypothetical persons[47] are defined in Table 27 and the labor income of hypothetical person middle is normalized (i.e. 33.994 euro will be normalized to 1) for simplicity. It will be shown to which labor income category the participant belongs (low (< 33.994), middle (33.994-100.000), high (> 100.000)). The main purpose of this section is to help the stakeholders (participants, pension funds, the government, social partners and the regulator) to interpret the results obtained in Section 3 and 4 for the optimal pension contract properly[48].

Table 27: Hypothetical persons

Persons	Normalized labor income	Labor income category
Hypothetical person low	0.5	Low
Hypothetical person middle	1	Middle
Hypothetical person middle +	1.5	Middle
Hypothetical person high	4	High

In Subsection 5.1, we will discuss the analytical implications of including the relative importance of the state pension in determining the optimal pension contract (in the second pillar) for different labor income categories. The different labor income categories are represented by the hypothetical persons in Table 27. The optimal pension contract for this newly identified heterogeneity characteristic will be derived as a starting point in Subsection 5.1.1. The implications of taking into account the relative importance of the state pension will be highlighted for a limited number of heterogeneity characteristics and life events in Subsection 5.1.2 and Subsection 5.1.3.

In Subsection 5.1 the focus will be on deriving the optimal pension contract in the second pillar, taking into account the relative importance of the state pension. Welfare effects of a suboptimal contribution scheme and suboptimal asset allocation for heterogeneous agents accommodating for life events could be affected[49] in this more realistic setting. Quantifying these welfare effects was needed in analyzing whether product innovation based on the heterogeneity characteristics[50] and life events is attractive for all stakeholders. Welfare effects are quantified in this more realistic setting as well.

[46]The hypothetical persons introduced in Table 27 will be based on a full-time employment. In case a different hypothetical person will have a part-time labor agreement and for example the wage coincides with the wage of hypothetical person low, the implications for the optimal pension contract in the setting of this section will be different. This is because, both persons will have a different pension base since the AOW franchise will be different for full-time and part-time workers.

[47]Hypothetical person high ++ will be the person where the relative importance of the state pension is small.

[48]The labor income of hypothetical person middle is set to 33.994 euro because AOW premiums are mandatory until this income level. The labor income of hypothetical person high is set to 100.000 since fiscally facilitated accumulation of pension wealth is possible until this income level. This might be a natural choice in case of extensions of this project.

[49]For the average participant (i.e. hypothetical person middle), half of the pension income in retirement will be the AOW level and the other half consists of second pillar pension income. This can be verified later on in Figure 36*b*. A first educated guess will be that welfare effects defined in a setting where the relative importance of the state pension is small (Section 3 and 4) will be overestimated by a factor 2 for the average participant represented by hypothetical person middle.

[50]Observe that the income distribution could be an important heterogeneity characteristic by taking into account the relative importance of the state pension.

5.1 Analytical implications of the relative importance of the state pension for the optimal pension contract

In this subsection, the analytical implications of including the relative importance of the state pension in the model will be discussed for the optimal pension contract in the second pillar. The optimal pension contract could be derived for all heterogeneity characteristics and life events, although a limited number will be analyzed. Given the heterogeneity characteristics and life events, the optimal pension contract will be analyzed for all the hypothetical persons defined in Table 27.

The value of the risk free state pension G_t with pension payments AOW (i.e. 0.41 normalized[51]) can be defined in the following way (see van Ool (2016) [30])[52]:

$$G_t = \begin{cases} e^{-r(T-t)} \cdot \frac{AOW}{r}\left(1-e^{-r(D-T)}\right) & t \leq T \\ \frac{AOW}{r}\left(1-e^{-r(D-t)}\right) & t > T \end{cases} \tag{40}$$

A graphical illustration of the value of the state pension, defined in equation (40) and the value of the mandatory premiums for the state pension will be presented in Figure 35 for the hypothetical persons[53].

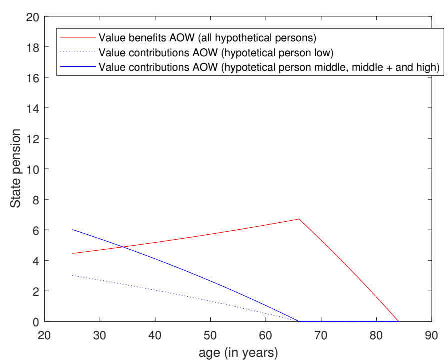


Figure 35: Value of the state pension

It can be concluded that the value of the state pension will increase over time until the retirement age (i.e. less discounting is needed). In retirement the value will decrease because of the payments (i.e. pension income) of the state pension. As a consequence of defining G_t in this way, the formula for human capital should be adjusted for the mandatory AOW premiums, $AOW_{premium}$[54], as follows:

$$\hat{H}_t = \begin{cases} \frac{L-AOW_{premium}}{r}\left(1-e^{-r(T-t)}\right) & t \in [0,T] \\ 0 & t \in [T,D] \end{cases} \tag{41}$$

[51] For an overview of the AOW income, see https://www.belastingdienstpensioensite.nl/Overzicht%20AOW-inbouw%20en%20AOW-franchise.htm

[52] Assuming that a person aged 25 has a complete claim on all AOW benefits might be an overestimation of the risk free asset for this person (i.e. move to another country after 5 years). AOW rights were accumulated over a 50 year period if the person lives in the Netherlands. To be concrete, AOW rights are accumulated in the age period 17-67 (i.e. the first years are not in the model since the life cycle starts at the age of 25. The value of G_{25} in line with formula (40) will be far above the level of what is accrued in practice at this age). To conclude, G_t (in this thesis) increases by a lower discounting. However, in practice new accrued AOW rights will increase the value of the state pension as well. Still for simplicity we will take this definition.

[53] The value of the contributions of the state pension will take into account the intragenerational solidarity of the state pension. This will not be discussed in further details.

[54] In the notation used, a different mandatory premium for different income levels is not taken into account.

This yields the following change in the optimal asset allocation that will include the value of the state pension and the adjusted representation for human capital as derived by Gollier (2008) [26]:

$$\bar{f_t^*} = \frac{\lambda}{\sigma\gamma}(1 + \frac{\hat{H}_t + G_t}{F_t}) \tag{42}$$

From the optimal asset allocation in formula (42), it can be noted that it is no longer optimal to have a constant optimal asset allocation in retirement. This is because the agent still has a claim on a risk free asset that is not depleted at the retirement age (i.e. recall that in the standard setting the risk free asset, human capital, was completely depleted at the retirement age).

Including the state pension in the model, yields the following change in the (approximated) optimal consumption (hence optimal contribution scheme):

$$\bar{C_t^*} = \frac{W_t}{g(t)} = \frac{\hat{H}_t + F_t + G_t}{g(t)} \tag{43}$$

The (approximated) optimal pension contract in the second pillar, corrected for the implications of the relative importance of the state pension, will be derived in terms of optimal contribution scheme and optimal asset allocation. To ensure the practical relevance and avoid an unrealistic pension contract, a few assumptions will be explicitly made:

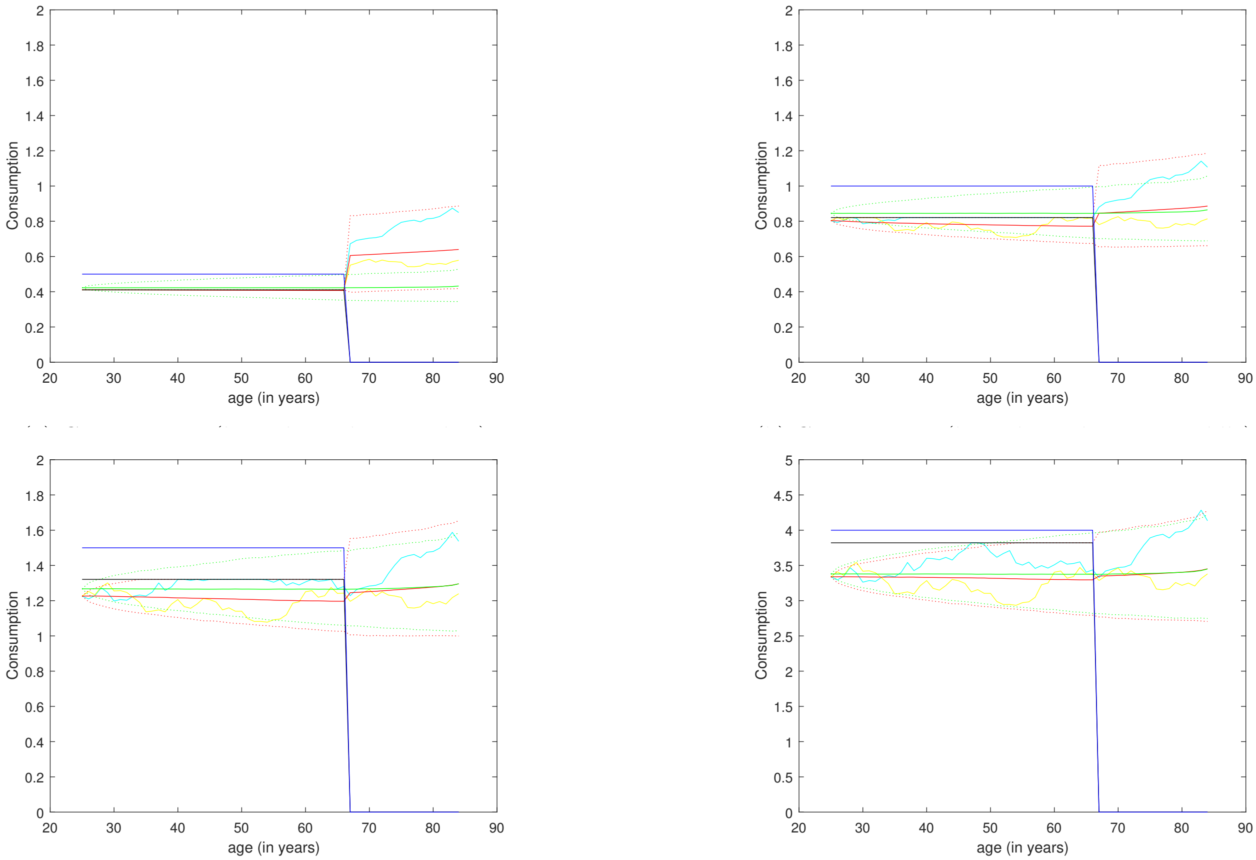
- In the setting of this thesis, the participant can dynamically contribute to the individual pension account. Incorporating the mandatory (deterministic) accumulation of pension wealth via the state pension will be a challenging task. To avoid the unrealistic assumption, that depletion of pension wealth will occur to finance the mandatory state pension premiums, the stochastic consumption (in the accumulation phase) will be maximized (for each sample path at each point in time) at the labor income minus the mandatory premiums for the state pension. This implies that the participant is to some extent restricted in adjusting the contributions to realized asset returns.
- In the setting of this thesis, it was not restricted that the individual can borrow against the human capital. In the calculation of the optimal contribution scheme and the welfare effects, no exogenous borrowing constraint will be imposed. To ensure the practical relevance, the truncated asset allocation strategy is presented.

The remainder of this section will discuss the implications of including the relative importance of the state pension in the model[55] for the standard contract and for heterogeneous agents accommodating for life events.

[55]In case of borrowing constraints, the optimal contribution rates and optimal asset allocation can no longer be determined analytically, so that we must rely on numerical simulations (see e.g. Koijen et al. (2007)) as is stated in Bovenberg et al. (2007). In Vila and Zariphopoulou (1991), it was concluded that optimal consumption policy can still be expressed as feedback functions of current wealth (i.e. as in standard life cycle model). If we want to determine the optimal pension contract in this section, as concluded we should switch to numerical techniques. In the setting of this thesis, it will be restricted to determine the approximate optimal pension contract. In determining the optimal contribution scheme over the life cycle and the welfare effects, there will be no exogenous borrowing constraint added to the model. In determining the optimal asset allocation over the life cycle, the asset allocation will be truncated.

5.1.1 Standard optimal pension contract

In this subsection, the implications for the optimal pension contract (i.e. second pillar) of including the relative importance of the state pension in the standard setting (Subsection 2.1) will be presented. The implications will be shown for all the hypothetical persons introduced in Table 27. Recall that in the Merton setting, where the relative importance of the state pension is small, an identical investment strategy will be optimal for the different labor income categories. The pension contributions in each path will be the multiple of the higher income, as argued in the introduction of this section. In Figure 40 the optimal contribution scheme for the hypothetical persons is presented that we will explain in more detail.



(c) Consumption (hypothetical person middle +)

(d) Consumption (hypothetical person high), Adjusted scale vertical axis

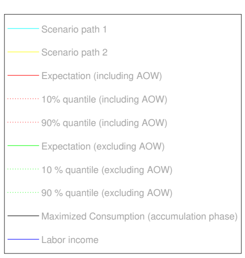
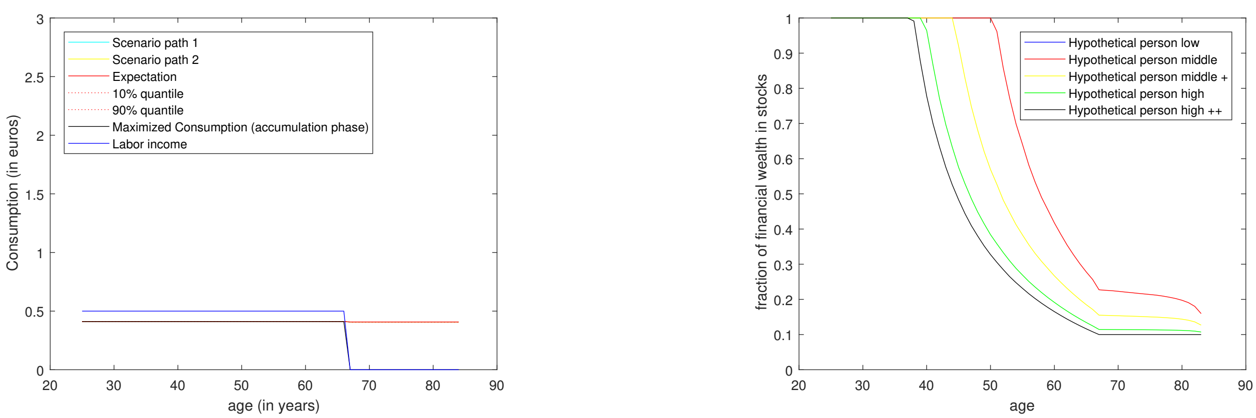


Figure 36: Consumption (optimal standard pension contract)

In each Subfigure (36*a*, 36*b*, 36*c* and 36*d*), two sample paths, the expectation and the 10 % and 90 % quantile for consumption (including the state pension) is presented. In addition to this, the labor income, the maximized consumption (during the accumulation phase, to guarantee the payment of the mandatory premiums) and the expected consumption in the standard life cycle model[56] (including the quantiles) is presented. Observe the adjusted scale (i.e. vertical axis) for the optimal contribution scheme of hypothetical person high in Figure 36*d*.

The difference between the blue line (labor income) and the black line (maximized consumption) can be interpreted as the mandatory contributions for the state pension. The difference (in the accumulation phase) between the black line (maximized consumption) and the red line (expected consumption, including the state pension) can be interpreted as expected pension contributions in the second pillar. The comparison between the green line(s) and the red line(s) (representing the expectation and the quantiles) will show the implications for the optimal contribution scheme of including the relative importance of the state pension in the standard life cycle model. The sample paths clearly indicate that the person is able and to some extent restricted in adjusting the contributions to realized asset returns. By taking into account the state pension in the optimal pension contract, hypothetical persons middle, middle + and high will no longer reach the target of flat expected consumption over the life cycle[57].

In Figure 36*a*, it is observed that hypothetical person low has a consumption standard (in retirement) above the level of the state pension without any contributions in the second pillar (i.e. AOW franchise exceeds the labor income). This higher consumption standard is explained by implementing no borrowing constraint in determining the optimal contribution scheme (see footnote 55 for the argumentation). If an investment strategy that automatically incorporates a borrowing constraint (for example a constant asset allocation of 50 % in stocks) is assumed, hypothetical person low only has the state pension income in retirement. This is because no contributions are made in the second pillar. This more realistic contribution scheme for hypothetical person low is presented in Figure 37*a*.



(a) Consumption (hypothetical person low) with constant asset allocation strategy

(b) Asset allocation (optimal standard pension contract)

Figure 37

In figure 37*b* the optimal asset allocation strategies are presented for the hypothetical persons. Observe that hypothetical person low will not accumulate pension wealth in the second pillar[58]. Hence the asset allocation

[56]This is denoted by hypothetical person high ++

[57]The contribution rate in the pension contract can be adjusted to obtain a flat expected consumption pattern. The corresponding expected consumption pattern will be lower compared to the standard life cycle. This is explained by the actuarial unfairness of the state pension that can be derived from a comparison between the red and green lines. Also, this can be explained by the fact that the participant is restricted in completely adjusting the pension contributions on realized asset returns.

[58]This is because the income level is below the AOW franchise

strategy in the second pillar is not defined. It can be concluded that the lower the income category is, the higher the equity exposure in the second pillar will be. Observe that we can see convergence to the optimal asset allocation in the Merton model (i.e. relative importance state pension is low) for the higher income categories.

In the above, the optimal pension contract was defined by including the relative importance of the state pension for different labor income categories. In this setting, the welfare effects can be calculated by neglecting the relative importance of the state pension in the optimal pension contract for the hypothetical persons.

Table 28: Welfare effects ignoring AOW in Contribution scheme and Asset allocation

	Welfare effect Suboptimal contribution[59]	Welfare effect Suboptimal Asset allocation
Hypothetical person low	+2.2 %	X[60]
Hypothetical person middle	-5.4 %	+0.2 %
Hypothetical person middle +	-3.5 %	+0.1 %
Hypothetical person high	-1.3 %	+0.0 %

This first column of Table 28 concerns welfare effects of being unable to completely dynamically contribute to the individual pension account. This is because the relative importance of the state pension is taken into account. Therefore, the participant will lose some flexibility in adjusting the pension contributions based on realized returns. By taking into account the relative importance of the state pension, the intragenerational solidarity is automatically present. Observe the welfare gain for hypothetical person low which is because of the intragenerational solidarity of the state pension. The effect will be largest for hypothetical person middle. This person will be worst of with the state pension in terms of actuarial unfairness. Also, this person is relatively most restricted (compared to hypothetical person middle + and person high) in being able to adjust the pension contributions to realized returns. For higher income categories (i.e. hypothetical person high ++) the welfare loss will converge to zero. This is because the relative importance of being restricted in adjusting pension contributions to realized asset returns and the actuarial unfairness of the state pension is small.
Welfare gains of including the value of the state pension in the asset allocation are minimal. For hypothetical person high, the effect has disappeared.

The welfare losses for the simplest case as discussed in Van Ewijk et al. (2017) [24] were replicated. This is done with the purpose to obtain some intuition for the impact of the relative importance of the state pension on the welfare effects. No particular extension will be made here. The replicated results concern welfare losses of undersaving percentages of 10, 20, 30, 40 and 50 %. In Figure 38, the replicated welfare losses are plotted for the standard person with a labor income of 40.000, by including versus excluding the relative importance of the state pension. In addition to this, the analysis was replicated by taking into account the relative importance of the state pension for a person with a labor income of 25.000 and 100.000.

From Figure 38 it can be concluded that by excluding the relative importance of the state pension for a person with a labor income of 40.000 in the analysis the welfare losses will be significantly higher (i.e up to 6 times for 50 % undersaving)[61]. Based on this analysis, the state pension can be interpreted as some guarantee that the participant will mandatory save for retirement, which is apparently beneficial in case of a suboptimal pension contract (i.e. suboptimal contributions). It is concluded from figure 38 that the welfare losses of undersaving will be significantly more for higher income categories (100.000) than for lower income categories (25.000). This can be explained by a decreasing relative importance of the state pension for higher

[59] These numbers do not split the effect of the intragenerational solidarity and the extent to which the hypothetical persons are restricted to adjust contributions to realized asset returns.

[60] This number will have no interpretation in a more realistic setting. This is because the labor income of hypothetical person low is below the AOW franchise. Hence, in reality there will be no accumulation of financial wealth in the second pillar. Therefore it is not a relevant question whether or not to include to state pension in the investment strategy (in the second pillar) for this person is welfare enhancing.

[61] Observe that the effects (for hypothetical person with an income of 40.000) are relatively high compared to reality.

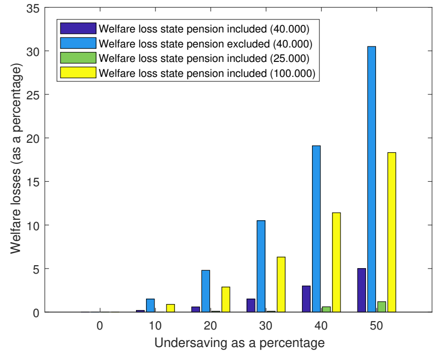


Figure 38: Welfare losses undersaving as in Van Ewijk et al. (2017) [24] for different labor income categories and including/excluding the relative importance of the state pension

income categories. Higher income categories will need accumulation of pension wealth in the second pillar to maintain the consumption standard in retirement. All these results can be found in Van Ewijk et al. (2017) [24] in Table 7a (first column) and Table 7c (second, fourth and sixth column).

In the above analysis the implications for the optimal pension contract (in the second pillar) of including the state pension was presented. It is decided to highlight the implications of including the relative importance of the state pension for one heterogeneity characteristic (case incorrect risk preferences, see Subsection 5.1.2) and one life event (case housing wealth, see Subsection 5.1.3). These cases will be extremely relevant in the pension debate and it enables us to make the comparison and extension with respect to the results of Van Ewijk et al. (2017) [24].

5.1.2 Optimal pension contract based on heterogeneity characteristic: mismatch risk preferences

This section highlights the implications of incorporating the relative importance of the state pension for heterogeneity characteristic 'mismatch risk preferences' (Section 3.1). It was concluded that a suboptimal pension contract, in terms of suboptimal contribution scheme and suboptimal asset allocation, will yield substantial welfare losses. Therefore, it will be increasingly more relevant to investigate whether pension product innovation based on this conclusion might be attractive in this more realistic setting for the participant and pension funds.

In the remainder of this subsection, three topics will be discussed:

- The optimal pension contract for optimal levels of risk aversion 5, 10 and 20 could be highlighted for the hypothetical persons defined in Table 27. The derivation of the optimal pension contract (i.e. second pillar) will take into account the relative importance of the state pension. In Subsection 5.1.1 the optimal pension contract was already presented for optimal risk aversion of 10 because this is the benchmark in this thesis. In a similar way, the optimal pension contract taking into account the relative importance of the state pension can be presented for optimal levels of risk aversion of 5 and 20. This we will not present.

- The welfare gains of tailoring the asset allocation to the risk preferences of the participant, in the setting of Van Ewijk et al. (2017) [24], can be extended to the standard setting where the relative importance of the state pension is small. In addition to this, welfare effects can be quantified for the hypothetical persons introduced in their setting, where the relative importance of the state pension is taken into account. The welfare gains were obtained in a setting were the contributions cannot be adjusted to realized returns. The welfare gains obtained are presented in Figure 39.

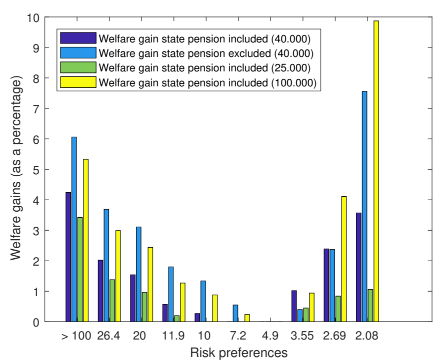


Figure 39: Welfare gains risk preferences as in Van Ewijk et al. (2017) [24] for different labor income categories and including/excluding the relative importance of the state pension

By assuming that the relative importance of the state pension is small, for hypothetical person 40.000, the welfare gains will increase for almost all suboptimal risk aversion levels[62].

- The (relatively high) welfare losses found in Table 5 and 6 will be quantified in a setting where the relative importance of the state pension is taken into account. The welfare losses obtained in the setting of this thesis, by extending the model with the relative importance of the state pension, are presented in Table 29 and Table 30 for hypothetical person middle[63]. For this person we are able to compare the welfare losses with respect to the setting where the relative importance of the state pension is small.

[62] Recall that in this setting only deviations from one risk profile (γ=5) were considered

[63] Recall that in the standard setting the labor income was normalized to 1.

Table 29: Welfare losses (asset allocation), incorrect risk preferences, hypothetical person middle

	Defensive (Optimal)	Neutral (Optimal)	Offensive (Optimal)
Defensive	0 %	-1.0 %	-2.6 %
Neutral	-2.8 %	0 %	-0.1 %
Offensive	-23.2 %	-5.7 %	0 %

Table 30: Welfare losses (contribution strategy), incorrect risk preferences, hypothetical person middle

	Defensive (Optimal)	Neutral (Optimal)	Offensive (Optimal)
Defensive	0 %	-0.5 %	-1.5 %
Neutral	-1.3 %	0 %	-0.6 %
Offensive	-7.3 %	-1.3 %	0 %

In particular, a comparison between the third column of Table 29 and the results of Van Ewijk et al. (2017) [24] can be made. This is because the same risk aversion level is assumed to be optimal. Observe that the results in the setting of this thesis, where the participant can adjust the contributions to realized returns, will be higher compared to the DC setting in Van Ewijk et al. (2017) [24]. Also, from these results, it can be concluded that including the relative importance of the state pension in the analysis yields smaller welfare losses. Smaller welfare losses of deviations from the neutral and defensive risk profile were found in this more realistic as well.

Welfare losses of a suboptimal contribution scheme, based on the risk preferences, tend to be smaller if the state pension is included.

5.1.3 Optimal pension contract based on life event: housing wealth

This section highlights the implications of including the relative importance of the state pension in the model for life event 'housing wealth' (Section 4.2). It was concluded that adjusting the contribution scheme on the accumulation of housing wealth yield significant welfare gains. After the attainment of housing wealth, reduced pension contributions were desirable (i.e. accumulation of additional pension wealth or changed consumption pattern). Unambiguous results were found if the investment strategy was adjusted on the accumulation of housing wealth.

In the remainder of this subsection, three topics will be discussed:

- The optimal pension contract conditioned on the attainment of housing wealth[64], where the relative importance of the state pension is taken into account, can be highlighted for the hypothetical persons defined in Table 27. The optimal pension contract can be presented under the assumption that housing wealth has the dynamics of a risk free asset or a geometric Brownian motion. Here, we will restrict to present the optimal pension contract under the assumption that housing wealth has the dynamics of a geometric Brownian motion[65]. In retirement it is assumed that housing wealth is depleted via a reverse mortgage and financial wealth is converted into a pension income stream via a fixed annuity. The optimal pension contract is defined for start value of housing wealth is 2.31 (i.e. normalized with respect to ALI of hypothetical person middle).

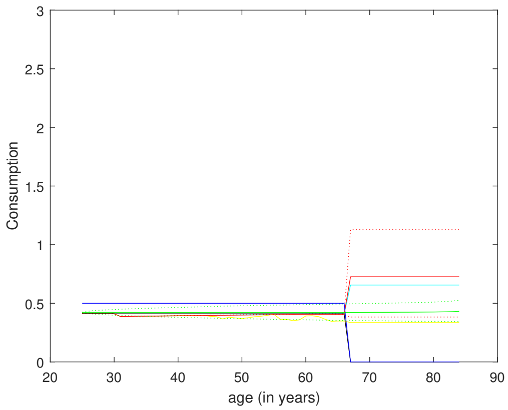
  The conclusions regarding the optimal pension contract, in case housing wealth is accumulated, are mainly similar to the the standard case (i.e. where the relative importance of the state pension was included). Important differences will be that interest payments reduce consumption directly and in retirement a fixed annuity is bought. For hypothetical person middle and middle + the target of flat consumption can no longer be attained. This is explained by the presence of the state pension that no longer enables the participant to have contribution scheme completely based on realized returns (i.e. a complete dynamic contribution scheme). Also, observe a higher replacement rate compared to the standard setting (i.e. compare Figure 36b, 36c with Figure 40b, 40c). For hypothetical person high, the consumption pattern will be approximately flat. This is because the relative importance of the state pension is small. For hypothetical person low, a pension income stream above the state pension income is observed without making contributions to the second pillar. An explanation for this was presented in the standard optimal pension contract (see subsection 5.1.1)[66].

  The optimal fraction of financial wealth in stocks is presented until the age of 67 in Figure 40e. This is because a fixed annuity is chosen in retirement. Indeed a higher equity exposure to the risky asset is found for lower income categories. Note that hypothetical person low accumulates no pension wealth in the second pillar. Therefore, the investment strategy for hypothetical person low is not defined. In the first years, hypothetical person middle will accumulate pension wealth in the second pillar. After the housing wealth is bought, hypothetical person middle will no longer accumulate pension wealth in the second pillar (i.e. and dissave the accumulated pension wealth). Therefore, the investment strategy is no longer defined a few years after the house is bought.

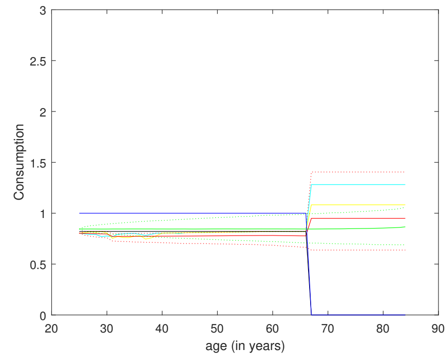
[64] This section will assume $Maint_t = ARC_t = 0$.

[65] If the state pension is assumed to be risk free, and we abstract from intragenerational solidarity in the first pillar, the welfare gains of optimally adjusting the contribution rate in the pension contract will change marginally,as can be found in subsection 4.2.2.

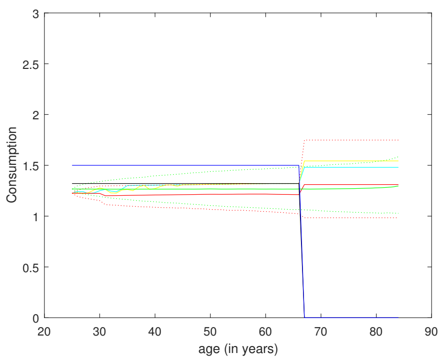
[66] In Figure 40a, the consumption of hypothetical person low is below the maximized consumption. At first sight this could be interpreted that this person contributes to the individual pension account. This is not the case and this can be explained by the fact that interest payments will reduce consumption (by assumption).



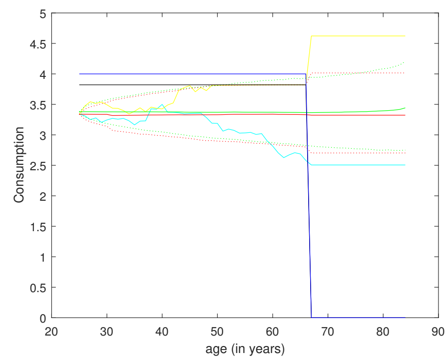
(a) Consumption (hypothetical person low)



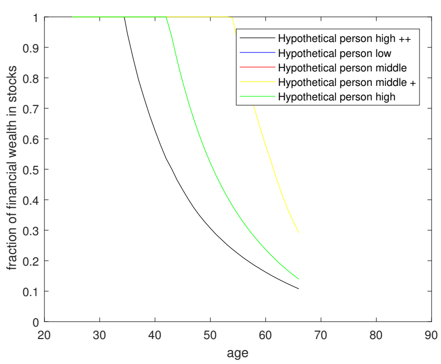
(b) Consumption (hypothetical person middle)

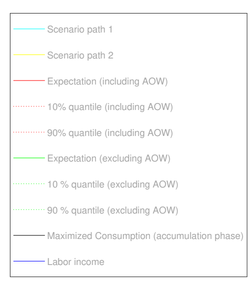


(c) Consumption (hypothetical person middle +)



(d) Consumption (hypothetical person high), Adjusted scale vertical axis





(e) Asset allocation

Figure 40: Optimal pension contract (housing wealth geometric Brownian motion)

- In Figure 41, the (robustness) results of optimally adjusting the contribution scheme on the accumulation of housing wealth in the setting of Van Ewijk et al. [24] is replicated[67]. This concerns a robustness check by taking into account the relative importance of the state pension for hypothetical person 25.000 and hypothetical person 100.000. Also, the welfare gains will be calculated where the

[67]Note that the percentage oversaving to obtain a terminal value of housing wealth at retirement age have changed with respect to changing the social security level and/or the labor income. In the analysis where the results were replicated based on the undersaving percentages, exactly the opposite was done (i.e. do not change the undersaving percentage, though the nominal undersaving amount has changed).

relative importance of the state pension is assumed to be small for hypothetical person 40.000.

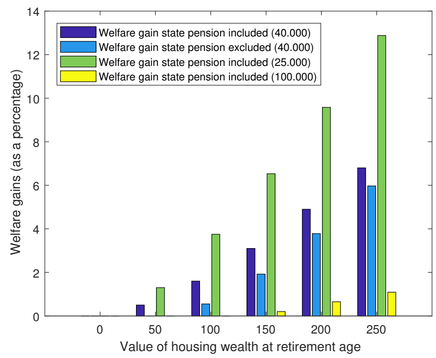


Figure 41: Welfare gains housing wealth as in Van Ewijk et al. (2017) [24] for different labor income categories and including/excluding the relative importance of the state pension

Observe that in the setting of Van Ewijk et al. (2017) [24] the welfare gains will increase in the more realistic setting by including the state pension. This can be concluded for a person with a labor income of 40.000. This contrasts the hypothesis that including the relative importance of the state pension yields lower welfare gains. In their setting contributions cannot be adjusted on realized asset returns. The results can be verified in Van Ewijk et al. (2017) [24] Table 7a (column 1), Table 7c (column 1, 3 and 5).

- In principle, all welfare gains found in Subsection 4.2 can be quantified in the situation where the relative importance of the state pension is taken into account. Welfare gains of being able to adjust the contribution rate in the pension contract will be calculated in the setting where housing wealth has the dynamics of a geometric Brownian motion. In retirement it is decided to deplete housing wealth via a reverse mortgage and the accumulated pension wealth is converted into a fixed annuity. The results are presented in Table 31[68]. Firstly, observe that the welfare gains will increase for lower income categories and decrease for higher income categories. This is because the value of housing wealth is defined as the ratio of housing wealth to ALI of hypothetical person middle.

Table 31: Welfare gains (Acc, **Opt. C** + RM versus Acc, **Subopt. C** + RM), AOW included, for hypothetical persons

Start value housing wealth (normalized w.r.t the ALI person middle), at age 30	Person low	Person middle	Person middle +	Person high
0.58	+3.1 %	+1.6 %	+1.2 %	+0.5 %
1.16	+6.1 %	+3.2 %	+2.4 %	+0.9 %
1.74	+9.1 %	+4.9 %	+3.7 %	+1.4 %
2.31	+11.8 %	+6.6 %	+4.9 %	+1.9 %
2.89	+13.2 %	+8.4 %	+6.2 %	+2.4 %

In line with Van Ewijk et al. (2017) [24] it can be concluded that adjusting the contribution rate based on the accumulation of housing wealth is beneficial for the participant. Conditioning the pension contract on the purpose at retirement age is important, although a robustness check against this conclusion was not performed.

[68] To be able to compare directly with the results of Van Ewijk et al. (2017) [24], this thesis abstracted from taxes here.

To end this subsection, some general remarks will be made. In this section, it is not considered that lower income categories could have different risk preferences (i.e. more risk averse) than the higher income categories. From the results of Alserda et al. (2016) [3], no convincing evidence was found of heterogeneity in the risk preferences based on the income distribution. Still, for a detailed description of the implications for the heterogeneity in risk preferences one can see Subsection 3.1.

In this section, the implications of intragenerational solidarity for the optimal pension contract were not taken into account. These supplements (i.e. compensation for costs of health care and rental costs) can be modeled as a risk free claim over the life cycle. In particular, this could change the desired optimal contribution scheme and asset allocation in the second pillar for hypothetical person low.

6 Adjusting the contract to realized asset returns

It will be investigated to what extent it is to desirable to offer flexibility in the pension contract based on realized asset returns (i.e. accumulated pension wealth). One could think of adjusting the asset allocation (i.e. change to a more defensive or offensive risk profile) or change the contribution strategy (contribute more or less to the pension account) for the remaining life cycle. The analysis for this event will be restricted to the accumulation phase. Therefore, the focus will be on the distribution of available pension wealth at the retirement age (i.e. proportional to a pension income stream). In the standard life cycle model discussed in Subsection 2.1, it is ex ante known that for example a strategy with a preference for guarantees is suboptimal. A different utility function might be needed (i.e preference for guarantees) to show the optimality of a pension contract incorporating guarantees in the setting of Subsection 2.1.

In Subsection 6.1, the case of underperformance of the stock market will be analyzed[69]. In Subsection 6.2, the case of overperformance of the stock market will be analyzed.

6.1 Underperformance

The case of underperformance of the stock market will be considered for a person aged 45 years. This implies that a deterministic amount of pension wealth is accumulated. By assumption the accumulated pension wealth is lower than the expected accumulated pension wealth at this age. The investor might reconsider the optimal pension contract in line with the following options in terms of adjusting the asset allocation strategy and the contribution strategy.

The implications of three asset allocation strategies on the distribution of available pension wealth will be considered:

- Guarantee asset allocation strategy: In this strategy no rebalancing (based on underperformance of the stock market) takes place and we keep the numerical amount of stocks. This implies that the fraction of financial wealth which is allocated to stocks is decreased due to low returns (compared to the optimal Merton rebalancing strategy, where it does depend on the performance of the stock market). This strategy implies some guaranteed pension wealth (i.e. guarantees) since no rebalancing (based on underperformance of the stock market) takes place. In this case we will gradually reduce (i.e. rebalance independent of the shock) the allocated fraction to stocks as the remaining investment horizon will become shorter.

- Rebalancing asset allocation strategy: The second strategy is what the Merton model prescribes as optimal. Rebalancing (based on underperformance of the stock market) takes place to retain the optimal exposure in terms of financial wealth. This implies that additional stocks need to be bought. This strategy prescribes a life cycle as well, in which the optimal allocated fraction will be decreased (i.e. rebalance independent of the shock) as the remaining investment horizon will be shorter.

- Gambling for resurrection asset allocation strategy: In this strategy additional exposure to stocks is chosen. This implies that additional stocks should be bought (rebalance based on underperformance of the stock market) to end up with the optimal rebalancing asset allocation strategy. On top of this, additional stocks have to be bought in order to increase the allocation to stocks in terms of financial wealth. Again, we will incorporate a decreased asset allocation (i.e. rebalance independent of the shock) as the remaining investment horizon will be shorter.

It was argued in Dahlquist et al. (2016) [22] that accounting for past stock market returns have positive implications for the distribution of available pension wealth at retirement. Flexibility regarding the pension contributions could be considered as well, which was not considered in Dahlquist et al. (2016) [22]. For the pension contributions we could be restricted to a preset deterministic contribution scheme. This implies

[69] By the final check of all the results I found out that in the benchmark setting of this section I have assumed a risk aversion level of 5 (and parameter for the time preference to have constant expected contributions). Initially, a risk aversion level of 5 was the benchmark in this thesis.

that this person will retain to these mandatory contributions after the shock. In case of a deterministic contribution scheme, the participant could be allowed to reoptimize the optimal constant contributions after the shock. Still, there will be constant contributions. Observe, that the optimal constant contributions could be determined in line with Bovenberg et al. (2007) [12] (Figure 11). In their setting, the optimal constant contribution scheme is defined where the welfare losses of being restricted to a constant contribution scheme are minimized compared to the dynamic setting. Still, we could allow for a dynamic contribution scheme[70]. This implies that the participant can adjust the pension contributions each year on the realized asset returns.

In the above, some intuition was presented on how the contribution strategy and asset allocation strategy in the pension contract could be adjusted based on the realized asset returns. In the setting of underperformance, it will be assumed that the participant is behind schedule at the age of 45. The expected available pension wealth, in financial terms, will be in the order of 6 at this age and it is assumed that the participant is 25 % behind schedule.

Since the pension contributions are defined in the context of the Merton setting (see Subsection 2.1) it might be reasonable to use the optimal asset allocation (rebalancing strategy) in this model as the starting point. In line with this, a more aggressive allocation strategy (gambling for resurrection) should be defined. It might be intuitive to consider an ad hoc asset allocation strategy that will achieve in expectation the targeted replacement rate. In the situation where the numerical number of stocks will be retained (i.e. guarantee strategy), no changes in the asset allocation will be the appropriate asset allocation strategy (no rebalancing based on underperformance of the stock market).

In the situation where we prefer the guarantees it is assumed that we do not rebalance (based on underperformance of the stock market). Rebalancing would imply a higher allocation to the risky asset in terms of financial wealth and this is not preferable in this strategy. Therefore, the initial asset allocation in terms of financial wealth (used if we were not behind schedule!) will be retained. As a consequence, we will act in line with this median (rebalancing) asset allocation path. Over time the exposure to the risky asset will be decreased (life cycle idea, independent of the shock). Conditionally, this strategy implies that there will be a higher guaranteed pension income for the older workers.

In the ad hoc strategy that will be used in gambling for resurrection, it is assumed that we will evaluate the accumulation of pension wealth at the age of 45. This implies that the participant has a remaining investment horizon of 22 years. By construction it is assumed that the pension wealth is insufficient compared to the standard setting (i.e. no underperformance of the stock market). The participant has 4.5 times the normalized labor income accumulated in financial wealth, whereas it was expected that already 6 times the normalized labor income was accumulated. This implies that we are 25 % behind schedule and there are 22 years until retirement to recover from the underperformance of the stock market. It is needed to recover 1 % each year approximately. Hence it is a natural approximation to increase the allocation in the upcoming periods by 25 %, keeping in mind that the equity premium is assumed to be 4 %.

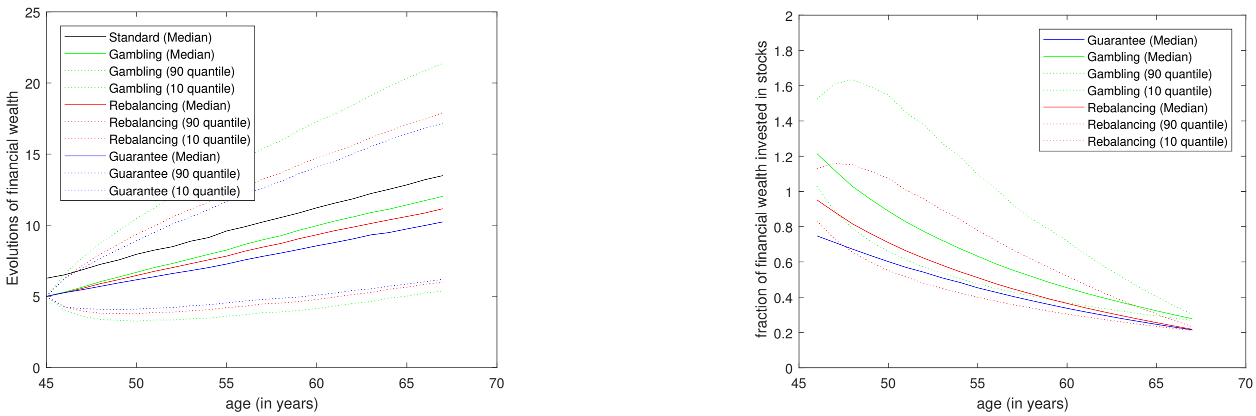
The available pension wealth will be evaluated each year from the age of 45 onwards to derive the appropriate asset allocation strategy and pension contributions. In line with Basu et al. (2009) [6] a stochastic amount of pension wealth will be available at the retirement date. The strategies will be evaluated by observing the distribution of the available pension wealth by looking at the quantiles as in Basu et al. (2009) [6].

Subsection 6.1.1 starts to analyze the impact of constant pension contributions optimized at the start of the life cycle in combination with the proposed asset allocation strategies on the distribution of available pension wealth. After this, the same analysis will be done for a constant contribution scheme optimized after the shock at the age of 45. Observe that the optimal pension contributions will be significantly higher. In Subsection 6.1.2, it will be analyzed to which extent a dynamic contribution scheme in combination with the three defined asset allocation strategies will influence the available pension wealth at retirement.

[70] To be able to conclude on the effect of adjusting the contribution rate on realized asset returns, it is decided to take the optimal (constant) expected pension contributions before and after the shock for the two constant contribution schemes respectively.

6.1.1 Constant pension contributions

In Figure 42 the median, 10 % and 90 % quantile for the implied evolutions of financial wealth and the asset allocation strategies are presented. The constant contribution scheme will be referred to as 'optimal constant pension contributions start' and will equal 0.07[71].



(a) Evolutions of financial wealth

(b) Asset allocation

Figure 42: Underperformance asset returns (optimal constant pension contributions start)

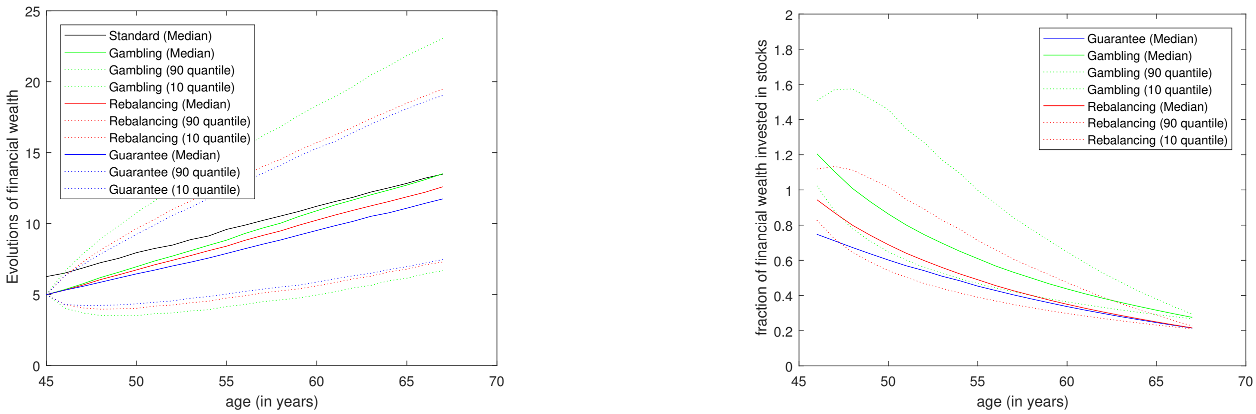
From Figure 42*a*, it is observed that in all asset allocation strategies, we will be further away from the initial Merton target (i.e. no underperformance, referred to as 'Standard') in terms of the quantiles. From Figure 42*b*, significant higher allocations to the risky asset were observed compared to the standard Merton setting. The alert reader may notice that (by construction) each quantile for the gambling for resurrection will be a constant fraction higher than the corresponding quantile for the rebalancing strategy. By assumption, the quantiles for a specific asset allocation strategy coincide at the age of 45 (in Figure 42*b*, the asset allocation is presented from the age of 46 onwards). As a consequence, in the first years an increased asset allocation path will be observed for the upper quantiles in the rebalancing strategy and the gambling for resurrection strategy.

From Figure 42*a* and 42*b*, it can be concluded that by increasing the equity exposure and by keeping the initial contribution scheme the targeted available pension wealth will not be approached [72]. This implies that additional contributions to the pension account could be desirable. More freedom of choice regarding pension contributions (i.e. customization of pension contributions adjusted to capital accumulation) implies higher welfare gains compared to freedom regarding the investment policy (Van Ewijk et al. (2017) [24]). In Bovenberg et al. (2007) [12], see Figure 11 and 12, it was concluded that an optimal contribution scheme will be a dominant factor over the asset allocation strategy in terms of tailoring the optimal pension contract. In this specific setting, the contribution scheme will be a crucial factor in the realization of available pension wealth at retirement.

[71] For a risk aversion level of 5 this will equal 0.07 and for a risk aversion level of 10 this will equal 0.15

[72] In this setting, the gambling for resurrection asset allocation strategy, yields an ad hoc chosen 25 % higher asset allocation compared to the rebalancing strategy in each sample path. By introducing an additional gambling for resurrection asset allocation strategy, that increases the equity exposure even more, it will be possible to achieve the expected targeted available pension wealth. As a consequence of increasing the equity exposure even more, there will be significantly more uncertainty regarding the available pension wealth at retirement age.

In Figure 43 the median, 10 % and 90 % quantile for the implied evolutions of financial wealth and asset allocation strategies are presented. The constant contribution scheme will be referred to as 'optimal constant pension contributions after shock' and will equal 0.13[73].



(a) Evolutions of financial wealth

(b) Asset allocation

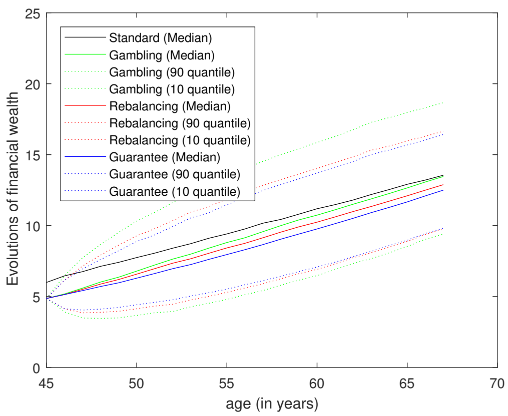
Figure 43: Underperformance asset returns (optimal constant pension contributions after shock)

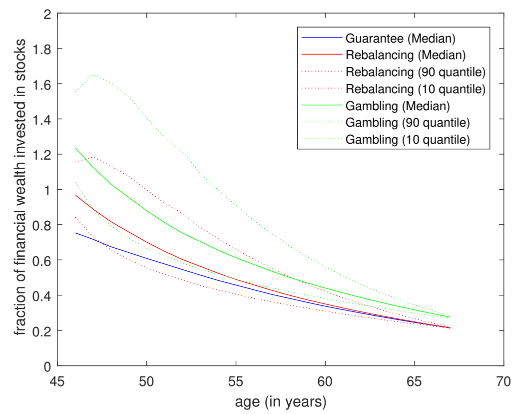
From Figure 43*a*, it can be observed that by allowing for higher pension contributions over the life cycle, the target (Merton under median circumstances) will be approached more (in median) in the three asset allocation strategies. This is in line with the hypothesis that pension contributions will be the mechanism to approach the targeted available pension wealth at retirement age. In the situation of higher pension contributions, a higher accumulation of financial wealth will be attained (ceteris paribus). This implies that in Figure 43*b*, a more defensive asset allocation strategy will be observed in each quantile, for the three asset allocation strategies respectively. These small differences between Figure 42*b* and 43*b* are difficult to see since a relatively large scale was used.

[73]For a risk aversion level of 5 this will equal 0.13 and for a risk aversion level of 10 it equals 0.20

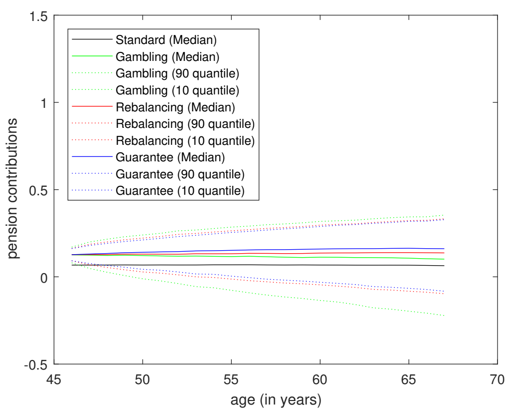
6.1.2 Dynamic pension contributions

In Figure 44 the median asset allocation strategies, the expected pension contributions, both determining the median evolutions of financial wealth are plotted. Observe that in this part optimal consumption (hence pension contributions) will be defined in line with the standard life cycle model (i.e. dynamic pension contributions). As a general remark, observe that 'Standard' in these figures refers to the situation in which there is no underperformance of the stock market. By assumption it was chosen that we are 25% behind schedule at the age of 45. This dynamic contribution scheme will be referred to as 'optimal dynamic pension contributions'.





(b) Asset allocation



(c) Pension contributions

Figure 44: Underperformance asset returns (optimal dynamic pension contributions)

Observe from Figure 44*a* that the asset allocation gambling for resurrection approaches the terminal value of financial wealth in the median in the case no underperformance took place. In all the strategies the target will be approached, as was the case with deterministic optimal pension contributions calculated after the shock. Still, the advantage of a dynamic contribution scheme, will be less uncertainty over the outcome of available pension wealth at retirement age. This can be clearly seen if the distribution of available pension wealth at retirement age from Figure 43*a* and 44*a* is compared. From Figure 44*b*, it is observed that an increased allocation to the risky asset in terms of financial wealth will be chosen in the gambling for resurrection strategy (by assumption a constant fractional increase of the asset allocation in the rebalancing strategy). Observe that in the guaranteed setting, no rebalancing (based on underperformance of the stock market) took place. Therefore it is assumed that we will allocate financial wealth in line with the initial setting. From Figure 44*c*, it is observed that in case of gambling for resurrection we will contribute additionally to the pension scheme in the first years. This is because of the lower consumption standard in the initial years, from which we are expected to recover. Additional pension contributions will be optimal in the case of guarantees as well since a decreased consumption pattern will be optimal. This is due to the consumption out of total wealth. Moreover the accumulation of financial wealth is lower (and will remain lower in expectation) compared to the base case. We are not expected to increase the consumption standard. In the rebalancing case, the consumption standard is adjusted downwards, therefore we will have constant pension contributions (in expectation).

To summarize, it was shown that by implementing the gambling for resurrection strategy defined in the setting of this thesis and maintaining the preset contribution scheme, the targeted available pension wealth will not be reached in the median. Additional pension contributions will be desirable to approach the targeted pension wealth at retirement age in case of underperformance of the stock market. Being able to contribute via a dynamic contribution scheme will reduce the risk in the realization of the available pension wealth at retirement age, measured by the quantiles, significantly.

6.1.3 Other interpretation

A different interpretation of this case could be that the participant enters the pension scheme at a later point in time. Also, the participant could be unemployed for a few years (see Subsection 4.3). Therefore, less pension contributions are the main reasons that these persons are behind schedule, instead of underperformance of the stock market. For these persons, it could be interesting to deviate from the standard contribution scheme and asset allocation strategy as will be defined in the standard pension contract.

6.1.4 Technical details

The main purpose of this subsection will be to elaborate on the technical details. In particular, for sample paths in terms of asset allocation and contribution strategy, where we are no longer behind schedule.

By construction, it is assumed that we are behind schedule and in terms of the asset allocation decision we are faced with three possibilities. In this illustrated case the investor could deviate to a strategy in which guarantees are preferred, or a strategy called gambling for resurrection. Recall that the optimal asset allocation strategy in line with the Merton model will be the rebalancing strategy.

We will compare the current financial wealth level by assumption each year and for each sample path, to the median financial wealth level under 'median' circumstances (of the stock market). If the answer will be that we are still behind schedule, we will not change the asset allocation strategy. The question will now be what will happen if we are no longer behind schedule, but exactly at scheme. In case the guarantee asset allocation strategy was preferred, by construction of this strategy, we will act in accordance with the rebalancing strategy. For the rebalancing strategy there is no need to deviate, since the dynamics of financial wealth will take automatically care of this. In case of gambling for resurrection we should be a little bit more careful. By construction, an allocation with a fixed percentage higher equity exposure compared to the rebalancing strategy was chosen. It might be natural to return to the rebalancing strategy in this case. It might be natural to allow again to switch to the gambling for resurrection strategy if we end up in a situation where we are again behind schedule at a later point in time.

An additional concern might be the implications for the sample paths if we end up at a specific point in time in the situation in which we have more financial wealth as expected. It might be natural (since this case focuses on the situation where we are behind schedule) that we will act in line with the rebalancing strategy.

Observe that the contribution strategy was retained independent of the evolutions of financial wealth in Subsection 6.1.1 and 6.1.2, where a constant and stochastic contribution scheme was in place respectively.

6.2 Overperformance

Completely analogue to the situation of underperformance, it could be argued that we enter a situation in which the investor has accumulated more pension wealth than expected. Due to a time constraint, this case will not be analyzed in this thesis. Still, some intuition will be provided. In terms of the asset allocation strategy, this person could reduce the equity exposure in the pension contract. Changing to a more defensive asset allocation strategy, the targeted replacement rate (i.e. the constant expected consumption pattern) will be achieved (in expectation) and the risk is reduced. In this situation we end up with some soft guarantees. Still, less pension contributions might be reasonable in the situation of overperformance as well.

It was concluded that mathematically analyzing the case of overperformance is equivalent to the case of underperformance. Therefore, a detailed analysis will be omitted. Still, the practical implications in the optimal pension contract based on the institutional setting and available pension products might differ. Some intuition is presented.

6.3 Implications realized asset returns for optimal pension contract

In the setting of this thesis, it was concluded that adapting the contribution strategy is the dominant factor in the realization of the available pension wealth. In the case of underperformance, it might be attractive to have additional contributions to the pension account. This is fiscally facilitated possible in either the second (depending on the exact contract agreements) or the third pillar in line with the Witteveenkader. Having less pension contributions than agreed on by the social partners in the second pillar will not be possible. This could be attractive in the case of overperformance. Switching to a different asset allocation strategy could be interesting to some extent. Some pension providers offer several risk profiles. Therefore, it could be possible to switch to a different risk profile based on the accumulated pension wealth (i.e. underperformance and overperformance). This would imply that the participant changes the risk attitude in the pension product.

7 Robustness

In this subsection, it is shown to what extent the parameter values defined in Table 1 influence the welfare effects and the optimal pension contract in terms of contribution strategy and asset allocation strategy. It is not considered how robust the results are to sensible changes in the model specification. Extensions to the model are proposed at the end of this subsection.

The effect of assuming incorrect economic parameter values will be investigated. One could think of assuming an incorrect risk free rate and an incorrect equity premium. In Ilmanen, Rauseo, and Truax (2014) [27] it is showed that overestimating the return on the portfolio by 2 percent point, yields 8 to 15 percent point more contributions to the DC pension scheme to obtain the targeted replacement rate (ceteris paribus).

In addition to a check against the economic parameter values (i.e. risk free rate and equity premium), we perform a robustness check against the preference parameters (i.e. time preference, risk aversion).

In Section 5 an implicit robustness check was performed against the income parameters. The analyses was extended by taking into account the relative importance of the state pension for hypothetical persons differing in the annual labor income.

In Van Ewijk et al. (2017) [24] a similar approach in the robustness analyses (of the welfare effects) was used. A limited number of cases (i.e. undersaving and oversaving) was discussed in their robustness check. In this thesis the robustness check will be restricted to the heterogeneity in the risk aversion (see Subsection 3.1) and the life event housing wealth (see Subsection 4.2). The set up is to recall the benchmark welfare losses of a suboptimal contribution scheme in the first three columns of table 32a (i.e. recall the welfare losses found in Table 6). In column 4-6, the analysis will be performed for an interest rate equal to 2 %. In column 7-13 a robustness check is performed against the equity premium. In column 1 and 2 of Table 32b, the welfare gains of adjusting the contribution rate in the pension contract for the case where housing wealth was assumed to be risk free (first column) and assumed to have the dynamics of a geometric Brownian motion (second column)[74] is presented. This thesis only performs a robustness check for the situation where the participant will deplete housing wealth via a reverse mortgage at retirement age. In column 3 and 4 the analysis is performed for an interest rate corresponding to 2 %. A robustness check against different values of the equity premium is presented in the remaining columns in a similar way.

In Table 33, a robustness check of welfare effects from a suboptimal asset allocation strategy will be performed for the economic parameters. A similar setup will be used as in Table 32. Observe that no robustness check against the welfare effects of adjusting the investment strategy in case housing wealth is accumulated will be presented.

[74] This thesis did not perform a robustness check against the parameters that prescribe the dynamics of the housing wealth asset (i.e. the parameters of the geometric Brownian motion)

[75] On the rows the optimal risk preference is defined, whereas in the column the risk preference used in the pension contract is defined. This notation is applicable for the other tables in this section as well.

Table 32: Robustness check economic parameters (Contribution scheme)

(a) Case (Risk preferences)

	Benchmark			r=2 %			λ=3 %			λ=5 %		
Risk[75]	Def	Neut	Off	Def	Neut	Off	Def	Neut	Off	Def	Neut	Off
Def	0 %	-0.5 %	-2.0 %	0 %	-0.3 %	-1.0 %	0 %	-0.4 %	-1.4 %	0 %	-0.8 %	-2.9 %
Neut	-1.2 %	0 %	-0.9 %	-0.6 %	0 %	-0.4 %	-0.8 %	0 %	-0.6 %	-1.8 %	0 %	-1.3 %
Off	-8.0 %	-2.1 %	0 %	-4.6 %	-1.0 %	0 %	-6.0 %	-1.4 %	0 %	-11.1 %	-3.1 %	0 %

(b) Case (Housing wealth)

	Benchmark		r=2 %		λ=3 %		λ=5 %	
Start value housing wealth (normalized w.r.t. ALI) at age 30	RF (risk free)	GBM	RF	GBM	RF	GBM	RF	GBM
0.58	+0.7 %	+0.8 %	+1.1 %	+1.2 %	X	+1.0 %	X	+0.7 %
1.16	+2.6 %	+2.5 %	+3.5 %	+3.3 %	X	+2.8 %	X	+2.4 %
1.74	+5.0 %	+4.5 %	+6.5 %	+5.5 %	X	+4.8 %	X	+4.3 %
2.31	+7.8 %	+6.4 %	+9.6 %	+7.3 %	X	+6.6 %	X	+6.3 %
2.89	+10.7 %	+8.4 %	+13.0 %	+9.1 %	X	+8.9 %	X	+8.1 %

Table 33: Robustness check economic parameters (Asset allocation)

(a) Case (Risk preferences)

	Benchmark			r=2 %			λ=3 %			λ=5 %		
Risk	Def	Neut	Off	Def	Neut	Off	Def	Neut	Off	Def	Neut	Off
Def	0 %	-1.2 %	-5.4 %	0 %	-1.0 %	-4.8 %	0 %	-0.6 %	-2.9 %	0 %	-1.7 %	-8.2 %
Neut	-3.1 %	0 %	-2.3 %	-2.8 %	0 %	-1.9 %	-1.8 %	0 %	-1.2 %	-4.5 %	0 %	-3.3 %
Off	-25.9 %	-6.1 %	0 %	-24.6 %	-5.4 %	0 %	-16.9 %	-2.8 %	0 %	-31.7 %	-7.1 %	0 %

A robustness check will be performed against preference parameters for welfare effects of a suboptimal pension contract in Table 34 and 35 in an analogue way as was performed for the economic parameters.

Table 34: Robustness check preference parameters (Contribution scheme)

(a) Case (Risk preferences)

	Benchmark			Time pref. 1 %			Time pref. 2 %		
Risk	Def	Neut	Off	Def	Neut	Off	Def	Neut	Off
Def	0 %	-0.5 %	-2.0 %	0 %	-0.1 %	-0.4 %	0 %	-0.3 %	-0.9 %
Neut	-1.2 %	0 %	-0.9 %	-0.2 %	0 %	-0.1 %	-0.6 %	0 %	-0.4 %
Off	-8.0 %	-2.1 %	0 %	-1.5 %	-0.3 %	0 %	-4.0 %	-0.9 %	0 %

(b) Case (Housing wealth)

	Benchmark		Time pref. 1 %		Time pref. 2 %		Risk aversion 5		Risk aversion 15	
Start value housing wealth (normalized w.r.t ALI) at age 30	RF	GBM	RF	GBM	RF	GBM	RF	GBM	RM	GBM
0.58	+0.7 %	+0.8 %	X	+0.8 %	+0.7 %	+0.8 %	+0.4 %	+0.5 %	+1.1 %	+1.1 %
1.16	+2.6 %	+2.5 %	X	+2.6 %	+2.6 %	+2.6 %	+1.4 %	+1.8 %	+3.4 %	+3.0 %
1.74	+5.1 %	+4.5 %	X	+4.7 %	+5.2 %	+4.7 %	+3.1 %	+3.7 %	+6.2 %	+5.1 %
2.38	+7.8 %	+6.4 %	X	+6.8 %	+7.7 %	+6.7 %	+5.1 %	+5.5 %	+9.0 %	+6.9 %
2.89	+10.7 %	+8.4 %	X	+8.7 %	+10.6 %	+8.6 %	+7.5 %	+7.7 %	+12.0 %	+8.4 %

Table 35: Robustness check preference parameters (Asset allocation)

(a) Case (Risk preferences)

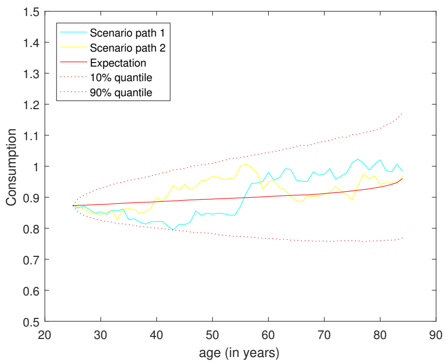
	Benchmark			Time pref. 1 %			Time pref. 2 %		
Risk	Def	Neut	Off	Def	Neut	Off	Def	Neut	Off
Def	0 %	-1.2 %	-5.4 %	0 %	-1.2 %	-5.6 %	0 %	-1.2 %	-5.5 %
Neut	-3.1 %	0 %	-2.3 %	-3.0 %	0 %	-2.2 %	-3.0 %	0 %	-2.3 %
Off	-25.9 %	-6.1 %	0 %	-25.7 %	-6.1 %	0 %	-25.7 %	-6.0 %	0 %

A general observation is that under alternative assumed parameter values, the size of the welfare effects remain substantial. This indicates robustness in the results. An overview of the main conclusions regarding the robustness results are presented.

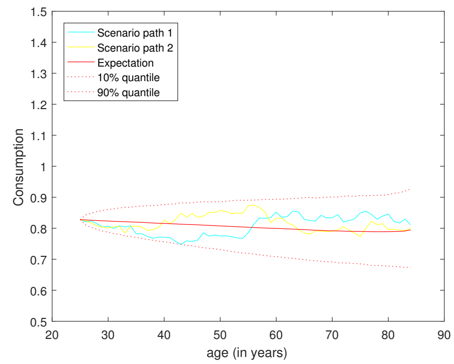
- Risk aversion: By making the risky asset less attractive compared to the risk free asset, the welfare losses will decrease for both a suboptimal asset allocation and a suboptimal contribution scheme. A lower time preference parameter will increase the welfare losses of a suboptimal contribution scheme significantly, whereas welfare losses of a suboptimal asset allocation yield insignificant deviations.
- Contribution Housing wealth: In case of a higher interest rate, adjusting the contribution rate on the accumulation of housing wealth is more beneficial. A similar effect is found by decreasing the equity premium. Observe that a higher equity premium yields lower welfare gains. Less preference for early consumption yields somewhat higher welfare gains. A lower risk aversion yields lower welfare gains, whereas a more risk averse person can attain higher welfare gains by adjusting the contribution rate.

Also note that the welfare effects were calculated in this section with some of the benchmark parameter values as assumed in Van Ewijk et al. (2017) [24]. For example, their benchmark is a time preference of 2 %, interest rate of 2 % and risk aversion level of 5.

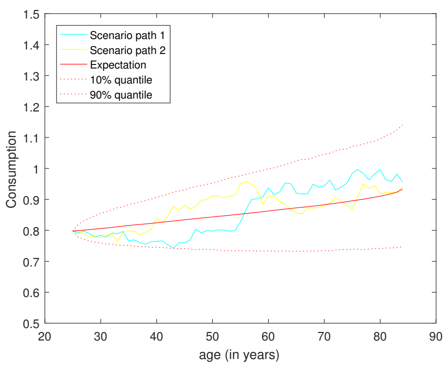
In principle, for each different assumption regarding the economic and preference parameters, the optimal pension contract can be presented. We will restrict in determining the optimal pension contract for hypothetical person middle for a limited number of different assumed parameter values. The optimal pension contract is defined for an interest rate of 2 %, equity premium of 3 %, a time preference parameter of 0.01, and a risk aversion level of 5. The optimal pension contract is calculated in the absence of heterogeneity and life events.



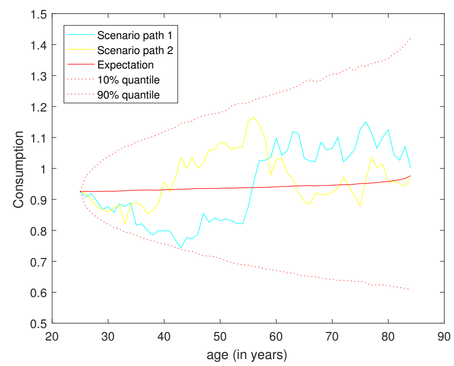
(a) Interest rate of 2 %



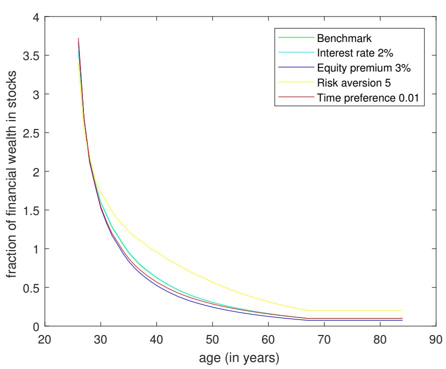
(b) Equity premium of 3 %



(c) Time preference parameter of 0.01



(d) Risk aversion level of 5



(e) Asset allocation

Figure 45: Pension contract (Robustness)

The optimal dynamic contribution scheme in the pension contract is discussed for each robustness check separately:

- Increasing the interest rate yields an increasing consumption standard as can be derived from equation (18) and therefore the flat expected contribution scheme is no longer optimal. In a dynamic contribution scheme, a higher risk free rate leads to less pension contributions over the life cycle. Note that an interest rate of 2 % is assumed in Bovenberg et al. (2007) [12] and Van Ewijk et al. (2017) [24].
- Decreasing the equity premium yields a decreasing consumption standard as can be derived from equation (18). In a dynamic contribution scheme, a lower equity premium leads to more pension contributions over the life cycle. Again, the flat expected contribution scheme is no longer optimal.
- Decreasing the time preference yields an increasing consumption standard as can be derived from equation (18). The effect on the contribution scheme is similar to an increase in the interest rate.
- Decreasing the risk preferences yields a higher initial consumption level (explained by the EIS), whereas the expected contribution scheme is nearly flat over the life cycle.

In terms of the optimal asset allocation, a lower allocated fraction to the risky asset is observed for an interest rate of 2% and an equity premium of 3 %. In both cases, the attractiveness of the risk free asset increases compared to the risky asset. Observe that a lower time preference parameter decreases the optimal equity exposure as well. For sure, a person that is less risk averse will have a higher allocated fraction to the risky asset.

This section had performed a robustness check against the parameter values in the model only. Extending the analysis by including more realistic assumptions in the model specification is desirable. This in order to formulate well motivated policy implications. A robustness check regarding the following assumptions is important.

- **CRRA utility function**: It is well known that investors utility cannot be described by a CRRA utility function, although it is a reasonable starting point. It is important that the pension product offered, reflects the preferences (i.e. guarantees, heterogeneity in the replacement rates) of the investor accurately.
- **Risk free human capital**: In the ideal state, all participants have a job. Still, this is far from reality, see Subsection 4.3. The modeling of human capital could include correlation with macro economic conditions, that will be typically stochastic.
- **Constant interest rate**: In a DC pension scheme the interest rate is less important compared to a DB pension scheme (see the discussion on funding ratio). Still accurately reflecting uncertainty in the interest rate is necessary in the optimal pension contract.
- **Deterministic date of death**: Including (heterogeneous) survival probabilities will be an important challenge in the development of pension products.

8 Conclusions and Recommendations

This thesis derived the optimal pension contract for several illustrated cases for heterogeneous agents accommodating for life events. The benefits of individualizing the life cycles were quantified via welfare effects. The results obtained in this thesis can contribute in defining the exact conditions of the new pension contract. Below, an overview of the most important conclusions of this thesis are presented.

Social partners typically agree on a suboptimal contribution scheme.
The current institutional setting regarding taxes and legislation in the pension domain was taken into account to indicate where customization in the current pension contract is already possible. In the current Dutch setting it is not the case that the law (i.e. Witteveenkader) is restrictive in adjusting the contribution rates for heterogeneous agents accommodating for life events. In reality, social partners set contribution rates that are close to the upper bound of the Witteveenkader. It is not possible to have less pension contributions than agreed on by the social partners. For example, this implies that the current pension contract (i.e. agreements of the social partners) cannot distinguish between the optimal contribution scheme of houseowners and renters.

Pension reforms, as announced by the current cabinet, should lead to an optimal pension contract for heterogeneous agents accommodating for life events
In the Dutch setting, pension schemes are surely moving from DB pension schemes towards DC pension schemes. The current cabinet soon hopes to reach an agreement to develop personal pensions with collective risk sharing. This transition, technological developments and market size could provide pension funds the opportunity of further individualizing the life cycles for their participants. This would imply that pension funds could offer heterogeneous agents accommodating for life events a customized, flexible pension contract, in terms of contribution scheme and asset allocation.[76]

Heterogeneity

Tailoring the pension contract on the heterogeneity in the risk preferences is more welfare enhancing in the setting where contributions can be adjusted to realized returns compared to the DC setting of Van Ewijk et al. (2017) [24].
In line with the products offered by NN, a defensive, a neutral and an offensive risk profile was defined. A similar set up was used in defining the contribution scheme differentiated on the risk preferences. In line with the results of Van Ewijk et al. (2017) [24] it can be concluded that an accurate reflection of the risk preferences in the asset allocation will avoid significant welfare losses. This thesis found higher effects compared to the DC setting defined in Van Ewijk et al. (2017) [24]. An explanation for this is the fact that this thesis allowed to adjust contributions to realized asset returns.
It can be concluded that a contribution scheme differentiated on the risk preferences is attractive as well. The effect is smaller compared to the welfare losses found for inaccurate reflection of the risk preferences in the asset allocation.

Adjusting the contribution scheme on the assumed wage profile is desirable. This can lead to the absence of pension contributions in the first of the accumulation phase in case of a high assumed wage profile.
In the setting where an assumed (risk free) wage profile is present, contributing to the pension account via a different wage profile can lead to significant welfare losses. For example if, in the contribution scheme, it is assumed that the wage will increase each year by 1.5 %, whereas the wage actually will increase by 1 % a welfare loss of 4 % is present. In particular, if a high assumed wage profile is present in the risk profiles it can be attractive to have no pension contributions in the first years of the life cycle. If the underlying wage profile is chosen in a clever way, welfare losses of an inaccurate reflection of the assumed wage profile in the investment strategy are minimal. An asset allocation that assumes a wage increase of 1 % each year might be a reasonable choice in the setting of this thesis.

[76] It will be a challenge for pension funds to take into account the behavioral imperfections of pension plan participants. This bounded rationality was not taken into account in this thesis.

Life events

Higher pension contributions after a bequest received is preferred to maintain the consumption standard in retirement and insure the micro longevity risk.
In the setting of this thesis it is optimal to contribute part of the bequest received to the individual pension account. Consuming the bequest within 20 years, where the agent is assumed to pass away in 40 years, leads to serious welfare losses. For example, a bequest of 3.3 times the normalized labor income leads to a welfare loss of 5.4 %. The welfare losses will increase in the numerical amount of the bequest and the shorter the consumption smoothening period. These results were obtained in a setting in which the participant is not restricted in fiscally facilitated contributing to the pension account (i.e. 'no Witteveenkader') to insure the micro longevity risk.
Adjusting the asset allocation in the pension contract after a bequest received will be a higher order effect.

Adjusting the contribution rate on the accumulation of housing wealth prevents oversaving in an extended setting compared to Van Ewijk et al. (2017) [24].
In Van Ewijk et al. (2017) [24] it was concluded that adjusting the contribution scheme, in the case housing wealth is accumulated, yield welfare gains in the order of 0.5 % - 6.8 %. If the participant can adjust the contributions on realized returns, this thesis found welfare gains in the order of 0.7 % - 13 %. The exact welfare gains depend on the ratio of house value to the annualized labor income, the dynamics of housing wealth and the purpose of housing wealth at retirement age. Therefore, it can be concluded that it is optimal for houseowners to have a lower contribution scheme compared to renters. The optimal contribution scheme crucially depends on the purpose of housing wealth at retirement age. Welfare losses will arise if too much reduced pension contributions were allowed. Therefore, the pension contract should condition the contribution scheme on the purpose regarding housing wealth during retirement.
In line with the conclusions of Olear et al. (2017) [31], it is not beneficial to take into account the housing wealth in the optimal asset allocation.

Consumption out of pension wealth will be helpful in maintaining the consumption standard during unemployment years.
In the unpleasant situation that an agent will become unemployed, the participant will have some choice regarding the contribution of pension wealth. Although it is not possible in the Dutch setting, it can be attractive to deplete part of the pension wealth in the unemployment period to maintain the consumption standard. Correcting the investment strategy for the depletion of the labor income minus the unemployment benefit is not significantly beneficial.

Relative importance of the state pension for heterogeneous agents accommodating for life events

Welfare gains remain present in the standard life cycle model extended by the relative importance of the state pension.
The model was extended by taking into account the relative importance of the state pension. Taking into account the relative importance of the state pension, in terms of the asset allocation yields limited welfare gains.
For a limited number of cases (i.e. risk preferences and housing wealth) the implications of the relative importance of the state pension were analyzed.

In this more realistic setting it can be concluded, that welfare losses of not accurately reflecting the risk preferences in the contribution scheme and the asset allocation have decreased. Though policy recommendations of tailoring the pension contract to the risk preferences remain valid in this more realistic setting.

In this more realistic setting, it can be concluded that adjusting the contributions on the accumulation of housing wealth should be incorporated in the pension contract. The results were quantified under the assumption that housing wealth is depleted during retirement via a reverse mortgage. In the setting, where the the relative importance of the state pension was included and the contributions can be adjusted to realized asset returns, welfare gains remain significant. The exact welfare gains increase for a higher ratio of housing wealth to ALI. Observe that the same start value of housing wealth was assumed for the hypothetical persons differing in their annual labor income.

Realized asset returns

Adjusting the contribution rate is preferred over more equity exposure in reaching the targeted replacement rate.
Realized asset returns are, in addition to pension contributions and investment strategy, an important determinant of the pension outcome. During the life cycle, the participant can already anticipate on the sufficiency of the accumulated pension wealth if there is a strong preference for a targeted replacement rate. In case of underperformance of the stock market, it can be attractive to reconsider the pension contract in terms of contribution scheme and asset allocation. It was shown that adjusting the contributions based on the realized asset returns (each year) is more efficient than increasing the equity exposure to reach the desired available pension wealth at retirement age.

Robustness
A general observation is that under alternative assumed parameter values, the size of the welfare effects remain substantial. This indicates robustness in the results.
Welfare gains of adjusting the contribution scheme, in case housing wealth is accumulated, is highest for a risk averse person in a financial market with a low equity premium. Welfare losses of tailoring the pension contract, differentiated on the risk preferences, will be lower in a financial market with a low equity premium.

Recommendations for future research
The standard life cycle model was used as the benchmark model in the analysis. Although some of the fundamental assumptions have been relaxed, it will remain a question for future research to do the analysis in an even more realistic setting. A list of useful suggestions for future research will be provided.

The implications of time varying interest rates and inflation rates and predictable returns.
This thesis can be extended to dynamic portfolios strategies that hedge real interest rate risk and time-varying risk premia (Koijen et al. (2017) [29], Van Binsbergen and Koijen (2017) [9]). Moreover, the findings in the literature can be extended to the case where the preferred future exposures are to be chosen in a later stage (as in the new Dutch DC legislation).

The impact of uncertainty in future employment and income profiles as well as differences between wage and price inflation.
A main extension to the current literature can be to relate the literature on employment dynamics with that on optimal pension design. The probability to get unemployed is likely to have a major impact on optimal pension saving and investing, because a spell of unemployment will require flexibility to adjust pension savings. The impact of disability will probably be even larger. Also unexpected positive career developments (promotion or finding a better job) require adjustments of the pension plan.

The impact of uncertainty in future life events, such as marriage, divorce, purchase of a house, unexpected health costs etc.
The existing literature can be extended by analyzing how uncertainty on future life events can best be addressed, by analyzing for which individuals flexibility in pension contributions and drawdown will be particularly important as well as how to nudge individuals towards attractive choices.

Implications of more general preference assumptions, including habit formation, loss aversion and a preference for guarantees.
As an extension to the thesis, the focus can be on the implications of a preference for (nominal) guarantees as is apparent from many survey studies. The existing literature can be extended in particular by introducing formal welfare comparisons using well calibrated non-standard utility functions as well as uncertainties about future events from extensions two and three and the options to deal with them e.g. through precautionary

savings or adjustments in labor supply.

Biometric risk sharing and heterogeneity in risk sharing pools.
An important extension of this thesis can be to analyze how to deal with heterogeneity in survival probabilities and with heterogeneity in asset allocations in sharing biometric risks. Model risk in determining the projected increase in life expectancy and the biometric returns due to macro longevity risk can also be investigated.

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A Appendix

In Appendix $A.1$, several important results and implications in the standard life cycle model will be derived. In Appendix $A.2$, an overview of the notation used in this thesis will be presented. The abbreviations used in this thesis will be clarified in Appendix $A.3$.

A.1 Derivations in Merton model

In Appendix $A.1.1$, we will start by providing a derivation of the optimal asset allocation and optimal consumption pattern in the standard life cycle model. For the sake of completeness, in Appendix $A.1.2$ analytical solutions are presented for the stock and the bond. In Appendix $A.1.3$, the risk aversion will be derived and in Appendix $A.1.4$ the elasticity of intertemporal substitution will be derived. The dynamics of financial wealth will be derived in Appendix $A.1.5$. In Appendix $A.1.6$ the life cycle optimization problem is formulated to be able to solve it via the martingale method. In Appendix $A.1.7$ the certainty equivalent will be derived in the setting of this thesis, whereas in Appendix $A.1.8$ equivalence will be shown with the certainty equivalent found in Bovenberg et al. (2007) [12]. In Appendix $A.1.9$, some additional intuition will be provided about the double role for the parameter γ in the standard life cycle model.

A.1.1 Asset allocation and Consumption derived using HJB

In this part the optimal asset allocation and the optimal consumption pattern will be derived using the HJB solution approach. The function $J(W_t, t)$ has to be maximized with respect to f_t and C_t:

$$J(W_t, t) = \max_{f_t, C_t} E_{W_t,t}\left(\int_t^D e^{-\delta(s-t)} U(C_s) ds\right) \tag{44}$$

Now we will assume that strategies after $t + \Delta t$ are known (and optimal). The idea is that we will keep in mind that a small value for Δt is taken. Also, it is assumed that a constant strategy f_t is chosen over the interval $[t, t + \Delta t)$. Then for Δt goes to zero we can rewrite expression (44) in the following way:

$$J(W_t, t) = \sup_{f_t, C_t} U(C_t)\Delta t + e^{-\delta \Delta t} E_{W_t,t}\Big(J(W_{t+\Delta t}, t + \Delta t)\Big) \tag{45}$$

Now some terms from expression (45) will be rearranged in the following way:

$$\frac{e^{\delta \Delta t} - 1}{\Delta t} J(W_t, t) = \sup_{f_t, C_t} e^{\delta \Delta t} U(C_t) + \frac{1}{\Delta t} E_{W_t,t}\Big(J(W_{t+\Delta t}, t + \Delta t) - J(W_t, t)\Big) \tag{46}$$

If Δt go to zeros, the first two terms of expression (46) converge as follows:

$$\delta J(W_t, t) = \sup_{f_t, C_t} U(C_t) + ... \tag{47}$$

For the third term of (46) it is not so trivial to see the convergence. At this point in time we will make use of a lemma (illustrated for the stochastic process Y_t) that states the following:

If we have a stochastic process Y_t defined in the following way:

$$dY_t = m(t)dt + s(t)dZ_t \tag{48}$$

Then the following convergence is obtained:

$$\lim \Delta t \downarrow 0 = \frac{E\Big(Y_{t+\Delta t} - Y_t\Big)}{\Delta t} = m(t) \tag{49}$$

From this we can learn that the third term of expression (46) will converge to the drift term of the stochastic process $J(W_t, t)$. This implies that we need to derive the stochastic process for $J(W_t, t)$. It is known that

Ito's lemma will give us stochastic differential equations. Therefore it is good to recall the dynamics of the total wealth W_t.

$$dW_t = \Big(W_t(r + f_t\sigma\lambda) - C_t\Big)dt + W_t f_t \sigma dZ_t \tag{50}$$

Please note that we will only need the terms in front of the dt, therefore terms in front of the Brownian motion can be neglected in the derivation of $dJ(W_t, t)$. Also observe that some terms will be equal to zero and therefore omitted.

$$dJ(W_t,t) = J_t(W_t,t)dt + J_{W_t}(W_t,t)dW_t + \frac{1}{2}J_{tt}(W_t,t)d[t,t] + \frac{1}{2}J_{W_tW_t}(W_t,t)d[W_t,W_t] + J_{W_tt}(W_t,t)d[W_t,t] \tag{51}$$

$$= J_t(W_t,t)dt + J_{W_t}(W_t,t)\Big(\Big(W_t(r+f_t\sigma\lambda) - C_t\Big)dt + W_t f_t\sigma dZ_t\Big) + 0 + \frac{1}{2}J_{W_tW_t}(W_t,t)W_t^2 f_t^2\sigma^2 dt + 0 \tag{52}$$

$$= \Big(J_t(W_t,t) + J_{W_t}(W_t,t)\Big(W_t(r+f_t\sigma\lambda) - C_t\Big) + \frac{1}{2}J_{W_tW_t}(W_t,t)W_t^2 f_t^2\sigma^2\Big)dt + (...)dZ_t \tag{53}$$

Combining everything, expression (46) converges as follows:

$$\delta J(W_t,t) = \sup_{f_t,C_t} U(C_t) + \Big(J_t(W_t,t) + J_{W_t}(W_t,t)\Big(W_t(r+f_t\sigma\lambda) - C_t\Big) + \frac{1}{2}J_{W_tW_t}(W_t,t)W_t^2 f_t^2\sigma^2\Big) \tag{54}$$

Now we will include terms in Φ_C if the term in formula (54) depends on C_t and in Φ_f terms that depend on f_t as follows:

$$\Phi_C = \sup_{C_t \geq 0} U(C_t) - C_t J_{W_t}(W_t,t) \tag{55}$$

$$\Phi_f = \sup_{f_t} J_{W_t}(W_t,t)W_t f_t\sigma\lambda + \frac{1}{2}J_{W_tW_t}(W_t,t)W_t^2 f_t^2\sigma^2 \tag{56}$$

Combining this, the following rewriting of equation (54) is obtained:

$$\delta J(W_t,t) = \Phi_C + \Phi_f + J_t(W_t,t) + J_{W_t}(W_t,t)W_t r \tag{57}$$

Now we have to maximize Φ_C and Φ_f with respect to C_t and f_t respectively. This is done by taking first order conditions:

$$U^{'}(C_t) - J_{W_t}(W_t,t) = 0 \tag{58}$$

$$U^{'}(C_t) = J_{W_t}(W_t,t) \tag{59}$$

$$\tag{60}$$

To write out the C_t, the inverse of $U^{'}(C_t)$ will be defined as $I_u(.)$. Then an expression of the consumption in terms of this unknown indirect utility of consumption can be found.

$$C_t = I_u(J_{W_t}(W_t,t)) \tag{61}$$

The first order condition with respect to the optimal asset allocation (f_t) will be taken.

$$J_{W_t}(W_t,t)W_t\sigma\lambda + J_{W_tW_t}(W_t,t)W_t^2 f_t\sigma^2 = 0 \tag{62}$$

$$f_t = -\frac{J_{W_t}(W_t,t)W_t\sigma\lambda}{J_{W_tW_t}(W_t,t)W_t^2\sigma^2} \tag{63}$$

For the optimal asset allocation, the optimal fraction (f_t) is expressed as a function of partial derivatives function of this unknown $J(W_t,t)$ function. The last step will be to come up with an analytical expression

for $J(W_t, t)$.
In the Merton setting CRRA utility preferences are assumed. For the sake of completeness these preferences could be defined in the following way:

$$U(C_t) = \begin{cases} \frac{C_t^{1-\gamma}}{1-\gamma} & \gamma \neq 1 \\ \log(C_t) & \gamma = 1 \end{cases} \tag{64}$$

In the upcoming derivations, the case in which $\gamma \neq 1$ will be considered.

Recall that in this setting the asset allocation is independent of the initial wealth level. Therefore the following conjecture seems reasonable:

If the strategy (C_t^*, f_t^*) is optimal for wealth level W_t, the strategy (kC_t^*, f_t^*) is optimal for initial wealth level kW_t. This is mathematically presented as follows:

$$J(kW_t, t) = E_{W_t,t}\bigg(\int_t^D e^{-\delta(s-t)}\frac{(kC_s^*)^{1-\gamma}}{1-\gamma}ds\bigg) \tag{65}$$

$$= k^{1-\gamma}E_{W_t,t}\bigg(\int_t^D e^{-\delta(s-t)}\frac{(C_s^*)^{1-\gamma}}{1-\gamma}ds\bigg) \tag{66}$$

$$= k^{1-\gamma}J(W_t, t) \tag{67}$$

Now we will reparametrize as follows, $k = \frac{1}{W_t}$ and $g(t)^\gamma = (1-\gamma)J(1,t)$. Next we will substitute k and $g(t)$ in (the above derived) equation (68):

$$J(kW_t, t) = k^{1-\gamma}J(W_t, t) \tag{68}$$

Then we end up with a more concrete form of $J(W_t, t)$.

$$J(W_t, t) = \frac{g(t)^\gamma W_t^{1-\gamma}}{1-\gamma} \tag{69}$$

Directly from formula (69) the following (partial) derivatives of $J(W_t, t)$ are needed to solve the optimal C_t and optimal f_t. In addition to this, we still need to come up with an explicit expression for the time dependent function $g(t)$. The necessary partial derivatives of $J(W_t, t)$ are as follows:

$$J_{W_t}(W_t, t) = g(t)^\gamma W_t^{-\gamma} \tag{70}$$

$$J_{W_tW_t}(W_t, t) = -\gamma g(t)^\gamma W_t^{-\gamma-1} \tag{71}$$

$$J_t(W_t, t) = \frac{\gamma}{1-\gamma}g(t)^{\gamma-1}g^{'}(t)W_t^{1-\gamma} \tag{72}$$

Plugging in the (partial) derivatives of $J(W_t, t)$ and the optimal solutions for C_t and f_t in the HJB, an ordinary differential equation involving $g(t)$ and $g^{'}(t)$ arises:

$$0 = \bigg(\bigg(\frac{\delta}{1-\gamma} - r - \frac{1}{2\gamma}\lambda^2\bigg)g(t) - \frac{\gamma}{1-\gamma} - \frac{\gamma}{1-\gamma}g^{'}(t)\bigg)g(t)^{\gamma-1}W_t^{1-\gamma} \tag{73}$$

In equation (73) we can forget about $g(t)^{\gamma-1}W_t^{1-\gamma}$ since we can divide by it. If a closer look is taken at this differential equation, the differential equation can be recognized as of the following type (with a and b are constants):

$$g^{'}(t) = ag(t) + b \tag{74}$$

A standard solution to such a differential equation in equation (74) is the following (with k a constant, depending on a given condition):

$$g(t) = \frac{-b}{a} + ke^{at} \tag{75}$$

The differential equation, defined in (73), with terminal condition $g(D) = 0$ derived from formula (69) can be solved in the general way as prescribed by formula (75):

$$g(t) = \frac{1}{A}\left(e^{A(D-t)} - 1\right) \tag{76}$$

$$A = \frac{(1-\gamma)r - \delta}{\gamma} + \frac{(1-\gamma)\lambda^2}{2\gamma^2} \tag{77}$$

Recall that we are trying to come up with a concrete solution for the optimal consumption pattern (C_t^*) and optimal asset allocation (f_t^*). In the previous steps we were mainly concerned with founding a concrete expression for $J(W_t, t)$ and we managed to do this. By substitution a final formula for the optimal consumption and optimal asset allocation can be obtained.

For the optimal consumption the following is obtained:

$$C_t = I_u(J_{W_t}(W_t, t)) \tag{78}$$

$$= I_u(g(t)^\gamma W_t^{-\gamma}) \tag{79}$$

$$= \left(g(t)^\gamma W_t^{-\gamma}\right)^{-\frac{1}{\gamma}} \tag{80}$$

$$= \frac{W_t}{g(t)} \tag{81}$$

For the optimal asset allocation the following is obtained:

$$f_t = -\frac{J_{W_t}(W_t, t) W_t \sigma \lambda}{J_{W_t W_t}(W_t, t) W_t^2 \sigma^2} \tag{82}$$

$$= -\frac{g(t)^\gamma W_t^{-\gamma} W_t \sigma \lambda}{-\gamma g(t)^\gamma W_t^{-\gamma-1} W_t^2 \sigma^2} \tag{83}$$

$$= \frac{\lambda}{\gamma \sigma} \tag{84}$$

This finishes the proof of the optimal consumption pattern and the optimal asset allocation in the Merton model, solved by the HJB method, in case $\gamma \neq 1$.

Until this point in time we abstract from the case that γ is equal to 1. This can be derived as well, where the steps will be completely analogue to the case in which $\gamma \neq 1$. Therefore, this will be omitted. If we would have done this, please note that we could copy until equation (65) and from that point onwards changes arises by imposing a different utility function.

A.1.2 Analytical solutions Asset prices

For the sake of completeness, the analytical solutions of the stock price (1) and the bond price (2) at time t are provided.

$$S_t = S_0 \exp\left(\left(\mu - \frac{\sigma^2}{2}\right)t + \sigma Z_t\right) \tag{85}$$

$$B_t = B_0 \exp(rt) \tag{86}$$

A.1.3 Deriving risk aversion in CRRA utility

In this subsection, the risk aversion in the CRRA setting will be derived. It is known that the risk aversion is defined as follows:

$$\text{Relative Risk aversion} = \frac{-c \cdot U''(C_t)}{U'(C_t)} \tag{87}$$

The case for $\gamma \neq 1$ will be consider first. We start by calculating the first and second order derivative of $U(C_t)$.

$$U^{'}(C_t) = C_t^{-\gamma} \tag{88}$$
$$U^{"}(C_t) = -\gamma C_t^{-\gamma-1} \tag{89}$$

So the calculation of the risk aversion will be as follows:

$$\text{Relative Risk aversion} = \frac{-C_t U^{"}(C_t)}{U'(C_t)} \tag{90}$$
$$= \frac{-C_t - \gamma C_t^{-\gamma-1}}{C_t^{-\gamma}} \tag{91}$$
$$= \frac{\gamma C_t^{-\gamma}}{C_t^{-\gamma}} \tag{92}$$
$$= \gamma \tag{93}$$

Now, the case for $\gamma = 1$ will be considered. Again we start by calculating the first and second order derivative of $U(C_t)$.

$$U^{'}(C_t) = \frac{1}{C_t} \tag{94}$$
$$U^{"}(C_t) = \frac{-1}{C_t^2} \tag{95}$$

So the calculation of the risk aversion will be as follows:

$$\text{Relative Risk aversion} = \frac{-C_t \cdot U^{"}(C_t)}{U'(C_t)} \tag{96}$$
$$= \frac{-C_t \cdot -\frac{1}{C_t^2}}{\frac{1}{C_t}} \tag{97}$$
$$= 1 \tag{98}$$

Note that in both cases, the risk aversion will be equal to the parameter value of γ.

A.1.4 Derivation of Elasticity of Substitution

In this subsection the EIS will be derived. The EIS will be defined as follows:

$$\text{EIS} = \frac{d\ln(\frac{C_{t+1}}{C_t})}{dr} \tag{99}$$

From the no arbitrage condition in financial markets, an expression for the return can be obtained.

$$R = \frac{U^{'}(C_t)}{\beta U'(C_{t+1})} \tag{100}$$

By taking the logarithm of formula (100), we can interpret r as a percentage.

$$r = \ln(R) = -\ln\left(\frac{U^{'}(C_{t+1})}{U'(C_t)}\right) - \ln(\beta) \tag{101}$$

After substitution equation (101) into (99), the EIS could be presented as follows:

$$\text{EIS} = -\frac{d\ln\left(\frac{C_{t+1}}{C_t}\right)}{d\ln\left(\frac{U'(C_{t+1})}{U'(C_t)}\right)} \tag{102}$$

Now we should recall that CRRA utility is imposed and the numerator and denominator of formula (102) can be rewrited (and related to each other) as follows:

$$\ln\left(\frac{U^{'}(C_{t+1})}{U^{'}(C_t)}\right) = \ln\left(\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}\right) = -\gamma\ln\left(\frac{C_{t+1}}{C_t}\right) \tag{103}$$

$$\ln\left(\frac{C_{t+1}}{C_t}\right) = \ln\left(\left(\frac{C_{t+1}}{C_t}\right)^{\frac{-\gamma}{-\gamma}}\right) = -\frac{1}{\gamma}\ln\left(\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}\right) = -\frac{1}{\gamma}\ln\left(\frac{U^{'}(C_{t+1})}{U^{'}(C_t)}\right) \tag{104}$$

To combine everything the final formula, after substitution the equivalence in formula (102), could be derived:

$$\text{EIS} = -\frac{d\ln\left(\frac{C_{t+1}}{C_t}\right)}{-\gamma d\ln\left(\frac{C_{t+1}}{C_t}\right)} = \frac{1}{\gamma} \tag{105}$$

A.1.5 Dynamics financial wealth

It might be interesting to present some additional intuition regarding the dynamics of the financial wealth as well. The dynamics of financial wealth were presented as follows:

$$dF_t = F_t\left((r + f_t^*\sigma\lambda)dt + f_t^*\sigma dZ_t\right) + (L - C_t)dt \tag{106}$$

At the beginning of the life cycle there is no financial wealth. So, if one starts working, labor income is received. In this model, one can either consume this or one could save it $(L - C_t)$. This saving is how the accumulation of financial wealth starts, presented by the second term of formula (9). The first term has to do with the investment decision. At this point in time, note the subtle difference between f_t, which is the fraction of total wealth invested in stocks, whereas f_t^* is the fraction of financial wealth that is invested in stocks.The first part of the first term of expression (9) consists of a risk free return (r) and a risk premium $(\sigma\lambda)$ depending on the allocation to risky assets. Note that λ is the return on stocks in excess of the risk free rate standardized by σ $(\frac{\mu-r}{\sigma})$. The second part of the first term of expression (9) is the stochastic part and presents the exposure of financial wealth to the standard Brownian motion.

A.1.6 Formulating life cycle problem for martingale method

Formulating the life cycle optimization problem in the following way allows for the martingale method as the appropriate solution approach.

$$\text{Sup}_{C_t,W_t} E_t\left(\int_t^D \frac{e^{\delta(s-t)}}{1-\gamma} C_s^{1-\gamma} ds\right) \tag{107}$$

$$\text{s.t. } E\left(\int_t^D M_t C_t dt\right) \leq W_0 \tag{108}$$

Observe that M_t in the constraint is the stochastic discount factor.

A.1.7 Derivation certainty equivalent

In this subsection the derivation of the certainty equivalent presented in equation (20) is illustrated. Expression (19) is the starting point in the derivation of the certainty equivalent. Recall that this was presented in the following way:

$$E_t\left(\int_t^D e^{-\delta(s-t)} \cdot u(C_s)ds\right) = \int_t^D e^{-\delta(s-t)} \cdot u(ce)ds \tag{109}$$

The first term of equation (19) is equal to a constant (since it is an expectation) and we call this term $USCS$ (Utility Stochastic Consumption Stream). Now we are able to present an exact definition of the certainty equivalent.

$$USCS = \int_t^D e^{-\delta(s-t)} \cdot u(ce) ds \tag{110}$$

$$USCS = \int_t^D e^{-\delta(s-t)} \cdot \frac{ce^{1-\gamma}}{1-\gamma} ds \tag{111}$$

$$\frac{USCS \cdot (1-\gamma)}{\int_t^D e^{-\delta(s-t)} ds} = ce^{1-\gamma} \tag{112}$$

$$ce = \left(\frac{USCS \cdot (1-\gamma)}{\int_t^D e^{-\delta(s-t)} ds} \right)^{\frac{1}{1-\gamma}} \tag{113}$$

We can work out the integral in the denominator of the certainty equivalent in equation (113):

$$\int_t^D e^{-\delta(s-t)} ds \tag{114}$$

$$= e^{\delta t} \int_t^D e^{-\delta s} ds \tag{115}$$

$$= e^{\delta t} [-\frac{1}{\delta} e^{-\delta s}]_{s=t}^{s=D} \tag{116}$$

$$= \frac{e^{\delta t}}{\delta} \Big(-e^{-\delta D} + e^{-\delta t} \Big) \tag{117}$$

$$= \frac{1 - e^{-\delta(D-t)}}{\delta} \tag{118}$$

Substitution of equation (118) in equation (113) yields the final equation for the certainty equivalent as presented in (20). For the sake of completeness this formula is presented here once more.

$$ce = \left(\frac{USCS \cdot (1-\gamma)}{\frac{1-e^{-\delta(D-t)}}{\delta}} \right)^{\frac{1}{1-\gamma}} \tag{119}$$

Still, observe that it could be more appropriate to change the integral to a sum in a discrete setting.

A.1.8 Equivalence certainty equivalent this thesis and panel paper Netspar

It is important to relate the expression for the certainty equivalent in equation (20) to the certainty equivalent in equation (21) found in Bovenberg et al. (2007) [12]. Their starting point is as follows:

$$\frac{W_0^{1-\gamma}}{1-\gamma} g(D)^\gamma = \int_0^D e^{-\delta s} \cdot u(ce) ds \tag{120}$$

On the right hand side of expression (120), the utility from the certainty equivalent consumption stream is presented. Difference with respect to the definition used in this thesis is that this thesis sets the lower bound equal to t instead of putting it equal to zero.

The left hand side of expression (120) is at first sight different from the definition that this thesis introduced. Recall that in this thesis, the starting point is the expected utility of consumption smoothened optimally over the life cycle, which we denoted for convenience by $J(W_t, t)$ presented for the first time in equation (6):

$$J(W_t, t) = E_t \Big(\int_t^D \frac{e^{-\delta(s-t)}}{1-\rho} C_s^{1-\gamma} ds \Big) \tag{121}$$

Again, note that t should be set to zero to have similar results and for convenience this will be done. Then we obtain:

$$J(W_0, 0) = E\left(\int_0^D \frac{e^{-\delta s}}{1-\rho} C_s^{1-\gamma} ds \right) \tag{122}$$

At this point in time, many steps in between will be omitted, however from the derivations in subsection $A.1.1$ it is known that $J(W_0, 0)$ should have a specific parametric form as follows:

$$J(W_0, 0) = \frac{W_0^{1-\gamma}}{1-\gamma} g(D)^\gamma \tag{123}$$

This shows the equivalence of the left hand side of expression (19) and expression (120).

Indeed, the certainty equivalent can be written out of equation (120) and we obtain the following expression for the certainty equivalent:

$$ce = \left(W_0^{1-\gamma} g(D)^\gamma \frac{\delta}{(1 - e^{-\delta D})} \right)^{\frac{1}{1-\gamma}} \tag{124}$$

A.1.9 Parameter γ in consumption level

In this part, it will be shown to what extent the consumption level depends on the parameter γ in the absence of equity exposure ($\lambda = 0$ and $f_t = 0$). Observe that it was shown that in the CRRA setting, γ has a double interpretation. On the one hand, it represents the risk aversion (γ) directly, on the other hand it represents the EIS by ($\frac{1}{\gamma}$) indirectly.

As a starting point, the general formula for C_t, will be repeated:

$$C_t = \frac{W_t}{g(t)} \tag{125}$$

$$C_t = \frac{W_t}{\frac{1}{A}\left(e^{A(D-t)} - 1 \right)} \tag{126}$$

$$C_t = \frac{W_t}{\left(\frac{1}{\frac{(1-\gamma)r-\delta}{\gamma} + \frac{(1-\gamma)\lambda^2}{2\gamma^2}} \right) \left(e^{\left(\frac{(1-\gamma)r-\delta}{\gamma} + \frac{(1-\gamma)\lambda^2}{2\gamma^2} \right)(D-t)} - 1 \right)} \tag{127}$$

$$\tag{128}$$

Under the absence of an equity premium (i.e. $\lambda = 0$), we can rewrite C_t as follows:

$$C_t = \frac{W_t}{\left(\frac{\gamma}{(1-\gamma)r-\delta} \right) \left(e^{\left(\frac{(1-\gamma)r-\delta}{\gamma} \right)(D-t)} - 1 \right)} \tag{129}$$

$$\tag{130}$$

In addition to this, one can assume that $\delta = r$ and we can rewrite C_t as follows:

$$C_t = \frac{W_t}{\left(\frac{\gamma}{(1-\gamma)\delta-\delta}\right)\left(e^{\left(\frac{(1-\gamma)\delta-\delta}{\gamma}\right)(D-t)} - 1\right)} \tag{131}$$

$$C_t = \frac{W_t}{\left(\frac{\gamma}{\delta-\delta\gamma-\delta}\right)\left(e^{\left(\frac{\delta-\delta\gamma-\delta}{\gamma}\right)(D-t)} - 1\right)} \tag{132}$$

$$C_t = \frac{W_t}{\left(\frac{\gamma}{-\delta\gamma}\right)\left(e^{\left(\frac{-\delta\gamma}{\gamma}\right)(D-t)} - 1\right)} \tag{133}$$

$$\tag{134}$$

In the last step, it is observed that the γ will be canceled out in the consumption level in the following way:

$$C_t = \frac{W_t}{-\frac{1}{\delta}\left(e^{-\delta(D-t)} - 1\right)} \tag{135}$$

To conclude, it should be noted that in the absence of investment risk (i.e. $\lambda = 0$ and $f_t = 0$), the γ will still play a role in the initial consumption level. This will be explained by the fact that γ will play a double role in the CRRA setting. Not only it represents the risk aversion (γ), in addition it represents the EIS by ($\frac{1}{\gamma}$). If we restrict to the setting of δ=r, the parameter γ will no longer influence the initial consumption level.

A.2 Overview notation

In this subsection an overview of the notation used in this thesis is presented.

Table 36: Overview notation

Parameter	Definition
A	Constant appearing in C_t (Merton model)
AOW	Pension income from the state pension
$AOW_{premium}$	Mandatory AOW premiums
a	Speed of mean reversion interest rate (Vasicek model)
α	Parameter indicating importance consumption at time t in exogenous reference level in (Van Bilsen et al. (2014) [8])
b	Long term mean interest rate (Vasicek model)
B_t	Bond price at time t
β	Parameter indicating importance exogenous reference level at time t in exogenous reference level in (Van Bilsen et al. (2014) [8])
C_t	Consumption at time t
C_t^*	Consumption at time t, AOW in the model
ce_t	Certainty equivalent at time t
D	Deterministic date of death
δ	Time preference parameter
H_t	Human capital at time t
$\hat{H}_t$	Human capital at time t, AOW in the model
F_t	Financial wealth at time t
f_t^*	Allocation fraction in terms of financial wealth
$\bar{f}_t^*$	Allocation fraction in terms of financial wealth, AOW in the model
f_t	Allocation fraction in terms of total wealth
Φ_C	All terms of HJB equation containing C_t
Φ_f	All terms of HJB equation containing f_t
G_t	Value of the state pension at time t
$g(t)$	Time dependent function appearing in C_t (Merton model)
γ	Risk aversion
γ^G	Curvature parameter utility function above reference level (Van Bilsen et al. (2014) [8])
γ^L	Curvature parameter utility function below reference level (Van Bilsen et al. (2014) [8])
$I_u(.)$	Inverse of the derivate of the utility function
$J(W_t, t)$	Indirect utility from consumption
$J_{W_t}(W_t, t)$	First order derivative indirect utility from consumption w.r.t W_t
$J_t(W_t, t)$	First order derivative indirect utility from consumption w.r.t t
$J_{tt}(W_t, t)$	Second order derivative indirect utility from consumption w.r.t t
$J_{W_t t}(W_t, t)$	Second order derivative indirect utility from consumption w.r.t t first and W_t second
$J_{W_t W_t}(W_t, t)$	Second order derivative indirect utility from consumption w.r.t W_t
κ	Loss averse parameter (Van Bilsen et al. (2014) [8])
L	Constant labor income
L_t^{max}	Maximum amount by which consumption at time t can fall (Van Bilsen et al. (2014) [8])
λ	Risk premium standardized by volatility

Parameter	Definition
M_t	Stochastic discount factor at time t
μ	Drift term of stock price
PC_t	Pension contributions at time t
RR_t	Replacement rate at time t
p_t	Survival provabilities at time t
r	Constant interest rate
r_t	Interest rate at time t
S_t	Stock price at time t
σ	Volatility term of stock price
T	Retirement date
θ_t	exogenous reference level consumption at time t (Van Bilsen et al. (2014) [8])
$U(.)$	Utility function
$U^{'}(.)$	First order derivative utility function
$U^{"}(.)$	Second order derivative utility function
W_t	Wealth level at time t
X	Standard normal random variable
Z_t	Brownian motion value at time t

A.3 Abbreviations

In this subsection an overview of the abbreviations used in this thesis is presented.

Table 37: Overview of the hypothetical persons (for each heterogeneity characteristic and life event)

Notation (separated for the cases)	Definition
Case mismatch risk preferences	
Opt. C	Optimal contribution scheme (in the calculation of welfare effects from a suboptimal investment strategy)
Opt. (.) A	Optimal investment strategy, in line with risk preferences (.) (in the calculation of welfare effects from a suboptimal investment strategy)
Subopt. (.) A	Suboptimal investment strategy, in line with risk preferences (.) (in the calculation of welfare effects from a suboptimal investment strategy)
Opt. A	Optimal asset allocation (in the calculation of welfare effects from a suboptimal contribution strategy)
Opt. C (.)	Optimal contribution strategy, in line with risk preferences (.) (in the calculation of welfare effects from a suboptimal contribution strategy)
Subopt. C (.)	Suboptimal contribution strategy, in line with risk preferences (.) (in the calculation of welfare effects from a suboptimal contribution strategy)
Case assumed wage profile	
Wage profile 1 %	Each year, the labor income will increase by 1 %
Wage profile 1.5 %	Each year, the labor income will increase by 1.5 %
Wage profile 3-2-1-0 %	The first ten years, the labor income will increase by 3 %, the next ten years by 2 %, and so on..
Opt. C (wage profile x)	Optimal contribution scheme assuming wage profile x
Opt. A (wage profile y)	Optimal investment strategy assuming wage profile y
Case bequest received	
C_{10}	Consume the bequest within 10 years
C_{20}	Consume the bequest within 20 years
C_{30}	Consume the bequest within 30 years
C_{40}	Consume the bequest within 40 years
Opt. A	Investment strategy that will rebalance after the bequest received
Opt. A unchanged	Investment strategy that will not take into account the bequest received
100-age	Investment strategy with an allocation to stocks of 100-age
Case Housing wealth	
Acc	Indicating that the focus is on the accumulation phase
Opt. C	Optimal contribution scheme taking into account the accumulation of pension wealth via the house
Subopt. C	Suboptimal contribution scheme not taking into account the accumulation of pension wealth via the house
Opt. A	Optimal investment strategy taking into account the exposure to the (risky) housing wealth
Subopt. A	Suboptimal investment strategy not taking into account the exposure to the (risky) housing wealth
RM	Reverse mortgage
VA	Variable annuity
not use HW	Not convert the accumulated housing wealth to generate a pension income stream (i.e. leave the house as a bequest)

Notation (separated for the cases)	Definition
Case temporary unemployment	
Opt. Mert.	Consumption out of total wealth as in the standard setting
Unempl. Ben.	Consumption will be restricted to the unemployment benefit
Unempl. Ben. min PC	Consumption will be restricted to the unemployment benefit minus pension contributions
Opt. A	Investment strategy that will take into account the depletion of human capital
Old A	Investment strategy that will not take into account the depletion of human capital
Case Relative importance state pension	
Hypothetical person low	Person with a (normalized) income level of 0.5
Hypothetical person middle	Person with a (normalized) income level of 1
Hypothetical person middle +	Person with a (normalized) income level of 1.5
Hypothetical person high	Person with a (normalized) income level of 4
Hypothetical person high ++	Person with a (normalized) income level where the relative importance of the state pension is small

Table 38: Abbreviations

Acronym	Definition
ALI	Annualized Labor Income
AOW	De Algemene Ouderdomswet
BM	Brownian motion
BC	Borrowing constraint
CDC	Collective Defined Contribution
CRRA	Constant Relative Risk Aversion
DC	Defined Contribution
DB	Defined Benefit
EIS	Elasticity of Intertemporal Substitution
GBM	geometric Brownian motion
HJB	Hamilton-Jacobi-Bellman
IDC	Individual Defined Contribution
PAYGO	Pay As You GO
USCS	Utility Stochastic Consumption Stream