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Antoon Pelsser
Ahmad Salahnejhad
Ramon van den Akker

DESIGN 63

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DESIGN PAPER 63

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Frans Kooymans
Netspar

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Affiliations

Antoon Pelsser – Maastricht University
Ahmad Salahnejhad – Maastricht University
Ramon van den Akker – SNS / Tilburg University

MARKET-CONSISTENT VALUATION OF PENSION LIABILITIES

Summary

Pension funds and life insurance companies have liabilities on their books with extremely long-dated maturities that are exposed to non-hedgeable actuarial risks and also to market risks. In this paper, we show that it is computationally feasible to price pensions contracts in an incomplete market setting with time-consistent and market-consistent (TCMC) pricing operators. Furthermore, we compare the TCMC prices for life-insurance and pension contracts to alternative pricing methods that are currently used for pricing pension and life-insurance liabilities: the best estimate pricing method which is typically used for pension liabilities, and the EIOPA's risk margin method that is used under Solvency II to value life-insurance liabilities. We show that the best estimate pricing method completely ignores the uncertainty in the non-hedgeable risks. We also show that the risk margin method is a significant step in the right direction to reflect most of this uncertainty in the pricing. However, the risk margin price still ignores some uncertainties, and is therefore not fully time-consistent. For long-dated contracts this effect should not be ignored.

1. Introduction

Pension funds and life insurance companies have liabilities on their books with extremely long-dated maturities. People typically start saving for their pension at age 25 with the build-up phase lasting until the retirement age and thus extending for a period of 40 up to years. Then they enter into the decumulation phase, which typically lasts for 20 years, but this can last up to 50 years, because their life expectancy is 85 years, with the oldest people living to age 115. Hence, pension funds and life insurance companies face contractual obligations with maturities of up to 90 years. The valuation and risk management of these extremely long-dated contracts is therefore a major and challenging problem.

Due to the long maturity of these contracts, a pension fund or life-insurance company is exposed to actuarial risks (such as longevity risk) and also to market risks (such as interest rate risk and inflation risk). In particular for the actuarial risks, such as longevity risk, it is generally impossible to hedge these risks since hardly any contracts traded in financial markets that can be used to hedge these risks. In addition, these long-dated contracts have significant exposure to market risks such as interest rate and inflation risk. The pricing and risk management of pension liabilities therefore requires valuation methods that take both financial risks and non-financial risks into account.

The European Insurance and Occupational Pensions Authority (EIOPA) has recognised the importance of valuation methods that take both financial and non-financial risks into account. Under the Solvency II framework, insurance companies are required to use so-called *market-consistent* valuation methods to price their liabilities which explicitly consider non-market risks. Furthermore, EIOPA has formulated in the IORP Directive the ambition to assess the risk position of pension funds also in a market-consistent framework[1]. In particular, EIOPA has proposed that the embedded options in pension contracts should be explicitly valued using a Holistic Balance Sheet approach.

From a theoretical perspective, the problem of pricing a long-dated pension liability is a pricing problem in an *incomplete market*, since a pension contract is exposed to hedgeable financial risks, as well as non-hedgeable actuarial risks. When faced with an incomplete market, the standard machinery of risk-neutral Black-Scholes pricing breaks down, because it is no longer possible to construct a perfect replicating portfolio that hedges all risks. We therefore need to consider pricing methods that explicitly take the non-hedgeable risks into account, but that remain *market-consistent* in the sense that the prices of “pure” financial contracts are still consistent with risk-neutral pricing.

Another important requirement for a pricing operator is *time-consistency*. When calculating the price of a contract, we cannot simply price a contract at $t = 0$ and then “forget” about the contract. Instead, we should follow the contract over time and, when new information about the financial market or actuarial risks arrives, then update our pricing and the hedging position.

In this paper, we want to show that it is computationally feasible to price pension contracts in an incomplete market setting with time-consistent and market-consistent (TCMC) pricing operators. Furthermore, we compare the TCMC prices for life insurance and pension contracts against alternative pricing methods that are currently used to price pension and life insurance liabilities:

1 For more information we refer to Netspar Design Paper 04 “European supervision of pension funds: purpose, scope and design” and Design Paper 06 “The design of European supervision of pension funds”.

- the Best Estimate pricing method, which is currently the usual method to value pension liabilities;
- EIOPAs Risk Margin pricing method, which is used under Solvency II to value life-insurance liabilities.

2. Time- and Market-Consistent Valuation

As stated in the introduction, the problem of pricing a long-dated pension liability is that it involves an *incomplete market*, since a pension contract is exposed to hedgeable financial risks as well as to non-hedgeable actuarial risks. We therefore look for pricing operators that are both *market-consistent* and *time-consistent*.

Before we proceed, let us introduce some notation. We will denote the collection of market risks by the vector-valued stochastic process x_t. The collection of non-market (or actuarial) risks is denoted by the vector-valued stochastic process y_t. We will denote a general pension contract payoff at time T as a function $f(x_T, y_T)$, where the argument (x_T, y_T) denotes that the contract value at time T may depend on the whole path of the processes $\{x_t\}_{0 \leq t \leq T}$ and $\{y_t\}_{0 \leq t \leq T}$.

A pricing operator is denoted by $\Pi[t, X_T]$. This means that it assigns to any payoff (i.e. a random variable) observable at time T an amount of money ("the price") that is computable at time t given the state of the world (x_t, y_t).

Many of the pricing operators that we will discuss can be expressed in terms of conditional expectation operators. We will make use of the notation: $\mathbb{E}^{\mathbb{P}}[X_T]$, $\mathbb{E}^{\mathbb{Q}}[X_T]$ to denote an expectation of the random variable X_T under the real-world measure $\mathbb{P}$ or the risk-neutral measure $\mathbb{Q}$. In many cases, we want to condition on the market information available at time t. In that case we use the notation $\mathbb{E}[X_T|x_t]$. Similarly, the notation $\mathbb{E}[X_T|y_t]$ means that we condition on the non-market information available at time t.

A **market consistent** pricing operator has the property that for any "pure financial" payoff $f(x_T)$ we get the same value as the Black-Scholes price. Using our notation we can express this as

$\Pi[t, f(x_T)] = e^{-r(T-t)}\mathbb{E}^{\mathbb{Q}}[f(x_T)|x_t]$.

A **time consistent** pricing operator has the property that the price at time t for any payoff X_T that is held to maturity T is equal to the price at time t of the same position that is held until time s and then sold at the then-current s-price. Using our notation, we can express this as: for all $t \leq s \leq T$ we have
$\Pi[t, X_T] = \Pi\big[t, \Pi[s, X_T]\big]$.

The risk-neutral pricing operator of (Black and Scholes, 1973) given by $\Pi^{\text{BS}}[t, f(x_T)] = e^{-r(T-t)}\mathbb{E}^{\mathbb{Q}}[f(x_T)|x_t]$ is an example of a time-consistent and market-consistent pricing operator for a complete market. The time consistency of the Black-Scholes price arises from the "tower property" of the conditional expectation operator $\mathbb{E}[f(x_T)|x_t]$. The market-consistency arises from the risk-neutral probability measure $\mathbb{Q}$.

In the remainder of this section we will discuss two types of pricing operators used by practitioners: Best Estimate valuation and valuation with a Risk Margin. We will also introduce the time consistent and market consistent (TCMC) pricing operator of Pelsser and Stadje (2014).

2.1. Best Estimate Valuation

A pricing operator widely used to price pension liabilities is the *Best Estimate* pricing operator. This pricing operator is constructed as follows: the actuarial risks (i.e. the non-market risks) are projected with the best possible model to obtain projected cash flows for the contract. These projected cash flows are then priced using the risk-neutral Black-Scholes pricing operator. For fixed cash flows this boils down to discounting the projected cash flows with the risk-free term-structure of interest rates observed in the market.

Using our notation, we can define the best estimate pricing operator as

$$\Pi^{\text{BE}}[t, f(x_T, y_T)] = e^{-r(T-t)}\mathbb{E}^{\mathbb{Q}}\left[f\left(x_T, \mathbb{E}^{\mathbb{P}}[y_T|y_t]\right) \;\middle|\; x_t\right]. \quad (1)$$

Note that we have nested two expectation operators inside each other. The inner expectation operator performs the projection of actuarial risks under the real-world measure $\mathbb{P}$. The result of the inner expectation operation is a set of cash flows that depend only on the market-risks x_T. In the outer expectation, these market-risks are then priced with the market consistent Black-Scholes pricing operator $\Pi^{\text{BS}}[]$.

The advantage of the best estimate pricing operator is that it is easy to evaluate. We replace the uncertain actuarial random variable y_T with its best estimate projection $\mathbb{E}^{\mathbb{P}}[y_T|y_t]$. Such a replacement is routinely performed by actuaries when they use a mortality table to project cash flows, instead of a stochastic mortality process to project stochastic cash flows.

The disadvantage of the best estimate approach is that the inherent uncertainty of the actuarial risks y_T is swept under the rug whenever the random variable y_T is replaced by its best estimate projection $\mathbb{E}^{\mathbb{P}}[y_T|y_t]$. Hence, the uncertainty arising from the non-market risks y_T is not reflected by the best estimate pricing operator.

2.2. Valuation with a Risk Margin

In the design of the Solvency II supervisory framework, the need to reflect the uncertainty arising from non-market risks is explicitly incorporated in the pricing methodology. Under Solvency II, the pricing of liabilities consists of two components: a best estimate and a risk margin.

The best estimate component is identical to the pricing operator $\Pi^{BE}[]$ discussed in the previous sub-section.

The risk margin component was introduced as an adjustment to the best estimate price to cover the uncertainty arising from the non-hedgeable risks in the liabilities. The calculation of the risk margin is based on a cost-of-capital argument.

The only way that non-hedgeable risks can be absorbed is by maintaining a buffer capital in the balance-sheet. The buffer capital has been provided by external stakeholders (e.g. the shareholders in the case of a publicly listed insurance company). The capital providers know that they are investing risk capital they are willing to provide such capital because they receive compensation in the form of a higher return than the risk-free rate. The return in excess of the risk-free rate is called the *cost-of-capital*.

Based on this argument, we can quantify the risk margin as the NPV of all cost-of-capital payments that need to be made to the capital-providers during the life of the liability. To make the calculation explicit, we need to determine the size of the buffer capital and the cost-of-capital percentage. EIOPA has set the following rules:

- The buffer capital is calculated as the one-year Value-at-Risk (for the non-hedgeable risks) with a confidence level of 99.5%.

- The cost-of-capital is set at 6%.

The Value-at-Risk (VaR) is the 99.5% worst-case quantile of the distribution of the non-hedgeable risks. For a normal probability distribution, the one-year 99.5% VaR is equal to 2.58 times the per-annum standard-deviation.

We can express the risk margin pricing operator as:

$$\Pi^{\mathrm{RM}}[t, f(x_T, y_T)] = \Pi^{\mathrm{BE}}[t, f(x_T, y_T)] + \sum_{s=t+1}^{T} e^{-r(s-t)}\gamma \mathrm{VaR}_{0.995}\left[\Pi^{\mathrm{BE}}[s, f_T] \,\middle|\, \mathbb{E}^{\mathbb{P}}[y_{s-1}|y_t]\right], \quad (2)$$

where the parameter γ denotes the cost-of-capital percentage. The summation term can be interpreted as follows. The summation takes annual time-steps from $t+1$ until the maturity T of the liability. For each time-step s, we consider the one-year VaR along the best estimate path of y, hence the VaR-operator is conditioned on the value $\mathbb{E}^{\mathbb{P}}[y_{s-1}|y_t]$. Taking this projected value of y_{s-1} as a starting-point, we then consider the impact of a 99.5% worst-case shock in the non-market risks y_s on the best estimate price $\Pi^{\mathrm{BE}}[s, f(x_T, y_T)]$ at time s. This is the projected buffer capital for time s. Over this buffer capital we have to pay the cost-of-capital γ to the capital-providers. The discounted sum of all these cost-of-capital payments is the risk margin.

Although the EIOPA risk margin pricing operator is calculated in a multi-period setting, it is *not* a time consistent pricing-operator. The risk margin pricing operator does take the uncertainty arising from the non-market risks y_T on the best estimate price into account. However, there is a "second-order" effect: the uncertainty arising from the non-market risks y_T on the future buffer capitals. What EIOPA's risk margin pricing operator therefore ignores is the "capital-on-capital" effect that a fully time consistent operator would take into account.

2.3. Time- and Market Consistent Valuation

The question of how to build a pricing operator that is fully market consistent and time consistent is addressed in the paper by Pelsser and Stadje (2014). They extend the "backward induction" method proposed by Jobert and Rogers (2008) for creating time consistent pricing operators.

The backward induction method can be explained as follows. If we have a liability with a maturity T, then one year before the maturity (at $T-1$) we have a one-year contract. For this one-year contract, the EIOPA risk margin pricing operator will yield the correct price. After all, there is no "capital-on-capital" problem as the contract expires one year later at time T.

So, for each state of the world (x_{T-1}, y_{T-1}) we can compute the price $\Pi[T-1, f(x_T, y_T)]$. For the one-step case, the EIOPA risk margin pricing operator simplifies to

$$\Pi^{\text{RM1}}[T-1, f(x_T, y_T)] = \Pi^{\text{BE}}[T-1, f(x_T, y_T)] + \gamma e^{-r} \text{VaR}_{0.995}\left[f(x_T, y_T) \mid y_{T-1}\right]. \quad (3)$$

Note that the pricing operator $\Pi^{\text{RM1}}[]$ prices the market risks in a market consistent way, due to the embedded $\mathbb{Q}$-expectation in the first $\Pi^{\text{BE}}[]$ term. The price of the non-market risks is reflected in the second term.

To emphasise the dependence on the state of the world, we will denote the price at time $T-1$ by $\pi(T-1, x_{T-1}, y_{T-1})$. We could sell the liability at time $T-1$ for the price $\pi(T-1, x_{T-1}, y_{T-1})$ in the state of the world (x_{T-1}, y_{T-1}). Hence, we can also interpret the price $\pi(T-1, x_{T-1}, y_{T-1})$ as a new liability with maturity $T-1$.

Therefore, we can take another one-year step, where we compute the one-step price at time $T-2$ of the liability $\pi(T-1, x_{T-1}, y_{T-1})$. We continue this procedure until we reach $t=0$.

The contribution of Pelsser and Stadje (2014) is that they prove that every time consistent and market consistent pricing operator can be obtained by applying the backward induction technique to a “simple” one-period pricing operator. For further details, we refer to Pelsser and Stadje (2014) and also to Pelsser and Salahnejhad (2016).

3. Valuation of Unit-linked Contract

In this section we introduce a simplified example, in which we explicitly calculate the three pricing operators that we have introduced in the previous section. The example is that of a unit-linked contract for an insurance company, or equivalently a stylised DC pension contract without any form of guarantee or indexation.

We are going the model the financial risk as a stock-price S_t. Please note, that we do not use x_t to denote the financial market process here, but instead the more familiar notation S_t. We assume that the stock-price S_t follows a Geometric Brownian Motion under the real-world measure $\mathbb{P}$:

$$dS_t = \mu S_t\, dt + \sigma S_t\, dW_t^S, \tag{4}$$

where the growth rate μ of the stock and the volatility σ are constants. We also assume that the risk-free interest-rate r is constant. Under these assumptions, we find that the stock-price process under the risk-neutral measure $\mathbb{Q}$ is given by

$$dS_t = rS_t\, dt + \sigma S_t\, dW_t^{S\mathbb{Q}}. \tag{5}$$

The actuarial risk is modelled as follows. We assume that y_t denotes the number of participants alive at time t. To obtain a tractable example, we model the number of surviving participants as a Geometric Brownian Motion (GBM) under the real-world measure $\mathbb{P}$:

$$dy_t = -ay_t\, dt + by_t\, dW_t^y, \tag{6}$$

where the mortality rate a of the survivors and the volatility b are constants.[2] We also assume that the financial market process S_t and the actuarial process y_t are independent processes.

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2 Note, that this way of modelling the number of survivors is not very realistic,

We now introduce the payoff of our contract. At the maturity date T each surviving participant receives the value of the stock-price S_T. Hence, the total liability at time T is given by the formula:

$$f(S_T, y_T) = S_T y_T, \quad (7)$$

which is the stock-price times the number of surviving participants.

3.1. Best Estimate Valuation

First, we consider the price of this liability at time $t = 0$ using the best estimate price operator, defined in equation (1).

For the best estimate valuation, we first project the number of survivors as $\mathbb{E}^{\mathbb{P}}[y_T] = y_0 e^{-aT}$. The projected cash flow at time T is therefore: $S_T\ y_0 e^{-aT}$. This projected cash flow is then calculated in a market consistent way under the risk-neutral measure $\mathbb{Q}$ as

$$\Pi^{\text{BE}}[0, S_T y_T] = e^{-rT}\mathbb{E}^{\mathbb{Q}}[S_T] y_0 e^{-aT} = S_0 y_0 e^{-aT}. \quad (8)$$

Note, that this best estimate price is the same when we would pay S_T to the deterministic number of survivors $y_0 e^{-aT}$. Hence, the uncertainty in the number of survivors y_T at time T is not reflected by the best estimate pricing operator.

3.2. Valuation with a Risk Margin

Second, we consider the price of this liability at time $t = 0$ using the risk margin price operator, defined in equation (2).

To compute the risk margin, we need to consider the one-year Value-at-Risk for each year t between 0 and T. Since we have assumed the number of survivors to follow a GBM, we know that

because a GBM process can have paths that increase over time, even when the mortality trend is negative.

the one-year probability distribution of y_t given y_{t-1} is a log-normal distribution with drift term $-a$ and volatility b. Therefore, the 99.5% worst-case value of y_t is given by $y_t^{\text{WC}} = y_{t-1}e^{-a+2.58b}$. Note that, for our particular contract, the worst-case scenario is that *more* people than expected survive. For this reason we take the upward shock $+2.58b$.

We now have to compute the Value-at-Risk for each year along the best estimate path. The best estimate value for y_{t-1} is equal to $\mathbb{E}^{\mathbb{P}}[y_{t-1}] = y_0 e^{-a(t-1)}$. We then apply the worst-case one-year shock: $y_t^{\text{WC}} = y_0 e^{-a(t-1)} e^{-a+2.58b} = y_0 e^{-at} e^{2.58b}$. Using this shocked value at time t, we then project the number of survivors at time T, this leads to $y_0 e^{-at} e^{2.58b} e^{-a(T-t)} = y_0 e^{-aT} e^{2.58b}$. The Value-at-Risk for time t is the difference between the best estimate price $S_0 e^{rt} y_0 e^{-aT}$ and the price for the shocked projection. Hence, we can express the VaR for year t as $S_0 e^{rt} y_0 e^{-aT}(e^{2.58b} - 1)$.

When we substitute these VaR expressions for each year t into the risk margin pricing formula, we obtain

$$\Pi^{\text{RM}}[0, S_T y_T] = S_0 y_0 e^{-aT} \left(1 + \gamma T(e^{2.58b} - 1)\right). \qquad (9)$$

In the summation of the VaR-terms we encounter T times the same term $S_0 y_0 e^{-aT}(e^{2.58b} - 1)$. Therefore the risk margin term simplifies to $\gamma T \, S_0 y_0 e^{-aT}(e^{2.58b} - 1)$.

3.3. Time- and Market Consistent Valuation

Finally, we consider the price of this liability at time $t = 0$ using the TCMC price operator.

When we apply the one-year risk margin pricing formula at $T-1$ we obtain the price $\Pi^{\text{RM1}}[T-1, S_T y_T] = S_{T-1} y_{T-1} e^{-a} \left(1 + \gamma(e^{2.58b} - 1)\right)$, which is the one-year best estimate price times the factor $\left(1 + \gamma(e^{2.58b} - 1)\right)$.

Note that, for this particular example, the factor is independent from S_{T-1} and y_{T-1}. Hence, if we apply the one-year risk margin pricing formula once more to obtain the price at time $T-2$, then we get the best estimate price times the factor $\left(1+\gamma(e^{2.58b}-1)\right)^2$.

After T backward induction steps, we find at time $t=0$ the TCMC price of

$$\Pi^{\text{TCMC}}[0, S_T y_T] = S_0 y_0 e^{-aT} \left(1+\gamma(e^{2.58b}-1)\right)^T . \qquad (10)$$

When we compare the TCMC price to EIOPA's risk margin price, we clearly see the "capital-on-capital" effect that is missing from the risk margin price, and that is included in the TCMC price.

The risk margin price is equal to the best estimate price times the factor $\left(1+\gamma T(e^{2.58b}-1)\right)$, whereas the TCMC price is equal to the best estimate price times the factor $\left(1+\gamma(e^{2.58b}-1)\right)^T$. For $T=1$ both factors are the same, but for $T>1$ the TCMC factor is larger than the risk margin factor, and the gap widens for larger values of T.

3.4. Numerical Illustration

In this subsection, we compare the different prices for specific values of the model-parameters. In our example we use for the stock-price process the values $S_0=1$, $r=4\%$ and $\sigma=16\%$. For the actuarial risk process we take $y_0=1000$, $a=1\%$ and $b=7\%$.

We report the three different prices for a range of maturities from $T=1$ to $T=30$. The contract values under the three different pricing operators are shown in Figure 1. The best estimate prices are labeled as "BestEst". EIOPA's risk margin prices as "EIOPA", and the TCMC-price as "TC-MC".

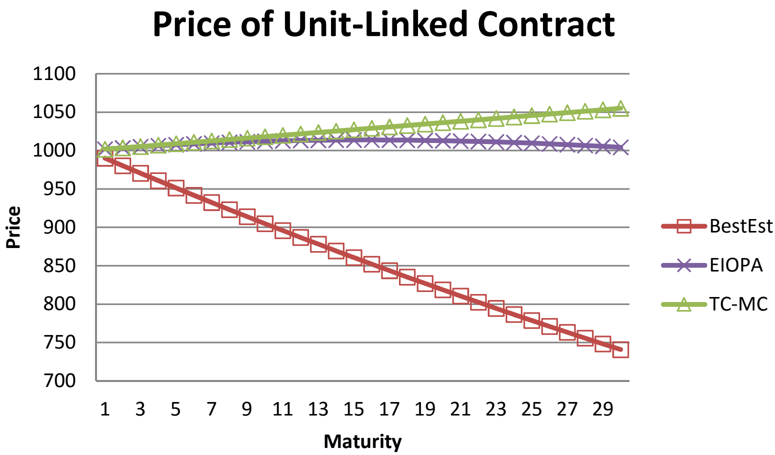

Figure 1: Comparison of Prices for a Unit-Linked Contract

We see that the best estimate prices simply reflect the expected number of survivors with a 1% mortality rate, ranging from 990 for $T = 1$ to 741 for $T = 30$. The uncertainty surrounding this projected number of survivors is not reflected in the best estimate price.

The risk margin prices do reflect the uncertainty surrounding the projected number of survivors, hence the EIOPA prices are higher than the best estimate prices. For longer-dated contracts, the unhedgeable uncertainty becomes larger; therefore the gap with the best estimate prices becomes consistently larger.

The TCMC prices reflect the "capital-on-capital" effect, and are therefore still a bit higher than the EIOPA prices. We also see that the risk margin prices do reflect a significant adjustment in the right direction. Hence, the remaining gap to the TCMC price is relatively small. However, for longer-dated contracts the capital-on-capital effect becomes relatively more important.

4. Time- and Market Consistent Pension Valuation

The example from the previous section allowed us to calculate all prices explicitly. But the payout and the modelling of the mortality process were not very realistic. In this section we demonstrate that we can compute our pricing operators also for a realistic type of contract. The purpose of this section is twofold. First, we wish to show that the computation of a time consistent and market consistent pricing operator is feasible for a realistic contract. Second, we wish to compare the prices of the three pricing operators for a realistic contract.

4.1. Indexation Mechanism

We focus our attention on a path-dependent contract with profit-sharing (i.e. indexation) introduced by Grosen and Jorgensen (2000). This contract is similar to a typical Dutch pension contract with conditional indexation.

The Grosen and Jorgensen (2000) indexation mechanism works as follows. At time $t = 0$ a participant obtains one unit of the contract with nominal value P_0. The pension fund invests the full amount in the financial market. Let S_t be the market value of the invested amount in the financial market and let P_t be the nominal pension claim in year t.

At the beginning of each year t, the nominal pension claim of each participant grows by the following formula:

$$P_t = P_{t-1}\left(1 + max\left\{r_G, \alpha\left(\frac{S_{t-1}}{P_{t-1}} - (1+\beta)\right)\right\}\right). \quad (11)$$

This formula can be interpreted as follows. The ratio S_{t-1}/P_{t-1} is the ratio of the assets over the nominal pension claims, hence this ratio can be interpreted as the nominal funding ratio of the pension fund. Each year, the ratio S_{t-1}/P_{t-1} is compared to a target funding ratio $(1+\beta)$. Also each year, a fraction α of the excess funding ratio $S_{t-1}/P_{t-1} - (1+\beta)$ is credited to the participants. However, if the credited amount falls below the minimum guarantee r_G, then each participant receives the minimum guarantee. This crediting mechanism is comparable to the conditional indexation mechanism used by Dutch pension funds.

4.2. Lee-Carter Model

To model the evolution of the survival probabilities in a realistic way, we use the model introduced by Lee and Carter (1992). In this model, the force-of-mortality $m_{k,t}$ for age k at time t is given by

$$\ln m_{k,t} = \alpha_k + \beta_k \kappa_t \tag{12}$$

where κ_t is the stochastic mortality trend, α_k is the average age-specific mortality and β_k is the age-specific sensitivity of the mortality to change of κ_t. The stochastic process κ_t is a latent process to model the mortality trend specified by

$$d\kappa_t = \mu_\kappa dt + \sigma_\kappa dW_t^\kappa, \tag{13}$$

where W_t^κ a standard Brownian Motion under the real-world measure $\mathbb{P}$.

To implement a Lee-Carter model, we must estimate the parameter vectors α_k and β_k for all ages k, and the parameters $\kappa_0, \mu_\kappa, \sigma_\kappa$ from historical mortality data.

4.3. Contract Payoff

We assume that the financial market process S_t follows the GBM process defined in (5) under the risk-neutral measure $\mathbb{Q}$.

Figure 2 illustrates five simulated paths of the market value of invested assets S_t and the policy reserve P_t for a 20-year pension

Figure 2: Simulation of the Asset Value S_t and Policy Reserve P_t of the Pension Contract. Parameter set: $S_0 = P_0 = 100$, $r = 4\%$, $\sigma_S = 0.15$, $r_G = 2\%$.

contract with guaranteed interest rate $r_G = 2\%$. We see that the Grosen-Jorgensen crediting mechanism smoothes the development of the policy reserve. We add the actuarial risk of mortality/longevity to the above financial setting to get a realistic pension payoff at time T. This requires that the policyholder is alive to get the policy reserve P_T.

When we use the Lee-Carter to model the force of mortality, the number of survivors N_T at time T is given by a Poisson process with stochastic intensity $-m_{k,t}$ at time t, where the jump-process is assumed to be independent of the Brownian Motions W^S and W^κ.

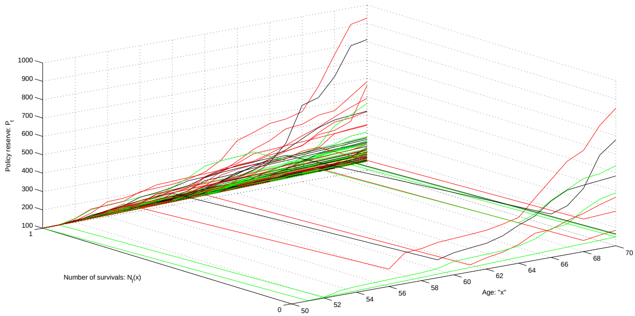

Figure 3: Simulation of the Policy reserve P_t and Survival event $\mathbb{1}_{\{T_x > T\}}$ for an individual of age 50 for maturity $T = 20$. Other parameters: $P_0 = 100$, $r = 4\%$, $\sigma_S = 0.15$, $r_G = 2\%$.

The final contract payoff will be

$$f(S_T, \kappa_T) = P_T(S_T)N_T(\kappa_T), \tag{14}$$

where use S_T to denote the entire history of the financial risk process, and κ_T to denote the entire history of the actuarial risk process.

Figure 3 shows a simulation of the policy reserve and the mortality events for an individual with age $k = 50$, up to retirement age 70 where $T = 20$ and with guaranteed interest rate $r_G = 2\%$. The simulation is performed for 100 scenarios in some of which the death event shifts the evolution of P_t from the left side of the graph to the right side. In each of those cases the payoff at age 70 is zero.

4.4. Numerical Computation

For a realistic payoff, we can no longer use analytical pricing formulas, as in Section 3; instead we have to use numerical approximation methods. To implement the calculations, we use Monte-Carlo simulation to generate paths for the financial and actuarial risk drivers. To simulate paths for the financial risk process, we use equation (5). To simulate paths for the mortality risk driver we use equation (13) combined with a simulation for the Poisson process N_t. Along each path, we can move forward in time and update P_t using the Grosen-Jorgensen crediting formula (11). Then, for each path, we determine the payoff $f(S_T, \kappa_T)$ at time T using the payoff formula (14). We now have for each simulated path the contract payoff at time T.

The next step is to evaluate our pricing operators. For the best estimate pricing operator, we can simply take the discounted average of all payoffs at time T to compute the Monte-Carlo approximation of the price.

For the risk margin and the TCMC pricing operators, we need to evaluate numerically the conditional expectation operators at each annual time-point t. An efficient method to perform these computations is the Least Squares Monte Carlo (LSMC) method. LSMC was introduced by Carriere (1996) and Longstaff and Schwartz (2001) to price American-style options. The LSMC method uses regressions across all simulated paths to estimate the conditional expectations at all the time-points. The conditional expectation at time t of any general payoff $\pi(t+1, S_{t+1}, \kappa_{t+1})$ at time $t+1$ can be approximated by a series of basis functions in S_t and κ_t as,

$$\mathbb{E}[\pi(t+1, S_{t+1}, \kappa_{t+1}) \mid S_t, \kappa_t] = \sum_{i,j=0}^{K} a_{tij}\, e_i(\kappa_t) e_j(S_t) \quad (15)$$

where we can choose different types of the basis functions such as $e_i(z) = z^i$. The coefficients a_{tij} are then estimated for time t by regressing across all simulated paths the dependent variable $\pi(t+1, S_{t+1}, \kappa_{t+1})$ onto the explanatory variables $e_i(\kappa_t)e_j(S_t)$.[3] The numerical estimate for the conditional expectation is then obtained by evaluating the right-hand side of (15) with the estimated coefficients $\widehat{a}_{tij}$.

To evaluate the risk margin and the TCMC pricing operator, we also need to evaluate numerically the conditional Value-at-Risk at different points in time t. For our calculations, we approximate[4] the VaR as 2.58 times the conditional standard deviation. The conditional standard deviation is the square-root of the conditional variance. The conditional variance can be computed from the conditional expectation

$$\mathbb{E}\left[\pi(t+1, S_{t+1}, \kappa_{t+1})^2 \mid S_t, \kappa_t\right] = \sum_{i,j=0}^{K} b_{tij}\, e_i(\kappa_t)e_j(S_t), \tag{16}$$

where the coefficients b_{tij} for time t can be estimated from a cross-sectional regression.

4.5. Comparison of the Different Pricing Methods

In this subsection, we compare the different prices for the pension contract. We consider a cohort of 1000 participants aged $k = 40$, and compute the value of the pension contract for a range of maturities from $T = 1$ to $T = 30$. The Grosen-Jorgensen crediting parameters are $\alpha = 0.50$, $\beta = 0.15$, hence the target funding ratio is $1 + \beta = 1.15$, and the minimum guarantee is set

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3 Due to the path-dependency of the contract, we can also add additional explanatory variables, such as $P_t(S_t)$ or $N_t(\kappa_t)$, that capture the history of the processes S_t and κ_t.

4 In principle, it is possible to estimate the VaR directly from the cross-sectional sample by using quantile regressions.

at $r_G = 2\%$. The financial market process has parameters $S_0 = 100$, $r = 4\%$, $\sigma = 15\%$. The parameters of the Lee-Carter model are estimated from Dutch mortality data, which are available from `www.mortality.org`.

We compare the prices for the pension contract using the three different pricing operators:

- the best estimate price;
- EIOPA's risk margin price;
- the Time Consistent and Market Consistent (TCMC) price.

The contract values under the three different pricing operators are shown in Figure 4. The best estimate prices are labeled as "Expected". EIOPA's risk margin prices as "EIOPA" and the TCMC-price as "Time Consistent".

We see that the best estimate prices simply reflect the expected number of survivors in the Lee-Carter model, ranging from €97 500 for payoff at age 41 to €91 700 for payoff at age 70. The uncertainty surrounding the projected number of survivors is not reflected in the best estimate price.

The risk margin prices do reflect the uncertainty surrounding the projected number of survivors, hence the EIOPA prices are higher than the best estimate prices, ranging from €97 500 for payoff at age 41 to €106 400 for payoff at age 70. For longer-dated contracts, the unhedgeable uncertainty becomes larger, and therefore the gap with the best estimate prices becomes ever larger. At payoff age 70 the EIOPA price is 16% higher than the EIOPA price.

The TCMC prices reflect the "capital-on-capital" effect, and are therefore still somewhat higher than the EIOPA prices. We also see

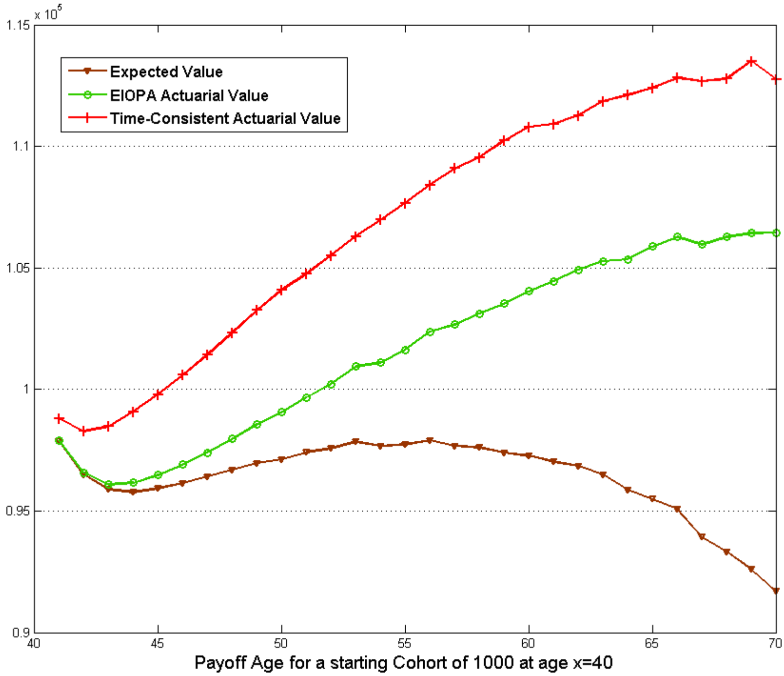

Figure 4: Comparison of Prices for a Pension Contract.

that the risk margin prices do reflect a significant adjustment in the right direction. Hence, the remaining gap to the TCMC price is relatively small. However, for longer-dated contracts the capital-on-capital effect becomes relatively more important. At payoff age 70 the TCMC price is €112 700, which is 6% higher than the EIOPA price.

5. Conclusions

The pricing of long-dated pension liabilities is an significant problem. In order to reflect both market and non-markets risks in the pricing operator in an arbitrage-free way, we need pricing operators that are both time consistent and market consistent (TCMC). However, current pricing methods for pension liabilities do not properly reflect the non-market risks.

In this paper, we have demonstrated that it is computationally feasible to price pension contracts in an incomplete market setting with TCMC pricing operators. Furthermore, we have compared the TCMC prices for life-insurance and pension contracts to alternative pricing methods that are currently used for pricing pension and life-insurance liabilities:

- the Best Estimate pricing method, which is used by most pension funds;
- EIOPA's Risk Margin pricing method, which is used for the pricing of life-insurance liabilities.

Our main findings can be summarised as follows:

- Best estimate prices simply reflect the expected value of the actuarial risk drivers. The uncertainty surrounding this projection is not incorporated in the best estimate price.
- Risk margin prices do reflect the uncertainty surrounding the projected actuarial risk drivers, hence EIOPA prices are higher than best estimate prices. However, the uncertainty in the projected capital requirements is not incorporated in the risk margin price. This implies that risk margin prices are not fully time consistent.

- TCMC prices do reflect the “capital-on-capital” effect, and are therefore fully time- and market consistent. Hence, TCMC prices are higher than risk margin prices.

- Risk margin prices do represent a significant adjustment in the right direction over the best estimate price, as the remaining gap to the TCMC price is relatively small. However, for longer-dated contracts the capital-on-capital effect becomes relatively more important.

- Using the Least-Squares Monte-Carlo method it is computationally feasible to compute TCMC prices for realistic pension and life-insurance contracts.

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Market-consistent valuation of pension liabilities

Due to the long maturity of its contracts, a pension fund or life-insurance company is exposed to actuarial risks such as longevity risk and also to market risks such as interest rate risk and inflation risk. The insurance and pensions regulator in Europe (EIOPA) has also recognized the importance of valuation methods that take financial risks and non-financial risks into account. In this paper, Antoon Pelsser, Ahmad Salahnejhad (both UM) and Ramon van den Akker (SNS/TiU) want to show that it is computationally feasible to price pensions contracts in an incomplete market setting with time-consistent and market-consistent (TCMC) pricing operators.

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P.O. Box 90153
5000 LE Tilburg
the Netherlands
Phone 013 466 2109
E-mail info@netspar.nl
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