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The ultimate forward rate: time for a step backwards?

Michel Vellekoop

OPINION 68

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OPINION PAPER 68

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Forward, not permanent,
sweet, not lasting.

The perfume and suppliance of a minute.
No more.

Hamlet, William Shakespeare

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THE ULTIMATE FORWARD RATE: TIME FOR A STEP BACKWARDS?

1. Introduction

Every financial institution that wishes to use a discount curve to estimate the current value of future cash flows has to decide how to construct such a curve from information that can be found in the fixed income markets. Since cash flows are always tied to a limited number of dates, the definition of a discount curve will require interpolation. If the financial institution needs to assess the value of cash flows for maturities that exceed the maturities of observable market instruments, or when the market information for these longer maturities is known to be unreliable, it is also necessary to extrapolate the curve. This latter problem naturally arises for many pension funds and insurance companies, since by the very nature of their business, they have liabilities that stretch out much further into the future than those of the average financial institution. At the same time, regulatory frameworks such as Solvency II for European insurers stress the importance of market-consistent pricing of liabilities. This raises the question of how one can use available market information in a way that is as transparent and objective as possible, while recognizing that it is unavoidable that subjective choices will have to be made for maturities in between or beyond those that can be directly related to available market instruments.

Interpolation problems for yield curves have been well studied in the scientific literature; see Nelson & Siegel [1987] and Svensson [1994] for important examples of possible methods and the book by Campbell, Lo, & MacKinlay [1996] for an extensive overview of term structure estimation problems. There is an abundance of studies that deal with *interpolation*, but the *extrapolation* problem for term structures has received considerably less attention until recently. The introduction of Solvency II has changed this, because the term structures that it prescribes for solvency capital calculations by insurance companies incorporate the assumption that forward interest rates must converge to an a priori specified value for a certain specified maturity. This concept of an *ultimate forward rate* (UFR) and the accompanying inter- and extrapolation methods were first introduced by EIOPA, and many modifications have been suggested since their original proposal. But the fundamental idea that forward interest rates for long maturities are relatively stable over time is a common assumption in the original and all the later methods.

However, there is no empirical evidence for such an assumption and no theoretical justification either. In fact, a quick glance at the data shows that forward rates for the long maturities that are relevant for insurance companies or pension funds have not been constant. In this NEA paper, it will therefore be argued that term structure extrapolation methods that artificially reduce the observed volatility in long-term rates should be avoided. This is relevant for insurance companies, but equally important for pension funds, where interest rate risks are shared collectively by participants from different generations. Model assumptions that lead to more optimistic or pessimistic scenarios for long-term fixed income returns can have a substantial influence on the solvability ratio of an insurance company or the funding ratio of a pension fund. Management decisions such as the amount of dividends paid to a company's shareholders or the level of indexation used for a fund's retired participants are based on such ratios. Moreover, the sensitivities with respect to long-term forward rate modifications will differ substantially across the generations that participate in a pension fund. This will be an important argument in the conclusion formulated at the end of this paper, which states that extrapolation of interest rates should not be based on a priori assumptions that cannot be justified theoretically or empirically, since this creates the possibility of wealth redistribution among stakeholders in a manner that is not objective or transparent.

In this NEA paper, we first discuss discount curve modelling in general in the next section. Then, the different existing methods that include an UFR assumption are introduced in Section 3, and their properties are discussed. Section 4 gives an overview of the main advantages and disadvantages of these methods, which leads to the presentation of a conclusion in the last section.

2. Discount curve modelling

Characterization of interpolation methods

Traditional interpolation methods for discount curves can be classified in terms of the coordinates used to characterize the discount curve. These coordinates can be the discount rates themselves, the spot rates (i.e., zero coupon yields), or the forward rates.[1] Interpolation techniques then assume that these rates can be described by a functional form based on a handful of parameters (which is the case for the well-known Nelson-Siegel approach, for example) or by a spline function, which may involve even more parameters than the number of data points that have to be fitted. Usually, splines allow a perfect fit for prices of fixed income products observed in the markets, while the extra parameters can be used to control the smoothness of the discount curve. The term structure that is found by applying a Nelson-Siegel procedure will also be infinitely smooth,[2] but since it involves only a few parameters, market prices cannot be fitted exactly.

The unavoidable tradeoff

The tradeoff between the quality of the fit and the smoothness of the interpolating function characterizes all interpolation problems. One can always use a piecewise linear function to connect a given set of data points for different maturities, but the result will generally not be smooth. On the other hand, one can fit any polynomial (e.g., a straight line) through all data points if the resulting approximation error is not restricted and end up with a

1 When forward rates are mentioned in this paper, it refers to 1-year forward rates (i.e., the rates over the smallest time interval considered). All other rates can then be written in terms of these "smallest elements" in term structure modelling.

2 In the sense that its n-th order derivative function exists for all integers n.

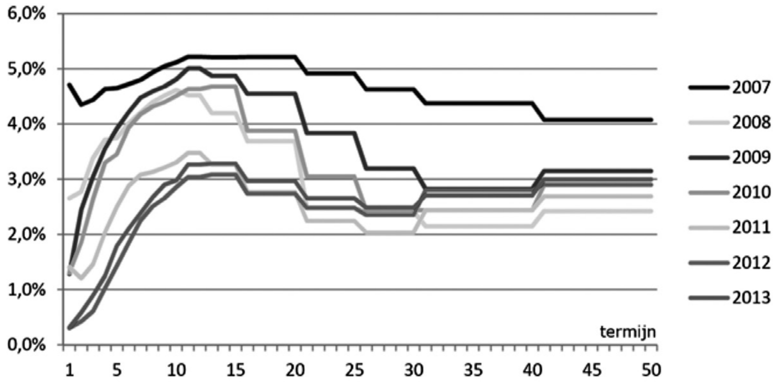
very smooth representation of the dataset. The choice of an interpolation method for term structures thus requires a subjective choice for the coordinates that are used to perform the interpolation, as well as an assumption about the smoothness over the different maturities that one expects in those coordinates. Spot rates can be interpreted as averages over different forward rates and will therefore be smoother than forward rates. Discount rates and spot rates can be transformed directly into one another and their smoothness (in terms of the order of differentiability) will be the same.

An earlier method

A common choice, used for example in the past by the Dutch Central Bank (DNB), is the assumption that forward rates are constant between the maturities for which data are available. This implies that forward rates jump to a new value at such maturities, that the logarithm of the discount curve is piecewise linear, and that spot rates are continuous.

A natural way to extrapolate the forward curve, and hence the whole term structure, simply assumes that the last forward rate that is observed continues to hold after that maturity. Not only is this rule easy to implement but it is also rather flexible, in the sense that it can still be used if the last maturity for which reliable market prices can be found in fixed income markets changes. This maturity may have to be chosen at a lower value if markets are in distress, but that simply means that the forward rate is then extrapolated from earlier maturities. Those who claim that there exists an ultimate forward rate which is constant over time and that forward rates for the highest observable maturities will always be close to this constant value, can rest assured. This extrapolation method then simply defines that value to hold

Forward rates per year, ultimo and June 2013



Source: Report of the *Commissie UFR*, version 10-11-2013

beyond the observable maturities. There was therefore no need to switch from the existing DNB extrapolation method to a new one to implement the belief in a constant ultimate forward rate. However, forward rates for high maturities do not seem to have remained close to 4.2% since the financial crisis of 2008, as shown in the figure above.

Moreover, the mere existence of an asymptotic value in forward rates is not the main issue when considering the use of term structures by insurance companies, pension funds, or regulators. It is very well possible, but hardly relevant, that forward rates will start to move towards a certain value at maturities far beyond the end of the current century. The most important aspect of the UFR methodology proposed by EIOPA is not so much the *ultimate* value of the forward rate, but the way it changes the discount curve for the relevant maturities between 20 and 50 years. To illustrate this, the original specifications of the UFR method defined by EIOPA in 2010 are given in the next section, along with the various modifications that have been proposed since.

3. Discount curve modelling with asymptotic constraints the EIOPA proposal

The original document that contains the EIOPA proposal for the introduction of the ultimate forward rate dates from 2010.[3] It states that "*the overall aim is to construct a stable and robust extrapolated yield curve which reflects current market conditions and at the same time embodies economical views on how unobservable long-term rates are expected to behave. Macroeconomic extrapolation techniques assume a long-term equilibrium interest rate. A transition of observed interest rates of short-term maturities to the assessed equilibrium interest rate of long-term maturities takes place within a certain maturity spectrum.*" The value of the ultimate forward rate was set at 4.2% for a range of currencies which included the euro and the dollar. It was defined as the sum of a standard inflation rate of 2.0%, an expected real rate of interest equal to 2.2%, a term premium of 0.0%, and a convexity adjustment of 0.0%. The term premium was set at zero since there are no empirical data on long-term premiums, so "*a practical estimation of the term premium is not undertaken.*"[4] At the same time EIOPA claims, with a level of confidence that is somewhat surprising for people who have looked at the data, that "*from a macro-economical point of view it seems consistent to expect broadly the same value for the UFR*

3 EIOPA (2010), QIS 5: Risk-free interest rates – Extrapolation Method. Available as: *https://eiopa.europa.eu/fileadmin/txdam/files/consultations/QIS/QIS5/ceiops-paper-extrapolation-risk-free-ratesen-20100802.pdf.*

4 EIOPA used research results from Barrie & Hibbert in which the term premium and convexity added 1.1% to the UFR to arrive at 5.3%. See Barrie & Hibbert (2008), A framework for estimating and extrapolating the term structure of interest rates, available as: *http://www.barrhibb.com/documents/downloads/A_Framework_for _Estimating_and_Extrapolating_the_Term_Structure.pdf.*

around the world in 100 years." In 2015 EIOPA reported that the level of the UFR would remain at the value of 4.2%, as proposed in the earlier study from 2010.

Speed of convergence

One of the currencies for which a different UFR is proposed at the moment (of 5.2% in fact) is the South African Rand. This is important since the EIOPA documents show that the assumed value for the speed of convergence parameter α, which determines how quickly forward rates converge to the UFR, is entirely based on a study (in a master's thesis) of South African yield curves between 2000 and 2007.[5] The authors of that study[6] state that "*a qualitative assessment of the model's performance against actual historical data indicated that a consistently smooth set of forward rates could be achieved by choosing α equal to 0.1. For the* purposes of illustration *we have therefore assumed α equal to 0.1 for the remainder of this paper.*"[7]

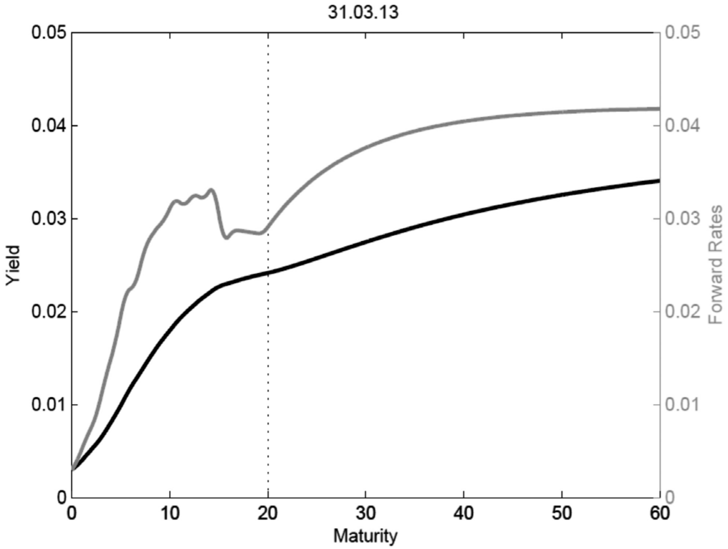
If extrapolated rates differ by more than 0.03% from the UFR value at a maturity of 90 years, then the value of α is, in the

5 As far as I know, this fact was first pointed out in a blog by Marco Folpmers (see *http://folpmers.wordpress.com/2012/07/19/meer-bezwaren-tegen-het-gebruik-van-de-ultimate-forward-rate-ook-de-weg-ernaartoe-is-wankel/*). Folpmers also remarks that the EIOPA document mistakenly mentions Australian yield curves as the underlying dataset, when in fact South African yield curves were used.

6 Thomas & Maré (2007), Long term forecasting and hedging of the South African Yield Curve, Presentation at the Convention of the Actuarial Society of South Africa. Available as: *http://www.actuarialsociety.org.za/Portals/2/Documents/Convention-HedgingOfSAYieldCurve-MT-EM-2007.pdf*; Results are based on Thomas (2008), Long term extrapolation and hedging of the South African Yield Curve, M.Sc. thesis (*http://upetd.up.ac.za/thesis/submitted/etd-06172009-085254/unrestricted/dissertation.pdf*).

7 In 2002, inflation in South Africa was 9.5%, according to the OECD, and in 2009 (the last year of data used by EIOPA in setting the UFR), it was 7.2%. The long-term rate of inflation chosen by EIOPA for South Africa is 3.0%.

An example of extrapolated forward rates beyond 20 years
Based on Euroswap data for March 3, 2013, and the original EIOPA UFR method for insurance companies



words of EIOPA, "*recalibrated*" to make sure the difference will become at most 0.03%.[8] The word "recalibrated" is perhaps a bit suggestive, since the South African study involved only six years of yield curve data and its authors never claimed that any calibration had taken place.

The value of the parameter α is just as important as the value of the ultimate rate in determining discount rates for high maturities. For example, when starting the extrapolation from a value of 2%, there is hardly any difference between an ultimate forward rate of 4.2% and a convergence parameter α set at 0.10

8 This rule has been changed since then. Alpha is now chosen as the lowest value that makes the forward rate at a maturity of 60 years differ by less than 1 basis point from the UFR of 4.2%, with a lower bound of 0.05.

and an ultimate forward rate of 5.2% and a convergence parameter of 0.05; the difference is less than 15 basis points in the first 20 years.

In the framework proposed by EIOPA the shape of the term structure thus depends on market data but also on

(1) The proposed level of the ultimate forward rate;
(2) The maturity at which one starts to replace market data by extrapolated values (often called the "last liquid point"); and
(3) The speed of convergence for forward rates beyond the last liquid point towards their ultimate value.

The last liquid point (LLP) is taken to be 20 years for the euro curve. This is based on the "Residual Volume Criterion," which means that the LLP is the maturity level such that 94% of all outstanding bond volumes in government and corporate bonds fall below this maturity.[9] Why a level of 94% was chosen and why risky bonds have been included in the liquidity considerations for the risk-free curve is not clear.

Although there has been some debate about the level of the ultimate forward rate and the last liquid point, the alpha parameter seems to have received much less attention. EIOPA acknowledges that "*objective criteria must be developed for setting the value of alpha in order to avoid that expert judgment is needed in all cases*" and a footnote in the Technical Appendix acknowledges that *"More work has to be done in order to see*

9 See EIOPA (2015), Technical Documentation Risk-free Interest Rate, paragraph 31: "*When determining the last maturity for which markets for bonds are not deep, liquid, and transparent anymore, in accordance with Article 77a Directive 2009/138/EC, the market for bonds denominated in euro [sic] should not be regarded as deep and liquid where the cumulative volume of bonds with maturities larger than or equal to the last maturity is less than 6 per cent of the volume of all bonds in that market.*" (*https://eiopa.europa.eu/Publications/Standards/EIOPA_RFR_Technical_Documentation.pdf*).

if a lower value of alpha than 0.1 could be more appropriate as starting value, as the resulting curves could be deemed to be more objective and market consistent." As far as I know, such a follow-up study has never been attempted by EIOPA.[10]

Regulatory Curves in Different Countries

Once the three values mentioned above have been specified, interpolation and extrapolation are implemented using the Smith-Wilson method.[11] Although EIOPA involved different currencies in its calibration efforts and considered a UFR of 4.2% to be appropriate for most of them, regulators in countries that have their own currency have implemented changes. The discount curve for Danish insurers uses the same UFR value but a much higher speed of convergence. In Sweden, the last liquid point was taken to be a maturity of 10 years and a different (linear) method of interpolation is used for forward rates. The Swiss financial regulating agency uses a slightly lower UFR, due to the lower expected inflation for the Swiss franc, and they take the last liquid points for the Swiss franc and the euro to be 15 years and 30 years, respectively.

The Dutch regulating body uses different curves for insurance companies and pension funds.[12] The EIOPA curve for insurers

10 In an empirical study based on a Vasicek model, Balter et al. (2014) estimate alpha to be around 0.02, with standard deviation of 0.01. See also Wahlers (2013).

11 See the EIOPA documentation mentioned above; the original document by Smith and Wilson does not seem to be available in the public domain. The method boils down to fitting scaled versions of the exponential tension splines that were introduced in Schweikert (1966). Such splines have been used before as interpolating functions for term structures. The earliest reference that I have found for this is Barzanti & Corradi (1998).

12 This may change in the future since the Upper House of the Dutch Parliament has decided that there should be a "level playing field" for insurance companies and pension funds. This will be discussed later in the paper.

assumes convergence of forward rates after 20 years towards a UFR of 4.2% at a maturity of 60 years.[13] But a different method is used for pension funds. As mentioned before, until 2012, the Dutch Central Bank used market information until maturity 50 years and applied interpolation and extrapolation based on the assumption that forward rates are piecewise constant.[14] Since September 30, 2012, a weighted combination of market-consistent rates and the ultimate forward rate has been used for maturities between 20 and 60 years.[15] These weights are constant in time. The original Smith-Wilson method defines forward rates for those maturities as a weighted combination of the observed forward rate at 20 years and the assumed rate at 60 years (which equals the UFR of 4.2%). Dutch regulators make this a weighted combination (with weights the same as in the original Smith-Wilson method) of the observed market rate at that maturity and the UFR.[16] The incorporation of market data beyond the last liquid point reduces the extreme sensitivity to the 20-year forward rate, and this is the main reason for the modification.[17] The funding ratio of

13 See *http://www.toezicht.dnb.nl/3/50-226257.jsp*.

14 To be precise, Bloomberg's London Composite Rates (CMPL) bid prices for contracts which swap 6-month EURIBOR for maturities 1,2,3,4,5,6,7,8,9,10,12,15,20, 25,30,40 and 50 years are used to construct zero coupon rates. Both interpolation between two such maturities and extrapolation beyond the maturity 50 are based on the assumption that forward rates remain constant for intervals between maturities (or beyond the last maturity of 50 years) and equal the forward rate at the beginning of the interval.

15 After January 1, 2012, zero coupon interest rates were first averaged over a window of three preceding months, but this has no longer been the case since January 1, 2015.

16 For details on, and the values of, the weights, see *http://www.toezicht.dnb.nl/binaries/50-226788.pdf* and *http://www.toezicht.dnb.nl/en/binaries/51-212329.pdf*. In the first document Footnote 4 suggests that the convergence parameter must be increased by 0.10 until convergence is reached. In the light of later documents from EIOPA, this is probably a misprint.

17 Warnings about the extreme sensitivity with respect to 20-year forward rates and the original proposal for the modification implemented by Dutch

pension funds in the Netherlands increased by 3% on average as a result of the introduction of the UFR. In July 2014 the effect had increased to 5%.[18]

UFR Committee

In early 2015, a new regulatory framework was announced for pension funds in the Netherlands. One of the important changes was that nominal discount curves for pension funds would be calculated using an algorithm proposed by the Commission UFR. This Commission had been set up by the Dutch government in late 2012 and charged with the task[19] of formulating recommendations for possible adjustments to the existing algorithm used to generate discount curves that incorporate ultimate forward rates. It was also asked to provide a method that could be used to assert that application of the UFR was, and would remain, realistic.

regulators can be found in Kocken, Oldenkamp, & Potters (2012), An alternative model for extrapolation, Insurance Risk, August 2012. A group of academics proposed the use of the Cardano Modification during one of the EIOPA consultation rounds (see *https://eiopa.europa.eu/Publications/Comments/CP-12-003_QIS_IORP_II_Academic_Community_Group.pdf*), but the proposal was not adopted by EIOPA. See also the presentation by Lord (2012), The ultimate forward rate – Background, issues and impact, available on www.rogerlord.com.

18 The increase is mentioned on Page 10 of the *Nota naar aanleiding van het verslag inzake het wetsvoorstel aanpassing financieel toetsingskader* (Memorandum in response to the report on the bill to modify the financial assessment framework [FTK]), September 23, 2014. See *https://www.rijksoverheid.nl/documenten/kamerstukken/2014/09/23/nota-naar-aanleiding-van-het-verslag-inzake-het-wetsvoorstel-aanpassing-financieel-toetsingskader*.

19 The text specifying the precise task of the committee, *Instellingsbesluit Commissie UFR* (Decree establishing the UFR Committee, December 21, 2012), mentions as one of the premises an UFR which "*forms the best possible approximation of the risk-free interest rate which can be expected in the long term*" ("*die een zo goed mogelijke benadering vormt van de risicovrije rente die op lange termijn mag worden verwacht*"). Since the UFR does not deal with the values of interest rates in the long term, but with current forward rates for long *maturities*, it seems the Dutch government has applied what's known as the "expectations hypothesis."

The Committee's final report made at least two very important contributions to the debate about the ultimate forward rate. First, it explicitly states that there is not sufficient justification for the assumption that the ultimate forward rate is constant and equal to 4.2%. In the report a different methodology is therefore proposed, which consists of a monthly updated 120-month average of a collection of 20-year forward rates. These forward rates are based on market information for bonds with maturities of up to 50 years, which is considered to be the last maturity for which reliable information can be found in the market. Secondly, the Committee chose a convergence parameter which has a different interpretation than in earlier models,[20] and explicitly states that this is "*a first considerable step in the direction of a more market consistent parameter*" but that it "*deems it prudent to make this first step not too large*" and that another Commission should perform further research to arrive at "a representative [convergence] parameter[21]". The Committee mentions that in the empirical literature[22] smaller values for the convergence parameter have been reported and that studies often suggest a value very close to zero; that is, that there should not be convergence to a fixed forward rate at all. They also summarize the results of a consultation among five international scientific experts on the use of an ultimate forward rate. The majority suggests that "the

20 If p(t,T) is the bond price at time t for a maturity T, then both the earlier method and the method proposed by the Committee satisfy p(t,20+h)=p(t,20) exp(-h*UFR)Q(t,h) for a transition function Q(t,h) which equals one for h=0 and becomes a constant which may depend on t for very large h. In the earlier method, Q(t,h)=1+b(t)(1-exp(-αh)); while in the new method, Q(t,h)=exp(c(t) (1-exp(-αh))) for a convergence parameter α and certain functions b(t) and c(t).

21 UFR Committee Report, version 9-10-2013, Page 50. From Page 80 of the report, one can conclude that this parameter equals the mean reversion parameter in a Vasicek model with zero volatility in the short rate.

22 See De Jong (2000) and Babbs & Nowman (1999).

best option would be to follow market observations up until a last liquid point and to extrapolate the forward rates or keep them flat afterwards." The experts also mention that it is probably hard to indicate what the last liquid point should be[23] and state that they are concerned that the use of a UFR curve would be sensitive to political intervention.

For maturities up to 50 years, the spot rate curve which was found by applying the method proposed by the Committee hardly differed from the curves used by EIOPA and the Dutch Central Bank at the time. But there is a noticeable change in forward rate curves, and the recalculation of the technical provisions, which were based on the total Dutch pension sector liabilities in 2012, showed a reduction of 1.1% compared to the existing method.

A motion in the Dutch Lower House of Parliament in October 2014[24] to bring the UFR for pension funds "*more in line with*" EIOPA's 4.2% was defeated, but it led to a request from the Dutch

23 UFR Committee Report, version 9-10-2013, Page 29.

24 Krol motion, October 15, 2014: "*De Kamer, gehoord de beraadslaging, overwegende dat de UFR voor pensioenfondsen met 3,6% ruim een half procentpunt lager is dan de UFR voor verzekeraars, die op 4,2% ligt, terwijl voor verzekeraars een zekerheidsgraad van 99,5% wordt gehanteerd en voor pensioenfondsen wordt uitgegaan van 97,5%; [...] verzoekt de regering, het oorspronkelijke advies van de Commissie UFR spoedig te laten actualiseren, zodat rekening wordt gehouden met ontwikkelingen in Europa en de UFR voor pensioenfondsen meer in lijn wordt gebracht met de UFR voor Nederlandse verzekeraars [...].*" (The House, having heard the deliberations, in consideration of the fact that the UFR for pension funds, at 3.6%, is over half a percentage point lower than the UFR for insurers, which is 4.2%, while insurers use a degree of certainty of 99.5%, whereas pension funds assume one of 97.5%; [...] hereby requests that the government update the original recommendation of the UFR Committee soon so that it takes into account developments in Europe, and the UFR for pension funds is brought more in line with the UFR for Dutch insurers [...].) Full text: *http://www.eerstekamer.nl/behandeling/20141015/motie_van_het_lid_krol_over/document3/f=/vj01hhwwzdzt.pdf*. A UFR of 3.6% would be the result of the calculation method used by the UFR Committee. Considerations of the Upper House of Parliament can be found here: *https://www.eerstekamer.nl/nieuws/20141217/wet_aanpassing_financieel*.

Upper House of Parliament to allow the Dutch Central Bank to decide on this matter after the level of the UFR for insurers had been established by EIOPA. On March 17, 2015, EIOPA announced that the level would remain at 4.2%, and on July 14, 2015, DNB decided that the term structure for pension funds would be calculated according to the proposals by the UFR Committee, with a slight modification.[25]

Principles for Discount Curve Extrapolation

The considerations that led to the newly proposed method and the long-term effects of the choices that have been made are perhaps more important than the numerical values that will eventually be decided upon. The Committee defines a term structure that is updated every month based on market information up until the maturity of 50 years. This means that it is not assumed to be constant, and it also implicitly suggests that information for maturities between 20 and 50 years is reliable enough to be incorporated in models for the discount curve. The report explicitly mentions the lack of empirical evidence or theoretical macroeconomic justification for the assumption that there exists a limit to which forward rates must converge. By choosing alpha, the parameter that controls the speed of convergence, equal to half of the value which best corresponds to the existing EIOPA method, calibration is still implicitly based on South African yield curves during a relatively small period of time. But the report also states that the speed of convergence must be reexamined in the future, which has yet to be done.

25 This slight modification concerns the start of the extrapolation at maturity 20, not the level of the UFR or speed of convergence. See *http://www.dnb.nl/nieuws/nieuwsoverzicht-en-archief/dnbulletin-2015/dnb324317.jsp*.

In its report, the Committee defines a number of principles that underpin the choices that have been made.[26] They state that methods should be transparent and consistent with financial markets, the academic literature, and the approaches used in other countries; should not lead to market distortions; and should result in stable parameters. In the next section, these points will be discussed, together with other considerations that can help identify the desirable and undesirable effects of any given method to extrapolate discount curves.

26 Some of these are earlier recommendations made by the Dutch Actuarial Society in the reports "Principles for the Ultimate Forward Rate" (March 2013) and "Principles for the Term Structure of Interest" (October 2009).

4. Perceived advantages and disadvantages

In this section, some possible advantages and disadvantages of the introduction of an ultimate forward rate will be discussed.

No Theoretical or Empirical Foundation for a Constant UFR So It Could Be Changed Later

From a purely theoretical point of view, there is no reason to assume that forward rates for long maturities converge to a value that does not change over time. There are results by Dybvig, Ingersoll and Ross which characterize the behavior over time of an ultimate forward rate *under the assumption that such a rate exists*. But those results tell us nothing about the question whether there is indeed such an ultimate forward rate or not[27]. Moreover, as was stressed earlier in this paper, the ultimate value of forward rates does not need to imply anything about the behavior of the relevant forward rates between 20 and 50 years. The connection between those rates and the ultimate rate critically depends on the choice of the last liquid point and the speed of convergence. EIOPA, the UFR Committee, and the Dutch government all mention that the speed of convergence parameter must be studied more extensively in the future, thus suggesting it may eventually have to be changed. The possibility of such a change, or a change in the location of the last liquid point,

27 See Dybvig, Ingersoll, & Ross (1996). This paper and later papers, such as the ones by Hubalek, Klein, & Teichmann (2002) and Kardaras & Platen (2012), ***assume*** that the limit of long-term rates does exist. It has also been shown (see Goldammer & Schmock, 2012) that the results of Dybvig, Ingersoll, & Ross can be extended to the case when the long-term rate does not exist, by considering the smallest upper bound instead of the limit for long rates. But these theoretical results assume the existence of bonds of all (i.e. unlimited) maturities, and no conclusion can be drawn for cases where that assumption is not satisfied.

creates a new risk, and an actual change may undermine the credibility of the regulatory framework.

Significantly More Complex Risk Management

Very soon after the introduction of an ultimate forward rate was proposed in the Solvency II regulatory design process, the severe consequences for hedging interest rate risks were pointed out. The introduction of a UFR means that the discount curve between maturities 20 and 50 no longer conforms to actual market prices or conventional asset pricing theory. This raises the question of whether interest rate hedging programs should be based on the real curve, meaning that such programs could be implemented in practice, or the virtual curve prescribed by regulators, which makes actual implementation impossible. Indeed, the justification usually given for replacing fixed income market information at large maturities with new, artificially distorted values is that liquidity at such maturities is too small to provide reliable estimates. But the estimates that they are replaced with are based on an ad hoc economic outlook that cannot be objectively substantiated. And, most importantly, these new rates which should reduce the problem of illiquid maturities have, of course, ***zero*** liquidity themselves.

It also becomes more complicated to define time-consistent interest rate models for maturities between 20 and 50 years. Standard interest rate models that explicitly model the uncertainty in future rates can be made consistent with currently observed market prices, whether forward rates are converging or not.[28] However, even if one assumes that current forward rates will follow the Smith-Wilson parametrization after the last liquid point, this property of the term structure may not be preserved at later times. Simulation studies that start with a term structure that is consistent with extrapolation based on a particular UFR assumption may thus generate scenarios for later term structures which are *not* consistent with that extrapolation. Insurance companies and pension funds must therefore decide whether to apply the UFR-based inter- and extrapolation techniques to their entire scenario set for future interest rates or only to the current term structure. The Dutch Central Bank has announced that it is not allowed to assume that interest rates always converge to the UFR in scenarios that are used for the valuation of contingent claims, such as implicit or explicit guarantees on investment returns.[29]

For Pension Funds, Distortions Propagate to Shorter Maturities

Dutch regulation for pension funds stipulates that certain bounds on the parameters used to generate stochastic economic scenarios have to be chosen by a commission (the *Commissie Parameters*). The calibration process for stochastic interest rate models used by this Commission is based on cash flows and prices from the fixed income markets and will thus not necessarily conform to the

28 The UFR Committee refers to the Vasicek model for interest rates in their report. In that model, forward rates always converge to a fixed value. But it cannot fit a given observed term structure of interest, apart from a few very limited cases. The *extended* version of the Vasicek model proposed by Hull and White (or the popular G2++ model) can fit *any* initial term structure without making prior assumptions as to the existence or absence of a UFR and is therefore a more natural starting point for term structure analysis. Note that EIOPA prescribes that the current market prices of instruments in the liquid part of the curve have to be fitted ***exactly***, so the Vasicek model cannot be used. Since the Extended Vasicek model makes a certain fixed parameter in the Vasicek model deterministically time-varying, this is an interesting illustration of the fact that the theoretical distinction between *parameter risk* and *model risk* can be hard to make in practical cases.

29 See *http://www.toezicht.dnb.nl/3/50-229621.jsp*.

assumptions made about the forward curve after extrapolation. This was evident in the latest version of the official parameter values. The Commission has decided to replace the estimated parameters that characterize interest rate risk premiums with different values, since the estimates that were based on a calibration of bond and inflation data did not lead to the desired value of the UFR. But since the entire term structure is sensitive to the value chosen for the risk premiums, this leads to a distortion for all maturities, including the shortest ones.

Note that the curve for pension funds, as proposed by the UFR Committee and adopted by DNB, uses a starting point for extrapolation at maturity 20 that depends on the 50-year market yield, while the forward rates at very high maturities (such as 50 years) equal a historical average of just the 20-year forward rates. This is not very intuitive.

Stability of Funding Ratios for Pension Funds

There is no doubt that the value of the liabilities of pension funds and insurance companies will often look more stable if market information concerning the long end of the term structure is replaced by constants or by weighted combinations of market information and such constants. This was one of the main reasons for introducing a UFR for pension funds. In the *Memorie van Toelichting Nieuwe Wet FTK* (Explanatory Memorandum on the New FTK Act), the Dutch government mentions that application of a UFR makes "*valuation of pension liabilities more stable and reliable*" and that the price to be paid is that "*hedging [...] interest rate risk [...] becomes slightly more complex*."[30] In reality, interest rate hedging will simply become impossible even under the calmest market conditions whenever market discount curves between maturities of 20 and 50 years are replaced by artificially constructed alternatives. Counterparties will simply refuse to trade at rates which would imply that insurance companies and pension funds have to pay less for cash flows than all other market participants. There will thus be a mismatch between the values of, for example, a cash flow in 30 years on the asset side and on the liability side of the balance sheet.

This also shows that it need not always be the case that funding ratios for pension funds or solvency capital requirements for insurance companies become less volatile. Both rely on the combined valuation of available assets and liabilities, and the valuation of assets still needs to be based on available market prices. If cash flows are well matched for maturities beyond the last liquid point (or first smoothing point), the volatility in the sum of their offsetting values will increase when interest rates for the liability side are changed, while asset prices remain unchanged.[31] In this light, it is interesting to see that in new regulations proposed for 2015, the Dutch government used the

30 Full text: "*De invoering van de ufr-methodiek is bedoeld om onzekerheid en onbetrouwbaarheid van marktinformatie over zeer lang lopende rentes te mitigeren. Het voordeel van toepassing van deze methodiek is dat de waardering van pensioenverplichtingen hierdoor stabieler en betrouwbaarder wordt. De prijs die hiervoor wordt betaald, is dat het afdekken van het renterisico dat pensioenfondsen lopen op hun verplichtingen enigszins complexer wordt.*" (Implementation of the UFR method is intended to mitigate the uncertainty and unrealiability of market information for very long-term rates. The advantage of applying this method is that it makes the valuation of pension liabilities more stable and reliable. The price paid for this is that hedging the interest rate risk pension funds run with regard to their liabilities becomes slightly more complex.)

31 Calculations for an explicit case study can be found in Duyvestein, Martens, Molenaar, & Steenkamp, *De schijnveiligheid van de Ultimate Forward Rate* (The false security of the Ultimate Forward Rate), *http://www.robeco.com/images/ultimate-forward-rate-2013-02-25.pdf* (February 2013).

advantage of an equal treatment of assets and liabilities on the balance sheet as an argument to discontinue the use of averaging interest rates over a three-month period. Apparently, this discrepancy is seen as less problematic when it concerns long-term maturities.

But most importantly, it is not clear why a reduction of volatility in, for example, funding ratios would necessarily be an advantage. The idea behind active risk management is not that certain key indicators remain conveniently stable under all circumstances, but that they only remain stable when no risk-mitigating actions are required. Removing the batteries from the smoke alarm in your house has a stabilizing effect on the level of noise it produces during its lifetime. It is not, however, encouraged from a risk management perspective.

If volatility in the value of liabilities would be problematic in itself, it would be better to discount liabilities of all maturities with a fixed value of, say, 4%.[32] But if the overall aim is to get an accurate estimate of the funding required to make sure, with a sufficiently high probability, that retirement provisions will turn out to be enough in years to come, then volatility simply indicates the inherent uncertainty in such estimates. Obviously, this does not mean that far-reaching actions need to be taken after every movement in a funding ratio or a solvency capital requirement. But there is an important difference between, for example, averaging funding ratios over a certain period of time to reduce the *impact* of short-term movements on such decisions, on the

32 This was in fact the proposal of the Dutch political parties PVV and 50+ in their 2012–2017 election programs. For the rather interesting transcript of the corresponding debate in the Dutch parliament, see *http://publitiek.nl/debat/korten_op_pensioenen_31-01-2012/1*.

one hand,[33] and changing the *measurement* of forward rate volatilities by assuming that certain parts of the term structure will behave differently than the markets suggest, on the other. In the second approach, objective information from the markets is mixed with purely subjective considerations. This will benefit certain stakeholders in a pension fund or insurance company while being detrimental to others.[34]

Dampening of Pro-cyclical Effects

Under the current difficult circumstances in the fixed income markets, the UFR implements an implicit optimism about long-term fixed income returns. If this actually were to lead to improvements in such rates, that would constitute a beneficial anti-cyclical effect. At the very least, it might make the balance sheets of insurance companies and pension funds look better, which buys them some time to restructure after a difficult period. But whereas this time can be useful if it is indeed used to make structural changes, it should not allow these organizations to avoid facing serious problems in the conviction that all will be well if they just wait long enough. There are economists who claim that interest rates cannot stay at their current low levels, but the recent experience in Japan shows that rates can be very low for a very long time.

33 In the new proposals for the regulation of Dutch pension funds, funding ratios will be averaged over twelve months.

34 The focus is on funding ratios here, but note that the introduction of the UFR will have an even larger dampening effect on the actuarially fair premium that needs to be charged for future entitlements. The duration of new entitlements is 30 years in the Netherlands, on average, while the duration over all liabilities involved in funding ratio calculations is around 15 years. See CPB (2012), *Generatie-effecten Pensioenakkoord* (Generational effects of the Pension Agreement), Memorandum for Ministry of SZW, May 23, 2012: *http://www.rijksoverheid.nl/onderwerpen/pensioen/documenten-en-publicaties/kamerstukken/2012/05 /30/cpb-notitie.html*.

Insurance companies have stakeholders who gain and stakeholders who lose by changes in discount curves, since dividends paid out to shareholders reduce the capital used to protect policyholders. And postponing difficult policy decisions can be even more controversial for collective pension funds. Investors in the insurance sector can always invest elsewhere if the growth in dividends is not to their liking, and policyholders can choose to buy contracts from other insurers with a higher capitalization if they think the underlying economic outlook of the regulatory framework is too optimistic. But for many participants in collective pension funds, there are no such alternatives.

Insurance companies and pension funds who hold on to assets that all other market participants are selling have a stabilizing effect on financial markets, but this may increase the risk for the fund participants or the companies' policy- and shareholders. If the asset cash flows are totally risk-free and perfectly matched to liabilities there is no risk, but in that case discounting with or without the UFR would not make any difference. And if cash flows are not perfectly matched, bonds or swaps need to be bought and sold in the market at prices that will not be consistent with the UFR approach.

Less Transparent Intergenerational Sharing of Risks

It is well known that it can be beneficial for different generations to share their financial risk collectively in a pension fund, since an a priori fair tradeoff can often be made. This tradeoff implies that payouts to pensioners do not have to be immediately reduced when funding ratios fall if they share investment risks with the working generations. Pensioners pay a price for this since their entitlements do not grow faster than inflation even when a fund's returns might make this possible, because they are used instead to build up buffers for the younger generations. Any assumption about long-term forward rates that is overly optimistic thus benefits the older generation and any assumption that is overly pessimistic benefits the younger generation. This does not mean that it is impossible to share risk when information about rates for long-term maturities is imperfect. But it does imply that assumptions made about such maturities must be as realistic and objective as possible if one wants to avoid a debate about fair implementation of intergenerational solidarity.

This point of view used to be the one held by the Dutch government. In early 2013, in the response to a proposal made by a Dutch member of parliament to use a fixed interest rate of 4% for *all* maturities, the secretary of state replied that "*proposals for a higher interest rate which are only aimed at avoiding pension cuts for the current generations of pensioners contain the risk that in the long term even more stringent actions are required. Future pension payments will be strongly dependent on realized high rates of return and failure to do so will immediately lead to reductions. An adjustment of interest rates that are used for the calculations is not appropriate for entitlements which are certain. It would put younger generations at a disadvantage and puts pressure on the solidarity in our pension system.*"[35]

I fully agree. And what's more, I see no reason to restrict the validity of this statement to maturities under 20 years.

35 Letter from the State Secretary of Social Affairs and Employment [SZW], *Beantwoording vragen van de leden van de vaste commissie voor SZW over rekenrente en uitvoeringskosten* (Answer to questions posed by the SZW standing committee on interest rates and implementation costs), documents of the Lower House, 2012-2013 session, House minutes 32 043-151.

5. Summary and outlook

There is no empirical evidence that euro forward rates for maturities beyond 20 years converge to 4.2%, or to the 20-year forward rate, or to the 120-month historical average of 20-year forward rates. There is no theoretical reason to assume that they will do so either. Making such an assumption makes interest rate risk management more complicated and creates a discount curve that is inconsistent with what is observed in the markets for maturities between 20 and 50 years. This leads to a direct mismatch between assets and liabilities on the balance sheet.

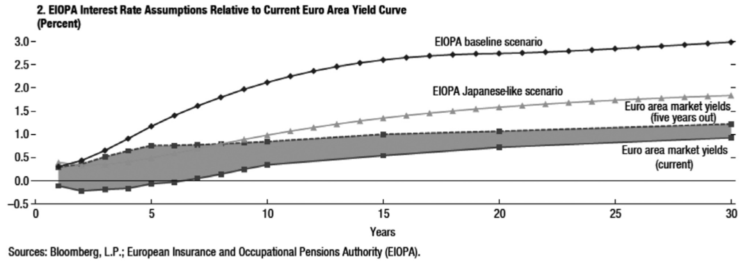
EIOPA's criterion for market instruments that can be used to determine the risk-free discount curve is abbreviated as DLT: they should be deep, liquid and transparent. But the introduction of the UFR suggests that existing market information for fixed income products beyond 20 years is so unreliable that is better to use adjusted prices that have zero depth, zero liquidity, and involve subjective choices that are far from transparent.

The Dutch Central Bank worries about the discrepancy between the observed market curves and the UFR curve. A report published in early 2015 states that this discrepancy "*may lead to expectations for participants which are too high, promises to policyholders which are unrealistic and distorted incentives for institutions.*"[36] This statement, and the warning that for a maturity of 60 years the difference between the curves has grown to 1.60%, suggests that DNB does not consider the UFR curve to be a better estimate for maturities between 20 and 50 years than the curve suggested by the market.

A similar sentiment can be found in the Global Financial

36 *Overzicht Financiële Stabiliteit* (Overview of Financial Stability), De Nederlandsche Bank (Dutch Central Bank), Spring 2015.

EIOPA Stress Test Scenarios for the euro and current yields (early 2015)



IMF Global Financial Stability Report 2015, Page 23

Stability Report 2015 from the International Monetary Fund, which contains a warning that the business model of European life insurers is unsustainable in a low-interest-rate environment. It notes that 24% of insurance companies were unable to meet their Solvency Capital Requirements under EIOPA's "Japanese-like scenario" in the 2014 stress test and that interest rates are currently significantly lower than in that scenario (see figure). It concludes that "*midsize insurers in Europe face a high and rising risk of distress. The failure of one or more midsize insurers could trigger an industry-wide loss of confidence if the failure is believed to reflect a generalized problem*."

These statements by the Dutch Central Bank and the IMF concern insurers. Similar worries have been expressed with regard to pension funds by the Bureau for Economic Policy Analysis (CPB) of the Netherlands[37]: "*The UFR of 4.2% is based on a level of inflation of 2% and a real interest rate of 2.2%. The level for the real interest rate is rather high, especially in terms of current standards. When interest rates remain at a low level for a long*

37 Memorandum for Dutch Ministry SZW, May 23, 2012, *Generatie-effecten Pensioenakkoord* (Generational effects of the Pension Agreement).

time (a Japan scenario) and the UFR is not adjusted, a fund may present itself as richer than it is in reality. This may result in a situation where nominal reductions can be postponed and new premiums can be kept below nominal sustainable levels."

This suggests that DNB, IMF and CPB do not believe that forward rates of around 4.2% at maturity 60 are realistic. But extrapolation beyond the last maturities for which reliable data can be found will always be necessary for pension funds and insurance companies. So what are the alternatives?

A Step Backwards: The Simple Earlier Alternative

Any value for a long-term forward rate that cannot be observed in the markets should be consistent with forward rates that can be observed. It therefore seems logical to define the UFR as a weighted average over such observed rates. An example was given in Section 2: the Dutch Central Bank used to take the last liquid forward rate and simply assumed it would also hold for all higher maturities. This seems a very sensible choice. It would mean that the forward rate used for extrapolation could change when the distribution of liquidity over the maturities changes, for example when markets are distressed.[38] Moreover, hedging programs for cash flows in the distant future would concentrate on buying the longest bonds available,[39] which seems preferable to buying lots

38 We remark that the possibility of a change in the point at which extrapolation is started poses a risk in itself, which this method shares with any other method that involves extrapolation beyond a certain maturity.

39 This may make the bonds with long maturities more expensive: if people in the wake of the financial crises want more security about future cash flows, the financial markets will show this by offering such risk-free cash flows at higher prices. If liquidity becomes less due to a drop in demand, prices can be expected to go down, and if it is due to a drop in supply, prices can be expected to go up. I suspect there is less supply than demand for the certainty that 30-year bonds can provide.

of 20-year bonds because of the special status granted to what is a priori declared to be the "last liquid point." And people who continue to believe that long-term rates will always tend to be around 4.2% have nothing to worry about if this rule is reinstated. If the market's last liquid forward rates are indeed roughly 4.2% under normal circumstances, then this will be the constant that will be used for the extrapolation.

An (almost) constant forward rate beyond the last liquid forward rate corresponds to choosing a very high value for alpha, the speed of convergence parameter in the original proposals of EIOPA and the UFR Committee. EIOPA, the UFR Committee, and the Dutch government have all suggested that the speed of convergence parameter must be studied more extensively in the future but none of them has commissioned such a study yet. In the meantime, they all continue to use a value that is based, directly or indirectly, on a master's thesis that considered fewer than seven years of data for a currency at the other side of the globe.

Making the ultimate forward rate equal to the last liquid forward rate creates an extrapolation method which is transparent, easy to implement, does not distort fixed income markets and does not create a misguided sense of stability by making liabilities with maturities between 20 and 50 years cheaper than the corresponding assets while they define exactly the same riskfree cash flows.

Weighted Schemes

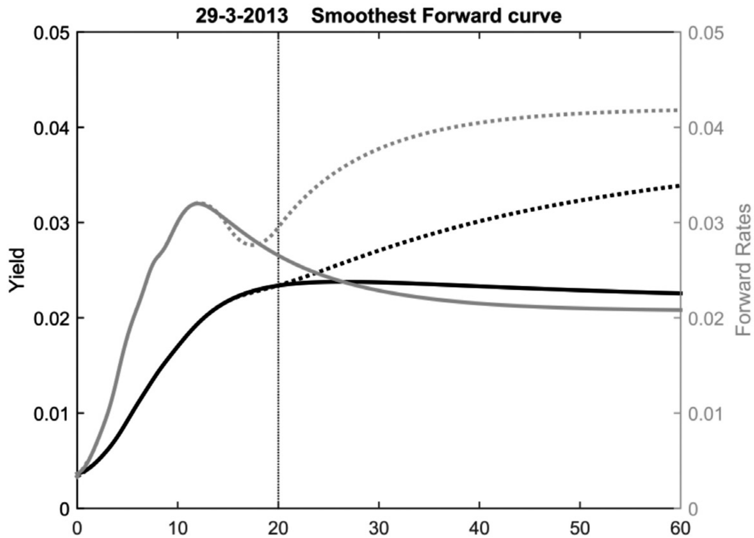
If forward rates are believed to be smooth, taking them piecewise constant between maturities and constant after the last maturity may be undesirable. One may then want to apply another weighting scheme on existing rates. Which weights are chosen is a subjective choice but if one takes the existing Smith-Wilson framework as a starting point, there is an objective and consistent way to let observed market data determine the UFR. It turns out that the Smith-Wilson method can be interpreted as the solution to an optimization problem where the convergence parameter determines how smooth the forward curve is going to be. Smith and Wilson took a given value for the UFR and then solved the optimization problem for this value but one could also solve the problem without defining the UFR a priori. The optimization program then **determines** the UFR, as the level that makes the discount curve, yield curve, or forward curve as smooth as possible. An example is given in the figure on the next page.

One could show that the value of the UFR that achieves this smoothest yield or forward curve will always be a weighted average over observed market yields for different maturities. This makes it a parameter with an easy interpretation,[40] which is based on a criterion that is a direct consequence of the Smith-Wilson inter- and extrapolation approach.

Note that the proposal from the UFR Committee also defines the UFR as a weighted combination of observed market rates, but they take it to be a historical average over 120 months. This makes no difference if the weighted combination of market rates is almost constant over time, since one then takes averages over values which hardly change. But that turns out not to be the case and the algorithm therefore artificially reduces the volatility of the discount curve for long maturities. Uncertainty in the valuation of cash flows in the distant future is thus made smaller than the uncertainty for cash flows in the near future, which seems not very realistic.

40 Details of this alternative method for determining the UFR are given in a separate Netspar Design Paper.

An example: The dashed lines represent the current method of EIOPA (Smith–Wilson with an UFR of 4.2%). When one chooses a UFR that makes the resulting curve as smooth as possible, the solid line is found instead. The kink at maturity 18 years in the forward rate curve disappears and a lower value for the UFR is found. Calculations were based on Euroswap data for March 29, 2013.



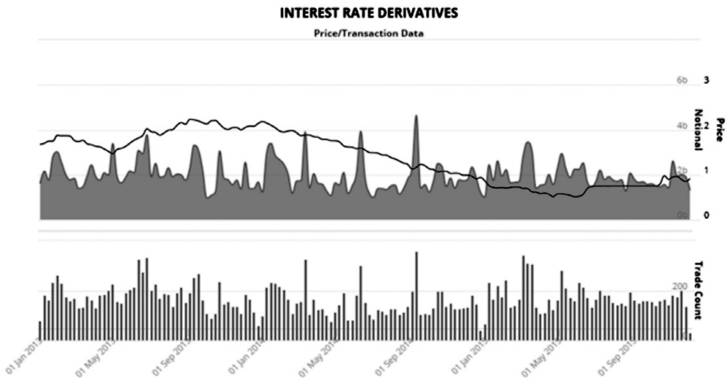
Subjective Policy Decisions, Fed by Objective Information

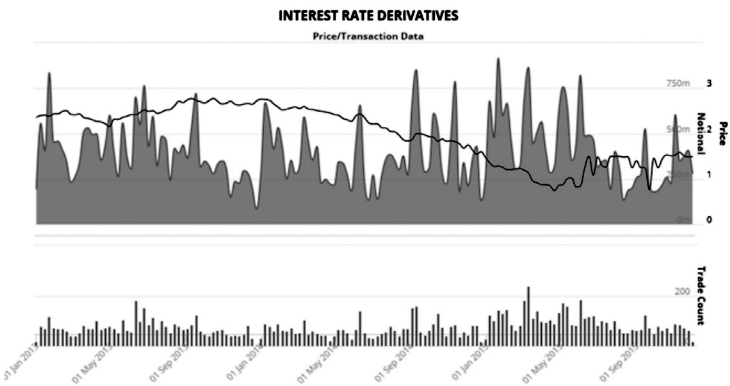
The introduction of the UFR is a modification without a solid foundation, which may yet bring rewards in the form of a positive macroeconomic effect created through temporary optimism on balance sheets under difficult circumstances. But it also masks the difficult decisions that need to be taken if interest rates remain low for a long time. And, perhaps most importantly, it may undermine the distinction between objective economic facts and subjective choices that is essential for the confidence in institutions that are entrusted to manage our retirement provisions. This is undesirable for insurance companies since the discount curve influences the essential tradeoff between shareholders and policyholders that such companies must always make when deciding how much capital is kept in reserve for bad scenarios and how much is paid out in dividends. For pension funds it is even less desirable, since the trade-off involves different generations and participants often do not have the freedom to choose an alternative pension fund.

This does not mean that the boards of insurance companies or pension funds should not be allowed to make subjective decisions about this trade-off. It is, for example, only natural that policy decisions by the board of a pension fund are not just based on the current funding ratio but on its recent history. The board may thus base its decisions on a version of the funding ratio that is an average over historical values, to avoid that decisions with long-term consequences are based on short-term effects. But the interest rates themselves are never averaged over time for short maturities and we should not do so for higher maturities either. Those who see a pension contract as a "social contract" instead of a "financial contract"[41] may maintain that pension fund boards should decide this for themselves, since boards have the right to use the UFR to transfer wealth from one generation to another. But such decisions become less transparent and harder to defend when one introduces unfounded assumptions about interest rates that make the current funding ratio look better (or worse) than the economic data imply.

41 For a discussion of the two approaches, see the Netspar Design Paper by Broeders, de Jong and Schotman (2014).

Depository Trust and Clearing Corporation (DTCC) plain vanilla euro swap trade volumes for maturity 10 years (top) and 30 years (bottom) in the last two years. The typical number of trades per day is 150 for the first maturity and 50 for the last one, with total notional values of 1500 million and 200 million, respectively. The rates are currently around 1.00% and 1.50%.





Source: International Swaps and Derivatives Association

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Every financial institution that wishes to use a discount curve to estimate the current value of future cash flows has to decide how to construct such a curve from information that can be found in the fixed income markets. The definition of a discount curve will require interpolation methods. For many pension funds and insurance companies, it is also necessary to extrapolate the curve. In this NEA paper by Michel Vellekoop (UvA) discount curve modelling is discussed in general. Different existing methods including an UFR assumption are introduced with their respective (dis)advantages.

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