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# Default life-cycles for retirement savings

*Anna Grebentchikova  
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DESIGN 70



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# DEFAULT LIFE-CYCLES FOR RETIREMENT SAVINGS

## Summary

This paper discusses optimal allocations to stocks and bonds during the contribution and retirement phases in a life-cycle optimization context. We recall known results from the literature and indicate where optimality results are available, and where they become model-dependent. In particular, we show that often-used assumed interest rates in the Dutch pension practice are suboptimal under standard financial market and preference assumptions. Moreover, we show that default life-cycles with respect to equity exposure perform fairly well, from the individual point of view. The default life-cycles should be adjusted for alternative components in the total wealth of an individual. Optimal interest rate exposure is difficult to derive and becomes model-dependent. We reference some results on robustness in that domain.

## 1. Introduction

Historically, Dutch pension funds have provided little life-cycle investment for their participants. The main reasons for this were the promised nominal guarantee and the so-called *doorsneesystematiek*, which implied a uniform premium, uniform indexation, and a uniform reduction mechanism over all participants. As a result, pension funds invested on behalf of the collective by investing for the “average” participant. With the growth of defined-contribution (DC) schemes, which provided individual life-cycle investing, questions arose about the performance of defined-benefit (DB) schemes. This, among other reasons, has led to a discussion in the Netherlands that has essentially yielded a hybrid scheme that combines elements from both pure DB and pure DC.

The present paper focuses on specification of “good” default life-cycles for an individual in a pension scheme. We use the word “default” to denote an option offered to an individual participant in case he or she does not express a choice. The default choice may be made dependent on observed individual characteristics of the participant. We do not consider “duty of care” aspects in this paper. Since the majority of participants often opt for the default in both contribution rate and asset allocation, the design of a “good” default has become even more important. We also document that a “good” default life-cycle depends on the risk preferences of the individual, without discussing how these preferences can be assessed. For this latter question, see Alserda, Dellaert, Swinkels, and van der Lecq (2015) who discuss this problem in the (Dutch) pension context.

Life-cycle models consider three different trade-offs: portfolio choice, savings rate and labor supply. This paper discusses only



portfolio choice. Given an amount of financial wealth, portfolio choice deals with the allocation of wealth over asset classes with different risk and return characteristics, and asset classes with different levels of liquidity. Life-cycle models also make predictions on the amount of savings and labor supply. In our setting, savings are fixed as a given percentage of labor income and fully paid as pension contribution. Similarly, labor income is exogenously given and is not part of household decision making. An important part of the labor supply decision is the retirement date, which we take as being fixed at age 67. Ignoring labor supply and savings decisions limit the flexibility of individuals to mitigate financial risks. For example, if investment returns are poor, an individual could decide to save more, *i.e.*, to make a larger contribution to the pension account, in order to keep pension wealth close to a desired level. Individuals may thus choose to give up some current consumption in order to bring their future consumption during retirement in greater agreement with the consumption over their entire life. Another way to reduce the impact of a financial shock is by either postponing retirement (or early retirement in case of unexpectedly large financial returns) or increasing working hours. Our comparison of different strategies may therefore overstate the riskiness inherent to portfolio choice.

The paper offers both a theoretical analysis and an empirical analysis. Section 2 discusses results available for the standard Merton model, which features a constant investment opportunity set and individuals with power utility preferences. It is well known that in this setting utility loss from “somewhat” suboptimal investment decisions is limited with respect to utility loss from suboptimal saving and dis-saving rates; see, e.g., Calvet, Campbell, and Sodini (2007). Therefore, we pay detailed attention to the

latter by deriving the optimal assumed interest rates for the decumulation phase in (12). In particular, these optimal rates differ substantially from the expected return on the underlying investment portfolio. We consider several practical extensions of this baseline model and show their implications.

Sections 3 and 4 consider standard life-cycle strategies from an empirical and simulation-based point of view in more realistic settings. This analysis uses financial market parameters commonly used in the analysis of Dutch pension plans: those prescribed by the so-called “Commissie Parameters” (Parameters Committee). Even though this committee only prescribes maximum parameters for, e.g., expected returns on stock portfolios, it is common practice to use this maximum outright as the expected return. The scenario set is thereby the one prescribed by the Dutch central bank (DNB), which is also the Dutch pension fund regulator. See Koijen, Nijman, and Werker (2010) for more information.

We briefly state three policy recommendations:

- “Good” default life-cycles depend more on individual heterogeneity than on financial market states. Important aspects of individual heterogeneity are unemployment spells, housing/mortgage wealth, and private savings.
- Current practice with respect to mitigating interest rate risk seems hard to beat.
- Some risk taking after retirement, for example in the form of variable annuities, is always beneficial.

Apart from portfolio choices a flexible retirement age can also be an effective tool to hedge against unfavorable retirement outcomes.

## 2. Theoretical life-cycle investment results

This section reviews briefly results in the literature concerning optimal investment and consumption during the contribution and retirement phases. These results are not new, but it is convenient to have an overview. The setting we consider is that of the “standard” Merton model. This term is sometimes used to refer only to the financial market being considered (a market with constant investment opportunity sets as described in Section 2.1). However, we will take it to mean additionally that preferences are described by expected Constant Relative Risk Aversion (CRRA) (Section 2.2).

Although this setting is very simple, it yields right away some important insights. Nevertheless, some extensions are of first-order importance to be included for the present Dutch pension debate. We briefly list them now.

**Human capital.** Introducing human capital to the model is the prime ingredient to turn the Merton model into a life-cycle investment model (Section 2.5).

**The first pillar.** In the retirement phase, a possible first-pillar pension is an important source of income that affects the results (Section 2.6).

**Longevity risk.** Idiosyncratic longevity risk turns out to be irrelevant for optimal savings and portfolio allocation decisions (Section 2.8).

**Interest rate risk.** Section 5 discusses the possibility that interest rates change over time. We refer to the academic literature for the consequences of this situation, but also argue in the

rest of this paper that taking these into account is less important than the heterogeneity of agents, on which we focus.

**Inflation.** A situation of constant inflation is easily handled (Section 2.10). Time-varying inflation has effects very similar to those of time-varying interest rates.

**Parameters.** Section 2.11 discusses the latest model used by the Dutch Committee Parameters and how it relates to the results presented in this paper.

**Habit formation.** The introduction of habit-formation preferences has important qualitative consequences for optimal savings/decumulation decisions, and optimal investment strategies. Numerical results are non-trivial, but we indicate the consequences qualitatively (Section 2.12).

## 2.1. The standard Merton financial market

In the standard Merton model there is a single risky investment opportunity and no interest rate risk. The interest rate is constant over time and maturity and is denoted by  $r$ .<sup>1</sup> The risky investment opportunity has a constant expected return  $\mu$  and constant volatility  $\sigma$ .<sup>2</sup> Usually, the risky investment opportunity is referred to as a stock index, but it can also be a portfolio of various liquid investments.

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<sup>1</sup>For now it is irrelevant whether  $r$ , and any other parameter, refers to real or nominal interest rates. The effect of inflation will explicitly be addressed in Section 2.10.

<sup>2</sup>All returns and interest rates in this paper are continuously compounded or “geometric”.

The compensation for risk (or the “price-of-risk”) is a key parameter in the model, and in this case equals the Sharpe ratio of the risky investment

$$\lambda = \frac{\mu - r}{\sigma}. \tag{1}$$

As both  $\lambda$  and  $r$  are constant, this is also the case for the investment opportunity set. There is, thus, no such thing as “diversification over time”. Mathematically the above can be written such that the stock index evolves as

$$dS_t = (r + \lambda\sigma) S_t dt + \sigma S_t dZ_t, \tag{2}$$

where  $Z$  denotes a standard Brownian motion. The price-of-risk  $\lambda$  is then the price-of-risk of this Brownian motion  $Z$  as a systematic risk factor. Equation (2) has a simple interpretation<sup>3</sup>: for each unit of risk  $\sigma$ , an investor receives a compensation  $\lambda$  in terms of expected return above the risk-free rate  $r$ .

**2.2. The standard Merton investment problem**

Given the simple financial market introduced, we consider an agent that wishes to invest in such a way that expected (CRRA or power) utility of wealth at horizon  $T$  is maximized. We discuss this basic problem first, also because it is needed to solve the optimal decumulation problem for the retirement phase (Section 2.3). The investment problem is thus to maximize

$$E_0 \left[ \frac{W_T^{1-\gamma}}{1-\gamma} \right],$$

where  $\gamma$  denotes the agent’s risk aversion that is commonly assumed to be in the range  $2 \leq \gamma \leq 10$ . Note that  $1 - \gamma$  is negative.

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<sup>3</sup>This is sometimes referred to as the “beta pricing” or factor investing.

We rely on the martingale method to solve the optimal investment problem. This technique is recalled in Appendix A. This results in an optimal investment strategy that is given by

$$dW_t^* = \left( r + \frac{\lambda^2}{\gamma} \right) W_t^* dt + W_t^* \frac{\lambda}{\gamma} dZ_t \quad (3)$$

$$= W_t^* \left( 1 - \frac{\lambda}{\gamma\sigma} \right) r dt + W_t^* \frac{\lambda}{\gamma\sigma} \frac{dS_t}{S_t}. \quad (4)$$

Both equations are mathematically equivalent, but have different interpretations. Equation (3) states that the optimal exposure to the risk factor  $Z$  is given by  $\lambda/\gamma$ . Equivalently, the exposure to the stock is  $\lambda/(\gamma\sigma)$ . The corresponding risk compensation is then  $\lambda \times \lambda/\gamma = \lambda^2/\gamma$ . This interpretation is in line with the factor investing literature. Alternatively, Equation (4) gives the optimal risky portfolio as

$$w = \frac{\lambda}{\gamma\sigma} = \frac{1}{\gamma} \frac{\mu - r}{\sigma^2}. \quad (5)$$

The remainder  $(1 - w)$  is invested in the risk-free asset. This shows that the optimal risky asset weight is independent of wealth  $W_t$ , independent of time  $t$ , and independent of the horizon  $T$ . In particular, it equals the classical mean-variance allocation. For typical values  $\lambda = 20\%$  and  $\sigma = 20\%$ , the optimal risky asset exposure varies from  $w = 50\%$  to  $w = 10\%$  for risk aversion varying from  $\gamma = 2$  to  $\gamma = 10$ . Additional sources of income may lead to higher risky allocations; see Sections 2.5 and 2.6.

For later use, we also need the optimal utility following from the

optimal investment strategy. We find (Appendix A):

$$E_0 \left[ \frac{(W_T^*)^{1-\gamma}}{1-\gamma} \right] = \frac{W_0^{1-\gamma}}{1-\gamma} \times \exp \left( (1-\gamma)rT - \frac{1}{2} \left(1 - \frac{1}{\gamma}\right) \lambda^2 T \right). \quad (6)$$

This simple, but standard, setting allows us to draw a few important conclusions.

- The optimal investment mix is independent of both time  $t$  and horizon  $T$ .
- The optimal utility is again of the CRRA form, with unchanged risk-aversion parameter. Optimal utility increases if the investment opportunity set “improves”, i.e., if  $r$  increases or if  $\lambda$  increases.

It is useful to relate the above also to the recent legislation proposed in the Netherlands to allow risk taking also after retirement.<sup>4</sup> The utility of a fully risk-free investment at horizon  $T$  would be

$$\frac{W_0^{1-\gamma}}{1-\gamma} \exp((1-\gamma)rT). \quad (7)$$

Rewriting (6) leads to

$$\frac{E_0 [(W_T^*)^{1-\gamma}]}{1-\gamma} = \frac{W_0^{1-\gamma}}{1-\gamma} \exp \left( (1-\gamma) \left( r + \frac{\lambda^2}{2\gamma} \right) T \right). \quad (8)$$

The certainty equivalent utility loss of a fully risk-free investment per unit of time thus equals  $\lambda^2/(2\gamma)$ . For the benchmark

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<sup>4</sup>Commonly known as “Wetsvoorstel doorbeleggen”.

parameters  $\lambda = 20\%$  this loss thus amounts to between 1% and 0.2% annually for risk aversions varying between  $\gamma = 2$  and  $\gamma = 10$ .

### **2.3. Optimal consumption and investment in the retirement phase**

The optimal consumption problem in the retirement phase consists of two (as we will see, independent) problems. The agent has to both determine the optimal allocation of initial retirement *wealth* to each of the pension *payments* and decide on the investment strategy. We will see that the optimal investment strategy, in the present setting, does not change. This holds even for the contribution phase; see Section 2.4. However, Sections 2.5 and 2.6 discuss how a first-pillar pension and human capital do affect the optimal investment plan.

We still assume that the agent has CRRA preferences over pension payments during the retirement phase, with risk aversion  $\gamma$  and time preference parameter  $\beta$ . We begin by formalizing this problem.

The conceptual idea behind allocating the available total retirement wealth to individual pension payments possibly may require more explanation. We consider the situation where the present value of all future pension payments equals the total available pension wealth. That is, there are no ex-ante transfers of pension wealth from one individual to another. In such a situation, one can think of the total available pension wealth consisting of the present value of the first pension payment, the second pension payment, and so forth. Equivalently, one may consider the total pension wealth, at each point of time, to be split in an amount needed to finance the first payment, an amount needed for the second payment, and so forth. That provides a convenient way to



think about allocation and smoothing of consumption over time.

We thus start with the allocation of total retirement wealth to payments for the individual retirement years. Denote the retirement year by  $T$ . Think of the pension *wealth* to be used to finance individual pension *payments* for the years  $T, \dots, T + h - 1$  for fixed  $h$ . For now, we ignore longevity risk, which will be discussed in Section 2.8.

To formalize the above, we split the available initial wealth  $W_0$  into  $h$  portions that are used to finance retirement consumption in period  $T + j, j = 0, \dots, h - 1$ . The optimization problem then becomes, for given intertemporal discount factor  $\beta$ ,<sup>5</sup>

$$\begin{aligned} & \max_{W_{0j}: j=0, \dots, h-1} \sum_{j=0}^{h-1} e^{-j\beta} \frac{\mathbb{E}_0 [(W_{T+j}^*)^{1-\gamma}]}{1-\gamma} \\ \text{s.t.} \quad & \sum_{j=0}^{h-1} W_{0j} = W_0, \end{aligned} \quad (9)$$

where  $W_{T+j}^*$  denotes the optimal achievable wealth at time  $T + j$  given initial wealth  $W_{0j}$ . In view of (6) we thus need to maximize

$$\begin{aligned} & \sum_{j=0}^{h-1} e^{-j\beta} \frac{W_{0j}^{1-\gamma}}{1-\gamma} \\ & \times \exp \left( (1-\gamma)r(T+j) - \frac{1}{2} \left(1 - \frac{1}{\gamma}\right) \lambda^2 (T+j) \right). \end{aligned} \quad (10)$$

This optimization problem, solved in Lemma 1, leads to

$$W_{0j} \propto \exp \left( -\frac{\beta}{\gamma} + \left( \frac{1}{\gamma} - 1 \right) \left( r + \frac{\lambda^2}{2\gamma} \right) \right)^j. \quad (11)$$

---

<sup>5</sup>The problem here is actually formulated for a deferred annuity, but that will not affect the results, as we shall see.

The optimal allocation of wealth over the payments at the various horizons is thus geometric in the horizon, with a coefficient that depends on  $\beta$ ,  $\gamma$ ,  $r$ , and  $\lambda$ . In this standard Merton case, a variable annuity with *fixed*, but suitably chosen, assumed interest rate (AIR) is optimal, as  $W_{0,j+1}/W_{0j}$  does not depend on  $j$ . Note that (11) only determines the relative allocations for pension payments at several maturities. The scale factor is determined by the available total wealth  $W_0$  and changes over time due to actual pension payments and financial market returns.

**Remark 1.** *The notion of assumed interest rate (AIR)<sup>6</sup> is most easily explained in this framework of the section as the way to distribute available pension wealth over the various pension payments.*

*Suppose that the available pension wealth at any point during retirement is  $W_0$ , and that we wish to allocate this wealth over payments for  $j = 0, \dots, h - 1$ . An AIR of  $a_0 = 2\%$  would mean that we reserve 2% less for the payment at date  $j + 1$  than for the payment at date  $j$ . In general, the assumed interest rates could be given by a term structure, i.e.,  $a_0$  depends on the horizon  $j$ . In the simplest setting in this paper the optimal AIR is horizon independent.*

*The intuition is that you may want to reserve less money for more distant pension payments, as that pot may still grow due to financial market returns. Taking an AIR equal to the risk-free interest rate, and investing all pension wealth risk-free would lead to a fixed annuity payment.*

*Finally, it is important to observe that the AIR may be changed over time, if one wishes to do so. However, the budget constraint*

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<sup>6</sup>The current Dutch pension debate features the term “projectierendement”. This has the same meaning as AIR. Alternatively, some people use the term “Assumed Rate of Return”.

*dictates (in the absence of the possibility of intergenerational risk sharing) that the amounts of money reserved for each future pension payment must add up to the available total pension wealth.*

**Remark 2.** *We use the term annuity for a life-long payment. For a nominal annuity, this payment is constant in nominal terms. For a variable annuity, the payment may vary over time. We use the term standard variable annuity for a variable annuity based on a constant assumed interest rate. In the present setting, the optimal AIR indeed turns out to be constant (both over time and horizon), but that will not be the case in other settings.*

*The above terminology does not reflect the underlying mechanism that is used to achieve these payments. Insurance companies often provide the variable annuity payments, but these could also be achieved by a so-called PPR (Personal Pension with Risk-sharing) type collective mechanism without external equity holders. The differences between these two systems are not relevant for the present paper.*

Summarizing, in the present setting the optimal consumption strategy is to use a standard variable annuity with (constant) assumed interest rate (AIR) given by

$$\text{AIR} = r + \frac{1}{\gamma} (\beta - r) - \frac{1}{2\gamma} \left( \frac{1}{\gamma} - 1 \right) \lambda^2. \quad (12)$$

We conclude the following:

- This optimal AIR, for  $\gamma > 1$ , increases in  $r$ , in  $\beta$ , and in  $\lambda$ .
- The optimal investment strategy is as in (3): an optimal exposure of  $\lambda/\gamma$  to the risk  $Z$ . In particular the optimal strategy depends on the preference parameters  $\gamma$  and  $\beta$ .

- In this setting with CRRA preferences, it is suboptimal to smooth retirement consumption by averaging past returns. Such a method, often advocated, proposes not reducing pension payments by an amount of the same magnitude as the shock that hit the pension wealth. This essentially increases the AIR after bad investment returns (and decreases it after good returns). See Section 2.12 for a discussion on habit formation preferences.
- The allocation of retirement wealth over the various retirement payments leads to a standard variable annuity with an explicit (constant) AIR. The underlying investment strategy is again independent of time, wealth, and horizon.

It is sometimes advocated that a proper AIR would equal the expected return on the underlying investment strategy, *i.e.*, in this case  $r + w^* \lambda \sigma$ . For the case  $\beta = r$ , one easily verifies that the optimal AIR is always smaller than the expected return on the underlying portfolio. The risk premium  $w^* \lambda \sigma$  needs to be multiplied by  $\frac{1}{2}(1 - \gamma^{-1})$  to obtain the optimal assumed interest rate. The relation between expected return and AIR is illustrated in Table 1.

Another common proposal is to use the nominal interest rate (curve) as the AIR in decumulation decisions. The rationale is then often given as that ‘expected inflation and risk premiums cancel’. Also this reasoning is not rooted in optimality arguments. In order to remain close to current practice, we will consider these suboptimal AIR’s in Section 3.

#### **2.4. Optimal investment in the contribution phase**

Plugging (11) into (10), we find the value function to be optimized during the contribution phase. It is obvious that the initial wealth

Table 1: Optimal AIR

$\gamma$	2	5	10
AIR	1.50%	1.32%	1.18%
$\mu_p$	3.00%	1.80%	1.40%

Entries show the expected return  $\mu_p = (1 - w)r + w\mu$  and the corresponding optimal assumed interest rate AIR in the Merton model. Benchmark parameters are  $r = \beta = 1\%$  and  $\lambda = 20\%$ .

when entering the retirement phase affects the value function through  $W_T^{1-\gamma}/(1 - \gamma)$ . As a result, the standard Merton investment solution holds during the contribution phase. Even stronger, we have a complete separability of the contribution and retirement phase. This property, which is convenient for implementation of optimal strategies, is lost under more complicated financial market models and/or preference structures.

### 2.5. Human capital

The Merton model is still applicable in a situation where agents have human capital, but then, wealth  $W$  should be interpreted as total wealth, that is, the sum of financial wealth and human capital. The latter is the present value of future labor income. Over the course of the life cycle, human capital generally decreases while financial wealth increases. In that case, a constant risky allocation to total wealth translates into a decreasing allocation to financial wealth. This is the standard life-cycle investment strategy used, with some variations, in practice; this will be the focus of

Section 3, where we assume that human capital is (almost) risk free. This is the most common assumption in the life-cycle literature, and is based on the low contemporaneous correlation between labor income growth and financial returns. Cocco, Gomes and Maenhout (2005), for example, estimate the correlation between permanent labor income shocks and stock returns at about  $-0.01$  and not statistically significantly different from zero. Some studies argue that human capital is far more risky. Examples are Benzoni, Collin-Dufresne and Goldstein (2007) and Lynch and Tan (2011). Benzoni et al. (2007) assume that the ratios of capital and labor relative to national income have a constant mean, which implies that human capital and financial capital have a very strong long-run correlation (they are cointegrated in formal terms). Such a high correlation implies that human capital is more like equity than like bonds, in the long-run, and hence leads to substantially lower optimal investment in equity at a young age. Lynch and Tan (2011) look at the business cycle frequency between labor and equity to reach a similar conclusion.

## **2.6. The first pillar**

For most Dutch pension participants, the not-means-tested salary-independent first pillar pension (AOW) is an important part of the total retirement provision. The average Dutch worker has, currently, about equal wealth in the first and second pillars. We now explore how this affects the analysis.

We consider the first-pillar pension to be a risk-free investment in (real) bonds. As a result, it can be considered as similar to human capital. The optimal strategy discussed thus far is then considered optimal for total wealth, defined as the present value of first-pillar entitlements and second-pillar wealth. We consider

utility to be defined over total consumption.<sup>7</sup> We discuss the consequences of the first-pillar pension in terms of both optimal asset allocation and optimal decumulation.

With respect to asset allocation, the optimal risky investment derived previously must be considered to refer to total wealth. The available first-pillar pension will thus lead to a larger risky investment of second-pillar pension wealth.

Over time, the ratio of first- and second-pillar wealth will change due to investment returns for the second-pillar part that may either be above or below those of the first-pillar part. As mentioned before, we assume the latter to be the risk-free return. In case the second-pillar return happens to equal the first-pillar return, there is no reason to adapt the investment mix. In case the second-pillar pension return lies above (below) the risk-free rate, the second-pillar pension becomes a larger (smaller) share of total pension wealth. As a result, the optimal investment weight in the risky asset for the second-pillar is reduced (increased). Note that this effect is actually opposite to what we will find in Section 2.12 concerning habit formation. In that case, a high (low) return leads to an increase (decrease) in risk taking.

Summarizing, the existence of a first-pillar pension is no reason to directly reduce risk-exposure for the second or third pillar with age, but it is a reason to reduce risk exposure after positive excess returns. Consequently, in expectation, there will be a decreasing allocation to risky assets with respect to age.

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<sup>7</sup>In case one considered utility to be defined over second-pillar pension payments only, the previous results would obviously be unaffected by the presence of a first-pillar pension.

## 2.7. Consumption before retirement

It is important to note that we consider a setting where contributions/labor supply of individuals is exogenously given. It is to be expected that large welfare gains are possible in case agents can increase premiums before retirement in the event of low financial market returns. As a result, they anticipate future drops in consumption. There may be other institutional circumstances which actually limit premium flexibility.

## 2.8. Idiosyncratic longevity risk

The optimization problem now becomes

$$\begin{aligned} & \max_{W_{0j:j=0,\dots,h-1}} \sum_{j=0}^{h-1} \beta^j \frac{E_0 (W_{T+j}^*)^{1-\gamma}}{1-\gamma} p(0:T+j) \\ \text{s.t.} \quad & \sum_{j=0}^{h-1} W_{0j} p(0:T+j) = W_0, \end{aligned} \tag{13}$$

where  $p(0:T+j)$  denotes the (expected) survival probability of the individual from time 0 to time  $T+j$ . Note that this implies, in line with much of the literature, that we assume that future utility is discounted with survival probabilities and that idiosyncratic longevity risk is shared in a large pool of identical agents. Such a view may be contested. However, in this case, Lemma 1 in appendix A.2 implies that idiosyncratic longevity risk does not change the optimal allocation of total initial wealth to payments for periods  $T, \dots, T+h-1$ . The optimal assumed interest rate and the optimal underlying portfolio thus do not depend on survival probabilities of individuals, as long as these are independent of financial markets.

Systematic longevity risk, in the sense that economic and macro longevity developments are dependent, is not taken into account



here. Such a correlation can have non-trivial consequences for the optimal allocations.

## 2.9. Interest rate risk

Interest rate risk makes the investment opportunity set time-varying, which leads to the inclusion of hedge demands in the optimal portfolio allocation. Optimizing in a richer model incorporating interest rate risk is possible. Examples abound in the rich literature on strategic asset allocation; see Campbell and Viceira (2001) for an early overview. In general, the allocation to equity and bonds will become time-varying. In most cases solutions must be obtained numerically. These solutions show volatile optimal allocations for both equity and the composition of the bond portfolio, reacting to every change in the shape of the term structure. The volatile behaviour of the equity allocation is mostly driven by a time-varying equity premium in this model.

The Campbell and Viceira (2001) and Brennan and Xia (2002) models remain closest to the Merton model analysed in section 2.2, since they assume constant risk premiums on equity and nominal bonds. With these restrictions the — perhaps somewhat surprising — result is that the optimal allocation to risky assets and bonds will remain time-independent, although it does become age-dependent. The actual level of the interest rate is thus irrelevant for the optimal allocation of wealth to stocks and bonds. However, the horizon  $T$  does become relevant. The optimal allocations will, next to the horizon, depend on the actual prices of risk, volatilities and preferences of the agents.

Both models assume that nominal interest rates are driven by two risk factors: the real rate of interest and inflation. Given assumptions and estimates for the time series processes for inflation and the short-term nominal interest rate, bond yields  $R_t^{(n)}$

of maturity  $n$  follow as

$$R_t^{(n)} = a_{0n} + a_{1n}\pi_t + a_{2n}r_t, \quad (14)$$

where  $\pi$  is expected inflation and  $r$  is the real rate of interest. The coefficients are maturity-specific and depend on the parameters of the time series model for the short-term interest rate and inflation. The coefficients are important for deriving the optimal allocations to long-term bonds. Exact derivations can be found in Brennan and Xia (2002), but it does not seem possible to transform these into an easily implementable setting concerning the current pension debate.

If inflation-protected bonds are available at all maturities, these are the best instruments for highly risk averse investors. When only nominal bonds are available, the optimal allocation strongly depends on the estimated parameters, particularly on the relative importance of inflation and interest rate risk. When inflation risk is large, investors should mainly buy short-term bonds. When interest rate risk dominates, long-term nominal bonds are favoured more. The equity weight is still very much determined by the equity premium. With a sufficiently large equity premium, the optimal allocation for a young person with low-risk human capital will still be 100%.

Based on the extensive empirical evidence (*e.g.*, Fama (1984), Campbell and Shiller (1991), Cochrane and Piazzesi (2005)), more elaborate models allow for a time-varying risk premium on bonds. Sangvinatsos and Wachter (2005) and more recent studies therefore derive the portfolio implications using the essentially affine term structure model of Duffee (2002). The solution and estimates in Sangvinatsos and Wachter (2005) exhibit the typical volatile behavior of optimal allocations.

In our simulations we explore the value of hedging interest rate risk by following a simplified hedging strategy. The bond investment according to the Merton model is invested in deferred annuities that start at the retirement date.

We do show (in Section 5) that the certainty equivalent losses from suboptimal interest rate exposure can be fairly large. Again, noting that optimal allocations are very model-dependent, it is difficult to provide practical advice for “the optimal interest rate exposure”, even in purely individual contracts.

### **2.10. Inflation**

Inflation plays an important role in long-term investment problems. To the extent that inflation is constant and money-illusion on behalf of the agents is ignored, the results derived in this paper are still valid as long as  $r$  is interpreted as the real interest rate.

When inflation is time-varying, and possibly commands a risk premium, analytical results are possible along the lines of Brennan and Xia (2002), under the assumption that prices-of-risk are still constant. That situation parallels that of time-varying interest rates. For the same reasons as mentioned before we will not go into this.

### **2.11. Committee parameters model**

The Dutch Parameters Committee essentially used a model that allows for state-dependent prices of risk. As a result no analytical results concerning the optimal investment and premium/consumption strategy are possible. Analytical results on the valuation of financial assets are possible, but are not needed for the rest of this paper; see Kojien, Nijman, and Werker (2010).

## 2.12. Habit formation

Habit formation is often assumed to be a relevant feature of an individual's preferences. A complete analysis of the numerical consequences of habit formation preferences is beyond the scope of this paper, but the interested reader is referred to Zhou (2014) and van Bilsen (2015) where optimal investment and consumption strategies are derived, with a particular focus on saving for retirement. As far as currently known, no explicit analytical solutions to the investment and consumption problem are available.

Habit can either be external or internal. External habit formation has to do with preferences based on the relative consumption of an individual with respect to his or her peers. This is sometimes known as 'keeping up with the Joneses'. Internal habit formation refers to non-time separability of preferences and is generally modeled as preferences defined relative to an individual's own prior consumption. Generally speaking, papers tend to focus only on one of these two forms of habit formation.

Focusing on internal habit formation, the academic results essentially state that the investment portfolio contains two parts: one relatively riskfree part that is used to ensure that the habit consumption level is achieved in the future and one more risky part that is used to profit from the equity premium. The exact allocations over both parts will be time-, state-, and horizon-dependent. Internal habit formation induces an agent to smooth returns; shocks in pension wealth are thus not translated immediately into a shock of the same size in consumption. As an example, suppose that with three more pension payments ahead, an individual's pension wealth drops by 10%. A CRRA agent would keep the AIR constant and, thus, reduce consumption for each of

the next three years by 10%. Under habit formation, an agent would perhaps reduce expected consumption next year by only 5% but in three years by 15%.<sup>8</sup> This is achieved by actually increasing the AIR after this negative shock. At the same time, optimal investment strategies will also adapt to these shocks: a negative return will lead to a more cautious strategy in the future.

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<sup>8</sup>In the Dutch pension debate this is known as “uitsmeren van schokken”.

### 3. Life-cycle investment results

We consider a number of benchmark optimized strategies from a stylized Merton life-cycle model, and compare these strategies with the composition of a standard life-cycle fund with a linear decreasing exposure to equity. Construction of the optimized portfolios requires assumptions on the number of available assets, their risk and return parameters, and assumptions on the income and contributions of participants.

For the portfolio construction we consider the stylized Merton model with equity and real risk-free bonds. Parameters are consistent with the table in 'Advies Commissie Parameters' (2014), implying a 7% geometric expected nominal return on equity with a volatility of 20%. Since a real risk-free rate does not exist, we assume that the bond earns the return of an investment in a nominal bond with a 10-year duration, which we assume to be 3.5%, consistent with the current term structure (geometric mean, Fall 2015). For the purpose of the portfolio optimization the bonds are assumed to be risk-free. To remain close to the stylized life-cycle model, we keep the portfolio choice limited to bonds versus equity, without distinguishing different bond maturities or making a distinction between nominal and inflation-linked bonds. The main purpose is to find typical exposures to equity over the life-cycle.<sup>9</sup>

An individual pays an annual pension contribution that is

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<sup>9</sup>Brennan and Xia (2002), Campbell and Viceira (2001) and Sangvinatsos and Wachter (2005) are typical examples that have explored the optimal allocation to different types of bonds in addition to equity in a life-cycle model with inflation and interest rate risk. Allowing for different types of bonds leads to interesting patterns which bonds an investor should choose, but has little effect on the equity weight.

Table 2: Pension premium percentage

age	25-29	30-34	35-39	40-44	45-49
percentage	5.7%	6.9%	8.4%	10.2%	12.5%
age	50-54	55-59	60-64	65-66	
percentage	15.4%	18.9%	23.6%	27.7%	

The entries in the table show the percentage of labor income (minus franchise) paid as a contribution to the pension system.

invested in both equity and bonds. We assume that contributions are exogenously fixed and independent of realized returns on the investment portfolio. The percentage pension premium is also independent of labor income. We assume that the percentage contribution increases with age. For young participants at age 27, the premium is 5.7% of their labor income. The premium increases to 27.7% at age 66 according to the schedule in Table 2. As an alternative we also consider a fixed premium of 12.5% of income, which leads to a faster build-up of pension wealth.

Portfolios are constructed to maximize the expected utility of pension wealth  $W_T$ . The strategies are optimized for investors with different levels of risk aversion and different age-income profiles. Labor income growth is assumed to be either low or high, with real growth rates taken from a study by Knoef and Been (2015). High growth means real income growth of 8.5% at age 25, slowing down to less than 1% after age 43. With low income growth the growth rate is initially 2.5%, increases to about 4% and then declines to zero at age 48. We assume initial income at age 25 equal to  $k\in\text{€}25$  in both cases. Apart from the age-related income growth there is a macro wage growth equal to inflation. For the

portfolio optimisation we estimate the equity exposure of labor income to be  $\beta_W = 0.015$ ,<sup>10</sup> which means that it is almost riskfree. The value of human capital at each time is computed as the present value of expected future labor income discounted at the riskfree rate plus  $\beta_W$  times the equity premium. Because human capital is such a large proportion of total wealth at a young age, the optimal investment in equity will often exceed 100% of financial wealth. In our simulations we never allow such leveraged positions for the optimized portfolios.

In the optimization we differentiate between homeowners and renters. A homeowner is assumed to buy a house at age 37 at a price of k€200. The decision to buy a house is completely exogenous in our model.<sup>11</sup> The house is treated as a financial asset that may change in value. Its return is assumed to be correlated with the stock market with a beta of  $\beta_H = 0.2$ . The purchase of the house is financed by a mortgage that is amortized over 30 years, implying that the house is debt-free when the individual reaches the age of 67. At retirement the individual sells the house and adds its value to pension wealth.<sup>12</sup> Since the house is a portfolio of implicit exposures to stocks and bonds, it affects the

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<sup>10</sup>All betas are relative to the return of the S&P500.

<sup>11</sup>See Cocco (2005) for a model with endogenous homeownership and a discussion of its implications for optimal investment of financial wealth.

<sup>12</sup>Since the mortgage is completely paid off during the working life, we implicitly assume that a homeowner spends less on consumption than a renter does, and therefore saves more for retirement. As a result, the homeowner will have a greater pension wealth. Pension contributions are the same as those of a renter, but on top of this the homeowner has the value of the house. Since in practice pension contributions for homeowners and renters are the same, we optimize portfolios under this assumption. This means, however, that we cannot compare the pension benefits of homeowners and renters in our life-cycle model without taking into account intermediate consumption during working life.



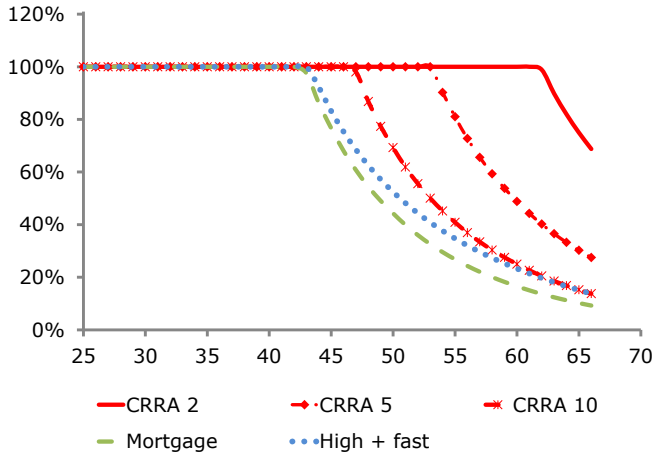
optimal investment of the pension contributions. In particular, the implicit equity exposure of the house enables investors to increase the equity share to be above 100% of financial wealth.

Figure 1 shows the optimized portfolios under different conditions. All optimized portfolios have financial wealth fully invested in equity up to age 45. The equity share is at 100% because of the equity premium and because labor income is considered to be a safe asset. Without the upper bound of 100%, the model implies that more than 100% of liquid wealth should be invested in equity in order to obtain the desired overall risky share. Upon reaching middle age individuals should start decreasing their investment in equity, since their human capital is diminishing. The lower the risk aversion, the later the downward slope of the equity exposure starts. For individuals with low risk aversion it is optimal to remain fully invested in equity until their sixties.

Homeowners are the first to reduce their equity holdings, since their house involves an implicit exposure to equity, while the mortgage is a short position in bonds. The line in the figure is for homeowners with high risk aversion ( $\gamma = 10$ ). We also find a decrease in equity holdings around age 45 in case we assume a flat pension contribution of 12.5% and high risk aversion. This flat rate leads to a faster growth of pension wealth and hence to a larger total investment in equity.

Figure 1 does not show every combination of income growth, risk aversion and homeownership. Alternative configurations exhibit the same pattern, however. Initially the equity investment is 100%, and depending on various characteristics the equity investment starts to decrease between the ages 45 and 62. Portfolios for persons with high or low income growth are almost indistinguishable. Given the assumptions on the equity premium it

Figure 1: Optimized portfolio strategies



The figure shows the percentage of financial wealth invested in equity (vertical axis) against age (horizontal axis). Different lines refer to different strategies. Three strategies shown differ only by level of risk aversion (2, 5, 10); two other strategies assume risk aversion level 10 in addition to homeownership financed by a mortgage or a flat contribution rate of 12.5%.

remains optimal to have some exposure to equity until and after retirement. The same patterns also emerge for alternative assumptions on the equity premium. In the Merton model the weight of equity is determined by the Sharpe ratio divided by the risk aversion coefficient. A higher assumed equity premium is equivalent to a lower CRRA parameter. Similarly, for investors that exhibit ambiguity with respect to uncertainty in the equity premium, results in Maenhout (2004) imply that one should replace the CRRA coefficient with the sum of risk aversion and ambiguity aversion. Again this would not change the pattern in figure 1. We therefore consider this set of strategies as representative for the heterogeneity in preferences and other characteristics.

The optimized strategies deviate from standard linear strategies. Linear strategies have an equity exposure that decreases linearly with age. Optimal portfolios in the Merton model are first flat at 100% and then decrease steeply to a low equity position at retirement.

In the life-cycle model with constant investment opportunities, it is optimal to keep the equity weight constant. The portfolios that we discussed are time-invariant in the sense that the equity share in the portfolio is fixed as a proportion of *total* wealth, which we define here as the sum of financial wealth plus human capital.<sup>13</sup> Even in the model with homeownership the portfolio has fixed weights, since the house is seen as a portfolio consisting of 80% investment in the risk-free asset plus a 20% equity stake, financed

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<sup>13</sup>Human capital itself includes the present value of future labor income and the present value of AOW benefits. Excluding the first pillar has only a minor effect on the optimal equity share, but it does of course heavily affect the evaluation of the benefits during retirement.

by a short position in risk-free debt. Constant weights are optimal, since the expected return and risk of bonds and equity are assumed to be constant over time. The extensive literature on strategic asset allocation (see Campbell and Viceira (2002) for an introduction) allows for time-varying investment opportunities. In that case the portfolio weights at a given time will depend on state variables such as the price-dividend ratio, the level of interest rates, inflation and credit risk and business cycle indicators. Even average shares in equity will depend on any auxiliary assumptions regarding the mean reversion of stock returns, inflation and interest rates, and the strength of the predictability of stock and bond returns. All these auxiliary parameters are highly uncertain and also not part of the standard parameter set in the 'Advies Commissie Parameters'. For that reason, and because we wish to focus on a typical life-cycle portfolio, we ignore the complications from time-varying investment opportunities in optimizing the life-cycle portfolios. We will, however, consider a richer set of financial and economic risks in our evaluation of the alternative portfolio strategies.

## 4. Evaluation of strategies

### 4.1. Distribution of pension result

To evaluate the strategies we rely on simulation. Scenarios in the simulation are as provided by DNB.<sup>14</sup> These scenarios do not necessarily have the same mean and variance as in the 'Advies Commissie Parameters'. The scenarios involve inflation and interest rate risk. Since our bond investment refers to a nominal bond with 10-year duration, returns are subject to interest rate risk. That means that the optimized portfolios are not necessarily optimal.

We compare the optimized portfolios to three linear life-cycle products. At age 25 these products start with an initial allocation between equity and bonds. Each year the allocation to equity is reduced by a fixed percentage until at age 67 a final low allocation to equity is reached. The three linear strategies differ in their initial and final allocations to equity. The *aggressive* strategy starts with an initial allocation of 120% to equity, which is decreased to 60% at age 65. The initial allocation involves some leverage. We include this product, because the standard Merton model often leads to optimal portfolios that are highly leveraged at young age. An individual investor will have limited means to create such an investment, but as a managed investment within a pension fund it can be created. The leverage is small, since invested wealth in the product is limited at a young age. The *flat* strategy always invests a constant proportion of pension wealth in equity, which we assume to be 50%. The *medium* strategy is meant to be an intermediate life-cycle portfolio with an initial allocation of 80% at age 25, which

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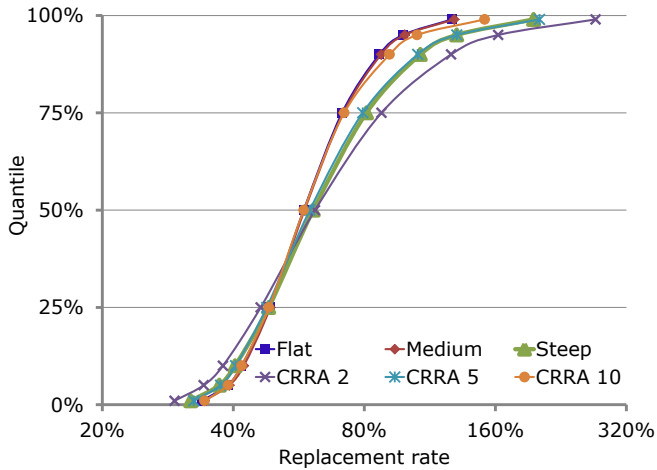
<sup>14</sup>See <http://www.toezicht.dnb.nl/3/50-233690.jsp>. In the actual simulations we simulate additional scenarios with a longer horizon using the specification of the underlying data generating process.

gradually decreases to 40%.

Figure 2 shows the distribution of pension benefits under the different strategies. The distributions differ according to the background characteristics on human capital and employment history. The case shown in the figure refers to an individual with high income growth who has worked full time during his or her entire working life. For the figure we assume that the benefits are converted to a nominal annuity at retirement. As a normalisation we show the distribution in the form of the replacement ratio at retirement including first-pillar AOW benefits. Given our contribution policy and the equity premium parameter, the median pension is between 58% and 62% of the last earnings, with the higher values for the strategies with more equity exposure. Overall, median pension results are very similar under all strategies.

The differences are in the dispersion of the pension result. All of the distributions cross each other somewhere, implying that none of the strategies is completely stochastically dominated by one of the others. By design the most extreme distribution is for a strategy optimized for the least risk-averse individual. In that case the 1% lower bound of the replacement ratio is barely 30%, whereas the upper 99% quantile is close to 270% of final income. The lower 1% quantiles of the other strategies are very similar, all in the range of 32%-34% replacement ratios. They differ in the upside: the CRRA-5 and the steep life-cycle product imply an almost identical pension distribution. The other strategies, CRRA-10 and the two less aggressive life-cycle products, form another group with a very similar pension distribution. The two groups differ at the high end of the distribution, where the more aggressive strategies produce better results. The 95% percentiles of the more aggressive strategies are all above 130% replacement

Figure 2: Distribution of pension results



The figure shows quantiles of the benefit payments. The horizontal axis is the replacement ratio (logarithmic scale) of the first retirement benefit, assuming that benefits are paid as a nominal annuity. The vertical axis shows quantiles of the distribution. Different lines refer to different strategies. The figure shows strategies optimized under three levels of risk aversion ( $\gamma = 2, 5, 10$ ) plus three linear life-cycle products.

rates.

Another way to illustrate the large dispersion in pension results is by looking at the results for successive cohorts, who differ just a single year in their retirement date. With the aggressive CRRA-2 strategy there is a 10% probability that successive generations differ by as much as 30% in their replacement ratio, even though they have almost completely overlapping return histories. With a large allocation to equity, the final year can make a huge difference in the outcome. The other strategies are not that extreme, but a 5% difference in pension result for successive generations is always within the 90% confidence region.

#### **4.2. Certainty Equivalents**

For a more formal comparison of strategies we compute certainty equivalents. The objective in constructing the optimal life-cycle strategies is the utility of pension benefits during retirement age. We distinguish between two different aspects of the pension design: the pre-retirement portfolio strategy and the post-retirement annuity conversion. Within the Merton model with constant investment opportunities the accumulation stage and the annuity conversion can be separated. The ordering of strategies should not be affected by different annuitisation schemes and should be the same as the ordering of the certainty equivalents of pension wealth. The portfolio strategies in Figure 1 are defined for the pre-retirement period. How pension wealth  $W_T$  at retirement is allocated over the remaining lifetime in the post-retirement period will not affect the optimality of pre-retirement portfolio choice under the assumption of constant investment opportunities. Expressions and numerical values for these were discussed in Section 2.3. However, since the actual scenarios in our simulations exhibit interest rate risk and inflation



risk, the separation may not hold. We therefore evaluate the portfolio strategies based on realized utility of the actual benefits they produce. For the benefits we consider two annuitisation schemes: nominal and variable annuities. Variable annuities allow a continuation of the equity exposure after retirement and should be the optimal design in the stylised Merton model. Regarding the variable annuity the equity weight after retirement is kept at the same level as at the retirement age 67.<sup>15</sup>

The certainty equivalent is defined as the level of certain annuity income during retirement that would make an individual indifferent between that level and the random annuity income resulting from a particular annuity scheme and (pre-retirement) portfolio choice. Utility during retirement is computed by averaging the realised utility for many different possible outcomes regarding pension wealth at retirement and the resulting annual income stream generated by that pension wealth. We weight the utility for each annual retirement benefit with the survival probabilities in the official mortality tables of the 'Actuariële Genootschap'.

To evaluate post-retirement annuity choice we compute the certainty equivalent using two different specifications of utility. The first specification is the standard power utility function with constant relative risk aversion, assuming all annual benefits are

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<sup>15</sup>This is not necessarily always the optimal weight. As discussed in section 2.3, the optimal weight may vary over time depending on the realized returns. AOW payments and annuity benefits determine the value of the remaining wealth in the first and second pillars. The optimal equity weight depends on the sum of the two components.

consumed. Realised utility for a single scenario  $\ell$  is calculated as

$$U_\ell = \sum_{t=T}^{\tau} \beta^t p_t \frac{b_{t,\ell}^{1-\gamma}}{1-\gamma}, \quad (15)$$

where  $b_{t,\ell} = B_{t,\ell}/\Pi_{t,\ell}$  are real benefits, with  $B_{t,\ell}$  nominal benefits at age  $t$  and  $\Pi_{t,\ell}$  the consumer price level;  $T$  is the retirement age,  $p_t$  the survival probability;  $\gamma$  the coefficient of relative risk aversion; and  $\beta = 0.98$  the time preference parameter. The benefits can be from either a nominal or a variable annuity. Expected utility is estimated by averaging over a large number of scenarios. Certainty-equivalent real benefits  $b$  are then defined as the constant  $b = B/\Pi$  that sets the right-hand side of (15) equal to average utility, *i.e.*,

$$b^{1-\gamma} = \frac{1}{M} \sum_{t=T}^{\tau} w_t \sum_{\ell=1}^M b_{t,\ell}^{1-\gamma} \quad (16)$$

with  $w_t = \beta^t p_t / \sum_s \beta^s p_s$ .

For the second specification we consider the replacement ratio, which scales the benefits by the final real earnings before retirement ( $Y_T/\Pi_T$ ),

$$R_t = \frac{B_t/\Pi_t}{Y_T/\Pi_T}. \quad (17)$$

Using the replacement ratio provides a simple statistic to compare pension results for individuals with different income levels.

Assuming power utility for the replacement ratio, the certainty-equivalent replacement ratio follows as

$$R^{1-\gamma} = \frac{1}{M} \sum_{t=T}^{\tau} w_t \sum_{\ell=1}^M R_{t,\ell}^{1-\gamma}. \quad (18)$$

If the final real wage  $Y_T/\Pi_T$  would be independent of the benefits, it would just be a scaling factor and not affect the evaluation of the different portfolio strategies. Since in the scenario data there is a covariance between the real benefits and the real final wage, average utilities in the two specifications will differ by more than a scaling factor and can affect the ordering of the strategies. When financial returns are correlated with real income, the optimal portfolio will be affected and should have contained a hedge demand to insure against changes in real income at retirement. We interpret the certainty equivalents for the replacement ratio as a robustness check on the different life-cycle portfolio strategies. These strategies have been optimized assuming power utility over real final wealth, and not wealth relative to real final earnings. How do the strategies perform when they are evaluated by individuals with different preferences than we assumed in the optimisation?

We evaluate the certainty equivalent measures  $b$  and  $R$  for each of the portfolio strategies  $i$ , for both annuity schemes, at three different levels of risk aversion ( $\gamma = 2, 5, 10$ ) and for two different assumptions on income growth. Since we consider 8 portfolio strategies (5 optimized strategies plus 3 linear products) we obtain a total of  $8 \times 3 \times 2 \times 2 = 96$  certainty equivalents. Obviously, people with higher income or higher income growth, receive on average higher pension benefits. Certainty equivalents will also be lower for higher values of  $\gamma$ , since individuals with higher risk aversion impose a stronger penalty on the same risky distribution of outcomes.

To facilitate the comparison of the portfolio strategies we normalise the certainty equivalents. For each particular strategy  $i$  we compute the certainty equivalent benefits  $b_i$  as it is evaluated

by an individual with risk aversion  $\gamma$ , holding annuity type and income growth constant. Among the strategies is the strategy that has been optimized for this level of risk aversion level (say  $b_1$ ), which we therefore would expect to have the highest certainty equivalent. As a normalisation, all certainty equivalents are expressed as the percentage gain relative to the strategy that would be optimal for this individual,

$$b_i^* = \frac{b_i}{b_1} - 1. \quad (19)$$

By definition  $b_1^* = 0$ . If the ‘optimized’ strategy is indeed the optimal strategy, we would expect  $b_i^* \leq 0$  for all other strategies. When measuring utility by the replacement ratio, which already scales all outcomes by income level, we normalise the certainty equivalent by taking the difference between the optimized and alternative certainty equivalent ( $R_i^* = R_i - R_1$ ).

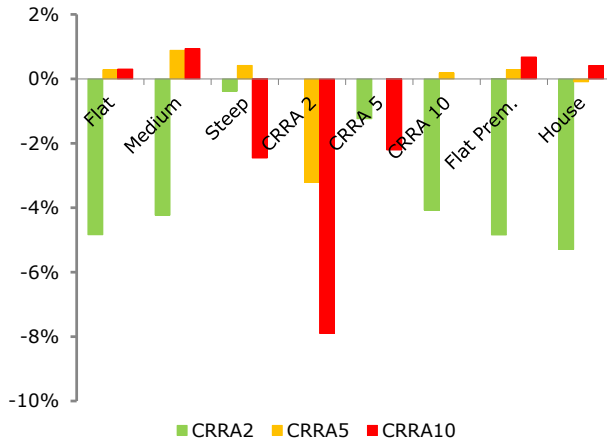
Figure 3 shows the normalised certainty equivalents, assuming benefits are paid as a nominal annuity. In the figure all green bars are negative. For an individual with risk aversion  $\gamma = 2$  the optimized strategy is therefore best among the 8 strategies we compare. The individual with very low risk aversion experiences (sometimes large) losses if his contributions are invested more cautiously using any of the other strategies. The only strategy that is perceived as equally attractive as the optimized portfolio is the aggressive linear strategy that starts with 120% in equity. Losses relative to most other strategies are in the order of 4% of real annual benefits, with the loss relative to the medium risk averse CRRA-5 strategy being only about 1%.

The aggressive equity strategy is far from optimal for more risk-averse individuals: the red and orange bars are firmly negative for the CRRA-2 strategy. Interestingly, two of the linear strategies

perform very well, even slightly better than the optimized strategy, for individuals with medium and high risk aversion. In the figure positive entries are possible for three reasons. First, the strategies have been optimised given the parameters laid down in the report of the 'Advies Commissie Parameters', but the model that generates the scenarios for the evaluation have different properties as they also exhibit interest rate risk and inflation risk. Second, we restricted the equity position in the optimal portfolios at a maximum of 100%, whereas one of the fixed life-cycle products starts with a leveraged position of 120% equity. Third, the nominal annuity is not the optimal benefit rule. Very risk-averse individuals are indifferent about almost all investment strategies, except for the aggressive strategy optimized for the lowest level of risk aversion. In the figure the red bar points sharply negative for the CRRA-2 strategy. Evaluating from the perspective of an individual with a medium level of risk aversion the losses are smaller, but again the biggest loss occurs for the aggressive portfolio strategy.

Figure 4 shows the certainty-equivalent gains of replacement ratios, for which benefits are scaled by final earnings. The pattern is similar to what we discussed before, except for the different scaling of the vertical axis. Taking both panels of the figure, the CRRA-5 strategy appears to be the most robust. It is the minimax strategy with the lowest maximum loss. Among the linear strategies, the minimax solution is the most aggressive strategy. It is almost optimal for individuals with either low or medium risk aversion. With high risk aversion there is a cost of about 1% replacement ratio relative to the strategy that is optimized for risk aversion  $\gamma = 10$ .

Figure 3: Certainty equivalent gains: real benefits

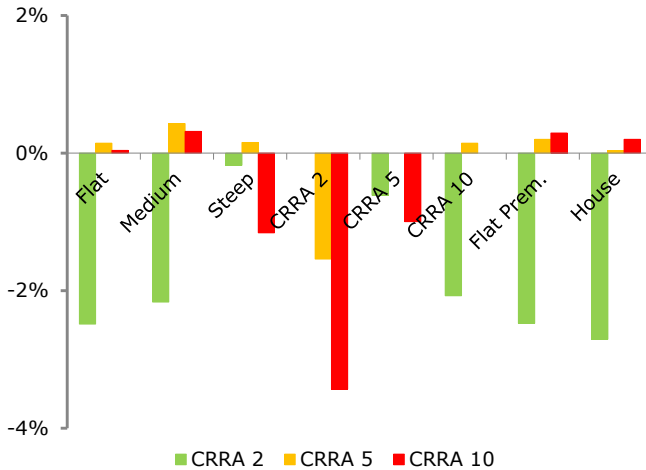


The figure shows certainty equivalent annual benefits from a nominal annuity if during working life contributions are invested according to the strategy label on the horizontal axis. Evaluation is according to individuals with three levels of risk aversion indicated by the colored bars. Utility is measured in terms of real benefits (see (15)). Certainty equivalents are normalised to gains relative to the optimized strategy.

### 4.3. Variable annuities

To evaluate the choice between nominal and variable annuities we compare for each portfolio strategy  $i$  the certainty equivalent of nominal and variable annuities:  $b_{iV}/b_{iN} - 1$  and  $R_{iV} - R_{iN}$ , where  $V$  and  $N$  refer to the variable and nominal annuity, respectively. According to theory variable annuities should be preferred. In the simulations they are indeed always preferred over nominal annuities, regardless of the portfolio strategy and the

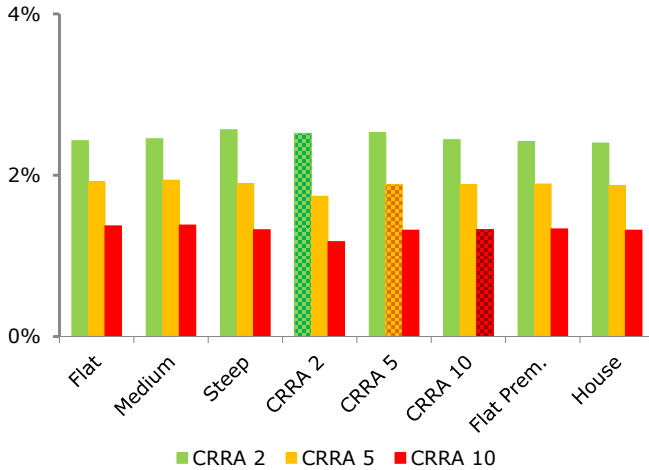
Figure 4: Certainty equivalent gains: replacement ratio



The figure shows certainty equivalent annual benefits from a nominal annuity if during working life contributions are invested according to the strategy label on the horizontal axis. Evaluation is according to individuals with three levels of risk aversion indicated by the colored bars. Utility is measured as a fraction of final earnings (see (18)). Certainty equivalents are normalised to gains relative to the optimized strategy.

level of risk aversion. Figure 5 shows the differences between the certainty equivalents for the variable and nominal annuities ( $R_{iV} - R_{iN}$ ). The gains are largest for less risk-averse individuals, but even for the most risk-averse individuals the cost of nominal annuities is about 1.5% in the replacement ratio. The case for variable annuities is the clearest message from the analysis.

Figure 5: Variable versus fixed annuities



The figure shows certainty-equivalent annual gains in replacement ratio from moving from a nominal annuity to a variable annuity. The difference in certainty equivalent replacement ratio is shown for each portfolio strategy (X-axis labels) and evaluated from the perspective of individuals with three levels of risk aversion represented by colored bars.



## 5. Interest rate risk

For the optimized strategies it was assumed that the interest rate was riskfree, resulting in a flat term structure. In the simulations interest rate risk is obviously present. Optimizing in a richer model with interest rate risk is possible; see *e.g.* Campbell and Viceira (2001), Brennan and Xia (2002) and Sangvinatsos and Wachter (2005). Contrary to the standard Merton model the allocation to equity and bonds will become time-varying, since a time-varying level and slope of the term structure represent time-varying investment opportunities. In these models the life-cycle portfolios in figure 1 will become time-varying and depend on the term structure. The general solution and estimates in Sangvinatsos and Wachter (2005) show that both the equity allocation and the composition of the bond portfolio become very volatile, reacting to every change in the shape of the term structure.

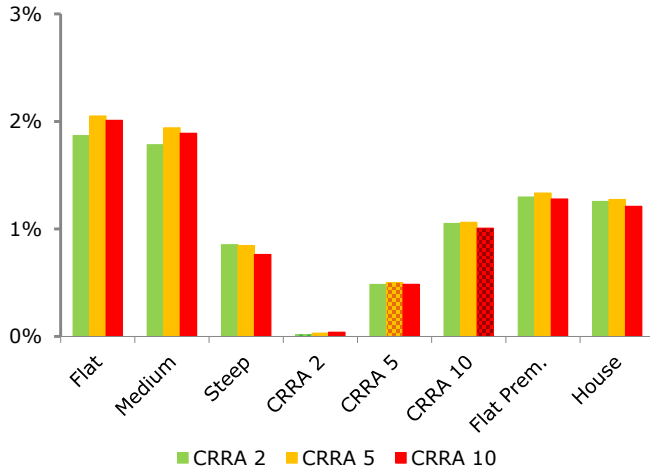
Empirically, interest rate risk has little effect on the equity weight, but strong effects on the optimal composition of the bond portfolio. The equity weight is still very much determined by the equity premium. With a sufficiently large equity premium, the optimal allocation for a young person with low-risk human capital will still be 100% on average. In models with a time-varying equity premium the allocation to equity can become very volatile. In the more restricted models of Campbell and Viceira (2001) and Brennan and Xia (2002) both the equity premium and the price of interest rate risk are constant. With these restrictions the optimal portfolio will become time-invariant (not age-invariant). In the Campbell and Viceira (2001) model bond prices are driven by two factors: the real interest rate and inflation. What kind of fixed income instruments are optimal in this model depends on the relative importance of inflation and interest rate risk. If inflation

protected bonds are available at all maturities, these are the best instruments for very risk-averse investors. When only nominal bonds are available, the optimal allocation strongly depends on the estimated parameters. When inflation risk is large, investors should mostly buy short-term bonds. When interest rate risk dominates, long-term nominal bonds are favoured more.

In our theoretical overview of the Merton model we assumed a constant real risk-free rate. Since this rate does not exist, the simulated strategies used the return on a 10-year nominal discount bond as the fixed income investment. This is suboptimal, but finding an optimal strategy is not straightforward, since this requires a view on the relative importance of interest rate risk and inflation risk over different investment horizons. When inflation risk is low and the bond risk premium is also small, a near optimal strategy will be to invest in nominal bonds with maturity equal to the expected remaining lifetime of the investor. The bond investments have payoffs that coincide with the timing of the benefit payments. We evaluate the value of hedging interest rate risk by following a simplified hedging strategy. The bond investment according to the Merton model is invested in deferred annuities that start at the retirement date.

Results for this investment strategy are shown in figure 6. The figure shows the gains of replacing the 10-year bond by the maturity matched bonds. For most life-cycle strategies this leads to a certainty equivalent gain of 1-2% in the replacement ratio. The only exception is the aggressive life-cycle strategy for the CRRA-2 investor. Here the interest rate hedge does not create value, since this investor is almost fully invested in equity during most of his or her working life. The gains are sizable, because the scenarios in the evaluation are generated by a model in which inflation risk is low.

Figure 6: Variable versus fixed annuities



The figure shows certainty-equivalent annual gains in replacement ratio from moving from the standard Merton allocation to an allocation in which all fixed income investments are in bonds with maturity equal to the average remaining lifetime of the investor. The difference in certainty equivalent replacement ratio is shown for each portfolio strategy (X-axis labels) and evaluated from the perspective of individuals with three levels of risk aversion represented by colored bars.

## 6. Discussion

In interpreting the results it is useful to note that we have only considered portfolio choice, keeping labor supply and contributions exogenously fixed. Flexibility in the retirement age or additional savings can mitigate the risk compared to our simulations. Blake, Wright and Zhang (2014) jointly determine the portfolio strategy and the optimal contributions a DC pension account. Flexibility in contributions appears to be an important element of optimal life-cycle strategy. In that sense we may have overestimated risk.

Our evaluations may also understate the risks. In computing the certainty equivalents, we assume that the risk and return parameters in our model are correct. In the simulations it is assumed that whatever parameter values have been decided by a committee, are the true values. In reality the expected returns are estimated with considerable uncertainty. This uncertainty increases the riskiness of the returns, especially over longer horizons. If it is assumed that expected return on equity is 7% with a standard deviation of 18%, then getting the average return wrong by 1 or 2 percent will not greatly affect the risk on an annual horizon. But if the same error is made year after year, the cumulative effect over a 40-years investment horizon can be huge.

Our evaluation of strategies assumes that preferences can be described by a utility function with constant relative risk aversion. We consider three levels of risk aversion, but do not evaluate portfolio strategies under alternative assumptions about preferences. The behavioral literature has identified preference orderings that either suggest different functional forms or are at odds with expected utility. Such alternative preferences are usually elicited from experimental or survey evidence. Since the experiments involve small stakes, applying these results in

evaluating pension outcomes is not straightforward. For example, Van Bilsen, Laeven and Nijman (2014) consider a utility function with a reference level of wealth  $H$  and preferences specified in terms of gains or losses relative to this reference level  $x - H$ . Treating gains and losses differently results in a 'kinked' utility function. It will exhibit loss aversion if at the kink there is a sharp discontinuity in marginal utility. Counterintuitively, the specification in Van Bilsen, Laeven and Nijman (2014), using with their suggested parameter values, results in a high certainty equivalents for the more risky strategies. Individuals strongly dislike small losses relative to the benchmark, but when losses get bigger the utility function does not penalise further losses as heavily as a utility function with a moderate level of risk aversion. Given the relatively large volatility of the pension outcomes in a DC system, the big losses have the largest influence on the overall expected utility. A similar result obtains from the loss aversion utility function estimated in Gaudecker, Van Soest and Wengström (2011). Their specification starts with constant absolute risk aversion (CARA) and adds a discrete jump to marginal utility at the reference point. Since CARA utility implies increasing relative risk aversion, really bad outcomes are not as heavily penalised as in a CRRA function. Applying a CARA function to pension outcomes therefore reveals a preference for risky strategies.

A different approach to alternative preferences is habit formation or the introduction of a subsistence level or some other form of lower bound on consumption (benefits). For example, one could see the first pillar as a lower bound subsistence level or habit. This suggests a utility function with  $B_t - A_t$ , total benefits minus first-pillar benefits, as its argument. This will likely lead to more risk aversion.

A more fundamental difference in the evaluation of portfolio strategies would be brought about by moving away from expected utility evaluations. This opens up a wide array of potential evaluations that is outside our current scope.

## 7. Conclusions

We have considered “optimal” life-cycle investment strategies. This topic is widely studied in the academic literature. Practitioners have, especially in the Netherlands, quite some experience with this type of strategy. Under the traditional assumptions of the Merton model life-cycle strategies are easily implemented. Such an approach implies 100% exposure of pension savings to equity risk up to an age around 40-45 with a steadily decreasing exposure at a later age. To deal with interest rate risk there also exist analytical results, but its implications are more complicated. From our simulation study we conclude that current practice of dealing with interest rate risk seems hard to beat.

The paper offers two other main conclusions. Disutility from suboptimal saving and dis-saving decisions is generally much larger than disutility from suboptimal investment strategies. Efforts to entice people into ‘wise’ behavior may therefore be more effective when such efforts address the savings rate and put less emphasis on the investment allocation. Heterogeneity of agents plays an important role here that has to be addressed adequately.

## Appendices

### A. The martingale method

We reconsider the optimal investment strategy for an investor that wants to maximize expected power utility of wealth at a fixed horizon. For this, we adopt the martingale method. We assume throughout the absence of arbitrage and, thus, the existence of a pricing kernel/stochastic discount factor process  $M_t$ , with  $M_0 = 1$ . The pricing equation states that an asset with payoff  $X_T$  at time  $T$  has price  $M_t^{-1} E_t [X_T M_T]$  at time  $t$ .

In case of a final wealth problem with horizon  $T$ , the agent wishes to maximize  $E_0 u(W_T)$  with  $u(x) = x^{1-\gamma}/(1-\gamma)$  for a given risk-aversion parameter  $\gamma > 0$  and given initial wealth  $W_0$ . The martingale method, assuming a complete market, rewrites the dynamic optimization problem as

$$\begin{aligned} & \max_{W_T} E_0 u(W_T) \\ \text{s.t. } & E_0 W_T M_T = W_0. \end{aligned} \quad (20)$$

Standard calculations using the Lagrange method lead to the optimal final wealth

$$W_T^* = [u']^{-1}(\eta M_T), \quad (21)$$

where  $\eta$  is the Lagrange multiplier for the budget constraint. Given the assumed power utility preferences, we have  $u'(x) = x^{-\gamma}$  and  $[u']^{-1}(y) = y^{-1/\gamma}$ , whence

$$W_T^* = \eta^{-1/\gamma} M_T^{-1/\gamma}. \quad (22)$$

From the budget constraint  $E_0 W_T^* M_T = W_0$ , we find the Lagrange multiplier as

$$\eta^{-1/\gamma} = \frac{W_0}{E_0 M_T^{1-1/\gamma}}. \quad (23)$$



The resulting optimal wealth path,  $W_t^*$ , follows from the pricing equation as

$$\begin{aligned} W_t^* &= \frac{E_t W_T^* M_T}{M_t} \\ &= \frac{W_0}{E_0 M_T^{1-1/\gamma}} \frac{E_t M_T^{1-1/\gamma}}{M_t}. \end{aligned} \quad (24)$$

Observe already that optimal wealth  $W_T^*$  scales linearly in initial wealth  $W_0$ . Finally, note that the optimal utility level is given by

$$\begin{aligned} E_0 \frac{(W_T^*)^{1-\gamma}}{1-\gamma} &= \frac{W_0^{1-\gamma}}{1-\gamma} \frac{E_0 M_T^{1-1/\gamma}}{[E_0 M_T^{1-1/\gamma}]^{1-\gamma}} \\ &= \frac{W_0^{1-\gamma}}{1-\gamma} [E_0 M_T^{1-1/\gamma}]^\gamma. \end{aligned} \quad (25)$$

### A.1. The martingale method in the Merton model

In the Merton model, the stochastic discount factor takes the form

$$dM_t = -rM_t dt - \lambda M_t dZ_t, \quad (26)$$

which implies

$$M_t = \exp\left(-\left[r + \frac{1}{2}\lambda^2\right]t - \lambda Z_t\right). \quad (27)$$

The Stochastic Discount Factor thus is log-normally distributed, which becomes a convenient fact later on. From (22)–(23) we find that the optimal final wealth of this investor is given by

$$W_T^* = W_0 \frac{M_T^{-1/\gamma}}{E_0 M_T^{1-1/\gamma}}. \quad (28)$$

The optimal final wealth *path* is then obtained as

$$\begin{aligned}
 W_t^* &= \frac{W_0 E_t M_T^{1-1/\gamma}}{M_t E_0 M_T^{1-1/\gamma}} \\
 &= W_0 M_t^{-1/\gamma} \exp \left( \left[ 1 - \frac{1}{\gamma} \right] \left[ r + \frac{1}{2} \lambda^2 \right] t - \frac{1}{2} \left[ 1 - \frac{1}{\gamma} \right]^2 \lambda^2 t \right) \\
 &= W_0 M_t^{-1/\gamma} \exp \left( \left[ 1 - \frac{1}{\gamma} \right] r t + \frac{1}{2} \frac{1}{\gamma} \left[ 1 - \frac{1}{\gamma} \right] \lambda^2 t \right). \tag{29}
 \end{aligned}$$

In SDE terms, this can be written as

$$dW_t^* = \left[ r + \frac{\lambda^2}{\gamma} \right] W_t^* dt + W_t^* \frac{\lambda}{\gamma} dZ_t. \tag{30}$$

The optimal utility then follows using the log-normality. We have

$$\begin{aligned}
 &\frac{E_0 (W_T^*)^{1-\gamma}}{1-\gamma} \\
 &= \frac{W_0^{1-\gamma}}{1-\gamma} \frac{E_0 M_T^{1-1/\gamma}}{\left( E_0 M_T^{1-1/\gamma} \right)^{1-\gamma}} \\
 &= \frac{W_0^{1-\gamma}}{1-\gamma} \left( E_0 M_T^{1-1/\gamma} \right)^\gamma \\
 &= \frac{W_0^{1-\gamma}}{1-\gamma} \exp \left( -\left( 1 - \frac{1}{\gamma} \right) \left( r + \frac{1}{2} \lambda^2 \right) T + \frac{1}{2} \left( 1 - \frac{1}{\gamma} \right)^2 \lambda^2 T \right)^\gamma \\
 &= \frac{W_0^{1-\gamma}}{1-\gamma} \exp \left( \left( 1 - \gamma \right) r T - \frac{1}{2} \left( 1 - \frac{1}{\gamma} \right) \lambda^2 T \right). \tag{31}
 \end{aligned}$$

It is useful to observe that this optimal utility in itself is again of the CRRA form with respect to initial wealth  $W_0$ . For  $\gamma > 1$  we see

that the optimal utility level is increasing in the interest rate  $r$  and the prices of risk  $\lambda$ .

In order to assess the effect of suboptimal asset allocation choices, observe that a risky exposure of  $w$  to the stock leads to a wealth

$$dW_t^*(w) = [r + w\sigma\lambda] W_t^*(w)dt + w\sigma W_t^*(w)dZ_t, \quad (32)$$

so that

$$\frac{W_T^*(w)}{W_0} \sim LN \left( (r + w\sigma\lambda)T - \frac{1}{2}w^2\sigma^2 T; w^2\sigma^2 T \right). \quad (33)$$

Consequently, the derived utility would be

$$\begin{aligned} E_0 \left[ \frac{(W_T^*(w))^{1-\gamma}}{1-\gamma} \right] &= \frac{W_0^{1-\gamma}}{1-\gamma} \exp \left( (1-\gamma)(r + w\sigma\lambda)T \right. \\ &\quad \left. - (1-\gamma)\frac{1}{2}w^2\sigma^2 T + \frac{1}{2}(1-\gamma)^2 w^2\sigma^2 T \right) \\ &= \frac{W_0^{1-\gamma}}{1-\gamma} \exp \left( (1-\gamma)rT + (1-\gamma)w\sigma\lambda T \right. \\ &\quad \left. - \frac{1}{2}\gamma(1-\gamma)w^2\sigma^2 T \right). \end{aligned} \quad (34)$$

## A.2. A useful lemma

This appendix states a useful lemma to solve for the optimal assumed interest rates.

**Lemma 1.** *Let  $\gamma > 0$ ,  $p(j) > 0$ ,  $j = 0, \dots, h-1$ , and  $f(j) > 0$ ,  $j = 0, \dots, h-1$ . Then the function*

$$\sum_{j=0}^{h-1} \frac{W_{0j}^{1-\gamma}}{1-\gamma} f(j)p(j) \quad (35)$$

is maximized subject to the constraint  $\sum_{j=0}^{h-1} W_{0j} p(j) = W_0$  by

$$W_{0j} \propto f(j)^{+1/\gamma}. \quad (36)$$

*Proof.* This follows easily from the Lagrange optimization principle. □

## References

- ALSERDA, G., B. DELLAERT, L. SWINKELS, AND F. VAN DER LECQ (2015): "Risk Preference Heterogeneity and Optimal Pension Asset Allocation," Preprint Erasmus University.
- BENZONI, L., P. COLLIN-DUFRESNE, AND R. GOLDSTEIN (2007): "Portfolio Choice over the Life-Cycle when the Stock and Labor Markets Are Cointegrated," *Journal of Finance*, 62, 2123–2167.
- BLAKE, D., D. WRIGHT, AND Y. ZHANG (2014): "Age-Dependent Investing: Optimal Funding and Investment Strategies in Defined Contribution Pension Plans when Members are Rational Life Cycle Financial Planners," *Journal of Economic Dynamics and Control*, 38, 105–124.
- BRENNAN, M., AND Y. XIA (2002): "Dynamic Asset Allocation under Inflation," *Journal of Finance*, 57, 1201–1238.
- CALVET, L., J. CAMPBELL, AND P. SODINI (2007): "Down or Out: Assessing the Welfare Costs of Household Investment Mistakes," *Journal of Political Economy*, 115, 707–747.
- CAMPBELL, J., AND L. VICEIRA (2002): *Strategic Asset Allocation: Portfolio Choice for Long-Term Investors*. Oxford University Press.
- CAMPBELL, J. Y., AND R. J. SHILLER (1991): "Yield Spreads and Interest Rate Movements: A Bird's Eye View," *Review of Economic Studies*, 58, 495–514.
- CAMPBELL, J. Y., AND L. M. VICEIRA (2001): "Who Should Buy Long-Term Bonds?," *American Economic Review*, 91, 99–127.

- COCCO, J., F. GOMES, AND P. MAENHOUT (2005): "Consumption and Portfolio Choice over the Life Cycle," *Review of Financial Studies*, 18, 491–533.
- COCHRANE, J. H., AND M. PIAZZESI (2005): "Bond Risk Premia," *American Economic Review*, 95, 138–160.
- DUFFEE, G. R. (2002): "Term Premia and Interest Rate Forecasts in Affine Models," *Journal of Finance*, 57, 405–443.
- FAMA, E. F. (1984): "The Information in the Term Structure," *Journal of Financial Economics*, 13, 509–528.
- KNOEF, M., AND J. BEEN (2015): "A panel data sample selection model with part-time employment: The differences in full-time and part-time wages over the life-cycle," working paper, Leiden University.
- KOIJEN, R., T. NIJMAN, AND B. WERKER (2010): "When Can Life-cycle Investors Benefit from Time-varying Bond Risk Premia?," *Review of Financial Studies*, 23, 741–780.
- LANGEJAN, T., G. GELAUFF, T. NIJMAN, O. SLEIJPEN, AND O. STEENBEEK (2014): "Advies Commissie Parameters,"  
[www.rijksoverheid.nl/documenten/rapporten/2014/03/21/advies-commissie-parameters](http://www.rijksoverheid.nl/documenten/rapporten/2014/03/21/advies-commissie-parameters).
- LYNCH, A., AND S. TAN (2011): "Labor Income Dynamics at Business-Cycle Frequencies: Implications for Portfolio Choice," *Journal of Financial Economics*, 101, 333–359.
- SANGVINATOS, A., AND J. A. WACHTER (2005): "Does the Failure of the Expectations Hypothesis Matter for Long-Term Investors?," *Journal of Finance*, 60, 179–230.

- VAN BILSEN, S. (2015): “Essays on Intertemporal Consumption and Portfolio Choice,” Ph.D. thesis, Tilburg University.
- VAN BILSEN, S., R. LAEVEN, AND T. NIJMAN (2014): “Consumption and Portfolio Choice under Loss Aversion and Endogenous Updating of the Reference Level,” Netspar DP 112014-048.
- VON GAUDECKER, H.-M., A. VAN SOEST, AND E. WENGSTRÖM (2011): “Heterogeneity in Risky Choice Behavior in a Broad Population,” *American Economic Review*, 101, 664–694.
- ZHOU, Y. (2014): “Essays on Habit Formation and Inflation Hedging,” Ph.D. thesis, Tilburg University.

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## Default life-cycles for retirement savings

This paper discusses optimal allocations to stocks and bonds during the contribution and retirement phases in a life-cycle optimization context. We show that often-used assumed interest rates in the Dutch pension practice are suboptimal under standard financial market and preference assumptions. Moreover, we find that default life-cycles with respect to equity exposure perform fairly well, from the individual point of view. The default life-cycles should be adjusted for alternative components in the total wealth of an individual.

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