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Bayesian Model Selection

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Exchange Rate Predictability:
Bayesian Model Selection

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1. Introduction

During the last several decades, exchange rate movement has become an important subject of macroeconomic analysis. However, despite its importance and much effort at constructing models, forecasting the exchange rate is still a challenge for academics and market practitioners.

The collapse of the Bretton Woods system of fixed exchange rates among major industrial countries marked the beginning of the Floating Exchange Rate Regime and exchange rate movement forecasting. However, empirical results stemming from various models in the literature, based on either fundamental economic principles or sophisticated statistical construction, could hardly satisfy and convince academics. Mussa (1979) argues that the spot exchange rate approximately follows a random walk process and most changes in exchange rates are in fact unexpected. Besides, the seminal result of Meese and Rogoff (1983) shows that none of the structural exchange rate models used in their paper could significantly outperform a simple driftless random walk model in both short and medium terms. Even when the in-sample prediction of exchange rate performs well, the out-of-sample forecast is always disappointing when compared to that of random walk model.

In recent years, substantial progress in econometric technique has made it possible to forecast exchange rate using large datasets, especially in the framework of Bayesian econometrics. Bayesian Model Averaging (BMA) is one of the most appealing methods and has received considerable attention in both statistics and econometrics literature. Instead of finding out the 'true' model, the BMA admits many possible models and assumes that one of these models is 'true', although the researcher does not know which one is. The forecast is then a weighted average from all the possible models according to their posterior probabilities, that is, each model contributes information about the parameters of interest, and all these pieces of information are combined according to our trust in each model, which is based on the data and prior beliefs. Wright (2003) applies BMA in exchange rate forecasting and argues that the prediction using Bayesian Model Averaging gives slightly smaller out-of-sample mean square error than the random walk benchmark.

In this paper, our main purpose is to propose two improved methods based on Bayesian Model Averaging to forecast exchange rates.

One approach is Bayesian Model Winner (BMW). It is in fact a single model rather than model averaging. Just as in the traditional econometric framework, the BMW also assumes that the right model is included in our models and we should use this single 'true' model to predict rather than average all the possible models. However, the BMW relaxes the assumption that the 'true' model is stable. In other words, the single 'true' model may change over time. It is quite intuitively appealing, since we have good reason to doubt whether the right model in 1970s is different from that in 2000s because the economic condition has changed significantly. Also, for example, the essential factors in recessions might be quite different from those in booms.

Another approach suggested in our paper is Bayesian Model Averaging Using Predictive Likelihood (BMAP). While the previous studies in exchange rate forecasting all apply BMA in a standard fashion using marginal likelihood, in our paper, we propose the method of using predictive likelihood. Originally, the conversion of prior model probabilities into posterior probabilities is determined by the marginal likelihood. Conditional on the true model being included in the set of models, it will lead to optimal forecasts. However, it raises the possibility that the forecast combination is adversely affected by in-sample overfitting of the data. This problem sometimes might seem to be counter intuitive because the marginal likelihood is commonly interpreted as an out-of-sample or predictive measure of fit. In fact, the interpretation as a predictive measure relies on the assumption that the prior has a predictive content, say, it is informative. However, in most cases of large scale model selection or model averaging, it is not possible to provide well thought out priors for all models. Besides, researchers are attempting to avoid any skepticism of introducing subjective prior to get a seemingly good result. Therefore, uninformative priors such as the prior suggested by Fernandez et al. (2001) are widely used, and, as a result, the marginal likelihood essentially reduces to an in-sample measure of fit. Moreover, in practice, we cannot expect the true model to be included in the set of the models we consider. Therefore, BMAP becomes a natural solution. Firstly, by using predictive likelihood, the weights will have better small sample properties than the traditional in-sample marginal likelihood when uninformative priors are used.

This improvement in small sample performance is mainly because that the predictive likelihood measure considers both in-sample fit and out-of-sample predictive performance which protects against in-sample overfitting of the data. Secondly, the BMAP has much better performance even when the true model is not included in the model set. Eklund and Karlsson (2007) show, when there is a true model, the predictive likelihood will select the true model asymptotically, but will converge more slowly to the true model than the marginal likelihood. It is the slower convergence and the protection against overfitting by considering the out-of-sample predictive ability that lead to the better performance of the predictive likelihood when the true model is not in the model set.

The remainder of the paper is organized as follows. Part 2 presents theories for Bayesian econometrics. We derive the formula for standard Bayesian Model Averaging while taking fixed variables into account, Bayesian model winner and Bayesian model averaging using predictive likelihood. In part 3, we explain the details about exchange rate forecasting and model comparison criteria. Part 4 displays the empirical framework, where we explain our out-of-sample results with respect to each model. A comparison among BMA, BMW and BMAP is made at the end of this Part. Part 5 illustrates an investment comparison based on the models discussed in our paper. The conclusion is presented in part 6.

2. The Theoretical Framework

We start from the linear regression model. Admittedly, the model does not have to be linear regression model, but, as a start, here we shall henceforth assume it is, because we do not have any good reason to accept any other special model structure instead of this most general and widely used linear model.

$$y = X\beta + \varepsilon, \quad \varepsilon \sim N(0, \sigma^2)$$

where y is the vector of observations. X is the matrix of regressors including 1 in the first column. β is a vector of unknown parameters. ε is i.i.d.

One step further, following the idea in Magnus (2010), we extent this model to distinguish two kinds of variables.

$$y = X_1\beta_1 + X_2\beta_2 + \varepsilon, \quad \varepsilon \sim N(0, \sigma^2)$$

The main reason for dividing and distinguishing between X_1 and X_2 is that we may fix some variable which we want in the model on theoretical or other grounds. For instance, considering the exchange rate theory, people may think that relative interest rate and relative price level should always be the explanatory variables in forecasting exchange rate. Therefore, X_1 contains k_1 variables called 'focus' regressors and X_2 contains k_2 variables called 'auxiliary' regressors which we are less certain, where $k_1 \geq 0, k_2 \geq 1$.

Apparently, there are 2^{k_2} possible models to consider, and each variable in X_2 can be chosen to either appear in the model or not. Denote the i-th model as

$$y = X_1\beta_1 + X_{2i}\beta_{2i} + \varepsilon$$

where $\mathrm{X_{2i}}$ is an $\mathrm{n \times k_{2i}}$ matrix containing a subset of k_{2i} column of X_2. β_{2i} denotes the corresponding subvector of β_2.

2.1 Standard Bayesian Model Averaging

The idea of Bayesian Model Averaging was firstly proposed by Learner (1978). Recently, the application of the BMA in out-of-sample forecast of output growth, stock return and exchange rate has been found to have some success in improving the predictive performance, and thus received increasing attention in the statistical literature.

The main issue of BMA is how to choose the correct model given the model uncertainty. Contrast to traditional econometrics, which select only one single model for forecasting, BMA stands one step back to consider the tremendous uncertainty the researcher has about the correct model. The BMA has a mechanism to combine all the possible models based on the performance over time and the advantages of BMA in model uncertainty have been testified by Raftery, Madigan and Hoeting (1997) with linear regression model. Besides, Wright (2003) shows that the application of the BMA method to the exchange rate forecast gives comparably favorable out-of-sample results as compared to the benchmark random walk model.

Consider a set of *n* models, which are $M_1, M_2, \dots . M_n$, a researcher does not know which one is the true model. However, the researcher has prior beliefs about the probability that the i-th model is the true one, which is $P(M_i)$. After the researcher observes the data and updates his beliefs, she may compute the posterior probability of the i-th model:

$$p(M_i|y) = \frac{p(y|M_i)P(M_i)}{\sum_{j=1}^{n} p(y|M_j)P(M_j)}$$

Where

$$p(y|M_i) = \int p(y|\theta^i, M_i)p(\theta^i|M_i)d\theta_i$$

Here θ contains all the parameters for a model which seeks to explain y.

Then, the BMA forecast is just the weighted (according to the model posterior probability) average of forecasts from each possible model. More specifically, the procedure of calculating is as follows:

The likelihood function

Assume that ε has a multivariate Normal distribution with mean 0_n and covariance matrix $\sigma^2 I_n$

For any realized data, using the definition of the Normal density, we obtain the likelihood of the data under model M_i:

$$p(y|\beta_1, \beta_{2i}, \sigma^2, M_i)$$
$$= \left(\frac{1}{\sqrt{2\pi\sigma^2}}\right)^{n/2} exp\,(-\frac{(y - X_1\beta_1 - X_{2i}\beta_{2i})'(y - X_1\beta_1 - X_{2i}\beta_{2i})}{2\sigma^2})$$

where $p(y|\beta_1, \beta_{2i}, \sigma^2, M_i)$ is the likelihood function of data conditional on the parameters and model M_i.

The prior

Priors are meant to reflect any information the researcher has before seeing the data which she wishes to include and therefore can take any form. However, in order to interpret and make computation easier, natural conjugate prior Normal-Gamma distribution is widely chosen by researchers because when combined with the likelihood, it yields a posterior that falls in the same class of distributions. Then we have

$$p(\sigma^2|M_i) \propto \frac{1}{\sigma^2}$$
$$p(\beta_1|\sigma^2, M_i) \propto 1$$
$$\beta_{2i}|\beta_1, \sigma^2, M_i \sim N(0, \sigma^2 V_{0i})$$

It remains to choose V_{0i}. We use the so-called g-prior, which was introduced in Feernandez, Ley and Steel (2001b), as follows

$$g_r = \begin{cases} 1/k_2^2 & if\ \ n \le k_2^2 \\ 1/n & if\ \ n > k_2^2 \end{cases}$$

and

$$V_{0i} = (g_r X_r' X_r)^{-1}$$

Then, the joint prior distribution is:

$$p(\beta_1, \beta_{2i}, \sigma^2|M_i) \propto (\sigma^2)^{-(k_{2i}+2)/2} exp\,(-\frac{\beta_{2i}' V_{0i}^{-1} \beta_{2i}}{2\sigma^2})$$

The posterior

Combining the prior with the likelihood gives the posterior

$$p(\beta_1, \beta_{2i}, \sigma^2|y, M_i)$$
$$\propto (\sigma^2)^{-(n+k_{2i}+2)/2} exp\,[-\frac{\beta_{2i}' V_{0i}^{-1} \beta_{2i} + (y - X_1\beta_1 - X_{2i}\beta_{2i})'(y - X_1\beta_1 - X_{2i}\beta_{2i})}{2\sigma^2}$$

Then, we arrive at (See appendix A for details):

$$p(\beta_1, \beta_{2i}, \sigma^2|y, M_i) \propto (\sigma^2)^{-(n+k_{2i}+2)/2} exp\,(-\frac{\varphi_i + \tau_i}{2\sigma^2})$$

where

$$\varphi_i = \begin{pmatrix} \beta_1 - b_1 \\ \beta_{2i} - b_{2i} \end{pmatrix}' V_i^{-1} \begin{pmatrix} \beta_1 - b_1 \\ \beta_{2i} - b_{2i} \end{pmatrix}$$

$$V_i = \begin{pmatrix} (X_1'X_1)^{-1} + (X_1'X_1)^{-1}X_1'X_{2i} & -(X_1'X_1)^{-1}X_1'X_{2i}V_{2i} \\ -V_{2i}X_{2i}'X_1(X_1'X_1)^{-1} & V_{2i} \end{pmatrix}$$

$$V_{2i}^{-1} = V_{0i}^{-1} + X_{2i}'M^*X_{2i}$$

$$\tau_i = y'(M^* - M^*X_{2i}V_{2i}X_{2i}'M^*)y$$

b_1 and b_{2i} are the estimator of β_1 and β_{2i} respectively

Hence, the posterior density of the parameters is the familiar normal-inverse-gamma distribution.

Marginal likelihood of model $\mathrm{M_i}$

From the marginal density of y in model $\mathrm{M_i}$ as

$$p(y|M_i) = \iiint p(y|\beta_1, \beta_{2i}, \sigma^2, M_i)p(\beta_1, \beta_{2i}, \sigma^2|M_i)d\beta_1 d\beta_{2i} d\sigma^2$$

$$= c\frac{|V_{0i}^{-1}|^{1/2}}{|V_{2i}^{-1}|^{1/2}}\tau_i^{-(n-k_1)/2}$$

where

$$c = \frac{\pi^{-\frac{n}{2}}\Gamma(\frac{n-k_1}{2})}{|X_1'X_1|\Gamma(-\frac{k_1}{2})}$$

Then, with the expression of τ_i and V_{2i} and some simplification, the marginal density can be written as:

$$p(y|M_i) = c\frac{|V_{0i}^{-1}|^{\frac{1}{2}}}{|V_{0i}^{-1} + X_{2i}'M^*X_{2i}|^{\frac{1}{2}}}(y'M^*S^*M^*y)^{-(n-k_1)/2}$$

where

$$S^* = (M^* - M^*X_{2i}(V_{0i}^{-1} + X_{2i}'M^*X_{2i})^{-1}X_{2i}'M^*)$$

(See Appendix B for further details)

If we let $p(M_i)$ denote the prior probability that M_i is the true model, and $p(M_i|y)$ is the posterior probability for model M_i , then

$$p(M_i|y) = \frac{p(y|M_i)P(M_i)}{\sum_{j=1}^{2^{k_2}} p(y|M_j)P(M_j)}$$

One thing left here is the prior probability for model M_i. Many researchers feel that simpler models should be preferred to more complex ones. Durlauf et al.(2005), on the other hand, find the idea of promoting parsimonious models through the priors unappealing. Brock and Durlauf(2001) raise objections against uniform priors on the model space because of the implicit assumption that the probability one regressor appears in the model is independent of the inclusion of others, whereas, in fact, regressors are typically correlated. They suggest a hierarchical structure for the model proxies for the model prior. However, as stated in Eicher et al.(2007b), the agreement on which kind of prior to choose is usually not within reach. Here, we follow Cremers(2002) to use a flexible prior probability for an individual model as follows:

$$P(M_i) = \rho^{k_{2i}}(1-\rho)^{k_2-k_{2i}}$$

With certain value of the parameter ρ, the formula may represent any case of the prior probability. For example, the choice of $\rho = 0.5$ would assign equal prior probability to all models considered, while if ρ is less than 0.5, a model including less explanatory variables is more likely than a model including more variables.

Combining the equations above,

$$p(M_i|y) = \frac{p(y|M_i)P(M_i)}{\sum_{j=1}^{2^{k_2}} p(y|M_j)P(M_j)} \propto p(y|M_i)P(M_i)$$

$$p(M_i|y) \propto c\left(\frac{g_i}{1+g_i}\right)^{k_{2i}/2}(y'M^*S^*M^*y)^{-(n-k_1)/2}\rho^{k_{2i}}(1-\rho)^{k_2-k_{2i}}$$

where

$$S^* = \left(M^* - \frac{1}{1+g_i}M^*X_{2i}(X_{2i}'M^*X_{2i})^{-1}X_{2i}'M^*\right)$$

Now, considering all the possible models that we have, following the basic idea of Bayesian Model Averaging, we may average the models and set the weight for each model to be equal to the posterior probability. Then

$$p(\beta_1, \beta_2, \sigma^2|y) = \sum_{i=1}^{2^{k_2}} p(\beta_1, \beta_{2i}, \sigma^2|y, M_i)p(M_i|y)$$

and

$$E(\beta_1, \beta_2, \sigma^2|y) = \sum_{i=1}^{2^{k_2}} E(\beta_1, \beta_{2i}, \sigma^2|y, M_i)p(M_i|y)$$

where we have

$$E(\beta_2|y, M_i) = b_{2i} = \frac{1}{1+g_i}(X'_{2i}M^*X_{2i})^{-1}X'_{2i}M^*y$$

$$E(\beta_1|y, M_i) = b_{1i} = (X'X)^{-1}X'_1(y - X'_{2i}b_{2i})$$

Finally, we calculate the parameter for forecasting from general Bayesian Model Averaging

$$\hat{\beta}_2 = \sum_{i=1}^{2^{k_2}} E(\beta_2|y, M_i)p(M_i|y)$$

$$\hat{\beta}_1 = \sum_{i=1}^{2^{k_2}} E(\beta_1|y, M_i)p(M_i|y)$$

$$\hat{\beta}_{BMA} = \begin{bmatrix} \hat{\beta}_1 \\ \hat{\beta}_2 \end{bmatrix}$$

2.2 Bayesian Model Winner

Based on the derivation of Bayesian Model averaging, Bayesian Model Winner is only one step further. Instead of averaging all the models, Bayesian Model Winner chooses the model with largest posterior.

Same as above

$$p(M_i|y) = \frac{p(y|M_i)P(M_i)}{\sum_{j=1}^{2^{k_2}} p(y|M_j)P(M_j)} \propto p(y|M_i)P(M_i)$$

$$p(M_i|y) \propto c\left(\frac{g_i}{1+g_i}\right)^{k_{2i}/2}(y'M^*S^*M^*y)^{-(n-k_1)/2}\rho^{k_{2i}}(1-\rho)^{k_2-k_{2i}}$$

where

$$S^* = \left(M^* - \frac{1}{1+g_i}M^*X_{2i}(X'_{2i}M^*X_{2i})^{-1}X'_{2i}M^*\right)$$

If $p(M_j|y) \geq p(M_i|y)$, for any $i \neq j, i \in [1, 2^{k_2}]$

We would think that the j-th model can fit the data and our prior the best and we naturally consider it as the 'true' model. One thing worth mention here is that no matter we use expanding window or rolling window, the best model can change to fit the new added data. For example, if the present 'true' model explains the 'future' data very well among all the models, as time moves forward, these 'future' data would be added into 'history' and further enlarge the posterior probability of our present 'true' model. On the other side, if the 'true' model 'predicts' less well than other models, newly included data would increase the posterior probability of other models more than the 'true' model. Therefore, our 'true' model can change over time to best fit the data.

$$\hat{\beta}_2 = b_{2j} = \frac{1}{1+g_j}\left(X_{2j}'M^*X_{2j}\right)^{-1}X_{2j}'M^*y$$

$$\hat{\beta}_1 = b_{1j} = (X'X)^{-1}X_1'\left(y - X_{2j}'b_{2j}\right)$$

$$\hat{\beta}_{BM-winner} = \begin{bmatrix}\hat{\beta}_1\\ \hat{\beta}_2\end{bmatrix}$$

2.3 Bayesian Model Averaging Using Predictive Measures

Two main problems that the standard BMA faces are the overfitting of data and the absence of true model. Intuitively, a feasible solution for the problem is to take into account the predictive, or out-of-sample, performance of the models--namely, Bayesian Model Averaging Using Predictive Likelihood. We average the model using weights based on the predictive performance rather than the marginal likelihood.

The first step is to split the sample y into two parts, with m and l observations and $n = m + l$. The first part of the data is used to convert the parameter priors $p(\theta_i|M_i)$ into the posterior distributions, and the second part of the sample is used for evaluating the model performance. Clearly, there is a trade-off involved in the choice of l. In order to calculate the weights for the models more precisely, l need to be set as large as possible. But, at the meantime, the number of observations available for estimation is thus reduced from n to m. In other words, as l increases, the predictive measure becomes more precise while estimation, on the other hand, is performed without taking the most recent observations into account, which might have a detrimental effect.

Similar to the formula we derive for classic Bayesian Model Averaging, the posterior predictive density of $\tilde{y} = (y_{m+1}, y_{m+2}, \dots, y_n)'$, conditional on $y^* = (y_1, y_2, \dots, y_m)'$ and model M_i is

$$p(\tilde{y}|y^*, M_i) = \int p(\tilde{y}|\theta_i, y^*, M_i)p(\theta_i|y^*, M_i)d\theta_i$$

where $p(\theta_i|y^*, M_i)$ is the posterior distribution of the parameters and $p(y_2|\theta_i, y^*, M_i)$ is the likelihood of observations in hold-out sample conditional on the previous observation and specific model. The posterior predictive density clearly gives us a description on future observations, given then observed sample y_1. Thus, this density gives the models, which can be used to better predict y_2, a larger predictive likelihood.

The next step is quite natural, simply replacing the marginal likelihood by our marginal predictive likelihood. In order to distinguish from the posterior probability in classic Bayesian Model Averaging, we denote the posterior predictive probability, which is actually a measure of predictive performance, as

$$p_{pre}(M_i|y^*, \tilde{y}) = \frac{p(\tilde{y}|y^* M_i)P(M_i)}{\sum_{j=1}^{2^{k_2}} p(\tilde{y}|y^*, M_j)P(M_j)}$$

With the same assumption on the prior probability of parameter

$$\beta_{2i}|\beta_1, \sigma^2, M_i \sim N(0, \sigma^2(g_r X_r' X_r)^{-1})$$

$$p(\sigma^2|M_i) \propto \frac{1}{\sigma^2}$$

This still yields the normal-inverse-gamma distribution, and the marginal predictive density of $\tilde{y}$ conditional on y^*, M_i is

$$p_{pre}(\tilde{y}|y^*, M_i) = \iiint p_{pre}(\tilde{y}|\beta_1, \beta_{2i}, \sigma^2, y^*, M_i) p(\beta_1, \beta_{2i}, \sigma^2|M_i) d\beta_1 d\beta_{2i} d\sigma^2$$

With similar calculation as before, we may get the formula of posterior probability for hold-out sample conditional on the training sample and model i. Specially, for simplicity, we set X_1 to be null set, which means we do not fix any variable and all the variables are freely chosen according to the posterior probability of the model (however, in our matlab code, we still allow to set any number of 'focus' variables). As a result, $X_2 = X$, and $k_2 = k$.

$$p_{pre}(\tilde{y}|y^*, M_i) \propto \left(\frac{y^{*\prime} S_{pre}^* y^*}{m}\right)^{-l/2} \frac{\left|(1+g_i) X_i^{*\prime} X_i^*\right|^{1/2}}{\left|(1+g_i) X_i^{*\prime} X_i^* + \tilde{X}_i' \tilde{X}_i\right|^{1/2}} \left[m + \frac{m}{y^{*\prime} S_{pre}^* y^*} (\tilde{y} - \tilde{X}_i b_i^*)' \left(I + \tilde{X}_i \frac{1}{1+g_i} \left(X_i^{*\prime} X_i^*\right)^{-1} \tilde{X}_i'\right)^{-1} (\tilde{y} - \tilde{X}_i b_i^*)\right]^{-n/2}$$

where

$$S^*_{pre} = \left(I - \frac{1}{1+g_i} X^*_i \left(X^{*\prime}_i X^*_i\right)^{-1} X^{*\prime}_i\right)$$

$$b^*_i = \frac{1}{1+g_i} \left(X^{*\prime}_i X^*_i\right)^{-1} X^{*\prime}_i y^*$$

(See appendix C for details)

This predictive likelihood can be decomposed into three components:

1. *The in-sample fit of the training sample.*

$$\left(\frac{y^{*\prime} S^*_{pre} y^*}{m}\right)^{-l/2}$$

This in-sample fit is large if the i-th model match the training sample data very well. The relative in-sample fit of two models is measured by:

$$\left(\frac{y^{*\prime} S^*_{pre\_i} y^*}{y^{*\prime} S^*_{pre\_j} y^*}\right)^{-l/2} = \left(\frac{y^{*\prime}\left(I - \frac{1}{1+g_i} X^*_i \left(X^{*\prime}_i X^*_i\right)^{-1} X^{*\prime}_i\right) y^*}{y^{*\prime}\left(I - \frac{1}{1+g_j} X^*_j \left(X^{*\prime}_j X^*_j\right)^{-1} X^{*\prime}_j\right) y^*}\right)^{-l/2}$$

It is clear that the effect of differences in fit is increasing in the size of hold-out sample.

2. *The penalty for the size of the model.*

$$\frac{\left|(1+g_i) X^{*\prime}_i X^*_i\right|^{1/2}}{\left|(1+g_i) X^{*\prime}_i X^*_i + \tilde{X}^{\prime}_i \tilde{X}_i\right|^{1/2}}$$

We have $\tilde{X}^{\prime}_i \tilde{X}_i \approx \frac{l}{m} X^{*\prime}_i X^*_i$, which gives

$$\frac{\left|(1+g_i) X^{*\prime}_i X^*_i\right|^{1/2}}{\left|(1+g_i) X^{*\prime}_i X^*_i + \tilde{X}^{\prime}_i \tilde{X}_i\right|^{1/2}} \approx \left(\frac{1+g_i}{1+g_i+m/l}\right)^{k_i/2} \approx \left(\frac{l}{m+l}\right)^{k_i/2}$$

This penalty of size also increases in hold-out sample size

3. *The out-of-sample forecasting accuracy.*

$$\left[m + \frac{m}{y^{*\prime} S^*_{pre} y^*} \left(\tilde{y} - \tilde{X}_i b^*_i\right)^{\prime} \left(I + \tilde{X}_i \frac{1}{1+g_i} \left(X^{*\prime}_i X^*_i\right)^{-1} \tilde{X}^{\prime}_i\right)^{-1} \left(\tilde{y} - \tilde{X}_i b^*_i\right)\right]^{-n/2}$$

Apparently, it is a measure of out-of-sample prediction performance. The larger the forecasting error is, the smaller the likelihood would be.

The rest of calculation is to take the prior weight on each model and to calculate the posterior probability:

$$p_{pre}(M_i|y^*,\tilde{y}) = \frac{p(\tilde{y}|y^*,M_i)P(M_i)}{\sum_{j=1}^{2^k} p(\tilde{y}|y^*,M_j)P(M_j)}$$

$$\propto \left(\frac{y^{*\prime}S_{pre}^{*}y^{*}}{m}\right)^{-l/2} \frac{\left|(1+g_i)X_i^{*\prime}X_i^{*}\right|^{1/2}}{\left|(1+g_i)X_i^{*\prime}X_i^{*}+\tilde{X}_i^{\prime}\tilde{X}_i\right|^{1/2}} \left[m + \frac{m}{y^{*\prime}S_{pre}^{*}y^{*}}(\tilde{y}-\tilde{X}_i b_i^{*})^{\prime}\left(I+\tilde{X}_i\frac{1}{1+g_i}\left(X_i^{*\prime}X_i^{*}\right)^{-1}\tilde{X}_i^{\prime}\right)^{-1}(\tilde{y} - \tilde{X}_i b_i^{*})\right]^{-n/2} \rho^{k_i}(1-\rho)^{k-k_i}$$

(Also see Appendix C)

This predictive weight has several appealing properties in addition to providing a basis for meaningful marginalization with respect to the auxiliary variables in the model.

1. Proper prior distributions are not required for all the parameters any more. In contrast to the marginal likelihood, the predictive likelihood is well defined as long as the posterior distribution of parameters conditioned on the training sample is proper.

2. The predictive likelihood is not only an absolute measure of forecasting performance. Instead it is relative to the precision of forecasts implied by the model and models with a good in-sample fit are penalized when a 'good' and 'bad' model both show poor predictability. Figure 2.3.1 illustrates the overall behavior of a model with good in-sample fit and corresponding small forecast error variance, and a model with poor in-sample fit and large forecast error variance. If the forecast error is modest, as can be expected from a model with good in-sample fit, the 'good' model is preferred. If, on the other hand, the forecast error is larger than what can be expected from the 'good' model which means the model is overfitting the data, then the 'bad' model is favored. The predictive weights will thus be small for models that overfit the data.

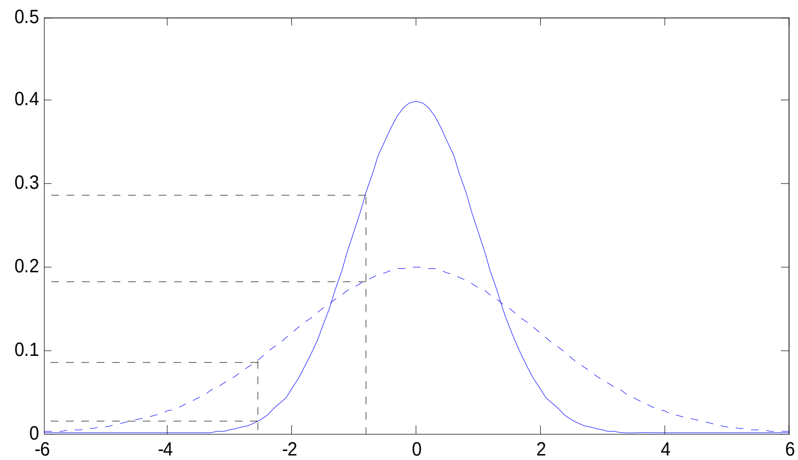

Figure 2.3.1 Predictive likelihood for models with small and large prediction error variance.

After that, the distribution of parameter is derived through model averaging

$$p(\beta, \sigma^2|y) = \sum_{i=1}^{2^k} p(\beta, \sigma^2|y, M_i) p_{pre}(M_i|y^*, \tilde{y})$$

and

$$E(\beta, \sigma^2|y) = \sum_{i=1}^{2^k} E(\beta, \sigma^2|y, M_i) p_{pre}(M_i|y^*, \tilde{y})$$

$$\hat{\beta} = \sum_{i=1}^{2^k} b_i * p_{pre}(M_i|y^*, \tilde{y})$$

here, b_i is different from b_i^*

$$b_i = \frac{1}{1+g_i}(X_i{}'X_i)^{-1}X_i{}'y$$

It is especially noteworthy that the parameter b_i^* estimated from training sample is only used to calculate the model posterior probability. After the weight of each model is decided, the coefficient of each variable is in fact computed using all the samples. Therefore, according to the model averaging formula, we yield $\hat{\beta}_{BMA-pre} = \hat{\beta}$

3. Exchange Rate Forecasting

3.1 Forecasts and Dataset

Turing to the exchange rate forecasting, we consider the bilateral exchange rate of the Pound, Yen and Mark/Euro, relative to the US Dollar. Consider the exchange rate forecasting model of the form

$$e_{t+h} - e_t = X_t\beta + \varepsilon_t = X_{1,t}\beta_1 + X_{2,t}\beta_2 + \varepsilon_t$$

where, e_t denotes the log exchange rate, h is the forecasting horizon and X_t denotes the vector of regressors in time period t. $X_{1,t}, X_{2,t}$ represent the 'focus' and 'auxiliary' variables respectively. β is the vector of coefficients of all the predictors used to forecast the exchange rate and β_1, β_2 are the coefficients of the corresponding variable set. ε_t is i.i.d.

If $\tilde{\beta}$ denotes the posterior mean of β and is derived from all the possible models using our three different methods, that is, $\tilde{\beta}$ will be $\hat{\beta}_{BMA}$, $\hat{\beta}_{BM-winner}$ and $\hat{\beta}_{BMA-pre}$ respectively, then the corresponding forecasting return of exchange rate would be $X_t\tilde{\beta}$.

The models in the model set contain k_1 fixed variables and all possible combinations of k_2 potential predict variables, including all of these 'auxiliary' predictors and none of them.

Thus, the i-th model would be:

$$e_{t+h} - e_t = X_{i,t}\beta + \varepsilon_t$$

In special case, if $k_1 = k_2 = 0$, the forecasting model is simply a diftless random walk model

$$e_{t+h} - e_t = \varepsilon_t$$

Following to the previous literature on the exchange rate forecast, we select 12 variables including both the economic and financial variables as follows:

1. growth rate of money supply[1]

[1] All the variables are those of foreign country relative to those of the US

2. short term interest rate
3. long term interest rate
4. growth rate in CPI
5. GDP
6. growth in GDP
7. international reserve
8. change in international reserve
9. change in industrial production
10. change in reserve minus gold
11. labor productivity
12. relative change rate in stock price

Our data cover the time periods from 1973 Q1 to 2010 Q1 including the exchange rate of the four currencies, Euro (EUR), British Pound (GBP), Japanese Yen (YEN) and US Dollar (USD) and the national economic and financial variables in these four countries. The detail of data and resources are listed in table 3.1.1. The descriptive statistics for all the variables and exchange rate returns are listed in the table 3.1.2, table 3.1.3 and table3.1.4 in appendix (See Appendix D).

Table 3.1.1 The detail of the data and sources

Variables	Sources and Details
Exchange Rates	*Since the EUR/USD is not available until 1999 Q1, the series from 1973 Q1 to 1998 Q4 is traced back using the series of the Deutsche mark against the USD (DEM/USD) with a conversion factor stated in IFS World and Country Notes as 1.95583 DEM per USD. The series of EUR/USD, DEM/USD, GBP/USD and YEN/USD from 1973 Q1 to 2010 Q1 are from the International Financial Statistics (IFS) of the International Monetary Fund (IMF)*
Money Supply	*The series of money supply growth of UK, Japan, Genmany and US, form 1973 Q1 to 2010 Q1 are from IFS*
Short Term Interest Rate	*The financing bill rate from Datastream is used for Japan, the Treasury Bill rate from Datastream is used for the Germany UK and the US.*
Long Term Interest Rate	*The series of government bond yields are from IFS*
Consumer Price Level	*The consumer price indices (2000 = 100) are from IFS.*
Real GDP	*The series of seasonally adjusted nominal GDP and GDP deflator (2000=100) are from IFS.*
International Reserve	*The series of international reserve from 1973 Q1 to 2010 Q1 measured in SDRs are from IFS*
Industrial Production	*The series of industrial production from 1973 Q1 to 2010 Q1 measured in SDRs are from IFS*
Reserve Minus Gold	*The series of reserve minus gold measured in SDRs are from IFS*
Labor Productivity	*The series of labor productivity are from Datastream*
Stock Price	*The series of share price index are from Datastream*

3.2 Out-of-sample Forecast Evaluation Criteria

For the purpose of comparison and assessment of the model performance, we employ two benchmarks in this paper. One is a simple random walk model suggested by Meese and Rogoff (1983), who find that no estimated renowned theoretical model could outperform the random walk. The driftless random walk model for an exchange rate can be expressed as:

$$e_{t+h} = e_t + \varepsilon_t$$

The other benchmark which is also widely used in literature is the historical average return.

$$e_{t+h} = \frac{1}{t} \sum_{t} e_t$$

Also, two measures are employed to evaluate the forecast accuracy of the models. One measure is the Ratio of the Root Mean Squared Forecast Error (RMSFE) of each model to that of benchmarks over time. If the model manages to predict with a smaller error compared to benchmark model, the ratio would be less than one. The other measure is the Ratio of Right Sign Forecast (RSF) given by the forecast model. A one will be assigned for each period when the model forecasts correctly the direction of real exchange rate, and zero otherwise. Then, the RSF is the proportion of 'ones' over all the periods. As the random walk cannot predict the direction of the exchange rate movement, a RSF larger than 0.5 means our model forecasts the future movement of exchange rate better than random walk. As to using historical average return as benchmark, the measure of RSF is a little different. Since historical average return also gives sign of future exchange rate movements, the RSF is the ratio of 'ones' given by our model to those given by historical average return model. That is, a RSF larger than 1 means the model is better than historical average return in predicting movements of exchange rate.

4. The Empirical Framework

In this part, we display the empirical results of the models. In the first section, we explain the out-of-sample results of each model, given expanding window or rolling window and various forecasting horizons. In the second section, we make a fare comparison among these three models.

Our out-of-sample forecasts start from 1993 Q1 and end at 2010 Q1. The sample before 1993 Q1 is used for estimating the models and once a one period ahead forecast is obtained, the sample is updated with one more period information for the next forecast (in the rolling window, one period of the oldest sample in order to keep the sample size whenever new period of data is obtained). The horizons examined in our paper are one, two, four and eight quarters, representing short, medium and long horizons

4.1 Bayesian Model Averaging

The results of Bayesian model averaging relative to driftless random walk model and historical average return model are shown in the table 4.1.1 and table 4.1.2, in expanding window and rolling window respectively. In the expanding window, generally, BMA forecasts less well than a random walk model in short horizon and only slightly better at medium and long horizons. This result is very much consistent with Wright (2003), who found that BMA forecasts are not necessarily very different from random walk in short and medium horizon. Since most economists believe that the exchange rate is very well approximated by a random walk, this is a reassuring feature of the BMA procedure.

In Table 4.1.1, results are shown under each kind of prior weight on models. $\rho = 0.5$ means all the models have equal prior weight while ρ smaller than 0.5 means simple models are preferred, and this preference is increasing as ρ decreases. For Yen and Mark/Euro, although BMA is worse in short term forecasts, it manages to outperform random walk in long term, especially in the prediction of Yen. The BMA performs very well in Japanese Yen in medium and long horizon and reduces the prediction error by up to 6% relative to random walk in two years ahead forecast. For

Mark/Euro, BMA predicts at an error of 6.2% more than that of random walk model in short term forecast while 0.8% less in long term. For Pound, however, the out-of-sample RMSFE is uniformly larger than 1. It means BMA performs worse than random walk under the measure of prediction error at every horizon. Besides, as the horizon being longer, the RMSFE increases and BMA forecasts even worse relative to random walk, from 1.0584 to 1.2160 for $\rho = 0.5$.

We also concern about the relationship between prior weight on models and model performance. Based on the comparison among different values of ρ, we find that RMSFE increases with ρ if RMSFE is above 1, and decreases with ρ if it is below 1. For example, in the 1 quarter prediction of Yen, BMA underperforms random walk, with a RMSFE of 1.0128, 1.0116 and 1.0113 for ρ equals 0.5, 0.3 and 0.1. However, in the 4 quarters ahead forecast, where BMA is shown to outperform random walk model, the RMSFE is 0.9451, 0.9565 and 0.9704. In other words, if models with smaller number of variables are preferred, the RMSFE always lies between the RMSFE of equal weight model and that of random walk model. It is reasonable since random walk is in fact an extreme case of model with small number of variables (simply, if $\rho = 0$, the BMA degenerates to a random walk model, because any model with the number of variables larger than 0 has a prior weight of 0).

Another question of much interest is whether BMA predicts the direction of exchange rate movements correctly. RSF shows the proportion of right sign predictions. As can be seen from the table, unlike RMSFE, RSF illustrates that BMA is better than random walk in predicting future movements of exchange rate. Under many conditions, RSF is above 0.5, and it means that BMA predicts correctly more than half of the time. In the table, it is ambiguous which is better in predicting British Pound. But for Mark/Euro and Yen, BMA generally outperforms random walk model except for some horizon-parameter combinations. For Yen, the RSF is above 0.5 even in 1 quarter forecast horizon and increases with forecast horizon. In the 4 quarters ahead forecast, the RSF climbs up to more than 0.7, which means BMA predicts direction of 1 year exchange rate movement 40% better than random walk. As to Mark/Euro, although most of the RSF is above 0.5, the BMA is not proved to be significantly better until the forecast horizon was extended to 2 years.

Table 4.1.1 Out-of-sample results for Bayesian model averaging-Expanding window

Benchmark:		Random Walk						Historical Average Return					
Currency	Horizon	RMSFE			RSF			RMSFE			RSF		
		ρ = 0.5	ρ = 0.3	ρ = 0.1	ρ = 0.5	ρ = 0.3	ρ = 0.1	ρ = 0.5	ρ = 0.3	ρ = 0.1	ρ = 0.5	ρ = 0.3	ρ = 0.1
						Expanding window							
Pound	1 quarter	1.0584	1.0111	1.0016	0.5147	0.4853	0.5000	1.0487	1.0018	0.9925	1.0000	0.9429	0.9714
	2 quarters	1.1813	1.1584	1.0590	0.5075	0.4925	0.4925	1.1634	1.1409	1.0430	1.0968	1.0645	1.0645
	4 quarters	1.2093	1.2021	1.1833	0.5231	0.5231	0.5231	1.1745	1.1675	1.1492	1.1724	1.1724	1.1724
	8 quarters	1.2160	1.2192	1.2060	0.4426	0.4426	0.4426	1.1517	1.1546	1.1422	1.1739	1.1739	1.1739
Yen	1 quarter	1.0128	1.0116	1.0113	0.5441	0.5000	0.5000	1.0058	1.0047	1.0043	1.1563	1.0625	1.0625
	2 quarters	0.9787	0.9802	0.9910	0.6119	0.6119	0.5682	0.9662	0.9676	0.9783	1.2424	1.2424	1.1515
	4 quarters	0.9451	0.9565	0.9704	0.7385	0.7385	0.6769	0.9211	0.9322	0.9457	1.4545	1.4545	1.3333
	8 quarters	0.9411	0.9566	0.9796	0.7049	0.7049	0.6885	0.8714	0.8857	0.9070	1.3438	1.3438	1.3125
Mark/Euro	1 quarter	1.0623	1.0534	1.0473	0.5441	0.5441	0.5147	1.0546	1.0457	1.0397	1.2333	1.2333	1.1667
	2 quarters	1.0375	1.0332	1.0304	0.5075	0.5075	0.4776	1.0327	1.0285	1.0257	1.0000	1.0000	0.9412
	4 quarters	1.0059	1.0034	1.0028	0.5231	0.5538	0.5385	1.0042	1.0016	1.0011	0.9444	1.0000	0.9722
	8 quarters	0.9924	0.9951	0.9979	0.6066	0.5246	0.5246	0.9908	0.9935	0.9963	1.1212	0.9697	0.9697

Note: This table shows the results of Bayesian Model Averaging relative to those of two benchmarks (random walk and historical average return) respectively, using expanding window. ρ is parameter of prior weight on models. $\rho = 0.5$ means all the models receive equal prior weight while ρ smaller than 0.5 means models with less variables are preferred, and the smaller the ρ is, the more preferred the models with less variables are.

The right hand side of the Table 4.1.1 displays the results when historical average return model is chosen as a benchmark. In general, the results are consistent with those of using random walk model as benchmark. The RMSFE is a little smaller than that compared to random walk model. However, it is mainly because the prediction error of historical average return is slightly larger than that of random walk. The RSF here is the ratio of correct predictions of BMA to those of historical average return model, and BMA outperforms benchmark if the RSF is larger than 1. Therefore, the figures in the table demonstrates that BMA forecasts better under most conditions but only consistently outperforms historical average return in Yen.

In conclusion, either using random walk model or setting historical average return model as benchmark, the data dose not suffice to prove that BMA using expanding window is overall significantly better than the benchmarks according to our two measures. A safe conclusion might be that BMA, considering the model and parameter uncertainty, is slightly better than benchmark models under some currency-horizon pairs and, for Yen, BMA outperforms benchmarks especially in medium and long horizons.

One possible improvement we may make is to employ rolling window instead of expanding window since the data too long ago might be much less useful than the most recent data. A rolling window only takes the most recent data, which are supposed to be most relevant, into account and the estimation is thought to be most closely related to present. However, there is also a trade off because the quarterly data is not abundant and it increases the probability of making mistakes by not using sufficient data. The results of rolling window are displayed in table 4.1.2.

Table 4.1.2 Out-of-sample results for Bayesian model averaging-Rolling window

Benchmark:		Random Walk						Historical Average Return					
Currency	Horizon	RMSFE			RSF			RMSFE			RSF		
		ρ = 0.5	ρ = 0.3	ρ = 0.1	ρ = 0.5	ρ = 0.3	ρ = 0.1	ρ = 0.5	ρ = 0.3	ρ = 0.1	ρ = 0.5	ρ = 0.3	ρ = 0.1
		Rolling window											
Pound	1 quarter	1.0684	1.0242	1.0131	0.5735	0.5735	0.6471	1.0586	1.0148	1.0038	1.1143	1.1143	1.2571
	2 quarters	1.1247	1.1125	1.0623	0.4776	0.5522	0.6418	1.1077	1.0956	1.0462	1.0323	1.1935	1.3871
	4 quarters	1.1743	1.1721	1.1629	0.5385	0.5385	0.5077	1.1404	1.1382	1.1294	1.2069	1.2069	1.1379
	8 quarters	1.1272	1.1206	1.1188	0.4426	0.4262	0.4262	1.0675	1.0613	1.0596	1.1304	1.1304	1.0870
Yen	1 quarter	1.0132	1.0100	1.0107	0.5735	0.5441	0.5441	1.0062	1.0030	1.0038	1.2155	1.1563	1.1563
	2 quarters	0.9317	0.9374	0.9557	0.7463	0.7313	0.7164	0.9198	0.9254	0.9435	1.5152	1.4848	1.4545
	4 quarters	0.8391	0.8481	0.8619	0.8308	0.8308	0.8154	0.8178	0.8266	0.8400	1.6364	1.6364	1.6061
	8 quarters	0.7707	0.7708	0.7756	0.7213	0.7213	0.7213	0.7136	0.7137	0.7181	1.3750	1.3750	1.3750
Mark/Euro	1 quarter	1.0555	1.0377	1.0223	0.5000	0.5147	0.4853	1.0478	1.0303	1.0148	1.1333	1.1333	1.1000
	2 quarters	1.0743	1.0643	1.0483	0.5522	0.5224	0.4925	1.0291	1.0594	1.0435	1.0000	1.0294	0.9706
	4 quarters	1.0473	1.0599	1.0688	0.5077	0.5385	0.5692	1.0455	1.0581	1.0669	0.9167	0.9722	1.0278
	8 quarters	0.9502	0.9621	0.9944	0.6393	0.6721	0.6557	0.9487	0.9605	0.9928	1.1818	1.2424	1.2121

Note: This table shows the results of Bayesian Model Averaging relative to those of two benchmarks (random walk and historical average return) respectively, using rolling window. ρ is parameter of prior weight on models. $\rho = 0.5$ means all the models receive equal prior weight while ρ smaller than 0.5 means models with less variables are preferred, and the smaller the ρ is, the more preferred the models with less variables are.

Relative to the random walk benchmark, rolling window does not show much improvement in British Pound prediction. The RMSFE in rolling window is only slightly smaller in medium and long horizon. However, this improvement is quite small, which is only around 2 % or 3%, and in short term forecasting, the prediction error is even larger in rolling window than that in expanding window.

A similar result can be found in Mark/Euro prediction. Using rolling window, a 2% reduction in RMSFE is realized in 1 quarter ahead forecasting while an up to 5% rise in RMSFE is found in 2 quarters and 4 quarters prediction. In the 2 years ahead forecast, the RMSFE again decreased by up to 4%.

Interesting conclusion would be made if we compare the results of Japanese Yen. Except for the 1 quarter ahead prediction with $\rho = 0.5$, the RMSFE in rolling window is significantly less than that in expanding window. There is 1% reduction in 1 quarter prediction, 4% drop in RMSFE in 2 quarters prediction, almost 10% decline in 4 quarters prediction and nearly 18% in 8 quarters forecast. That is to say, for Yen, using rolling window does help to reduce the forecasting error of BMA and to get a much more precise prediction of future exchange rate. One step further, Japanese Yen is the only currency in which BMA shows significant priority in prediction relative to random walk benchmark. In conclusion, rolling window improves the preciseness of BMA only slightly. But, in those currency-horizon combinations where BMA shows significant forecast ability, the reduction in prediction error using rolling window is much more significant.

On the other hand, the improvement is much more apparent in forecasting signs as is shown by RSF. In expanding window, 10 pairs of currency-horizon-ρ combination are equal or less than 0.5 while there are only 6 pairs in rolling window. For Yen, from 2 quarters to 8 quarters forecast, BMA consistently predicts more than 70% of the signs correctly in rolling window. The results are nearly the same when historical average return is considered as a benchmark and BMA outperforms historical average return in sign prediction almost in all situations.

The last issue about BMA which we would like to explain is whether, by fixing some special variables on theoretical ground, we can improve BMA. In fact, some simple regression models derived from basic economic theories are proved to have some degree of predict ability, such as the Purchasing Power Parity (PPP) model, Uncovered Interest Rate Parity (UIP) model, and Sticky Price Monetary (SP) model of Dornbusch (1976) and Frankel (1979). Enlightened by these models, we select four variables as the 'focus' variables to be fixed in the model.

Relative change in CPI: theoretically, the change in the countries' price levels is a major factor of exchange rate movements between two economies' currencies. The goods-market arbitrage mechanism will move the exchange rate to equalize prices in the two economies. For example, if the US goods are more expensive than those in Japan, consumers in the US and Japan may tend to purchase more Japanese goods. As a result, the increased demand for Japanese goods drives the Japanese yen to appreciate with respect to the US dollar until the dollar denominated prices of the US goods and Japanese goods are equalized.

Relative interest rate: relative interest rate is another potential factor in explaining exchange rate. Ignoring transaction cost and liquidity constraints, an arbitrage mechanism drives the exchange rate to a value that equalizes the returns on the same assets in different economies. Here we use both short term interest rate and long term interest rate to represent the returns.

Relative money supply: relative money supply is used in SP model and also proved to be useful in describing exchange rate by Frankel (1979).

Table 4.1.3 Models specifications

variables	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
constant	--	focus	--	--	--	focus
growth rate of money supply	--	--	focus	--	--	focus
short term interest rate	--	--	--	focus	--	focus
long term interest rate	--	--	--	focus	--	focus
change in CPI	--	--	--	--	focus	focus
GDP	--	--	--	--	--	--
growth in GDP	--	--	--	--	--	--
international reserve	--	--	--	--	--	--
change in international reserve	--	--	--	--	--	--
change in industrial production	--	--	--	--	--	--
change in reserve minus gold	--	--	--	--	--	--
labor productivity	--	--	--	--	--	--
growth rate of stock return	--	--	--	--	--	--

Notes: all the variables except the constant are relative to those of the US. '--' represents the auxiliary regressors in each model.

The Out-of-sample results are shown in Table 4.1.4. Compared to Model 1 (BMA without 'focus' variables), other models (BMA with some fixed variables) are equivalent to adding some restrictions to Model 1. The more variables are fixed, the more restrictions are in fact added to our model. Theoretically, this kind of restriction serves to reduce our in-sample fit to some extent. However, as is shown in the table, it is not always the case. In fact, these variables chosen according to some basic economic theory are helpful in reducing forecasting error in some cases especially in short term forecasts. For instance, in the 1 quarter forecast for Pound, Model 1 has a RMSFE of 1.0684 while model with a fixed constant has 1.0272. If relative growth of money supply is fixed, the RMSFE is 1.0238. For model 4, where short term interest rate is held, the RMSFE is 1.0188. The least RMSFE, 1.0142, is in model 5, when relative growth rate of CPI is set to be the 'focus' variable. Even when all the selected variables are held in the model, the RMSFE is still less than that of Model 1. It is also the case in 2 quarters forecasts. However, in medium and long term, these 'focus' models can hardly outperform Model 1. Similar results can be found in Yen. In the very short horizon, 1 quarter, some of the models manage to outperform general BMA, and again, from 2 quarters to 8 quarters, BMA appears to be the best model. Furthermore, those 'focus' variables are currency specified. That is, some variables are proved to be useful on one currency but might be useless on another currency. For instance, relative growth of CPI, if fixed in the model, reduces the forecasting error of Pound both in 1 quarter and 2 quarters prediction. But, for Yen, the short interest rate seems to be most helpful in reducing the prediction error.

The result is counter-intuitive at the first glance. Theoretically, a Bayesian model average is already the best model, more precisely, the best model combination, to fit the data. After all, in the procedure of Bayesian model averaging, parameters and models are selected wholly based on their ability to fit samples. However, in-sample fitting and out-of-sample forecast are not always the same. When we try to find a 'best' model to fit the in-sample data, we are usually disappointed about its out-of-sample performance-that is, the problem of overfitting. This seemingly 'counter-intuitive' result is apparently evidence: some variables from the theories, which are ignored under the in sample fit procedure, on the contrary, have better out-of-sample fit. One may argue that, considering the importance of these variables, we may simply fix some of the variables to improve out-of-sample performance. Unfortunately, in order to improve forecasting, selecting and fixing some variables in the model is, although appealing, not always an easy task because generally these variables are currency and horizon specified. It is also one of the reasons that we propose BMAP in forecasting exchange rate.

Table 4.1.4 Out-of-sample results for model with 'focus' variables

		RMSFE						RSF					
		Model 1	Model 2	Model 3	Model 4	Model 5	Model 6	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
Random walk as benchmark													
Pound	1 quarter	1.0684	1.0272	1.0238	1.0188	1.0142	1.0532	0.5735	0.5000	0.4265	0.5000	0.4412	0.4853
	2 quarters	1.1247	1.1055	1.1053	1.0969	1.0944	1.1163	0.4776	0.4925	0.5075	0.6119	0.4776	0.4478
	4 quarters	1.1743	1.1846	1.2026	1.1894	1.1823	1.1813	0.5385	0.5231	0.5846	0.5385	0.5231	0.5385
	8 quarters	1.1272	1.1270	1.1219	1.1265	1.1187	1.1114	0.4426	0.4098	0.4098	0.4590	0.4426	0.4590
Yen	1 quarter	1.0132	1.0152	1.0144	0.9997	1.0022	1.0360	0.5735	0.4706	0.5735	0.5882	0.4853	0.4559
	2 quarters	0.9317	0.9635	0.9786	0.9586	0.9741	1.0097	0.7463	0.6866	0.5682	0.6866	0.6418	0.5224
	4 quarters	0.8391	0.8466	0.8481	0.8399	0.8342	0.8749	0.8308	0.8308	0.8308	0.8308	0.8308	0.7846
	8 quarters	0.7707	0.7849	0.8023	0.7663	0.7685	0.7914	0.7213	0.7213	0.7213	0.7213	0.7213	0.7213
Historical average return as benchmark													
Pound	1 quarter	1.0586	1.0178	1.0144	1.0095	1.0049	1.0436	1.1143	0.9714	0.8386	0.8468	0.8571	0.9429
	2 quarters	1.1077	1.0888	1.0886	1.0803	1.0778	1.0994	1.0323	1.0645	1.0968	1.3226	1.0323	0.9677
	4 quarters	1.1404	1.1505	1.1680	1.1551	1.1483	1.1472	1.2069	1.1724	1.3103	1.2069	1.1724	1.2069
	8 quarters	1.0675	1.0673	1.0625	1.0669	1.0595	1.0526	1.1304	1.0870	1.0870	1.2174	1.1739	1.2174
Yen	1 quarter	1.0062	1.0082	1.0074	0.9928	0.9953	1.0289	1.2155	1.0000	1.2188	1.2500	1.0313	0.9688
	2 quarters	0.9198	0.9512	0.9661	0.9463	0.9617	0.9968	1.5152	1.3939	1.1515	1.3939	1.3030	1.0606
	4 quarters	0.8178	0.8251	0.8265	0.8186	0.8130	0.8527	1.6364	1.6364	1.6364	1.6364	1.6364	1.5455
	8 quarters	0.7136	0.7267	0.7428	0.7095	0.7115	0.7327	1.3750	1.3750	1.3750	1.3750	1.3750	1.3750

Notes: this table shows the comparison among each model when some of the variables are fixed. All these ratios are calculated using rolling window and with $\rho = 0.5$. Model 1 is BMA model averaging without fixed variables.

4.2 Bayesian Model Winner

Bayesian Model Winner, just as classical econometrics, is in fact a single 'true' model at each time rather than model combination. However, we use Bayesian method for our selection of this single 'true' model. The results are displayed in Table 4.2.1 and Table 4.2.2 for expanding window and rolling window respectively.

The data demonstrates that, no matter expanding window or rolling window is applied, and no matter random walk model or historical average return is set as a benchmark, BMW underperforms benchmarks in prediction error almost in all situations. A poor predictability can be also found in the forecasts of exchange rate movements.

It is not surprising considering the great effort economists have made in using single model for predicting and little consensus across the articles on what kind of model works well. Our result is also a reassurance of the widely accepted opinion that: no single model could significantly outperform random walk in out-of-sample test, even though they have good in-sample fit. In fact, this is also the main reason for the increasing attention to model combination in recent years. A further comparison between BMW and model combination will be made in the next section.

Table 4.2.1 Out-of-sample results for BMW in expanding window

Benchmark:		Random Walk						Historical Average Return					
Currency	Horizon	RMSFE			RSF			RMSFE			RSF		
		ρ = 0.5	ρ = 0.3	ρ = 0.1	ρ = 0.5	ρ = 0.3	ρ = 0.1	ρ = 0.5	ρ = 0.3	ρ = 0.1	ρ = 0.5	ρ = 0.3	ρ = 0.1
Expanding window													
Pound	1 quarter	1.1374	1.0383	1.0055	0.4559	0.5000	0.5441	1.1270	1.0288	0.9963	0.8857	0.9714	1.0571
	2 quarters	1.7251	1.8151	1.7251	0.4776	0.4776	0.4776	1.6989	1.6989	1.6989	1.0323	1.0323	1.0323
	4 quarters	1.9245	1.9245	1.9245	0.4615	0.4615	0.4615	1.9245	1.9245	1.9245	1.0345	1.0345	1.0345
	8 quarters	2.1827	2.1827	2.3509	0.4098	0.4098	0.3934	2.0671	2.0671	2.2264	1.0870	1.0870	1.0435
Yen	1 quarter	1.0202	1.0202	1.0202	0.4265	0.4265	0.4265	1.0132	1.0132	1.0132	0.9063	0.9063	0.9063
	2 quarters	1.4790	1.4790	0.9954	0.4925	0.4925	0.4925	1.4601	1.4601	0.9827	1.0000	1.0000	1.0000
	4 quarters	1.9247	1.9247	1.9247	0.5077	0.5077	0.5077	1.8758	1.8758	1.8758	1.0000	1.0000	1.0000
	8 quarters	3.2302	3.2302	3.2302	0.4918	0.4918	0.4918	2.9908	2.9908	2.9908	0.9375	0.9375	0.9375
Mark/Euro	1 quarter	1.0622	1.0622	1.0622	0.5294	0.5294	0.5294	1.0545	1.0545	1.0545	1.2000	1.2000	1.2000
	2 quarters	1.0436	1.0436	1.0436	0.4925	0.4925	0.4925	1.0389	1.0389	1.0383	0.9706	0.9706	0.9706
	4 quarters	1.0252	1.0252	1.0252	0.4769	0.4769	0.4769	1.0234	1.0234	1.0234	0.8611	0.8611	0.8611
	8 quarters	1.0150	1.0150	1.0150	0.4918	0.4918	0.4918	1.1758	1.0134	1.0134	1.0000	0.9091	0.9091

Notes: this table shows the expanding window out-of-sample forecasting results of BMW relative to random walk model and historical average return model. The results for all currencies and horizons are listed here.

Table 4.2.2 Out-of-sample results for BMW in rolling window

Benchmark:		Random Walk						Historical Average Return					
Currency	Horizon	RMSFE			RSF			RMSFE			RSF		
		$\rho = 0.5$	$\rho = 0.3$	$\rho = 0.1$	$\rho = 0.5$	$\rho = 0.3$	$\rho = 0.1$	$\rho = 0.5$	$\rho = 0.3$	$\rho = 0.1$	$\rho = 0.5$	$\rho = 0.3$	$\rho = 0.1$
Rolling window													
Pound	1 quarter	1.0741	1.0668	0.9812	0.5735	0.5588	0.6029	1.0643	1.0570	0.9722	1.1143	1.0857	1.1714
	2 quarters	1.3365	1.2501	1.1058	0.4478	0.6619	0.6567	1.3162	1.2312	1.0891	0.9677	1.3226	1.4194
	4 quarters	1.5126	1.5111	1.5018	0.4923	0.4769	0.5077	1.4690	1.4675	1.4586	1.1034	1.0690	1.1379
	8 quarters	1.3574	1.3450	1.2607	0.4262	0.4426	0.3770	1.2852	1.2738	1.1940	1.1304	1.1739	1.0000
Yen	1 quarter	1.0474	1.0828	1.0828	0.4118	0.4265	0.4265	1.0401	1.0753	1.0753	0.8750	0.9063	0.9063
	2 quarters	1.0532	0.9744	1.0200	0.6269	0.6866	0.5970	1.0397	0.9619	1.0070	1.2727	1.3939	1.2121
	4 quarters	0.9647	0.9710	0.9277	0.7692	0.7692	0.7846	0.9402	0.9463	0.9041	1.5152	1.5152	1.5455
	8 quarters	1.5031	1.2027	1.2027	0.6393	0.6393	0.6393	1.3917	1.1135	1.1135	1.2188	1.2188	1.2188
Mark/Euro	1 quarter	1.0944	1.0586	1.0586	0.5147	0.5588	0.5588	1.0864	1.0509	1.0509	1.1667	1.2667	1.2667
	2 quarters	1.1568	1.1824	1.0979	0.5970	0.4776	0.4478	1.1515	1.1770	1.0929	1.1765	0.9412	0.8824
	4 quarters	1.0951	1.0814	1.0823	0.6000	0.5846	0.5846	1.0932	1.0795	1.0804	1.0833	1.0556	1.0556
	8 quarters	1.1194	1.1158	1.0270	0.6230	0.6557	0.7049	1.1176	1.1141	1.0254	1.1515	1.2121	1.3030

Notes: this table shows the rolling window out-of-sample forecasting results of BMW relative to random walk model and historical average return model. The results for all currencies and horizons are listed here.

Despite the poor results we get from BMW, we still believe that the ideas behind BMW are quite convincing: the 'true' model-if there is one-might change overtime, and the factors might be different when the country is in different stages of the economic cycle. Therefore, we still expect to reveal some relationships between those models and macro-economy.

Here, we redo our BMW regression and prolong the out-of-sample test periods, starting from 1983 Q1 to 2010 Q1 (from the 41st period). We set it like this for two reasons. Firstly, 10 years (in-sample time period) is a sufficient long time period for defining a 'economic stage', and when we use rolling window, it is more likely that only the sample in the same 'stage' is used to select the best model. Secondly, we need a sufficient long out-of-sample time span to let the model change.

Besides, we only employ rolling window here because if using expanding window, as moving forward, it would be growingly difficult to 'change' a model-an old model may accumulate too much weight from the previous data. Therefore, we start from the model which best explains the first 10 years' one quarter ahead exchange rate using realized data. This model is selected according to the posterior probability. Then, we use this model and the already known economic and financial information on the 40th period to predict the exchange rate of the 41st period. After that, one period later, we 'know' the next period's information including the realized exchange rate and other fundamental information. Next step is to check whether the model and the variables included in the model are still suitable by using data from the 2nd period up to the 41st period to calculate the model posterior. If the previously selected model explains new data well and still gains the highest posterior probability, it should be used to forecast the exchange rate of the 42nd period. If the model does not perform well, it might be replaced by another 'true' model with other variable combination. Actually, we do not expect the 'true' model to change immediately, however, the larger the forecasting error is, the sooner a new model will be selected.

The results are shown in table 4.2.3

Table 4.2.3 The 'true' model specification in each currency and period

	constant	growth rate of money supply	short term interest rate	change in CPI	GDP	growth in GDP	international reserve	growth rate of stock return
Pound								
1983.Q1-1985.Q1	+		+		+		+	
1985.Q2-1999.Q3	+		+	+	+		+	
1999.Q4-2000.Q1				+				
2000.Q2-2004.Q4	+		+	+	+		+	
2005.Q1-2010.Q1				+				
Yen								
1983.Q1-1991.Q2						+		
1991.Q3-1994.Q2								+
1994.Q3-2010.Q1	+							
Mark/Euro								
1983.Q1-1985.Q1								+
1985.Q2-1987.Q1		+						
1987.Q2-1998.Q3	+	+						+
1998.Q4-2010.Q1								+

Notes: those variables which do not appear in any of the currencies and periods are not listed in this table.

We choose Yen to explain our interesting findings. From 1983 to 2010, three single models are chosen as the 'true' model, dividing the whole time period into three sub-periods. The first one is from 1983 to 1991. A single model with only the relative growth rate in GDP as explanatory variable best fits the data and is used to predict future exchange rate. The second sub-period starts from 1991 to 1994 and the relative growth rate of stock return is most powerful in forecasting exchange rate. During the third sub-period, since 1994, the exchange rate movement is best described by a simple drift random walk.

Considering the history of Japan, during the period from 1970s to 1980s, the GDP was growing at an average of 6% annually, which is a part of the so-called "Japanese miracle". The fast growing GDP is a proper indicator of the overall economy of Japan during this time. Therefore, the model with relative growth rate of GDP as a predictor is suggested by BMW. After that, because of the after-effects of Japanese asset price bubble and domestic policies intended to wring speculative excesses from the stock and real estate markets, growth rate slowed down remarkably at the beginning of

1990s which is called the 'Lost Decade'. In 1992, the growth rate of GDP was only 0.4%. The rate was 0.5% in 1993 and 0.6% in 1994. At the same time, the stock prices plunges more than 60%. The crash of the stock market indicates the collapse of Japan's economy as a whole and becomes a main predictor of the exchange rate. After 1995, the economy enters into a very low growing period. From 1995 to 2005, the average growth rate of GDP was only 1% and the exchange rate was relatively quite stable. It is reported that during 1985 to 1995, the exchange rate of Yen has climbed up by more than 200% and since 1995, which is called 'stable period' by economists, the exchange rate has only declined by 8.7% with very little fluctuations. At this time period, BMW employs a single drift random walk model to forecast future exchange rate. Therefore, although it is not exactly matched, the BMW generally manages to choose the major factor of the country's economy to predict future exchange rate in each sub-period.

The change of model and variable clearly illustrates that different model is required at each specific time period and the good match between major variables used in the model and macro-economic conditions offers a reasonable interpretation of our 'true' model. Besides, this result might also be an explanation of the puzzle that no one single model has consistent out-of-sample performance and some models generate good out-of-sample forecasting performance only when applied to a particular currency and particular subsample.

4.3 Bayesian Model Averaging Using Predictive Likelihood

As we discussed before, given the problems of overfitting of data and the absence of true model, Jana Eklund (2007) suggests that BMAP need to be applied in forecasting combination. Replacing marginal likelihood by predictive likelihood, our Bayesian model average procedure contains predictive content. One major issue left here is how to divide our sample into two parts-training sample and hold-out sample. On one hand, the hold-out sample needs to be large to calculate a less erratic predictive measure. On the other hand we lose more recent data for estimation as hold-out sample increases. Eklund and Karlsson (2007) use Monte Carlo study to investigate some aspects of the small sample performance of forecast combinations based on the predictive likelihood as well as the traditional in-sample marginal likelihood. Their research shed some light on two issues: the appropriate choice of training sample and hold-out sample and how the procedures cope with the likely case that the true model is not included in the set of models considered. Based on their simulation, they propose that 70% of the data for hold-out sample seems to be appropriate for sample size between 100 and 400.

In our study, we test five different divisions, setting the hold-out sample from 30% to 70%. We denote φ as the proportion of sample allocated to hold-out sample. Our BMAP out-of-sample forecast also follows the same way used in BMA, where the dataset is sequentially updated. That is, the first forecast for $t = 81$ is based on the first 80 observations which are split into $(1-\varphi)*80$ training sample and $\varphi * 80$ hold-out sample. The training sample is used to convert the prior into a posterior and the predictive likelihood is calculated for the hold-out sample. The model averaged forecasts are then computed using the model posterior probability. One thing worth mention is that, although predictive likelihood is derived from only the hold-out sample, the forecast from each model is based on the parameter posterior from the full sample of 80 observations. That is, the sample split is only used for the purpose of calculating the predictive weights. For the next forecast for $t = 82$, observation 81 is added to the data and the procedure is repeated. In the expanding window, there are now 81 observations for calculation. As moving forward, both the training sample and hold-out sample increase. In the rolling window, the training sample and hold-out sample are held constant for $(1-\varphi)*80$ and $\varphi * 80$ observations respectively.

All the results for both expanding window and rolling window are shown in tables below. Table 4.3.1, table 4.3.2, table 4.3.5 and table 4.3.6 display the results of RMSFE in both expanding window and rolling window. The other tables showing the RSF results are listed in the appendix (See appendix E). Firstly, we make a comparison between these two data processing methods. Then we illustrate the detail of the results.

Unlike those results from BMA and BMW, where rolling window proved to be at least as good as expanding window, the result of BMAP shows little preference on either of the two methods. For example, in expanding window, relative to random walk, if $\rho = 0.5$, $\varphi = 0.3$, the RMSFE of 1 quarter for Pound and Yen is 1.0171 and 1.0237.

The same RMSFE in rolling window is 1.0414 and 1.0313, which demonstrates that expanding window is better than rolling window. However, the RMSFE of 2, 4 and 8 quarter forecast for Yen, 0.9652, 0.8984 and 0.8668, is higher than those in rolling window, which is 0.9577, 0.8182 and 0.7205. Therefore, no consistent conclusion can be made. One possible explanation might be that there is a tradeoff between them. A rolling window takes the most relevant sample into account, however, at the same time, the sample size is limited to 80 observations. An expanding window, on one hand, uses the sample long period ago which may be less relevant, on the other hand, it has an average sample size of 110 observations. It is not clear which one plays a more crucial role. Only one thing seems to be clear to some extent: if BMAP predicts better than random walk, the prediction error would be smaller when rolling window is applied.

Here, we use the rolling window results to illustrate BMAP. As a whole, BMAP outperforms random walk nearly 1/2 of the time. Even in those combinations where BMAP underperforms benchmark, almost all the RMSFE is less than 1.1. On the other side, BMAP outperforms benchmark 30% at best. On average, the RMSFE of BMAP is 0.9717 for all currencies and horizons. This is a good result, especially when compared to that of BMA and BMW. A further comparison will be discussed in the next section.

There is another issue we might be more interested in: what is the appropriate proportion for hold-out sample. Labeling those RMSFE which is the lowest in each group, we realize that the lowest prediction errors concentrate in the first and second columns, when $\varphi = 0.3$ and $\varphi = 0.4$. It means when 30%-40% of the observations are chosen as the hold-out sample, BMAP performs best. In the expanding sample, the lowest prediction errors appear more often when $\varphi = 0.5$. It is possible that when the sample size increases, more observations could be allocated to hold-out sample without affecting the estimation of model parameters. Comparing the results of various value of ρ, if $\rho = 0.1$, the RMSFE is slightly lower than that in other values of ρ. If $\rho = 0.1$ and $\varphi = 0.4$, 70% of the RMSFE relative to random walk and 80% of that relative to historical average return is less than 1, which means, under this combination, BMAP is significantly better than these benchmark models in prediction error. So is in the sign prediction. BMAP consistently

Table 4.3.1 Expanding window ratios of the root mean squared forecast error for BMAP relative to random walk model

		RMSFE														
		$\rho = 0.5$					$\rho = 0.3$					$\rho = 0.1$				
		$\varphi = 0.3$	$\varphi = 0.4$	$\varphi = 0.5$	$\varphi = 0.6$	$\varphi = 0.7$	$\varphi = 0.3$	$\varphi = 0.4$	$\varphi = 0.5$	$\varphi = 0.6$	$\varphi = 0.7$	$\varphi = 0.3$	$\varphi = 0.4$	$\varphi = 0.5$	$\varphi = 0.6$	$\varphi = 0.7$
Benchmark: Random walk model																
Pound	1 quarter	1.0171	1.0236	**1.0124**	1.0149	1.0219	1.0077	1.0103	**1.0019**	1.0023	1.0058	1.0018	1.0023	0.9991	**0.9987**	1.0010
	2 quarter	1.0262	1.0386	**1.0192**	1.0435	1.0430	1.0102	1.0176	**1.0061**	1.0210	1.0200	1.0024	1.0039	**0.9995**	1.0015	1.0002
	4 quarter	1.0433	1.0751	**1.0160**	1.0647	1.0543	1.0167	1.0527	**1.0115**	1.0674	1.0468	1.0025	1.0156	1.0144	1.0296	1.0465
	8 quarter	**0.9980**	1.0241	1.0105	1.0445	1.0354	**0.9758**	1.0334	1.0087	1.0608	1.0906	0.9618	1.0248	**0.9944**	1.0876	1.0684
Yen	1 quarter	**1.0237**	1.0247	1.0266	1.0241	1.0249	1.0151	1.0178	1.0167	**1.0147**	1.0149	1.0142	1.0114	1.0099	1.0095	**1.0093**
	2 quarter	**0.9652**	0.9657	0.9658	0.9691	0.9753	0.9883	0.9834	**0.9767**	0.9792	0.9791	0.9917	0.9885	**0.9796**	0.9821	0.9806
	4 quarter	**0.8984**	0.9301	0.9525	0.9393	0.9091	**0.8924**	0.9292	0.9538	0.9382	0.9274	**0.9106**	0.9204	0.9368	0.9191	0.9478
	8 quarter	**0.8668**	0.8961	0.9284	0.9284	0.9556	**0.8691**	0.9113	0.9387	0.9365	0.9700	**0.9079**	0.9254	0.9669	0.9327	0.9949
Mark/Euro	1 quarter	1.0475	1.0555	1.0535	1.0467	**1.0424**	**1.0273**	1.0373	1.0365	1.0325	1.0276	**1.0117**	1.0202	1.0195	1.0173	1.0126
	2 quarter	1.0361	1.0377	1.0421	1.0483	**1.0322**	**1.0206**	1.0337	1.0361	1.0414	1.0222	**1.0063**	1.0261	1.0260	1.0278	1.0092

4 quarter	1.0022	**0.9918**	1.0221	1.0201	0.9948	1.0007	**0.9923**	1.0115	1.0126	0.9946	1.0003	0.9960	1.0043	1.0067	**0.9959**
8 quarter	**0.9513**	0.9549	1.0329	1.0154	1.0580	**0.9634**	0.9645	1.0120	1.0144	1.0422	0.9836	**0.9826**	1.0114	1.0099	1.0186

Notes: this table shows the results of each ρ-φ-horizon-currency combination. The figures in bold are the smallest RMSFE within each value of φ.

Table 4.3.2 Expanding window ratios of the root mean squared forecast error for BMAP relative to historical average return model

		RMSFE														
		$\rho = 0.5$					$\rho = 0.3$					$\rho = 0.1$				
		$\varphi = 0.3$	$\varphi = 0.4$	$\varphi = 0.5$	$\varphi = 0.6$	$\varphi = 0.7$	$\varphi = 0.3$	$\varphi = 0.4$	$\varphi = 0.5$	$\varphi = 0.6$	$\varphi = 0.7$	$\varphi = 0.3$	$\varphi = 0.4$	$\varphi = 0.5$	$\varphi = 0.6$	$\varphi = 0.7$
Benchmark: Historical average return																
Pound	1 quarter	1.0078	1.0143	**1.0032**	1.0057	1.0125	0.9985	1.0011	**0.9927**	0.9932	0.9966	0.9927	0.9931	0.9900	**0.9896**	1.0010
	2 quarter	1.0106	1.0228	**1.0038**	1.0277	1.0272	0.9949	1.0020	**0.9909**	1.0055	1.0045	0.9872	0.9887	**0.9844**	0.9863	0.9850
	4 quarter	1.0132	1.0441	**0.9867**	1.0341	1.0125	0.9874	1.0224	**0.9823**	1.0366	1.0167	**0.9736**	0.9863	0.9852	0.9999	1.0163
	8 quarter	**0.9452**	0.9699	0.9570	0.9892	0.9745	**0.9241**	0.9786	0.9553	1.0046	1.0328	**0.9108**	0.9706	0.9417	1.0298	1.0118
Yen	1 quarter	**1.0167**	1.0176	1.0195	1.0171	1.0174	1.0081	1.0108	1.0097	**1.0077**	1.0079	1.0092	1.0098	1.0054	1.0026	**1.0024**
	2 quarter	**0.9528**	0.9534	0.9538	0.9567	0.9628	0.9757	0.9708	**0.9642**	0.9667	0.9666	0.9791	0.9758	**0.9670**	0.9695	0.9680
	4 quarter	**0.8756**	0.9065	0.9283	0.9155	0.8859	**0.8697**	0.9056	0.9296	0.9144	0.9039	**0.8875**	0.8970	0.9130	0.8958	0.9237
	8 quarter	**0.8025**	0.8297	0.8556	0.8596	0.8848	**0.8046**	0.8438	0.8691	0.8671	0.8981	**0.8406**	0.8534	0.8952	0.8636	0.9211
Mark/Euro	1 quarter	1.0399	1.0478	1.0458	1.0391	**1.0348**	**1.0198**	1.0297	1.0290	1.0250	1.0201	**1.0044**	1.0128	1.0121	1.0099	1.0052
	2 quarter	1.0314	1.0329	1.0373	1.0435	**1.0284**	**1.0159**	1.0290	1.0314	1.0366	1.0175	**1.0017**	1.0214	1.0213	1.0232	1.0046

4 quarter	1.0004	**0.9901**	1.0203	1.0184	0.9930	0.9990	**0.9906**	1.0098	1.0108	0.9929	0.9986	**0.9942**	1.0025	1.0049	**0.9942**
8 quarter	**0.9498**	0.9533	1.0312	1.0138	1.0531	**0.9619**	0.9629	1.0214	1.0128	1.0374	0.9821	**0.9808**	1.0098	1.0083	1.0139

Notes: this table shows the results of each ρ-φ-horizon-currency combination. The figures in bold are the smallest RMSFE within each value of φ.

Table 4.3.5 Rolling window ratio of the root mean squared forecast error for BMAP relative to random walk model

		RMSFE														
		$\rho = 0.5$					$\rho = 0.3$					$\rho = 0.1$				
		$\varphi = 0.3$	$\varphi = 0.4$	$\varphi = 0.5$	$\varphi = 0.6$	$\varphi = 0.7$	$\varphi = 0.3$	$\varphi = 0.4$	$\varphi = 0.5$	$\varphi = 0.6$	$\varphi = 0.7$	$\varphi = 0.3$	$\varphi = 0.4$	$\varphi = 0.5$	$\varphi = 0.6$	$\varphi = 0.7$
Benchmark: Random walk model																
Pound	1 quarter	1.0404	**1.0205**	1.0274	1.0236	1.0406	1.0267	**1.0121**	1.0142	1.0155	1.0179	1.0119	**1.0046**	1.0065	1.0132	1.0132
	2 quarter	1.0396	**0.9470**	0.9574	1.0414	1.0780	1.0152	**1.0041**	1.0206	1.0175	1.0361	0.9994	0.9938	**0.9928**	0.9968	1.0027
	4 quarter	1.0716	**1.0654**	1.1088	1.1590	1.1641	1.0405	**1.0364**	1.0729	1.1288	1.1423	1.0056	**0.9907**	1.0309	1.0725	1.0946
	8 quarter	**1.0375**	1.0914	1.0921	1.1446	1.1249	**1.0184**	1.0666	1.0818	1.1401	**1.1219**	**0.9940**	1.0115	1.0590	1.1354	1.1252
Yen	1 quarter	1.0301	1.0247	1.0177	**1.0139**	1.0179	1.0172	1.0124	1.0105	**1.0047**	1.0065	1.0106	1.0081	1.0071	1.0038	1.0033
	2 quarter	0.9577	0.9470	0.9470	**0.9397**	0.9450	0.9565	0.9411	0.9419	**0.9376**	0.9453	0.9817	0.9474	0.9491	**0.9464**	0.9541
	4 quarter	0.8182	**0.8122**	0.8280	0.8349	0.8536	0.8182	**0.8098**	0.8292	0.8406	0.8687	0.8475	**0.8314**	0.8516	0.8584	0.8888
	8 quarter	**0.7205**	0.7531	0.7568	0.7596	0.7779	**0.7246**	0.7553	0.7535	0.7670	0.7898	**0.7434**	0.7723	0.7694	0.7912	0.8148
Mark/Euro	1 quarter	**1.0471**	1.0490	1.0481	1.0520	1.0501	**1.0275**	1.0301	1.0302	1.0338	1.0333	**1.0108**	1.0126	1.0138	1.0156	1.0158
	2 quarter	**1.0337**	1.0422	1.0479	1.0564	1.0580	**1.0186**	1.0255	1.0342	1.0428	1.0422	**1.0040**	1.0115	1.0176	1.0269	1.0186

4 quarter	**0.9762**	0.9872	1.0210	1.0636	1.0638	**0.9783**	0.9849	1.0122	1.0542	1.0578	**0.9854**	0.9927	1.0047	1.0352	1.0443
8 quarter	**0.8879**	0.9311	1.0021	1.0695	1.0554	**0.9165**	0.9279	0.9929	1.0544	1.0555	**0.9579**	0.9584	0.9827	1.0255	1.0473

Notes: this table shows the results of each ρ-φ-horizon-currency combination. The figures in bold are the smallest RMSFE within each value of φ.

Table 4.3.6 Rolling window ratio of the root mean squared forecast error for BMAP relative to historical average return model

		RMSFE														
		$\rho = 0.5$					$\rho = 0.3$					$\rho = 0.1$				
		$\varphi = 0.3$	$\varphi = 0.4$	$\varphi = 0.5$	$\varphi = 0.6$	$\varphi = 0.7$	$\varphi = 0.3$	$\varphi = 0.4$	$\varphi = 0.5$	$\varphi = 0.6$	$\varphi = 0.7$	$\varphi = 0.3$	$\varphi = 0.4$	$\varphi = 0.5$	$\varphi = 0.6$	$\varphi = 0.7$
Benchmark: Historical average return																
Pound	1 quarter	1.0309	**1.0112**	1.0180	1.0142	1.0311	1.0173	**1.0028**	1.0050	1.0062	1.0086	1.0027	**0.9954**	0.9973	1.0040	1.0015
	2 quarter	1.0396	0.9451	**0.9349**	1.0256	1.0617	0.9998	**0.9889**	1.0051	1.0021	1.0204	0.9843	0.9788	**0.9777**	0.9817	0.9875
	4 quarter	1.0407	**1.0344**	1.0769	1.1256	1.1306	1.0105	**1.0062**	1.0420	1.0963	1.1094	0.9767	**0.9622**	1.0012	1.0416	1.0631
	8 quarter	**0.9825**	1.0336	1.0343	1.0840	1.0654	**0.9645**	1.0101	1.0245	1.0797	1.0625	**0.9414**	0.9579	1.0029	1.0752	1.0656
Yen	1 quarter	1.0230	1.0176	1.0107	**1.0070**	1.0109	1.0109	1.0054	1.0036	**0.9977**	0.9996	1.0036	1.0011	1.0001	0.9969	**0.9964**
	2 quarter	0.9454	0.9349	0.9349	**0.9277**	0.9329	0.9443	0.9291	0.9299	**0.9256**	0.9332	0.9692	0.9353	0.9369	**0.9344**	0.9417
	4 quarter	0.7975	**0.7916**	0.8070	0.8137	0.8320	0.7974	**0.7892**	0.8081	0.8193	0.8466	0.8260	**0.8103**	0.8300	0.8366	0.8662
	8 quarter	**0.6671**	0.6982	0.7007	0.7033	0.7213	**0.6709**	0.6993	0.6976	0.7102	0.7312	**0.6883**	0.7151	0.7123	0.7325	0.7545
Mark/Euro	1 quarter	**1.0395**	1.0414	1.0405	1.0443	1.0433	**1.0200**	1.0226	1.0227	1.0263	1.0258	**1.0034**	1.0052	1.0064	1.0082	1.0084
	2 quarter	**1.0290**	1.0374	1.0431	1.0516	0.0531	**1.0140**	1.0209	1.0295	1.0380	1.0374	**0.9994**	1.0069	1.0129	1.0222	1.0139

4 quarter	**0.9745**	0.9854	1.0192	0.0617	1.0619	**0.9766**	0.9821	1.0104	1.0524	1.0559	**0.9837**	0.9910	1.0030	1.0334	1.0425
8 quarter	**0.8865**	0.9296	1.0005	1.0678	1.0537	**0.9150**	0.9265	0.9914	1.0527	1.0539	**0.9564**	0.9569	0.9811	1.0239	1.0456

Notes: this table shows the results of each ρ-φ-horizon-currency combination. The figures in bold are the smallest RMSFE within each value of φ.

outperforms benchmark in the future movements of exchange rate, no matter random walk model or historical average is selected as the benchmark.

In conclusion, when proper split is conducted, BMAP shows superior predictability not only under the measure of prediction error, but also under the measure of sign forecast. It consistently outperforms both the random walk and historical average return.

4.4 Comparison

In this part, we make a fair comparison among BMA, BMW and BMAP. For BMAP, we set $\varphi = 0.4$. To be fair, we display the results in three categories with ρ equals 0.5, 0.3 and 0.1 respectively. Based on the table below, four major conclusions can be made.

1. Model combination consistently outperforms single model BMW.

Although we apply a flexible model BMW which allows not only parameters but also model to change over time, and in the out-of-sample test, the best suitable model is chosen and used to forecast. It relaxes the traditional econometrics assumption that one 'true' model does exist and consistently to be 'true'. However, this single model hardly outperforms benchmarks under the measure of RMSFE. The RMSFE for BMW is always larger than one, and is also much larger than that of BMA and BMAP. The average RMSFE of BMW for all the currencies and horizons is 1.2136 under $\rho = 0.5$, while it is 1.0856 and 1.0489 for BMA and BMAP.

2. BMW, although has the largest prediction error, predicts the movement of exchange rate well, especially when small number of variables is preferred.

When $\rho = 0.1$, model with very few variables are strongly preferred, BMW predicts the future movements of exchange rate much better than benchmarks and only a little worse than BMA and BMAP. BMW predicts an average of 57.4% of the movements correctly, and this result is still only slightly worse than that of BMA and BMAP, whose RSF are 60.2% and 63.3%. This is a surprising result considering its large prediction error. One possible explanation is that some explanatory variables

Table 4.4.1 Comparison of BMA, BMW and BMAP using random walk as benchmark

currency	horizon	$\rho = 0.5$			$\rho = 0.3$			$\rho = 0.1$		
		BMA	BMW	BMAP	BMA	BMW	BMAP	BMA	BMW	BMAP
RMSFE										
Pound	1 quarter	1.0684	1.0741	1.0205	1.0242	1.0668	1.0121	1.0131	0.9812	1.0046
	2 quarter	1.1247	1.3365	0.9470	1.1125	1.2501	1.0041	1.0623	1.1058	0.9938
	4 quarter	1.1743	1.5126	1.0654	1.1721	1.5111	1.0364	1.1629	1.5018	0.9907
	8 quarter	1.1272	1.3574	1.0914	1.1206	1.3450	1.0666	1.1188	1.2607	1.0115
Yen	1 quarter	1.0132	1.0474	1.0247	1.0100	1.0828	1.0124	1.0107	1.0828	1.0081
	2 quarter	0.9317	1.0532	0.9470	0.9374	0.9744	0.9411	0.9557	1.0200	0.9474
	4 quarter	0.8391	0.9647	0.8122	0.8481	0.9710	0.8098	0.8619	0.9277	0.8314
	8 quarter	0.7707	1.5031	0.7531	0.7708	1.2027	0.7553	0.7756	1.2027	0.7723
Mark/Euro	1 quarter	1.0555	1.0944	1.0490	1.0377	1.0586	1.0301	1.0223	1.0586	1.0126
	2 quarter	1.0743	1.1568	1.0422	1.0643	1.1824	1.0255	1.0483	1.0979	1.0115
	4 quarter	1.0473	1.0951	0.9872	1.0599	1.0814	0.9849	1.0688	1.0823	0.9927
	8 quarter	0.9502	1.1194	0.9311	0.9621	1.1158	0.9279	0.9944	1.0270	0.9584
RSF										
Pound	1 quarter	0.5735	0.5735	0.5882	0.5735	0.5588	0.6618	0.6471	0.6029	0.6471
	2 quarter	0.4776	0.4478	0.6866	0.5522	0.6619	0.5970	0.6418	0.6567	0.6269
	4 quarter	0.5385	0.4923	0.6000	0.5385	0.4769	0.5846	0.5077	0.5077	0.5538
	8 quarter	0.4426	0.4262	0.5082	0.4262	0.4426	0.4918	0.4262	0.3770	0.5246
Yen	1 quarter	0.5735	0.4118	0.5441	0.5441	0.4265	0.5588	0.5441	0.4265	0.5441
	2 quarter	0.7463	0.6269	0.6866	0.7313	0.6866	0.6567	0.7164	0.5970	0.6269
	4 quarter	0.8308	0.7692	0.8308	0.8308	0.7692	0.8308	0.8154	0.7846	0.8154
	8 quarter	0.7213	0.6393	0.7541	0.7213	0.6393	0.7705	0.7213	0.6393	0.7869
Mark/Euro	1 quarter	0.5000	0.5147	0.5147	0.5147	0.5588	0.5588	0.4853	0.5588	0.5735
	2 quarter	0.5522	0.5970	0.5970	0.5224	0.4776	0.5970	0.4925	0.4478	0.5970
	4 quarter	0.5077	0.6000	0.5846	0.5385	0.5846	0.5846	0.5692	0.5846	0.6154
	8 quarter	0.6393	0.6230	0.7049	0.6721	0.6557	0.6721	0.6557	0.7049	0.6885

Notes: BMA, BMW and BMAP represent Bayesian Model Averaging, Bayesian Model Winner and Bayesian Model Averaging Using Predictive Likelihood.

Table 4.4.1 Comparison of BMA, BMW and BMAP using historical average return as benchmark

currency	horizon	ρ = 0.5			ρ = 0.3			ρ = 0.1		
		BMA	BMW	BMAP	BMA	BMW	BMAP	BMA	BMW	BMAP
RMSFE										
Pound	1 quarter	1.0586	1.0643	1.0112	1.0112	1.0148	1.0570	1.0038	0.9722	0.9954
	2 quarter	1.1077	1.3162	0.9451	0.9451	1.0956	1.2312	1.0462	1.0891	0.9788
	4 quarter	1.1404	1.4690	1.0344	1.0344	1.1382	1.4675	1.1294	1.4586	0.9622
	8 quarter	1.0675	1.2852	1.0336	1.0336	1.0613	1.2738	1.0596	1.1940	0.9579
Yen	1 quarter	1.0062	1.0401	1.0176	1.0176	1.0030	1.0753	1.0038	1.0753	1.0011
	2 quarter	0.9198	1.0397	0.9349	0.9349	0.9254	0.9619	0.9435	1.0070	0.9353
	4 quarter	0.8178	0.9402	0.7916	0.7916	0.8266	0.9463	0.8400	0.9041	0.8103
	8 quarter	0.7136	1.3917	0.6982	0.6982	0.7137	1.1135	0.7181	1.1135	0.7151
Mark/Euro	1 quarter	1.0478	1.0864	1.0414	1.0414	1.0303	1.0509	1.0148	1.0509	1.0052
	2 quarter	1.0291	1.1515	1.0374	1.0374	1.0594	1.1770	1.0435	1.0929	1.0069
	4 quarter	1.0455	1.0932	0.9854	0.9854	1.0581	1.0795	1.0669	1.0804	0.9910
	8 quarter	0.9487	1.1176	0.9296	0.9296	0.9605	1.1141	0.9928	1.0254	0.9569
RSF										
Pound	1 quarter	1.1143	1.1143	1.1429	1.1143	1.0857	1.2857	1.2571	1.1714	1.2571
	2 quarter	1.0323	0.9677	1.3939	1.1935	1.3226	1.2903	1.3871	1.4194	1.3548
	4 quarter	1.2069	1.1034	1.3448	1.2069	1.0690	1.3103	1.1379	1.1379	1.2414
	8 quarter	1.1304	1.1304	1.3478	1.1304	1.1739	1.3043	1.0870	1.0000	1.3913
Yen	1 quarter	1.2155	0.8750	1.1563	1.1563	0.9063	1.1875	1.1563	0.9063	1.1563
	2 quarter	1.5152	1.2727	1.3939	1.4848	1.3939	1.3333	1.4545	1.2121	1.2727
	4 quarter	1.6364	1.5152	1.6364	1.6364	1.5152	1.6364	1.6061	1.5455	1.6061
	8 quarter	1.3750	1.2188	1.4375	1.3750	1.2188	1.4688	1.3750	1.2188	1.5000
Mark/Euro	1 quarter	1.1333	1.1667	1.1667	1.1333	1.2667	1.2667	1.1000	1.2667	1.3000
	2 quarter	1.0000	1.1765	1.1765	1.0294	0.9412	1.1765	0.9706	0.8824	1.1765
	4 quarter	0.9167	1.0833	1.0556	0.9722	1.0556	1.0556	1.0278	1.0556	1.1111
	8 quarter	1.1818	1.1515	1.3030	1.2424	1.2121	1.2424	1.2121	1.3030	1.2727

Notes: BMA, BMW and BMAP represent Bayesian Model Averaging, Bayesian Model Winner and Bayesian Model Averaging Using Predictive Likelihood.

have predictability. These few variables, although not sufficient to make precise prediction, are capable of forecasting the trend of exchange rate movement. After all, predicting the sign of exchange rate movements is always easier than predicting the exact return.

3. BMAP has the smallest prediction error among the three models (BMA, BMW and BMAP) we examine.

Almost in every scenario the RMSFE of BMAP is smaller than that of BMA and BMW. Especially in Pound prediction, BMAP is the only model which performs better than random walk model at some horizons. Besides, BMAP beats the random walk almost half of the time and outperform random walk 30% in forecast error at most but underperform random walk no more than 10%.

4. BMAP predicts the movements of exchange rate better than the other two models and significantly better than benchmarks.

In fact, the RSF for BMAP is always above 0.5 and sometimes more than 0.8. BMAP outperforms BMA for every currency and for Pound, where BMA shows little predictability relative to random walk model, BMAP predicts correctly nearly 60%. On average, if $\rho = 0.5$, BMAP predicts 63.3% of the exchange rate movements, higher than that of BMA and BMW.

In conclusion, BMAP outperforms the other two methods (BMA and BMW) not only under the measure of prediction error but also the RSF. BMAP also clearly outperforms those two benchmarks especially in the forecasting of exchange rate movements. Therefore, BMAP, avoiding in-sample overfitting, is the best model we can choose in exchange rate forecasting among these three methods.

5.Investment Performance

5.1 Investment Strategy

Based on our results displayed in the previous part, we illustrate that BMAP is an improvement of BMA and it is better than random walk model and historical average return model. However, without a real investment test to check the performance, this conclusion is still unconvincing. In general, a small prediction error and a high ratio of sign forecasting is not equivalent to the ability of making money in real investment, although there might be some positive correlations. In this part, in order to compare our models from another perspective, we simulate a real investment environment, and consider an investor in the US who makes investment decisions solely based on models.

A simple investment strategy is employed here. It is a naive strategy where if we forecast a positive return, buy it; otherwise, sell it. We also take the interest rate into to account. That is, when calculating the return of an investment in a foreign currency, both the return from exchange rate movements and the return from the difference of interest rates among those countries are considered. Starting from one quarter ahead forecast, the detail of this procedure, take Pound for example, is as follows:

At the beginning of Q2 1993, an investor knows the short term interest rate (Treasury bill rate) for both the UK and the US. Based on the in information from Q1 1993 and earlier and all these exchange rate forecast models (including BMA, BMW,BMAP, Random Walk model and Historical Average Return model), the investor can predict the exchange rate at the end of Q2 1993. Then, the investor confronted with two choices: one is to keep his original 10,000 US dollar, and receive the US short term interest; the other choice is to exchange his US dollar into British Pound, and then get the short term interest from the UK and at the end of Q2, exchange the Pound he has back into US dollar. The investor uses the expected exchange rate from the models to evaluate his two choices and make a decision to maximize his total wealth. At the beginning of Q3 1993, with new information and updated models, the investor follows a similar procedure to invest, and so on. At the end of Q1 2010, the total amount of money the investor obtains by using different models will be compared in order to evaluate these models. All the transaction costs are ignored in our analysis.

Since the random model gives no suggestion on future movements of exchange rate, we use two basic strategies instead. One is always holding foreign currency (AFC), the other is never buying any foreign currency (NFC) and just claim for the short term interest from US Treasury bill. The models examined are BMA, BMW, BMAP, historical average return model (HAR), AFC and NFC.

5.2 Investment Results Using Short Term Forecast

The results for one quarter ahead forecast for Pound, Yen and Mark/Euro are displayed in figures below.

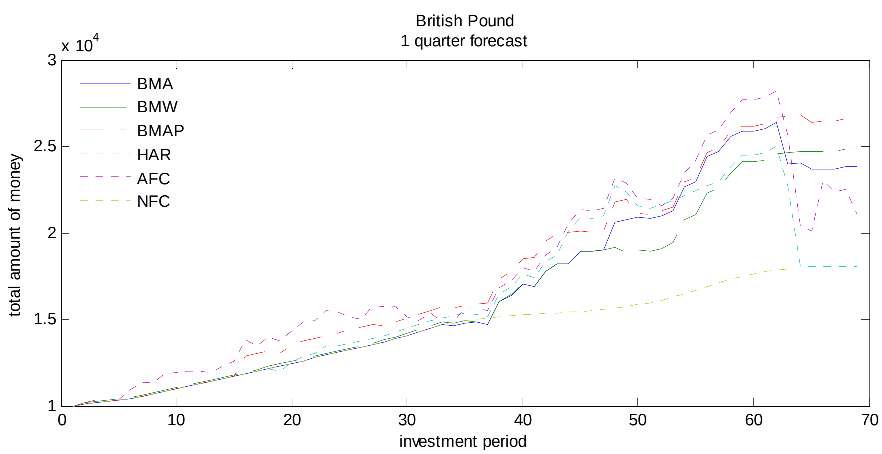

Figure 5.1 Investment results using 1 quarter ahead forecast models for British Pound

For British Pound, according to AFC and NFC, 21051.67 and 17953.84 dollar remain in the US bank account. It means a 110.52% total return is yielded if holding the British Pound all over the period and 79.54% is received if never buying the British Pound. The return for historical average return model is about 80.67%. The other three models seem to be much better than the benchmarks. The best model is BMAP, which yields a 166.1% return at the end of investment period. BMW, although not good at reducing forecasting error, performs much better than benchmark and obtain 148.88% return. BMA also manages to make a profit of 138.56%. However, a further look at the returns, we find that before 2008, the differences between models are not that large, although BMA and BMAP are still better than other models. A sharp decrease in benchmarks appears around 2008 and 2009, which means the benchmarks make some major investment mistakes in the period when exchange rate fluctuates heavily and lose a lot of money.

In the forecasting of Japanese Yen, the historical average return model is proved to be the best model with a return of 136.4%. In addition, BMAP and BMA are still better than AFC and NFC. For Mark/Euro, BMAP, with a return of 149.8%, is obviously the best model an investor could choose and both BMAP and BMA manage to outperform the benchmarks.

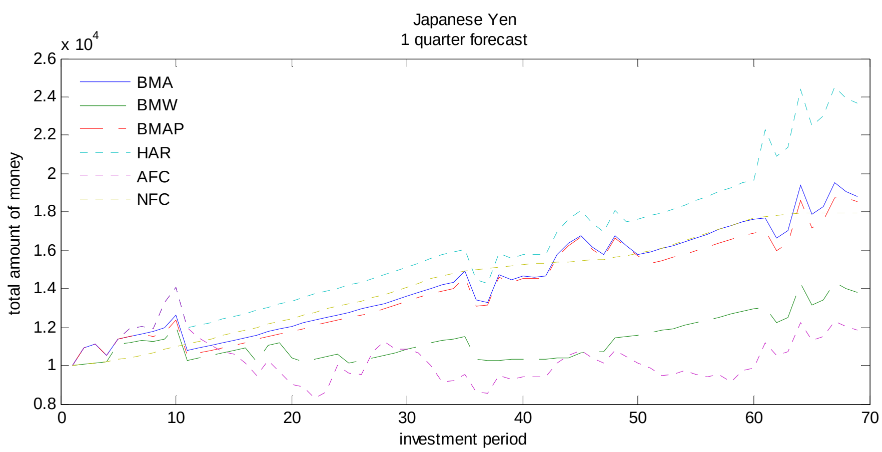

Figure 5.2 Investment results using 1 quarter ahead forecast models for Yen

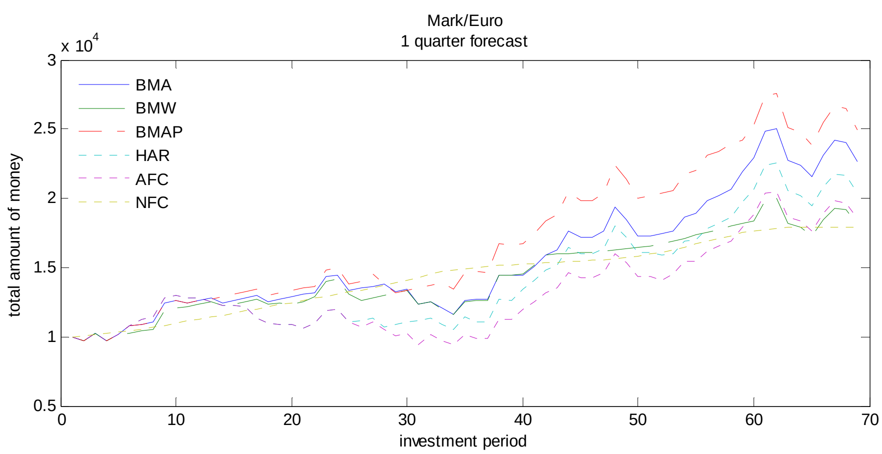

Figure 5.3 Investment results using 1 quarter ahead forecast models for Mark/Euro

In order to examine the performance of long horizon forecasts, such as 2 years ahead forecast, we have two alternatives. One is to invest every 2 years while doing nothing between the two investments. However, it is a waste of our effort in forecasting because less than 10 investments are made from 1993 to 2010, and it is far less than sufficient for us to check the performance of the models. Besides, it is not realistic since investors do not have to hold any currency for a long time, if he has a new expectation of that currency after obtaining updated information. In other words, if investor has some additional information 1 quarter later and changes his original expectation, he will probably change his investment rather than hold it till the end. The other alternative is to make long horizon investment decision every quarter; however, if new information is obtained, it is still allowed to change the investment at each quarter. It is assumed that, at the end of each quarter, the investor makes a long horizon forecast and an investment decision. Next quarter, he can make new decisions if necessary and one quarter's return is realized. That is, although investment decisions are made according to long term forecasting, changes could still be made at short term. In our analysis, we apply the second method.

Take the 2 quarters forecast for example, also, at the beginning of Q2 1993, the investor forms his expectation of exchange rate movement during the next 2 quarters. Together with the information of interest rate in each country, the investor makes an investment decision. Next quarter, Q3 1993, if the original expectation is maintained, the investment will be maintained; otherwise, new investment decision will be made and one quarter return will be realized.

The figures below shows the results of 2 quarters ahead forecast for the three currencies.

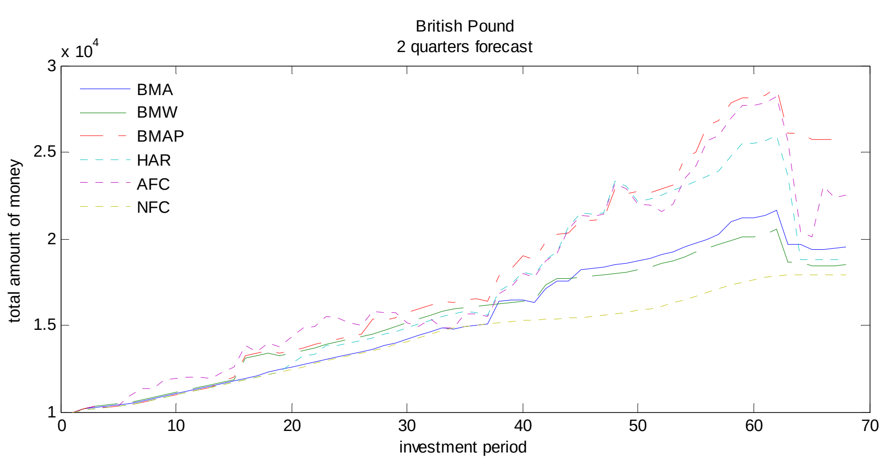

Figure 5.4 Two quarters ahead forecast models for British Pound

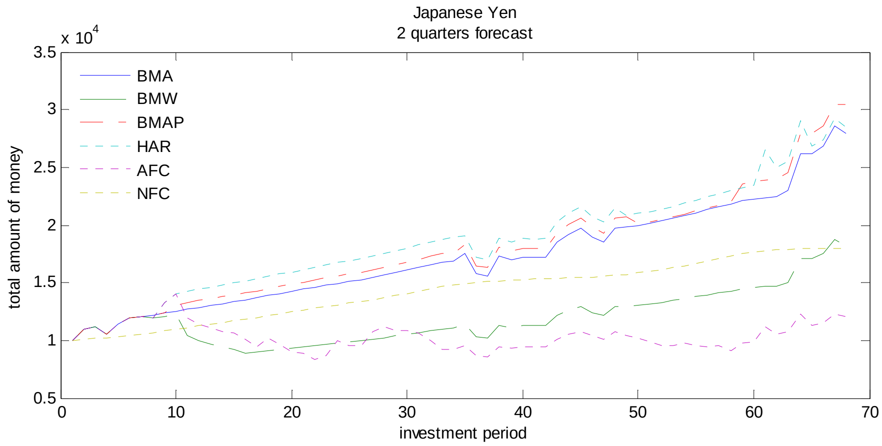

Figure 5.5 Two quarters ahead forecast models for Yen

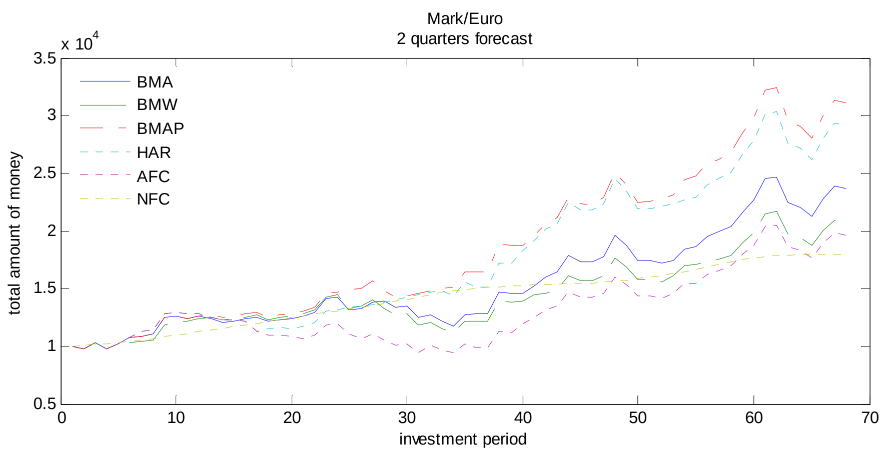

Figure 5.6 Two quarters ahead forecast models for Mark/Euro

For all these three currencies, BMAP clearly outperforms any other model including the benchmarks. However, BMA generally underperforms historical average return model and sometimes it is even not as good as simply holding a foreign currency all over the period.

5.3 Investment Results Using Medium and Long Term Forecast

In medium term forecasting, the difference between the models at the end of investment period tends to be much smaller, which means in the medium term most of the models have similar forecasts. Although slightly, the BMAP still out performs historical average return and other models except in the case of Pound AFC appears to be the best model. BMA, as in the 2 quarters forecast, underperforms historical average return in each of the three cases. BMW, on the other hand, only proved to be better than the simplest strategy.

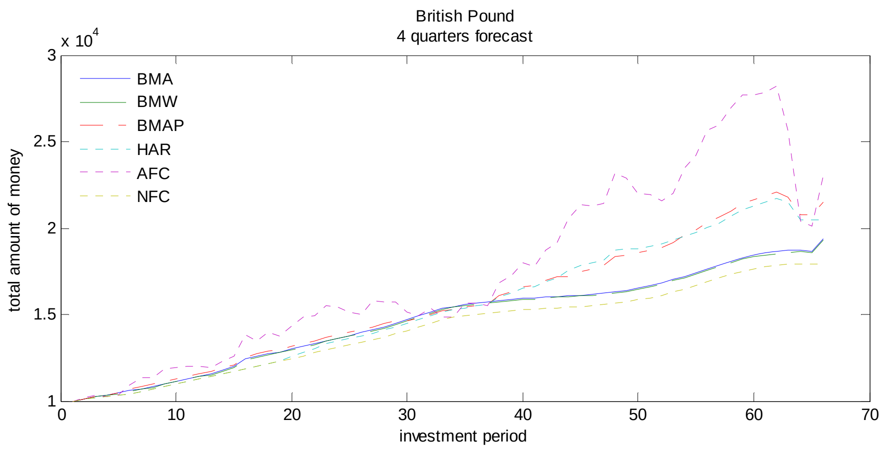

Figure 5.7 One year ahead forecast models for Pound

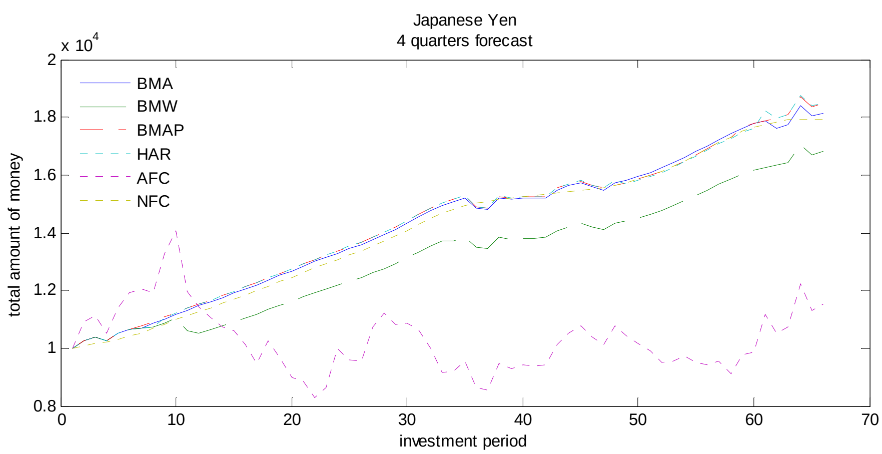

Figure 5.8 One year ahead forecast models for Yen

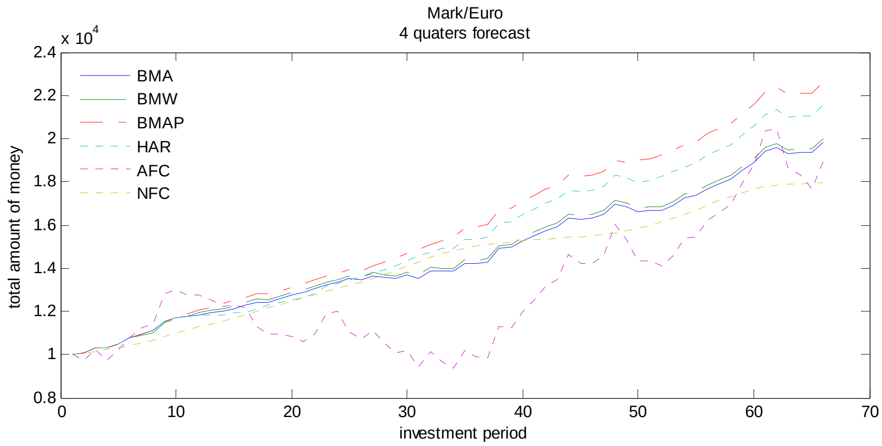

Figure 5.9 One year ahead forecast models for Mark/Euro

In the long term forecasting, model difference tends to be even smaller. Most of models suggest moderate strategy-hold US dollar at most of the time. Therefore, the trends of BMA, BMW, BMAP and HAR are not very different from that of NFC. It is not too surprising since in long term, US dollar is the most strong and safest currency in the world.

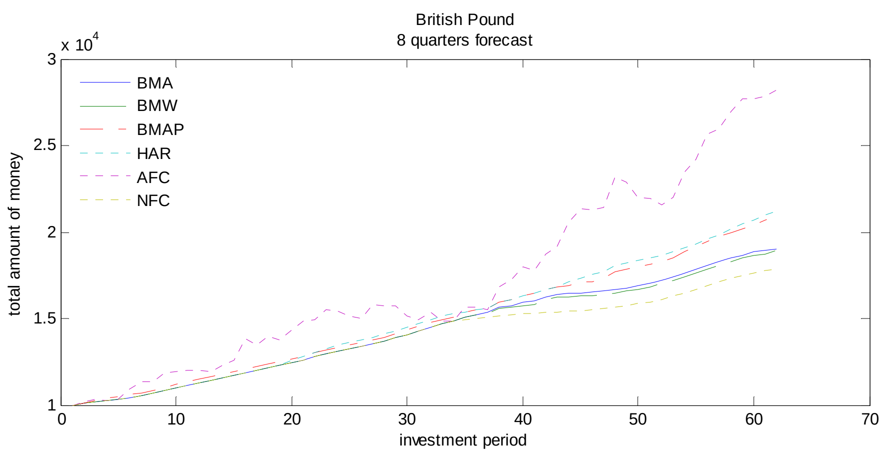

Figure 5.10 Two years ahead forecast models for Pound

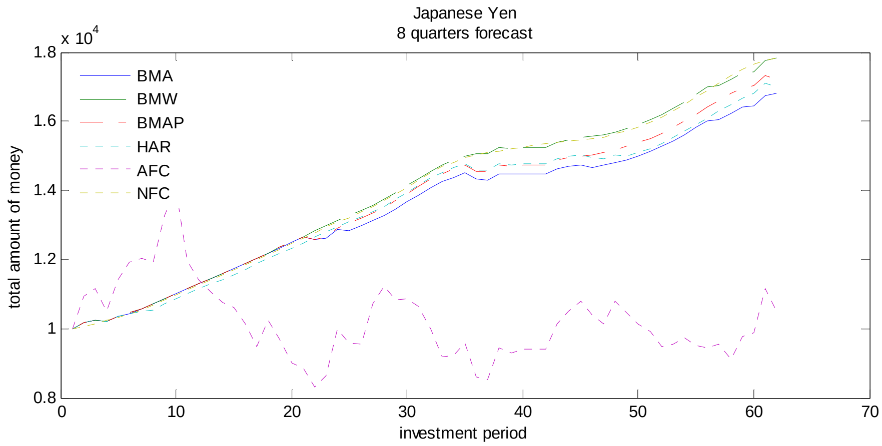

Figure 5.11 Two years ahead forecast models for Yen

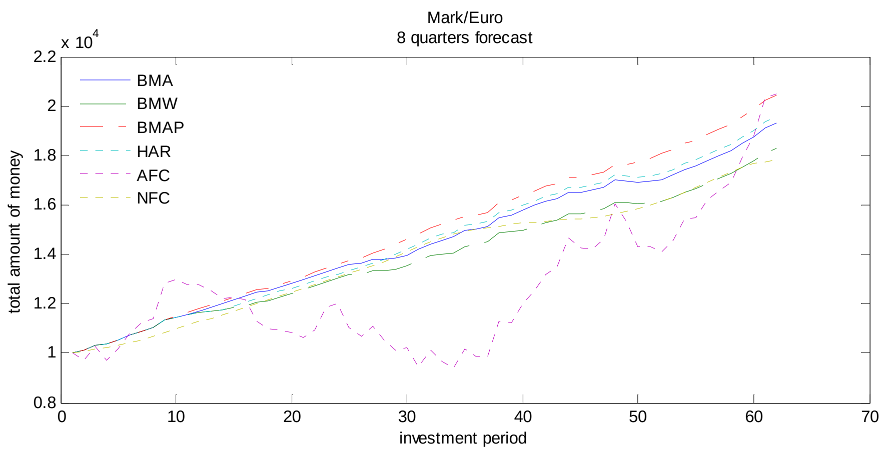

Figure 5.12 Two years ahead forecast models for Mark/Euro

5.4 Forecast Horizon Comparison

At last, we are still interested in which forecast horizon is most profitable within each currency. Comparing the model performance among different horizons (see Appendix F), it is clear that as the increasing of forecast horizon, BMA, BMW, BMAP and HAR converge from AFC to NFC. However, the short term forecast, especially the 2 quarters ahead forecast, is proved to be most profitable. In our paper, we are trying to distinguish between model profitability and model accuracy. In part 4, we argue that the models show better predictability in medium and long horizons under the measure of forecast accuracy, but here this more accuracy leads to lower profitability.

In conclusion, based on the model comparison in a real investment performance, BMAP, not only outperforms benchmarks and BMA from the perspective of model accuracy, but also shows its superiority in the real investment and is proved to be the most profitable model. BMA, although yields root mean square error much lower than historical average return model, generally performs worse than this benchmark. In contrast, BMW, whose prediction error is quite large relative to benchmarks, obtains better results in investment analysis than what we expected in some cases.

6.Conclusion

In this paper, we have considered two specific approaches to forecast exchange rate, namely Bayesian Model Winner (BMW) and Bayesian Model Averaging Using Predictive Likelihood (BMAP). We compare the two proposed models to general Bayesian Model Averaging (BMA) and some benchmark models, such as random walk model and historical average return model, not only in the out-of-sample forecasting test but also in a simulated real investment environment. We also made some improvements in BMA to allow for 'focus' variables on theoretical and other grounds and argued that some variables chosen according to fundamental theory are helpful in reducing the forecasting error in some specific currency-horizon pairs. This seemingly counter-intuitive result is also a proof of the existence of overfitting in most econometric methods, including BMA.

BMW, as in fact a single model, generally underperforms BMA and the two benchmarks (random walk and historical average return). Despite the intuitive appealing, BMW yields mean square forecasting error larger than random walk and provides another evidence for the argument made by Meese and Rogoff (1983) that all the single exchange rate models do less well in out-of-sample forecasting than a simple driftless random walk. Besides, with the comparison to BMA, BMW also shows that single model performs much worse than model combination, such as BMA, which performs slightly better than random walk in forecasting error (this result is consistent with Wright (2003)). However, despite the poor performance under the measure of RMSFE, BMW exhibits its excellent ability in capturing the major factors of exchange rate movements with respect to the specific economic conditions the country is in. In future work, it would be possible and interesting to include nonlinear models, such as those considered by Engel and Hamilton (1990) and Kilian and Taylor (2003). These nonlinear models are proved to be better than linear regression models and BMW can be expected to be improved by including these models.

Another more important focus in our paper is on testing whether Bayesian Model Averaging Using Predictive Likelihood could manage to beat general BMA and random walk in out-of-sample forecasts and real investment. BMAP overcomes two major problems that BMA faces: firstly, it is possible that the forecast combination is adversely affected by in-sample overfitting of the data; secondly, the key assumption that the true model to be included in the set of considered models is not always reliable in practice. Using predictive likelihood, BMAP protects in-sample fitting and enables the model to contain predictive content, even uninformative default priors such as the ones suggested by Fernandez, Ley, and Steel (2001) are used. Moreover, the slower convergence to the true model and the protection against overfitting by considering the out-of-sample predictive ability lead to the better performance of the predictive likelihood when the true model is not in the model set. The empirical results indicate that the weights based on the predictive likelihood have better small sample properties than the traditional in-sample marginal likelihood used in BMA. When proper split is applied, BMAP consistently outperforms BMA under both the measure of RMSFE and that of RSF. Compared to random walk, BMAP gives promising results in out-of-sample forecasting. BMAP yields mean square prediction errors lower than random walk for most situations and predicts the future exchange rate movements better in more than 80% of the cases. In the real investment, especially when short horizon forecast is used, BMAP manages to earn a higher return and beat all other models including BMA and those benchmarks and clearly shows its superiority in the real investment.

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Appendix

Appendix A

Combining the prior with the likelihood gives the posterior

$$p(\beta_1, \beta_{2i}, \sigma^2 | y, M_i)$$
$$\propto (\sigma^2)^{-(n+k_{2i}+2)/2} exp\,[-\frac{\beta_{2i}' V_{0i}^{-1} \beta_{2i} + (y - X_1\beta_1 - X_{2i}\beta_{2i})'(y - X_1\beta_1 - X_{2i}\beta_{2i})}{2\sigma^2}$$

It can be shown that

$$(y - X_1\beta_1 - X_{2i}\beta_2)'(y - X_1\beta_1 - X_{2i}\beta_2)$$
$$= [(y - X_1b_1 - X_{2i}b_{2i}) - (X_1(\beta_1 - b_1) - X_{2i}(\beta_{2i}$$
$$- b_{2i}))]'[(y - X_1b_1 - X_{2i}b_{2i}) - (X_1(\beta_1 - b_1) - X_{2i}(\beta_{2i} - b_{2i}))]$$

b_1 and b_{2i} are the estimator of β_1 and β_{2i} respectively

Then, we arrive at

$$p(\beta_1, \beta_{2i}, \sigma^2 | y, M_i) \propto (\sigma^2)^{-(n+k_{2i}+2)/2} exp\,(-\frac{\varphi_i + \tau_i}{2\sigma^2})$$

Where

$$\varphi_i = \begin{pmatrix} \beta_1 - b_1 \\ \beta_{2i} - b_{2i} \end{pmatrix}' V_i^{-1} \begin{pmatrix} \beta_1 - b_1 \\ \beta_{2i} - b_{2i} \end{pmatrix}$$

$$V_i^{-1} = \begin{pmatrix} X_1'X_1 & X_1'X_2 \\ X_{2i}'X_1 & X_{2i}'X_{2i} + V_{0i}^{-1} \end{pmatrix}$$

$$\tau_i = y'y - y'(X_1\, X_{2i})V_i(X_1\, X_{2i})'y$$

Hence, the posterior density of the parameters is the familiar normal-inverse-gamma distribution.

Furthermore, if we let

$$M^* = I_n - X_1(X_1'X_1)^{-1}X_1'$$

And

$$V_{2i}^{-1} = V_{0i}^{-1} + X_{2i}'M^*X_{2i}$$

A little algebra gives

$$V_i = \begin{pmatrix} (X_1'X_1)^{-1} + (X_1'X_1)^{-1}X_1'X_{2i} & -(X_1'X_1)^{-1}X_1'X_{2i}V_{2i} \\ -V_{2i}X_{2i}'X_1(X_1'X_1)^{-1} & V_{2i} \end{pmatrix}$$

And

$$\tau_i = y'(M^* - M^*X_{2i}V_{2i}X_{2i}'M^*)y$$

Appendix B

From the marginal density of y in model M_i as

$$p(y|M_i) = \iiint p(y|\beta_1, \beta_{2i}, \sigma^2, M_i)p(\beta_1, \beta_{2i}, \sigma^2|M_i)d\beta_1 d\beta_{2i} d\sigma^2$$

$$= \iiint \left(\frac{1}{\sqrt{2\pi\sigma^2}}\right)^{\frac{n}{2}} exp\left(-\frac{(y - X_1\beta_1 - X_{2i}\beta_{2i})'(y - X_1\beta_1 - X_{2i}\beta_{2i})}{2\sigma^2}\right)$$

$$* (\sigma^2)^{-(n+k_{2i}+2)/2} exp\,(-\frac{\varphi_i + \tau_i}{2\sigma^2})d\beta_1 d\beta_{2i} d\sigma^2$$

$$= \frac{\pi^{-\frac{n}{2}}\Gamma(\frac{n-k_1}{2})}{|X_1'X_1|\Gamma(-\frac{k_1}{2})} \frac{|V_{0i}^{-1}|^{1/2}}{|V_{2i}^{-1}|^{1/2}} \tau_i^{-(n-k_1)/2}$$

$$= c\frac{|V_{0i}^{-1}|^{1/2}}{|V_{2i}^{-1}|^{1/2}} \tau_i^{-(n-k_1)/2}$$

$$c = \frac{\pi^{-\frac{n}{2}}\Gamma(\frac{n-k_1}{2})}{|X_1'X_1|\Gamma(-\frac{k_1}{2})}$$

Then, with the expression of τ_i and V_{2i} we have

$$p(y|M_i) =$$

$$c\frac{|V_{0i}^{-1}|^{\frac{1}{2}}}{|V_{0i}^{-1} + X_{2i}'M^*X_{2i}|^{\frac{1}{2}}}[y'M^*(M^* - M^*X_{2i}(V_{0i}^{-1} + X_{2i}'M^*X_{2i})^{-1}X_{2i}'M^*)M^*y]^{-(n-k_1)/2}$$

To simplify, let

$$S^* = (M^* - M^*X_{2i}(V_{0i}^{-1} + X_{2i}'M^*X_{2i})^{-1}X_{2i}'M^*)$$

The marginal density can be written as:

$$p(y|M_i) = c\frac{|V_{0i}^{-1}|^{\frac{1}{2}}}{|V_{0i}^{-1} + X_{2i}'M^*X_{2i}|^{\frac{1}{2}}}(y'M^*S^*M^*y)^{-(n-k_1)/2}$$

Appendix C

$$p_{pre}(\tilde{y}|y^*, M_i) = \iiint p_{pre}(\tilde{y}|\beta_1, \beta_{2i}, \sigma^2, y^*, M_i) p(\beta_1, \beta_{2i}, \sigma^2|M_i) d\beta_1 d\beta_{2i} d\sigma^2$$

$$\beta_{2i}|\beta_1, \sigma^2, M_i \sim N(0, \sigma^2(g_r X_r' X_r)^{-1})$$

$$p(\sigma^2|M_i) \propto \frac{1}{\sigma^2}$$

This is still a normal-inverse-gamma distribution, we get:

$$p_{pre}(\tilde{y}|y^*, M_i) = \frac{\Gamma\left(\frac{m+l-k_1}{2}\right)\left(y^{*\prime} M^* S_{pre}^* M^* y^*\right)^{m/2}}{\pi^{1/2}\Gamma\left(\frac{m-k_1}{2}\right)\left|I_l + \widetilde{M}^*\tilde{X}_{2i}(V_{0i}^{-1} + X_{2i}^{*}{}' M^* X_{2i}^*)^{-1}\tilde{X}_{2i}'\widetilde{M}^*\right|^{1/2}|X_1'X_1|^{1/2}}\Bigg[y^{*\prime} M^* S_{pre}^* M^* y^* + \left(\tilde{y} - \tilde{X}_{2i} b_{2i}^*\right)' \widetilde{M}^* \left(I_l + \widetilde{M}^* \tilde{X}_{2i} \frac{1}{1+g_i}\left(X_{2i}^{*}{}' M^* X_{2i}^*\right)^{-1} \tilde{X}_{2i}' \widetilde{M}^*\right)^{-1} \widetilde{M}^* (\tilde{y} - \tilde{X}_{2i} b_{2i}^*)\Bigg]^{-(n-k_1)/2}$$

Where

$$S_{pre}^* = \left(M^* - \frac{1}{1+g_i} M^* X_{2i}^* \left(X_{2i}^{*}{}' M^* X_{2i}^*\right)^{-1} X_{2i}^{*}{}' M^*\right)$$

$$b_{2i}^* = \frac{1}{1+g_i}\left(X_{2i}^{*}{}' M^* X_{2i}^*\right)^{-1} X_{2i}^{*}{}' M^* y^*$$

Thus,

$$p_{pre}(\tilde{y}|y^*, M_i) \propto \left(\frac{y^{*\prime} M^* S_{pre}^* M^* y^*}{m}\right)^{-l/2} \frac{\left|(1+g_i) X_{2i}^{*}{}' M^* X_{2i}^*\right|^{1/2}}{\left|(1+g_i) X_{2i}^{*}{}' M^* X_{2i}^* + \tilde{X}_{2i}' \widetilde{M}^* \tilde{X}_{2i}\right|^{1/2}} \Bigg[m + \frac{m}{y^{*\prime} M^* S_{pre}^* M^* y^*}\left(\tilde{y} - \tilde{X}_{2i} b_{2i}^*\right)' \widetilde{M}^* \left(\widetilde{M}^* + \widetilde{M}^* \tilde{X}_{2i} \frac{1}{1+g_i}\left(X_{2i}^{*}{}' M^* X_{2i}^*\right)^{-1} \tilde{X}_{2i}' \widetilde{M}^*\right)^{-1} \widetilde{M}^* \left(\tilde{y} - \tilde{X}_{2i} b_{2i}^*\right)\Bigg]^{-(n-k_1)/2}$$

With the prior probability $P(M_i) = \rho^{k_{2i}}(1-\rho)^{k_2 - k_{2i}}$, posterior probability for each model is calculated as follows

$$p_{pre}(M_i|y^*,\tilde{y}) = \frac{p(\tilde{y}|y^*,M_i)P(M_i)}{\sum_{j=1}^{2^{k_2}} p(\tilde{y}|y^*,M_j)P(M_j)}$$

$$\propto \left(\frac{y^{*\prime}M^*S_{pre}^*M^*y^*}{m}\right)^{-l/2} \frac{\left|(1+g_i)X_{2i}^{*}{}'M^*X_{2i}^*\right|^{1/2}}{\left|(1+g_i)X_{2i}^{*}{}'M^*X_{2i}^* + \tilde{X}_{2i}'\widetilde{M}^*\tilde{X}_{2i}\right|^{1/2}} \Bigg[m$$

$$+ \frac{m}{y^{*\prime}M^*S_{pre}^*M^*y^*}\left(\tilde{y} - \tilde{X}_{2i}b_{2i}^*\right)'\widetilde{M}^*\Bigg(\widetilde{M}^*$$

$$+ \widetilde{M}^*\tilde{X}_{2i}\frac{1}{1+g_i}\left(X_{2i}^{*}{}'M^*X_{2i}^*\right)^{-1}\tilde{X}_{2i}'\widetilde{M}^*\Bigg)^{-1}\widetilde{M}^*\big(\tilde{y}$$

$$- \tilde{X}_{2i}b_{2i}^*\big)\Bigg]^{-(n-k_1)/2} \rho^{k_{2i}}(1-\rho)^{k_2-k_{2i}}$$

Specially, for simplicity, we set X_1 to be null set, which means we do not fix any variable and all the variables are freely chosen according to the posterior probability of the model. As a result, $X_2 = X$.

$$p_{pre}(\tilde{y}|y^*,M_i)$$

$$\propto \left(\frac{y^{*\prime}S_{pre}^*y^*}{m}\right)^{-l/2} \frac{\left|(1+g_i)X_i^{*\prime}X_i^*\right|^{1/2}}{\left|(1+g_i)X_i^{*\prime}X_i^* + \tilde{X}_i'\tilde{X}_i\right|^{1/2}} \Bigg[m$$

$$+ \frac{m}{y^{*\prime}S_{pre}^*y^*}\left(\tilde{y} - \tilde{X}_ib_i^*\right)'\left(I + \tilde{X}_i\frac{1}{1+g_i}\left(X_i^{*\prime}X_i^*\right)^{-1}\tilde{X}_i'\right)^{-1}\big(\tilde{y}$$

$$- \tilde{X}_ib_i^*\big)\Bigg]^{-n/2}$$

And similarly,

$$p_{pre}(M_i|y^*,\tilde{y}) = \frac{p(\tilde{y}|y^*,M_i)P(M_i)}{\sum_{j=1}^{2^k} p(\tilde{y}|y^*,M_j)P(M_j)}$$

$$\propto \left(\frac{y^{*\prime}S_{pre}^{*}y^{*}}{m}\right)^{-l/2} \frac{\left|(1+g_i)X_i^{*\prime}X_i^{*}\right|^{1/2}}{\left|(1+g_i)X_i^{*\prime}X_i^{*}+\tilde{X}_i^{\prime}\tilde{X}_i\right|^{1/2}} \Bigg[m$$

$$+ \frac{m}{y^{*\prime}S_{pre}^{*}y^{*}}\left(\tilde{y}-\tilde{X}_i b_i^{*}\right)^{\prime}\left(I+\tilde{X}_i\frac{1}{1+g_i}\left(X_i^{*\prime}X_i^{*}\right)^{-1}\tilde{X}_i^{\prime}\right)^{-1}(\tilde{y}$$

$$- \tilde{X}_i b_i^{*})\Bigg]^{-n/2} \rho^{k_i}(1-\rho)^{k-k_i}$$

where

$$S_{pre}^{*} = \left(I - \frac{1}{1+g_i}X_i^{*}\left(X_i^{*\prime}X_i^{*}\right)^{-1}X_i^{*\prime}\right)$$

$$b_i^{*} = \frac{1}{1+g_i}\left(X_i^{*\prime}X_i^{*}\right)^{-1}X_i^{*\prime}y^{*}$$

Appendix D:

Table 3.1.2 Descriptive Statistics for UK

	Minimum	Maximum	Mean	Std. Deviation	Skewness	Kurtosis
exchange rate return	-0.2107	0.1439	-0.0033	0.0536	-0.444	1.490
growth rate of money supply	-5.2808	22.8622	5.9052	5.9009	0.422	-0.252
short term interest rate	-3.2367	9.4100	2.3657	2.1252	0.450	0.141
long term interest rate	-1.9267	8.8667	1.6609	2.1582	1.251	1.172
change in CPI	-1.8776	18.0665	1.9532	3.5340	2.521	7.398
GDP	-3.7141	-2.9801	-3.2584	0.1501	-0.349	-0.024
growth in GDP	-0.0373	0.0633	0.0041	0.0139	0.756	2.864
international reserve	-1.4657	0.0823	-0.6733	0.3062	0.574	0.342
change in international reserve	-0.2851	0.8111	0.0000	0.1174	2.586	16.071
change in industrial production	-0.1093	0.1289	-0.0039	0.0558	0.677	-0.573
change in reserve minus gold	-0.3803	0.9179	-0.0095	0.1440	1.976	12.285
labor productivity	-0.0216	0.0572	0.0082	0.0119	1.322	2.774
growth rate of stock return	-0.2156	0.3749	0.0020	0.0739	1.186	5.388

Notes: all these data, except for exchange rate return, are relative to those of the US. The variables, growth rate of money supply, short and long term interest rates and change in CPI, are in percentage.

Table 3.1.3 Descriptive Statistics for Japan

	Minimum	Maximum	Mean	Std. Deviation	Skewness	Kurtosis
exchange rate return	-0.1501	0.1698	0.0071	0.0598	0.443	-0.024
growth rate of money supply	-13.5881	12.1326	-0.5784	5.4606	-0.141	-0.528
short term interest rate	-9.4097	1.0810	-3.0103	2.2387	-0.232	-0.249
long term interest rate	-6.5300	1.9933	-2.7565	1.8605	0.744	0.253
change in CPI	-8.1617	13.3111	-1.6718	3.4034	2.068	6.253
GDP	-1.2221	-0.2318	-0.8660	0.2595	0.492	-0.903
growth in GDP	-0.1529	0.1694	0.0013	0.0620	0.394	-0.075
international reserve	-0.4226	2.5677	0.7384	0.9660	0.712	-0.956
change in international reserve	-0.4554	0.2979	0.0122	0.0991	-0.859	3.944
change in industrial production	-0.2252	0.0851	-0.0010	0.0367	-1.34	8.389
change in reserve minus gold	-0.5278	0.3209	0.0020	0.1261	-1.118	4.055
labor productivity	-0.0247	0.1001	-0.0014	0.0150	3.055	15.836
growth rate of stock return	-0.2621	0.2048	-0.0097	0.0891	-0.119	0.132

Notes: all these data, except for exchange rate return, are relative to those of the US. The variables, growth rate of money supply, short and long term interest rates and change in CPI, are in percentage.

Table 3.1.4 Descriptive Statistics for Germany/Europe

	Minimum	Maximum	Mean	Std. Deviation	Skewness	Kurtosis
exchange rate return	-0.1391	0.1574	0.0045	0.0598	-0.01	-0.181
growth rate of money supply	-8.1801	14.7237	-0.1041	4.7020	1.205	1.669
short term interest rate	-6.3000	5.5700	-0.9428	2.1380	0.391	0.63
long term interest rate	-5.0133	3.0390	-0.9099	1.7085	-0.299	0.01
growth in GDP	-0.0457	0.1481	-0.0047	0.0173	4.557	40.909
international reserve	-0.7148	1.0633	0.1515	0.4315	0.212	-0.983
change in international reserve	-0.2869	0.5746	-0.0103	0.0882	1.705	13.473
change in IND production	-0.1378	0.1386	-0.0020	0.0591	0.157	-0.24
change in reserve minus gold	-0.4085	0.6076	-0.0203	0.1167	0.579	7.642
labor productivity	-0.0353	0.0544	0.0014	0.0132	0.348	1.636
growth rate of stock return	-0.2817	0.2492	0.0011	0.0869	-0.412	0.825

Notes: all these data, except for exchange rate return, are relative to those of the US. The variables, growth rate of money supply, short and long term interest rates and change in CPI, are in percentage.

Appendix E:

Table 4.3.3 Expanding window ratio of right sign forecast for BMAP

		RSF														
		$\rho = 0.5$					$\rho = 0.3$					$\rho = 0.1$				
		$\varphi = 0.3$	$\varphi = 0.4$	$\varphi = 0.5$	$\varphi = 0.6$	$\varphi = 0.7$	$\varphi = 0.3$	$\varphi = 0.4$	$\varphi = 0.5$	$\varphi = 0.6$	$\varphi = 0.7$	$\varphi = 0.3$	$\varphi = 0.4$	$\varphi = 0.5$	$\varphi = 0.6$	$\varphi = 0.7$
Benchmark: Random walk model																
Pound	1 quarter	0.5588	0.5294	0.5000	0.5294	0.5147	0.5735	0.5735	0.5588	0.5882	0.5294	0.5147	0.5147	0.5882	0.5735	0.5294
	2 quarter	0.4925	0.5522	0.5373	0.5224	0.4925	0.5373	0.5075	0.5224	0.5373	0.5075	0.4925	0.4627	0.5075	0.5522	0.4627
	4 quarter	0.4154	0.5077	0.5538	0.5231	0.5231	0.4462	0.4923	0.5385	0.5231	0.5231	0.4308	0.5385	0.5231	0.1923	0.5231
	8 quarter	0.4754	0.4918	0.4590	0.4590	0.4590	0.5082	0.4426	0.4590	0.4426	0.4426	0.5085	0.4562	0.4426	0.4426	0.4426
Yen	1 quarter	0.5294	0.5588	0.5588	0.5294	0.5294	0.5147	0.5735	0.5441	0.5294	0.5000	0.5574	0.5574	0.5735	0.5735	0.4853
	2 quarter	0.5970	0.6119	0.6119	0.5970	0.5970	0.5672	0.5970	0.5970	0.6119	0.5970	0.5672	0.5522	0.6418	0.6269	0.5821
	4 quarter	0.7231	0.7231	0.7077	0.7231	0.7692	0.7385	0.7231	0.7077	0.7538	0.7692	0.6769	0.6923	0.6923	0.7231	0.7077
	8 quarter	0.7213	0.7213	0.7049	0.7213	0.7049	0.7541	0.7213	0.7049	0.7213	0.7049	0.7213	0.7213	0.6721	0.7377	0.7049
Mark/Euro	1 quarter	0.4853	0.5147	0.5147	0.5147	0.5294	0.4853	0.5294	0.5148	0.5000	0.5294	0.4706	0.5147	0.4853	0.4853	0.4706

2 quarter	0.5821	0.5821	0.5821	0.5522	0.5970	0.5373	0.5672	0.5821	0.5373	0.5682	0.5522	0.5224	0.4925	0.5522	0.4925
4 quarter	0.5385	0.5385	0.5077	0.5077	0.5538	0.5385	0.5077	0.5231	0.5077	0.5538	0.4923	0.4923	0.5385	0.5231	0.5538
8 quarter	0.6393	0.6230	0.5574	0.5246	0.5970	0.6230	0.6230	0.5246	0.5246	0.5821	0.6230	0.6066	0.5574	0.5574	0.5522

Table 4.3.4 Expanding window ratio of right sign forecast for BMAP

		RSF														
		ρ = 0.5					ρ = 0.3					ρ = 0.1				
		φ = 0.3	φ = 0.4	φ = 0.5	φ = 0.6	φ = 0.7	φ = 0.3	φ = 0.4	φ = 0.5	φ = 0.6	φ = 0.7	φ = 0.3	φ = 0.4	φ = 0.5	φ = 0.6	φ = 0.7
Benchmark: Historical average return																
Pound	1 quarter	1.0857	1.0286	0.9714	1.0286	1.0000	1.1143	1.1143	1.0857	1.1429	1.0286	1.0000	1.0000	1.1429	1.1143	1.0286
	2 quarter	1.0645	0.1935	1.1613	1.1290	1.0645	1.1613	1.0968	1.1290	1.1613	1.0968	1.0645	1.0000	1.0968	1.1935	1.0000
	4 quarter	0.9310	1.1379	1.2414	1.1724	1.1724	1.0000	1.1034	1.2069	1.1724	1.1724	0.9655	1.2069	1.1724	1.1034	1.1724
	8 quarter	1.2609	1.3043	1.2174	1.2174	1.2174	1.3478	1.1739	1.2174	1.1739	1.1739	1.3478	1.1304	1.1739	1.1739	1.1739
Yen	1 quarter	1.1250	1.1875	1.1875	1.1250	1.1250	1.0938	1.2188	1.1563	1.1250	1.0625	1.0303	1.0303	1.0303	1.2188	1.0313
	2 quarter	1.2121	1.2424	1.2424	1.2121	1.2121	1.1515	1.2121	1.2121	1.2424	1.2121	1.1515	1.1212	1.3030	1.2727	1.1818
	4 quarter	1.4242	1.4242	1.3939	1.4242	1.5152	1.4545	1.4242	1.3939	1.4848	1.5152	1.3333	1.3636	1.3636	1.4242	1.3939
	8 quarter	1.3750	1.3750	1.3438	1.3750	1.3438	1.4375	1.3750	1.3438	1.3750	1.3438	1.3750	1.3750	1.2813	1.4063	1.3438
Mark/Euro	1 quarter	1.1000	1.1667	1.1667	1.1667	1.2000	1.1000	1.2000	1.1667	1.1333	1.2000	1.0667	1.1667	1.1000	1.1000	1.0667
	2 quarter	1.1471	0.1471	1.1471	1.0882	1.1765	1.0588	1.1176	1.1471	1.0588	1.1176	1.0882	1.0294	0.9706	1.0882	0.9706

4 quarter	0.9722	0.9722	0.9167	0.9167	1.0000	0.9722	0.9167	0.9444	0.9167	1.0000	0.8889	0.8889	0.9722	0.9444	1.0000
8 quarter	1.1515	1.1515	1.0303	0.9697	1.1765	1.1515	1.1515	0.9697	0.9697	1.1471	1.1515	1.1212	1.0303	1.0303	1.0882

Table 4.3.7 Rolling window ratio of right sign forecast for BMAP

		RSF														
		$\rho = 0.5$					$\rho = 0.3$					$\rho = 0.1$				
		$\varphi = 0.3$	$\varphi = 0.4$	$\varphi = 0.5$	$\varphi = 0.6$	$\varphi = 0.7$	$\varphi = 0.3$	$\varphi = 0.4$	$\varphi = 0.5$	$\varphi = 0.6$	$\varphi = 0.7$	$\varphi = 0.3$	$\varphi = 0.4$	$\varphi = 0.5$	$\varphi = 0.6$	$\varphi = 0.7$
Benchmark: Random walk model																
Pound	1 quarter	0.5735	0.5882	0.6029	0.5882	0.5735	0.6471	0.6618	0.6324	0.6176	0.5882	0.6324	0.6471	0.6029	0.6029	0.6324
	2 quarter	0.5672	0.6866	0.6716	0.5672	0.5522	0.5821	0.5970	0.5821	0.6119	0.5821	0.6269	0.6269	0.6716	0.6269	0.6567
	4 quarter	0.5692	0.6000	0.5846	0.4769	0.5385	0.5846	0.5846	0.5692	0.5231	0.5385	0.5231	0.5538	0.5385	0.5077	0.5385
	8 quarter	0.5738	0.5082	0.4590	0.4426	0.4426	0.5410	0.4918	0.4590	0.4262	0.4590	0.5246	0.5246	0.4754	0.4262	0.4754
Yen	1 quarter	0.5441	0.5441	0.5882	0.5882	0.5735	0.5294	0.5588	0.5735	0.5882	0.5588	0.5147	0.5441	0.5000	0.5588	0.5147
	2 quarter	0.6418	0.6866	0.6716	0.7164	0.7313	0.6567	0.6567	0.6866	0.7463	0.7313	0.6119	0.6269	0.6716	0.7313	0.7164
	4 quarter	0.8154	0.8308	0.8308	0.8308	0.8308	0.8308	0.8308	0.8308	0.8308	0.8308	0.7846	0.8154	0.8462	0.8462	0.8308
	8 quarter	0.7705	0.7541	0.7377	0.7377	0.7213	0.7705	0.7705	0.7377	0.7541	0.7213	0.7869	0.7869	0.7541	0.7541	0.7377
Mark/Euro	1 quarter	0.5147	0.5147	0.5147	0.4853	0.4559	0.5147	0.5588	0.5000	0.5147	0.4853	0.5588	0.5735	0.5147	0.5294	0.4853
	2 quarter	0.5821	0.5970	0.5970	0.5821	0.5970	0.5373	0.5970	0.5522	0.5821	0.5821	0.5522	0.5970	0.5373	0.5373	0.5522

4 quarter	0.6154	0.5846	0.5846	0.5538	0.4769	0.6154	0.5846	0.5538	0.5385	0.4923	0.6154	0.6154	0.5538	0.5538	0.5077
8 quarter	0.7049	0.7049	0.6885	0.5902	0.6393	0.7213	0.6721	0.6557	0.5902	0.6066	0.6230	0.6885	0.6066	0.5738	0.5902

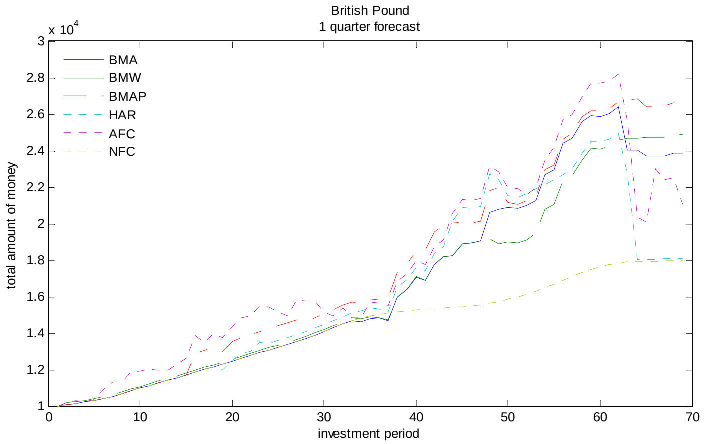
Table 4.3.7 Rolling window ratio of right sign forecast for BMAP

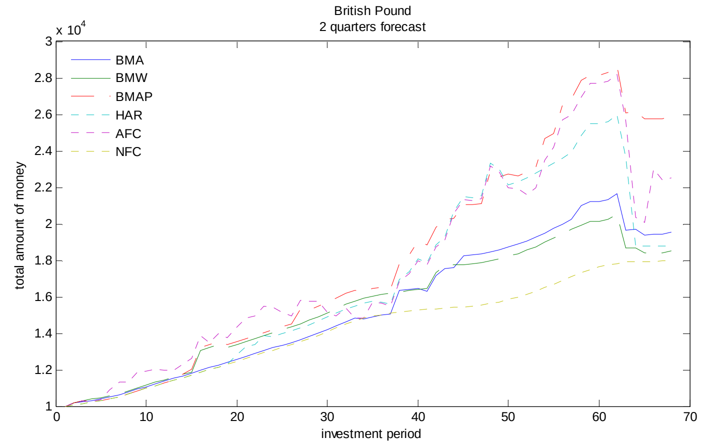
		RSF														
		ρ = 0.5					ρ = 0.3					ρ = 0.1				
		φ = 0.3	φ = 0.4	φ = 0.5	φ = 0.6	φ = 0.7	φ = 0.3	φ = 0.4	φ = 0.5	φ = 0.6	φ = 0.7	φ = 0.3	φ = 0.4	φ = 0.5	φ = 0.6	φ = 0.7
Benchmark: Historical average return																
Pound	1 quarter	1.1143	1.1429	1.1714	1.1429	1.1143	1.2571	1.2857	1.2286	1.2000	1.1429	1.2286	1.2571	1.1714	1.1714	1.2286
	2 quarter	1.2258	1.3939	1.3636	1.2258	1.1935	1.2581	1.2903	1.2581	1.3226	1.2581	1.3548	1.3548	1.4516	1.3548	1.4194
	4 quarter	1.2759	1.3448	1.3103	1.0690	1.2069	1.3103	1.3103	1.2759	1.1724	1.2069	1.1724	1.2414	1.2069	1.1379	1.2069
	8 quarter	1.5217	1.3478	1.2174	1.1739	1.1739	1.4348	1.3043	1.2174	1.1304	1.2174	1.3913	1.3913	1.2609	1.1304	1.2609
Yen	1 quarter	1.1563	1.1563	1.2500	1.2500	1.2188	1.1250	1.1875	1.2188	1.2500	1.1875	1.0938	1.1563	1.0625	1.1875	1.0938
	2 quarter	1.3030	1.3939	1.3636	1.4545	1.4848	1.3333	1.3333	1.3939	1.5152	1.4848	1.2424	1.2727	1.3636	1.4848	1.4545
	4 quarter	1.6061	1.6364	1.6364	1.6364	1.6364	1.6364	1.6364	1.6364	1.6364	1.6364	1.5455	1.6061	1.6667	1.6667	1.6364
	8 quarter	1.4688	1.4375	1.4063	1.4063	1.3750	1.4688	1.4688	1.4063	1.4375	1.3750	1.5000	1.5000	1.4375	1.4375	1.4063
Mark/Euro	1 quarter	1.1667	1.1667	1.1667	1.1000	1.0333	1.1667	1.2667	1.1333	1.1667	1.1000	1.2667	1.3000	1.1667	1.2000	1.1000
	2 quarter	1.1471	1.1765	1.1765	1.1471	1.1765	1.0588	1.1765	1.0882	1.1471	1.1471	1.0882	1.1765	1.0588	1.0588	1.0882

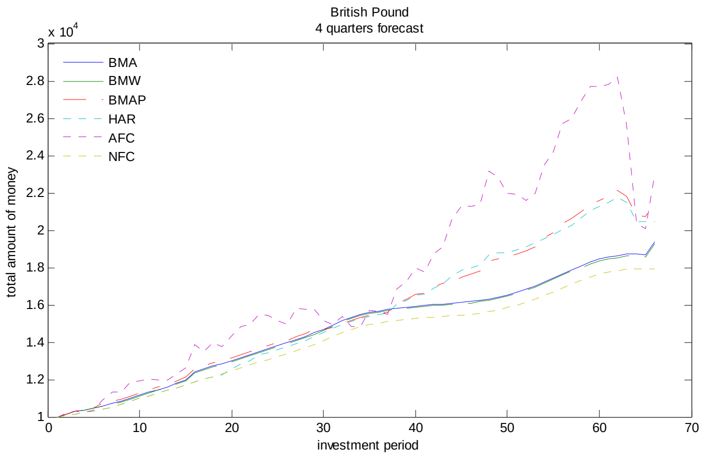
4 quarter	1.1111	1.0556	1.0556	1.0000	0.8611	1.1111	1.0556	1.0000	0.9722	0.8889	1.1111	1.1111	1.0000	1.0000	0.9167
8 quarter	1.3030	1.3030	1.2727	1.0909	1.1818	1.3333	1.2424	1.2121	1.0909	1.1212	1.1515	1.2727	1.1212	1.0606	1.0909

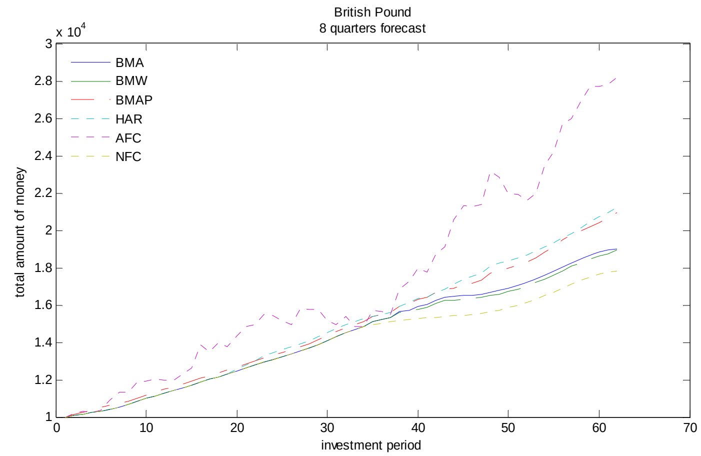
Appendix F: comparison of investment performance at various horizons

Invest performance for British Pound

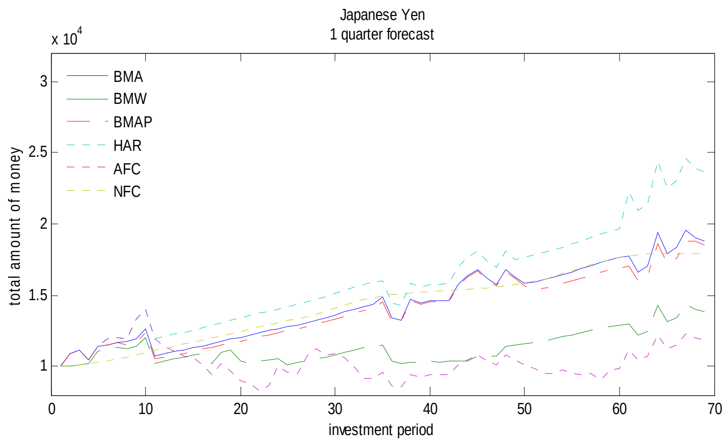


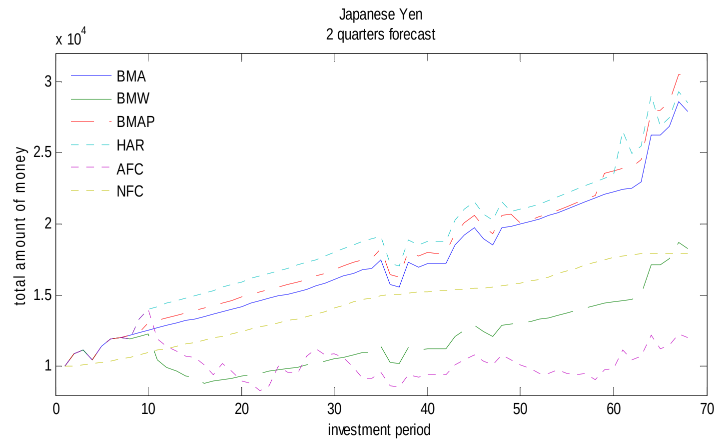


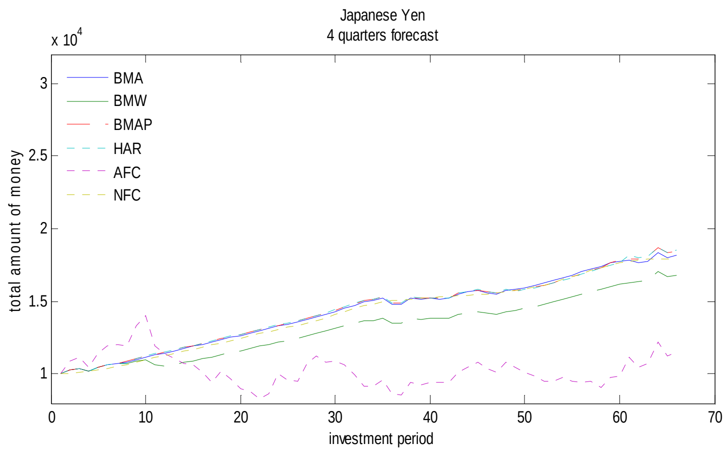


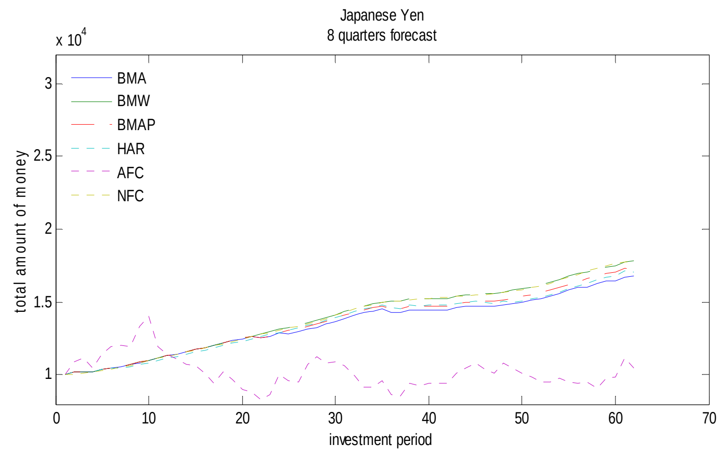


Invest performance for Yen









Invest performance for Mark/Euro