Value-based Generational Accounting on National Old-age Pension in the Netherlands

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Abstract

In response to the large-scale demographic ageing, alternative policy reforms with respect to the national old-age pension have effects on generational accounts in the Netherlands. Traditional methodology calculates implicit tax and implicit tax rate in a deterministic manner. In this paper, we introduce the so-called stochastic discount factor and thereafter perform valuation in a stochastic setting. The basic idea is that future cash flows in different scenarios should be discounted to the present value differently. Two policy reforms are investigated, increasing retirement age and adjusting pension benefits and contributions, including pension benefits cut, higher payroll contributions and the mixed strategy.

Key words: generational accounting, intergenerational effects, implicit tax, implicit tax rate, stochastic discount factor, national old-age pension

1 Introduction

Throughout most of the OECD countries, large-scale demographic ageing has an impact on PAYG (pay-as-you-go) national old-age pension schemes. Ageing arises from two common trends. One is a typical swing in lower fertility rate, a trend to having fewer children. The other one is a long-term increase in life expectancy. Net immigration can mitigate this shift in the population structure, but the effect is rather small.

A widely used indicator for ageing is called elderly dependency ratio, which is defined as the ratio of the number of people aged 65 and older to that of working population. In
the Netherlands, it was just 21% in 2004 but will increase dramatically in the coming decades and peak just before 2040 to 43%.

The demographic ageing creates a problem to the fiscal sustainability in the Netherlands. The National Old-age pension (AOW) appears to be one of the most stable schemes in the Dutch social security system. Since it is financed on a PAYG base, it is vulnerable to a significant increase in the elderly dependency ratio, which implies a burden to the future generations and government budget. However, we should notice that the problem became international as all industrialized countries encounter the ageing of the population and adopt the PAYG mode. In fact, fiscal sustainability is less problematic in the Netherlands as it experiences a relatively modest demographic ageing and many pension funds exist, compared to other OECD countries.

A couple of appropriate policy reforms may restore the sustainability of the national old-age pension, such as increasing retirement age, pension benefits cut and higher payroll contributions, which will be examined in this paper. Government policies affect the welfare of current and future generations differently. The well-known generational accounting proposed by Auerbach et al. (1991) allows one to investigate welfare change of each cohort due to policy reforms.

Nevertheless, traditional generational accounting includes all the government budget items and considers the interaction of all government policies, but our main purpose is to analyze the intergenerational effects of different policy reforms with respect to the national old-age pension in the Netherlands. Thus we employ the concept of implicit tax as it usually applies to analyze micro effects in public pension scheme. Implicit tax is the difference in present value terms between life-time contributions and pension benefits received later on.

Several other studies have focused on generational accounting for public finance or implicit tax for public pension scheme, but none of them incorporated uncertainty into the problem; they dealt with it in a deterministic way. This paper stands out in the sense it introduces a stochastic framework to the valuation of pension benefits and contributions. In line with the principle that future cash flows should be evaluated and discounted differently in good and bad scenarios, we will perform the valuation of implicit tax in a stochastic setting, by using the so-called stochastic discount factor (SDF). Another advantage that we should mention is that introduction of stochastic discount factor into our model excludes arbitrage opportunity.

In order to simulate SDF, we specify state variables in a commonly used first-order vector autoregressive model (VAR(1)). SDF can be written as a function of state variables. By estimation of parameters and projection of state variables, we are able to
simulate SDF for the coming years. Discounting the pension benefits and contributions by stochastic discount factor gives the fair option value. To eliminate the horizon effect, we calculate the implicit tax rate, defined as the ratio of implicit tax to the life-time labor income. Hence the comparison of current generations and future generations becomes more reasonable.

We will also look at several discussion items after main simulation results. Behavioral effects may play an important role on the intergenerational effects, but is excluded from the paper for sake of simplicity. More state variables such as additional asset classes and macro risk factors may improve the calculation of SDF. An alternative assumption on productivity and wage growth better captures the spirit of the stochastic framework. We discuss other possibilities to eliminate the horizon effects, though implicit tax rate appears to be quite a successful indicator. The last but not the least, Sensitivity to parameter and model specification should be kept in mind.

This paper is structured as the following. Section 2 performs a comprehensive literature review. Not only literatures that we directly referred to are discussed, but also those came up with alternative methodologies are mentioned. Section 3 gives an overview of the Dutch AOW and summarizes policy reforms from current debate, earlier research and past evidences. Section 4 constructs the model, introducing concepts to measure intergenerational effects of different policy reforms and illustrating the way to calculate implicit tax in a stochastic framework. Section 5 describes the whole simulation procedure and interprets the results. Discussions and conclusions are provided in section 6 and 7.
2 Literature review

Netherlands as well as most OECD countries are experiencing rapid ageing of the population. As a consequence, higher expenditure of social security put a burden on the fiscal system. On the other side, a smaller contribution base makes this problem even worse. A lot of debates are centered on the sustainability of Dutch public finance, such as increasing the retirement age, simulating labor participation and redesigning taxation policies. Apparently the AOW (Dutch old-age pension) will be affected by these policy alternatives. It is interesting to investigate these effects in a generational accounting perspective, which has been developed by Auerbach, et al. (1991), in order to examine the intertemporal effects of government policies.

Harry ter Rele (Generational accounts for the Dutch public sector, 1997) made a number of extensions to the standard practice of generational accounting methodology to make calculations concerning how fiscal policy and future development interact in affecting the welfare of generations and the sustainability of the public finance. He made the assumption that net benefit of future generations is calculated residually from the intertemporal budget constraint of the government, so policies that benefit current generation impose a burden on future generations. To put it more precisely, the sum of discounted value of the net benefits of all generations is equal to the ‘inherited capital’.

According to the ter Rele’s simple model, the net benefits of a representative agent grows in line with the productivity growth, thus the future ratio between the net benefits remains constant, and the ratio of total net benefits in a period only changes in response to changes in demographic composition. Net benefits are calculated out of government revenue and expenditure.

Ter Rele investigated distribution of net benefits over age in a baseline case. Benefits generally rise with age mainly due to AOW and health care expenditure, while taxes vary with age since labor incomes first rise with age until 50 and thereafter gradually decrease due to a lower participation in labor force. Taxes paid are extrapolated into the future in line with productivity growth, labor participation rate and age-earnings profile. Benefits are extrapolated by taking into account the participation growth and size of public capital stock. Combining the two components, age profile of net benefits can be shown.

Furthermore, Ter Rele accessed intergenerational impact of present fiscal policies. First, an increase in labor-force participation; second, a rise in pension contributions; third, the maturing of private, funded pension; and fourth, a flattening of the age-earnings profile.
In the baseline case, accounts are in surplus for the young and the age group over 50 years-old, and in deficit for middle group. The increase of participation and the rise of pension incomes both raising the level of taxes, turn out to have a remarkable negative effect on the net benefits of existing generations. The increase of pension contribution rates and the flattening of the age profile of wages only have a small positive effect.

This paper gives interesting results about different policy reforms and their intergenerational effects. However, the net benefits consisting all the social security payment are derived out of public budget, while our study focus on the national old-age pension. Thus the generational accounting methodology applied here serves as a starting point to build our model. We calculated net benefits of national old-age pension differently, as suggested below.

Robert Fenge and Martin Werding (Ageing and fiscal imbalances across generations: concepts of measurement, 2003) pointed out that four major classes of concepts have been developed to measure the impact of ageing on unfunded pension schemes, namely, net pension liabilities, general government fiscal balances, generational accounting, and implicit tax. One major distinction is between concepts that measure on aggregate level and those on individual level; the other distinction is between concepts that concentrate on pension system alone and those expand the view to general government revenue and expenditure. We focus on two measures: net pension liabilities and implicit tax.

Net pension liabilities of unfunded pension schemes is defined as the net present value of outstanding claims of pensioners and future pension benefits of the active population. The function which links pension benefits to contributions can be a very odd function. In our case, pension benefits are proportional to the minimum wage. As we have already known, total contributions must equal to total benefits during the same period for a unfunded pension scheme. Thus by relating the pension benefits to the life-time contributions, one may calculate the debt implied in unfunded pension schemes, both in absolute terms and relative terms. It can be shown that the size of net pension liabilities is mainly determined by the population size, outstanding pension benefits and interest rate. For given value of population size and interest rate, manipulating contribution rate and pension benefits function are ways of solving ageing problems.

In consistence with the concept of net pension liabilities, implicit tax is a useful tool to understand the intergenerational distribution in a PAYG unfunded pension scheme. The concept of implicit tax comes from the fact that usually pension benefits received fall short of actuarial returns to contributions paid while working. By definition, the implicit tax involved in unfunded pension scheme is given by the difference of net present value
of life-time contributions and pension benefits accruing later on. For unfunded pension schemes, benefits have to be paid out of contributions, thus can be written as a function of contributions. Implicit tax is therefore determined by contribution rates, wage growth, population growth and interest rates. The last three components appear in the famous 'Non-Aaron condition', which states that if the product of population growth and wage growth falls short of the gross return in capital market, PAYG pensions involve a burden for their participants.

Adjustments in contribution rates and benefit levels or prefunding that aims to reduce net pension liabilities affect the profile of implicit tax rates across different generations. For instance, cutting down the pension benefits relatively hurts the elderly, while raising the contribution rates hits young participants the most. If public unfunded pension schemes do not adjust their policies, implicit tax rate will increase sharply for the current young generations. Simulations for the size of implicit tax over a long sequence of birth cohorts, both in absolute terms and relative terms, illustrate the effects of different policy measures under the ageing pressure. While employing the concept of implicit tax, we have to notice that in reality total contributions usually fall short of total benefits in the same period for the unfunded pension scheme.

Furthermore, Robert Fenge and Martin Werding (Ageing and the tax implied in public pension schemes: simulations for selected OECD countries, 2003) explained the concept implicit tax more in detail and implemented it to analyze intergenerational imbalance for seven OECD countries.

Fenge and Werding ran the simulation by using CESifo Pension Model which was based on a simple accounting approach suited to forecast PAYG pension budgets. Unlike projections based on general equilibrium model, this model does not incorporate the rational behavior of individuals. Population, labor force participation, employment rate, contribution rates and benefits are projected into the future, while productivity growth, wage growth and real interest rate are set to be constants. An individual with a stylized biography and working-career is constructed as a representative agent in each age cohort. They concluded as the following:

'It should have become apparent that the notion of an implicit tax is closely linked to the basic theory of unfunded pensions – in fact, it is located at the very heart of the pay-as-you-go mechanism. At the same time, it is easy to apply to an empirical context. Being a well-defined measure for the impact exerted by current public pension schemes on a micro level, it can also be used for doing various kinds of behavioral analyses, proceeding to both efficiency and equity considerations. For instance, one can look at the inter-generational
pattern of full life-time tax rates for a number of real-world examples, illustrating the effects of current pension reforms (Fenge and Werding, 2003);

alternatively, one can spell out the structure of annual, or ‘marginal’, tax rates that are implicitly levied over a given individual’s life cycle, highlighting the impact on periodic labor supply decisions from both a theoretical and an empirical perspective (Fenge et al., 2002).

Different from the aforementioned papers, Casper van Ewijk, Nick Draper, Harry ter Rele and Ed Westerhout (Ageing and the Sustainability of Dutch Public Finances, 2006) introduced GAMMA (Generational Accounting Model with Maximizing Agents), which was first developed for analyzing long-term issues such as ageing, pension issues and structural policy reforms. This model distinguishes different generations and goes beyond the traditional generational accounting models by taking into account important aspect of economic behavior of households, firms and private pension funds, that households choose optimal labor supply and savings rate, firms choose optimal demand for labor and capital, and pension funds choose optimal contributions and benefits. GAMMA is a new member of the family of overlapping generation (OLG) models.

Several policy options are analyzed. We list the options that are most relevant to our study below.

Raising taxes as an instrument to realize sustainable public finance. Raising indirect tax and labor income tax both produce behavioral effect of declined labor supply and outcome, which means a more substantial change in tax rate is required in order to achieve sustainability. For the former practice, elderly are affected negatively. Overall, life-time welfare declines also for all cohorts younger than 65. For the latter, the elderly are beneficiaries from the policy change. The net benefits for the cohorts between birth years 1965 and 2000 are negative. For the very young and unborn cohorts, net benefits are slightly positive.

Raising the statutory retirement age. This policy invariant not only reduces the pension payments but also broadens the taxation base. However, the delay of retirement reduces the subsidy implicit in second-pillar pensions, therefore reduces labor supply. This effect is very small, though. This policy change benefits the elderly who have already retired, since they escaped from the consequence of raising the retirement age. The burden is fully borne by the middle-aged and younger groups. Net benefits of future generations are very small.

As one can imagine, solving GAMMA needs a considerable amount of work, since there are multiple optimization problems involved. Also we can hardly separate old-age
pension from the whole public sector, as GAMMA investigates the multiple effects of policy change. Nevertheless, results from GAMMA could serve as support and guideline for our study.

**Dirk Krueger and Felix Kubler** (Pareto-improving social security reform when financial markets are incomplete, 2005) studied whether unfunded social security system enhances welfare of the society in the sense of Pareto-improving by constructing an overlapping generation model with stochastic production and incomplete markets. Uncertainty enters by means of stochastic return of capital, wage and depreciation. Similar with GAMMA, households make optimal decisions regarding consumption and savings while labor supply is assumed to be constant. Firms decide on capital and labor demand. Government has to make sure that tax is equal to benefits in the PAYG social security system. When general equilibrium of the market is obtained, the crowd-out effect of payroll taxes offsets the gain from intergenerational risk sharing.

The problem is that they considered social security benefits as a whole, thus it provides little help of accessing the policy effect on AOW pension. Also it demands a great deal of computational work, such as Smolyak’s algorithm. To apply such methodology in a generational accounting perspective renders it even more complicated, hence unrealistic to solve.

**Roy P.M.M Hoevenaars, Eduard H.M. Ponds** (Valuation of intergenerational transfers in funded collective pension schemes, 2006) developed value-based ALM tool as an extension to the classical ALM. Value-based generational accounting enables us to rewrite the pension as an aggregate of embedded generational options, namely, net benefit option and residual option. Each cohort has its specific embedded generational pension options, and the sum of all the generational options is equal to zero, which reflects the zero-sum game property of generational accounting in public finance.

Instead of the traditional ALM, value-based ALM takes into consideration the risk adjustment according to different valuations in good and bad scenarios. Indexation cut and contribution raise that often occur at bad scenarios have a stronger impact on option values. They introduced SDF for valuating the generational options and a VAR model describing the return dynamics (i.e. affine term-structure model). Moreover, Monte-Carlo simulation is applied to project the key state variables into the future, i.e. stock and bond returns, nominal interest rates, inflation (wage growth), other valuation ratio, funded ratio, contributions and indexations.

Initially this method was proposed to evaluate different policy designs for pension funds. However, it is broadly applicable to the evaluation of institutional arrangements that imply intergenerational redistribution and risk sharing. Contributions and pension
benefits of Dutch national old-age pension is evaluated differently in good and bad scenarios, thus they should be discounted to the present value by using a stochastic the SDF. As was pointed out in their paper:

‘Usually generational accounting is performed in a deterministic setting. However, future projections of tax revenues and government outlays are subject to uncertainty. Generational accounts typically use a real rate of discount rate that exceeds the government short-term rate, to adjust for the relative risk of future cash flows. Sensitivity analysis for various discount rates can be carried out to analyze the impact on generational accounts of different degree of risk. However this approach disregards the differences in relative risks of government cash flows...We think the use of a stochastic framework and value-based pricing with a specification of macroeconomic risks (productivity, real income growth) and financial market risks (term structure of interest rates, asset pricing) would improve generational accounting in the field of public finance.’
3 National old-age pension in the Netherlands

3.1 Overview

The National Old-age pension (AOW) appears to be one of the most stable schemes in the Dutch social security system since it was introduced in 1956, despite the fact that the basic principles of the Act have frequently come under fire.

The Dutch old-age pension system is based on three pillars. The AOW belongs to the first pillar, old-age pension. Every resident of the Netherlands is covered by this scheme. The second pillar consists of supplementary or non-statutory pensions arising from a public or private employment relationship. The third pillar comprises private pension provisions in the form of annuity insurance that individuals can make independent decisions on the insurance market irrespective to their labor contract. Recently there have been talks of the fourth pillar, which consists of real estate and private capital saving as a retirement provision. The Sociale Verzekeringsbank (SVB) is responsible for implementing the AOW old-age pension. The amount of AOW pension depends on three factors: the number of years under insurance, the level of minimum wage, and the domestic conditions.

- **The number of years under insurance.** Dutch old-age pension system is a scheme of accrued entitlement. Participants accrue entitlements by compulsory insurance, voluntary insurance or a transitional arrangement since 15 years-old. A minimum of one year under insurance gives entitlement to a person. One can be insured for a maximum period of 50 years, while each year of non-insurance reduces the pension by 2%.

- Minimum wage. In 1980, the Adjustment Mechanisms Act linked the minimum wage to the contract wage. It also regulated the linkage between the AOW rates and the contract wage. According to the Minimum Wage and Minimum Holiday Allowance Act, the amount of pension is based on the net minimum wage, which is defined as the gross minimum wage net of the social insurance contributions and health insurance contributions. The gross minimum wage applies for employees between 23 and 65, with a standard amount of 1284.60 Euro per month from July 1, 2009 to January 1, 2010.

- According to the point of view that AOW beneficiaries shared in the increasing prosperity, old-age pensions were indexed to adjust the purchasing power so as to keep pace with the increasing costs of living as well as the developing
prosperity. The AOW provided for an automatic adjustment of pensions each
time the wage index rose by at least 3% over a consecutive period of six months.

3.2 Funding of the AOW scheme

The funding of the AOW is based on a Pay-As-You-Go (PAYG) system, which creates
intergenerational solidarity. The current working populations cover the expenditure of
pensions paid to the retirees. An alternative system to this is the capital funding scheme
that apparently adopted by the pension funds in the Netherlands. There has been debate
about whether the public pension system should switch to a capital funding or partial
funding scheme, such as DB and DC scheme. Here we only list three reasons why Dutch
government initially introduced PAYG schemes in 1957.

First, PAYG system can proof against inflation. Pensions have to rise for protecting their
values when prices rise sharply. Government has learned lessons from the Invalid Act
(IW) and voluntary old-age pension insurance (VOV), which is based on capital funding
system. Value of these benefits rapidly declined under IW and VOV. Secondly, a large
savings surplus that demanded for full funding has a negative effect on investment
returns and consequently on the climate for private investment. Third, it takes very long
time to build a capital fund.

However, the long-run return to PAYG pension schemes depends on the labor income
growth-defined as the sum of growth of productivity and labor participation growth-
that determines the growth of the contribution base. The return on funded pension
schemes, in contrast, depends on the return of financial assets, which is closely related to
the situation of capital market. Hence it might turn out that funding can offer higher
retirement payment if the return of financial assets exceeds the labor income growth, in
the long run. Therefore, PAYG schemes are relatively attractive to funded schemes if a
high wage growth implies a high rate of return on human capital while financial assets
only offer low returns.

Because of demographic ageing and change of market conditions, this question needs to
be reconsidered. We will come back to this discussion briefly in the following chapters,
in forms of Aaron condition, which states that the return of financial assets, the
productivity growth and the labor participation growth together determine the relative
merits of PAYG versus funded pension schemes. A.L. Bovenberg and A.S.M van der
Linden from CPB (Netherlands Bureau for Economic Policy Analysis) developed a
scenario-based methodology to investigate the results of Aaron conditions. Two
scenarios are presented, namely, market scenario and intergenerational solidarity
scenario. These scenarios made different assumptions concerning the return on capital,
productivity growth and labor participation growth. In consequence, different policy reforms should be carried out.

The Old-age pension has been financed by contributions and government grants. Wage and income tax earmarked for AOW contributions finds its way into financing AOW through tax authorities. Contributions are paid by people younger than 65. Since 1999, the contribution rate for the AOW scheme has kept constant at 17.9 percent of income which falls into the lowest two tax brackets.

The new tax system introduced in 2001 abolished the tax-free amount and replaced a number of tax-deductible items by tax credits. This reform reduces the AOW contributions. Before 2001, the tax-free amount was taken from the higher tax brackets on which no contribution was collected. However, the tax credits offsets tax and contributions payable, especially those within the lowest two brackets. In addition, no contributions are collected from capital income. Thus despite the fact that total amount of income liable for contributions increases, AOW contributions decreases. Central government pay contributions in forms of BIKK(Bijdragen in de Kosten van de Kortingen) and central government grants financed by the national budget.

3.3 Policy reform

Growing concerns of achieving sustainability of current public pension system under the demographic ageing pressure, as a consequence of sharply declining fertility combined with ever-increasing life-expectancy, have fuelled an ongoing debate for policy reform in Dutch old-age pension system. Fenge and Werding (2003) considered three policy reforms: increasing the future contribution rate, lowering the benefit level, increasing contribution rate at an earlier stage of the ageing process in order to nourish some way of pre-funding. They noticed that all of these pure strategies should be better combined to form more complex reform packages, which vary on the current situation of different countries.

Future is fundamentally uncertain, especially over the long run. In the light of Bovenberg and Linden’s scenario-based analysis, policy makers can take action in two main ways: investing in human capital and investing in non-human capital. Investing in human capital works well in the intergenerational solidarity scenario while investing in non-human capital is more favorable in the market scenario. Most of the policy reforms for public sector can also be applied to the national old-age pension system. The impact of each policy measure is different over generations. Here we list several policy options

3.3.1 Increasing the statutory retirement age
Investing in the human capital of the elderly not only reduces social spending but also broadens the contribution base. Therefore they rely less on the solidarity of the young and more on their own human capital. The trend of ageing makes it possible to increase the retirement age. Indexing the retirement age to life expectancy is the most natural way to insure society against a longer average life of its citizens so that people spend part of their longer life in work and part in retirement.

In Germany, the government has announced its intention to increase the statutory retirement age from 65 to 67 (beginning in 2010 until 2035). France linked the number of working years necessary to receive a full pension to life expectancy from 2008. In several other OECD countries where the retirement age was low in the past, government raised it to 65. The Dutch government proposed to raise the retirement age gradually. We will illustrate this policy measure in more details later.

3.3.2 Strengthening the link between contributions and benefits

By building a stronger link between individual contributions and benefits, the government may be able to reduce the overall marginal tax rate. Consider the policy combination of increasing the maximum number of contribution years required to receive a full pension and changing the weight given to years of payments. A closer correspondence between life-time contributions and pension benefits enhances the sustainability of the public pension system by lowering the entry pension level of future pensioners.

The other way to tighten the link is to separate pension schemes in a part focusing on poverty alleviation and a part dealing with old-age insurance. As the elderly population grows more heterogeneous, the net contribution to the budget should be based less on age and more on income. This policy measure can improve the risk-sharing within generations rather than across generations.

3.3.3 Investing in human capital of the young

As part of an implicit intergenerational contract, the young receive education financed by the elderly, and later on the young pay for the elderly when they get retired. Since contribution is linked to wage, helping the young to accumulate more human capital strengthens the contribution base for PAYG schemes. But we also need to notice the fact that the elderly fully share in the productivity gains if pension benefits are indexed to wages.

3.3.4 Stimulating labor participation

Meanwhile, stimulating labor market participation can create a stronger contribution base. Increasing the rewards to work by tightening social security benefits and reducing
the tax wedge may stimulate labor supply of all generations. Replacing payroll taxes by consumption taxes alleviates the tax burden on the workers by shifting it in part to those outside the labor force, such as the retired.

3.3.5 Replacing payroll taxes by consumption taxes

Because of the fact that consumption spreads over the life-cycle more evenly than labor income, the idea of substituting payroll contributions of old-age pension by public subsidies financed out of consumption based taxes may be appealing. To maintain the balance of the base-year pension insurance budget, same amount of consumption taxes is collected to reduce payroll contributions. Relying on consumption taxation to finance social security gives pensioners a share in financing pensions (reducing the net transfer to the old), which could render pension finances less prone to demographic ageing.

3.3.6 Switching to a partial funded scheme

To alleviate the demographic pressure on PAYG system, a number of countries have taken steps forward to funded or partial funded system. The capital accumulation in good market timing can meet the sharply increased demand of old-age pensions. From a social welfare perspective, the transition to a (partial) funded system can be pareto-superior if distortions from the old system are eliminated. The current generations have to forgo part of their income to accumulate the capital stock and finance the expiring PAYG system in the mean time. However, dynamic gains may suffice to compensate the living generations who bear this burden. It is obviously to see that young and future cohorts benefit the most from a prosperous capitalized market.

The transition can be achieved by several policy variants: to accumulate assets by raising payroll contributions, by pension cuts, or a mixed strategy. The government might want to choose the mixed strategy which spreads the burdens relatively evenly among all cohorts, for sake of political feasibility.
4 Model

Similar with most of the public pension system, Dutch old-age pension is based on PAYG financing in an explicit way. In unfunded schemes, no capital stock or funds exist. Contributions of today’s employees finance the benefits received by today’s retirees. Thus revenue and expenditure (in other terms, contributions and benefits) have to balance in each period. Demographic ageing implies that the old-age dependency ratio, which is defined as the ratio of elderly people over working age population, is increasing over time. Consequently the so-called system dependency ratio- the ratio of pensioners over active contributors is very likely to increase. Without any policy adjustment, current contributions will start to fall short of pensions.

4.1 Concepts of measurement

Now we introduce several approaches that can measure the imbalance of burden sharing between different generations. According to Robert Fenge and Martin Werding (2003), one distinction is between concepts that measure on aggregate level and on individual level; the other distinction is between concepts that concentrate on pension system alone and those expand this view to general government budget. It gives rise to four concepts, namely, implicit tax, net pension liabilities, generational accounting, and general government fiscal balances.

- The implicit tax is the difference in present value terms between life time contributions and pension benefits received later on. This concept is applied to pension scheme study to analyze the intergenerational effects of old-age security system.

- Net pension liabilities is defined as the net present value of current and future entitlements. Net of existing assets and future contributions. As an extension of implicit debt, this concept measures on aggregate level of the public pension scheme and is based on long-term projections of revenues and expenditures.

- Generational accounting extends the concept of net pension liabilities. All fiscal activities and the taxes and transfers that they involve are taken into account. This concept is developed to study the effects of different policy reforms on each cohort. It implies that all expenditures must be covered by government net wealth or present and future generations in forms of taxes or social security contributions.
• General government fiscal balances extends the idea of measuring implicit public
debt to a larger range of fiscal activities. But unlike Generational accounting, it
measures on aggregate level and the results are not broken down to individual
level.

Table 1: concepts of measurement

<table>
<thead>
<tr>
<th>Public pension schemes</th>
<th>General government budget</th>
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<tbody>
<tr>
<td>Net pension liabilities (macro-level effect)</td>
<td>General government fiscal balances (macro-level effect)</td>
</tr>
<tr>
<td>Implicit tax (micro-level effect)</td>
<td>Generational accounting (micro-level effect)</td>
</tr>
</tbody>
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In this paper, we employ net pension liabilities and implicit tax for modeling, simulation
and empirical work since our main purpose is to investigate how different policy
reforms affect Dutch national old-age pension.

4.1.1 Implicit debt

Start from a rather simple concept, implicit debt of unfunded pension scheme. It is
defined as the present value of outstanding claims of current old-age pensioners and
future benefits of the active population who have paid the past and current
contributions. Consider a simple case, a series of successive ‘overlapping generations’
numbered by \( t \in \{0,1,2,\ldots\} \) in which they enter the labor market, each generation lives
for three period, two for working and one of retirement. Labor supply is inelastic and
normalized to unity. Each individual has to pay contributions to the old-age pension
and he receives pension payment after retirement.

Total contributions of the young generations consist of the contribution rate and the
wage sum

\[
\Psi_t = \theta_t W_t = \theta_t \left[ \sum_{N_{t-1}} w_{i,t} + \sum_{N_t} w_{i,t} \right]
\]

This equation can also be written as

\[
\Psi_t = \theta_t (N_{t-1} + N_t) w_t
\]

• \( N_t \): population size of generation \( t \)
• \( \theta_t \): contribution rate at period \( t \) which is assumed to be invariant over different individual \( i \)
• \( w_{i,t} \): wage rate accruing to individual \( i \in \{1, 2, \ldots, N_t\} \) at period \( t \)
• \( w_t \): the average wage at period \( t \), i.e. \( W_t = (N_{t-1} + N_t)w_t \)

Total pension benefits equal the pension level times the number of retirees

\[
P_t = \sum_{N_{t-2}} p_{i,t} = N_{t-2}p_t
\]

• \( p_{i,t} \): pension benefit paid to individual \( i \in \{1, 2, \ldots, N_t\} \) at period \( t \)
• \( p_t \): the average pension benefit at period \( t \)

In general, pension benefit is a function of the contributions. i.e. \( p_{i,t+2}(\theta_{t}w_{i,t}, \theta_{t+1}w_{i,t+1}) \). This function links the benefits to contributions can have various forms. It can jump from zero to a full flat-rate amount when one pays contributions for a certain number of years. It may be related to contributions or wages either in a linear or a more complex nonlinear way. Dutch national old-age pension is linked to the minimum wage, thus remains constant across different individuals in the same period.

In our simple framework, the implicit debt is given by

\[
D_t = \frac{p_{t+1}(\theta_{t-1}w_{t-1}, \theta_{t}w_{t})N_{t-1}}{R_{t+1}} + \frac{p_{t+2}(\theta_{t}w_{t})N_t}{R_{t+1}R_{t+2}}
\]

• \( R_{t+1} \): interest rate factor used for discounting future cash flows to present value in the deterministic setting. \( R_{t+1} = 1 + r_{t+1} \)

Implicit debt in relative terms—the ratio of implicit debt over total wage income may give us a better instruction of the unfunded pension scheme

\[
d_t = \frac{D_t}{W_t} = \left( \frac{p_{t+1}(\theta_{t-1}w_{t-1}, \theta_{t}w_{t})N_{t-1}}{R_{t+1}} + \frac{p_{t+2}(\theta_{t}w_{t})N_t}{R_{t+1}R_{t+2}} \right) / (N_{t-1} + N_t)w_t
\]

In practice, the calculation is relatively easy for the retirees, but more complex for those current workers. To arrive at the exact value of the implicit debt, we need to take into account the detailed policies governing the Dutch national old-age pension.

We notice that the size of implicit debt in absolute terms (relative terms) has three influencing factor, the number of active contributors, pension benefits (replacement rate: \( p/w \)) and interest rate. The pension benefit level depends on the rules regarding how to link pension benefits to wage income.

\[
D_t = D(N, p, R)
\]
\[
d_t = d(N, p/w, R)
\]
In the context of ageing, change in the size of the implicit debt is ambiguous. It may change in either direction. More information on productivity, labor participation, wage income and the relative change of size of each cohort is required to make accurate estimation. The size of $D_t$ and $d_t$, or their development over time are not good indicators for the impact of ageing on unfunded pension schemes. Nevertheless, the burden on current or future contributors gets larger no matter what the precise pattern of demographic ageing is.

### 4.1.2 Net pension liabilities

OECD economists van den Noord and Herd (1993) developed the concept of net pension liabilities as an extension of implicit debt. This new concept captures the present value of current and future pension benefits, net of existing assets and future contributions. Assuming that current worker continue to work thus pay contributions and acquire pension claims, and future generations (the unborn) will also pay contributions and acquire pension claims when entering the labor market, future contributions reduce the current debt while future pension benefits resulting from future contributions increase it, based on the projection for the future development of the pension scheme.

Based on the assumption that each generation lives for three period, two for working and one of retirement, we derive the expression for net pension liabilities in a time horizon from period $t$ to $t + T (T > 2)$.

Contributions are paid till $t + T - 2$

$$\frac{\Psi_{t+1}}{R_{t+1}} + \frac{\Psi_{t+2}}{R_{t+1}R_{t+2}} + \cdots + \frac{\Psi_{t+T-2}}{\prod_{i=1}^{T-2} R_i}$$

Pension benefits are paid till $t + T$, at period $t$,

$$\frac{R_{t+1}}{R_{t+1}} + \frac{R_{t+2}}{R_{t+1}R_{t+2}} + \cdots + \frac{p_{t+T-1}}{\prod_{i=1}^{T-2} R_i} + \frac{P_{t+T}}{\prod_{i=1}^{T-1} R_i}$$

Net pension liabilities

$$NL_t = \frac{p_{t+1}}{R_{t+1}} + \frac{p_{t+2}}{R_{t+1}R_{t+2}} + \cdots + \frac{p_{t+T-1}}{\prod_{i=1}^{T-1} R_i} + \frac{P_{t+T}}{\prod_{i=1}^{T} R_i}$$

$$-\left(\frac{\Psi_{t+1}}{R_{t+1}} + \frac{\Psi_{t+2}}{R_{t+1}R_{t+2}} + \cdots + \frac{\Psi_{t+T-3}}{\prod_{i=1}^{T-3} R_i} + \frac{\Psi_{t+T-2}}{\prod_{i=1}^{T-2} R_i}\right)$$

The budget of an unfunded or PAYG pension system requires that contributions equal benefits for each period $t$

$$\Psi_t = P_t$$
So the contributions cancel out benefits with continuous policy adjustment, and the net pension liabilities is equal to the present value of implicit debt at period $t + T - 2$ (such as in France, Germany and Italy)

$$ NPV(D_{t+T-2}) = \frac{P_{t+T-1}}{\prod_{t+1}^{t+T-1} R_i} + \frac{P_{t+T}}{\prod_{t+1}^{t+T} R_i} $$

However, when performing our calculation, this is not necessarily the case that net pension liabilities and implicit debt coincide. We rely on status-quo projections using a combination of policy parameters concerning retirement age, contributions rate and benefits level. Contributions and benefits need not cancel out in this case. Net pension liabilities often tend out to be substantially larger than the present value of implicit debt at period $t + T - 2$, in context of demographic ageing.

Since we also have the relation

$$ P_t = p_t N_{t-2} $$

$$ \Psi_t = \theta_t (N_{t-1} + N_t) w_t $$

Rewrite the net pension liabilities as

$$ NL_t = \frac{p_{t+1} N_{t-1}}{R_{t+1}} + \frac{p_{t+2} N_t}{R_{t+1} R_{t+2}} + \cdots + \frac{p_{t+T-1} N_{t+T-3}}{\prod_{t+1}^{t+T-1} R_i} + \frac{p_{t+T} N_{t+T-2}}{\prod_{t+1}^{t+T} R_i} $$

$$ -\frac{\theta_{t+1} (N_t + N_{t+1}) w_{t+1}}{R_{t+1}} + \frac{\theta_{t+2} (N_{t+1} + N_{t+2}) w_{t+2}}{R_{t+1} R_{t+2}} + \cdots + \frac{\theta_{t+T-2} (N_{t+T-3} + N_{t+T-2}) w_{t+T-2}}{\prod_{t+1}^{t+T-2} R_i} $$

We summarize by reviewing the comments on implicit debt of Fenge and Werding:

Net pension liabilities extend the notion of an implicit debt to a longer time-horizon, taking into account potential future deficits of revenues over expenditures which may arise in unfunded pension schemes given current policy parameters. Thus, they offer a concept of measurement which is straightforward from the simple algebra of PAYG financing and is highly suited to capture the impact of ageing on current pension systems on an aggregate level. At the same time, it is less clear how these hidden liabilities will be perceived by current and future individuals and, in particular, how they may affect individual decisions. (Fenge and Werding (2003))

### 4.1.3 Implicit tax

To understand the intergenerational redistribution of PAYG system and the impact of different policy reforms on individuals who belong to different cohorts, we introduce the concept of implicit tax. Implicit tax, also known as the tax entailed in unfunded
pensions is defined as the difference between life-time contributions and old-age pensions at an individual level, both discounted (compounded) to present value terms.

In our stylized model, implicit tax is expressed as

\[
T_{t}^{t-2} = R_{t-1} R_{t} \theta_{t-2} w_{t-2} + R_{t} \theta_{t-1} w_{t-1} - p_{t}
\]

\[
T_{t}^{t-1} = R_{t} \theta_{t-1} w_{t-1} + \theta_{t} w_{t} - \frac{p_{t+1}}{R_{t+1}}
\]

\[
T_{t}^{t} = \theta_{t} w_{t} + \frac{\theta_{t+1} w_{t+1}}{R_{t+1}} - \frac{p_{t+2}}{R_{t+1} R_{t+2}}
\]

\[
T_{t}^{t+1} = \frac{\theta_{t+1} w_{t+1}}{R_{t+1}} + \frac{\theta_{t+2} w_{t+2}}{R_{t+1} R_{t+2}} - \frac{p_{t+3}}{R_{t+1} R_{t+2} R_{t+3}}
\]

Each generation survives for three periods, as described before. Subscript denotes the time of measurement while superscript denotes generation number.

Here we can clearly see the implicit tax is actually part of the generational accounts. Recall that generational accounts include all present and future taxes and contributions vs. all present and future benefits. As a consequence, the rich details of sub-modules and projections regarding the wide range of all fiscal activities lead to the inconvenience of setting up generational accounts. When it comes to specific policy implications on a single budget item, i.e. Dutch old-age pension, we are in favor of a more scope-restricted concept. Separating the unfunded pension system from the whole public sector, the concept of implicit tax gives us a measurement that concentrates on unfunded pension system and net pension liabilities alone.

Defining implicit tax on relative terms might be more instructive than absolute terms. We introduce the implicit tax rate \( \tau \) which is given by the ratio of implicit tax to life-time income of one generation.

Life-time income for generation \( t \) discounted to present value terms

\[
w_{t} + \frac{w_{t+1}}{R_{t+1}}
\]

Thus the implicit tax rate for generation \( t \) is

\[
\tau_{t} = \frac{T_{t}}{w_{t} + \frac{w_{t+1}}{R_{t+1}}} = \frac{\theta_{t} w_{t} + \frac{\theta_{t+1} w_{t+1}}{R_{t+1}} - \frac{p_{t+2}}{R_{t+1} R_{t+2}}}{w_{t} + \frac{w_{t+1}}{R_{t+1}}}
\]

Applying this definition to generation \( t-2, t-1, t \) yields \( \tau_{t-2}, \tau_{t-1}, \tau_{t+1} \) respectively.
For funded pension scheme, the internal rate of return is equal to the market rate of
return, thus implicit tax is zero. However this is not necessarily the case for unfunded
pension scheme. Since contributions are not invested in the capital market, the internal
rate of return may fall short of market rate of return. The size of implicit tax and implicit
tax rate depends on the difference of these two values. In terms of redistribution, a
perfectly actuarial system (funded pension scheme) does not involve any redistribution
across generations whereas there is redistribution in an unfunded or partial funded
pension scheme. In this sense, implicit tax reflects the actuarial fairness of the pension
system.

Now we examine the famous Aaron-condition in our simplified model. Assume that
that population growth rate and wage growth rate are constant over time, and policy
remains unchanged.

\[
\frac{w_{t+1}}{w_t} = G_t = \cdots = G
\]

\[
\frac{N_{t+1}}{N_t} = M_t = \cdots = M
\]

\[
\theta_t = \cdots = \theta
\]

Taking into account the budget constraint for unfunded pension system

\[
p_{t+2}N_{t+2} = \theta_{t+2}W_{t+2}
\]

We can write implicit tax rate as function of population growth rate, wage growth rate
and policy parameters

\[
\tau_t = \frac{\theta_t + \frac{\theta_{t+1}G_{t+1}}{R_{t+1}} - \frac{\theta_{t+2}G_{t+1}G_{t+2}M_{t+1}(1 + M_{t+2})}{R_{t+1}R_{t+2}}}{1 + \frac{G_{t+1}}{R_{t+1}}} = \theta[1 - \frac{(GM)^2 + G^2M}{R^2 + RG}]
\]

A little algebra can tell that

\[
GM < R \text{ leads to } \tau_t > 0
\]

Non-Aaron condition states that in an introductory stage, a positive implicit tax falls on
those who pay contributions over their life-cycle before they are entitled to receive
pensions because the internal rate of return is expected to fall short of the market rate of
return used for discounting. Empirical research also supports the fact that implicit tax is
likely to be positive for present and future generations. (Fenge and Werding 2003)
However, it may not be the case during a transition stage.
4.2 Extension to a stochastic setting

In the previous section we introduced implicit tax in a deterministic setting. However, contributions and pension benefits should be evaluated thus discounted differently in good and bad scenarios, in line with Hoevenaars and Ponds’s (2006) early work. So in this section we will perform the valuation of implicit tax in a stochastic setting, by using the so-called stochastic discount factor of pricing kernel. Moreover, introduction of stochastic discount factor into our model excludes arbitrage opportunity.

First we carry out the result of a first-order vector autoregressive (VAR(1)) model. Conditional on the estimation of VAR(1) coefficients we are able to write the interest rate as function of market price of risk, incorporating term structure models. By minimizing the square of fitting error, market risks of price can be estimated. Then we compute stochastic discount factor and discount all the contributions, benefits and wage incomes to present value, which gives rise to implicit tax in a stochastic setting.

4.2.1 Return dynamics

We describe return dynamics by a VAR(1) model. This type of specification has been employed by Hoevenaars, Molenaar and Ponds (2008), Campbell, Chan and Vicerira (2002), Ang, Piazzesi, Wei (2006), among others. Notice that the use of VAR(1) is not restrictive since any vector autoregressive model can be written as a VAR(1) by adding state variables if necessary.

In accordance with Hoevenaars et al.(2008), we specify state variables as the following

\[ z_t = [y_t^{(1)}, y_t^{(120)}, \pi_t, x_t, d_y_t]^T \]

- \( y_t^{(1)} \): the 1-month interest rate
- \( y_t^{(120)} \): the 10-year zero-coupon rate
- \( \pi_t \): price inflation
- \( x_t \): excess stock returns over the 1-month interest rate
- \( d_y_t \): the corresponding dividend yield.

The vector of state variables follows a Gaussian vector autoregression with one lag:

\[ z_{t+1} = \phi_0 + \phi_1 z_t + \Sigma \zeta_{t+1} \]

- \( \phi_0 \): vector of intercepts
- \( \phi_1 \): matrix of slope coefficients
- \( \Sigma \zeta_{t+1} \): shocks to the state variables

\[ \zeta_{t+1} \sim N(0, I). \]
Thus we allow the shocks to be cross-sectionally correlated, but independently and homoscedastically distributed over time. The assumption of homoscedasticity rules out the possibility that state variables can predict changes in the risk. This assumption makes sense as Campbell and Viceira (1999), Harvey (1989, 1991) have proved that changes in risk are not persistent enough to have effects on the intertemporal hedging demand, and the risk predictability of state variables is no more than modest. Given the specification of our VAR(1) model, the state vector $z_{t+1}$ inherits the normality of the shocks.

We assume risk premia on bonds are linear in state variables. The pricing kernel is log-normal:

$$-\log M_{t+1} = y_t^{(1)} + \frac{1}{2} \lambda_t' \lambda_t + \lambda_t' \zeta_{t+1}$$

$$M_{t+k} = M_{t+1} M_{t+2} \cdots M_{t+k}$$

$M_{t+k}$ can also be interpreted as the stochastic discount factor for valuation of future revenues and expenditures of unfunded pension schemes. Multiplication of the future payoffs $k$ periods ahead by the corresponding SDF $M_{t+k}$ and average over all scenarios gives the fair current price.

$$V_t[P_{t+k}] = E_t[M_{t+k}P_{t+k}]$$

$\lambda_t$ is the time varying market prices of risk for various risks. $\lambda_t$ is a linear function of the state variables.

$$\lambda_t = \lambda_0 + \lambda_t' z_t$$

Besides the fact that future outcomes can be discounted back to present value terms with an appropriate risk-adjusted discount rate, the pricing kernel specification has other merits, as depicted by Campbell et. al (2001): in the absence of a general equilibrium model for asset pricing, this setting enables us to capture the non-arbitrage properties. The parsimonious and flexible factor model allows us to match many stylized facts about return dynamics. Incorporating macro factors into the model further improves forecast of state variables.

The price of an $n$-period zero-coupon bond is determined by solving the equation recursively

$$P_t^{(n)} = E_t(M_{t+1}P_{t+1}^{(n-1)})$$

or in log terms

$$p_t^{(n)} = E_t(m_{t+1} + p_{t+1}^{(n-1)}) + \frac{1}{2} Var_t(m_{t+1} + p_{t+1}^{(n-1)})$$
with terminal condition $p_t^{(0)} = 0$

where $p_t^{(n)} \equiv \log p_t^{(n)}$ and $m_t \equiv \log M_t$

Since

$$p_t^{(n)} = E_t \left(M_{t+1} p_{t+1}^{(n-1)} \right) = \exp \left( E_t (m_{t+1} + p_{t+1}^{(n-1)}) + \frac{1}{2} \text{Var}_t (m_{t+1} + p_{t+1}^{(n-1)}) \right)$$

According to Nijman and Koijen (2006), the resulting bond price can be written as a linear function of the state vector

$$-p_t^{(n)} = A_n + B_n z_t$$

$A_0, B_0 = 0$

A one-period bond can be written as

$$-y_t^{(1)} = p_t^{(1)} = E_t (m_{t+1}) + \frac{1}{2} \text{Var}_t (m_{t+1})$$

The absence of arbitrage is imposed by computing these coefficients from the following equations

$$A_n = A_{n-1} + B'_{n-1} (\phi_0 - \Sigma \lambda_0) - \frac{1}{2} B'_{n-1} \Sigma' \Sigma' B_{n-1}$$

$$B_n = e_1 + (\phi_1 - \Sigma \lambda_1)' B_{n-1}$$

with initial condition $A_1 = 0, B_1 = -e_1$

The $n$-period zero-coupon yield $y_t^{(n)}$ are an affine function of the state vector

$$y_t^{(n)} = -\frac{p_t^{(n)}}{n} = \frac{A_n}{n} + \frac{B_n'}{n} z_t = a_n + b_n' z_t$$

The above equation allows us to model the entire yield curve. We also find that the constant risk premia parameter $\lambda_0$ only affects the constant yield coefficient $a_n$ therefore determines the average term spread and average bond excess returns, while the parameter $\lambda_1$ also affects the factor loading $b_n'$ and controls the time variation in term spread and expected returns.

Ang et al. (2006) used a two step procedure to estimate the model. Let $\theta_1 = \{\phi_0, \phi_1, \Sigma\}$ and $\theta_2 = \{\lambda_0, \lambda_1\}$ be the partitioned parameter space and $\theta = \{\theta_1, \theta_2\}$. In the first step, he fitted a VAR(1) process to the observed state variables and estimate $\theta_1$ using a standard SUR. In the second step, conditional on the value of $\theta_1$ from the first step estimation, he employed GMM approach to estimate $\theta_2$ by minimizing the sum of the squared fitting errors of the model. Since the value of $\theta_1$ and the initial condition of $A_1, B_1$ (therefore also $a_1, b_1$) are known, the $n$-period zero-coupon yield $y_t^{(n)}$ is a function of $\theta_2$. To be
precise, compute $\hat{y}_t^{(n)} = a_n + b_n z_t$ for given value of state vector, and solve the minimizing problem

$$\min_{\{a_n,b_n\}} \sum_{t=1}^{T} \sum_{n=1}^{N} (\hat{y}_t^{(n)} - y_t^{(n)})^2$$

$$y_t^{(n)} = \hat{y}_t^{(n)} + \epsilon_t$$

The measuring error $\epsilon_t \sim N(0,\Omega)$

According to Hoevenaars et al. the minimizing is based on yields with maturities from 1 to 15 years.

4.2.2 Implicit tax in a stochastic setting

First we extend the three period model to a more realistic setting. Assume agent lives for 85 years and then passes away. (This assumption is quite reasonable in the Netherlands, despite the fact that female is more likely to live longer than male, but we do not distinguish genders in this context) He starts to participate in the labor market since 25 years-old and retires at 65. During working years, he pays contributions to accrue entitlements for pension benefits, as a percentage of his income. After retirement, he receives a predetermined amount of pension benefits until death. Since we assumed full insurance, i.e. insured for 50 years, the amount of pension benefits only depends on minimum wage level and domestic situation.

$$T_t^* = \theta_t w_t + \frac{\theta_{t+1} w_{t+1}}{R_{t+1}} + \frac{\theta_{t+2} w_{t+2}}{R_{t+1} R_{t+2}} + \cdots + \frac{\theta_{t+39} w_{t+39}}{\prod_{i=1}^{t+39} R_i} - \frac{p_t}{\prod_{i=1}^{t+40} R_i} - \frac{p_{t+1}}{\prod_{i=1}^{t+41} R_i} - \cdots - \frac{p_{t+60}}{\prod_{i=1}^{t+60} R_i}$$

$$\tau_t = \frac{T_t^*}{w_t + \frac{w_{t+1}}{R_{t+1}} + \frac{w_{t+2}}{R_{t+1} R_{t+2}} + \cdots + \frac{w_{t+39}}{\prod_{i=1}^{t+39} R_i}}$$

Now we turn to analysis in value terms

$$V_t(T_t) = \theta_t w_t + E_t(M_{t+1} \theta_{t+1} w_{t+1}) + \cdots + E_t(M_{t+39} \theta_{t+39} w_{t+39})$$

$$-E_t(M_{t+40} p_{t+40}) - \cdots - E_t(M_{t+60} p_{t+60})$$

$$V_t(\tau_t) = \frac{V_t(T_t)}{w_t + E_t(M_{t+1} w_{t+1}) + E_t(M_{t+2} w_{t+2}) + \cdots + E_t(M_{t+39} w_{t+39})}$$

Applying this definition to each generation yields the age profile of implicit tax and implicit tax rate. Note that for present and future generations $\{t, t+1, t+2, \ldots\}$, all the contributions and benefits are included in the formula. However for generations numbered $\{t-1, t-2, \ldots, t-60\}$ (note that generation $t-60$ is 85 years old and about to pass away) we only consider their present and future contributions and benefits.
This calculation deviates from the initial definition of implicit tax. Recall that we defined implicit tax as the difference between life-time contributions and old-age pensions for each generation, both discounted (compounded) to present value terms. Nevertheless, here we exclude those realized values of contributions and benefits, based on the following two reasons

- We aim to investigate the effects on implicit tax of different policy reforms in response to the increased net pension liabilities, by comparing the difference of implicit tax of a) baseline case, and b) old-age pension under new policy. Each realized value in a) and b) cancels out each other.
- This considerably saves our calculation efforts.
5 Simulation results

5.1 Parameter specification

5.1.1 Wage growth and labor productivity growth

It is reasonable to assume that wage growth fully comes from labor productivity growth. In line with Casper van Ewijk et al. (Ageing and the Sustainability of Dutch Public Finances, 2006), we set the annual rate of labor-augmenting technological change (and the rate of age-specific labor productivity growth) equals to 1.7%.

The corresponding rate of labor productivity growth may deviate from this value due to demographic factors. James Feyrer (Demographics and Productivity, 2005) proved that change in the age structure of the workforce is found to be significantly correlated with changes in aggregate productivity. Productivity is age dependent, although some literatures pointed out it is not clear whether older workers really have higher productivity. However, since the effect of deviation in our calculation is modest, we ignore the impact of demographic change on labor productivity growth and wage growth.

5.1.2 Contribution rate

Agent pays contributions on part of his entire income defined by law. In 2001, new tax system introduced two tax credits, the general tax credit and the earned income tax credit, which mitigated the tax and national insurance burden (and the old-age pension contribution). Within the lowest two tax brackets, contribution burden increases rapidly as income rises, while the tax burden remains low. For incomes beyond the second tax bracket, in contrast, contribution burden declines as income rises, but this is offset by an increasing tax burden.

A person with annual income of up to 6150 Euro does not need to pay any contribution. The contribution rate increases gradually to 12.7 percent with income rising up to 30631 Euro. Afterwards, contribution rate starts to shrink slowly. A person with salary 100,000 needs to pay 5.1 percent of income as old-age pension contribution. The reason is no contribution needs to be paid out of income higher than 30631 Euro.

In the Netherlands, most people earn a salary higher than 30631 Euro. Thus, we assume, in the baseline case, agent pays 12.7 percent out of 30631 Euro as payroll contributions in the basic year. Contributions grow each year in line with wage growth, which is set to be 1.7%.
5.1.3 Pension benefits

Recall that according to the Minimum Wage and Minimum Holiday Allowance Act, the amount of pension is based on the net minimum wage, which is defined as the gross minimum wage net of the social insurance contributions and health insurance contributions. The gross minimum wage applies for employees between 23 and 65, with a standard amount of 1284.60 Euro per month from July 1, 2009 to January 1, 2010. From 1980 to 1994, Dutch old-age pension benefits more or less followed development in the gross minimum wage, but afterwards deviated from that. For sake of simplification, we assume gross minimum wage is proportional to the wage level, so pension benefits level should follow the development of wage level.

Now old-age pension benefits level is 70% of net minimum wage for single pensioner and 50% for pension beneficiary with a younger partner. The latter case refers to supplementary allowance. According to SVB, around 10 percent of the total number of old-age pension beneficiaries had a younger partner in 2006 and this number will rise to 14 percent in 2020.

In the baseline case, therefore, we assume old-age pension benefits level equals to 70 percent of the minimum wage and follows wage growth.

5.2 Simulate stochastic discount factor

We borrow estimation results from Hoevenaars et al. (2008) to make simulation of the SDF. In the following section, we will present all the necessary inputs (dynamics of state variables and prices of risk) in tables, generate new variable which we need for simulation and comments on each table.

Table 2: Summary statistics

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Std</th>
<th>Min</th>
<th>Max</th>
<th>Skewness</th>
<th>Xkurtosis</th>
<th>Views</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y_t^{(1)}$</td>
<td>5.31</td>
<td>2.53</td>
<td>2.00</td>
<td>13.10</td>
<td>0.93</td>
<td>0.05</td>
<td>3.75</td>
</tr>
<tr>
<td>$\pi_t$</td>
<td>2.88</td>
<td>0.91</td>
<td>-0.47</td>
<td>1.40</td>
<td>0.94</td>
<td>1.77</td>
<td>2.00</td>
</tr>
<tr>
<td>$y_t^{(120)}$</td>
<td>6.54</td>
<td>1.74</td>
<td>3.16</td>
<td>10.71</td>
<td>0.01</td>
<td>-0.82</td>
<td>4.50</td>
</tr>
<tr>
<td>$x_{st}$</td>
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<td>14.41</td>
<td>-23.15</td>
<td>11.84</td>
<td>-0.81</td>
<td>2.61</td>
<td>2.75</td>
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<tr>
<td>$DY_t$</td>
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<td>1.16</td>
<td>1.27</td>
<td>5.70</td>
<td>0.65</td>
<td>-0.85</td>
<td>2.25</td>
</tr>
</tbody>
</table>

Table 2 gives the summary statistics for the observations. Data covers the full period 1972:09 to 2008:08. Mean and standard deviation are annualized while min and max are on a monthly base. Except dividend yield, the statistics are presented in log term. Views are constructed by adjusting the constants in VAR model and $\lambda_0$ in the prices of risk. We
consider views as subjective long-term equilibrium value. Hoevenaars et al. did not show the estimation results of $\phi_0$. So we will compute it on the basis of views.

The condition of long-term equilibrium gives:

$$E_t z_{t+1} = z_t$$

Combining with:

$$z_{t+1} = \phi_0 + \phi_1 z_t + \Sigma t_{t+1}$$

$$z_t = \phi_0 + \phi_1 z_{t}$$

We find that:

$$\phi_0 = (I - \phi_1) z_t$$

| Table 3: VAR coefficient estimates |
|-------------------|-----------------|----------------|-----------------|----------------|
| $y_t^{(1)}$       | $\pi_t$         | $y_t^{(120)}$  | $x_s_t$         | $d_y_t$        |
| 0.95              | 0.03            | 0.04           | -0.00           | -0.01          |
| 61.10             | 3.89            | 1.48           | -0.56           | -1.04          |
| $\pi_t$           | 0.39            | 0.17           | 0.05            | 0.00           |
| 4.68              | 3.66            | 0.35           | 0.62            | 2.07           |
| $y_t^{(120)}$     | 0.01            | 0.00           | 0.98            | 0.00           |
| 1.69              | 1.02            | 79.20          | 1.02            | -0.16          |
| $x_s_t$           | -1.11           | -2.18          | -4.08           | 0.10           |
| -0.74             | -2.57           | -1.57          | 2.10            | 3.38           |
| $d_y_t$           | 0.00            | 0.02           | 0.03            | -0.00          |
| 0.19              | 2.52            | 0.97           | -1.39           | 129.46         |

| Table 4: Cross-correlations of the shocks |
|-------------------|-----------------|----------------|-----------------|----------------|
| $y_t^{(1)}$       | $\pi_t$         | $y_t^{(120)}$  | $x_s_t$         | $d_y_t$        |
| 0.04              | 0.03            | 0.19           | -0.04           | 0.06           |
| $\pi_t$           | 0.22            | 0.16           | -0.07           | 0.05           |
| $y_t^{(120)}$     | 0.02            | 0.12           | -0.10           | -0.91          |
| $x_s_t$           | 4.03            | 0.06           | -0.91           | 0.04           |

Table 3 and table 4 report the estimation results of VAR coefficient matrix and the correlation matrix of the shocks. In table 3, first row shows the coefficient value while
second row shows the t statistics. In table 4, volatility is given on the diagonal. All the estimations results are on a monthly base. We have three comments here:

First, VAR has mean-reversion property, which means in the long-run, shocks will dampen and hence the system is stable. We will show this property in appendix later by simulation.

Second, one month interest rate is significantly influenced by the lagged inflation. Excess stock return is significantly correlated with lagged inflation and dividend yield. A higher inflation also increases the future dividend yield, which will eventually boosts the stock price.

Third, stock return shows mean reversion due to the positive exposure to the lagged dividend yield and negative correlation with dividend yield of the same period. Thus a positive shock to dividend yield reduces the current stock return while at the same time drives a higher stock return in the future. This supports the fact revealed by early literatures (Campbell and Viceira, 2005, and Hoevenaars et al., 2005) that stocks and bonds are less risky in the long-run.

Table 5: Prices of risk

<table>
<thead>
<tr>
<th></th>
<th>$\lambda_0$</th>
<th>$y_t^{(1)}$</th>
<th>$\pi_t$</th>
<th>$y_t^{(120)}$</th>
<th>$xs_t$</th>
<th>$dy_t$</th>
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</thead>
<tbody>
<tr>
<td>$y_t^{(1)}$</td>
<td>0.14</td>
<td>140.00</td>
<td>303.61</td>
<td>135.54</td>
<td>5.48</td>
<td>11.91</td>
</tr>
<tr>
<td>$\pi_t$</td>
<td>0.00</td>
<td>4.79</td>
<td>10.39</td>
<td>4.64</td>
<td>0.19</td>
<td>0.41</td>
</tr>
<tr>
<td>$y_t^{(120)}$</td>
<td>0.14</td>
<td>143.55</td>
<td>37.60</td>
<td>126.17</td>
<td>2.31</td>
<td>5.79</td>
</tr>
<tr>
<td>$xs_t$</td>
<td>3.09</td>
<td>21.45</td>
<td>46.13</td>
<td>117.31</td>
<td>2.47</td>
<td>60.99</td>
</tr>
<tr>
<td>$dy_t$</td>
<td>6.83</td>
<td>65.57</td>
<td>133.00</td>
<td>208.24</td>
<td>5.60</td>
<td>137.78</td>
</tr>
</tbody>
</table>

Table 5 reports the parameter estimates of $\lambda_t = \lambda_0 + \lambda_1 z_t$. First column shows the value of vector $\lambda_0$, while the rest shows the slope matrix of prices of risk. From now on, we will use matlab to make simulation of SDF, based on the following set of equations.

\[-log(M_{t+1}) = y_t^{(1)} + \frac{1}{2} \lambda_t \lambda_t + \lambda_t \zeta_{t+1}\]

$\lambda_t = \lambda_0 + \lambda_1 z_t$

$M_{t+k} = M_{t+1}^1 M_{t+2}^1 \cdots M_{t+k}^1$

$V_t [P_{t+k}] = E_t [M_{t+k} P_{t+k}]$
Figure 1 shows expectations of SDF and the traditional deterministic discount factor (annual discount rate equals 4%) over 130 periods (in our model one period refers to one year). We use views of state variable as the initial condition. Expectation of SDF can be seen and applied as a normal discount factor due to our assumption that wage growth, pension benefits and contributions have no uncertainty involved. In a more general model, where productivity growth—thus wage growth is affected by shocks, we need to simulate future cash flow and SDF, calculate the product in each scenario and expectations of all products.

Here we can see that the expectation of SDF has similar shape with traditional discount factor over time. Both of them stay around one in the very short term and converge to zero in the very long run, which are consistent with common sense. However, the curve of SDF obviously remains below that of the traditional discount factor, which leads stochastic calculation in our model to noticeable deviation from the deterministic calculation. A high equity premium may well explain the difference. According to the standard Capital Asset Pricing Model (CAPM), the required rate of return is the sum of risk-free rate and a risk premium:

$$R = r^f + \delta$$

Recent studies suggest that the interest rate has dropped from 4% in the 1980s to slightly below 2% in 2004. More concerns are attached to the appropriate risk premium. As results depend on the model used and the time period considered, there is large literature on this topic. But we believe a modest risk premium between 3% and 4% is
reasonable. Therefore, using SDF to discount pension benefits and contributions to the present value seems more reliable. Forecasts for state variables in VAR(1) are given in Appendix A.

5.3 Simulation of the baseline case

Denotes the oldest generation who is going to pass away by $1$, the generation who just retired by $20$, the current generation who has just entered labor force by $60$, etc. up to the generation $130$. Since past values do not matter, we only consider current and future wage level. Denote the current minimum wage level by $w_1$, and the minimum wage level of the last period by $w_{130}$. Denote stochastic discount factor by $\{m_1, m_2, \ldots, m_{129}\}$.

![Figure 2: generation number](image)

We use the following set of equations for the baseline case simulation:

$V(T_1) = -pw_1$

$V(T_2) = -pw_1 - E[m_1pw_2]$

$V(T_3) = -pw_1 - E[m_1pw_2] - E[m_2pw_3]$

$\vdots$

$V(T_{20}) = -pw_1 - E[m_1pw_2] - E[m_2pw_3] - \cdots - E[m_{19}pw_{20}]$

$V(T_{21}) = con_1 - E[m_1pw_2] - E[m_2pw_3] - \cdots - E[m_{19}pw_{20}] - E[m_{20}pw_{21}]$

$V(T_{22}) = con_1 + E[m_1con_2] - E[m_2pw_3] - \cdots - E[m_{20}pw_{21}] - E[m_{21}pw_{22}]$

$\vdots$

$V(T_{60}) = con_1 + E[m_1con_2] + \cdots + E[m_{39}con_3] - E[m_{40}pw_{41}] - \cdots - E[m_{59}pw_{60}]$

$V(T_{61}) = E[m_1con_2] + \cdots + E[m_{40}con_{41}] - E[m_{41}pw_{42}] - \cdots - E[m_{60}pw_{61}]$

$\vdots$
\[ V(T_{129}) = E[m_{69} con_{70}] + \cdots + E[m_{108} con_{109}] - E[m_{109} pw_{110}] - \cdots - E[m_{128} pw_{129}] \]

\[ V(T_{130}) = E[m_{70} con_{71}] + \cdots + E[m_{109} con_{110}] - E[m_{110} pw_{111}] - \cdots - E[m_{129} pw_{130}] \]

### 5.4 Implicit tax

#### 5.4.1 Increase the retirement age

As we have briefly discussed before, increasing retirement age alleviates the burden of the net liabilities on old-age pension system by two effects. The elderly not only reduces the pensions they receive but also broadens the contribution base. The increased and still increasing life expectancy has already prompted a number of countries to take this policy measure. This can be achieved by raising the official retirement age or by removing incentives for early retirement. CPB proposed to carry out this plan in two steps, by a total of two years, in 2015 and 2025, to increase the retirement age for two years from 65 to 67.

Although the increase in retirement age itself could not restore intergenerational balance completely, but it helps to reduce the net pension liabilities and therefore rebuild the long-term stability of the Dutch public pension system, together with other policy reforms.

To model the impact of increasing effective retirement age on generational accounts, we assume that the original relative age profile of old-age pension is shifted forward by two years, and this measure takes place immediately from the basic year.

In the simulation we keep the relative payroll contributions unchanged as the incentives on active labor participation and the potential revenue of a prolonged working life are difficult to predict. We neglect the increase of pension benefits caused by a longer stay in labor force. Of course behavioral effects are excluded from our model (the increase of the retirement age makes pension less generous and reduces the subsidy implicit in second-pillar pension, which leads to a reduction of labor supply for all age groups, but this effect is very small according to CPB)

Apparently we expect that those cohorts younger than 65 at 2015 fully bear this burden, as the older cohorts escape the consequence of a higher retirement age. Bonin et al. proved that an increase in retirement age indeed significantly improves the sustainability of the pension system by the cost of raising the generational accounts of all living cohorts younger than 65 because of the prolonged working time and reduced benefits. Moreover they gave empirical results as follows:
‘As a consequence the intertemporal liabilities accumulated by the system fall by about 30 percentage to 146.5 percent of the 1996 GDP. While present newborns experience net life-time contributions of 1590 thousand pesetas, future generations are exposed to a 4813 thousand peseta higher burden, contributing 6403 thousand peseta net.’ (Beyond the Toledo agreement: the intergenerational impact of the Spanish pension reform, 1999)

**Figure 7: Intergenerational effects of increasing retirement age**

![Figure 7: Intergenerational effects of increasing retirement age](image)

Figure 7 shows us the simulation results. The red line represents the effects of increasing retirement age by two years on generational accounts, all cash flow discounted to the fair present value by SDF. The blue line represents the effects with cash flow discounted by traditional discount rate (r=4%). The red line falls below the blue line, because we are using a relatively low discount rate. The policy effects are expressed in terms of implicit tax.

From figure 7, we see that cohorts older than 65 escape from the policy change, as expected. Change in implicit taxes for all cohorts younger than 65 are positive. Financial burden of the national old-age pension system is borne by current middle-aged, young and future generations who have to face the direct consequences of a higher retirement age. The group aged 64 years experiences the highest loss, 28658 euro, and it gradually decreases for the younger age group, to 6789 euro for the cohort aged 25 and 461 euro to future generation aged -45, since they only need to face the impact of the new policy many years later in the future, and all the values are discounted into present value terms. Note that all numbers here are given in the SDF framework.
5.4.2 Adjust pension benefits and contribution

Now we focus on the immediate once-and-for-all policy invariants that eliminate the implicit debt by changing contributions rate or pension levels, thereby assuring intergenerational balance.

If government responds to demographic ageing by cutting down the level of pension benefits, the implicit tax for the old-aged will increase the most significantly, since they face a direct loss from a lower pension benefits. Building on Aaron (1966), a constant contributions scheme is usually regarded as a benchmark for running unfunded pension schemes. In this case, the internal rate of return is simply given by the rate of aggregate payroll growth and the implicit debt will not respond too much to the large-scale demographic ageing. Calculation of implicit tax is also quite straightforward in such case.

In many existing public pension schemes, the actual policy responses are more likely to associate with driving up contributions, especially for those operated on a defined benefits base. To some extent, this will offset the increase in implicit tax for older generations, and it will necessarily feed through to an increase in implicit tax for younger generations. Thus retirees escape the consequence of a higher contributions rate while the burden of financing old-age pension is largely passed on to current labor force and subsequent generations. In a sense, a policy of increasing contributions can be interpreted as building a new PAYG system on top of the old one. New inaugural gains accrue to the retirees that are just enough to reduce the burden they inherited from earlier generations and younger generations have to pay the bill.

As a political concern for the government, a mixed strategy that spreads the transitional burdens more evenly among all cohorts is more favorable, compared to the first two ‘pure strategies’. Less perceptible than the alternatives, this policy option may get more supporters from the majority of voters willing to preserve the social security insurance, but trying to minimize the share in the resulting fiscal burden. Assessing the merits of the mixed strategy and investigating the optimal setup are clearly beyond the scope of our study, though.

All of these strategies mentioned above can be combined with other policy measures to form complex reform packages, affecting the generational accounts in many different ways. In this paper, we want to analyze the effects of these pure strategies independently. Consequently, we consider three alternative scenarios: pension benefits cut of 20%, increasing contribution by 30%, and a liner combination of the first two extreme scenarios in which contributions are raised by 20% and pension benefits cut by
15%. To model these policy reforms, we assume that they will take place immediately from the basic year. The following graphs show the simulation results.

**Figure 8: Intergenerational effects of cutting off pension benefits**

Table 8 shows the effects of cutting off pension benefits by 20% on generational accounts. The red line represents the intergenerational effects with all cash flow discounted to the fair present value by SDF. The blue line represents the effects with cash flow discounted by traditional discount rate (r=4%).

Changes on implicit tax for all generations are positive. The current 65 years-old cohort is affected the most, with a loss of 32520 euro, since they start to receive the lowered pension benefits from the basic year until they pass away, that is, within 20 years.

From 65 to 84, the change on implicit tax declined sharply, because the older generations only have part of retirement years that they receive pension benefits ahead of them. The cohort aged 84 only has a loss of 2157 euro, which is the direct consequence of pension benefits cut for one year.

From age 65 to younger generations, the change on implicit tax also decreases, but rather slowly, to 7243 euro for the cohort aged 25 and 487 euro for future generation aged -45, since they only need to face the impact of the new policy many years later in the future, and all the values are discounted into present value terms.
Figure 9: Intergenerational effects of increasing contributions

Figure 9 shows the effects of driving up payroll contributions by 30% on generational accounts. We see from figure 9 that changes on implicit tax for all age groups are positive again. Cohorts aged older than 65 escape from the consequence of this policy reform, because they pay no contributions after retirement. The cohort aged 25, who has just joined the labor force, will face the strongest hit of 25079 euro. From 25 to 64 years-old, implicit tax declines sharply to 1167 euro, as the number of years they have to pay contributions decrease. It also gradually decreases from 25 years-old to the younger age groups, until it reaches 1574 euro for future generation aged -45, since they only need to face the impact of the new policy many years later in the future.

Figure 10: Intergenerational effects of mixed strategy
Figure 10 shows the effects of the mixed strategy on generational accounts. Contributions are raised by 20% and pension benefits are cut by 15%. Compared to the first two pure strategies, the burden to finance old-age pensions is shared more evenly among generations, especially for those in the labor force.

The cohort aged 65, who just retired from labor force, has to bear the highest loss of 23896 euro. Change on implicit tax decreases slowly to 21524 euro for the cohort aged 25, though there only exists slight difference among the working populations. It then falls down to 1338 euro rather sharply for the future generation aged -45, as the younger and future generations only need to face the consequence of this new policy a few years later in the future.

Here we discover that the traditional deterministic methodology gives quite a different picture that the change on implicit tax decreases from 25 to 65 years-old. It peaked for cohort aged 25. Indeed the result from value-based SDF methodology exhibits clear variation, despite of the size of change, though we leave the reason undetected in this paper. From 65 to 84 years-old, it decreases remarkably to 1617 euro.

Table 6: Intergenerational effects of different policy reforms

<table>
<thead>
<tr>
<th>age of the cohorts</th>
<th>45</th>
<th>0</th>
<th>25</th>
<th>45</th>
<th>64</th>
<th>65</th>
<th>84</th>
</tr>
</thead>
<tbody>
<tr>
<td>generation number</td>
<td>130</td>
<td>85</td>
<td>60</td>
<td>40</td>
<td>21</td>
<td>20</td>
<td>1</td>
</tr>
<tr>
<td>increase retirement age</td>
<td>462</td>
<td>2550</td>
<td>6789</td>
<td>14397</td>
<td>28658</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>pension benefits cut 20%</td>
<td>487</td>
<td>2727</td>
<td>7243</td>
<td>15528</td>
<td>31440</td>
<td>32520</td>
<td>2157</td>
</tr>
<tr>
<td>higher contributions 30%</td>
<td>1574</td>
<td>9248</td>
<td>25079</td>
<td>17238</td>
<td>1167</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>mixed strategy -15%p+20%c</td>
<td>1338</td>
<td>8008</td>
<td>21524</td>
<td>22362</td>
<td>23830</td>
<td>23896</td>
<td>1618</td>
</tr>
</tbody>
</table>

Table 6 summarizes the effects of different policy reforms on intergenerational effects. Notice here we adjust pension benefits and contributions with certain value, -20% pension benefits cut, 30% higher contributions and a combination of -15% and 20%, but these adjustments do not necessarily cancel out the net pension liabilities. Same principle can be applied to analyzing effects under other policy parameter values.

Holger Bonin et al. (1999) first calculated the parameter values that required to achieve partial funding, and then discussed the intergenerational effects conditional on these values. It is interesting for us to calculate parameter values required to eliminating net pension liabilities, thereby assuring intergenerational balance, though this analysis goes beyond concerns of this paper.
5.5 Implicit tax rate

Up till now we have performed the calculation of implicit tax under different policy reforms. However, there exists one potential shortcoming when using implicit tax as an indicator of intergenerational effects. All the future cash flows have been discounted to the present value, thus incurs an underestimation of changes in implicit tax for the very young and future generations.

To eliminate the horizon effects mentioned above, define implicit tax rate $\tau$ as the ratio of implicit tax to life-time income of one generation. Since both the nominator and the denominator are discounted (or compounded) to present value, we obtain a rather fair comparison between current and future generations. Recall the simple three-periods case in the our model, life-time wage income for generation $t$ discounted to present value is given by

$$w_t + \frac{w_{t+1}}{R_{t+1}}$$

Extend the life-time wage income to its value terms in this simple model

$$w_t + E[M_{t+1}w_{t+1}]$$

However, to calculate the life-time wage income for the very old generations, realized wage since long ago should be included. Define a wage vector $W$ which has a length of 169. The first component represents the wage income that generation 1 (aged 84) received 60 years ago when they were 25 years old. The 60 component represents the wage level of the basic year.

All the future cash flows are discounted to present value with stochastic discount factor as we did before, while the past cash flows are compounded to present value by an appropriate interest rate. We use the following set of equations for the life-time wage income:

$$V(\tau_1) = \frac{-pw_1}{W_1 (1 + r)^{59} + \cdots + W_{40} (1 + r)^{20}}$$

$$V(\tau_2) = \frac{-pw_1 - E[m_1pw_2]}{W_2 (1 + r)^{58} + \cdots + W_{41} (1 + r)^{19}}$$

$$V(\tau_3) = \frac{-pw_1 - E[m_1pw_2] - E[m_2pw_3]}{W_3 (1 + r)^{57} + \cdots + W_{41} (1 + r)^{18}}$$

$$\vdots$$

$$V(\tau_{20}) = \frac{-pw_1 - E[m_1pw_2] - E[m_2pw_3] - \cdots - E[m_{19}pw_{20}]}{W_{20} (1 + r)^{40} + \cdots + W_{59} (1 + r)}$$

40
\[ V(\tau_{21}) = \frac{c \cdot m_1 p \cdot w_2 - E[m_2 p w_3] - \cdots - E[m_{19} p w_{20}] - E[m_{20} p w_{21}]}{W_{21} (1 + r)^{39} + \cdots + W_{59} (1 + r) + W_{60}} \]

\[ V(\tau_{22}) = \frac{c \cdot m_1 p \cdot w_2 - E[m_2 p w_3] - \cdots - E[m_{20} p w_{21}] - E[m_{21} p w_{22}]}{W_{22} (1 + r)^{38} + \cdots + W_{60} + E[m_1 W_{61}]} \]

\[ \vdots \]

\[ V(\tau_{60}) = \frac{c \cdot m_1 p \cdot w_2 + \cdots + E[m_{39} c \cdot n]}{E[m_1 W_{61}] + E[m_2 W_{62}] + \cdots + E[m_{40} W_{100}]} - E[m_{40} p w_{41}] - \cdots - E[m_{59} p w_{60}] \]

\[ V(\tau_{61}) = \frac{E[m_1 c \cdot n] + \cdots + E[m_{40} c \cdot n]}{E[m_1 W_{61}] + E[m_2 W_{62}] + \cdots + E[m_{40} W_{100}]} \]

\[ \vdots \]

\[ V(\tau_{129}) = \frac{E[m_{69} c \cdot n] + \cdots + E[m_{108} c \cdot n]}{E[m_{69} W_{129}] + \cdots + E[m_{18} W_{168}]} - E[m_{109} p w_{110}] - \cdots - E[m_{128} p w_{129}] \]

\[ V(\tau_{130}) = \frac{E[m_{70} c \cdot n] + \cdots + E[m_{109} c \cdot n]}{E[m_{70} W_{130}] + \cdots + E[m_{119} W_{169}]} - E[m_{110} p w_{111}] - \cdots - E[m_{129} p w_{130}] \]

In order to choose an appropriate interest rate to compound past cash flows into the present value, we make plots by matlab, and observe which fixed interest rate gives rise to a discount factor curve that fits in the SDF curve the best. Details are provided in appendix B.

We assume wage income of the basic year is 30631 euro, and wage growth completely comes from productivity growth. We set interest rate to compound past cash flows equals to 5.5%. All the other parameters remain the same with previous specification. The following graphs show the simulation results.

**Figure 14 Effect of increasing retirement age on implicit tax rate**

![Graph showing effect of increasing retirement age on implicit tax rate](image-url)
Figure 14 shows change on implicit tax rate if retirement age is increased by two years immediately from the basic year. Same as before, cohorts aged 65 and above will escape from the consequence of this policy variant. Change on implicit tax rate peaks at 1.05% for the cohort aged 64. It decreases slightly to 0.99% for the future generation aged -45. The curve is almost flat for all the generations younger than 65.

It is interesting to see that deterministic calculation generates a curve that goes down from age 25 to 65, which implies that increasing retirement age has totally different effects on the working force of different age. Stochastic calculation says this is not the case, though.

Figure 15 Effect of cutting off pension benefits on implicit tax rate

Figure 15 shows change on implicit tax rate under the policy reform that cuts off pension benefits by 20%. The cohorts aged 65 experiences the highest change of 1.15%. It significantly falls down to 0.04% for the cohort aged 84, while slightly decreases to 1.05% for the cohort aged 25 and 1.06% for future generation aged -45.

We see distinctive variation of deterministic calculation and stochastic calculation. For young and future generations, change on implicit tax rate calculated deterministically is almost twice as big as that calculated stochastically, though it narrows down from age 25 on, especially for cohorts older than 65.
Figure 16 Effect of increasing contributions on implicit tax rate

Figure 16 gives the effect on implicit tax rate of increasing contributions by 30%. The cohorts aged 65 and above escaped from the policy consequence and those younger than 25 years-old face the identical 3.81% higher implicit tax rate, which shows a rather different pattern from the implicit tax, because we exclude the horizon effect. This number decreases sharply from 25 to 65 years-old. Stochastic calculation only exhibits little difference from the deterministic method.

Figure 17 Effect of mixed strategy on implicit tax rate

Figure 17 shows the effect on implicit tax rate of the mixed strategy, a combination of 20% higher contributions and 15% pension benefits cut. Clearly all generations have to share the burden. From 25 years-old forward, change on implicit rate decreases gradually to 0.84% for cohort aged 65 and 0.03% for cohort aged 84. Both curves remain
flat for cohorts younger than 25 years-old. Under stochastic calculation numbers are 3.27%, 3.30%, 3.24% for cohorts aged 25, 0, and -45 respectively. Modest difference can be found between stochastic and deterministic calculation.

Table 7: Intergenerational effects of different policy reforms

<table>
<thead>
<tr>
<th>age of the cohorts</th>
<th>-45</th>
<th>0</th>
<th>25</th>
<th>45</th>
<th>64</th>
<th>65</th>
<th>84</th>
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<tbody>
<tr>
<td>generation number</td>
<td>130</td>
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<td>60</td>
<td>40</td>
<td>21</td>
<td>20</td>
<td>1</td>
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<td>increase retirement age</td>
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<td>0.0095</td>
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<td>0.0105</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>pension benefits cut 20%</td>
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<td>0.0102</td>
<td>0.0106</td>
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<td>0.0115</td>
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<td>0.0126</td>
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<td>0.0163</td>
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</table>

Table 7 summarizes how different policy reforms affect implicit tax rate. Compared to the change on implicit tax, the burden distributed more evenly across generations when using implicit tax rate as the indicator. Sensitivity test on productivity growth can be found in appendix C.
6 Discussions

6.1 Behavioral effects

For sake of simplicity, our model assumed that different policy reforms have no impact on productivity growth and wage growth. Hence behavioral effects are excluded. However, in reality, individual behavior is influenced by policies with respect to labor income. Some rather complicated models such as GAMMA, are useful tools to analyze effects of different policy reforms, taking into account the behavioral effects.

Usually behavioral effects are expressed by maximization of rational agent. He makes optimal decisions on labor supply and its complement, leisure, to maximize his utility function. This implies that labor supply and leisure depend on the marginal reward of labor, also known as price of leisure. Apparently labor supply decreases as the price of leisure goes down.

GAMMA distinguishes three important channels through which the government influences household behavior: the labor income tax, the consumption tax and the capital tax on household savings. As labor supply depend on net wages, taxes on labor income and consumption reduce labor supply. Household also take account of the implicit subsidy or tax in mandatory contributions to supplementary pension schemes.

In our context, all of the four policy reforms have complex effects on life-time income, which is defined here as the sum of labor income and pension benefits. Two effects work on opposite direction. As we mentioned before, increasing the rewards to work by tightening social security benefits and reducing the tax wedge may stimulate labor supply of all generations. At the same time, a less generous pension depresses the incentive to work, though.

On one hand, from the individual’s perspective, his wage income and life-time income are affected. On the other hand, from the government’s perspective, a higher labor participation rate even strengthens revenues from new policy reforms, while a lower labor participation rate may lead to a partial offsetting reduction of the revenues.

A simple way to model the behavioral effects is to apply the rules of thumb, which comes from earlier literature. Using rules of thumb could be a good substitution for solving utility maximization. Take the policy measure of increasing retirement age as an
example, it makes pensions less generous and indirectly reduces the subsidy implicit in second-pillar pension schemes. This reduces labor supply for all groups, though the actual effect is quite small. Research on labor supply and wage change under this policy measure is available, as many OECD countries have carried out these policy reforms in response to the demographic ageing.

6.2 Which state variables?

We included five state variable in our model, which is in line with Hoevenaars et al.(2008). However, it is debatable which state variables should be taken into return dynamics.

Campbell et al. (2002) select the following state variables, the real interest rate in log term vector of excess returns of stocks and bonds, and vector of other state variables such as dividend yield.

Hoevenaars et al. (2005) took one step further by adding more state variables into the VAR(1). Other risky assets than stocks, bonds and cash are included, such as credits, commodities, hedge funds, listed real estate and the liabilities. To deal with the missing data and large dimension problem, they imposed a number of restrictions and make optimal use of the data information for estimating the dynamics of the series with shorter histories, which is helpful for further study of VAR(1).

A. Ang et al. (2006) jointly modeled return dynamic factors and economic growth. They only use GDP growth in the basic model as a macro factor. To improve the out-of-sample forecasting result, they also considered other variables such as inflations. Nevertheless this does not yield significantly better forecasts, because short rate and term spread already contain information about expectations of future inflation. We address their comment here in our paper again:

However, an unanswered question is whether we can improve on these yield curve forecasts by combining both term structure information and other macro variables.

In order to compute stochastic discount factor in the context of public finance, we may need a specification of macroeconomic risks and include these macro variables into the VAR(1). Possible candidates consist of representative variables of ageing, productivity, labor participation. Moreover we can test if macro variable are correlated with term structure variables (asset pricing forecasting variables), for example, if low asset returns and low productivity growth tend to occur simultaneously. But one encounters a problem here that monthly data about productivity growth and labor participation growth can be hardly collected directly. Moreover, most time series start from 1970s.
Alternative choices of state variables give rise to different computation of SDF. It might also be interesting to compare the results from different specifications of state variables. Since this paper aims to demonstrate the idea of Generational accounting under stochastic setting, we will not perform further computation here.

6.3 Shock to wage and productivity level

Recall the parameter specification, we set the annual rate of labor productivity growth equals to 1.7% and wage growth fully comes from labor productivity growth. For sake of simplification, we also assumed minimum wage is proportional to the wage level, so pension benefits level should follow the development of wage level. There is no uncertainty involved.

But in reality, labor productivity growth follows a stochastic process. We discuss two alternative approaches here. One is to assume it follows a Brownian motion (or Geometric Brownian motion), with predetermined drift term and volatility parameters. The other one is to include productivity growth into the VAR(1) as a macro risk factor, but turns out to be unrealistic because we do not have productivity growth sample size big enough to estimate.

Allowing uncertain productivity growth better captures the spirit of valuation in a stochastic setting, though calculation may become more complex.

6.4 Alternative ways to eliminate horizon effects

In this paper, we employed the concept of implicit tax rate to eliminate the sizable horizon effects, since the nominator and denominator are both expressed in present value terms. Now we will examine other approaches.

One way is to divide the change on implicit tax by the implicit tax in the old system. But we have to make sure implicit tax in the old system is positive for each generation. The other way is to separate the pension benefits and contributions, which makes sense even with negative implicit tax: we calculate the ratio of change in pension benefits to pension benefits received in old system, net of the ratio of change in contributions to contributions paid in old system for each cohort. Clearly these approaches involve more calculations.

6.5 Sensitivity to parameter and model specifications

We have displayed sensitivity test on productivity growth in appendix and found that implicit tax rate is sensitive to the change on productivity growth. However, we also made other parameter specifications that implicit tax rate might be sensitive to.
First, we used views of state variables as initial condition to simulate SDF. Views are interpreted as the long term equilibrium values. We believed the simulation results largely depend on the initial value we chose. Thus if the mean values of state variables are chosen instead as the initial value, the expectations of SDF as well as other variables will be quite different. For instance, a much higher long rate is implied. Views and means are constructed conditionally, thus results will change with different history information. With a prosperous market, it is more likely to get an optimistic forecast.

Second, in deterministic calculation, we set the fixed discount factor to be 4%, which is commonly used by actuarial practice. To compound the past cash flows into present value terms, we decided to use a fixed discount factor of 5.5% since it generates a curve that fits in the stochastic discount factor curve very well, though not perfectly. This specification implies inconsistency in discount factors hence turns out to be debatable. We think it is reasonable to use two fixed discount factors because 1) Realized values and uncertain future cash flows should be treated differently. 2) Size of the discount factor leads to the first-order effect on the simulation results, but hardly has any influence on the pattern. One may want to consider to use an identical discount factor, either implied by VAR(1) or expert opinions, instead.

Third, compared to parameter specification, model specification plays a more important role. We employed VAR(1) to describe return dynamics in the model. In fact, more elements of interests can be added, such as the regime-switching model, which reflects reality even better.
7 Conclusions

This paper has studied the intergenerational effects of different policy reforms. Large-scale demographic ageing questions the fiscal sustainability in the Netherlands. The national old-age pension that financed on a PAYG base, as one of the most stable social security schemes, is sensitive to the population structure.

To restore the sustainability of public finance, alternative policy reforms are proposed under two main categories, namely, investing in human capital and investing in non-human capital. We have introduced several policy reforms that attract most attention, increasing the statutory retirement age, strengthening the link between contributions and benefits, investing in human capital of the young, stimulating labor participation, replacing payroll taxes by consumption taxes, switching to a partial funded scheme. However, some of the policy effects cannot be properly modeled in our framework, thus we only focused on two policy reforms, increasing the retirement age and adjusting pension benefits and contributions, which of course show up only as starting points.

Unlike pension funds, assets and liabilities of national old-age pension are not well defined since capital stock does not exist, thus we have to choose another indicator to express generational accounts. We introduced four concepts of measurement: implicit tax, net pension liabilities, generational accounting and general government fiscal balances extends, based on two distinctions. First distinction is between concepts that measure on aggregate level and on individual level; the other is between concepts that concentrate on pension system alone and that expand this view to general government budget.

Defined as the difference in present value terms between life time contributions and pension benefits received later on, implicit tax is commonly applied to analyze the intergenerational effects of old-age security system, thus we decided to use it as an indicator for generational accounts. To state more precisely, implicit tax is actually part of the generational accounts because the latter also includes other budget items.

Implicit tax is usually calculated in a deterministic way that all future cash flows are discounted to present value terms by a predetermined interest rate. However, contributions and pension benefits should be evaluated differently in good and bad
scenarios. So we have performed the valuation of implicit tax in a stochastic setting, by using the stochastic discount factor. Moreover, introduction of SDF into our model excludes arbitrage opportunity.

We defined return dynamics by a first-order vector autoregressive (VAR(1)) model. Conditional on the estimation of VAR(1) coefficients we forecasted the state variables as well as market prices of risk. We computed stochastic discount factor and discount all the contributions, benefits and wage incomes into present value terms, which gives rise to implicit tax in a stochastic setting.

Though stochastic calculation mostly gives results of the same structure to deterministic calculation, it exhibits clear variation in the case of mixed strategy that change on implicit tax significantly falls down from 25 to 65 years-old under deterministic calculation, while stochastic calculation gives the opposite result; it slightly increases over cohorts aged between 25 and 65 under stochastic calculation. We have also discovered the mixed strategy indeed spreads the transitional burdens more evenly among all generations, compared to the other two ‘pure strategies’.

To eliminate the horizon effects, we defined implicit tax rate as the ratio of implicit tax to life-time income of one generation. We obtained a rather fair comparison between current and future generations by using implicit tax rate as the indicator for generational accounts, since both the nominator and denominator are discounted (or compounded) to present value terms.

We found that except the policy measure to increase contributions, stochastic calculation draws pictures of distinctive variation. For instance, under the policy measure to cut pension benefits by 20%, change on implicit tax rate calculated deterministically is almost twice as big as that calculated stochastically for young and future generations. Under traditional determinist calculation, increasing retirement age and cutting off pension benefits generate curves that go down from age 25 to 65, which implies that these two policy reforms both have totally different effects on the working populations of different age; younger workers get hurt worse than elder ones. However, stochastic calculation says this is not the case.

Moreover, compared to the change on implicit tax, the burden distributed more evenly across generations, when using implicit tax rate as the indicator, especially for young generations.

Our research has several limitations that should be kept in mind when evaluating the methodology. First, we considered a model excluding behavioral effects. This omission
can be remedied by using rules of thumb or labor supply elasticity. Second, we may add more state variables such as macro risk factors and returns of other assets to the VAR(1). Third, rethinking how to model productivity and wage growth improves the performance of stochastic calculation. Fourth, it may be interesting to take a look at alternative approaches that eliminate horizon effects. As our paper aims to demonstrate the stochastic valuation of generational accounts due to different policy reforms, we made the model rather simple. Fifth, sensitivity to parameter and model specification should also be noticed. To integrate all these factors into a theoretically implementable framework is a challenging task for future study.
Appendix A: Simulation of state variables

Figure 3: Expected 1-month interest rate

Figure 4: Expected inflation
Figure 3 to Figure 6 forecast the state variable in VAR(1), 1-month interest rate, inflation rate, 120-months interest rate and stock excess return respectively. All the state variables display mean-reverting property. Consistent with Hoevenaars et al. (2006), all state variables are annualized. Excess stock return shows a mean of 3.51%, which looks quite reasonable. We exclude the plot of dividend yield here because it goes to negative for
several years. This may due to the implicit drawback of VAR forecast itself; large dimension renders estimation and forecast difficult, and no restrictions are imposed on the state variables.

Appendix B: Finding an appropriate interest rate to compound past cash flows

Figure 11, 12 and 13 show the simulation results of discount factor curve, when interest rate equals 5%, 5.5%, 6%, respectively.

**Figure 11 Interest rate equals 5%**

![Figure 11 Interest rate equals 5%](image)

**Figure 12 Interest rate equals 5.5%**

![Figure 12 Interest rate equals 5.5%](image)
The curve generated by an interest rate of 5.5% fits in the stochastic discount factor curve very well, though still not perfectly. Therefore we conclude that 5.5% is a reasonable interest rate to compound the past wage income into the present value. (Individual invests their wealth in the capital market with a rate of return approximately 5.5%)

Appendix C: Sensitivity test for productivity growth

Table 8: Sensitivity of implicit tax on productivity growth

<table>
<thead>
<tr>
<th>productivity growth 1%</th>
<th>age of the cohorts</th>
<th>generation number</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-45</td>
<td>0</td>
</tr>
<tr>
<td>increase retirement age</td>
<td>197</td>
<td>1557</td>
</tr>
<tr>
<td>pension benefits cut 20%</td>
<td>200</td>
<td>1546</td>
</tr>
<tr>
<td>higher contributions 30%</td>
<td>934</td>
<td>7557</td>
</tr>
<tr>
<td>mixed strategy -15%p+20%c</td>
<td>773</td>
<td>6197</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>productivity growth 3%</th>
<th>age of the cohorts</th>
<th>generation number</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-45</td>
<td>0</td>
</tr>
<tr>
<td>increase retirement age</td>
<td>1717</td>
<td>5622</td>
</tr>
<tr>
<td>pension benefits -20%</td>
<td>2034</td>
<td>6501</td>
</tr>
<tr>
<td>Higher contributions +30%</td>
<td>4945</td>
<td>16524</td>
</tr>
<tr>
<td>mixed strategy -15%p+20%c</td>
<td>4823</td>
<td>15892</td>
</tr>
</tbody>
</table>
Comparison of table 8 and table 9 implies that change on implicit tax rate is more sensitive to productivity growth than change on implicit tax considering all cohorts. When using implicit tax rate as the indicator for generational accounts, results alter significantly with decision on productivity growth.

The only exception is change on implicit tax rate under higher contribution; productivity growth has very little effect on implicit tax rate, especially for cohorts younger than 25 years-old.

In addition we find that for very young generations, change on implicit tax turns out to be more sensitive because productivity growth dominates.
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