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Björn Wijbenga

Intergenerational Risk-Sharing and Changing Demographics in a Defined Benefit Pension Scheme

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MASTER'S THESIS ACTUARIAL STUDIES

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B.S. WIJBENGA

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Author: Björn Wijbenga, s1467328
Supervisor: dr. L. Spierdijk
Co-assessor: prof. dr. R.H. Koning
Coördinator: dr. C. Praagman

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Abstract

We investigate the effects of changing demographics on intergenerational risk-sharing in a defined benefit pension fund. The pension system in the Netherlands is briefly discussed. We show that the optimal risk allocation rules spread out the risk over many years. A 100% allocation to risky assets can be justified using IRS and the long term horizon of the pension fund. We demonstrate that applying these parameters to a real economy needs further study. Changing demographics, in this research the Dutch demographics, distort the intergenerational fairness. We illustrate that the impact is severe, and repairing the effects leads to high costs.

Keywords: intergenerational risk-sharing, demography, defined benefit, pension fund, baby boom.

Preface

This Master's thesis is the result of my Master's program in Econometrics, Operations Research and Actuarial Studies at the University of Groningen. After the Bachelor's program Econometrics, I chose the specialization Actuarial Studies, the field for which this thesis is written. Because this marks the end of my studies in Groningen, I would like to use this opportunity to express my gratitude towards a number of people.

First, I would like to thank the staff of the Econometrics department for their efforts in teaching me about their interesting subject. I experienced a good atmosphere during classes, but also outside lectures many of them were available to answer questions and provide guidance where necessary. In particular, I would like to thank Laura Spierdijk for her help during the process of writing this Master's thesis. She gave me much freedom in writing this thesis, but also gave me many good suggestions and comments. She was always available for help, and was very quick in reviewing my work. This made our cooperation very pleasant for me. I would also like to mention Ruud Koning here. First for being the co-assessor of this thesis, but foremost for the courses he gave me. During his lectures I learned a lot. Writing my Bachelor's thesis under his supervision taught me even more and helped me much in writing this thesis.

I would also like to thank my fellow students and friends in Groningen, who made living there a great pleasure. In my time in committees and the board of VESTING, I gathered many experiences which I deeply value. I want to thank Pieter Bultena here for his help during my thesis. Next to the helpful discussions about our lay-outs and programming code, he made his computer available to me to speed up my calculations a lot. My thanks also goes to everybody who read this thesis and gave me useful comments about spelling, style, and language.

Finally, I would like to express my deepest gratitude towards my parents, who made my studies possible. They supported me in every way they could during the entire process. They have been an indispensable factor for me to successfully complete my study of Econometrics in Groningen.

Björn Wijbenga
Groningen, October 2009

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Chapter 1

Introduction

1.1 Background

Pension funds invest contributions from individuals or their employers in order to pay out benefits when the participants retire. Investing your money using such a fund has advantages in comparison to investing individually. First of all, the fund guarantees to pay a contribution as long as you live. This takes away uncertainty about the age you will finally reach, and thereby about the amount of money you need to save. Secondly, due to the long term horizon of pension funds, it is always thought more affordable for such a fund to take on the risk of the equity market than for individual investors, who have a shorter time horizon. Barberis (2000) shows that the investment horizon is of great importance for investment decisions. The third argument is the fact that the costs of a pension fund as a percentage of the funds managed are lower. This is an advantage of the scale of the pension fund. A final argument is that a pension fund can spread out the investment risk over different generations, known as intergenerational risk-sharing (IRS), as discussed in e.g. Varian and Gordon (1988). Arguments that one lifetime is not long enough to handle the shocks of the market thereby disappear.

At this moment, all pension funds are recovering from a severe financial crisis. In the second half of 2008, stock markets collapsed, showing losses not seen since the great depression in the 1930s. Many pension funds, if not most, saw their funding ratios drop to well under 100%. Next to the investment returns, pension funds should worry about some other developments. The demographic conditions in the Western world are changing. A post war baby boom and an increasing life expectancy cause discussions about the pensionable age for the government pension, in the Netherlands known as the Old Age Pension Act (AOW)[1]. Due to the same developments, the employee pension plans also saw rising costs.

All these economic and demographic changes lead to a higher premium level and benefits that will not be indexed for a while. This affects the current workforce, as well as the retirees. The elderly complain that their pensions are affected, at a moment at which they have no means left to compensate this. The younger generations also complain, saying that they pay for the current retirees instead of saving for their own pension. An example of these arguments is that, due to good results in the past, pension funds in the recent past lowered premiums and were still able to make good returns. The elderly of today enjoyed the benefits of those premiums, but on the other hand are confronted with less indexation on their pension now. It may not be fair to let the current workforce pay for this, but the retired generations cannot change their choices or return to work either.

[1] "Algemene Ouderdomswet" in Dutch.

1.2 Research question

The goal of pension funds is to develop strategies that deal explicitly with many sources of uncertainty, making sure that the returns for their participants are as high as possible. These risks include, but are certainly not limited to, demographic uncertainties, future inflation, the uncertainty in the financial markets for stocks, bonds and other assets, and changes in the regulatory requirements for pension funds. The chosen strategy affects the allocation of risk to different generations, known as intergenerational fairness. In many studies, intergenerational risk-sharing is studied and researchers look for an optimal pension policy. For example, Beetsma and Bovenberg (2008) find that only a defined benefit (DB) pension scheme establishes optimal IRS. Often these researches make a lot of assumptions about the economy, and the distribution to different generations is left out of the research.

In this Master's thesis, we will investigate the distribution of wealth in a defined benefit pension fund with policy rules that allow for IRS. Starting point will be the model discussed in Cui, De Jong, and Ponds (2008), in which a defined benefit pension scheme is optimized using risk allocation rules. They find a set of parameters that lead to the highest certainty equivalent consumption which exhibits much IRS. We will repeat parts of their research here, to create our own version of a pension model. We will first optimize this model using the same assumptions in a stylized economy. Thereafter, we will change the assumptions in order to investigate the consequences for different generations. First we investigate a case study of a baby boom, and then we will introduce the Dutch demographics.

The main question in this research is: *"What is the effect of intergenerational risk-sharing on different generations?"* This research will show what the effects are, and that they are significant. Where other research finds that intergenerational risk-sharing is optimal over an individual plan, we note that this need not be the case for every generation when demographic conditions are not taken into account. The allocation of wealth to the generations in our model shows great differences. Some years enjoy the benefits of the demographic changes, others pay for this. Redistribution of the wealth in this model is expensive, which emphasizes that the effects are large.

1.3 Thesis outline

This research focuses on the Dutch pension system. It is divided into three chapters. In Chapter 2 we will present some aspects of pension funds and pension schemes. We will discuss the three pillar system that is used in the Netherlands and describe the most common types of pension policies. We observe that the funded defined benefit pension plan based on average wage is the most used pension plan in the Netherlands. In the same chapter we will describe the new Dutch pension act, the investment returns and funding ratios, the influence of the interest rate, and the demographic developments. This shows that pension funds have changing circumstances in which to work, which lead to changing outcomes.

In Chapter 3 we will start to investigate intergenerational risk-sharing in a stylized economy. The economy is based on an overlapping generations (OLG) model. Many assumptions about the investment returns, the interest rate, and consumer utility will be discussed. In this economy we design the DB pension model, using risk allocation rules that allow for IRS. The certainty equivalent consumption (CEC) will be our main interest, and we will optimize this quantity. The optimal parameters show interesting results. The investment strategy shows a 100% allocation to risky assets. Furthermore, the premium and benefit levels are expected to rise over the lifetime of the participants, due to the risk allocation. The optimal parameters give a very high CEC, which is greater than the CEC for an optimal individual model. The CEC is also higher than in restricted DB pension schemes.

In the same chapter, we also highlight some important numbers. We show that the risk allocation rules in the optimal scenario arrange that a surplus, positive or negative, is distributed in 14.5 years. This is not in line with the current regulations. We also see that the number of persons in the workforce for each retiree is equal to 2.67. This is not far from the true value at the moment, but this number is decreasing and that should be taken into account. This will be one of the subjects of the following chapter.

In Chapter 4, we will address some of the assumptions made in the previous chapter. We put our pension model is a more realistic environment. We will focus on adapting the demographic assumptions, such as the generation sizes and the life expectancy. We will start by studying a stylized case of the baby boom, where we will show different effects of the generation sizes. For the changes are different causes, i.e. the changing group over which to share risks and changing liability levels. We will finally introduce the complete Dutch demographics in our model. Next to the baby boom, we then also observe a decreasing trend upwards in the total population, and an increasing life expectancy. We will adjust our pension model, in order to capture all these differences in comparison to the previous chapter.

The new environment has large effects on the performance of our model. As in the case of the baby boom, the CEC for different cohorts changes a lot. The effects are in some ways comparable, but much larger. Interesting to see is the interaction between different changes. The increasing life expectancy, the aftermath of the baby boom, and other developments push the CEC above the average level for some generations, and below it for others. We will propose two ways to avoid this. These examples show that the effects are significant, and the costs of repairing this externally are high.

All in all, the demographic environment disrupts intergenerational fairness in the economy under study. The results we present in this research show that we need big adjustments to our current pension model to achieve intergenerational fairness. We therefore emphasize the need for a model that corrects for demographic effects in the risk allocation parameters. We will discuss more ideas for further research in the conclusion in Chapter 5.

Chapter 2

Pension funds and pension schemes

In this first chapter, we will present some aspects of pension funds and pension schemes. We will discuss the three pillar system used in the Netherlands, and describe the most common types of pension policies. We observe that the funded defined benefit pension plan based on average wage is the most used plan in the Netherlands. This is the most interesting pension fund to look at when studying intergenerational risk-sharing, as argued by Cui, De Jong, and Ponds (2008) and Gollier (2008). Here we will also describe the new Dutch pension act, the investment returns and funding ratios, the influence of the interest rate, and the demographic developments. This shows that pension funds have changing circumstances in which they need to work. In the following chapters we will show the effects of the circumstances.

2.1 Pension schemes

A pension is an income for people who do not receive a regular income from employment due to old age. Often there are provisions for the widow or widower of the participants, or for the orphans. Various special arrangements can be put in the pension contract for e.g. military service, pregnancy, and short unemployment. In the United States, pension schemes are usually called retirement plans. There are many different forms of pension plans. In this section we describe the five most common pension schemes, as discussed in Kakes and Broeders (2006). First, the difference between a funded and an unfunded pension plan is explained.

2.1.1 Funded and unfunded pension schemes

In a funded pension plan, the pension benefits are paid from the premiums that have been received in the past. These premiums, paid by the participants of the pension, are gathered in the pension fund. This amount of money is invested to generate returns. These assets, together with the investment return and the future premium payments, should be able to pay for the future benefits. In other words, the assets should match the liabilities. The investment returns are random. Pension funds often keep on more capital than is required to exactly pay the benefits, in order to take the investment risk into account. For this capital requirements regulations apply, and below we will discuss the situation in the Netherlands. The status of the funding is given in the funding ratio. This is defined as the total assets divided by the total discounted liabilities. For a funded pension scheme, this ratio should be somewhere around one or 100%. Examples of pension schemes that are (fully) funded are given below.

When a pension scheme is unfunded, the benefits are paid for by the current contributors to the pension fund. There are some countries in which companies run such a pension plan, but often this is less attractive due to tax rules. In some countries it is even forbidden to have an unfunded pension scheme, e.g. in Australia and the Netherlands. Therefore almost all privately held pension funds are funded. The best examples of unfunded pension policies are national pension funds run by governments. Most developed countries in the world have an unfunded national pension arrangement provided by the state. In this method of financing, the benefits for the current retirees are paid directly from taxes and contributions from the current workers. The government retirement plan in the USA and most European countries, including the Netherlands, is financed using this method. The unfunded method is also known as the pay-as-you-go (PAYG) method.

Individual defined contribution

Within an individual defined contribution (DC) pension scheme, individuals save for their own pension benefits. It is a fully funded pension strategy. During the working period, the participant will contribute to a fund. All the risks, such as investment risk, inflation risk, and longevity risk, are borne by the participant. When the person reaches the pensionable age, the assets of the fund at that time are converted to a benefit stream. This pension benefit stream is often bought at an insurance company in the form of a life annuity. From the time of retirement onwards, all the risk is borne by the insurance company instead of the participant.

The main advantage of this pension scheme is the fact that the premium can be kept at a constant level. This guarantees a high degree of stability over the years. The main disadvantage of this scheme is the fact that all the risk is borne by the participant individually. Although this can be a good thing when stock returns are good, the downside should not be neglected. More uncertainty comes from the volatile interest rate. When converting the assets into a life annuity at the pensionable age, the interest rate determines what the price of an annuity will be. Hence, for the participant this is a major source of uncertainty for their future benefit level. Another disadvantage is the costs of this arrangement. Due to the fact that it is an individual scheme and because there is often much choice in e.g. the risk profile, the costs of maintaining this fund are high. In Ambachtsheer (2005) we read that the costs of an individual DC arrangement in the USA can cost as much as 2.5% of the total wealth, were a collective DC scheme only costs 0.4%.

When individuals or their employers choose for an individual pension scheme, it is often the case that the participants do not fully realize how much money they will need in the future. In countries where the amount is determined by the participant only, it often occurs that he or she is underinsured. When reaching the pensionable age, this becomes painfully clear and may even mean the pension should be postponed and the person has to continue working. This is also a disadvantage of this type of pension plan and the last we will mention here. We will continue with the collective DC plan, which eliminates some of the disadvantages.

Collective defined contribution

The collective defined contribution pension scheme has the same fixed premium as the individual scheme, but the risk is carried by the participants before and after retirement. How this is arranged differs between the various particular plans, according to the predefined rules in the pension contract. The nice aspect of this plan compared to the individual plan is that the risk is spread out over more participants. For e.g. the longevity risk, this means we can average out the life expectancies and therefore reduce the risk. The inflation risk and investment risk can be spread over many generations.

This is a type of pension plan that can exhibit intragenerational risk-sharing and IRS.

These risk-sharing properties are some of the advantages. A constant premium is the same advantage as observed with the individual plan. It is often the case that the premiums are paid by the employer of the participant. For a company, the constant premium is even more interesting. The fact that almost all risk is borne by the employee is also an advantage for the company. For the participants, the benefits of the collective fund over the individual scheme are that risk is shared amongst participants. The costs, as we mentioned above, are also lower.

The disadvantages contain the same as the individual case. The higher costs do not apply here, although a defined benefit scheme is often less expensive. A disadvantage for the sponsoring company can be that when returns are low, their employees will be worse off. This is not in the best interest of a corporation, and it might be necessary to make a donation to the fund. In this way, some risk is still carried by the employer.

Notional defined contribution

A notional defined contribution (NDC) pension scheme is a pension scheme that is not fully funded, and has characteristics of a PAYG policy. The premiums of the participants are used to finance the benefits of the current retirees. Next to these payouts, the premiums are used to build up a fictional or notional fund. The fund is not invested, but indexed to obtain a higher value for the benefits. The indexation can be used to manage the fund, affecting both active and passive participants. At the pensionable age the fictional fund is converted into a life annuity as we saw before.

Advantages of this type of scheme are that it can exhibit IRS. It also has intragenerational risk-sharing, for e.g. the longevity risk. This comes from the fact that is has individual as well as collective properties. We also see that participants in this fund are not exposed to investment risk, since it only uses indexation. However, this may also be one of the disadvantages for the participants, since thes indexation can be adjusted, also when the participants are retired. The second disadvantage is that the PAYG scheme leads to discontinuity risks. When new entrants to the fund stop to come, there will be no premium out of which the benefits can be paid out. In some cases, this forces the pension fund to stop, and then it is not able to pay out benefits anymore. This is one of the reasons why some countries forbid such a scheme. In the Netherlands, the law stipulates that employee pension funds should be fully funded, and hence we do not see a NDC scheme here. Examples of countries that do allow such policies are Sweden and Italy.

Collective defined benefit average pay

In a collective defined benefit (DB) average pay pension plan, participants save collectively for a pension that is paid out as a life annuity. The main difference between DB and DC pension schemes is the moment at which the investments are converted to the life annuity. For a DC plan, this happens at retirement, where in a DB plan arrangements are already made at the beginning of participation. The life annuity in this particular form of DB plan is connected to the average pay. This means that people who make promotions early in their working life benefit from this. They also pay the most premium, since this often is a percentage of income. Hence, the participant, or the sponsoring company, pays for this extra pension income as well. The financing can theoretically be managed in a PAYG way, but in almost all cases these funds are fully funded.

In the first instance, depending on the precise contract rules, the pension fund is responsible for inflation, longevity, and investment risk. However, when for example returns are bad, the fund can raise premiums for the participants, ask the sponsor (most often the employer) for a contribution, or even adjust indexation. A combination of all these measures also is possible. Due to the fact that it has so many possible ways to manage the fund, such a pension policy can utilize IRS in a good way. We also observe good possibilities for intragenerational risk-sharing. In the fund, participants, future participants, and sponsors pay for the risks.

The first advantage of such a fund is that it can utilize IRS in a good way. The many generations in the fund are often also quite large, which also ensures the risk spreading of longevity risk. For the employer, benefits are that the premiums are most often a constant fraction of the salary, adjusting for the franchise. We discuss the franchise below. In this way, all participants pay for their own pension, which is very fair. For the employee the advantage is that he knows what he will get when retired, not depending on changing interest rates anymore. This can help his financial planning.

Disadvantages of such a pension scheme are the fact that it is a quite complex fund. This makes it complex to manage, since many risks are for the account of the fund, such as the longevity risk. When better information about mortality becomes available, the pension funds have to react to this. See Section 2.2 for more information on this. There we also see that a fund has many regulations to follow, but this is also the case for the DC funds.

In Ponds and Van Riel (2007) we see that the average pay DB pension plan is the most common pension plan in the Netherlands, with almost 75% of the total market under the active participants. The next DB pension plan we discuss below, based on final pay, was always the dominant pension type, with around 60% of the market in 2000. However, as argued by Ponds and Van Riel, this dropped to just over 10% in 2005 as a reaction to the fall in funding ratios at the beginning of this century. Nevertheless, unlike the rest of the world, the DB pension plan is still the most used scheme in the Netherlands. Over 90% of all policies of active participants are of this form. In other countries the shift from DB to DC was made due to the problems occurring due to the falling funding ratios, but this is clearly not the case in the Netherlands.

Collective defined benefit final pay

The DB final pay pension scheme has the same characteristics as the DB average pay plan. Instead of the average pay, the benefit level is based on the last earned income. In this way, it does not matter how fast you make a promotion. If you end up at the same level as someone who makes career much faster, you still end up with the same pension. This may look fair, but as mentioned before, the person who makes career faster also pays more premium. This kind of intragenerational risk-sharing is actually passing the costs of certain pensions on to others. It does make sure that the plan uses indexation for welfare by definition. It cannot suspend indexation for the active period, which has consequences for the management of the fund.

Advantages are the same as for the other DB plan. Additionally, we find the welfare improvement by definition and the fact that we do not have to worry about the speed in which to make career. This makes that the pension liabilities can rise sharply at the end of the working life of a participant. That makes the fund more difficult to manage, since liabilities are difficult to predict exactly. It also reduces the ways in which the fund can make up for bad returns.

This concludes our discussion of the different types of pension schemes available. These are the most common types of pension policies. We saw that a pension fund can be characterized by the way it is funded, i.e. fully funded or PAYG, or by the way the benefits are determined. A DC pension scheme can be divided into a collective or an individual scheme. We also saw a non-funded example in the NDC plan. The DB pension plans can be divided in the ones based on average pay and the ones based on final pay. In the next section, we will discuss how these schemes are used in the Netherlands.

We will treat the three pillar system, and show how important employee pension funds are in the Netherlands.

2.1.2 Pension system in the Netherlands

In the Netherlands, as well as in most developed countries, we have a pension system that is based on three pillars. The first pillar is the state pension, regulated in the Old Age Pension Act (AOW)[1]. It is the basic pension everybody[2] over the age of 65 receives. It is comparable to the Social Security in the US. The government pays this basic pension from the current premium incomes paid by the working generations in the forms of taxes and contributions. As mentioned before, it is a PAYG pension policy. The height of the benefit level is depending on your social status. If you are single or living alone, you receive €1,048.09 before taxes. If you have under-aged children, this increases to €1,321.13. When you are married or live together with a partner, you will receive €730.64[3].

The second pillar is an additional pension that you are obligated to save for when employed. Every employed person has a compulsory membership of the company's pension plan or the pension plan for his or hers particular industry or profession. This pension plan can have one of the forms described above and it is usually fully funded. The premium is connected to the wage of the participants, and the employer pays most of it. In the premium calculations, the first pillar pension is taken into account. This is called the franchise, and can be different for the various contracts. The income minus the franchise is the pension basis, on which the premium and later benefit level is based.

The third pillar of the old age provision consists of individual extra arrangements. Some persons do not find the first two pillars enough, or just want to decide as much as possible for themselves. You can do this in the third pillar pension arrangements. This is not obligatory. Most often third pillar contracts are arranged by insurance companies and take the form of a life annuity. These products are often tax deductible vehicles.

The relative size of the pillars and more information about the types of arrangements is discussed in Kakes and Broeders (2006). The relative sizes of the three pillars change throughout time. The first pillar is and always was the most important pillar, covering the biggest part of the yearly payments. However, in the past this pillar covered more than half of the total pension, but this is not the case anymore. The private second and third pillar became much more important. For the remaining yearly payments, the second pillar always covered the larger part, but this is now paid by the third, individual pillar. We also see that 85% of the pension schemes are arranged in an industry pension fund, and 14% by company funds. A percentage as big as 97% of the policies are defined benefit pension schemes. 79% of all pension plans are DB schemes based on average pay with indexation. Only 3.2% are DC schemes.

The Dutch pension system is one of the most elaborate in the world. The total investments in 2007[4] were €740 billion, which is more than the GDP. Over 90% of the working population is covered, which makes this a quasi mandatory system according to the OECD. In 2007 there were 713 pension funds.

Taxation levels depend on the type of pension scheme. The benefit levels will also change when accrual rates change. We give the figures for the most common DB plans. Final pay plans have a

[1] "Algemene Ouderdomswet" in Dutch.

[2] There are some restrictions for the AOW. You have to have lived in the Netherlands from age 16 to the moment you will receive these benefits to get the maximum amount.

[3] These are the monthly amounts for 2009, which can be found on `www.svb.nl`.

[4] Source: OECD Pension Country Profile: Netherlands, extracted from the OECD Private Pensions Outlook 2008.

maximum accrual rate of 2% per year, leading to a 70% replacement rate in 35 years. Average pay plans have a maximum of 2.25%. When the benefits succeedingly exceed 100% of the final pay, the surplus is taxed progressively.

This ends our discussion about the Dutch pension system and pension systems in general. In the next section, we will discuss some developments in the pension world that influence pension funds in the Netherlands and abroad. These developments are important for our research, but we will not take all of them into account. However, our research aims to help understanding the effects of different developments, such as the introduction of new regulations.

2.2 Regulations and developments in the Dutch pension sector

In this section, we will discuss some developments in the pension world that influence pension funds in the Netherlands and abroad. These developments are split into two subsections. In the first, we will discuss new regulations for the pension funds that is introduced or will be introduced in the near future. In the second subsection, we will describe some of the current developments that are important for our research. We will finish by discussing which developments are taken into account in this research, and which are not.

2.2.1 The new Pension Act and FTK

In the Netherlands, the Pension Act[5] was introduced in 2007. Due to the introduction in stages, this will be finished in 2009. The old law[6], dating back to 1954, was adjusted many times and finally replaced by this new law. In this law, the duties and responsibilities of employers, employees, and pension administrators are defined. The requirements of a pension contract are defined in the law, e.g. it is required that such a contract is placed at a recognized pension fund or pension insurer.

The new pension act is introduced to provide more transparency and certainty for the participants. It therefore contains regulations to improve the information stream to participants and imposes restrictions on the funding level. Most of the restrictions are defined in the Financial Assessment Framework[7] (FTK), which is part of the pension act. According to the Dutch Central Bank (DNB): "The Financial Assessment Framework is the part of the Pension Act that lays down the statutory financial requirements for pension funds." For more information we refer to the website[8] of DNB. We summarize some of the rules here.

One of the most discussed new rules is the fact that pension funds have to valuate their investments and pension obligations using market prices. For the investments, this was already the case, and the prices for e.g. stocks and bonds are readily available. However, the obligations were always discounted using a fixed actuarial discount rate of 4%. The new rules mean that these should now be discounted using the current nominal term structure of interest rates. This term structure is provided by the central bank, and has a direct influence on the funding ratio. The liabilities will change when this term structure changes. The method is called the fair value accounting method.

The market value of the obligations must be fully covered by the market value of the investments at all times. There are specific rules for this funding. DNB requires that "a pension fund must have

[5] Nederlandse Pensioenwet.

[6] Pensioen- en spaarfondsenwet.

[7] Financieel Toetsingskader.

[8] `http://www.dnb.nl` or `http://www.dnb.nl/openboek/extern/id/en/pf/41-194653.html` for the FTK regulations.

sufficient own funds to ensure, with a confidence level of 97.5%, that the value of the fund's investments will not be less than the level of the technical provisions within a period of one year." We thus discern two funding levels. The regulatory own funds (approximately 105% funding ratio) takes the risks of the specific funds into account. The minimum regulatory own funds (100% funding ratio) does not take the fund's risk into account. When a pension fund's assets drop below the first, we talk about a reserve deficit. When it drops under the minimum regulatory own funds, we talk about a funding shortfall. The minimum regulatory own funds requirement is defined as the lower limit of the regulatory own funds.

When a pension fund has a funding shortfall or reserve deficit, it must draw up a recovery plan. For a reserve deficit this should happen within three months, and the plan can be long-term. The reserve deficit should be steadily eliminated within fifteen years. When a fund reaches the more severe funding shortfall, the recovery plan must be ready within two months. It must outline how the shortfall will be recovered in three years time, and is must show three things. The likelihood of recovery must improve, the risks for entitlement beneficiaries and pension beneficiaries should not increase, and the likelihood of granting additional rights may not be adversely affected. When these three requirements are not met, the recovery should take place within just one year.

In this research, we do not take these new rules into account. The main reason is that these will make the model more complex. Making the model compliant with all these rules requires additional programming code. We already have many simplifications in our model and we would like to look at other factors of the pension model. Another reason is that the research focuses on the theoretical model for a DB pension scheme and IRS, and searches for the optimal scenario. We will show some general effects in our model, but we also show some shortcomings. This may help further research with the development of a model that is compliant with all the rules. It can also help policy makers when they are designing new pension regulations, and they may take the results of this research into account.

2.2.2 Current developments

In the recent past, many developments took place that had a major influence on pension funds. New regulations were implemented, stock markets declined very much twice, and the demographic developments became clearer every day. The consequences of these developments are often subject of discussion. The media devote much attention to these subjects, since it concerns the entire population. We will therefore treat the most important developments here shortly.

Investment returns

Stock developments often have a big impact on pension funds, since these funds invest an increasing part of their assets in stocks. According to Kakes and Broeders (2006), the investment in stocks and bonds rose from 23% in 1985 to 88% in 2005. This means that pension funds are taking more risks in the investments of their assets. When stock markets decline, this hurts the pension funds in their funding ratio. We saw this shortly after the turn of the millennium, when the markets collapsed. The internet bubble burst and caused stock markets to go down.

This gave way to a lot of new regulations, such as the new pension act and the financial assessment framework discussed above. Just when the ratios where stable again, another crisis hit the financial world in the latter half of 2008: the credit crunch and the subsequent deep financial crisis. It is the worst financial crisis since the depression of the 1930s. This again led to a fall in funding ratios.

This fall in funding ratios was even worse due to the financial assessment framework, which demands that the liabilities are calculated using the nominal term structure of interest rates. The crisis caused the interest to go down to historical low levels. This meant that, using the new fair value accounting method, the funding ratios were even lower. Where the assets went down due to the crises, the liabilities went up due to a decreased interest rate.

The interest rate

This interest rate is a risk in itself. In the 1990s, large hidden reserves became clear because the interest rate was at a very high level. At the same time, the actuarial discount rate of 4% was still used. The increased reserves were used to link the indexation of the benefits to wage inflation. However, the introduction of the fair value accounting method and a lower interest rate let the excess funding disappear again. Indexation is a nice feature of a pension fund, but calculations by the Netherlands Bureau for Economic Policy Analysis[9] (CPB) and other institutions showed that only 75% of the indexed pension liabilities were covered at the end of 2002: see Van Rooij, Siegmann, and Vlaar (2004). As mentioned before, the fair value accounting method amplifies the shocks in the interest rates. This again became clear in the financial crisis at the end of 2008. The underfunding of many small pension funds caused a wave of mergers and acquisitions in the Dutch pension system. The underfunding also resulted in increasing pension contributions, indexation adjustments, and a switch from DB pension schemes based on final pay to DB average pay plans. See for a discussion of these effects again Ponds and Van Riel (2007).

Demographic developments

The last developments we will mention here are demographic developments. After World War II, two major developments took place. First, there was a baby boom after the war, reaching its peak around 1965. This is one of the causes that the sizes of different age cohorts differ a lot at this moment. See Chapter 4 for the exact figures. Many persons which are part of this baby boom are currently working, but will retire within a few years time. This will increase the pressure on the current workforce. One example is given by the largest pension fund in the Netherlands, the ABP. The relative share of retirees in total liabilities will rise from under 40% in 2004 to around 70% in the year 2024. Since the main instrument to adjust the funding ratio are the premiums, this is a problem for future generations. The possibilities for intergenerational risk-sharing are affected.

Another demographic development is the rising life expectancy. People had a life expectancy of 71 in 1950, this number lies around 80 years in 2009. It is expected that this number will even further increase to 84 in 2050. This again increases the pressure on the working generations, since the number of retirees per worker will further increase. We will come back to this in Chapter 4. The consequences are visible in the second pillar pension funds such as the ABP, and again has consequences for IRS. The first pillar pension, the state pension, is also influenced by these developments. In fact, the problems there are even worse, since this is a non-funded PAYG system. The currently working generations pay for the retirees. This means that in the past years the pressure increased. This was first dampened by the increased working group due to the baby boom. However, the life expectancy is still rising and the baby boom will very soon amplify this effect instead of reducing it. This led to serious discussions amongst policymakers. Some politicians want to increase the pensionable age to reduce the effects and keep the state pension payable. Others are fiercely against these measures and want to finance the pension in another way.

[9] Centraal Planbureau.

In the Dutch media there was a lot of attention for the pension funds. For example, the chairman of the APG, the fund that manages and invests the funds of the ABP, wants to adjust the rules for pension funds after the financial crisis[10]. He thinks that the current rules trigger a reaction at participants, supervisors, and fund managers that is disproportional. The funding ratio fell from 144% in 2007 to well under 100% on average at the end of 2008 and the beginning of 2009. The four biggest pension funds in the Netherlands together had a funding ratio as low as 90%. The pension funds were all over the news with these low rates and had to hand in a recovery plan on the first of April 2009. However, on 16 September 2009, the same financial newspaper reports that on average, the funding ratio reached 103% with the help of rising stock prices. The chairman of the APG therefore argued that the long term horizon of the pension funds and the short term of the current regulations are not in accordance with one another.

2.3 Summary and conclusion

In this chapter, we presented some aspects of pension funds and pension schemes. We focused on the Dutch system, which is a three pillar system, with government, employee, and individual pension schemes. We described the five most common pension schemes, which are mostly funded, but unfunded so-called PAYG schemes also exist. The funded defined benefit pension scheme is mostly used in the Netherlands. Within the DB schemes, the currently most used type is based on the average wage, where this was the final wage in the past.

Pensions are currently a must discussed topic, due to changing regulations, demographic developments, and the stock market crises. We discussed the new Dutch Pension Act and the Financial Assessment Framework. The fair value accounting method is the most important new rule stated in the new framework. We also discussed some developments in the investment returns and funding ratios, the influence of the interest rate, and the demographic developments.

This chapter shows that pension funds are always interesting to study. The circumstances in which they operate continuously change. Changing conditions lead to very different results. The circumstances will be the subject of the following chapters in this research. We will take some of the developments of this chapter into account. In Chapter 3, we will treat intergenerational risk-sharing in a stylized setting. In Chapter 4, we will change the setting in which this model operates, taking the demographic developments into account. We thereby look at the effects of the changing cohort sizes and the effect of the life expectancy. We will not study the effects of a financial crisis, raising the retirement age, the volatile interest rate, or the regulations in this research.

[10] B. Zevenbergen, *Het Financieele Dagblad*, 26 August 2009.

Chapter 3

Intergenerational risk-sharing

Risk-sharing between generations is often a point of discussion when stock returns are low and funding ratios decline. Governments impose restrictions on pension funds when they are underfunded, in order to protect participants. This is a fairly short term vision, while pension funds have a very long term horizon. In this way, governments reduce the opportunities of pension funds to use intergenerational risk-sharing.

In this chapter, we will look at the optimal way a pension fund should apply intergenerational risk-sharing. We do not take financial regulations into account. In this way, we will try to find a global maximum in our model, not restricting the parameters. This is supported by Gollier (2008), in which it is argued that the regulations of the government lead to a second best optimal solution. The first best solution in his research is found when these rules are set aside.

The same conclusion can be inferred from Cui, De Jong, and Ponds (2008). In this article, the authors do not look at the probability of underfunding, but just look at the optimal way in which to allocate the downward (and upward) shocks to the assets of the fund. They find that it is better to share this over many generations than to restore the insolvency fast. Pension funds can use their funds to smooth the returns on the portfolio for their clients. They can do this in a better way than an individual could do this. A DB pension plan can also do this more efficient than a pension fund with less flexibility than an individual, like a defined contribution plan which uses a fixed contribution.

In this chapter, we will look into the model of Cui, De Jong, and Ponds (2008). The research is carried out here again and the results are interpreted. Our study will focus at the defined benefit pension scheme, since this is the most widely used pension plan in the Netherlands. It is also one of the pension schemes that is capable of utilizing the benefits of IRS. For the other policies, we refer to the original research. We chose this pension model because it is a simple model. The model gives a good representation of reality, and the entire process is modeled accurately. It is also a quite recent research, since it was part of a dissertation and promotion in 2009. New is the direct application of IRS in the pension scheme. It becomes immediately clear how much of surplus is recovered within a year, and it can also be calculated how long it takes to recover on average. In this research we will use the model and program it ourselves, which yields different results.

Another reason to choose this model is that it can easily be extended. In the following chapter, we will use this to incorporate the real demographic conditions. The model can be adjusted to be as realistic as necessary. New inputs will test whether the results found in this chapter will be directly applicable to reality. If this is not the case, we would like to know where the major adjustments should be made. We will show that the differences between the results in this chapter and the following will yield a few aspects that need further research.

In this chapter, we will define the economy in which this model works in the first section. The pension fund and its characteristics are defined in the section after that. We will thereafter look at the optimal risk allocation. A summary and conclusion will end this chapter.

3.1 The economy

In this research, we will use and extend the model found in Cui, De Jong, and Ponds (2008). In particular, we will adopt and research the model for the defined benefit pension fund. In their paper, Cui et. al. show that this model is the optimal way in which to invest pension assets. Another reason to adopt this specific model, is that we are able to manipulate the toy economy it works in. Some limitations of our model are discussed throughout the following research and in the conclusion. In the simple toy economy a lot of assumptions are made. To test whether our results are still optimal in a more realistic economy, we will alter some of these assumptions in the following chapter. The main adjustment will be in the generation sizes to allow for different demographic developments. See Chapter 4 for the results. We will now start with an explanation of the model used.

3.1.1 OLG models

In our research, we use an OLG model to represent the economy. The concept of an economic model containing overlapping generations is made popular by Samuelson (1958). Another early example can be found in Diamond (1965). Our model as well as the model presented in Gollier (2008) uses overlapping generations. An overlapping generations (OLG) model consists of two or more generations, which (partly) overlap one another. It is a very popular way of simplifying an economy with certain characteristics to which agents in every generation will be exposed. The lifetime utility of the agents is a function of their consumption in all periods.

In the model we use, the agents enter the economy at age of 25, when they are assumed to enter the workforce. They will work until they reach the age of 65, at which they retire and live on. Eventually every agent will die when he or she reaches the age of 80 years. These are the assumptions we will work with. They are not far from the real values, since the current life expectancy in the Netherlands is around 80 years[1]. People may join the workforce a little earlier than at age 25, but the assumption of 40 years of work before retiring is common in the Netherlands. Hence, the assumptions are reasonable for the current Dutch population.

3.1.2 Simple OLG model of the economy

In this section, we will explain the model and the assumptions we used. The model, based on Cui, De Jong, and Ponds (2008), is generalized here where possible, in order to be useful for other assumptions about the economy as well. We will adjust some of these assumptions in the next chapter. There we will discuss some other adjustments needed as well.

In order to study the effects of IRS, we use an OLG model. In this model, let t denote time. All agents in the model are assumed to start working at age 25 ($t = 0$). R denotes the retirement time in the model. Hence, after R years of work, they will retire at age $25 + R$. We set R to be 40 years in line with the above, which means the age is equal to 65 ($t = R = 40$). The person will eventually die at the end time T. We assume a person dies at age 80, hence T is set to 55 ($t = T = 55$). When an agent in the economy is working, he or she earns a flat real income of 1. In our economy, all amounts

[1] CBS, *Statistics Netherlands*, 2009.

are expressed in real terms, and wage inflation is assumed to equal price inflation. When an agent is retired, he or she will receive a pension. It depends on the way how this is invested how high this amount is. It will either come out of self-invested funds or out of funds invested in and managed by a pension fund. This can be in the form of a defined contribution or a defined benefit pension fund. In this research, we assume that the pension wealth will be managed in the form of a defined benefit pension fund. In the next section we will treat the different possible pension schemes and the choice for the DB pension strategy.

In this model, every individual receives an income of 1 for $0 \leq t < R$, which he will spend on two things: retirement savings in the form of premium, p_t, and consumption, c_t. After retirement, the consumption will be equal to the pension benefits, $c_t = b_t$ for $R \leq t < T$. We assume constant relative risk aversion (CRRA) for the individuals, which yields a power utility function as a function of wealth W and the risk aversion parameter γ:

$$u(W) = \frac{W^{1-\gamma}}{1-\gamma}. \tag{3.1}$$

An individual in our model will maximize its total utility, denoted by U. Let δ denote the subjective discount rate, and let γ be the risk aversion parameter for the individuals in our economy. We then get that the preferences of each individual in our economy are defined by

$$U = E\left[\int_0^T e^{-\delta t}\frac{c_t^{1-\gamma}}{1-\gamma}dt\right]. \tag{3.2}$$

The values for δ and γ are set to be equal to $\delta = 4\%$ and $\gamma = 5$.

For the investment opportunities, we assume there are just two assets traded in our economy, a risky asset and a risk-free asset. At time t, The pension fund invests a fraction ω_t of the total wealth W_t in the risky asset and $1 - \omega_t$ in the risk-free asset. We use a non-stochastic interest rate r for the risk-free asset or bond. The dynamics of the risky asset, or stock, and the bond are driven by the well-known Black-Scholes model. Let Z_t be a Brownian motion, that is $Z_0 = 0$, Z_t is almost surely continuous, and the increments follow a normal distribution, $Z_t - Z_s \sim N(0, t-s)$ for $t \geq s$. $N(\mu, \sigma^2)$ stands for a normal distribution with mean μ and variance σ^2. The stock or risky asset is denoted by S_t and the bond or risk-free asset by B_t.

The dynamics of these two product are now given by

$$dB_t = rB_t dt, \tag{3.3}$$

$$dS_t = \mu S_t dt + \sigma S_t dZ_t. \tag{3.4}$$

These equations are actually short for the integral equations

$$B_t = B_0 + \int_0^t rB_u du, \tag{3.5}$$

$$S_t = S_0 + \int_0^t \mu S_u du + \int_0^t \sigma S_u dZ_u. \tag{3.6}$$

The development of the stock and bond prices determine the total wealth in the portfolio, described by W_t. We find that the total wealth is described by the differential equation and the integral equation below:

$$dW_t = (r + \omega_t(\mu - r))\, W_t dt + \omega_t \sigma W_t dZ_t, \tag{3.7}$$

$$W_t = W_0 + \int_0^t (r + \omega_u(\mu - r))\, W_u du + \int_0^t \omega_u \sigma W_u dZ_u. \tag{3.8}$$

The default (real) values for the trend μ and volatility σ are taken to be $\mu = 6\%$ and $\sigma = 15\%\dot{T}$o find out whether these values are reasonable, we checked these values against the S&P 500 for the past 50 years. Investigating the yearly returns of this index, we find the values $\mu = 0.0540$ and $\sigma = 0.1574$. Because these values are very similar, and to be able to compare our results with other research, we keep these default values. Comparable values are also found in Cocco, Gomes, and Maenhout (2005). For the same reasons, and for simplicity, we assume that the real interest rate is constant and equal to $r = 2\%$.

The certainty equivalent consumption

Using this model, we will assess the impact of different pension plans in different circumstances. We will do this by means of the certainty equivalent consumption (CEC). This will be our measure to help decide which pension setup is performing best in our economy. In the next chapter, it will also give an indication of the impact of changes the assumptions, for e.g. the demographic conditions and the life expectancy. We will also use this measure to compare the expected welfare of different generations.

The CEC follows from

$$U = \int_0^T e^{-\delta t} \frac{CEC^{1-\gamma}}{1-\gamma} dt. \tag{3.9}$$

The CEC is the amount of money a person should get each year of his life with certainty to obtain the same utility of the random amount under the specified pension rules. This starts at age 25, and continues into the pensionable ages, up to the age of 80, or the life expectancy. Since a person earns money only in the first 40 years, we expect this quantity to be not higher than the income[2], which is normalized to 1. The CEC is easy to calculate using a computer program.

3.2 Pension scheme

In this research, we will focus on one pension scheme, the defined benefit pension scheme. We chose this pension strategy since it is shown to be the optimal pension scheme, capable of handling IRS. See again Cui, De Jong, and Ponds (2008) and Gollier (2008). The choice is also motivated by the fact that most pension plans in the Netherlands have this form, see Chapter 2 and Ponds and Van Riel (2007). In this paper, Ponds and Van Riel also discuss the switch from DB pension plans based on the final salary to DB plans based on the average wage. This switch is not common in the rest of the world. In e.g. the US and the UK we see a switch to DC plans, which do not allow for IRS. When IRS turns out to be profitable, the switch in the Netherlands is more fortunate than the switch to DC plans we observe abroad. Hence, we would like to see the effects of IRS in a DB pension plan. We would like to study the current situation in the Netherlands, and that is another reason we choose for a DB pension plan. We will not discuss average wage or final salary plans, since in our model the salary remains constant throughout the lifetime of the individual. We do not choose one of these policies. We use an actuarially fair benefit level, based on the premium paid. This premium depends on the income, which we will now discuss.

[2]Due to extremely high stock returns, the CEC can be greater than 1, since the benefits can be very large or the premium level negative. Also due to demographic changes, some generations will profit more from the investment return than others. We will not elaborate on this here, see Chapter 4.

3.2.1 Defined benefit pension scheme

In a DB pension scheme, we have a fixed benefit level for the participants when they retire. For this benefit level, the participants pay a premium each year. In this research, we do not consider the costs of the pension fund that might be added to the premium. The benefit and premium level can also be expressed in terms of respectively the replacement rate and the contribution rate. The contribution rate is the fixed fraction of the labor income during the working period that defines the height of the pension benefits. The replacement rate is a measure of effectiveness of the pension benefits to replace the income during the working life. Denote the benefit level by b, and the premium level by p. When y denotes the income of the participants, we define the contribution rate as p/y. Similarly, the replacement rate is defined as b/y. In our research, the income is normalized to 1, and hence we find that the contribution rate and the replacement rate equal respectively the premium and benefit level.

Under IRS it is the question whether the premium and the benefit should have a one-to-one relationship, but for the moment let us assume they have. It is actuarially fair that every generation pays for his own pension, and hence we find the following relation:

$$\int_0^R e^{-rs} p ds \quad = \quad \int_R^T e^{-rs} b ds. \tag{3.10}$$

Hence, in this way we choose a fixed actuarial discount rate of 2%. Note again that this is a real rate. This discount rate was previously set to 4% in the real world and in nominal terms. With an inflation rate of around 2%, these values are close together. Currently, pension funds have new rules that state that they cannot work with these fixed actuarial discount rates as discussed in Chapter 2. However, to keep the model simple, we maintain the assumption of a fixed discount rate. This means we calculate the liabilities using a discount rate of 2%. In our model, this is the minimal expected return, since we invest in the risk-free assets returning 2% and in the risky asset with an expected return of 6%.

We will choose the premium level with optimization. This optimal premium level determines the optimal benefit level via Equation (3.10). We mentioned that the benefit and premium levels are fixed in a defined benefit pension scheme. However, this will not turn out to be totally true, since we have risk in the investment of the premiums to deal with. In one of the following sections, we will explain the risk allocation rules in this pension fund. These rules allocate risk to the participants that are either paying premiums or receiving benefits, which means that the benefit level as well as the premium level can change over time.

Premiums will be invested to generate returns and pay out benefits. The invested premiums and returns constitute the assets of the pension fund. The liabilities are equal to the amount of money the fund needs to set aside now to be able to pay all the benefits it is obliged to, minus the premiums the fund will receive. All amounts are discounted using the interest rate of 2%. In the current setting, with constant demographics, wages, and benefit levels, the liabilities L are invariant over time, hence $L_t = L$ and

$$\begin{aligned} L = & \int_0^R \left(\int_R^T e^{-r(t-x)} b dt - \int_x^R e^{-r(t-x)} p dt \right) dx \\ & + \int_R^T \left(\int_x^T e^{-r(t-x)} b dt \right) dx. \end{aligned} \tag{3.11}$$

The fund's assets change over time, dependent on the investment choices. When more is invested in the risky asset, the assets have a higher variance. This investment risk is the only source of risk for the current model. We restrict ω to be constant over time as well as over the different ages for simplicity. We find for the dynamic development of the assets

$$dA_t = (r + \omega(\mu - r))\, A_t dt + \omega\sigma A_t dZ_t + (40p_t - 15b_t)dt, \tag{3.12}$$

$$A_t = A_0 + \int_0^t (r + \omega(\mu - r))\, A_u du + \int_0^t \omega\sigma A_u dZ_u + \int_0^t 40p_u - 15b_u du. \tag{3.13}$$

Note the fact that for the liabilities we take b and p, since the liabilities stay constant over time. The adjustments made to the premium and benefit level are not taken into account. For the assets we take the real contributions and benefits, p_t and b_t. The difference in assets and liabilities that arises in this way will make sure that both premiums and benefits will converge back to the originally intended, optimal levels.

3.2.2 Risk allocation rules

We now almost completely defined a DB pension model in our economy. We now need to define what happens when the assets are not equal to the liabilities. When assets are larger than the liabilities, we need to distribute the extra wealth over the participants. When assets are smaller that the liabilities, we need to raise extra funds to be able to cover the liabilities in the future. For this purpose we define risk allocation rules. These rules determine where the excess money goes or where extra funds are taken from. We only need these rules when our assets do not match our liabilities, that is $A_t \neq L$. However, due to the volatile nature of the stock prices, we expect that this is almost always the case. We define the surplus S at moment t as

$$S_t = A_t - L. \tag{3.14}$$

When we have a positive surplus, the funding ratio of the pension fund is more than 100%. When the surplus is negative, we are in a state of underfunding and hence the funding ratio is below 100%. We assume that the pension fund starts with a funding ratio of 100% at $t = 0$.

For $t > 0$ we have a certain value for the surplus. First assume this is a negative surplus, hence the pension fund needs more money than the current wealth to pay all future benefits minus future premiums. Simply hoping that the returns will be better next year could be one way to do this, but this is very uncertain. The government also demands from pension funds that their funding level is high enough[3]. Therefore, the fund needs to raise extra money to cover the mismatch. The fund has two ways to do this: it can either raise the premiums or lower the benefits. The second way shows that the *defined* benefit can also be adjusted. It depends on what the pension funds policy is.

The policy of our pension fund is defined in the risk allocation rules. These rules determine how the premium, the benefit, or both are adjusted to reach a satisfactory level of funding. Let α be the portion of the surplus distributed to the premium, and β the portion distributed to the benefits. We assume all premiums and all benefits are adjusted in the same way, not differentiating to age or anything else. We can choose for an adjustment of the premiums only, of the benefits only, or of both, by varying the parameters. Particularly of our interest is the case where we choose $\alpha + \beta \neq 1$. In this case, we do not reach the situation where the liabilities are matched by the assets after the payment of the premiums. In this way, we can spread out the recovery of underfunding or profit distribution over more than one year and more than the currently participating generations. Hence, these parameters determine the amount of IRS. When the sum of α and β goes to zero, we have a high degree of IRS.

[3] Actually, the government imposes all kinds of rules for the funding ratio of a pension fund, see Chapter 2. However, in this model we do not take these regulations into account and search for an optimal strategy.

When it approaches the risk free rate r, the current generations only pay for the interest on the deficit or receive the interest on the positive surplus. In this scenario we hope the stock market eventually will pay for the deficit. When the sum approaches one, we draw near to a funding ratio of one at the end of each year, when premiums are received and benefits paid.

Define the size of the working generations as G_{work}, and the size of the retired generations as G_{ret}. The total size of all generations over 25 years old is defined as G. We only consider full-time workers, hence these variables are integers. See Section 4.1 for the other definitions concerning the generations. For the moment we only need these variables. In the setting of this chapter, $G_{\text{work}} = 40$ and $G_{\text{ret}} = 15$. We already used these numbers in Equations (3.12) and (3.13).

We distinguish three types of policies within the DB pension scheme. When we choose to only adjust the premiums, we talk about the defined benefit scheme with contribution adjustment (DB_{CA}). Premiums are adjusted based on the level of surplus S_t and the parameter α:

$$p_t \quad = \quad p - \alpha S_t / G_{\text{work}}, \tag{3.15}$$

where p is the actuarially fair pension premium.

The second type of DB scheme we identify is the defined benefit scheme with benefit adjustment (DB_{BA}). The benefits are calculated in a similar way as the premiums in the (DB_{CA}) scheme. The benefit level is adjusted based on the level of surplus S_t and the parameter β:

$$b_t \quad = \quad b + \beta S_t / G_{\text{ret}}, \tag{3.16}$$

where b is the actuarially fair benefit level. In this scheme the shocks in the surplus are evenly distributed over all retirees, which makes the benefits dependent on the stock market return. This scheme compares to some extent with a defined contribution pension scheme, since the contribution level or premium stays fixed, but the benefit level is adjusted according to the results of the investment. The main difference is that the funding imbalance is shared among more generations in the DB_{BA} scheme.

The last scheme we define is a combination of the previous two schemes, namely the hybrid defined benefit (DB_H) pension scheme. In this policy the premium level and the benefit level can be adjusted. The risk is therefore shared amongst all generations that are active. The relative sizes of α and β determine if we burden the working generations or the retired generations more with today's surplus. The two schemes above can be seen as special cases of this scheme. The DB_{CA} is the same as the DB_H scheme with $\beta = 0$, and the DB_{BA} is the same as the DB_H scheme with $\alpha = 0$. We will now discuss how we calculated and optimized the pension scheme, and how we will look at the risk-sharing.

3.2.3 Calculation of the pension process

The DB pension model described in the previous sections will be optimized. In this way, we find the optimal risk-sharing parameters α and β, but we also optimize over the parameters p and ω, since we are free to choose these. We will optimize our model in the same way as described in Cui, De Jong, and Ponds (2008). All of the numerical work that is presented in this research is carried out using R, a software environment for statistical computing and graphics: R Development Core Team (2009).

We first fill in the assumed values for the different parameters. See Table 3.1 for all the values we defined. The risk-free interest rate, r, is fixed at 2%. The expected real return on the risky asset, μ, is set at 6%, with a volatility σ of 15%. The subjective discount rate δ is 4%, and we take the risk aversion parameter γ equal to 5. We denote the initial funding ratio by FR_0, and assume it is 1. This

parameter	value	description
r	0.02	interest rate
μ	0.06	mean stock returns
σ	0.15	volatility stock returns
δ	0.04	subjective discount rate
γ	5	risk aversion
FR_0	1	initial funding ratio
R	40	retirement age/time
T	55	end time / age of death
....		
ω	1	investment strategy
p	0.14	pension premium
α	0.045	risk allocation premium
β	0.02	risk allocation benefit

Table 3.1: *Parameter values.*

is the funding ratio when the person under study enters the workforce. The retirement age is 65, hence a person retires at time $t = R = 40$, because he enters the model at age 25. Following the same reasoning for the age of death, we find a time of death $t = T = 55$. Below the dotted line we show the optimal parameter values as found in Cui, De Jong, and Ponds (2008). These are the parameters we will optimize over.

Thereafter we create a matrix with generation sizes for all the years. All generations are set to equal one for this moment. We chose to create a matrix for this instead of just disregarding the sizes. In this way, we can adjust these sizes later. For the current setting, we find that $G = 55$, $G_{\text{work}} = 40$ and $G_{\text{ret}} = 15$. Hence, for every retired person, we have 2.67 individuals in the workforce. As mentioned in Chapter 2, this ratio is currently much discussed in the Dutch media. The increasing pressure on the workforce induces policy makers to search for solutions to this problem. The ratio we find here is a snapshot of reality. We will continue to work here with this constant ratio, but in the next chapter we will adjust this assumption and come back to this result.

For this research, we generate 10,000 paths for our stock price, using the parameters above. This is the only random part in our model. Hence, when we take expectations in this chapter, we take this expectation over the 10,000 outcomes of the model due to the stock price development. Throughout this research, the 10,000 simulated prices are kept constant in order to generate consistent results.

We define the pension scheme as a function of the different variables: ω, p, α, and β. The other variables of Table 3.1 are also given as input, with the mentioned starting values. When we apply this function to the variables, it first calculates the corresponding b, using Equation (3.10). It then calculates the initial liabilities, using the cohort sizes, and starting value of the assets, using the initial funding ratio. These define the surplus, which in turn determines the premiums and benefits. When we start at $t = 0$, we assume that the premiums have just been received and the benefits just been paid, and the funding ratio equals one.

For each turn of the year, it calculates the following in the given order. It first calculates the liabilities (which stay constant in this setting). It then calculates the wealth of the fund after one year of investment, and calculates the corresponding surplus using Equation (3.14). Using this surplus, the new premium and benefit levels are calculated using Equations (3.15) and (3.16). The new premium cannot be higher than one. Premiums that should be higher than one according to the model are set to one. Premiums can be negative, where the pension fund gives money to the participants. The benefits cannot be negative. Benefits lower than zero calculated here are set to zero. When we have the new premium and benefit levels, we can calculate the wealth of the pension fund at the start of the new year, which will be invested.

The function repeats this cycle for one lifetime of 55 years. It calculates the process 10,000 times at once, using the previously generated stock returns. It then calculates the corresponding simulations of consumption levels for one individual. The consumption is equal to the income of 1 minus the premium p_t for $0 \leq t < 40$, and equal to the benefit level b_t for $40 \leq t < 55$. These simulations of consumption levels yields this person's expected utility, using Equation (3.2). Finally, the expected utility is used to calculate the certainty equivalent consumption, using Equation (3.9). This is the outcome of the function, the quantity we are interested in.

In Listing B.1 in Appendix B we give the R code for this pension model. The steps that are described above are presented there in code. We can also represent this model in a flowchart, see Figure 3.1.

Optimization of the CEC

We are looking for the highest value of the CEC. This CEC is determined by the pension model, and influenced by four parameters: α, β, ω, and p. The optimization of the model is performed in a very straightforward way. We calculate the value of the CEC for all possible values of the parameters and simply choose the parameters that lead to the highest CEC. This brute force method is necessary, because the model is complex and there does not exist an optimization algorithm suitable for this model that we know of. Thanks to the increased computer power, such a grid search does not take as long as it would have taken a long time ago.

We perform the grid search by constructing a 4-dimensional array, one dimension for every parameter, and put the corresponding value of the CEC in this matrix. We first calculate the values accurate up to one decimal. Consequently we construct a smaller grid around the optimal values and calculate the values accurate up to two decimals. We need to check if this method really gives us the optimal values. Therefore we plot the CEC as a function of its parameters and look at the shape. We will do this in the next section, in which we will discuss the results. The fact that the results are close to the values found by Cui, De Jong, and Ponds (2008) also shows that our values are accurate.

3.3 The optimal risk allocation

In this section, we will present and discuss the results we found when investigating intergenerational risk-sharing in a defined benefit pension scheme. Following the optimization procedure described in the previous section, we attained the results presented in Table 3.2. The premium is equal to 0.147 with an income of 1, or 14.7% of the yearly income. The investment decision parameter ω is equal to one, which means we should invest 100% of the assets in the risky asset. The risk allocation parameter for the premiums is 5.2%, the risk parameter for the benefits is 1.7%. This means 6.9% of a deficit is paid for or 6.9% of a positive surplus is received by the participants of the pension fund in one year. More than 75% of this is paid or received by the working participants by means of a premium adjustment.

The first thing we treat here is the optimal investment mix ω. We find that the optimal ω is equal to one, which means a 100% investment in risky assets. This result is not what we see in practice. Pension funds often have a conservative investment policy to ensure their participants mantain a high

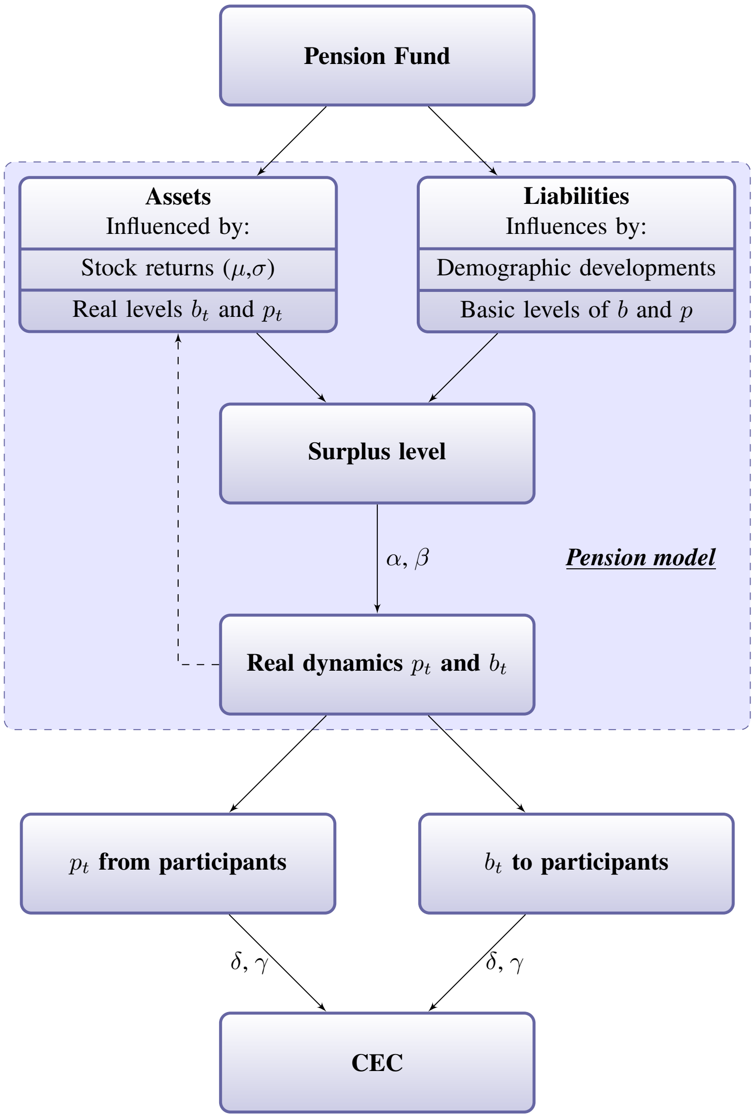


Figure 3.1: *Flowchart of the pension model.*

degree of certainty with respect to their future benefits. This is also due to government regulations which require the pension funds to keep their funding ratio high enough, taking the investment risk into account. In this research we do not take these requirements into account. We then see, taking the risk aversion of the participants into account, that the risk appetite of the participants is high. Gollier (2008) also makes notice of the result that a higher degree of intergenerational risk-sharing leads to an increase in the risk appetite of the pension fund. Merton (1969) also used a two asset model and constant relative risk aversion, but he ends up with increasing consumption near the end of the horizon. This means the investment in the risky asset should go down in the end. In our model, the investment in the risky asset is constant. This could mean this proportion is averaged, but we find it to equal 100%. Due to the short selling constraint, we conclude it is at 100% during the entire lifespan of the participants.

The 100% allocation to the risky asset might be due to the high expected return and the conservative discount rate and interest rate. The high expected return gives a high average return for the

parameter	value
ω	1
p	0.147
α	0.052
β	0.017
....	
CEC	0.9593

Table 3.2: *Optimization results.*

pension fund's assets, while the volatility is distributed over many generations. Since the high returns are not immediately distributed over the participants, these high average returns stay at the pension fund and can generate even more return, with an increased certainty. The certainty increases since the funding ratio is higher than one and hence the risk of underfunding decreases. The risks on the downside are well compensated by the high returns on the positive side.

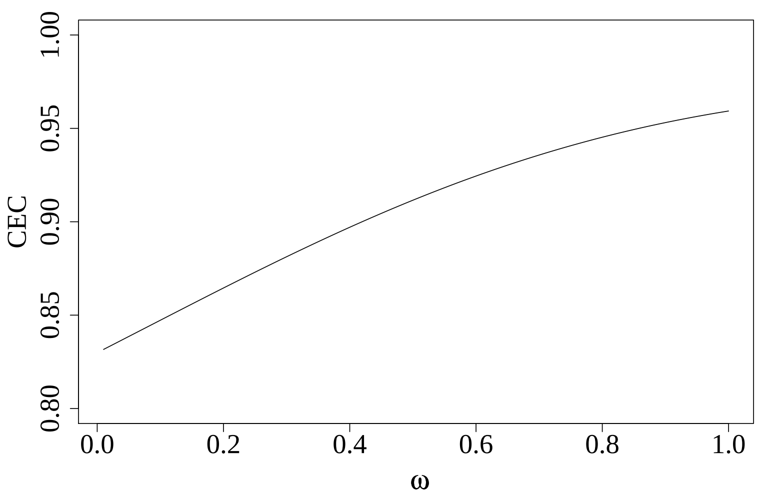
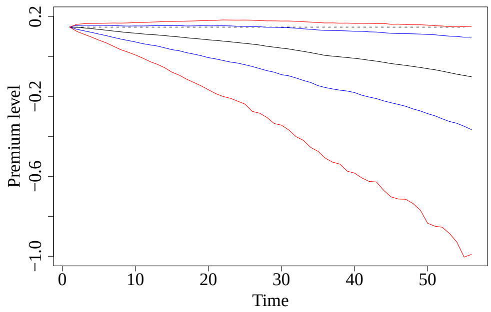


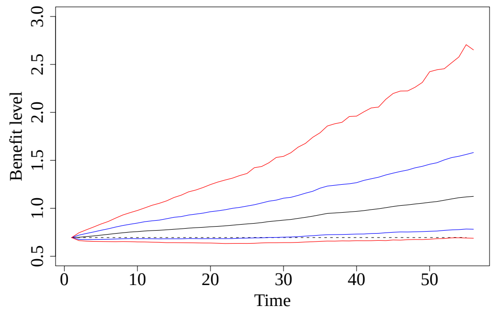
Figure 3.2: *The CEC as a function of the investment decision parameter ω. The other parameters are kept at their optimal level as given in Table 3.2.*

In this research, we focus on the strategy of IRS, and not on the investment policy in particular. Therefore we did not include many stock returns and more assets classes. We chose to incorporate one risky asset and one risk-free asset, with corresponding assumptions about their returns. In this way, we cannot use theories such as the mean-variance analysis or the Sharpe ratio to select the best investments. Since we have only two possible assets, we only have one investment decision parameter to play around with. The CEC automatically takes the risk following from this strategy into account. We plot the investment decision parameter ω against the CEC in Figure 3.2. We see there that the CEC is almost a linear function of the investment parameter.

The premium, or contribution rate, is found to be $p = 14.7\%$ in the optimal scenario. The benefit level that follows from Equation (3.10) is equal to $b = 69.5\%$. This is equal to the replacement rate in this setting. This means a person drops back from an income of 0.853 to 0.695, which means a drop of 18.5%. The benefit level compares very well to the Dutch average of 70%, see Van Duijn, Lindeboom, Lundborg, and Mastrogiacomo (2009). However, due to many shocks in the funding ratio, the premium and benefit levels are adjusted throughout time. Therefore, we plot the average premium and benefit levels against time. The surplus level determines the premium and benefit levels. Since this is random, the premium and benefit levels are also random. We plot the 90% and 99% confidence bounds in the same figure. To complete the picture, we also give the basis levels $p = 0.147$ and $b = 0.695$. See Figure 3.3 for the results.



(a) Premium level



(b) Benefit level

Figure 3.3: *The premium and benefit level in the pension model with the optimal parameters. We plotted the average of the 10.000 simulations with 90% (blue) and 99% (red) confidence levels. The dotted line is the basis level.*

We see that the average premium level goes down over time, and the average benefit level goes up. On average, the premium even reaches levels under zero after 55 years. The average benefit level passes one. A person that starts at time $t = 1$ will, on average, benefit from a higher replacement rate than 69.5%. Both developments are favorable, and are mainly due to the high expected stock returns. We also see that in 99% of the cases, the benefit and premium will eventually return to a level at least as high as the base level. The long term horizon of the pension fund makes sure that we can profit from the high expected return, with a lower long term risk for the low returns.

Finally, we take a closer look at the risk allocation parameters α and β. As follows from Table 3.2, we have a high degree of intergenerational risk-sharing. The risk of a mismatch between assets and liabilities is distributed over many years, because $\alpha + \beta < 1$. In fact, we see that $\alpha + \beta = 0.069$, which means we expect to pay for a deficit or share the benefits of a surplus over $\frac{1}{0.069} = 14.5$ years. This figure is a direct consequence of IRS, and the fact that this turns out to be the optimal value proves that IRS can improve welfare.

We already stated that only 6.9% of a deficit is paid or the same percentage of a positive surplus is received by the participants of the pension fund in one year. This means that the rest is left to the investment strategy and future generations to recover. This is risky, since we need a good return to make up for this. If the returns were 2%, the deficit would increase, because the liabilities are discounted with this value. Hence, we need a return over 2%. Since the expected real return of the risky asset, in which we invest, is 6%, we expect that it will be higher than 2% in most scenarios. Therefore, we think that this pension strategy is a good choice.

This pension strategy might be optimal, but it contradicts government policy, as argued in Chapter 2. A deficit is not recovered within a certain limit of time, which is required. Instead of recovering as soon as possible, we look at the optimal recovery parameters. Hence, following the rules in this scenario would lower the expected consumption possible for the participants. In further research, the allocation parameters for a deficit may be different than the ones for a surplus. This may lead to a more realistic model, that is compliant with the law.

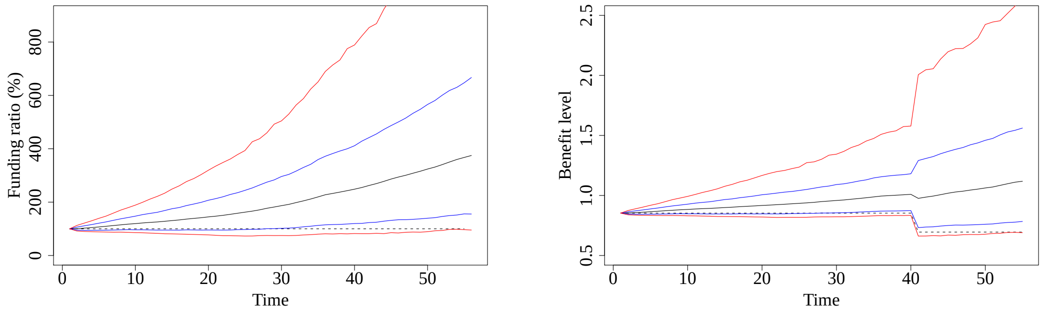
The risk allocation parameters allocate more risk to the working generations than to the retired generations. However, the total working population consists of more persons. In the current setting, we have 40 working persons and 15 retired persons. Hence, this means every age cohort that is working pays $\frac{\alpha}{G_{\text{work}}} = \frac{0.052}{40} = 0.0013$, or 0.13% of the total surplus. Every retired cohort pays $\frac{\beta}{G_{\text{ret}}} = \frac{0.017}{15} = 0.0011$, or 0.11% of the total surplus. Here we see that the risk of the investment strategy is spread almost evenly over working persons and retirees. However, these are nominal amounts with which the premium respectively the benefits are adjusted. Relatively these parameters allocate more risk to the retirees, since they have a lower disposable income than the workers. In other words, the amount they have to pay or receive for the surplus is a higher percentage of their disposable income than it is for the working generations. This is not in accordance with other research. For example, Bovenberg (2005) argues that investment risk for a pension should be higher for active members than for passive participants. We reach another conclusion here, due to the benefits of the IRS. The main reason for this is again the high expected stock returns and the long term horizon. The downside risk is well compensated and hence we can allow for a higher risk, even for the retirees.

The parameters above determine, together with the investment returns, the development of the pension fund over time. The most important outcomes are the consumption for the participants and the funding ratio, which is a measure of the financial health of the fund. The random investment returns also make that the funding ratio and the consumption outcomes are random. The average and basic levels of both are given in Figure 3.4, together with the 90% and 99% confidence bounds. In this figure, we see that the funding ratio is expected to increase, which has a direct impact on consumption. This observation is already explained above. The rising premium and benefit levels are due to the high expected real stock returns. We observe that 99% of the scenarios reach a funding ratio of almost 100%. That shows that in only 1% of our cases, the pension fund will be in a status of underfunding after 55 years. The 90% bound shows an even nicer picture. After 30 years, 90% of our scenarios reach a status of overfunding.

The high funding ratio is the main cause that the pension benefits and premiums can be at such good levels. A high funding ratio means a big positive surplus, which is distributed to the participants. The downside risk is quite low, whereas the good returns make sure that the CEC will be high. This is visible in the right panel of Figure 3.4. The jump at $t = 40$ is due to the switch from an income and premium payments to the retirement period where only benefits are received. The figure, and also Figure 3.3, shows that the benefit level is more volatile at all times than the premium level. This is due to the fact that the income from labor during the working period is constant, and hence the total income is less volatile. This explains the jump in the figure.

The certainty equivalent consumption (CEC)

Using the optimal parameters we found in our DB pension model, we attain a CEC of 0.9593. This is a lot higher than the basic consumption level in the active period, which is equal to $1 - p = 0.853$, and in the passive period, which is equal to $b = 0.695$. Hence, this pension scheme is capable to benefit from the high expected investment return, despite the high volatility or risk. The CEC takes the risk appetite of the participants into account. When we compare our results to the benchmark case in Cui, De Jong, and Ponds (2008), in which they calculated the CEC for an optimal *individual* pension strategy, we see that we are better off. The optimal individual case yields a CEC of 0.892 under the same conditions and assumptions we set here. This is mainly due to the fact that an individual consumer cannot have a positive or negative surplus in their investments. The results of investment are automatically carried by the person self, and cannot be shared by others.



(a) Funding status of the pension fund

(b) Consumption of one individual

Figure 3.4: *The funding status and individual consumption of the defined benefit pension fund with the optimal parameters. The solid black line is the mean, the dotted line is the basic level. The 90% (blue) and 99% (red) confidence bounds are also displayed.*

We find other results than the comparable *defined benefit* pension scheme in their research. As we look back at Table 3.1, we see that the optimal parameter values differ. We also reach another CEC in our research. Where they find a CEC of 0.912 in the optimal scenario, we find 0.9593. We were not able to find the differences by just reading their paper, so maybe further comparison in the future might show us where the differences arose. Since the optimal values are not too far apart, we continue the research with our model.

In Figure 3.5 we plot the CEC as a function of α and β. The parameters ω and p are fixed at their optimal level. From the shape of this plot we can infer a couple of things. First, the CEC is most sensitive in changes to the benefit adjustment through β. The shape is much skewer viewed from the β axis than it is from the axis of α. This is explained by the fact that the impact of β is higher due to the smaller group of retirees, G_{ret}. Secondly, the hill shape of the graph shows that we really reached an optimal point, and we do not expect the CEC to be higher elsewhere.

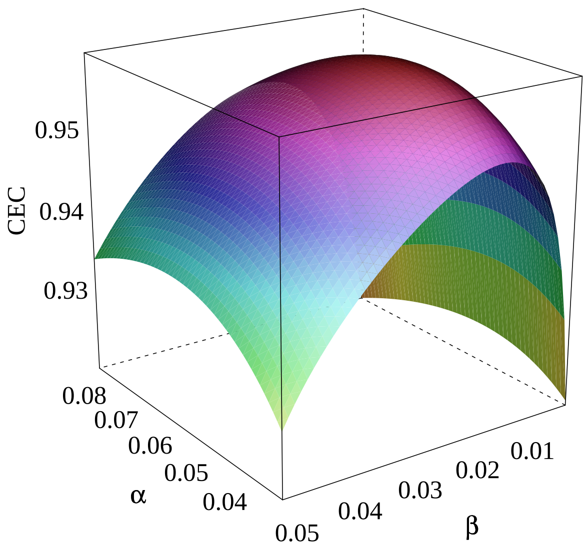


Figure 3.5: *The CEC as a function of α and β, keeping ω and p at their optimal levels.*

3.3.1 Other pension schemes

In the beginning, we distinguished three types of policies within the DB pension scheme: the hybrid scheme we discussed above, but also the contribution adjustment scheme and the benefit adjustment scheme. Above we let the parameters α and β take the optimal levels. However, there can be certain reasons to keep the benefit or contribution at a constant level. Regarding the premium, this can have advantages for the sponsors and participants. A varying premium level might be bad for their planning. For the benefit level, participants may like certainty better than a volatile income. Therefore, we also show the results of the DB_{CA} and DB_{BA} DB pension schemes. The results are given in Table 3.3.

DB_{CA}	
parameter	value
ω	1
p	0.175
α	0.06
β	-
....	
CEC	0.9283

DB_{BA}	
parameter	value
ω	0.81
p	0.11
α	-
β	0.025
....	
CEC	0.8884

Table 3.3: *Optimization results DB_{CA} and DB_{BA} pension schemes.*

We clearly see that the CEC in both cases turns out lower. This makes perfect sense, since when we optimize without restrictions, both turn out non-zero. The DB_{BA} gives a CEC of 0.8884, the DB_{CA} performs slightly better with a CEC of 0.9283. It is not in the interest of the participant to chose any of these schemes when the hybrid scheme is also available. The risk appetite is taken into account. However, if for some reason the risk appetite is lower, e.g. due to known circumstances after or before retirement, one of these schemes might be profitable.

We see that the parameters also change. We look at the DB_{CA} first. The investment decision parameter is still equal to 1. This is reasonable, since the high returns are still really interesting. The funding levels will not differ much from the ones mentioned before. That is due to the risk allocation parameter. In the hybrid case, we yearly distribute or make up for 6.9% of the total surplus. In the contribution adjustment policy this percentage is equal to 6%. Hence, almost the same amount of total risk is allocated, which is now entirely for the account of the active participants. They also should pay a higher premium of 17.5% of their income. For the benefit side, this leads to a basic benefit level of 0.8275. This is a higher basic level than in the hybrid scenario, but it is lower than the average level: see Figure 3.3(b).

For the DB_{BA} scheme, we see the opposite. The premium, which is constant throughout the active period of the participants, is lower. This yields extra consumption in this period. We also see that much less risk is distributed in total, which is now entirely for the account of the passive participants. The parameter β is higher than in the hybrid case, so per person the risk allocation to the retirees is higher than before, and even higher than to the workers in the DB_{CA} scheme. However, the total allocation is much lower. This means that a possible shortfall is not adjusted quickly. Only 2.5% of this is made up in one year. It is also true that only 2.5% of a positive surplus is distributed to retirees. This means that a surplus will be adjusted in 40 years, which is entirely against government regulations. Therefore we do not advise this pension scheme in practice.

3.4 Summary and conclusion

In this chapter, we discussed intergenerational risk-sharing in a defined benefit pension fund in a stylized economy. The economy was built using an OLG model and many assumptions about the investment returns, the interest rate, and consumer utility. In this economy we designed the DB pension model, using risk allocation rules that allow for IRS. This pension model is optimized, and the optimal parameters show interesting results. Using IRS, the risk appetite of a pension fund is large, with a 100% allocation to risky assets. Furthermore, the premium and benefit levels are expected to rise over the lifetime of the participants, due to the risk allocation. The optimal parameters give a very high CEC, which is greater than the CEC for an optimal individual model, showed in Cui, De Jong, and Ponds (2008). The CEC is also higher than in restricted DB pension schemes.

In this chapter, we also highlighted some important numbers. We showed that a positive surplus is distributed, or a negative surplus paid, in 14.5 years time. We also showed that 2.67 persons are active in the workforce for every retiree. The first number of 14.5 years does not satisfy the regulations of the Netherlands discussed in Chapter 2. Particularly, the number of years needed to make up for a deficit is too large. We leave it to further research to study the possibilities of complying with these rules, and distinguishing between negative surplus allocation and distribution of the good returns. In Chapter 2, we also discussed an increasing pressure on the workforce, which makes the second number of 2.67 an unrealistic assumption. This problem will be addressed in the next chapter.

In the following chapter, we will put this pension model in a more realistic environment. We will focus on adapting the demographic assumptions, such as the generation sizes and the life expectancy. We will start by studying a stylized case of the baby boom, where we will show different effects of the generation sizes. We will finally introduce the complete Dutch demographics in our model. The consequences differ much between generations and age cohorts. We propose two measures to dampen these fluctuations. These measures show how big the fluctuations are. They also emphasize the need for effective policy rules to change this.

Chapter 4

IRS and changing demographics

In the previous chapter, we discussed a model for researching intergenerational risk-sharing in a defined benefit pension scheme. The results showed that there is enough reason to select the optimal strategy that allows for IRS. In Chapter 2, we discussed the Dutch pension system. In this discussion, it became clear that the main pension strategy followed in the Netherlands is a defined benefit pension type. We are now inclined to advise this model for all pension plans and conclude that the choice for DB pension schemes in the Netherlands is favorable. However, we made some strong assumptions while constructing the model. These assumptions can be adjusted if we want to study a particular economy. In this chapter, we will adjust the assumptions with regard to demographics. We will show what the effects on the CEC are when we apply our model to Dutch demographics. Particular of our interest will be the differences for various generations.

The first and foremost assumption we will have to change is that we assume that all generations are of equal size. In the real world we see different generation sizes for each age cohort, due to changes in fertility rates, the effects of World War II, and migration. In our model, this would lead to different sized cohorts bearing the risks of investment. It will also lead to changing liabilities through time. Since these liabilities should match the assets, the surplus is influenced. The surplus is a main factor in determining the CEC. We will also see that the premium income will vary due to a changing size of the workforce, $G_{\text{work}} \neq 40$, while the benefit payments vary by the size of the retired population, $G_{\text{ret}} \neq 15$. The premium and benefit adjustments as given in Equations (3.15) and (3.15) still have the same size in total, since it is a fraction of total surplus. However, the allocation to the individual's premiums and benefits is influenced, since the size of the group over which this adjustment is spread out changes.

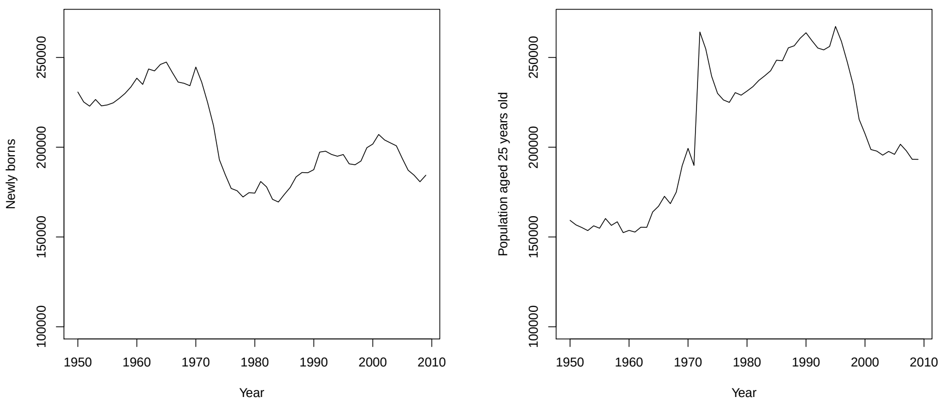
The second assumption we need to change is that of a constant life span. Since the year 1950, when people reached an average age of 71.4, the life expectancy rose with almost ten years in the period to 2009. In 2009, the average number of years a person lives is 80.5. For the future, this number is predicted to rise to 84.4 in 2050. This influences the benefits from a pension fund in a few ways. The fact that a person lives longer, increases his or her exposure to investment risk. This could change the investment strategy for the pension fund, since the investment horizon in crucial. See, for example, Barberis (2000). The other way it influences the benefits is through Equation (3.10). Due to the higher life expectancy, T should be higher, which leads to a lower benefit level. This is true intuitively because we have the same time to save for a pension that lasts longer, assuming we do not alter the retirement age.

In this chapter, we will investigate the influence of changing demographics in combination with intergenerational risk-sharing on the consumption level. We will look at how the optimal risk allocation rules distribute welfare over the generations. This research will show who are the net contributors and which generations are the net receivers. We can approximate the demographic developments in the past and see how this would have affected the risk allocation. In this chapter we do not only look at intergenerational risk-sharing, but also at intergenerational fairness.

In the first section, we give a short overview of the Dutch demographic developments in the past years. After that, we will use one of its characteristics, the baby boom, to study the consequences. We will then implement the entire demographic conditions to study the combined effects. The penultimate section will treat possible responses to these demographic changes. A summary and conclusion will finish this chapter.

4.1 Demographic developments in the Netherlands

In the Netherlands, we see more than one development at the same time. First, we see the baby boom in the period after World War II. This sudden raise in fertility rate is clearly visible in the demographic environment of the Netherlands and other European countries. Next to this baby boom, we see that the current workforce has a lower fertility rate, which means a lower workforce in the future. These two developments decrease the ratio of working people and the retired population. We also face a rising life expectancy, which further decreases this ratio. The decrease of this ratio means an increase in the pressure of the benefits on the pension fund. These effects are even worse for pay-as-you-go pension schemes, such as the government is running for the AOW. See Chapter 2 for a discussion of the different pension schemes.



(a) Population aged 0 (b) Population aged 25

Figure 4.1: *Population size of the Netherlands for ages 0 and 25.*

In Figure 4.1 the demographic developments in the Netherlands are plotted. In the left panel we see the newly borns, on the right the population aged 25. We can clearly see the post war baby boom and the effect of this on the workforce, starting at age 25. The baby boom lasts for 25 to 30 years. The population aged 25 starts at about 155.000 in 1950, and is gradually increasing to 200.000 in 2009. The baby boom adds an extra 50.000 to 60.000 in the years 1972 up to and including 1997. In 1998 and 1999 the baby boom effect is clearly over and the cohort of 25-year-olds drops back to just below 200.000. This number is higher than it was at the beginning of the baby boom. Hence, beside the baby boom, we see an upward trend in the fertility rate.

The fact that the baby boom is not apparent after 1970 anymore, can have different reasons. One of these reasons might be the introduction of the birth control pill in the Netherlands in 1962, but there could be many causes. In this research, we mainly focus on the effects of the changes, taking the changes themselves as facts.

Generations

In Chapter 3, we already defined some characteristics of the generations. We will expand this setting here, to generalize it some more and allow for the different sizes generations we observe in reality.

The total population size is given by G. We assume the population starts at age 25. The working population size, hence the population between ages 25 and 65, is given by G_{work}. The amount of retired persons is defined as G_{ret}. We expand this notation by denoting the size of each age cohort at each point in time by $g_{t,a}$. The variable t denotes the point in time. We will use both the calendar years as well as the time in our model, but the context will make clear which is used. The parameter a denotes the age cohort. Hence, as an example, $g_{1950,25}$ denotes the size of the age cohort 25 in 1950. For simplicity, we assumed in the previous chapter that all such age cohorts have a size of 1. This will be adjusted in this chapter.

In the following sections we will model the changing demographics. We will start with only the baby boom. In this section we will not discuss the adjustments which needed to be made to the model, it is just an example of the effects it has. Thereafter, we will incorporate the entire population demographics of the Netherlands, including the forecasts in Section 4.3. Before we do that, we need to change the model in a few ways, which will be discussed in Section 4.3.2.

4.2 The baby boom: a case study

In this section we will study the effects of a baby boom in our pension model. This section serves two purposes. First, it is a good case study for demographic changes, thereby allowing us to study how we might need to change our model. Secondly, it is based on the real baby boom observed in the Netherlands, hence it is a realistic scenario and the results could be useful for pension policy makers.

An increase in fertility leads to an increase of the workforce 25 years later. See for the observed effects Figure 4.1 again. We only look at the temporary increase, and do not consider the trend of an increasing population or the increasing life expectancy here. We assume that each age cohort drops back to its original level when the baby boom ends. Hence, the baby boom is created as a wave that goes through all cohorts, temporarily increasing the size.

Based on the data of the Netherlands from 1950 to 2009, we take some decisions about the size of the artificial increase. We see that the increase is about 50.000, taking the increasing population into account. Assuming the population stayed constant at about 150.000, we have an increase of 33%. We will assume that the increased fertility rate will hold on for 25 years, after which it will drop back to its original level immediately. We assume we have an original level of one, hence $g_{t,a} = 1$, for all t and $0 \leq a < 55$. The increase in fertility due to the baby boom is assumed to be 33%, has a duration of 25 years, and starts at time B. In Figure 4.1(a), we observe that B is somewhere in the interval 1945-1950. We see the first increase of the workforce at $t = B + 25$, since the baby boom starts at age 0 and the workforce at age 25. The generation sizes are modeled by the following equation:

$$g_{t,a} = \begin{cases} 1.33 & \text{if } B \leq t - a < B + 25; \\ 1.00 & \text{elsewhere.} \end{cases} \tag{4.1}$$

We will calculate the CEC for a person that will enter the model at age 25. We assume that the baby boom takes place in 1950, hence the effects in the workforce are visible for the first time in 1975. The CEC will be calculated for the years 1950 up to and including 2060. We further assume that the funding ratio is always 100% when a person enters the workforce. When generations are all equally sizes as before, we find a CEC of 0.959. Now we simulate the effects of a baby boom. The results are given in Figure 4.2.

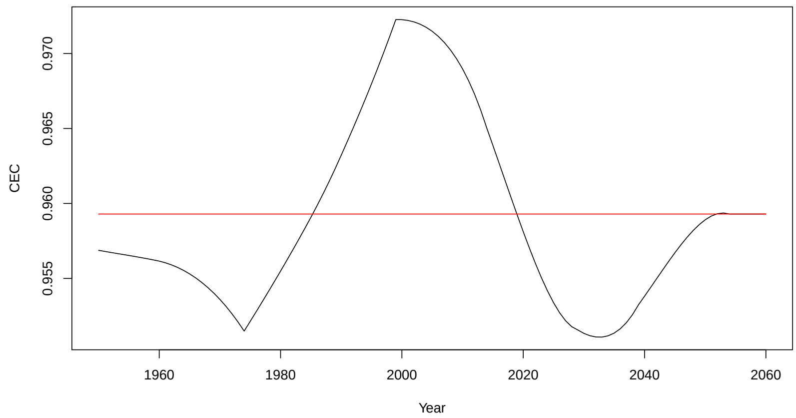


Figure 4.2: *The CEC for the years around a simulated baby boom. The black line represents the CEC for persons entering the workforce in a specified year. The red line is the average level.*

We see that the effects of the baby boom are visible for many years. This is what we should expect. If a baby boom reaches the workforce in 1975, the persons who started working in 1950 also notice the effects. This is because when they have been working for 25 years, the baby boom starts working. In fact, since they work for 40 years, and are retired for 15, the cohort that is born 54 years after or before the cohort under study affects the CEC. The closer the cohorts are to the one under study, the bigger the period is they overlap and the bigger the influence is. A similar argument goes for the other side. When a person is born in 2050, he will still feel the effects of the baby boom. This is because the generations that start working in 1975 up to and including 1999 have a size of 1.33 instead of the standardized 1. Hence, when you start to work in 2050, the generation that started in 1999 is now 75 years old and will stay in the model for 4 years, up to and including 2054. We will now explain that the CEC is affected due to two reasons: the risk-sharing and the development of the liabilities.

When you are active in a period in which the effects of a baby boom are noticeable, the risk will be shared amongst a bigger group of persons. Because the expected return is positive, this means we have to give part of our profits away. This is only the case when the baby boomers are entering the workforce when you are active, hence the persons that started before the baby boom. When the baby boom generations enter the workforce, the amount of persons in the population rises. Hence, the positive surplus that remained from earlier years due to IRS should now be distributed over more generations. This means they will have a lower CEC, and this is visible for the years 1950-1974 in the figure.

When the baby boomers are retired, their share of liabilities in the total liabilities is high. However, in the case of negative returns, the biggest share is covered by the working cohorts. Since the risk of these bigger generations is not proportionally carried by the active members, their CEC goes down. In this case, we see that the risk aversion is too high to let the positive returns make up for the negative returns. This is not entirely true, since there is another effect working at the same time.

When cohort sizes are equal, the liabilities are constant over time, as was the case in Chapter 3. When the sizes differ, the level of the liabilities changes. Hence, when a baby boom enters a pension fund, this influences the liabilities of a pension fund through time. This has serious consequences for the allocation of risk and the surplus level. Especially the relative surplus level is important. This is visible in the plot. For the generations that are born a long time before the baby boom generations enter the economy, we should not notice anything due to changing sizes. These cohorts pay for their own premiums, and the benefits are allocated the same excess return as before. However, it is the relative excess return that changes. When a bigger cohort enters the economy, the liabilities should stay the same. However, when there is a surplus, these persons share in this surplus because they will pay less premium due to the surplus and receive higher benefits. This is not taken into account, and lowers the CEC for the participants that experience the baby boom cohorts in the pension fund. The same is true at the other side. When the baby boomers are retired, the liabilities are expected to go down when they take their benefits. However, since there is likely a positive surplus, they take a greater share of these liabilities than before. This diminishes the total assets in a bigger way than expected, and influence the cohorts born after the baby boom.

However, there are also positive effects which can be mentioned. The cohorts that belong to the later part of the baby boom clearly enjoy a high CEC. This comes from the fact that the higher liabilities generate higher profits, and more risk-free investments due to this buffer of surplus. The generations just after the baby boom enjoy also a higher CEC than usual. Since liabilities are high, they benefit from these higher liabilities and their accompanying returns.

In this funded DB pension scheme, the effects are manageable. Only for a funded pension scheme the above results are true. However, one might imagine that the effects in an unfunded PAYG scheme are much bigger, which is what is currently happening in the state pension in the Netherlands and other Western countries. This is only due to the real sizes, and does not have anything to do with risk sharing. Also the liabilities argument is not applicable in the PAYG schemes.

The results we found in this section are interesting. We will continue this research in the following section and describe the entire Dutch population in our model. This will contain the baby boom. The fact that we know some things about this baby boom may help us to explain the observations for the more realistic model.

4.3 Applying the pension model to Dutch demographics

In this section, we will apply the pension model to Dutch demographics. We have two main concerns before we can do this. First, we have to model the demographics in a suitable way. Secondly, we need to adjust the model to be able to work with the different structure of generations. When we finished this, we will perform the same analysis as in the previous section, but now on a simplification of the entire real demographics and not just the baby boom. We will show that the effects are much bigger than with only a distortion in the cohort sizes. This is due to more demographic effects, such as a trend in cohort sizes we ignored in the previous section, and an increasing life expectancy.

4.3.1 Representing Dutch demographics

We would like to obtain an estimate of the CEC for a person in the Netherlands when our pension strategy is implemented. In order to do this, we need a representation of Dutch demographics. Up to now, we represented an entire population with different final ages by one representative with as final age the life expectancy. This representative was active in the population for 55 years and then died. However, in real life, the population starts with a certain amount of persons and during the way, some of these persons die, and others live on and surpass the life expectancy. The model as described above is not fit to handle such a population, but it can be adjusted such that it will be able to. We will do this in the following subsection. This gives us the freedom to feed the demographics in the Netherlands to the model in the way we did before.

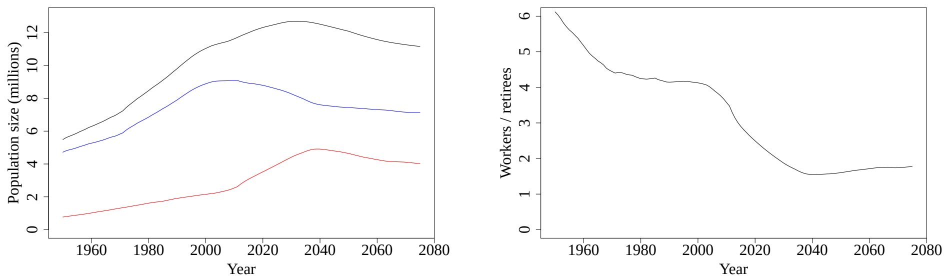
We use data from the CBS. For the years 1950 up to and including 2009, there is a detailed description of the demographics available. For each year and age we can give the amount of residents, e.g. we know that there are 198.198 persons aged 23 in 2009. For the years after 2009, we have no observations. The CBS does give an expected number of births per year for the years 2010 up to and including 2049. The CBS also provides a mortality table for the Dutch population. This table gives the probability that a person will die within the next year. This probability is given for a specific age, but also for a given period. We find the mortality rates for the years 1950 up to and including 2008. Finally, we used a table provided by the CBS with forecasts about the future population. It gives the number of persons between the ages 0-20, 20-65, and 65 and older. We will use this table, the births data, the mortality table, and the detailed information for the years 1950-2009 to construct a new table.

The new table contains detailed information for the years 1950 up to and including 2149. It is created in the following way. The predicted births are added to the detailed table of the demographics. To be able to predict even further, we repeat the last input of the births, 2049, a hundred times. In this way, we get a predicted number of births up to and including 2149. After that, the mortality rates are used to fill the empty spots in the table. Finally, the data are manipulated to fit the predicted proportions of the CBS.

We end up with a nice table, but this still is not big enough. To be able to calculate the CEC for the year 2049, we need the generation sizes in the years 2049 up to and including 2197. This is since we use the real demographic developments to calculate the liabilities. Hence, when the last person of the cohort 2049 is still alive in 2123, we need the development of the cohorts alive at that time as well. To complete the table, we assume the mortality rates stay the same. Notice that we only use the developments of the cohorts alive in 2023, the year in which the cohort under study receives his last pension benefit payment. Since the liabilities are calculated using the future of these cohorts, we need the extra 74 years.

We made a number of strong assumptions and adjustments here. However, the influence on the representation is limited. This is due to the nature of the pension plan. The model needs the demographics of more than just the generation under study. It also needs all generations older than the one under study that are alive at the same time. It also needs generations born after the one we research, and their future development. The last cohort that influences the CEC of the cohort under study is the one that enters the workforce a great number of years later. This number of years is equal to the life expectancy of the cohort under study. This last generation only has a minor influence, hence how further in the future we predict, the less influence it has. This is a good thing, since the further we predict, the less certain we are about the data we put into the model. The births that are assumed to be constant from 2049 onwards are an example of this, and this is also true for the constant mortality rate. A cohort born in 2050 enters the workforce for the first time in 2075, 25 years later. This reduces the influence of the forecasts we used on the predicted CEC. The fact that we manipulate the data to match the forecasts of the CBS also reduces the potential errors.

We end up with a detailed description of the Dutch demographics. The dataset, given in one table, gives the exact number of persons alive for a specific age and year. Hence, we can find each $g_{t,a}$. This is scaled down by dividing by the average input in this table[1]. In Figure 4.3 we show some statistics of this dataset.



(a) Population sizes in the dataset

(b) Ratio of workers and retirees

Figure 4.3: *In panel (a) we plot the total population sizes in the dataset. The black line is the total population over 25, the blue line is the size of the population under the age of 65 and the red line shows the amount of people over 65. In panel (b) we display the number of workers per retiree.*

We see in this figure that the total population size over 25 is rising. This is due to a trend in fertility rate after World War II. The effects of the baby boom are also visible. Around the year 1975 the total population over 25 is rising faster. This trend is continued, which may be caused by the increasing life expectancy. Starting around 2040, we see a small decline in the total size. The effects of the baby boom wear out at that time. The decline is not very big, and in line with the forecasts of the CBS.

The total population over 25 is split into the active participants, with an age under 65, and the passive participants, with an age over 65. We see the same developments in these groups, but at different times. The baby boom will pass through the active group earlier, where the effect wears out around 2040. The size after the baby boom is decreasing slowly. For the passive part of the population, the boom becomes visible around 2010, and stops as late as 2060. After that, the size is also declining. For both groups, disregarding the effect of the baby boom, we see that the trend is upwards in the beginning, and this is slowing at the end. In the passive group, the effect of the rising life expectancy plays a great role in the developments.

In the right panel of Figure 4.3, we show the ratio of the active and passive populations. The figures in the graph can be interpreted as the number of working persons per retiree. In Chapter 3, we calculated this number to be a constant 2.67. This is the same number as we observe around 2018 in the real demographics. Before that time, we observe a lower number. This is due to the baby boom, which influences this result starting in 1975. Also the lower life expectancy in the beginning is a cause of this. However, the number of workers per retiree is decreasing, reaching ratios well under 2 around 2040. This will have serious consequences for our results, as will discussed below.

4.3.2 Adjusting the model

To be able to work with the new data, we need to adjust the model. We will now discuss the adjustments we needed to make and the assumptions we make concerning the implementation of the data

[1] This manipulation is done for computational simplicity. Since the model was first used for generations of size one (see the previous chapter), this helped in the research when switching to the real demographics in this chapter.

in the model. The two main adjustments concern the generation sizes and the life expectancies of the different age cohorts.

Calculation of the liabilities for this model in Chapter 3 was quite simple. This was due to the constant life expectancy and the equally sized generations. It did not matter which generation sizes we took into account. However, when the generations change, we should take the correct generations into account. This requires some adjustment in the calculations.

The second new parameter we need to implement is the life expectancy. Due to this changing life expectancy, the benefit levels should change. If someone has the same time to save for a pension which has a longer duration, this should mean that the benefit level turns out lower. This is the fact since we use Equation (3.10). This means that we have a model with at any time 35 different benefit levels. Also for the calculations of the liabilities this has a big influence.

We choose to calculate the benefits using the life expectancies we got from the CBS. However, to calculate the liabilities, we use the real future developments, hence assuming perfect foresight for the demographic changes and mortality rates. This is not completely realistic, since the real world might turn out very different from the life expectancy we know at the moment of looking ahead. However, we assume this is a small difference, and does not influence our results too much. However, we might need to look into this some more, which we leave for further research.

The complete model, in the language of R (R Development Core Team (2009)), can be found in the appendix. In Listing B.2 in Appendix B we give the R code for this pension model. Main differences between this model and the one we presented in the previous chapter can be found there. After these adjustments, we can use the new model to calculate the CEC for the Dutch population when using this pension model. We will do this in the following section.

4.3.3 Results

We now calculate the CEC for the Dutch population for the different starting years. We use the pension model described in Chapter 3 with the adjustments made above. We calculated the CEC for the years 1950 up to and including 2049. We chose this number to get an even 100 years, but also because these are the main generations alive now and in the near future. The data for the distant future is worse, and hence we choose to predict not too far ahead.

We calculate the CEC for the parameters showed in Table 3.2. These are the values calculated under constant demographic conditions, which is not the case here, due to the varying fertility rate and life expectancy. However, these parameters cannot be optimized here, since we deal with a great number of starting years instead of one. Hence, the optimization would yield different parameters for each starting year. Because the current optimal parameters are approximately in the middle, we assume these level out in our case.

In Figure 4.4 the CEC for these years is plotted. Note that the CEC for a certain year, say 2009, is not the predicted CEC for all persons in that year, but it is the CEC for a person that enters the workforce in that year.

When looking at the plot, we see a different result from the previous section, Figure 4.2. We have a comparable mean as in that case, but with 0.950 slightly lower. We also see a decline before the baby boom in the year 1975, and a rise starting in that year. However, we do not see the same decline after the baby boom as we did observe before. Next to these differences, the effects are a lot bigger. We had a minimum of 0.951 and a maximum of 0.972 in the case of just a baby boom. Here we find a minimum of 0.814 and a maximum of 1.047. We deal with a lot more demographic changes here, not just one precisely defined baby boom. The life expectancy also changes for the different cohorts. This

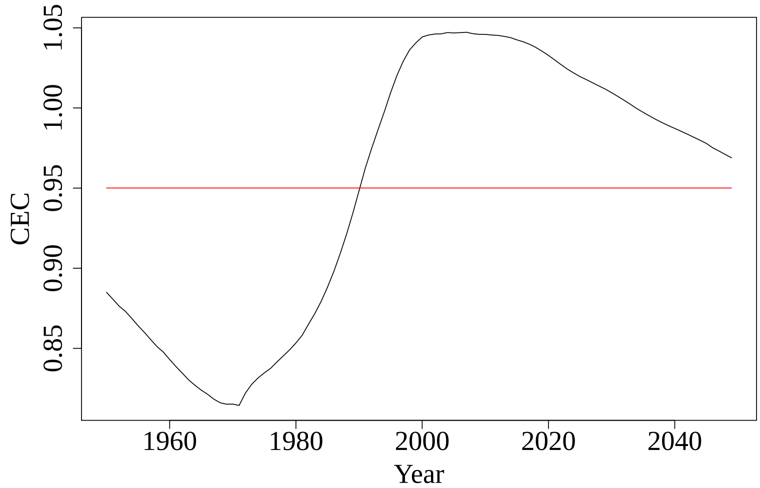


Figure 4.4: *The CEC of the optimal pension model of the previous chapter when applied to the real Dutch demographics. The black represents the CEC for persons entering the workforce in a specified year. The red line is the average level.*

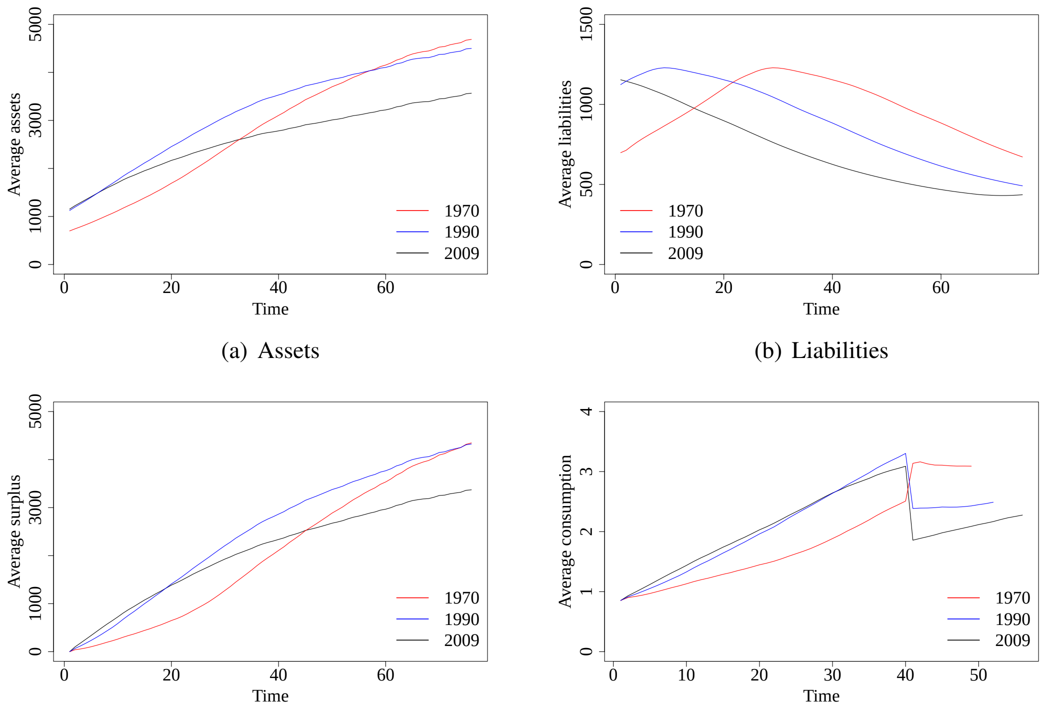
influences the benefit level, which is a main determinant of the total consumption and hence the CEC. Due to the combination of both effects, we cannot separate the influences of these developments.

We can take a look at the direct causes of the changing CEC. See Figure 4.5 for a plot of the average assets, liabilities, average surplus, and average consumption. Note that the scale of the x-axis is as follows. It is not the calendar year as before, but it is the time since the person under study entered the workforce. The time a person is under study lasts for 75 years in the first three figures. In the figure for the consumption, we plot the consumption until the person reaches its life expectancy. Hence, note that the first point in the graph, $t = 1$, is actually equal to 1970, 1990, or 2009.

The first thing that may be noticed from Figure 4.5(d), is the fact that the consumption for the year 1990 looks very much like the consumption of 2009, but the CEC differs a lot. In 2009 we find 1.046, in 1990 this is equal to 0.948. To be complete, in 1970 the CEC equals 0.815. The big difference between 2009 and 1990 should come from another statistic than the average consumption. When we look at some quantiles of all the yearly consumptions simulated, we find the values given in Table 4.1. We split the quantiles for the working period, where these are equal to the pension premiums, and the retired period, in which the consumption comes entirely from the benefits. We used the same 10.000 simulations for each year.

year	0%	25%	50%	75%	100%
1970 premiums	0.136	0.877	1.227	1.837	46.009
1990 premiums	0.183	1.059	1.571	2.430	75.654
2009 premiums	0.307	1.108	1.598	2.422	80.923
1970 benefits	0.499	1.834	2.552	3.692	73.613
1990 benefits	0.580	1.396	1.923	2.816	64.641
2009 benefits	0.479	1.115	1.577	2.388	101.470

Table 4.1: *The quantiles of the yearly consumption levels for a person entering the workforce in the years 1970, 1990, and 2009. The premiums stand for the consumption during the active period, where the benefits stand for the pension benefit which equals consumption.*



(a) Assets

(b) Liabilities

(c) Surplus

(d) Consumption

Figure 4.5: *Average assets, liabilities, average surplus, and average consumption for a person that enters the workforce in 1970, 1990 and 2009. The time in the figures starts at the moment the person enters the workforce. The time runs to the end of the simulated loop, which is* $t = 75$ *(which means an age of 99) in the first three figures. In the figure for the consumption, the time ends at the life expectancy for the different years.*

In this table, we see that the values for 1990 and 2009 are not so similar anymore. For the premiums, which form the part of the income that is discounted least, we see that the mean, the 25th percentile, and the 75th percentile are really close. However, the minimum value and the maximum are higher in 2009 than in 1990. Since the lower values are penalized strongly due to our risk averse consumer, the CEC is influenced negatively by this premium distribution. The benefits are smaller for all quantiles in 2009, except for the maximum, where 1990 is just over half the value we find in 2009. But due to the fact that these amounts are discounted more, this influences the CEC less.

Now we know where the lower CEC comes from. It is due to the distribution of the consumption levels. This goes for the premiums as well as the benefits. We will now look for the cause of this. The distribution of both levels is influenced by the difference in the 10.000 stock development, but these are fixed to be the same in all simulations for this research. Hence, the only cause for differences in the CEC can be found in the development of the level of liabilities and assets, which determine the surplus to be distributed. Note that the liabilities are the exact liabilities. The asset and surplus levels are simulated. What we see is that the liabilities are a determining factor in the CEC, just as in the baby boom scenario. Let us compare 1990 and 2009 again. For 1990, we see that the liabilities are first increasing. The investment is, due to the 100% allocation to stocks, expected to make a bigger return than the liabilities require, hence a surplus will occur in most cases. However, the surplus is divided amongst the current participants and the future ones. In the case of 1990, the surplus as a percentage of liabilities decreases, and hence the surplus per person decreases in the beginning. This leads to less allocation of the surplus to individuals. When the liabilities are decreasing, we see that the surplus as a percentage of liabilities is increasing. The persons that live now and in the near future profit most from this fact. This is exactly what we see for the year 2009, and the opposite is visible for 1970. Just as in the baby boom scenario, the liabilities are part of the explanation.

The other main determinant of the different CEC levels for the cohorts is the life expectancy. This differs a lot in the interval treated here. The life expectancy influences the benefit level. When a person lives longer, he has a lower basic benefit level. This can also be seen in Figure 4.5(d). The average level is lower for the 2009 case, and higher for the 1970 case, which is due to the life expectancy. In our model the premium is the same for all cohorts. In further research, it is a good idea to research the effects of adjusting the premium in order to keep the replacement rate the same. We continue with our model, in which the basic benefit level decreased over the years. This has a negative effect on the CEC.

Where it influences the CEC via the benefit level indirectly, the increasing life expectancy may also have a direct positive effect on the CEC. There are two reasons for this. The basic benefit levels are only theoretical. In Chapter 3 we already saw that these levels are easily surpassed due to high investment returns. In this model, the effects of the basic level after 40 years is reduced, and the premiums are closer together. This is the first reason the life expectancy can have a positive effect. Another reason is the fact that the person is a participant in the pension fund for a longer period of time. In Figure 3.4 we showed that the funding level of the pension fund and the accompanying consumption are expected to rise throughout the lifetime of a participant. When you live longer, you can take better benefits from this. The long term horizon of a pension fund becomes even longer. Hence, in two ways the life expectancy may also influence the CEC in a positive way.

The generations sizes and the life expectancy are the main differences between this setting and the setting in the previous chapter. How these interact precisely and influence the CEC is not really clear. The decrease in the CEC in the beginning might be the effect of the baby boom and other demographic developments. Next to the baby boom, the size of the population is rising. In this scenario the level of fertility does not drop back to the original level, but it remained higher than before the baby boom. This might contribute to the higher differences than before. It also contributes to the fact that we do not observe a fall in CEC a longer time after the baby boom, a fall that was visible in the stylized example. The positive effects of the life expectancy contribute to this.

All in all, we have a reasonable idea as to what causes the differences in CEC. We do not know how big the effects of each cause are, and do not know how they interact. We do observe what outcomes the model gives. The results show that the effects of the developments are severe. With a CEC fluctuating between a minimum of 0.814 and a maximum of 1.047, the effects should not be ignored. In fact, these results prove that for some participants, it would be more favorable for them to arrange their own pension in an individual pension scheme. For them, the benefits of IRS and intragenerational risk sharing do not exceed the disadvantages of the developments in the circumstances. This would even deepen the problems, since these are the smaller generations, which then become even smaller. The fact that the pension arrangements are compulsory partly solves this, but it is still not fair.

This section showed that the introduction of our pension model in the Dutch economy has different effects for different cohorts in the population. Our pension model is a DB pension scheme with risk allocation parameters that allow IRS. This model is showed to be optimal, but it does not reach the desired effects in its current form. There is no intergenerational fairness accomplished by the current model. Therefore, we will investigate what we can do about this in the next section.

4.4 Intergenerational fairness

In this section, we will show two measures for our pension model and their effects. These measures are meant to distribute surpluses more efficiently over the generations, leading to more intergenerational fairness. In the current model, we see that one generation is better off than another, due to the demographic conditions However, we do not know exactly how big the effects of the different factors are. We only observe the effects on the CEC and on other factors, such as the liabilities. These liabilities turned out to be a major direct determinant in the model. We already showed this for only a baby boom, ceteris paribus.

In order to distribute the surpluses more efficiently, the first opportunity you might think of is an adjustment of the risk allocation parameters. These parameters distribute the risk and hence must also be capable of distributing the consequences of the demographic developments. We might therefore make the risk allocation parameters dependent on the demographic conditions. The life expectancy should then also be taken into account. However, there are a few problems with this approach.

The easiest way to alter the parameters is to change the allocation to the different cohorts active in a certain year, keeping the amount of risk allocated in each year unchanged. However, when this is implemented in the model, it only has a minor effect. The effect of the individual risk allocation parameters is too low. This is unfortunate, because the model is capable of handling these changes and study the effects for all generations. This is true since this adjustment does not influence the long term development, because the total risk allocation each year, the sum of α and β, stays the same. Due to the low effect we cannot use this method. We are therefore inclined to investigate the effects of changing the total risk allocation parameters.

Changing the risk allocation parameters gives other problems. First, to reach the best solution, we should optimize again. In this process, we should take the demographics and life expectancy into account. This is quite complex, since the effects are not really discernible and requires further research. We have an indication for the effect of the demographics with the help of the baby boom case study, but the life expectancy and the trend in population size adds extra complexity. Second, when we succeed in finding new optimal parameters for all age cohorts, we encounter problems due to the design of our model. The model is not suitable to study long term effects of a varying risk allocation parameter. The changing risk allocation parameter influences the development of the assets and hence the surplus. Since we assume we have an initial funding ratio of one for each time we apply the model, these developments are ignored.

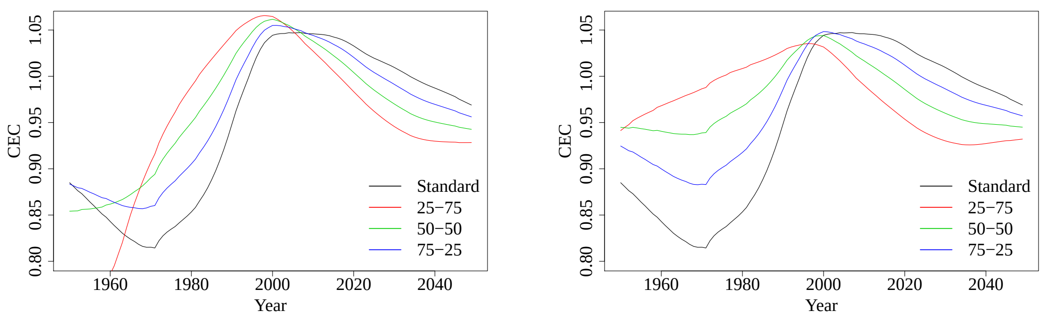
All in all, changing the risk allocation parameters is too complex and outside the scope of this research. In a different setting, this might be a good way to use IRS and reach intergenerational fairness. In the remainder of this research, we will search for other ways to alter the allocation of the pension wealth. Main question remains what we can do to make the pension policy more fair over the different generations. We will propose two new measures here. With the first we will change one of the direct causes of the change in consumption: the liabilities. The second measure will fight the consequences and level the different CEC results out over the years, using some sort of taxation and subsidy. Both measures have a very large impact on the entire population. We discuss them here as an example and as an indication for further research.

4.4.1 Changing the liabilities

The idea to change the liabilities comes from the fact that it is a direct cause of the changing surplus and consumption. Is also comes from an intuitive basis. When the risk allocation parameters are not good for all groups, we can try to let each group pay for (a percentage of) the risk of their own liabilities. Question arises whether this still means intergenerational risk-sharing or if this measure neutralizes IRS. This measure will never completely neutralize IRS, since there is only a percentage of the risk allocated each year. Hence, most risk is still spread over the generations. However, this measure also only partly offsets the big difference in CEC. The CEC is not influenced much by a small adjustment in the risk allocation parameters. We also encounter the same downsides to this approach as discussed above. Hence, we cannot let people pay for the risks of their own liabilities.

To overcome these problems, we will calculate the liabilities differently. In our pension model, we now assume the pension fund does not calculate the liabilities in the usual way, but levels it out over the years. That means, we assume that the pension fund first calculates the liabilities for that moment in time. Secondly, it calculates an estimate for the liabilities for the coming 50 or 100 years. It then takes the average of these two figures and treats this as the liabilities they need to work with. We also investigate the cases where the pension fund does not take the average but uses the real liabilities for 25% or 75%, and the future liabilities for the remaining part. Using this technique, the liabilities do not increase or decrease as fast as before. In this way, we eliminate one of the causes of the decreasing CEC. For the generations that previously experienced rising liabilities, this will mean the liabilities rise slower. A second effect is that due to the higher liabilities, the actual outflow is lower than expected. This increases the surplus for the future and hence increases the CEC for these cohorts.

We assume the pension fund followed this strategy before 1950, and hence the assets match the new liabilities in the beginning (the initial funding ratio is equal to one when the person under study enters the workforce). Using this measure, we find the new CEC's for the different years presented in Figure 4.6. In this figure, we find the original CEC in black, and the CEC for the liabilities calculated in different ways in the two figures. In the left panel, we used a mean liability level looking 50 years ahead, in the right panel we look further into the future, 100 years.



(a) 50 years ahead

(b) 100 year ahead

Figure 4.6: *The CEC with liabilities calculated using the average liabilities of the coming 50 and 100 years, using different proportions. The black line is the original CEC.*

We see that this measure distributes the CEC over the years in a more efficient way. We will first treat the difference between looking 50 years ahead and looking 100 years ahead, and then continue with the different weightings. Finally we will treat the advantages and the disadvantages of this measure.

In the figure, we see that looking 100 years ahead makes the CEC more constant than looking 50 years ahead. In fact, we see that looking only 50 years ahead can increase the intergenerational fairness. Hence, we are inclined to chose for the measure looking 100 years ahead. However, looking further into the future is also more difficult. The uncertainty increases, which makes the calculations of the future CEC levels less accurate. This might lead to more unfairness in the future when conditions turn out differently. Hence, it is not easy to choose between looking 50 of 100 years ahead.

The figure also shows that the different weightings, using 25%, 50%, or 75% current liabilities, provide different results. In the left panel, we see that only the method using 75% of the current liabilities yields more stability. The 50-50 scenario does not improve the situation much, and the 25-75 case seems to enlarge the problems we already have. The right panel shows a different view.The more we lay the emphasis on the future benefits, the better the CEC is distributed.

In almost all scenarios above, we observe a higher average CEC. Especially for the scenario where we look 100 years in the future and use the 25-75 method this is clear from the picture. This is due to the higher level of liabilities and hence assets in the beginning. This method almost does not lower the CEC for the good years. That is due to the fact that in the future, liabilities are quite constant due to our assumptions. The extra high levels of assets should be held by the pension funds, and have to be paid. In further research, we can look at how the CEC can be further leveled by letting the cohorts that enjoy the highest CEC pay for this extra funding. The method introduced in the next section might be a good way to do this.

This method seems to be capable of leveling the CEC more than our pension model could do before. The main advantage of this method is that we fight the symptoms at one of the sources. In previous sections we showed that the liabilities are a main factor in determining the CEC. There are also some disadvantages to this method. The level of liabilities for the coming years is difficult to predict. Therefore, the forecasts may change from year to year, which changes the CEC. There are also some costs to this method. These should be allocated in an efficient way. Another disadvantage is that the stock developments may turn out bad for a certain year. This might tempt the pension funds to adjust the method along the way. The costs might be shifted further into the future, or a switch to a more favorable calculation can be made. This method will therefore require more legislation. However, this legislation is already necessary, since the real funding ratios are not reached. The fact that we just take one of the source of the changing CEC into account is also a disadvantage.

4.4.2 Changing the consumption

Another way to change the resulting CEC is through a very direct way. The method we propose here calculates the relative CEC to the average CEC for the years under study. The measure multiplies the consumption with this ratio. It can therefore be interpreted as a tax or subsidy, imposed or granted by the government on the pension premium and benefit. Another way to do this is by immediately adjusting the premiums and benefits, disregarding the effects on the funding level of the pension fund. This is possible to implement in a government fund, but is more difficult for a privately held fund. The AOW, the first pillar pension, can be adjusted to eliminate the effects in the second pillar pension, but this would require extra regulation. The easiest interpretation is a tax or subsidy.

Using this new measure, we get the CEC for the different years as represented in Figure 4.7. This figure looks exactly as Figure 4.4, where the red line is the mean CEC. This means, despite its negative aspects, this measure is really effective in redistributing the welfare. This is of course what we would expect, since it directly adjusts the consumption, but it is comforting to see that it works. It also shows that our model performs correctly in these conditions.

Let us look at some examples for the average consumption adjustment. Figure 4.8(a) shows the average consumption adjustment for the years 1970, 1990 and 2009, to stay with the years we studied before. We see here that the year 1990 requires almost no adjustment. Since the CEC is almost at the average level, we see that persons who start participating in this year are not taxed or subsidized much. For the year 1970 we see a different picture. A person that starts in that year needs a lot of subsidy to get him or her on the average expected level. We see that the average adjustment rises to 40 percent of the yearly income for the working period, and even over 50 percent when the person

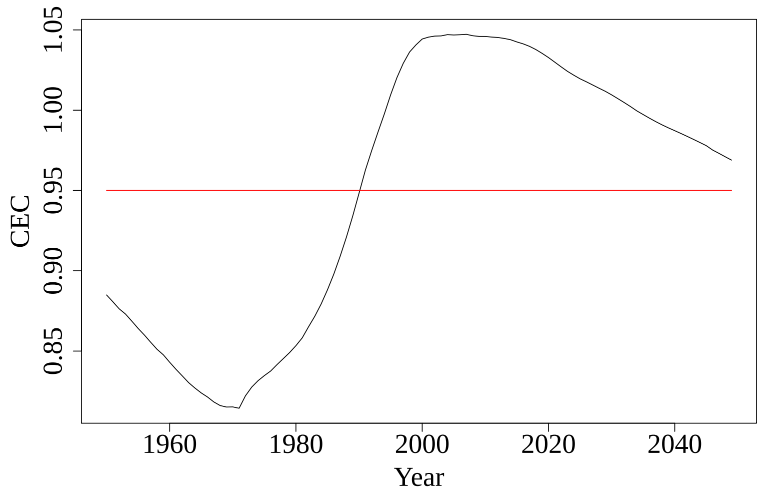
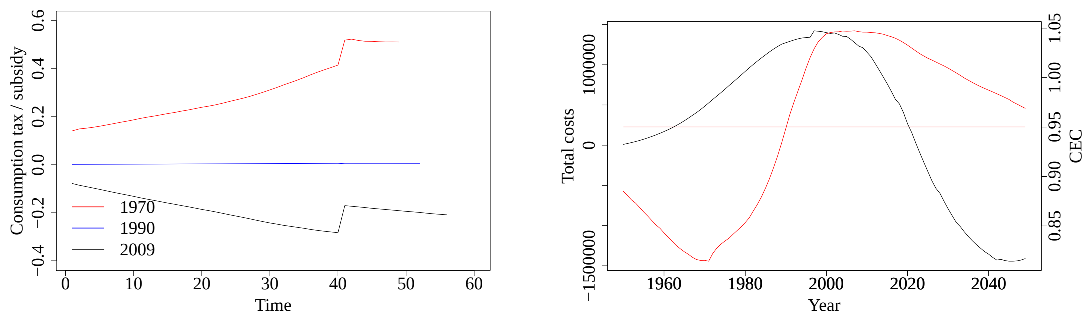


Figure 4.7: *The CEC in the standard scenario (black) and the CEC for the method of compensating consumption directly (red).*

is retired. This is a very big adjustment. On the other hand, for a person that starts in 2009, we will receive up to 30 percent of his or her salary. For an average salary of e.g. €40,000, this measure takes up to €12,000. This is a great amount of money. This emphasizes that the differences are really large, and the intergenerational fairness in this model is poor.



(a) Average consumption adjustments

(b) Total costs and CEC

Figure 4.8: *In panel (a), we plot the average consumption adjustments for a participant. We show the adjustments for three different starting years. In panel (b), we give the total costs of the consumption adjustment method (black line and left scale). The CEC in the original case before the adjustment (red varying line) and the CEC after the adjustment (straight red line) use the right scale.*

This method should not cost anything in the long run. It just takes money from one group and gives it to another, diminishing the effects of demographic developments. In the long run, the balance should end up at zero. However, during the years, this is a big investment which should be earned back. In Figure 4.8(b), we plot the total costs per year, compared to the CEC of a person that starts in that year. The total costs are calculated for the entire Dutch population older than 25 years, with a standardized wage of 1 each year. Hence, to calculate the true costs, we should multiply this with the average salary.

We see that the total costs per year rise very quickly in the beginning. That is due to the fact that many persons active at that moment have a lower CEC than average. When persons enter the workforce that have a CEC above average, in the beginning of the nineties, we see that the costs keep on rising for a few years and stay positive for an even bigger number of years. That can be explained by the fact that the persons from the earlier decades are still participating in the pension plan. Only after 2020, the costs are below zero, and hence the government or pension fund is earning back their investment. This means that there is a really big debt for the future generations. In our calculations, this ends up around 57 million times the salary in the average scenario. Again assuming an average wage of €40,000, we find a debt of €2,265 billion in the beginning of 2023. This is much higher than the current deficit of the Dutch government, which was €347 billion[2] at the end of 2008. All in all, this measure seems much too expensive. It shows how big the intergenerational differences caused by a pension model can be.

There are many ways in which this measure can be improved. We only looked at the average consumption adjustment. The remaining debt can be much higher or lower due to stock market developments. The current measure gives a subsidy to someone who started in a year with a low CEC, also when stock returns are high. The costs are higher when we do this, since we give a percentage of the consumption as subsidy, which makes this method pro-cyclical.

Another flaw of this model is the fact that we only look at a certain 100 years. We look far in the future, which means that we use an uncertain scenario to base very influential policy on. It also means that after these 100 years, we should start over again. This makes that the cohorts that become active one year after the 100 years we study, may be much better off. We again reach intergenerational differences. We could change the measure and only look at the persons that are currently active. This would mean less intergenerational transfers, which is cheaper but less powerful. It does make the model more dynamic, with different adjustments every year.

In this section, we proposed a way to improve intergenerational fairness. The measure presented uses the CEC to calculate a percentage of income that should be adjusted in order to compensate the positive or negative effects of the various circumstances. We note that the costs of such a measure are very high, which means the consequences of the current circumstances are substantial. The proposed measure and its costs and effects can be used by policy makers. They can alter the policy rules or impose a tax, a subsidy, or both. If this model would be used in the Netherlands, it shows that we currently are at a good time to implement such a system, since the generations that start working at this moment in time will be in good shape to start this new policy.

4.5 Summary and conclusion

In this chapter, we discussed IRS in a DB pension scheme in the Dutch economy. In contrast to the previous chapter, we implemented the real demographics, studying whether the assumptions we made before could be justified. In the first section, we briefly treated some of the developments in the Dutch demographics, and generalized the generation sizes. We took one particular feature, the baby boom, and studied its effects in a case study. We saw that the baby boom has different effects for the various cohorts influenced by the boom. In the section thereafter, we introduced the complete Dutch demographics into our model. Next to the baby boom, we also observe a decreasing trend upwards in the total population, and an increasing life expectancy. We needed to adjust our pension model, in order to capture all these differences relative to the previous chapter.

[2]CBS, *Statistics Netherlands*.

The new environment has large effects on the performance of our model. As in the case of the baby boom, the CEC for different cohorts changes a lot. The effects are in some ways comparable to the baby boom, but much larger. Interesting to see is the interaction between different developments. The increasing life expectancy, the aftermath of the baby boom, and other developments push the CEC above the average level for some generations, and below it for others. All in all, the demographic environment disrupts intergenerational fairness in the economy under study. In the last section, we proposed two ways to avoid this. These examples show that the effects are big, and the costs of repairing this externally are high. For further research, we therefore emphasize the need for a model that corrects for demographic effects in the risk allocation parameters.

Chapter 5

Conclusion

5.1 Thesis summary and conclusions

In this Master's thesis, we investigated intergenerational risk-sharing and changing demographics in a defined benefit pension scheme. We started by introducing the Dutch pension system, a three pillar system, with government, employee, and individual pension schemes. The funded defined benefit pension scheme is the most widely used pension plan in the Netherlands. We discussed the new Dutch Pension Act, the investment returns and funding ratios, and also the influence of the interest rate and important demographic developments.

We then introduced a pension model in a stylized economy in Chapter 3. We investigated IRS by using risk allocation parameters in our model and calculated their optimal levels. This yielded interesting results, but applicable in this simplified economy only. We investigated the assumptions of this simple economy in Chapter 4. We first looked at the baby boom in another stylized setting. Thereafter, we replaced this example by the real Dutch demographics, yielding different results than in the previous chapter. Finally, we gave two examples of measures to fight the effects of demographics on intergenerational fairness.

We found some interesting results in this research. We agree with earlier research that a defined benefit pension scheme provides excellent ways to share risk amongst different generation. We found that the certainty equivalent consumption, based on an income of 1 every year, is equal to 0.9593, almost as high as the income. This shows that investment risk can be absorbed by the pension strategy. An aggressive investment strategy in combination with optimal risk allocation rules provide a high income as well as certainty. We showed that a positive surplus is distributed, or a negative surplus paid, in 14.5 years time, due to the risk allocation parameters. 1.7% is paid out of the benefits, and 5.2% comes from higher premiums.

We used these optimization results in a representation of the real Dutch economy as well, but we found severe consequences. When applying the same pension model in a real economy, intergenerational fairness is affected. Where we used 2.67 persons in the workforce for every retiree in Chapter 3, we used the real assumptions in Chapter 4. We also used the real and increasing life expectancy. We found that the influences of the new assumptions are severe, and the two measures we proposed to make the intergenerational distribution of wealth more fair both mean high costs.

The main research question we aimed to answer is: *"What is the effect of intergenerational risk-sharing on different generations?"* In this thesis, we showed that the effects in our pension model are severe. It became clear that the assumptions made when designing the model are influential. The optimal parameters calculated using these assumptions lead to big differences between the generations in our economy. We would therefore advise to have a closer look at the effects of intergenerational risk-sharing in further research concerning pension funds. Demographic developments should be taken into account. We recommend to develop a model that really uses the characteristics of an OLG model, and can study the effects of the risk allocation parameters over more generations.

5.2 Recommendations for further research

In this research, we used the same model as proposed in Cui, De Jong, and Ponds (2008), both for the economy and the pension scheme. However, this model has a few shortcomings that we would like to discuss here and which lead to recommendations for further research. We advise policy makers to develop a new pension model that can be used to study the effects of the risk allocation parameters over more generations. In the development of that model, we can learn from our model and the results we presented, but some of the assumptions we made should be studied more before proceeding.

In the first place, the assumptions about the investment require further study. We assumed normality for the stock returns. This is not a really bad assumption for the yearly model we built, but better models can be introduced in order to make the investment simulation more realistic. It may also be a good idea to look at the conservative calculation of future benefits and liabilities. We here calculate the liabilities using a real discount rate of only 2%, where we assume an average return of our stock of 6%. This leads to great surpluses, which may lead to unrealistically high benefit levels. The fixed 2% is not in line with the fair value accounting method.

A second shortcoming of our model is that is does not comply with the regulations for pension funds. Regulations, such as described in Chapter 2, demand that the underfunding of a pension fund is recovered faster than it will be when using our model. Therefore, for further research we would advise to take this into account. For example, different parameters for downwards and upwards shocks can be implemented. This gives the parameters even more freedom, and we can comply to current regulations by adjusting the downwards risk allocation parameters. In this way, we look for a kind of second-best solution in the IRS setting.

The last assumptions we would like to mention here is the assumption of constant mortality in the future demographics. We point out the fact that many good models for mortality are available, such as discussed in e.g. Ballota and Haberman (2006), or Lee and Carter (1992). The mortality rate in the future also influences our model, since it is one of the risks a pension fund has to deal with. Using a stochastic mortality model can include this uncertainty in the model and hence improve the results for policy makers.

We did not look at the retirement age in this research. Currently, the pensionable age is a point of discussion and it is likely that this age will be increased to 67 in a few years time. Further research should also look at the consequences of this decision. As we showed in this thesis, the change in liabilities distorts intergenerational fairness. Since the pensionable age also affects the liabilities, we should investigate the effects of this decision on the fairness amongst different generations. Maybe it is necessary to compensate certain generations in the transition phase.

For further research, we once more emphasize the need for a model that corrects for demographic effects in the risk allocation parameters. Our research shows that the influence of demographics can not be ignored.

Appendix A

Research proposal

Research Proposal
Master's Thesis Actuarial Studies

B.S. WIJBENGA

June 26, 2009

Supervisor: Dr. L. Spierdijk

Coördinator: Dr. C. Praagman

Problem

Pension funds invest contributions from individuals in order to pay out benefits when they retire. Investing your money through such a fund has advantages over investing individually. First of all, the fund guarantees to pay a contribution as long as you live, which takes away uncertainty about the age you finally reach. Second, due to the long term horizon of the pension fund, it is always thought more affordable for a fund to take on the risk of the equity market than for short term or individual investors. In this way, a pension fund can spread out the investment risk onto different generations, known as intergenerational risk-sharing (IRS).

Due to the underfunded status of many pension funds at this moment and the increasing life expectancy, IRS and increasing the pensionable age is often subject of discussion. Due to the underfunding, the funds need to stop indexation, which makes the elderly complain they pay for the problems. The contributions might also rise, which makes the funds unattractive to younger participants. Due to good results in the past, the funds lowered premiums in the past and were still able to make good returns. The elderly of today enjoyed the benefits of those premiums, but on the other hand are confronted with less indexation on their pension now. It is maybe not fair to let the current workforce pay for this, but the retired generation cannot change their choices or return to work either.

Besides lower returns on the stock market and an underfunded status, pension funds also face the problem of longevity risk. In all developed countries in the world, the median age is rising. This puts a huge financial strain on the retirement system. The funds need more money to pay for longer lasting pensions. Most funds did see this problem coming, but the combination with a decreasing workforce paying for a growing group of pensioners, makes the room for errors in a funded fund smaller, and the pressure in a pay-as-you-go system on the workforce larger. The goal of pension funds is to develop strategies that deal explicitly with these many sources of uncertainty while doing so. These include, but are certainly not limited to, demographic uncertainties, future inflation, the uncertainty in the financial markets for stocks, bonds and other investment assets, and changes in the regulatory requirements for pension funds. The chosen strategy affects IRS.

Research Question

The reasoning above is the basis of this research proposal. The main goal of this research is to study the concept of IRS. We will try to measure the surplus for a person investing in a pension fund with IRS in comparison to the optimal individual investment for pension, without risk-sharing between the generations. We will do this by comparing the certainty equivalent consumption in both situations. We will focus on the defined benefit (DB) pension funds, which are proven to be optimal as argued in Beetsma and Bovenberg (2008).

We use an overlapping generations (OLG) model to quantify the consumptions of different generations. We will first follow Cui, De Jong, and Ponds (2008) in this, and derive an optimal solution by numerical methods. Investment is only possible in two assets, a risky and a risk-free one. However, instead of making assumptions about the returns of these assets, real returns from the past will be used to derive the optimal investment strategy, the optimal risk sharing rules and the optimal contribution rate. This also enables us to quickly asses what happened in the recent past, and hopefully shows us what the implications of IRS are for the current generations at work and the retirees.

After constructing this model, we will try to expand it by adding different sized generations. The OLG model in Cui, De Jong, and Ponds (2008) consists of equally sized generations, which reach the age of 80 and then die. The current situation in most countries is different, with more persons reaching the pensionable age each year. This might put a huge strain on the model. The risk sharing is carried out by gradually making up for an underfunded status by means of the risk sharing rules. This means that after a downward shock the extra capital needed is recovered over a period of time by adjusting the contribution and the benefits. This is the case in a hybrid defined benefit pension fund. When the sizes of the groups change, we expect this affects the optimal risk sharing rules and may make them dynamic. We also look into changing the pensionable age to reduce the strain on contributions.

Next to changing the sizes of the generations, we will look into longevity risk. In the current model, the generations are expected to all reach the average life expectancy, around 80 years. This is useful for studying a population with a life expectancy that is considered exact. However, this age is rising, and we might not be able to estimate how much exactly. The generations do not all reach the same age, as argued before, and the variance also differs for generations further from now. To quantify these risks, we will also try to incorporate this in the OLG model. We will use the well known Lee-Carter model, taken from Lee and Carter (1992), as the starting point to model mortality. Since the solutions in Cui, De Jong, and Ponds (2008) are analytically intractable, we rely on numerical optimization. We will use simulations using past data to form a distribution of returns.

The above discussion gives the following main research question:

What is the effect of intergenerational risk-sharing on different generations?

Next to this main question, we will try to answer the following ones:

1. *How did pension funds and IRS distribute capital amongst generations in the past?*
2. *What is the effect of changing demographics in the distribution of welfare by pension funds?*
3. *How does longevity risk influence IRS and welfare improvements?*

Literature Overview

A short overview of available literature:

- Cui, De Jong, and Ponds (2008) will be the starting point of this research. The model presented there will be expanded in different ways, described above.
- Lee and Carter (1992) provides us with a basis for the modeling of the mortality in our OLG model.
- Beetsma and Bovenberg (2008) investigated IRS in a two-pillar pension system, which will yield useful literature to compare different policies and structures (DC versus DB).
- Gollier (2008) shows a first-best and second-best intergenerational risk-sharing, where the second has more realistic assumptions and restrictions. Simulations show what the pension wealth of two different generations will be.
- Fehr and Haberman (2005) formulate in an unpublished paper a model with variable labor supply, which might be useful for our model.
- Various other papers, such as Siegmann (2007), Haberman and Sung (1994), Vigna and Haberman (2001) and Haberman, Khorasanee, Ngwira, and Wright (2003) give and describe models for optimal investment strategies, which can provide a benchmark to which our policy should be compared.
- Ballota and Haberman (2006) describe the stochastic mortality case, which also provides insights into our mortality modeling.

Appendix B

R code pension model

Listing B.1: *R code pension model Chapter 3*

```
# Define the generation sizes:
generations              <- matrix(1,1,T)
generations[1,]          <- seq(1,T)
colnames(generations)    <- seq(1,T)

generation_sizes                <- matrix(1,100,T)
colnames(generation_sizes)      <- seq(25,79)


# Define the function DB_H:
DB_H <- function(w,p=0.147,alpha=0.052,beta=0.017,Nsim=10000,FR_0=1,
         interest=0.02,gamma=5,delta=0.04,delay=0,life_exp=55){

t <- seq(0,T)

# Calculate b from the inputs:
b <- sum(exp(-interest*generations[1:R]))*p/
         (sum(exp(-interest*generations[(R+1):life_exp]))+
         exp(-interest*generations[ceiling(life_exp)])*
         (life_exp-floor(life_exp)))

# Construct matrices for the premiums and benefits:
p_t <- matrix(p,length(t),Nsim)
b_t <- matrix(b,length(t),Nsim)

# Calculate the first period liabilities:
L <- 0
for(x in 1:R){
         L <- L + sum(exp(-interest*(generations[(R+1):T]-x))*b*
         generation_sizes[1,(R+1):T]) -
         sum(exp(-interest*(generations[x:R]-x))*p*generation_sizes[1,x:R])
         }
for(x in (R+1):T){
         L <- L + sum(exp(-interest*(generations[x:T]-x))*b*
         generation_sizes[1,x:T])
         }
```

```
# matrix with the asset value including starting value for each year:
A <- matrix(NA, length(t),Nsim)
A[1,] <- FR_0 * L

# surplus for time and Nsim:
S_t <- matrix(NA, length(t),Nsim)
S_t[1,] <- A[1,] - L

# Start of main loop, filling the matrix with asset values
# and determining the new premium and benefit levels:
for(j in 2:length(t)){

        # Calculate the liabilities:
        L <- 0
        for(x in 1:R){
                L <- L + sum(exp(-interest*(generations[(R+1):T]-x))*b*
                generation_sizes[j,(R+1):T]) -
                sum(exp(-interest*(generations[x:R]-x))*p*
                generation_sizes[j,x:R])
                }
        for(x in (R+1):T){
                L <- L + sum(exp(-interest*(generations[x:T]-x))*b*
                generation_sizes[j,x:T])
                }

        # Calculate the value of the assets at the end of the year:
        A[j,] <- A[(j-1),]*(exp(interest) + w*
        (stocks[j,1:Nsim]/stocks[(j-1),1:Nsim] - exp(interest) ))

        # Calculate the surplus:
        S_t[j,] <- A[j,] - L

        # This new surplus determines the new premiums and benefits, which
        # will be payed and received before the beginning of the new year:
        p_t[j,] <- p - alpha*S_t[j,]/sum(generation_sizes[j,1:R])
        b_t[j,] <- b + beta*S_t[j,]/sum(generation_sizes[j,(R+1):T])

        # Checking for b < 0 and p > 1:
        p_t[j,][p_t[j,] > 1] <- 1
        b_t[j,][b_t[j,] < 0] <- 0

        # Determining the new value for the assets at the
        # beginning of the new year:
        A[j,] <- A[j,] + sum(generation_sizes[j,1:R])*p_t[j,] -
                sum(generation_sizes[j,(R+1):T])*b_t[j,]
}

# Calculate the expected consumption for someone who starts working
# at time t=0 + delay and lives for life_exp more years:
c_t                          <- matrix(NA,(ceiling(life_exp)),Nsim)
c_t[1:R,]                    <- 1 - p_t[(1+delay):(R+delay),]
c_t[(R+1):life_exp,]         <- b_t[(R+1+delay):(life_exp+delay),]
c_t[ceiling(life_exp),]      <- b_t[ceiling(life_exp),]

# Calculate this persons expected utility:
U <- sum(exp(-delta*t[1:(ceiling(life_exp))])*(c_t^(1-gamma))/(1-gamma)/Nsim)
```

```
# Calculate this persons Certainty Equivalent Consumption:
CEC <- (U/sum(exp(-delta*t[1:(ceiling(life_exp))]))*(1-gamma))^(1/(1-gamma))

# create a list and fill it:
B       <- list()
B$U     <- U
B$CEC   <- CEC
B
}

# Calling the function:
DB_H(w=1,p=0.147, alpha=0.052,beta=0.017,Nsim=10000,FR_0=1,interest=0.02,
        gamma=5,delta=0.04,life_exp=55)
```

Listing B.2: *R code pension model Chapter 4*

```
# Define the benefit level for all generations
B             <- matrix(NA,100,1)
rownames(B)   <- seq(1950,2049)

for(year in 1950:2049){
    generation_sizes     <-    generation_sizes_II[(year-1949):200,]
    life_exp             <-    expectancy[year-1949]
    B[year-1949]         <-    sum(exp(-interest*generations[1:R]))*p/
          (sum(exp(-interest*generations[(R+1):life_exp]))+
          exp(-interest*generations[ceiling(life_exp)])*
          (life_exp-floor(life_exp)))
}

B             <-    rbind(as.matrix(rep(B[1],100)),B)
B             <-    rbind(B,as.matrix(rep(B[200],200)))
rownames(B)   <-    seq(1850,2249)


# Define the function DB_H again:
DB_H <- function(w,p=0.147,alpha=0.052,beta=0.017,Nsim=10000,FR_0=1,
    interest=0.02,gamma=5,delta=0.04,delay=0,life_exp=55){

 t <- seq(0,T)

# The inputs should be {p,alpha,beta,w), from which b follows, using B:
b <- B[year-1849]

# p_t and b_t will be altered, but we start with the chosen value
# and the actuarially fair value b:
p_t <- matrix(p,length(t),Nsim)
b_t <- matrix(b,length(t),Nsim)
B_temp <- matrix(NA,length(B),Nsim)
B_temp[,1] <- B

# Calculate the first period liabilities, noting the cohort sizes:
L <- 0
for(x in 1:R){
    L <- L + sum(exp(-interest*(generations[(R+1):T]-x))*(B[(year-40-1849):
    (year-74-1849)]*generation_sizes[x,(R+1):T])) -
    sum(exp(-interest*(generations[x:R]-x))*p*generation_sizes[x,x:R])
    }
for(x in (R+1):T){
    L <- L + sum(exp(-interest*(generations[x:T]-x))*(B[(year-x-1849+1):
    (year-74-1849)]*generation_sizes[x,x:T]))
    }

# In this way we manipulate the liabilities to reduce the variation:
# L <- 0.75*L + 0.25*mean(liabilities[(year-1949):(year-1949+50)], na.rm=T)

# matrix with the asset value including starting value for each year:
A <- matrix(NA,length(t),Nsim)
A[1,] <- FR_0 * L



# surplus for time and Nsim:
```

```
S_t <- matrix(NA, length(t),Nsim)
S_t[1,] <- A[1,] - L

# Start of main loop, filling the matrix with asset values
# and determining the new premium and benefit levels:
# We need the liabilities in this loop now!
for(j in 2:length(t)){

        # Calculate the liabilities, noting the cohort sizes:
        L <- 0
        for(x in 1:R){
                L <- L + sum(exp(-interest*(generations[(R+1):T]-x))*
                (B[(year+j-1-40-1849):(year+j-1-74-1849)]*
                generation_sizes[(x+j-1),(R+1):T])) -
                sum(exp(-interest*(generations[x:R]-x))*p*
                generation_sizes[(x+j-1),x:R])
                }
        for(x in (R+1):T){
                L <- L + sum(exp(-interest*(generations[x:T]-x))*
                (B[(year+j-1-x-1849+1):(year+j-1-74-1849)]*
                generation_sizes[(x+j-1),x:T]))
                }

# In this way we manipulate the liabilities to reduce the variation:
# L <- 0.75*L + 0.25*mean(liabilities[(year-1949+j):(year-1949+j+50)], na.rm=T)

        # Adjustment pensionable age:
        # if(year > 2009 & (year + j - i) >= 2019){R <- 42}

        # Calculate the value of the assets at the end of the year:
        A[j,] <- A[(j-1),]*(exp(interest) + w*
        (stocks[j,1:Nsim]/stocks[(j-1),1:Nsim] - exp(interest)))

        # We find a new value for the surplus:
        S_t[j,] <- A[j,] - L

        # This new surplus determines the new p and b, which will be
        # payed and received before the beginning of the new year:
        p_t[j,] <- p - alpha*S_t[j,]/sum(generation_sizes[j,1:R])
        b_t[j,] <- b + beta*S_t[j,]/sum(generation_sizes[j,(R+1):T])
        for(i in (year+j-40-1849):(year+j-74-1849)){
        B_temp[i,] <- B[i,] + beta*S_t[j,]/sum(generation_sizes[j,(R+1):T])
        B_temp[i,][B_temp[i,] < 0] <- 0
        }

        # Checking for 0 and > 1:
        p_t[j,][p_t[j,] > 1] <- 1
        b_t[j,][b_t[j,] < 0] <- 0

        # Determining the value for the assets for the new year:
        A[j,] <- A[j,] + sum(generation_sizes[j,1:R])*p_t[j,] -
                apply(generation_sizes[j,(R+1):T]*
                B_temp[(year+j-40-1849):(year+j-74-1849),],2,sum)
}


# Calculate the expected consumption for someone who starts working
```

```
# at time t=0 + delay and lives for life_exp more years:
c_t                       <- matrix(NA,(ceiling(life_exp)),Nsim)
c_t[1:R,]                 <- 1 - p_t[(1+delay):(R+delay),]
c_t[(R+1):life_exp,]      <- b_t[(R+1+delay):(life_exp+delay),]
c_t[ceiling(life_exp),]   <- b_t[ceiling(life_exp),]


# The adjustment of the consumption is made in this way:
# c_t <- c_t * mean(CEC_year_final) / CEC_year_final[year-1949]

# Calculate this persons expected utility:
U <- sum(exp(-delta*t[1:(ceiling(life_exp))])*(c_t^(1-gamma))/(1-gamma)/Nsim)

# And finally, calculate this persons Certainty Equivalent Consumption:
CEC <- (U/sum(exp(-delta*t[1:(ceiling(life_exp))]))*(1-gamma))^(1/(1-gamma))

# create a list and fill it:

OUTCOME          <- list()
OUTCOME$U        <- U
OUTCOME$CEC      <- CEC
OUTCOME
}


# We need a bigger matrix for the generation_sizes:
R        <- 40
T        <- 75

generations               <- matrix(1,1,T)
generations[1,]           <- seq(1,T)
colnames(generations)     <- seq(1,T)

year     <- 1950

test             <- matrix(NA,150,75)
test[1:51,]      <- generation_sizes_II[200,]

generation_sizes <- rbind(generation_sizes_II[(year-1949):200,],test)

# Calling the function:
DB_H(w=1,p=0.147,alpha=0.052,beta=0.017,Nsim=10000,FR_0=1,interest=0.02,
        gamma=5,delta=0.04,life_exp=expectancy[year-1949])
```

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