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Growing old and staying young: population policy in an ageing closed economy

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Abstract This paper analyses the relation between public pensions, fertility and child care in a closed-economy overlapping generations model with endogenous fertility. It is shown that raising a child involves two social externalities and that it is optimal to introduce child allowances if the government redistributes income from the young to the old. The optimal child allowance rises when longevity increases. If the costs of raising children depend positively on the wage, a third externality arises and the returns to savings should be taxed.

Keywords Child allowances · Endogenous fertility · Pensions

JEL Classifications D10 · H55 · J13

1 Introduction

Population ageing raises widespread concern about the sustainability of existing social security programs, notably public pension systems that are run on a pay-as-you-go (PAYG) basis. The emphasis in the current debate among economists and politicians is, therefore, mostly put on the appropriate way of reforming the pension

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system so as to dismantle the 'demographic time bomb'. Though nobody doubts the necessity of adjusting pension arrangements to the changing social circumstances, it is striking that neither the cause of the low rate of fertility, which is one of the main true causes of ageing, nor measures to tackle it have received much attention in the economic literature. This paper aims to partly fill this gap by analysing the relation between social security, fertility and child care in a closed economy. Fertility is treated as an endogenous economic variable, that is, as the result of individuals' decisions on how many children to raise. The economic approach to fertility was introduced by Leibenstein (1957) and Becker (1960), who postulated that the number of children directly enters the utility function, just like consumption goods do.[1] The so-called old-age security hypothesis (Cigno 1992) stresses that, apart from giving instantaneous utility, offspring can also be viewed as a kind of capital good that enhances utility later in life by serving as an insurance against the risk of old-age dependency. However, in the second half of the last century, old-age care has been institutionalised in many developed countries through the installation and extension of public pension and health care arrangements. As a result, the old-age security hypothesis is less relevant: raising a child does not 'pay out' to the individual parent herself any more. This is one of the reasons for the dramatic drop in the fertility rate.[2] Instead of an individual return, children now have a return to society as a whole. This social externality is not taken into account when fertility choices are made (see for a review, e.g., Cigno (1993) and Folbre (1994), who typify offspring as a public rather than a private 'capital good'). A PAYG pension scheme, thus, creates an incentive for an individual to free ride on the public system by raising few children, while still being entitled to a retirement benefit (see, e.g., Sinn 2001, for a review). This distortion can be corrected by making pensions partly contingent upon fertility (Kolmar 1997; Abío et al. 2004; Sinn 2005) or by introducing child allowances. These policies have been analysed in different ways for a small open economy. van Groezen et al. (2003) investigate child allowances in such an economy model and show that PAYG pensions and child allowances are Siamese twins: if the government wants to redistribute from the young to the old, it is optimal to also introduce child allowances to correct the external effect caused by the PAYG scheme. In particular, they find that the optimal subsidy to parents is equal to the present value of a child's contribution to the intergenerational redistribution scheme during his working life. Fenge and Meier (2004, 2005) show that, with endogenous labour supply, such a system of child allowances is equivalent to fertility-related pensions, but only if the costs of raising a child involve a reduction of wage income. If these costs are modeled as a fixed amount per child, the equivalence result only holds if the fraction of general pensions is paid according to individual contributions. Another line of research (e.g., Cigno et al. 2004; Cremer et al. 2004) focuses on differences in parents' abilities to raise children and

[1] According to Zhang and Zhang (1998), this can be viewed as a weak form of altruism towards children. Much as in, e.g., Razin and Ben-Zion (1975) and Zhang (1995), parents could alternatively also derive direct utility from the utility of their offspring, which can be considered complete or strong altruism.

[2] This is confirmed by Cigno and Rosati (1996) and Cigno et al. (2003) who investigate the cases of Germany, Italy, UK, and USA. Another reason is that women got better labour force opportunities and (real) wages increased, so that the opportunity costs of raising children increased (see for a review, e.g., Becker and Barro 1988; Galor and Weil 1996.

concludes that, if there is asymmetric information concerning these abilities (or parents' actions), the government's best policy may be to differentiate child subsidies. In the current paper, we analyse the relation between social security, fertility and child care in a closed economy, which may, therefore, be viewed as a generalisation of van Groezen et al. (2003). We find that raising a child implies two externalities: not only will the child supply labour in the future and, thus, contribute to national production but (s)he will also use part of the capital stock to do so. The first effect implies that parents decide to raise too few children, whereas the second works in the opposite direction as parents will not increase their savings along with a higher fertility rate. It turns out that both effects cancel out if there is no PAYG scheme, but if such a pension scheme does exist, the net social externality of a child is positive. Hence, PAYG pensions and child allowances are Siamese twins also in a closed economy and the optimal level of the child allowance equals the child's contribution to the PAYG scheme discounted at the interest rate that results in the optimum. However, we also show that there is an important difference between a closed and a small open economy: in a closed economy, the optimal level of the PAYG tax and the accompanying child allowance fall when the social discount factor rises, whereas they increase in a small open economy. Furthermore, the conclusion that the optimal amount of child allowance equals the present value of the contribution to the PAYG scheme does not hold if the costs of raising children depend positively on the wage. In that case, a third social externality arises: when making their savings decision, individuals do not take into account that savings increase the future capital–labour ratio and, thereby, the costs of raising children. Hence, the returns to savings should be taxed, and the optimal amount of child allowances turns out to be higher than the present value of the contribution to the public pension scheme. The current paper is not merely a generalisation to a closed economy but it adds to that by analysing the effects of an increase in average life span, i.e. how one form of ageing affects the rate of fertility. It turns out that this effect is ambiguous: in a closed economy, the negative effect of increasing longevity on fertility that exists in an open economy (see, for a review, van Groezen et al. 2002) is reinforced by a positive income effect as the capital–labour ratio increases, but it may also be offset by the negative general equilibrium effects that this generates. Finally, we show that, provided a weak condition holds, increasing longevity implies a higher optimal level of child allowance and a higher optimal PAYG tax. The rest of the paper is organised as follows. In the section "The model", the model is presented. The section "Effects of increasing longevity" analyses the results of an increase in life span. In the section "The first-best solution", the first-best solution is derived and the section "Child allowances and public pensions" shows that this solution can be realised in a market economy using both child allowances and PAYG pensions (or a tax on children in combination with redistribution from the old to the young). Moreover, increasing longevity implies a higher optimal child allowance. The section "Modifications" dwells on the case of wage-dependent costs of raising children and discusses the similarities and differences between child allowances and fertility-related pensions. The "Conclusion" section concludes.

2 The model

Consider a closed economy where production is described by a standard neo-classical constant-returns-to-scale production function, $f(k_t)$, with k_t the amount of capital per young individual in period t. The interest rate and wage are given by, respectively, $r_t = f'(k_t) \equiv \frac{df(k_t)}{dk_t}$ and $w_t = f(k_t) - k_t f'(k_t)$. The economy is populated by a large number of individuals who live for at most two periods, such that in each period, both a young and an old generation are alive. Apart from age, individuals are identical. Every young person faces a probability ε of growing old, so $1-\varepsilon$ is the fraction of young that die after one period of life. All young individuals inelastically supply one unit of labour. The government imposes a constant proportional tax τ on labour income so as to finance public pension benefits received by the current elderly. The remainder of first-period income is either spent on material consumption (c^y) and offspring (n) or is saved for old age (s). Raising a child directly increases the individual's utility and also entails a cost p, which not only consists of commodities like food and clothes but also of services such as education and child care.[3] First-period consumption is consequently restricted by the following budget constraint,

$$c_t^y + pn_t = w_t(1-\tau) - s_t. \tag{1a}$$

If an individual survives to the second period of life, he is retired and derives utility from old-age (material) consumption (c^o),[4] which is financed from the return on first-period savings and a public PAYG pension benefit (η). We assume agents to have perfect foresight with respect to the level of the future public pension benefit. Savings are invested in annuities or through an actuarially fair pension fund. As only a fraction ε of young savers survive to the next period, the assets of those who passed away fall to surviving contemporaries. Therefore, the return on the savings, assuming full depreciation of capital after one period,[5] is equal to $\frac{r}{\varepsilon}$, so the second-period budget constraint can be written as

$$c_{t+1}^o = \frac{r_{t+1}}{\varepsilon} s_t + \eta_{t+1}. \tag{1b}$$

Young people derive utility from the number of children they raise (n_t) and material consumption (c_t^y), whereas the retired value material consumption (c_{t+1}^o) only, as mentioned above. For analytical convenience, we assume the expected utility function of a representative individual to be additively separable and the felicity functions to be logarithmic,

$$EU(c_t^y, n_t, c_{t+1}^o) = \log(c_t^y) + \gamma \log(n_t) + \beta\varepsilon \log(c_{t+1}^o), \tag{2}$$

[3] The case of an endogenous price of raising a child, which is a positive function of the actual wage, is analysed in the section "Modifications".

[4] If she derived utility from children, too, the main results would still hold as long as the marginal utility from children in the second period of life is not greater than in the first period of life. This is a reasonable assumption as the utility one derives from children depends to a great extent on the time spent together with them, which is most during the childhood years, i.e. when parents are young.

[5] Any rate of depreciation could be assumed but would not affect the results.

where β is the private discount rate and γ the utility weight of raising children relative to material consumption.[6]

The government runs the public pension system as a PAYG scheme, implying that

$$\varepsilon\eta_t = n_{t-1}\tau w_t. \tag{3}$$

As the public pension benefit is equal for all retired in a certain generation and, therefore, does not directly depend on the number of children one has raised, individuals do not take the government budget constraint into account when making their fertility decision. Put differently, the so-called old-age security hypothesis does not play a role in the individual's choice, though it still holds for society as a whole. As a consequence, this allows people to free ride on the system by rearing few children (or none at all) and still being entitled to a full pension benefit (see for a review, e.g., Cigno 1993; Folbre 1994).

Individuals maximise utility Eq. 2 subject to their budget constraints Eqs. 1a and 1b, which gives the following individual first-order conditions,

$$c^o_{t+1} = \beta r_{t+1} c^y_t, \tag{4a}$$

$$pn_t = \gamma c^y_t. \tag{4b}$$

According to Eq. 4a, households equate the marginal rate of substitution between current and future consumption to the rate of interest, and according to (4b), households choose the number of children and consumption in their first period of life such that the marginal rate of substitution between a child and current consumption equals the marginal cost of rearing an extra child. From Eqs. 1a, 1b, 3, 4a and 4b, we can derive the steady state,[7]

$$c^y = \frac{w(1-\tau)pr}{\Delta}, \tag{5a}$$

$$n = \frac{\gamma w(1-\tau)r}{\Delta}, \tag{5b}$$

$$s = \frac{w(1-\tau)(\beta\varepsilon pr - \gamma\tau w)}{\Delta}, \tag{5c}$$

with $\Delta \equiv (1+\gamma+\beta\varepsilon)pr - \gamma\tau w > 0$.

[6] This is a conventional way of modeling endogenous fertility (see for a review, e.g., Eckstein and Wolpin 1985; Galor and Weil 1996).

[7] Whenever time subscripts are omitted, we refer to the steady-state value of the respective variable.

3 Effects of increasing longevity

Nearly all industrialised countries will experience an ageing population caused by lower fertility rates and an increasing life span. This will not only have important implications for pension systems but also factor rewards and the rate of fertility itself will be influenced by the demographic change. The purpose of this section is to analyse how fertility behaviour changes in a closed economy where people expect to live longer. In a closed economy, savings are entirely invested domestically. Equilibrium in the capital market is, therefore, given by

$$s_t = n_t k_{t+1}. \tag{6}$$

So the capital–labour ratio is the result of both the individual savings and fertility choices. This, in turn, determines the wage and interest rate on which economic agents base their decisions. Because the expected life span is an important determinant of savings, increasing longevity affects several economic variables, as described by the following proposition.

Proposition 1 *If the* (*expected*) *life span increases in a closed economy, the long-run capital–labour ratio increases. The number of children may increase or decrease in the long run.*

Proof Inserting Eq. 5 into Eq. 1a and combining with Eq. 6 gives the implicit solution of k,

$$\beta\varepsilon pr - \gamma\tau w = \gamma r k.$$

Comparative statics gives $\frac{\partial k}{\partial \varepsilon} = \frac{\beta p(1+r)^2}{\gamma(r^2 - \tau k f''(k) r - \tau w f''(k))} > 0$. From this, it follows that $\frac{\partial r}{\partial \varepsilon} < 0$ and $\frac{\partial w}{\partial \varepsilon} > 0$. The change in the number of children due to a longer life span is given by

$$\frac{\partial n}{\partial \varepsilon} = \frac{n}{\Delta}\left(-\beta pr + (1+\gamma+\beta\varepsilon)pr\frac{\partial w/\partial\varepsilon}{w} - \gamma\tau w\frac{\partial r/\partial\varepsilon}{r}\right).$$

The first term between brackets is negative, the second and third terms are positive. Hence, the number of children can decrease or increase due to a longer life span.

Increasing longevity urges people to save more, which initially implies lower expenses on consumption when young and also on children. So, in the short run, fertility decreases. The higher savings are invested domestically so that, in subsequent periods, the capital stock increases. This, together with a smaller population growth, implies a higher capital–labour ratio. As a result, the wage increases, which causes a positive income effect on the number of children. Furthermore, the interest rate decreases, so the present value of the public pension benefit increases, making individuals save less and raise more children. The

general equilibrium effects may eventually dominate the direct negative effect,[8] though this is not very likely. In a small open economy, these general equilibrium effects do not occur, and the number of children unambiguously decreases due to an increasing life span (see, for a review, van Groezen et al. 2002).

4 The first-best solution

In the model of endogenous fertility presented above, the only reason for people to raise children is the direct utility they get from it. The fact that their children will also participate in the labour market one period later and consequently increase production is, thus, neglected. This section, therefore, explores the externalities that a child brings about for society as a whole by comparing the choices that individuals make in a market setting to the choice that a social planner would make.

Consider a social planner at time t whose objective function consists of the lifetime utilities of all current and future generations,

$$W_t = \sum_{i=t}^{\infty} \delta^{i-t} U(c_{i-1}^y, n_{i-1}, c_i^o), \tag{7}$$

where $\delta < 1$ is the social discount factor, i.e. the factor at which the planner discounts lifetime utility of future generations. In every period, total production is allocated to consumption of the young and old, the costs of rearing children and domestic investments. The social planner is, therefore, restricted by the economy's resource constraint,

$$f(k_t) = c_t^y + pn_t + \frac{\varepsilon c_t^o}{n_{t-1}} + n_t k_{t+1}. \tag{8}$$

Maximising Eq. 7 at time t subject to Eq. 8 results in the following first-order conditions for the command optimum,

$$\frac{c_{t+1}^{y*}}{c_t^{y*}} = \frac{\delta r_{t+1}^*}{n_t^*}, \tag{9a}$$

$$\frac{c_t^{y*}}{n_t^*} = \frac{p + k_{t+1}^*}{\gamma + \beta\varepsilon}, \tag{9b}$$

$$\frac{c_t^{o*}}{c_t^{y*}} = \frac{\beta n_{t-1}^*}{\delta}, \tag{9c}$$

[8] For instance, if the capital–labour ratio is initially quite low (e.g. due to an extensive unfunded pension scheme), the marginal product of capital will be high. A growing capital stock will consequently induce a substantial wage increase, boosting the fertility rate. The same applies if capital and labour are weak substitutes in the production process.

where x^* denotes the socially optimal value of variable x. Combining Eqs. 9a and 9c gives

$$\frac{c_{t+1}^{o*}}{c_t^{y*}} = \beta r_{t+1}^*. \tag{9d}$$

Comparing the first-order conditions of individual households and the social planner indicates several reasons why the actual number of children that households raise may not coincide with the socially optimal fertility rate. Equation 9b differs from Eq. 4b in two ways. First, a child will contribute to the economy's production when it has grown to maturity. This extra output will be shared with all people alive in that period, including the child's parents. However, the parents do not take this effect into account when they make their fertility decision, as they will only receive a small part of this production when they are retired. This positive social externality of a child, which can be called the *dependency-ratio effect*, implies that too few children are born. Second, however, an extra child also implies that a higher capital stock is required to keep future per capita production at the same level. As in a closed economy, the capital stock is financed with national savings, parents should save an extra amount for each additional child they raise. Put differently, every extra child implies a smaller capital–labour ratio. This *capital-dilution effect* can be considered a negative social externality that an individual does not consider and, therefore, would imply that fertility is above the social optimum. The number of children in a market economy may, therefore, be too low or too high as compared to the command optimum, depending on which of the effects dominate, and in general the government will need an instrument to correct this market failure.

5 Child allowances and public pensions

To analyse the policy instruments that the government needs to replicate the social optimum in a market setting, we will assume a Cobb–Douglas production function, $f(k) = k^\alpha$, so $r = \alpha k^{\alpha-1}$ and $w = (1-\alpha)k^\alpha$. If both offspring and the intergenerational allocation of consumption deviate from the social optimum, the government needs two instruments to correct this. As offspring can be considered a normal good, a decrease of its price due to a subsidy per child would induce people to have more children.[9] In the opposite case, it would need a tax on children so that the number of children that parents decide to raise decreases. Moreover, the intergenerational allocation can be influenced through a PAYG pension scheme. It is well known that an intergenerational transfer scheme affects savings.[10] However, if the cost of raising children is exogenous, the government does not need an additional instrument to realise the optimal level of saving.

[9] Empirical evidence suggests that child allowances indeed have a significant (though moderate) positive effect on fertility choice (see for a review, e.g., Blanchet and Ekert Jaffé 1994; Vallés Giménez and Zárate Marco 2002, for Spain). Laroque and Salanié (2005) find that financial incentives play a sizeable role in determining fertility decisions in France.

[10] Empirical evidence on the negative effect of PAYG pensions on savings can be found in, e.g., Feldstein (1996).

Suppose the government contributes a fraction φ towards the costs of raising children, which cannot only be thought of as a (direct) child benefit but also involves, e.g., subsidising child care, public education and student grants. These allowances are financed by a proportional wage tax of θ . The individual's first-period budget constraint then becomes

$$c_t^y + (1-\varphi)pn_t = (1-\tau-\theta_t)w_t - s_t, \tag{1a'}$$

and the government's budget constraint for the child allowance scheme is

$$\theta_t w_t = \varphi p n_t, \tag{10}$$

which agents do not take into account when deciding on the number of children due to their atomistic behaviour. First-order condition Eq. 4b becomes

$$\gamma c_t^y = (1-\varphi)pn_t. \tag{4b'}$$

A child allowance lowers the marginal costs of raising a child, so people will decide to have more children and save less; both effects decrease the capital–labour ratio. More specifically, the following proposition shows that the choice of policy instruments crucially depends on the social discount factor.

Proposition 2 *The government can let the market outcome coincide with the social optimum by introducing a child allowance equal to* $\varphi^* = \frac{(\gamma+\beta\varepsilon)(1-\delta\alpha)-\alpha(\delta+\beta\varepsilon)-\gamma}{(\gamma+\beta\varepsilon)(1-\delta\alpha)-\alpha(\delta+\beta\varepsilon)}$ *and setting the PAYG tax equal to* $\tau^* = \frac{\beta\varepsilon-\alpha(\delta+\delta\gamma+\delta\beta\varepsilon+\beta\varepsilon)}{\delta+\delta\gamma+\delta\beta\varepsilon+\beta\varepsilon}$. *Hence,*

- *no government intervention is required* ($\varphi^* = \tau^* = 0$) *if* $\delta = \delta_0 \equiv \frac{(1-\alpha)\beta\varepsilon}{\alpha(1+\gamma+\beta\varepsilon)}$
- *both a child allowance and PAYG pensions are needed if* $\delta < \delta_0$; *the child allowance amount corresponds to the present value of the public pension benefit, i.e.* $p\ \varphi^* = \tau^* w^*/r^*$
- *a tax on children and transfers from the old to the young are required if* $\delta > \delta_0$

Proof See "Appendix".

For a certain value of the social discount rate, government intervention is not required because the intergenerational allocation realised in the market coincides with that preferred by the social planner. Moreover, for this value of the social discount rate, the positive and negative social externalities of a child cancel out. If the discount rate is below that value, current young and future generations have a relatively low weight in the social welfare function compared to the currently retired so, for a given level of fertility, the government wants to increase old-age consumption relative to that of young workers (cf. Eq. 9c). To do so, a PAYG scheme is introduced. This implies a net positive social externality of a child, which the government internalises through a child allowance scheme. This raises the number of children and reinforces the initial effect that follows from Eq. 9c. The introduction of the PAYG pension scheme has a negative impact on savings which, in combination with the increase in fertility, implies a strong decrease in the capital–labour ratio. According to Eq. 9b, this implies an increase in fertility relative to young-age consumption, and this is exactly what is realised through the

child allowance scheme. An inspection of Eq. 9a implies that, in steady state, we have $\delta = n/r$. This corresponds to the well-known modified golden rule which basically says that the social planner equates the intertemporal marginal rate of substitution (δ) to the intertemporal marginal rate of transformation (n/r) which is affected by changes in fertility and the social return to capital. The severe drop in the capital–labour ratio implies that the interest rate rises sharply. It is this rather strong general equilibrium effect that causes a low value of δ to go along with a high fertility rate, so a child allowance scheme is required. This is opposite to the case of a small open economy (see, for a review, van Groezen et al. 2003). In such an open economy, the social planner wants to set the fertility rate such that $\delta = n/\bar{r}$, where $\bar{r}$ is the interest rate exogenously given for a small open economy. Hence, the relation between the social discount factor and the rate of fertility is straightforward: instead of child allowances and public pensions, the government needs a tax on children and lump sum transfers from the old to the young if the social discount rate is below the rate at which no intervention is required. If, however, the conclusion is that the government should stimulate fertility, then the optimal policy in both a small open economy and a closed economy is to implement a child allowance that is equal to the present value of the public pension benefit. In a small open economy, the introduction of such an allowance raises the utility of the currently young and all future generations (see, for a review, van Groezen et al. 2003). In a closed economy, however, introducing a child allowance is not Pareto improving: the utility of the currently young will be higher but, as the higher fertility rate goes along with a lower capital–labour ratio (and the economy is dynamically efficient), the utility of future generations will be lower. Social welfare as defined in Eq. 7 increases though.

5.1 Optimal policy in an ageing society

In light of the ageing of the populations in many countries, it is interesting to analyse not only how the optimal pension policy changes but also how the appropriate child allowance is affected. Because people live longer, the savings and fertility decision is affected, which subsequently has implications for optimal government policy, as stated by the following proposition.

Proposition 3 *Increasing longevity implies a higher optimal child allowance if* $1 - \alpha - \alpha\delta > 0$ *and a higher optimal PAYG tax.*

Proof Differentiating φ^* for ε gives $\frac{d\varphi^*}{d\varepsilon} = \frac{\gamma\beta(1-\alpha-\alpha\delta)}{((\gamma+\beta\varepsilon)(1-\alpha\delta)-\alpha(\delta+\beta\varepsilon))^2}$, which is positive if $1 - \alpha - \alpha\delta > 0$. Differentiating τ^* for ε gives $\frac{d\tau^*}{d\varepsilon} = \frac{\beta\delta(1+\gamma)}{(\delta+\delta\gamma+\delta\beta\varepsilon+\beta\varepsilon)^2} > 0$.

So, for realistic values of the parameter α, population ageing (caused by a longer life span) means that the optimal child allowance increases. Suppose that the PAYG tax, the child allowance and the capital–labour ratio are optimal for a given value of ε. Then, if ε rises, people live longer in retirement, so they decide to save more and have fewer children in their first period of life to prevent a too-large drop in old-age consumption. This implies that the capital–labour ratio rises and reaches a suboptimally high level. To correct this, the government both raises the PAYG tax

(which means lower savings) and increases the child allowance (and, thus, raises fertility).

6 Modifications

6.1 Endogenous price of raising children

When a parent decides to raise a child, this either implies that (s) he has to reduce labour supply or use child care facilities. In both cases, the process of upbringing a child is a labour-intensive activity, and the costs are to some extent directly related to the actual wage. Therefore, in this section, we explore how previous results on child allowances and public pensions change if the price of a child is a positive function of the wage, i.e. $p_t = p(w_t)$, with $p' \equiv \frac{dp}{dw} > 0$. In deriving the first-best solution, we have to take into account that the social planner's resource constraint is now given by

$$f(k_t) = c_t^y + p(w_t)n_t + \frac{\varepsilon c_t^o}{n_{t-1}} + n_t k_{t+1}. \tag{8'}$$

First-order condition Eq. 9a, therefore, changes to

$$\frac{c_{t+1}^{y*}}{c_t^{y*}} = \frac{\delta(r_{t+1}^* + n_{t+1}^* p'(w_{t+1}^*)k_{t+1}^* f''(k_{t+1}^*))}{n_t^*}, \tag{9a'}$$

which implies that

$$\frac{c_{t+1}^{o*}}{c_t^{y*}} = \beta(r_{t+1}^* + n_{t+1}^* p'(w_{t+1}^*)k_{t+1}^* f''(k_{t+1}^*)). \tag{9d'}$$

An endogenous price of raising children ($p' > 0$) does not change the first-order condition of the individual household concerning the intertemporal allocation (Eq. 4a). However, it does alter the social planner's first-order condition, as it changes from Eq. 9d to Eq. 9d', implying a third externality. If $p' > 0$, higher savings not only lead to a lower interest rate but also to a higher wage and, thus, to higher costs of raising a child. An individual does not take this effect into account when making his/her savings decision. Note that this does not occur in a small open economy where the capital–labour ratio is determined on the world capital market, implying an exogenous wage and child raising costs. Furthermore, as can be seen from Eq. 9a', the social optimum in steady state is achieved if $n = \delta r + \delta n p' k f''(k)$, which gives a different modified golden rule than in the standard case. The government, therefore, cannot confine itself to the two instruments discussed in the previous section but rather needs an additional tool to achieve the first-best outcome. As can be seen from Eq. 5, savings are a positive function of the interest rate if a PAYG pension scheme is present. By imposing a tax σ on the return to savings and using

the tax revenues for paying a lump sum transfer T, the additional externality can be internalised. The individual's second-period budget constraint then changes to

$$c^o_{t+1} = \frac{(1-\sigma)r_{t+1}}{\varepsilon} s_t + \eta_{t+1} + T, \tag{1b'}$$

with $T = \frac{\sigma r_{t+1} s_t}{\varepsilon}$. Consequently, first-order condition Eq. 4a becomes

$$c^o_{t+1} = \beta(1-\sigma) r_{t+1} c^y_t. \tag{4a'}$$

The optimal savings tax results from equating this to Eq. 9d':

$$\sigma^* = -\frac{\delta p'(w^*) k^* f''(k^*)}{1 - \delta p'(w^*) k^* f''(k^*)}.$$

This leads to the following proposition.

Proposition 4 *If the price of raising a child is a positive function of the actual wage, the government needs to impose a tax on the return to savings, next to a child allowance scheme and a PAYG pension scheme to achieve the first-best outcome. The optimal amount of child allowances is higher than the present value of the public pension benefit, i.e.* $p(w^*)\varphi^* > \tau^* w^*/r^*$*, whereas the optimal PAYG tax is the same as with an exogenous price of raising a child.*

Proof See "Appendix".

If due to an endogenous price of raising a child, savings are to be decreased, this will reduce the capital dilution effect, which by itself implied that the number of children that individuals raise is too high. However, the other social externality of a child, causing individuals to raise too few children, does not change in importance, which can be seen from the fact that the optimal PAYG tax is unchanged. So, the optimal fertility rate is higher than with an exogenous price of raising a child. The government, therefore, needs to pay a higher child allowance.

6.2 Fertility-related pensions

Another way to internalise the social externalities of a child would be to introduce fertility-related pensions, i.e. when an individual's public pension benefit directly depends on the number of children (s) he raised. Such pension schemes have been analysed by Kolmar (1997), Abío et al. (2004), Sinn (2005) and Fenge and Meier (2005). The latter find that fertility-related pensions and child allowances are equivalent in a small open economy. The following proposition states that the same holds for a closed economy.

Proposition 5 *A system of child allowances is equivalent to fertility-related pensions, where the individual public pension benefit completely depends on the number of children raised by the individual.*

Proof Without fertility-related pensions, but with the optimal child allowance as given by proposition 2, the resulting fertility rate is given by $n = \frac{\gamma(1-\tau)w}{(1+\gamma+\beta\varepsilon)(p-\tau w/r)}$. With fertility-related pensions and $\varphi = 0$, the individual takes the government's budget constraint Eq. 3 into account and the first-order condition Eq. 4b changes to $(p - \tau w_{t+1}/r_{t+1})n_t = \gamma c_t^y$, which results in the same fertility rate.

Hence, a system of child allowances can be replaced by fertility-related pensions to achieve the socially optimal situation, and the PAYG scheme itself would be the only instrument needed. This, however, only applies to the case of an exogenous price of raising children. If these costs are a positive function of the wage, the government, nevertheless, needs an additional instrument.

7 Conclusion

Dealing with the (adverse) consequences of population ageing involves more than merely reforming the public pension scheme. As a matter of fact, the most direct way would be to battle one of the causes of ageing itself: a lower fertility rate, which is clearly the result of individual deliberation. This paper had a closer look at fertility as the endogenous source of population growth and, thereby, of the implicit return of a PAYG-financed social security scheme, in a closed economy. It was found that one form of ageing, increasing longevity, may incite a lower number of children, thus aggravating the ageing problem, but that the general equilibrium effects of the resulting increase in the capital–labour ratio may lead to the opposite result. Furthermore, offspring causes several externalities on society that individuals do not take into account. First, a child increases future production, which can be shared among the then living generations; second, a child decreases the capital–labour ratio because individual parents do not increase their savings along with the upbringing of their children. A government can internalise these externalities by implementing a program that effectively lowers the individual costs of rearing children, such as a system of child allowances, study grants and child care, so as to bring the fertility rate to its optimal level. This assumes that individuals know the precise price of a child. In reality, this price can be misperceived. If people underestimate the true price, child subsidies should be lower than in the case where people know the correct price. But the actual price can also be overestimated. The government should then first make people more aware of the true parental costs of raising a child before deciding to increase subsidies. It was shown that, if the costs of raising are independent of the wage, PAYG pensions and child care are Siamese twins in a closed economy just like they are in an open economy. However, contrary to the open economy, both the optimal level of child care and the optimal PAYG tax fall if the social discount factor is increased. Furthermore, if the costs of raising a child depend positively on the wage, a third externality arises, as individuals do not take account of the positive effect of their savings on these costs. In that case, the return to savings should be taxed, and the optimal amount of child allowances exceeds the present value of the contribution to the PAYG pension scheme. In a closed economy, the introduction of such an allowance scheme is not a Pareto improvement: either a currently living generation is worse off or future generations suffer. Furthermore, a direct consequence of our result would be that if individuals differ in their abilities to raise children or earning

potentials of children vary; the amount of child allowance would also have to differ. In particular, if ability or the child's earning potential is positively correlated with the wage, those who earn a high wage contribute more to the future basis of the public pension scheme and should receive higher subsidies than those with low incomes. Although long-run efficiency would be improved, equity is reduced, which limits the political feasibility. An interesting topic for further research is, therefore, how to adapt the population policy in such a way so as to make it more acceptable. Nevertheless, implementing a program that lowers the costs of rearing children is an effective and relatively easy way of reducing the risk of the demographic time bomb in economies with extensive PAYG schemes. The need for an appropriate population policy (through a child allowance scheme or fertility-related pensions) becomes more urgent in a closed economy where people grow older. Neglecting this part of social security will, thus, result in greater welfare losses.

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Appendix

1. Proof of proposition 2

1.1 The market outcome

In steady state, the individual number of offspring and savings with a child allowance scheme follow from combining Eq.1a' with Eqs. 4a, 4b' and 10,

$$n = \frac{\gamma(1-\tau)wr}{(1+\gamma+\beta\varepsilon)p(1-\varphi)r+\gamma\varphi pr-\gamma\tau w},$$
$$s = \frac{(1-\tau)(w\beta\varepsilon p(1-\varphi)r-\gamma\tau w)}{(1+\gamma+\beta\varepsilon)p(1-\varphi)r+\gamma\varphi pr-\gamma\tau w}.$$

Capital market equilibrium implies $nk = s$. With $f(k) = k^{\alpha}$, the resulting equilibrium capital–labour ratio is

$$k = \frac{\alpha\beta\varepsilon p(1-\varphi)}{\alpha\gamma+(1-\alpha)\gamma\tau}. \tag{11}$$

1.2 The social optimum

The first-order conditions for a social planner follow from maximising the social welfare function (Eq. 7) subject to the resource constraint (Eq. 8), resulting in first-

order conditions (Eqs. 9a, 9b and 9c). Substituting these equations into the economy's resource constraint gives the optimal capital–labour ratio,

$$k^* = \frac{\alpha p(\delta + \delta\gamma + \delta\beta\varepsilon + \beta\varepsilon)}{(\gamma + \beta\varepsilon)(1 - \alpha\delta) - \alpha(\delta + \beta\varepsilon)}. \tag{12}$$

The policy instruments are chosen such that the first-order conditions in a market setting, as given by Eqs. 4a and 4b', are equal to those of a social planner. The optimal child allowance follows from equating Eq. 4b' to Eq. 9b. Combining this with Eq. 12 results in the optimal child allowance,

$$\varphi^* = \frac{(\gamma + \beta\varepsilon)(1 - \delta\alpha) - \alpha(\delta + \beta\varepsilon) - \gamma}{(\gamma + \beta\varepsilon)(1 - \delta\alpha) - \alpha(\delta + \beta\varepsilon)}. \tag{13}$$

Combining Eqs. 11, 12 and 13 gives the optimal PAYG tax,

$$\tau^* = \frac{\beta\varepsilon - \alpha(\delta + \delta\gamma + \delta\beta\varepsilon + \beta\varepsilon)}{\delta + \delta\gamma + \delta\beta\varepsilon + \beta\varepsilon}. \tag{14}$$

Using Eq. 12, it can easily be seen that $p\varphi^* = \frac{\beta\varepsilon p - \gamma k^*}{\gamma + \beta\varepsilon} = \tau^*(1 - \alpha)k^*/\alpha = \tau^* w^*/r^*$.

2. Proof of proposition 4

Equilibrium in the capital market is given by $nk = s$, which boils down to

$$\gamma rk + \gamma\tau w = \beta\varepsilon(1 - \sigma)p(w)(1 - \varphi)r. \tag{15}$$

The optimal child allowance is set such that

$$\frac{p(w^*)(1 - \varphi^*)}{\gamma} = \frac{p(w^*) + k^*}{\gamma + \beta\varepsilon}, \tag{16}$$

so the previous equation can be written as $\frac{\tau^* w^*}{r^*} = \frac{\beta\varepsilon p(w^*) - \gamma k^*}{\gamma + \beta\varepsilon} - \frac{\sigma^* \beta\varepsilon(p(w^*) + k^*)}{\gamma + \beta\varepsilon} = p(w^*)\varphi^* - \frac{\sigma^* \beta\varepsilon(p(w^*) + k^*)}{\gamma + \beta\varepsilon}$. Hence, if $p' > 0 \Longrightarrow \sigma^* > 0$, then $p(w^*)\varphi^* > \frac{\tau^* w^*}{r^*}$.

Combining the economy's resource constraint with the social planner's first-order conditions and the optimal savings tax gives

$$k^{*\alpha} = \frac{p(w^*) + k^*}{\gamma + \beta\varepsilon}(\delta + \delta\gamma + \delta\beta\varepsilon + \beta\varepsilon)r^*(1 - \sigma^*).$$

Together with Eqs. 15 and 16, this results in the optimal PAYG tax,

$$\tau^* = \frac{\beta\varepsilon - \alpha(\delta + \delta\gamma + \delta\beta\varepsilon + \beta\varepsilon)}{\delta + \delta\gamma + \delta\beta\varepsilon + \beta\varepsilon},$$

which is equal to the case of an exogenous price of raising children, as given by Eq. 14.

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