Pension Schemes with Smoothing Buffers

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Contents

- Return smooth with buffer
- Sustainable buffer
- “Model Free” upper and lower bounds
- Conclusion

Intergenerational Risk Sharing

- One of the alternatives for setting up intergenerational risk sharing within a pension fund system, is to establish an explicit buffer fund to smooth fluctuations in investments returns.
- This paper investigates the long-term viability of such a system.

Buffer Fund

- A buffer fund is a collective account that is financed by all participants in the pension scheme. All participants will also be able to finance their pension in certain scenarios from this buffer fund.
- A lower threshold and an upper threshold are set. The returns above the upper threshold finance the buffer fund.

Buffer Fund (2)

- In case the returns are below the lower threshold, the buffer fund will supplement the personal pension capital from the individual participant through the reduction of the funds in the buffer fund.
- Such an set-up combines elements from defined benefit contracts with defined contribution contracts.

Viability of Buffer Fund

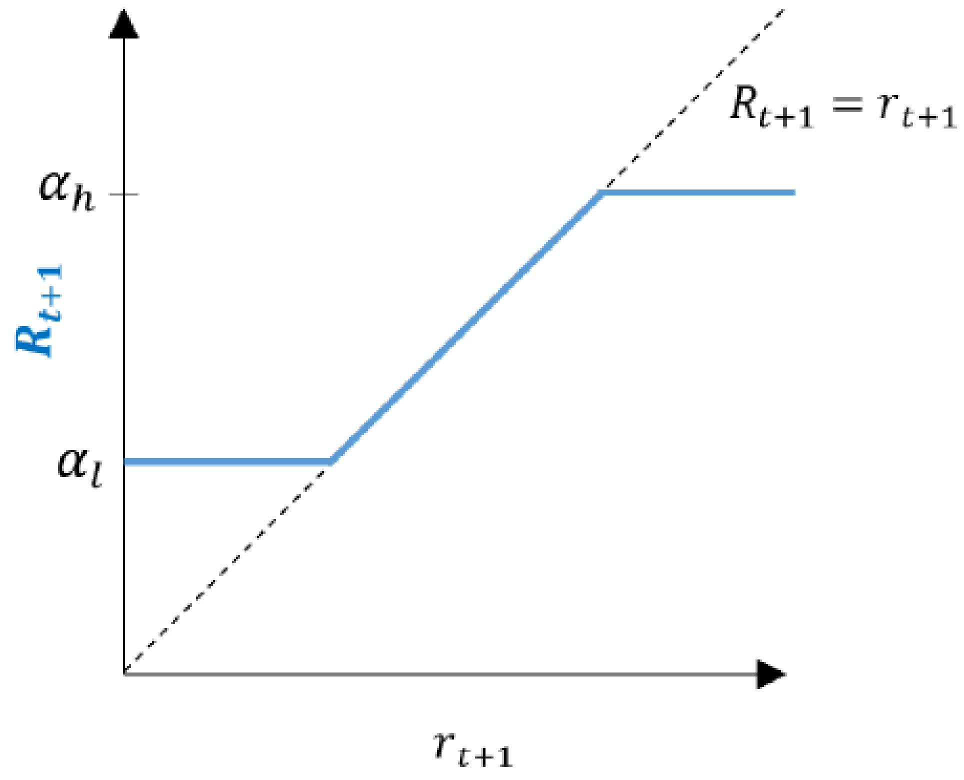
- However, to maintain a long-term viable pension system, the buffer fund should not be contributed to by outside parties, for example the government.
- The buffer fund should be self-financing.
- In this paper we investigate how to set up such buffer funds to be self-financing under a wide range of economic circumstances.

Bounds on Returns

- Consider investment portfolio S_t
- Consider annual returns:

  $1 + r_{t+1} = \frac{S_{t+1}}{S_t}$
- Cap investment returns at α_h
- Guarantee returns below α_l

Realized return of pension scheme



Bounds on Returns (2)

- Buffer fund can only be sustainable if:
  - Value of “cap” has same value as guarantee
- Otherwise: fund becomes “too rich” or gets depleted over time

- Implication: cannot choose α_h and α_l independent(!)
  - If you choose annual guarantee α_l,
  - Then α_h is fixed also to obtain “sustainable” buffer

Bounds on Returns (3)

- For annual returns:
  - At "1st of January" for each year t
  - Observe price of 1-year put-option with strike $K_l = (1 + \alpha_l)S_t$
  - Search for strike K_h such that call has same price as put
  - Then: $(1 + \alpha_h) = \frac{K_h}{S_t}$
- Theoretically: actually sell call & buy put
  - Perfect hedge for buffer fund

Bounds on Returns (4)

- For a fixed guarantee α_l the upper level α_h changes each year, due to changing market prices of calls and puts

- **Question:** can we say something (in general) about relation between α_l and α_h?
  - Yes, we can ☺
  - α_l and α_h (almost) symmetric around 1-year risk-free rate

Put and Call option prices (Black-Scholes)

- We could use the Black-Scholes formula for options:
  - $Call = e^{-r_1}[F_{t+1}N(d_1) - KN(d_2)]$
  - $Put = e^{-r_1}[KN(-d_2) - F_{t+1}N(-d_1)]$
  - F_{t+1} is 1-year forward price $F_{t+1} = S_t e^{r_1}$ and r_1 is 1-year rate
- But B-S model makes very strong assumptions
  - Constant interest rates, constant volatility

- Can we derive more general ("model free") result?

"Model Free" Option Prices

- General expression for option prices
  - Use 1-year discount bond $D_1 = e^{-r_1}$ as *numéraire*

$$Call = D_1 \mathbb{E}_t^{\mathbb{Q}}[(S_{t+1} - K)^+]$$

$$= D_1\left(\mathbb{E}_t^{\mathbb{Q}}\left[S_{t+1} \mathbb{I}_{S_{t+1}>K}\right] - \mathbb{E}_t^{\mathbb{Q}}\left[K\, \mathbb{I}_{S_{t+1}>K}\right]\right)$$

$$= D_1\left(F_{t+1} \Pr^{\mathbb{S}}[S_{t+1} > K] - K \Pr^{\mathbb{Q}}[S_{t+1} > K]\right)$$

  - Note: $F_{t+1} = \mathbb{E}_t^{\mathbb{Q}}[S_{t+1}]$ is the forward price
    - $\Pr^{\mathbb{Q}}[S_{t+1} > K]$ corresponds to B-S term $N(d_2)$
    - Extract $\mathbb{Q}$-distribution of S_{t+1} from option prices

"Model Free" Option Prices (2)

- Sensitivity of price to change in strike
  - $\frac{\partial Call}{\partial K} = -D_1 \Pr^{\mathbb{Q}}[S_{t+1} > K]$
  - Depends on $\mathbb{Q}$-probability of event $[S_{t+1} > K]$
- Negative sign means:
  - Higher strike *K* leads to lower price for call
- For put option we have
  - $\frac{\partial Put}{\partial K} = D_1 \Pr^{\mathbb{Q}}[S_{t+1} < K] = D_1\left(1 - \Pr^{\mathbb{Q}}[S_{t+1} > K]\right)$
  - Higher strike *K* leads to higher price for put

“Model Free” upper and lower bounds

- Define: $K_l = (1+\alpha_l)S_t = e^a F_{t+1}$ and $K_h = (1+\alpha_h)S_t = e^b F_{t+1}$
- We are searching for (implicit) function $b(a)$ such that put and call prices are always matched
  - Note: for $a = 0$ we have ATMF option with $K = F_{t+1}$
  - ATMF call has same price as ATMF put, hence: $b(0) = 0$
- General result:
  - $b(a) = -a\frac{\Pr^{\mathbb{Q}}[S_{t+1} > F_{t+1}]}{1-\Pr^{\mathbb{Q}}[S_{t+1} > F_{t+1}]} + \mathcal{O}(a^2) = -a\frac{N\left(\frac{1}{2}\sigma_{ATMF}\right)}{1-N\left(\frac{1}{2}\sigma_{ATMF}\right)} + \mathcal{O}(a^2)$
  - Factor “Pr/(1-Pr)” is (a bit) larger than 1, increasing in σ_{ATMF}

Values for upper bounds ("exact" values for B-S)

$r, \downarrow \sigma \rightarrow$	3%	4%	5%	6%	7%	8%	9%	10%	11%	12%	13%
0%	2,08%	2,09%	2,11%	2,13%	2,14%	2,16%	2,18%	2,20%	2,21%	2,23%	2,25%
0,50%	3,11%	3,13%	3,15%	3,17%	3,19%	3,21%	3,24%	3,26%	3,28%	3,30%	3,32%
1%	4,15%	4,18%	4,20%	4,22%	4,25%	4,28%	4,30%	4,33%	4,35%	4,38%	4,41%
1,50%	5,20%	5,23%	5,26%	5,29%	5,32%	5,35%	5,38%	5,41%	5,44%	5,47%	5,50%
2%	6,26%	6,30%	6,33%	6,36%	6,39%	6,43%	6,46%	6,50%	6,53%	6,57%	6,61%
2,50%	7,34%	7,37%	7,41%	7,44%	7,48%	7,52%	7,56%	7,60%	7,64%	7,68%	7,72%
3%	8,42%	8,46%	8,49%	8,54%	8,58%	8,62%	8,66%	8,71%	8,75%	8,80%	8,85%
3,50%	9,51%	9,55%	9,59%	9,64%	9,68%	9,73%	9,78%	9,83%	9,88%	9,93%	9,98%
4%	10,62%	10,66%	10,70%	10,75%	10,80%	10,85%	10,91%	10,96%	11,01%	11,07%	11,12%
4,50%	11,72%	11,77%	11,82%	11,88%	11,93%	11,98%	12,04%	12,10%	12,16%	12,22%	12,28%
5%	12,86%	12,90%	12,95%	13,01%	13,07%	13,13%	13,19%	13,25%	13,31%	13,38%	13,44%
5,50%	14,00%	14,04%	14,10%	14,15%	14,22%	14,28%	14,35%	14,41%	14,48%	14,55%	14,62%
6%	15,16%	15,19%	15,25%	15,31%	15,38%	15,44%	15,51%	15,59%	15,66%	15,73%	15,81%

Table 1: Value of the upper threshold, α_h, for a value of α_l of -2% for different values of the risk-free interest rate, r, and the volatility of the underlying investment portfolio, σ

Values for upper bounds (approx values)

- Compare approximation to exact values
  - (for r_1=0% and α_l = -2%)

	3%	4%	5%	6%	7%	8%	9%	10%	11%	12%	13%
Approx	2.09%	2.11%	2.12%	2.14%	2.16%	2.18%	2.19%	2.21%	2.23%	2.25%	2.27%
Exact	2.08%	2.09%	2.11%	2.13%	2.14%	2.16%	2.18%	2.20%	2.21%	2.23%	2.25%

  - ("Exact" values are using Black-Scholes)

Conclusions

- Intergenerational risk sharing via buffer fund to smooth fluctuations in annual investments returns
- Finance guarantee via “cap” on annual returns
- Only viable if cap-level matches guarantee-level
  - α_l and α_h (almost) symmetric around 1-year risk-free rate
  - “Model Free” derivation of upper-bound