Demand for annuities and long-term care insurance with recursive utility: Impact of housing

Mengyi Xu[*], Jennifer Alonso-García, Adam W. Shao, and Michael Sherris

School of Risk and Actuarial Studies and ARC Centre of Excellence in Population Ageing Research (CEPAR), UNSW Sydney, Australia.

September 11, 2017

Paper prepared for 2017 Netspar Pension Day

PLEASE DO NOT QUOTE OR CITE WITHOUT AUTHORS' PERMISSION

Abstract

Housing assets are often the single most important asset class in a retired homeowner's portfolio. They are rarely spent except for moving to long-term care facilities. This means home equities can be used to insure against uncertain out-of-pocket healthcare costs. In addition, retirees also face the risk of outliving their financial resources, which can be protected by life annuities. We use a lifecycle model to study the optimal consumption and product choice between long-term care insurance (LTCI) and life annuities for a retired homeowner whose preference is represented by a recursive utility. The retiree faces uncertain lifespan, uncertain out-of-pocket healthcare expenditure, and house price risk. Our results show that the presence of home equity generally increases the optimal annuitisation rate when retirees have no access to LTCI. The reason is that home equity serves as a form of precautionary savings to cover healthcare costs, and it can also be bequeathed to fulfil the bequest motive. When both products are available, the presence of home equity can make annuities more attractive provided that retirees have sufficient liquid assets. If the amount of liquid assets is low, the spending on LTCI can impair demand for life annuities. Given retirees tend to liquidate housing assets in the event of moving to a long-term care facility, home equity can weaken demand for LTCI.

Keywords: Housing, Life annuities, Long-term care insurance, Recursive utility, Lifecycle model

*[Corresponding author]. Email: m.xu@unsw.edu.au; Postal address: Australian Research Council Centre of Excellence in Population Ageing Research, UNSW Business School, NSW 2052, Australia; Phone: +61-2-9385 7008.

1 Introduction

Demographic changes and pension scheme transitions have exposed individuals to greater challenges in financing their retirement. Improved life expectancy at retirement has been observed across the globe. It is projected that by 2050, female life expectancy at 65 will exceed 30 years in countries like Japan and Singapore; even in less-developed countries like Afghanistan and Somalia, the figure will reach 15 years (He et al., 2016). Longer life expectancy at 65 means retirees face a harder time allocating their financial resources across time to avoid outliving their wealth. As life expectancy at older ages increases, individuals are also likely to spend more time in disability that requires expensive healthcare costs (see e.g. Crimmins and Beltrán-Sánchez, 2011). Uncertain out-of-pocket healthcare costs represent a key source of risk during retirement. Older Australians with five or more chronic conditions could spend almost six times on out-of-pocket healthcare costs as much as those without chronic conditions (McRae et al., 2012). The burden of retirement planning on individuals is further exacerbated by the shifts from defined benefit to define contribution plans, which significantly increase individual's responsibility in managing various retirement risks.

There has been growing interest in retirement products such as annuities, long-term care insurance to address the need in retirement. Life annuities are an effective instrument to hedge against the risk of outliving one's financial resources. Since the seminal work of Yaari (1965), a great amount of literature has been devoted to the role of life annuities in retirement planning (for a review, see Brown, 2009). In addition, long-term care insurance (LTCI) can alleviate the burden of healthcare costs. Since Pauly (1990) first raises the idea of combing life annuities with LTCI, a number of recent papers (see e.g. Murtaugh et al., 2001; Brown and Warshawsky, 2013; Wu et al., 2016) have further explored the idea.

Among the studies looking at optimal consumption and portfolio choice at retirement, only a handful of papers consider home equity. Yet the role of housing wealth among the elderly can hardly be overlooked. Retirees tend to have high home ownership rates. In both the U.S. and Australia, households headed by people aged 65 and over have one of the highest home ownership rates among all age groups over the past few decades (U.S. Census Bureau, 2017; Reserve Bank of Australia, 2015). Retired homeowners have a large fraction of household portfolio held in the form of home equity. The median ratio of home equity to all assets is estimated to be 0.56 among the elderly homeowners in the U.S. (Davidoff, 2009). Home equity is generally not reduced among people who continue to own their homes (Venti and Wise, 1990; Venti and Wise, 1991; Venti and Wise, 2004). The preserved home equity will be left to heirs. Selling the house is often associated with losing spouse or entering into a nursing home (Walker, 2004; Venti and Wise, 2004). This means home equity can supplement or replace LTCI to pay health expenses.

The present paper aims to study the impact of housing wealth on demand for life annuities and LTCI. We build a multi-period lifecycle model for a single retired homeowner who faces uncertain lifespan, uncertain out-of-pocket health expenditure, and house price risk. Individual preferences are modelled using the Epstein-Zin-Weil-type utility (Epstein and Zin, 1989; Epstein and Zin, 1991; Weil, 1989). The Epstein-Zin model is commonly used in the literature (see e.g. Pang and Warshawsky, 2010; Blake et al., 2014; Yogo, 2016) along with the power utility model. The Epstein-Zin model is preferred over the power utility model for its ability to separately identify the risk aversion and elasticity of intertemporal substitution (EIS). By contrast, the power utility model cannot distinguish the impact of these two factors since the model imposes that one is the inverse of the other. Individuals can choose between an ordinary life annuity and LTCI at the point of retirement. Both products have actuarially fair prices. Home equity will either be bequeathed or be liquidated at the point of moving into a long-term care facility. This assumption is based on the empirical evidence that home equity is rarely spent before death except for moving into a nursing home. Davidoff (2009) makes a similar assumption. The probabilities of health state transitions are calibrated to the data from U.S. Health and Retirement study, and the other parameters in the lifecycle model take commonly used values in the literature.

Our results show that the presence of home equity generally increases the optimal annuitisation rate when retirees have no access to LTCI. Prior literature has shown that bequest motive (Lockwood, 2012) or precautionary savings for healthcare costs (Sinclair and Smetters, 2004; Turra and Mitchell, 2008) can weaken demand for life annuities. For retired homeowners who tend to sell the property at the time of moving into a nursing home, home equity can serve as a bequest and be a form of precautionary savings. As a result, demand for annuities is enhanced in the presence of home equity. When both life annuities and LTCI are available, the presence of home equity can make life annuities more attractive provided that the retiree has sufficient liquid assets. If the amount of liquid assets is low, the spending on purchasing LTCI can impair demand for life annuities. Given retirees tend to liquidate housing wealth in the event of moving to a long-term care facility, home equity typically crowds out demand for LTCI. The sensitivity analysis on preference parameters shows the importance of separately identifying risk aversion and EIS. A higher degree of risk aversion and a lower level of EIS have opposite effects on demand for life annuities and LTCI. Since the power utility model imposes an inverse relationship on risk aversion and EIS, the Epstein-Zin model is more suitable to determine demand for life annuities and LTCI when individuals have various levels of risk aversion and EIS.

The rest of the paper is organised as follows. Section 2 briefly surveys the related literature. Section 3 discusses the lifecycle model in detail. Section 4 presents the findings from the base case analysis and sensitivity analysis on wealth endowment and preference parameters. Section 5 concludes.

2 Literature review

The present paper is related to several strands of literature. The first few strands of literature study demand for annuities and/or LTCI when individuals face uncertain healthcare cost. One major strand of literature tries to link the lack of annuitisation (i.e. the so-called 'annuitisation puzzle') to medical expenditures (see e.g. Sinclair and Smetters, 2004; Turra and Mitchell, 2008; Pang and Warshawsky, 2010; Peijnenburg et al., 2015). A growing strand of literature considers the interaction between annuities and LTCI, and a number of papers show annuities and LTCI are complements (see e.g. Ameriks et al., 2008; Wu et al., 2016). Our contribution is to include housing wealth in the household portfolios to examine its impact on demand for annuities and LTCI.

The next strand of literature studies the optimal portfolio choice in the presence of housing investment to explain the composition of household portfolios over the lifecycle (see e.g. Flavin and Yamashita, 2002; Cocco, 2004; Hu, 2005; Yao and Zhang, 2005). They usually focus on the sources of risk in the pre-retirement stage, e.g. labour income risk. Our paper considers the common risks in the post-retirement phase, including uncertain lifespan and health expenditure.

The last strand of literature studies the optimal consumption and portfolio choice for retired homeowners. Davidoff (2009) uses a two-period model to analyse joint demand for annuities and LTCI, and the role of home equity in determining the demand. He shows that illiquid home equity can crowd out demand for annuities and LTCI separately, and can reverse the complementarity between annuities and LTCI. Davidoff (2010) uses a one-period model to show that home equity can weaken demand for LTCI. Hanewald et al. (2016) compare the two home equity release products, reverse mortgage and home reversion plan, and conclude that reverse mortgage gives higher utility gains. Shao et al. (2017) build on the work of Davidoff (2010) and Hanewald et al. (2016) to investigate the complementarity between LTCI and reverse mortgage in a multi-period setting. Yogo (2016) extends the work in the previous strand of literature to study the optimal consumption, health expenditure, and portfolio allocation when households face health risk. Andréasson et al. (2017) study the optimal housing at retirement, optimal consumption and risky asset allocation over the course of retirement. They focus on the impact of means-tested public pension on the optimal decisions. Our contribution to this strand of literature is to use a recursive utility framework to study demand for annuities and LTCI.

3 Lifecycle model in retirement

We set up a discrete-time lifecycle model starting at retirement. The model consists of a series of one-year period that is indexed by $t \in \{1, 2, \cdots, T, T+1\}$[1]. The individual retires at $t = 1$ aged 65, and her maximum attainable age is 100, so $T = 36$. All variables are defined in real terms.

[1]Note that the latest possible consumption occurs at $t = T$. The last time index $T + 1$ is for the purpose of bequest only.

3.1 Health dynamics and costs

We follow Ameriks et al. (2011) to model the retiree's health status with states '1' (healthy), '2' (mildly disabled), '3' (severely disabled), and '4' (dead). The categorisation of the first three states is based on the number of difficulties in independently performing Activities of Daily Livings (ADLs). There are usually a total of six ADLs: dressing, walking, bathing, eating, transferring and toileting. Mildly disabled state is defined as having 1 – 2 ADL difficulties, and severely disabled state is defined as having 3 – 6 ADL difficulties. The health state at period t is denoted as s_t.

The health state transitions are modelled using a Markov process. Fong et al. (2015) shows a significant proportion of the elderly can recover from disabled state to healthy state. On the other hand, severe disability is usually chronic in nature that substantially reduces the possibility of recovery (Ferri and Olivieri, 2000; Olivieri and Pitacco, 2001). We therefore allow for transition from mildly disabled state to healthy state and do not allow for recoveries from severely disabled state. Figure 1 depicts the health state transitions, where σ_{jk} ($j \in \{1,2,3\}, k \in \{1,2,3,4\}$) denotes the transition intensity.

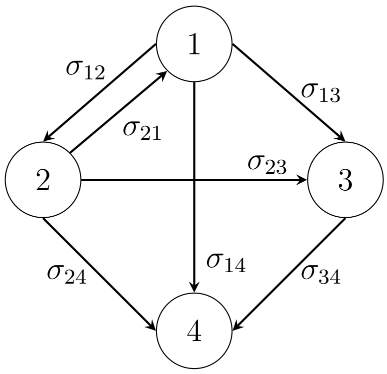

Figure 1. Four-state Markov process that models health state transitions.

Given the transition intensities, σ_{jk}, the transition probabilities, $\pi(k|j)$, can be solved through Kolmogorov equations, i.e.

$$\begin{pmatrix} \pi(1|1) & \pi(2|1) & \pi(3|1) & \pi(4|1) \\ \pi(1|2) & \pi(2|2) & \pi(3|2) & \pi(4|2) \\ \pi(1|3) & \pi(2|3) & \pi(3|3) & \pi(4|3) \\ \pi(1|4) & \pi(2|4) & \pi(3|4) & \pi(4|4) \end{pmatrix} = \exp\left[\begin{pmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} & \sigma_{14} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} & \sigma_{24} \\ 0 & 0 & -\sigma_{34} & \sigma_{34} \\ 0 & 0 & 0 & 0 \end{pmatrix}\right],$$

where $\sigma_{11} = -(\sigma_{12} + \sigma_{13} + \sigma_{14})$, $\sigma_{22} = -(\sigma_{21} + \sigma_{23} + \sigma_{24})$, and $\exp[\cdot]$ refers to the matrix exponential.

We follow Ameriks et al. (2011) to model the out-of-pocket health expenditure ($h_t \equiv h(s_t, t)$) as a deterministic process given the health state, s_t. Since the healthcare inflation usually exceeds that of the consumer price index, it is assumed that the relative price of healthcare increases at a rate of q per annum.

3.2 Housing and financial assets

Given that a large majority of retired homeowners have paid off their mortgages, the model assumes the individual lives in a mortgage-free home at retirement. In addition, empirical data shows that housing assets are rarely drawn upon unless the retiree moves to a long-term care facility (see e.g. Venti and Wise, 2004). It is assumed that the retiree will liquidate the house when she becomes severely disabled and subsequently moves to a nursing home. The house has a gross rate of return $R_{H,t+1}$ from time t to time $t+1$, where $\ln(R_{H,t+1})$ follows a normal distribution with mean μ_H and variance σ_H^2. The liquid assets earn a constant risk-free return of R_f. We abstract from the equity market.

3.3 Retirement products

At retirement, the individual has access to two types of retirement products, life annuities and long-term care insurance (LTCI), both of which are offered by private companies. The retiree decides the proportion (α) of liquid assets to annuitise and the percentage coverage (λ) of LTCI to purchase. The decisions are made at retirement only. The public offering of similar products is not explicitly considered in the model. Nevertheless, the individual's endowment at retirement can be perceived as including the expected present value of public pension paid during retirement, and the out-of-pocket health expenditure can be seen as net of any publicly funded schemes.

The life annuity is of an ordinary type that provides annual level payment for the remaining lifetime of the annuitant. The payment starts at the beginning of the first period. The annuity is charged at an actuarially fair price. Given an α proportion of liquid assets annuitised at retirement, the annual income from annuity is given by

$$\mathsf{Y} = \frac{\alpha \mathsf{B}}{\sum_{t=1}^{T} R_f^{-(t-1)} {}_{t-1}p_{65,s_1}}, \tag{1}$$

where B denotes the initial endowment of liquid assets, ${}_{t-1}p_{65,s_1}$ denotes the probability that a 65-year-old individual with health state s_1 will survive for the next $(t-1)$ years.

The LTCI covers healthcare costs when the policyholder is severely disabled (i.e. health state 3). The premium is assumed to be paid as a lump sum and to exclude any loadings on the product. The actuarially fair premium (P) for a full coverage LTCI policy is given by

$$\mathsf{P} = \sum_{t=2}^{T} R_f^{-(t-1)} \pi(s_t = 3|s_1) h(s_t = 3, t). \tag{2}$$

3.4 Budget constraints and wealth dynamics

In the first period, the retiree is endowed with liquid wealth of B and housing wealth of $\mathsf{W^H}$, and the retiree is in the healthy state (i.e. health state 1). She then decides the proportion of liquid assets to annuitise and the LTCI coverage to purchase. After that, she receives income from annuity (if any), incurs the healthcare cost, and decides how much to consume. Let B_1 denote the amount of liquid assets available after purchasing the retirement products. It is given by

$$B_1 = (1-\alpha)\mathsf{B} - \lambda \mathsf{P}, \; B_1 \geq 0. \tag{3}$$

Starting from the second period, the retiree enters the period t with health state s_t and wealth W_t, which consists of housing wealth W_t^H and liquid wealth B_t. Note that W_t, W_t^H, and B_t denote the amount available at the beginning of the period t (i.e. before any action is taken) except for B_1, which is specified otherwise in Equation (3). The timing of events is as follows.

1. If $s_t = 4$, the individual is deceased, so the wealth W_t is bequeathed.

2. If $s_t < 4$, one of the following events will occur.

   (a) If $s_t = 3$ and $s_{t-1} \in \{1, 2\}$, the individual will liquidate the home equity and move into a residential care facility.

(b) If $s_t = 3$ and $s_{t-1} = 3$, the individual will remain staying at the residential care.

(c) If $s_t < 3$, the individual will remain living at home.

3. If $s_t < 4$, the health costs h_t are incurred and a consumption decision (C_t) is made. The remaining liquid assets earn a risk-free return R_f.

The chosen consumption level must not fall below the consumption floor C^f to ensure a minimum standard of living. If the individual's budget cannot support the minimum consumption level, we assume the government will provide subsidy to increase the consumption level to C^f, and that the liquid wealth in the next period will be zero.

The budget constraint for liquid assets B is given by

$$\begin{aligned} &B_2 = \left(B_1 + \mathsf{Y} - h_1 - C_1\right)^+ R_f; \\ &\text{for } t \in \{2, 3, \cdots, T\}, \\ &B_{t+1} = \begin{cases} \left(B_t + \mathsf{Y} - h_t - C_t\right)^+ R_f & \text{if } s_t \in \{1, 2\} \\ \left(B_t + \mathsf{Y} + W_t^H \mathbb{1}_{\{s_{t-1} \in \{1,2\}\}} - (1-\lambda) h_t - C_t\right)^+ R_f & \text{if } s_t = 3 \end{cases}, \end{aligned} \tag{4}$$

where $(\cdot)^+$ is defined as $\max\{\cdot, 0\}$.

The budget constraint for total wealth W is given by

$$\begin{aligned} &W_2 = B_2 + W_1^H R_{H,2}, \quad \text{where } W_1^H = \mathsf{W}^\mathsf{H}; \\ &\text{for } t \in \{2, 3, \cdots, T\}, \\ &W_{t+1} = \begin{cases} B_{t+1} + W_t^H R_{H,t+1} & \text{if } s_t \in \{1, 2\} \\ B_{t+1} & \text{if } s_t = 3 \end{cases}. \end{aligned} \tag{5}$$

3.5 Preferences

Individuals in the model are assumed to have Epstein-Zin-Weil-type preferences (Epstein and Zin, 1989; Epstein and Zin, 1991; Weil, 1989) over non-housing consumption and a bequest. The housing service consumption is not directly included in the utility function. The housing wealth contributes to the utility through bequest or liquidation of housing that alleviates the budget constraint caused by excessive medical care costs.

The Epstein-Zin model generalises the power utility model in that it can separately identify the risk aversion and elasticity of intertemporal substitution (EIS). The two elements are intrinsically different. Risk aversion describes an individual's willingness to substitute consumption across different states of the world, whereas EIS describes an individual's willingness to substitute consumption over time. When the individual's EIS (ψ) is the reciprocal of the coefficient of relative risk aversion (γ), the Epstein-Zin model reduces to the power utility model.

The preferences are specified by

$$
\begin{aligned}
V_t &\equiv V(B_t, W_t^H, s_t, t) \\
&= \max_{O_t} \Bigg\{ (1-\beta) C_t^{1-\rho} + \beta \Bigg[\mathbb{E}_t \Bigg[\sum_{k \neq 4} \pi(s_{t+1} = k | s_t) V(B_{t+1}, W_{t+1}^H, s_{t+1} = k, t+1)^{1-\gamma} \\
&\qquad + \pi(s_{t+1} = 4 | s_t) b^{\gamma} W_{t+1}^{1-\gamma} \Bigg] \Bigg]^{\frac{1}{\theta}} \Bigg\}^{\frac{1}{1-\rho}}, \quad \theta = \frac{1-\gamma}{1-\rho}; \qquad (6)
\end{aligned}
$$

$$
O_t = \begin{cases} \{\lambda, \alpha, C_t\}, & \text{for } t = 1; \\ \{C_t\}, & \text{for } t = 2, \cdots, T. \end{cases}
$$

The notation V_t is the indirect utility value at time t, β the subjective discount factor, ρ the inverse of EIS (i.e. $\rho = 1/\psi$), $\mathbb{E}$ the expectation operator, b the strength of bequest motive. The subjective discount factor measures an individual's impatience to defer consumption. It takes values between zero and one, with a lower value meaning less willingness to postpone the consumption. The strength of bequest motive takes non-negative values, with a higher value meaning a stronger bequest motive.

3.6 Optimisation problem and solution method

Individuals optimise over consumption, annuitisation rate, and insurance coverage to maximise the expected lifetime utility in (6), subject to conditions (1) to (5). We set up grid points on liquid wealth, housing wealth, and current health state to solve the optimisation problem. The method of endogenous grid points (Carroll, 2006) is used to set up the grid points for the liquid assets. The grid points on housing wealth are given exogenously. The lognormal distribution of house price growth is discretised by Gauss-Hermite quadrature. The first-order condition for consumption can be solved analytically to speed up the solution process. The analytical form is derived in Appendix A.

The optimisation problem is solved backward, starting from the last period. For the points not lying on the grid, a hybrid interpolation method introduced in Ludwig and Schön (2016) is used to find the optimal consumption and the indirect utility value. Figure 2, adapted from Figure 5 of Ludwig and Schön (2016), illustrates the method. Both the before-consumption liquid wealth[2] and housing wealth are used to construct the grid points, denoted by $G^{\cdot,\cdot}$ in the figure. Given the before-consumption wealth w_{t+1} and housing wealth w^H_{t+1}, the following procedures are employed to find the interpolated value. First, locate the two rows $G^{\cdot,j}$ and $G^{\cdot,j+1}$ in the exogenous dimension (which is housing wealth) that form the most narrow bracket of w^H_{t+1}. Compute the weights (z and $1-z$) based on the relative distance to the two rows of grid points. Second, perform linear interpolation in each of the two rows given the before-consumption liquid assets, w_{t+1}. Finally, the interpolated value is the weighted average of the interpolated values from each row, using the weights found in the first step.

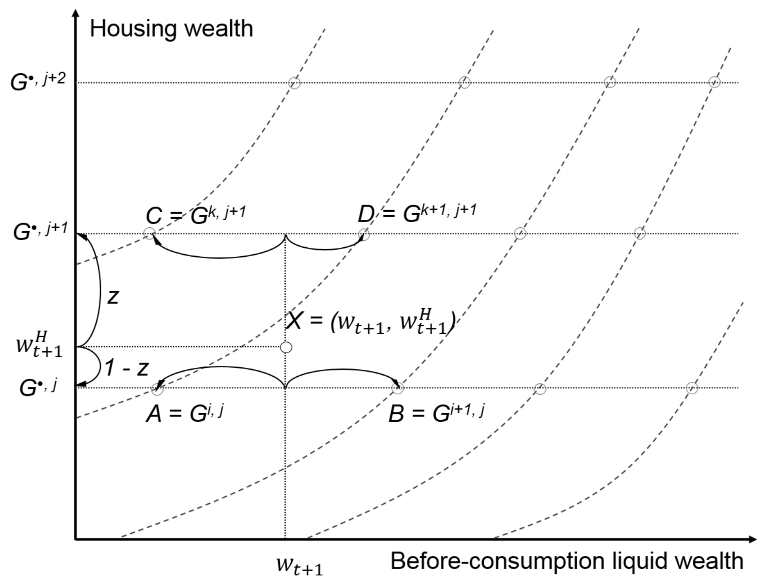

Figure 2. Illustration of the hybrid interpolation method.

The optimal annuitisation rate and LTCI coverage are solved in the first period using the following steps. First set up the grid points on annuitisation rate and LTCI coverage. On each

[2]Before-consumption liquid assets refer to the assets that are ready to be consumed, i.e. after any annuity income, any housing liquidation, and medical expenditure net of LTCI coverage.

grid point, solve the optimal consumption and indirect utility levels backwards from the last period to the first period. Given the initial liquid wealth and housing wealth, the indirect utility value in the first period for a healthy individual can be found through the hybrid interpolation method. The optimal annuitisation rate and LTCI coverage are found by searching for the grid point that gives the highest value of indirect utility.

3.7 Model parameterisation

3.7.1 Health dynamics

The health state transition is estimated using the data from U.S. Health Retirement Study (HRS). HRS surveys a nationally representative sample of Americans over age 50 every two years, starting from 1992. The data from 1998 to 2010 is used due to inconsistent question structure before 1998. The data for female is chosen to calculate the crude transition rates, which are then graduated using Poisson generalised linear model (Fong et al., 2015). We choose female data since they face greater challenges in retirement planning. Females have longer life expectancy than males, and they tend to spend more years in disabled state (Fong et al., 2015).

The estimation procedure begins with counting number of transitions and exposure years for each integer age between 50 and 100. The aggregate results in five-year interval are shown in Tables 1 and 2. The crude transition rates are then graduated using a generalised linear model (GLM) with the log link function. In particular, the number of transitions at age x is assumed to follow a Poisson distribution with mean (m_x) defined as a polynomial function of age with degree K

$$m_x = e_x \sum_{k=0}^{\mathsf{K}} \eta_k x^k, \tag{7}$$

where e_x is the central exposure to risk for x-year-old individuals, η_k the coefficients of the polynomial. The degree of polynomial is selected based on Akaike information criterion corrected for sample size (AICc), Bayesian information criterion (BIC), and the likelihood ratio test. Table 3 shows the results of selection criteria. The chosen degree of polynomial value is in bold for each set of nested models.

Table 1. Number of transitions between different health states.

	$1 \to 2$	$1 \to 3$	$1 \to 4$	$2 \to 1$	$2 \to 3$	$2 \to 4$	$3 \to 4$
50 – 54	67	21	8	52	13	2	4
55 – 59	280	40	55	212	69	27	16
60 – 64	458	74	114	436	129	37	36
65 – 69	553	112	193	474	147	86	79
70 – 74	575	107	226	441	178	97	86
75 – 79	579	144	257	349	157	116	171
80 – 84	570	162	315	338	190	166	242
85 – 89	445	172	302	235	211	212	312
90 – 94	218	92	160	86	156	172	296
95 – 100	52	24	51	18	76	75	174
Total	3,797	948	1,681	2,641	1,326	990	1,416

Note: '1' is healthy state, '2' mildly disabled state, '3' severely disabled state.

Table 2. Number of exposure years in healthy, mildly disabled, and severely disabled states.

	Healthy	Mildly disabled	Severely disabled
50 – 54	4,527.18	361.92	121.51
55 – 59	10,816.97	1,136.76	387.61
60 – 64	15,721.89	1,811.16	692.93
65 – 69	16,610.65	2,146.23	802.31
70 – 74	13,975.53	2,079.22	948.19
75 – 79	10,807.98	2,164.77	1,071.76
80 – 84	7,512.86	2,131.81	1,242.44
85 – 89	3,870.87	1,826.11	1,457.01
90 – 94	1,235.42	965.27	1,006.33
95 – 100	235.92	265.35	421.37
Total	85,315.27	14,888.60	8,151.45

Table 3. Model selection of the Poisson generalised linear model.

K	AICc	BIC	D_c	ΔD_c
		Disability		
σ_{12}: *healthy to mildly disabled*				
1	334.84	337.96	87.51	
2	304.56	**309.05**	**54.90**	**32.62*****
3	**303.87**	309.61	51.74	3.16*
σ_{13}: *healthy to severely disabled*				
1	260.49	263.60	64.61	
2	247.74	**252.23**	**49.53**	**15.08*****
3	**246.66**	252.40	45.99	3.54*
σ_{23}: *mildly disabled to severely disabled*				
1	316.44	319.55	100.70	
2	279.25	**283.74**	**61.17**	**39.52*****
3	**279.14**	284.88	58.60	2.57
		Recovery		
σ_{21}: *mildly disabled to healthy*				
1	301.16	304.27	73.30	
2	**292.57**	**297.06**	**62.38**	**10.92*****
3	294.97	300.72	62.32	0.06
		Mortality		
σ_{14}: *healthy to dead*				
1	272.53	275.64	51.01	
2	**265.01**	**269.50**	**41.16**	**9.85*****
3	267.02	272.77	40.71	0.45
σ_{24}: *mildly disabled to dead*				
1	246.79	249.90	45.02	
2	**243.68**	**248.18**	**39.58**	**5.44****
3	244.11	249.85	37.54	2.04
σ_{34}: *severely disabled to dead*				
1	**245.02**	**248.13**	**29.59**	
2	247.35	251.85	29.58	0.00
3	247.45	253.20	27.22	2.36

Note: D_c is the residual deviance statistics. ΔD_c denotes the test statistics for the likelihood ratio test. * is for statistic that is significant at the 10% level, ** at the 5% level, *** at the 1% level.

3.7.2 Other parameters

The other parameters used in the numerical simulation take the commonly used values in the literature. They are displayed in Table 4. The sources of the parameters, unless otherwise specified, are listed in the brackets.

Table 4. The parameter values used for the base case.

Parameter	Explanation	Value
Asset returns (Yogo, 2016)		
R_f	Risk free rate	1.025
μ_H	Parameters of the lognormal distribution	0.34%
σ_H^2	of house price growth	3.5%
Consumption floor (Ameriks et al., 2011)		
C^f	Floor for healthy and mildly disabled states	\$4,630
	Floor for severely disabled states	\$5,640
Health expenditure (Ameriks et al., 2011)		
$h(s_1, 1)$	Initial cost for healthy state	\$1,000
$h(s_2, 1)$	Initial cost for mildly disabled state	\$10,000
$h(s_3, 1)$	Initial cost for severely disabled state	\$50,000
$q^{\dagger}$	Health expenditure inflation in excess of CPI inflation	1.90%
Preference (Pang and Warshawsky, 2010)		
b	Strength of bequest motive	2
β	Subjective discount factor	0.96
γ	Coefficient of relative risk aversion	5
ψ	Elasticity of intertemporal substitution	0.5

† *Source*: Yogo (2016).

4 Results

4.1 Base case analysis

In the base case analysis the individual is endowed with \$220,000 liquid wealth and \$280,000 housing wealth at retirement. The \$220,000 liquid wealth is based on the median level of total wealth (consisting of pre-annuitised wealth and liquid financial wealth) for a single woman U.S. household in the HRS estimated by Peijnenburg et al. (2015). The \$280,000 housing wealth leads to a home-equity-to-all-assets ratio of 0.56, which is consistent with the median ratio among homeowners estimated by Davidoff (2009). The individual is healthy at retirement. Based on the estimated health transition probabilities, the annuity costs about \$14.89 for \$1 annual income, and the full coverage of LTCI costs \$94,752.31.

After solving the optimal decision rules defined on the state space, the time-series profiles of retiree's optimal consumption can be obtained through simulation. Specifically, we first simulate house price growths and health states, and then use the optimal policy rules to calculate the optimal consumption. The simulation is run for 200,000 times. Figure 3 shows the simulated housing wealth values in the absence of liquidation. Should the retiree fall into the severely disabled state, the amount of cash from liquidating housing asset alone can support the health expenditure for several years.

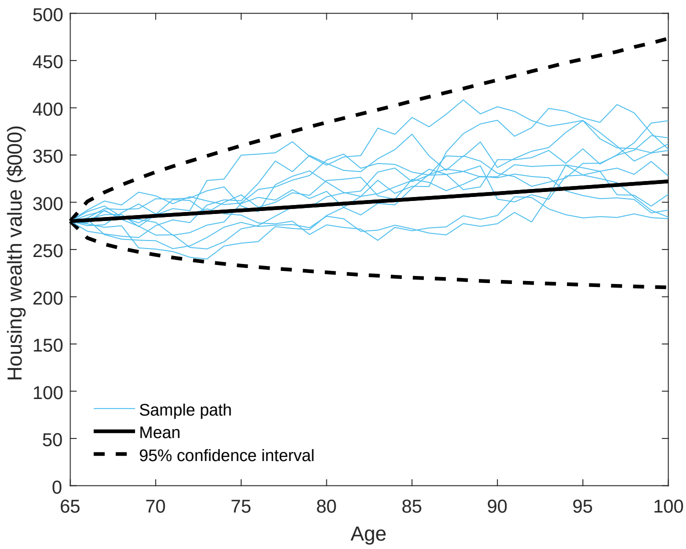

Figure 3. Simulated housing asset values in the absence of liquidation. The individual is endowed with \$280,000 housing asset at retirement.

Figure 4 shows the survival curve and the simulated proportions of survivors in each health state for individuals who are healthy at retirement. The estimated health transition probabilities predict that a 65-year-old healthy female has about 50% chance of living beyond age 85, and that the probability of becoming severely disabled increases exponentially after age 85. Table 5 summarises the number of years spent in each health state and the age of entering into each health state. Conditional upon becoming severely disabled, the average age of occurrence is around 82. The remaining life expectancy after becoming severely disabled is about two years.

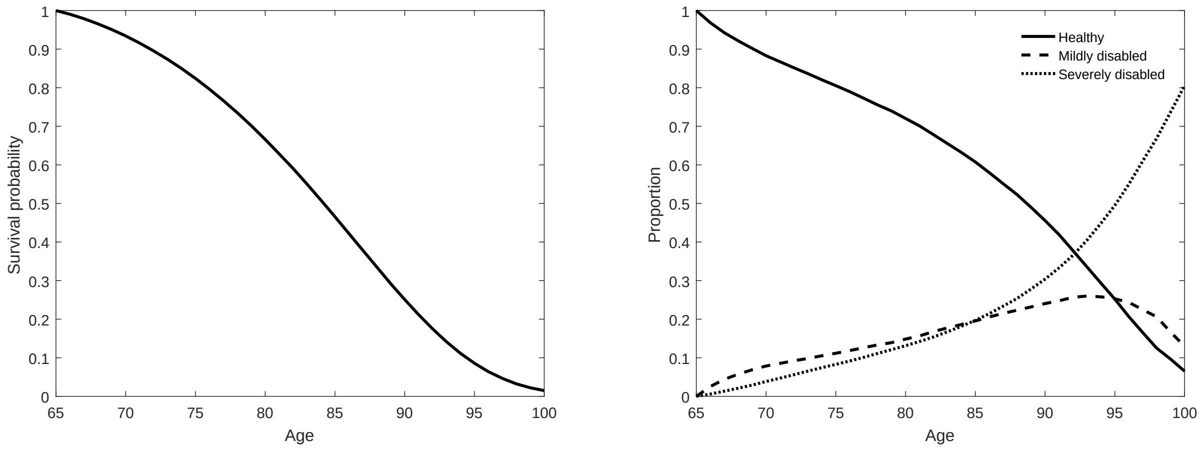

Figure 4. (Left Panel) Survival curve and (Right Panel) simulated proportions of survivors in each health state. Individuals are healthy at retirement.

Table 5. Number of years spent in each health state and age of entering into each health state conditional upon occurrence: mean, standard deviation (Std), and 95% confidence interval (CI).

Health state	Duration			Starting age		
	Mean	Std	95% CI	Mean	Std	95% CI
Healthy	14.9	7.5	(2, 29)	65	0	(65, 65)
Mildly disabled	2.3	3.3	(0, 11)	76.9	7.3	(66, 92)
Severely disabled	2.1	3.8	(0, 13)	81.8	8.2	(67, 96)

Table 6 shows the optimal annuitisation rate and the optimal LTCI coverage for the base case. For the purpose of comparison, the optimal product choices in the absence of housing wealth are also displayed. When annuities alone are available in the market, illiquid housing wealth significantly enhances the demand for annuities. The increased annuitisation rate is related to the dual role of housing wealth in the model. A large proportion of precautionary savings for healthcare costs are held in the form of home equity. If the wealth locked in the home equity is not released, it will be bequeathed to fulfil the bequest motive. Prior research has found that the need for liquidity to cover sizeable health expenditure (Sinclair and Smetters, 2004; Turra and Mitchell, 2008) and bequest motive (Lockwood, 2012) tend to limit demand for annuities. The presence of home equity therefore lowers the barrier to annuitisation. When LTCI alone is available in the market, illiquid housing wealth reduces demand for LTCI regardless of whether life annuities are available. This confirms the role of home equity as an insurance against healthcare costs.

When both products are accessible and retirees have no illiquid home equity, Table 6 shows that LTCI significantly increases demand for life annuities because the insurance reduces the need

Table 6. Optimal annuitisation rate as a proportion of liquid wealth (% Liquid) and as a proportion of total wealth (% Total), and optimal LTCI coverage (LTCI) for the base case.

Wealth ($000)		Single product			Both products		
		Annuity only		LTCI	Annuity		LTCI
Liquid	Housing	% Liquid	% Total	only	% Liquid	% Total	
500	0	0.30	0.30	0.93	0.71	0.71	0.92
220	280	0.94	0.41	0.89	0.65	0.29	0.81

to hold precautionary savings against uncertain healthcare cost. This result is in line with the prior research showing that including elements of LTCI to annuities can enhance the demand for standard life-contingent annuities (see e.g. Ameriks et al., 2008; Wu et al., 2016). When retirees have a significant proportion of wealth locked in illiquid home equity, however, LTCI reduces the optimal annuitisation rate. As a result, it seems that illiquid home equity reduces demand for life annuities when LTCI is also available in the market. In fact, as later to be examined in the sensitivity analysis, whether or not illiquid housing wealth reduces demand for annuities depends on the amount of liquid wealth. In the base case, the amount of liquid wealth available ($220,000) is relatively low, and retirees find it optimal to purchase a substantial coverage of LTCI (which costs about $76,749.37, or 35% of liquid wealth). Therefore the optimal proportion of liquid wealth to be annuitised is reduced.

Figure 5 shows the simulated average consumption in four different cases: *1*) no access to either LTCI or life annuities; *2*) access to life annuities only; *3*) access to LTCI only; *4*) access to both LTCI and life annuities. The consumption excludes the healthcare costs. As discussed in Figure 4, the likelihood of becoming severely disabled grows exponentially after age 85. The severely disabled state is associated with expensive healthcare costs which can constrain the consumption if LTCI is not accessible to retirees. On the other hand, purchasing LTCI involves a lump sum payment at retirement, which can reduce the consumption at early retirement. As a result of these two factors, Figure 5 shows two intersections at around age 85. Compare individuals with no access to either product (dotted line) to those with access to LTCI only (dash-dot line). The former group, on average, consumes more before 85 and consumes substantially less afterwards. The comparison between the rest two groups shows a similar pattern.

Figure 5 also shows a contrasting feature of consumption when no products are available and

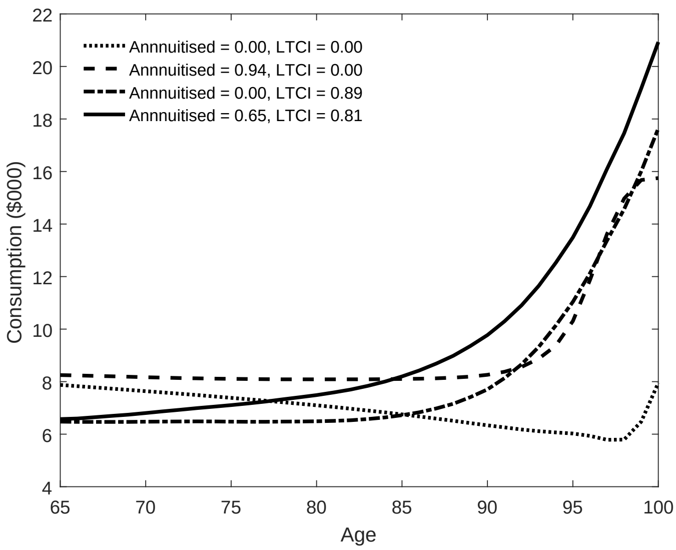

Figure 5. Simulated average consumption (excluding healthcare costs) paths at different annuitisation rates and LTCI coverage.

when at least one product is accessible to retirees. When no products are available, the average consumption shows a downward trend before increasing slightly after age 97. By contrast, when at least one product is accessible, the average consumption remains relatively flat before increasing significantly at early 90s. The home equity liquidation does not materially enhance the average consumption because of the excessive healthcare costs. Figure 6 compares the average consumption with and without healthcare costs assuming no products are available in the market. In the absence of healthcare costs, the average consumption increases substantially after around age 90 due to home equity liquidation. Figure 5 shows that life annuities alone can also improve the consumption at late retirement. This is due to the mortality premium, which is higher at more advanced ages.

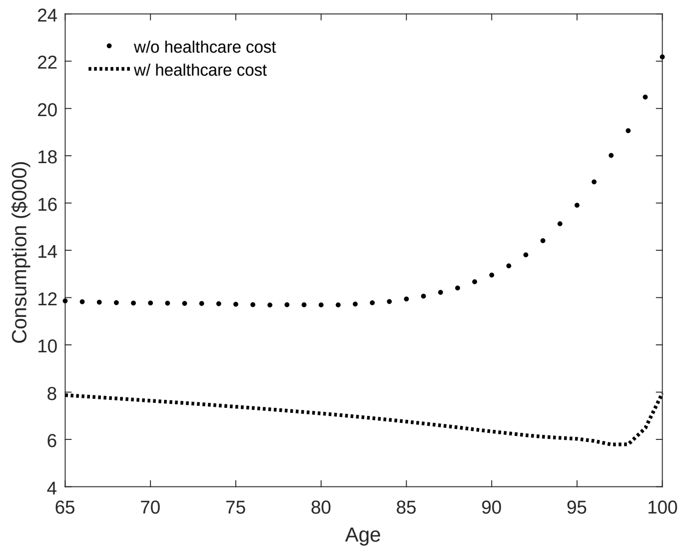

Figure 6. Simulated average consumption (excluding healthcare costs) paths with and without healthcare cost. Neither life annuity nor LTCI is available in the market.

Figure 7 shows the average liquid asset paths at different annuitisation rates and LTCI coverage. Compare the two cases where only the life annuity is available (dashed line) and where both products are available (solid line). Individuals, on average, accumulate more liquid assets in the former case for the purpose of precautionary savings to cover healthcare expenditure. In addition, the liquid assets tend to decrease at a faster rate when neither product is available (dotted line) compared to the case where only LTCI is available (dash-dot line) because the healthcare cost in the severely disabled state is partially covered by the insurance in the latter case.

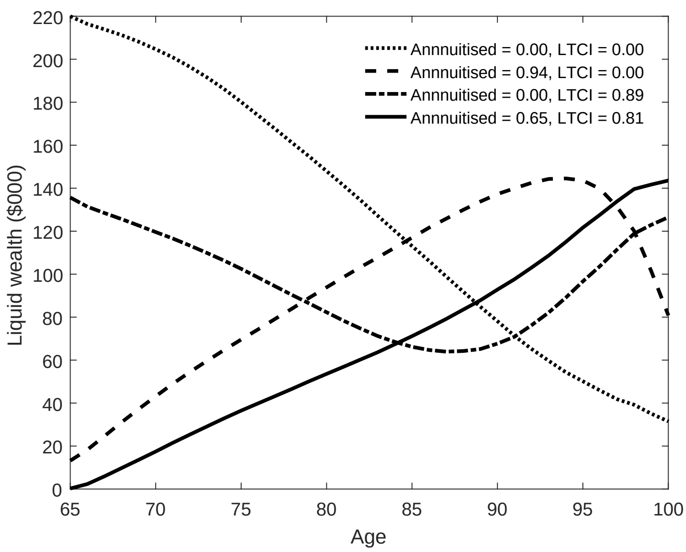

Figure 7. Simulated average liquid wealth paths at different annuitisation rates and LTCI coverage.

4.2 Sensitivity analysis: Wealth endowment

The section investigates the impact of wealth endowment and its composition on the optimal product choice. The wealth endowment doubles and halves compared to the base case, and the ratio of home equity to total wealth varies from less than 30% to over 80% to capture a wide range of household portfolio compositions. Table 7 shows the optimal annuitisation rate and LTCI coverage in each scenario. For comparison purposes, the case without illiquid home equity in each wealth level is also shown in the table.

Table 7. Optimal annuitisation rate as a proportion of liquid wealth (% Liquid) and as a proportion of total wealth (% Total), and optimal LTCI coverage (LTCI) for different wealth endowments.

Wealth ($000)			Single product			Both products		
			Annuity only		LTCI	Annuity		LTCI
Total	Liquid	Housing	% Liquid	% Total	only	% Liquid	% Total	
250	250	0	0.26	0.26	0.00	0.50	0.50	0.98
250	180	70	0.35	0.25	1.00	0.48	0.35	0.98
250	110	140	0.00	0.00	1.00	0.13	0.06	1.00
250	40	210	0.00	0.00	0.42	0.00	0.00	0.42
500	500	0	0.30	0.30	0.93	0.71	0.71	0.92
500	360	140	0.54	0.39	0.97	0.78	0.56	0.83
500	220	280	0.94	0.41	0.89	0.65	0.29	0.81
500	80	420	1.00	0.16	0.84	0.00	0.00	0.84
1,000	1,000	0	1.00	1.00	1.00	0.73	0.73	0.96
1,000	720	280	1.00	0.72	0.82	0.89	0.64	0.83
1,000	440	560	1.00	0.44	0.60	0.88	0.39	0.55
1,000	160	840	1.00	0.16	0.30	0.84	0.13	0.27

Housing wealth generally crowds out the demand for the LTCI except when the total wealth level is too low, e.g. $250,000. Compare the optimal LTCI coverage between the scenarios with and without home equity when the total wealth is $500,000 or $1,000,000. The presence of illiquid home equity reduces the optimal insurance coverage regardless of the availability of annuities. The only exception is when retirees are endowed with $500,000 total wealth and they can only purchase LTCI. The optimal insurance coverage increases slightly from 93% to 97% when the endowment includes $140,000 housing wealth. The small increment is related to the fact that the liquid wealth left after purchasing LTCI can also hedge against uncertain healthcare costs. When housing wealth endowment is relatively low, its hedging effectiveness is marginally inferior to that of liquid wealth. Retirees therefore want to purchase more LTCI coverage. Compare the optimal LTCI coverage among the scenarios with home equity when the total wealth is \$500,000 or \$1,000,000. It decreases as home equity endowment increases, except when the retiree is extremely cash poor and asset rich (e.g. endowed with \$80,000 liquid wealth out of \$500,000 total wealth).

When individuals are endowed with a relatively low level of total wealth, i.e. \$250,000, they purchase nearly the full LTCI coverage subject to their budget constraints[3]. The government subsidy that guarantees a minimum level of consumption plays a role in the high take-up of insurance. LTCI primarily severs to transfer the consumption from healthy state to severely disabled state. When the amount of liquid wealth is low, a higher LTCI coverage has no material impact on the consumption levels in the healthy state as they remain close to the consumption floor (the left panel of Figure 8). On the other hand, a higher insurance coverage can significantly improve the consumption level in the severely disabled state (the right panel of Figure 8), lifting the lifetime utility. As a result, retirees are willing to purchase a high insurance coverage when their liquid wealth is very limited.

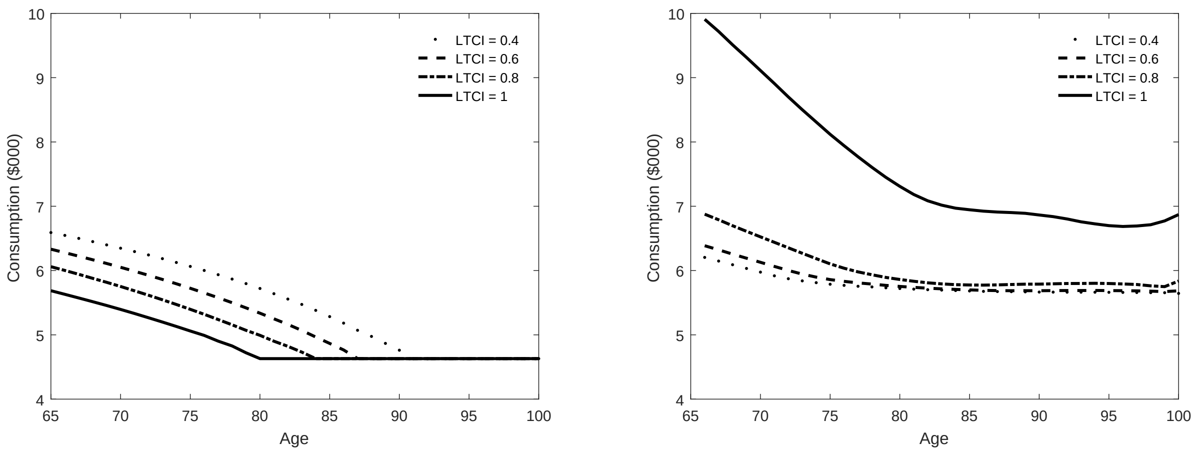

Figure 8. Simulated average consumption (excluding healthcare costs) paths of individuals at different health states: (Left Panel) healthy; (Right Panel) severely disabled. The annuitisation rates are zero in both panels. Individuals are endowed with \$180,000 liquid wealth and \$70,000 housing wealth.

In terms of demand for annuities, Table 7 shows that when life annuities alone are accessible to retirees, home equity increases the optimal proportion of liquid wealth to be annuitised unless the liquid wealth is too low (where no annuitisation is optimal) or total wealth is high

[3]When individuals are endowed with \$40,000 liquid wealth, 42% is the maximum LTCI coverage they can afford.

(where full annuitisation is optimal even without housing wealth). This finding is consistent with the base case analysis. When both annuities and LTCI are available in the market, home equity can increase or decrease demand for life annuities depending on the amount of liquid wealth. Compare the optimal annuitisation rates between scenarios with and without home equity. When liquid wealth is sufficiently high (e.g. $360,000 liquid wealth out of $500,000 total wealth), the presence of home equity increases the optimal proportion of liquid assets to be annuitised and vice versa. In the model the risk of uncertain healthcare cost is more severe than the risk of outliving one's financial resources, so retirees value LTCI more than life annuity. When allocating the liquid assets between annuities and LTCI, they are willing to satisfy the demand for LTCI at the cost of a lower annuitisation level. The presence of home equity for a given level of total wealth decreases the amount of liquid wealth available, so retirees might reduce the optimal annuitisation rate (as a percentage of liquid wealth) to fulfil demand for LTCI. Compare the demand for annuities among scenarios with home equity controlling for the level of total wealth. The optimal annuitisation rate decreases as home equity value increases. An increasing home equity reduces both the spending on LTCI and the amount of liquid wealth. The net effect is that the spending on LTCI as a proportion of liquid wealth increases, so the optimal annuitisation rate, as a percentage of liquid wealth, decreases.

4.3 Sensitivity analysis: Preference parameters

This section performs sensitivity analysis on the values of the parameters that determine an individual's preference. The optimal product choices are shown in Table 8. Overall the optimal annuitisation rate and LTCI coverage are relatively robust when both products are accessible. When life annuities alone are available, the optimal annuitisation rate is sensitive to different sets of parameters.

Table 8. Optimal annuitisation rate as a proportion of liquid wealth (% Liquid) and as a proportion of total wealth (% Total), and optimal LTCI coverage (LTCI) for different values of preference parameters.

	Single product			Both products		
	Annuity only		LTCI	Annuity		LTCI
	% Liquid	% Total	only	% Liquid	% Total	
Base case	0.94	0.41	0.89	0.65	0.29	0.81
Coefficient of relative risk aversion						
$\gamma = 2^{\dagger}$	1.00	0.44	0.85	0.68	0.30	0.74
$\gamma = 10$	0.27	0.12	0.91	0.64	0.28	0.83
Elasticity of intertemporal substitution						
$\psi = 0.2^{\dagger}$	1.00	0.44	0.85	0.67	0.29	0.76
$\psi = 0.7$	0.91	0.40	0.95	0.63	0.28	0.85
Strength of bequest motive						
$b = 1$	1.00	0.44	0.78	0.68	0.30	0.74
$b = 4$	0.20	0.41	0.94	0.63	0.28	0.85
Subjective discount factor						
$\beta = 0.93$	0.59	0.26	0.75	0.49	0.22	0.72
$\beta = 0.99$	1.00	0.44	0.96	0.63	0.28	0.85

† When $\gamma = 2$ or $\psi = 0.2$, the Epstein-Zin model defined in Equation (6) reduces to the power utility model.

The coefficient of relative risk aversion reflects an individual's attitude towards risk. A lower value means the individual is more risk tolerant, hence requiring less LTCI coverage. The EIS reflects an individual's willingness to substitute consumption over time. A higher value means the individual is less concerned about consumption smoothing year after year, and relatively more concerned about insuring against health risk. A higher value of EIS therefore leads to a stronger demand for the LTCI and weaker demand for life annuity.

The sensitivity analysis on γ and ψ highlights the advantage of the Epstein-Zin model over the power utility model in separating the risk aversion and EIS. The power utility model imposes that the coefficient of relative risk aversion is the inverse of the EIS, so a higher degree of risk aversion inevitably leads to a lower level of EIS. Table 8 shows that a higher degree of risk aversion and a lower level of EIS have opposite effects on the optimal LTCI coverage and the optimal annuitisation rate. The power utility model is therefore inadequate in determining the demand for annuities and LTCI when the individual's risk aversion does not coincide with the inverse of her EIS.

Purchasing LTCI transfers the wealth from early retirement to late retirement and to estates. Consequently, a stronger bequest motive leads to a higher LTCI coverage. A lower subjective discount factor means the individual is less willing to postpone the consumption. Since purchasing LTCI reduces the consumption at early retirement, a lower subjective discount factor reduces demand for LTCI.

5 Conclusions

The high home ownership rate among the elderly and the significance of home equity in household portfolios among retired homeowners suggest the importance of home equity in retirement planning. We study the impact of housing wealth on the demand for life annuities and LTCI in a lifecycle framework. The individual chooses an annuitisation rate and LTCI coverage at retirement, and consumption over the course of retirement. Upon becoming severely disabled, the retired homeowner will liquidate her home equity and move to a long-term care facility. We use the Epstein-Zin utility model to separately identify an individual's risk aversion and EIS. Retirees face multiple sources of risk from uncertain healthcare cost, uncertain life span, and house price shocks.

The results show that the presence of home equity typically increases the optimal annuitisation rate when life annuities alone are available in the market. Prior studies show that precautionary savings for sizeable health expenditures (Sinclair and Smetters, 2004; Turra and Mitchell, 2008) and bequest motive (Lockwood, 2012) are among a number of factors that can dampen demand for life annuities. For retired homeowners who tend to sell the property at the time of moving into a nursing home, home equity is both a form of precautionary savings and bequest. The presence of home equity therefore lowers the barrier to annuitisation. When retirees have access to both life annuities and LTCI, the presence of home equity can enhance demand for annuities if the retiree has sufficient liquid assets. Otherwise, the spending on LTCI can impair demand for life annuities. The demand for LTCI is generally crowded out by home equity since the liquidation of housing wealth tends to be highly correlated with the payment of LTCI.

We also show the effect of preferences on the optimal annuitisation rate and LTCI coverage. It is important to separately identify risk aversion and EIS. A higher degree of risk aversion and a lower level of EIS have opposite effects on the demand for life annuities and LTCI. Since the power utility model imposes an inverse relationship on risk aversion and EIS, the model is unable to disentangle the impact of these two factors. It is therefore more appropriate to apply the Epstein-Zin model to individuals with various levels of risk aversion and EIS.

Our research has practical implications on the offering of retirement products. Both life annuities and LTCI are effective instruments to manage post-retirement risks and to maintain living standard at retirement. For a given wealth level, the proportion of illiquid home equity in the portfolio can have a large impact on demand for annuities and LTCI. It is important to differentiate between homeowners and non-homeowners when providing the products.

Appendix

A First-order condition for consumption

This section derives the first-order condition for consumption given the LTCI coverage and annuitisation decisions have been made. The method of solving optimal annuitisation rate and LTCI coverage is discussed in Section 3.6. The techniques used below build on the derivations in Chapter 6 of Munk (2013) who solves the optimal consumption problem for an individual with no bequest motive or health risk.

The first-order condition for C_t implies that

$$\begin{aligned}(1-\beta)C_t^{-\rho} = \beta &\left\{\mathbb{E}_t\left[\sum_{k\neq 4}\pi(s_{t+1}=k|s_t)V_{t+1}^{1-\gamma}+\pi(s_{t+1}=4|s_t)b^{\gamma}W_{t+1}^{1-\gamma}\right]\right\}^{\frac{1}{\theta}-1} \\ &\times R_f\mathbb{E}_t\left[\sum_{k\neq 4}\pi(s_{t+1}=k|s_t)V_{t+1}^{-\gamma}\frac{\partial V_{t+1}}{\partial B_{t+1}}+\pi(s_{t+1}=4|s_t)b^{\gamma}W_{t+1}^{-\gamma}\right],\end{aligned} \tag{8}$$

where $\partial V_{t+1}/\partial B_{t+1}$ can be derived by taking the derivative on the Equation (6). For the optimal decision, the equation holds without the maximum, that is

$$\begin{aligned}V_t &\equiv V(B_t, W_t^H, s_t, t) \\ &= \left\{(1-\beta)\left(C_t^*\right)^{1-\rho}+\beta\left[\mathbb{E}_t\left[\sum_{k\neq 4}\pi(s_{t+1}=k|s_t)V(B_{t+1}^*, W_{t+1}^H, s_{t+1}=k, t+1)^{1-\gamma}\right.\right.\right. \\ &\qquad\left.\left.\left.+\pi(s_{t+1}=4|s_t)b^{\gamma}\left(W_{t+1}^*\right)^{1-\gamma}\right]\right]^{\frac{1}{\theta}}\right\}^{\frac{1}{1-\rho}},\end{aligned} \tag{9}$$

where C_t^* denotes the optimal consumption at time t, B_{t+1}^* and W_{t+1}^* denotes the next period liquid assets and total wealth, respectively, under the optimal consumption in period t.

Take the derivative of Equation (9) w.r.t. B_t.

$$
\begin{aligned}
\frac{\partial V_t}{\partial B_t} = V_t^{\rho} \Bigg\{ & (1-\beta)(C_t^*)^{-\rho} \frac{\partial C_t^*}{\partial B_t} \\
& + \beta \left[\mathbb{E}_t \left(\sum_{k \neq 4} \pi(s_{t+1} = k | s_t) V_{t+1}^{1-\gamma} + \pi(s_{t+1} = 4 | s_t) b^{\gamma} (W_{t+1}^*)^{1-\gamma} \right) \right]^{\frac{1}{\theta}-1} \\
& \times \mathbb{E}_t \left[\sum_{k \neq 4} \pi(s_{t+1} = k | s_t) V_{t+1}^{-\gamma} \frac{\partial V_{t+1}}{\partial B_{t+1}^*} \frac{\partial B_{t+1}^*}{\partial B_t} + \pi(s_{t+1} = 4 | s_t) b^{\gamma} (W_{t+1}^*)^{-\gamma} \frac{\partial W_{t+1}^*}{\partial B_t} \right] \Bigg\},
\end{aligned} \tag{10}
$$

where $\partial B_{t+1}^* / \partial B_t$ and $\partial W_{t+1}^* / \partial B_t$ can be derived from the budget constraints (4) and (5)

$$
\begin{aligned}
\frac{\partial B_{t+1}^*}{\partial B_t} &= \left(1 - \frac{\partial C_t^*}{\partial B_t}\right) R_f, \\
\frac{\partial W_{t+1}^*}{\partial B_t} &= \frac{\partial W_{t+1}^*}{\partial B_{t+1}^*} \frac{\partial B_{t+1}^*}{\partial B_t} = \frac{\partial B_{t+1}^*}{\partial B_t} = \left(1 - \frac{\partial C_t^*}{\partial B_t}\right) R_f.
\end{aligned} \tag{11}
$$

Substitute the Equation (11) into Equation (10) and then using the first-order condition (8)

$$
\begin{aligned}
\frac{\partial V_t}{\partial B_t} = V_t^{\rho} \beta R_f & \left[\mathbb{E}_t \left(\sum_{k \neq 4} \pi(s_{t+1} = k | s_t) V_{t+1}^{1-\gamma} + \pi(s_{t+1} = 4 | s_t) b^{\gamma} (W_{t+1}^*)^{1-\gamma} \right) \right]^{\frac{1}{\theta}-1} \\
& \times \mathbb{E}_t \left[\sum_{k \neq 4} \pi(s_{t+1} = k | s_t) V_{t+1}^{-\gamma} \frac{\partial V_{t+1}}{\partial B_{t+1}^*} + \pi(s_{t+1} = 4 | s_t) b^{\gamma} (W_{t+1}^*)^{-\gamma} \right].
\end{aligned} \tag{12}
$$

Consequently, the first-order condition for C_t can be re-written as

$$
\frac{\partial V_t}{\partial B_t} = (1-\beta) V_t^{\rho} C_t^{-\rho}. \tag{13}
$$

This is the envelope condition for the preferences defined in Equation (6).

Substitute the envelope condition (13) into Equation (8). The first-order condition for C_t can be re-stated as

$$
\begin{aligned}
(1-\beta)C_t^{-\rho} = \beta \left\{ \mathbb{E}_t \left[\sum_{k\neq 4} \pi(s_{t+1}=k|s_t) V_{t+1}^{1-\gamma} + \pi(s_{t+1}=4|s_t) b^\gamma W_{t+1}^{1-\gamma} \right] \right\}^{\frac{1}{\theta}-1} \\
\times R_f \mathbb{E}_t \left[(1-\beta) \sum_{k\neq 4} \pi(s_{t+1}=k|s_t) V_{t+1}^{\rho-\gamma} C_{t+1}^{-\rho} + \pi(s_{t+1}=4|s_t) b^\gamma W_{t+1}^{-\gamma} \right].
\end{aligned} \tag{14}
$$

Therefore, the optimal consumption in period t is given by

$$
\begin{aligned}
C_t^* = \Bigg\{ \beta R_f \left\{ \mathbb{E}_t \left[\sum_{k\neq 4} \pi(s_{t+1}=k|s_t) V_{t+1}^{1-\gamma} + \pi(s_{t+1}=4|s_t) b^\gamma W_{t+1}^{1-\gamma} \right] \right\}^{\frac{1}{\theta}-1} \\
\times \mathbb{E}_t \left[\sum_{k\neq 4} \pi(s_{t+1}=k|s_t) V_{t+1}^{\rho-\gamma} C_{t+1}^{-\rho} + \pi(s_{t+1}=4|s_t) \frac{b^\gamma}{1-\beta} W_{t+1}^{-\gamma} \right] \Bigg\}^{-\frac{1}{\rho}}.
\end{aligned} \tag{15}
$$

In the terminal period, $\pi(s_{T+1}=k|s_T)=0$ for $k \in \{1,2,3\}$ and $\pi(s_{T+1}=4|s_T)=1$, so the optimal consumption in period T becomes

$$
C_T^* = \left(\frac{1}{\beta} - 1 \right)^{\frac{1}{\rho}} \left\{ \mathbb{E}_T \left[b^\gamma W_{T+1}^{1-\gamma} \right] \right\}^{\frac{1}{\rho}-\frac{1}{\rho\theta}} \times \left\{ R_f \mathbb{E}_T \left[b^\gamma W_{T+1}^{-\gamma} \right] \right\}^{-\frac{1}{\rho}}. \tag{16}
$$

References

Ameriks, John, Andrew Caplin, Steven Laufer, and Stijn Van Nieuwerburgh (2008). “Annuity valuation, long-term care, and bequest motives”. In: *Recalibrating Retirement Spending and Saving*. Ed. by John Ameriks and Olivia S. Mitchell. New York, NY: Oxford University Press. Chap. 11, pp. 251–275.

Ameriks, John, Andrew Caplin, Steven Laufer, and Stijn Van Nieuwerburgh (2011). “The joy of giving or assisted living? Using strategic surveys to separate public care aversion from bequest motives”. *Journal of Finance*, vol. 66, no. 2, pp. 519–561.

Andréasson, Johan G., Pavel V. Shevchenko, and Alex Novikov (2017). “Optimal consumption, investment and housing with means-tested public pension in retirement”. *Insurance: Mathematics and Economics*, vol. 75, pp. 32–47.

Blake, David, Douglas Wright, and Yumeng Zhang (2014). “Age-dependent investing: Optimal funding and investment strategies in defined contribution pension plans when members are rational life cycle financial planners”. *Journal of Economic Dynamics and Control*, vol. 38, pp. 105–124.

Brown, Jason and Mark Warshawsky (2013). “The life care annuity: A new empirical examination of an insurance innovation that addresses problems in the markets for life annuities and long-term care insurance”. *Journal of Risk and Insurance*, vol. 80, no. 3, pp. 677–704.

Brown, Jeffrey R. (2009). “Understanding the role of annuities in retirement planning”. In: *Overcoming the Saving Slump: How to Increase the Effectiveness of Financial Education and Saving Programs*. Ed. by Annamaria Lusardi. Chicago, IL: University of Chicago Press, pp. 178–206.

Carroll, Christopher D. (2006). “The method of endogenous gridpoints for solving dynamic stochastic optimization problems”. *Economics Letters*, vol. 91, no. 3, pp. 312–320.

Cocco, João F. (2004). “Portfolio choice in the presence of housing”. *Review of Financial Studies*, vol. 18, no. 2, pp. 535–567.

Crimmins, Eileen M. and Hiram Beltrán-Sánchez (2011). “Mortality and morbidity trends: Is there compression of morbidity?” *Journals of Gerontology: Series B*, vol. 66B, no. 1, pp. 75–86.

Davidoff, Thomas (2009). "Housing, health, and annuities". *Journal of Risk and Insurance*, vol. 76, no. 1, pp. 31–52.

Davidoff, Thomas (2010). "Home equity commitment and long-term care insurance demand". *Journal of Public Economics*, vol. 94, no. 1-2, pp. 44–49.

Epstein, Larry G. and Stanley E. Zin (1989). "Substitution, risk aversion, and the temporal behavior of consumption and asset returns: A theoretical framework". *Econometrica*, vol. 57, no. 4, pp. 937–969.

Epstein, Larry G. and Stanley E. Zin (1991). "Substitution, risk aversion, and the temporal behavior of consumption and asset returns: An empirical analysis". *Journal of Political Economy*, vol. 99, no. 2, pp. 263–286.

Ferri, Stefano and Annamaria Olivieri (2000). "Technical bases for LTC covers including mortality and disability projections". In: *Proceedings of the XXXI International ASTIN Colloquium*. Porto Cervo, Italy, pp. 295–314.

Flavin, Marjorie and Takashi Yamashita (2002). "Owner-occupied housing and the composition of the household portfolio". *American Economic Review*, vol. 92, no. 1, pp. 345–362.

Fong, Joelle H., Adam W. Shao, and Michael Sherris (2015). "Multistate actuarial models of functional disability". *North American Actuarial Journal*, vol. 19, no. 1, pp. 41–59.

Hanewald, Katja, Thomas Post, and Michael Sherris (2016). "Portfolio choice in retirement – What is the optimal home equity release product?" *Journal of Risk and Insurance*, vol. 83, no. 2, pp. 421–446.

He, Wan, Daniel Goodkind, and Paul Kowal (2016). *An Aging World: 2015*. U.S. Census Bureau, International Population Reports, P95/16-1. Washington, DC: U.S. Government Publishing Office.

Hu, Xiaoqing (2005). "Portfolio choices for homeowners". *Journal of Urban Economics*, vol. 58, no. 1, pp. 114–136.

Lockwood, Lee M. (2012). "Bequest motives and the annuity puzzle". *Review of Economic Dynamics*, vol. 15, no. 2, pp. 226–243.

Ludwig, Alexander and Matthias Schön (2016). "Endogenous grids in higher dimensions: Delaunay interpolation and hybrid methods". *Computational Economics*. DOI: 10.1007/s10614-016-9611-2.

McRae, Ian, Laurann Yen, Yun-Hee Jeon, Menaka Herath, and Beverley Essue (2012). "The health of senior Australians and the out-of-pocket healthcare costs they face". Report. National Seniors Productive Ageing Centre, Melbourne.

Munk, Claus (2013). *Financial Asset Pricing Theory*. 1st ed. Oxford, United Kingdom: Oxford University Press.

Murtaugh, Christopher M., Brenda C. Spillman, and Mark J. Warshawsky (2001). "In sickness and in health: An annuity approach to financing long-term care and retirement income". *Journal of Risk and Insurance*, vol. 68, no. 2, pp. 225–253.

Olivieri, Annamaria and Ermanno Pitacco (2001). "Facing LTC risks". In: *Proceedings of the XXXII International ASTIN Colloquium*. Washington, DC.

Pang, Gaobo and Mark Warshawsky (2010). "Optimizing the equity-bond-annuity portfolio in retirement: The impact of uncertain health expenses". *Insurance: Mathematics and Economics*, vol. 46, no. 1, pp. 198–209.

Pauly, Mark V. (1990). "The rational nonpurchase of long-term-care insurance". *Journal of Political Economy*, vol. 98, no. 1, pp. 153–168.

Peijnenburg, Kim, Theo Nijman, and Bas J. M. Werker (2015). "Health cost risk: A potential solution to the annuity puzzle". *The Economic Journal*. DOI: 10.1111/ecoj.12354.

Reserve Bank of Australia (2015). "Home ownership rates". Submission to the Inquiry into Home Ownership. House of Representatives Standing Committee on Economics, June.

Shao, Adam W., Hua Chen, and Michael Sherris (2017). "To borrow or insure? Long term care costs and the impact of housing". Working Paper 2017/11. ARC Centre of Excellence in Population Ageing Research.

Sinclair, Sven and Kent Andrew Smetters (2004). "Health shocks and the demand for annuities". Technical Paper Series 2004-9. Congressional Budget Office.

Turra, Cassio and Olivia S. Mitchell (2008). "The impact of health status and out-of-pocket medical expenditures on annuity valuation". In: *Recalibrating Retirement Spending and Saving*. Ed. by John Ameriks and Olivia S Mitchell. Oxford: Oxford University Press. Chap. 10, pp. 227–250.

U.S. Census Bureau (2017). "Current population survey/housing vacancy survey". Report. U.S. Census Bureau, Washington, DC.

Venti, Steven F. and David A. Wise (1990). "But they don't want to reduce housing equity". In: *Issues in the Economics of Aging*. Ed. by David A. Wise. Chicago, IL: University of Chicago Press. Chap. 1, pp. 13–32.

Venti, Steven F. and David A. Wise (1991). "Aging and the income value of housing wealth". *Journal of Public Economics*, vol. 44, no. 3, pp. 371–397.

Venti, Steven F. and David A. Wise (2004). "Aging and housing equity: Another look". In: *Perspectives on the Economics of Aging*. Ed. by David A Wise. Chicago, IL: University of Chicago Press, pp. 127–180.

Walker, Lina (2004). "Elderly households and housing wealth: Do they use it or lose it?" Working Paper No. 2004-070. University of Michigan, Michigan Retirement Research Center.

Weil, Philippe (1989). "The equity premium puzzle and the risk-free rate puzzle". *Journal of Monetary Economics*, vol. 24, no. 3, pp. 401–421.

Wu, Shang, Hazel Bateman, and Ralph Stevens (2016). "Optimal portfolio choice with health-contingent income products: The value of life care annuities". Working Paper 2016/17. ARC Centre of Excellence in Population Aging Research.

Yaari, Menahem E. (1965). "Uncertain lifetime, life insurance, and the theory of the consumer". *Review of Economic Studies*, vol. 32, no. 2, pp. 137–150.

Yao, Rui and Harold H. Zhang (2005). "Optimal consumption and portfolio choices with risky housing and borrowing constraints". *Review of Financial Studies*, vol. 18, no. 1, pp. 197–239.

Yogo, Motohiro (2016). "Portfolio choice in retirement: Health risk and the demand for annuities, housing, and risky assets". *Journal of Monetary Economics*, vol. 80, pp. 17–34.