Optimal portfolio choice with health-contingent income products: The value of life care annuities

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Outline

1 Motivation and research questions

2 The life-cycle model

3 Results

4 Conclusion
Background

**Health-contingent income products**: a lump sum → regular incomes, amount depending on health status

**Advantages**: e.g.,
- smooth consumption when facing out-of-pocket healthcare expenditure (HCE)
- cash flow is attractive: small ↓ in regular income, large ↑ in bad-health income
Life care annuity—introduction

Product feature: the life care annuity (LCA) (Murtaugh et al., 2001)
- payment of two income streams:
  - survival-contingent income (annuity segment)
  - health-contingent income (long-term care insurance (LTCI) segment):
    - 2+ limitations in activities of daily living (ADLs); and/or
    - cognitive impairment.

Policy discussion:
- the Netherlands: choice between a lowered pension (with additional payments when need LTC) and a ‘normal’ pension at a fixed rate (Ooijen et al., 2014)
- the U.S.: offer LCA in the private market (Warshawsky et al., 2002; Spillman et al., 2003)

Why bundling? → a potential solution for low voluntary annuitization and LTCI take up
Life care annuity—how it works

**Rationale:** bundling \(\rightarrow\) minimal underwriting \(\rightarrow\) inclusion of most of the currently rejected for LTCI (Murtaugh et al., 2001)

- for life annuities:
  - reduce adverse selection
  - (partially) hedge health costs risk
- for LTCI:
  - increase market potential
  - single premium: no lapses & dynamic selection

**Availability of minimal underwriting (for LTCI):** single premium

- single premium and lifetime cover: YES
  (Finkelstein and McGarry, 2006; Brown and Warshawsky, 2013)
- periodic level-premium: NO
  - survival-contingent premium
Research question and contribution

1. LCA against conventional life annuities:
   - impact on annuitization choices for different health groups
     ▶ from insurance feature (additional LTCI)
     ▶ from risk pooling pricing
   - impact on optimal consumption, savings and risky asset holdings.
   - how much is the welfare gain?

2. Impact of out-of-pocket HCE: allowing for end-of-life HCE
   - mixed results for effects of HCE on annuitization choices: e.g., Pang and Warshawsky (2010); Pashchenko (2013); Reichling and Smetters (2015); Peijnenburg et al. (2015)
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Preference

Single individuals: age 65 at retirement ($t = 0$), maximum age 100 ($T = 35$), obtain utility from

- consumption: time-separable CRRA

$$u(C_t) = \frac{C_t^{1-\gamma}}{1 - \gamma},$$

where $\gamma = 5$ is the relative risk aversion, exponential discounting by $\beta = 0.96$.

- leaving bequests:

$$v(W_t) = \frac{(bW_t)^{1-\gamma}}{1 - \gamma},$$

where $b = 0.17$ for strength of bequest (Yogo, 2009), discounted by $\beta$. 
Decisions

At retirement:
- retirement wealth $W = 500,000$ (Dushi and Webb, 2004)
- one-off and irreversible annuitization, $x$: standard life annuities or LCA
  → income $Y_t$ (health-contingent for LCA)

In each period:
- liquid wealth $W_t$, receive income $Y_t$, pay (exogenous) out-of-pocket
  HCE, $M_t$
- if cash-on-hand $≡ W_t + Y_t - M_t <$ wealth floor $\underline{W}$ → subsidy
  $G_t = \max\{0, \underline{W} - (W_t + Y_t - M_t)\}$, where
    - $\underline{W} = 22,000$
    - $C^\ast$ with $\underline{W}$ in line with the consumption floor in Ameriks et al. (2011) and
      Peijnenburg et al. (2015)

- decisions:
  - consumption $C_t \rightarrow$ after consumption wealth $\bar{W}_t$
  - allocation to the risky asset $\alpha_t \rightarrow$ risk-free, $1 - \alpha_t$
Health dynamics

Heath states: Brown and Warshawsky (2013)

- 11 health states ($H_t$) by
  - self-reported health
  - history of illness
  - cognitive impairment and ADL

- roughly
  - 1-4: 0-1 ADLs, eligible for LCA
  - 5-7: 2-3 ADLs
  - 8-10: 4-5 ADLs
  - 11: death

- estimated to the U.S. Health and Retirement Study (HRS) data from 1998 to 2008
Out-of-pocket healthcare expenditure

For each gender and health state: a mixture distribution with

- **bottom**: no HCE

\[ M_t = \begin{cases} 
0, & \text{if } I_t = 1; \\
m_t, & \text{if } I_t = 0, 
\end{cases} \]

where \( I_t \sim \text{Bernoulli}(p) \) an indicator for zero HCE

- **main**: truncated lognormal distribution

- **tail**: exponential distribution

\[
\begin{cases}
  m_t \sim \text{Truncated LN } (\mu, \sigma, a), & \text{if } m_t < a; \\
  (m_t - a) \sim \text{Exp } (\lambda), & \text{if } m_t \geq a.
\end{cases}
\]

- **‘The cost of dying’**: different parameters for survivors and the deceased

**Reasons:**

- **bottom**: cluster of zero health costs
- **main**: LN fits body well, in line with the literature (De Nardi et al., 2010)
- **tail**: LN does not fit the tail well: a fat tail at very large percentiles
### Calibration

<table>
<thead>
<tr>
<th>Moments</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5 to 7</th>
<th>8 to 10</th>
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<td>Health state</td>
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<td>Panel A: HCE from HRS (’000 $)</td>
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<tr>
<td>Mean</td>
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<td>4.2</td>
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<td>8.7</td>
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<td>12.1</td>
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<td>17.9</td>
<td>48.9</td>
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<td>Panel B: Lognormal fit</td>
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<td>43.9</td>
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<tr>
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<td>10.3*</td>
<td>9.4</td>
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<td>9.4*</td>
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<td>Panel C: Mixture distribution</td>
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<td>8.5</td>
<td>16.9</td>
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<tr>
<td>Std</td>
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<td>10.5*</td>
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<td>48.9</td>
</tr>
<tr>
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<td>60.8</td>
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<td>135.7</td>
<td>124.8</td>
<td>334.5</td>
<td>530.0</td>
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</tbody>
</table>

* outside 95% CI
Financial assets

At retirement: Brown and Warshawsky (2013)

- conventional life annuities: adverse selection – the most healthy group ($H_0 = 1$)
- LCA: pooling transition probabilities of eligible health groups ($H_t \in [1, 4]$)
  - $1$ lifetime income
  - $2$ additional income when need aged care ($H_t \in [5, 10]$)

In each period: Pang and Warshawsky (2010)

- a bond: $R^f = 1 + r_f$ with $r_f = 3\%$ risk-free rate;
- a stock: gross return $\tilde{R}_{t+1} \sim LN (\mu_S, \sigma_S)$ with $\mu_S = 0.065$ and $\sigma_S = 0.161$.
- portfolio return: $R_{t+1} = (1 - \alpha_t)R^f + \alpha_t\tilde{R}_{t+1}$
The optimization problem

Objective function:

\[
V(W, H_0) = \max_{\{x, c_t, \alpha_t\}_{t=0}^T} \left\{ \mathbb{E}_0 \left[ \sum_{t=0}^{T-1} \sum_{j=1}^{10} \pi_t^0(H_0, j) \cdot \beta^t \cdot [u(C_t, H_t) + \beta \cdot \pi_t(j, 11) \cdot v(W_{t+1})] \right] \right\},
\]

Subject to:

\[
\begin{align*}
\bar{W}_t &= W_t + Y_t - M_t + G_t - C_t \geq 0 \quad \text{(budget constraint);} \\
0 &\leq \alpha_t \leq 1 \quad \text{(no short selling);} \\
W_{t+1} &= \bar{W}_t \cdot R_{t+1} \quad \text{(wealth dynamics).}
\end{align*}
\]

for all \( t \in [0, T - 1] \).

Sensitivity analysis: for most parameters (see paper).
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Effect of LCA on optimal annuitization choice

Health state 1 at retirement, males

Health state 4 at retirement, males

Certainty equivalent consumption (in $000)

Annuitization level (in %)

Optimum
LCA at fair price
Life annuity at fair price
LCA with risk pooling (benchmark case)
Life annuity with adverse selection (benchmark case)
On optimal wealth and consumption paths

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**Health state 1 at retirement, males**

- Liquid wealth (in $000)
  - Age 65: 0
  - Age 70: 50
  - Age 75: 100
  - Age 80: 150
  - Age 85: 200
  - Age 90: 250
  - Age 95: 300

- Consumption (in $000)
  - Age 65: 0
  - Age 70: 5
  - Age 75: 10
  - Age 80: 15
  - Age 85: 20
  - Age 90: 25
  - Age 95: 30

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**Optimal LCA (=69%)**

**Optimal life annuity (=57%)**

**No annuity**

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Shang Wu (UNSW Australia)

The value of life care annuities

19 January, 2017
On optimal asset allocations

[Graphs showing asset allocation by age and health state]

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On individual welfare

Life annuity → LCA: Higher consumption in retirement

Willingness to pay for LCA: at retirement, an individual
- with $550,000 & standard life annuities $\equiv$ with $500,000 & LCAs
- retirees in health state 1 (4) is willing to pay a loading up to 16% (21%) for LCAs $\equiv$ with $500,000$ with standard life annuities
Effect of out-of-pocket HCE and end-of-life HCE

- On annuitization choice

<table>
<thead>
<tr>
<th>Optimal annuitization level in percent</th>
<th>H1</th>
<th>H4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Panel A: Standard life annuity</td>
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<tr>
<td>Without HCE</td>
<td>66</td>
<td>53</td>
</tr>
<tr>
<td>HCE without costs of dying</td>
<td>70</td>
<td>53</td>
</tr>
<tr>
<td>HCE with costs of dying</td>
<td>57</td>
<td>41</td>
</tr>
</tbody>
</table>

- On consumption and saving behavior: additional precautionary savings & less consumption

- On allocation to the stock: increase proportion of total wealth in the risky asset
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Conclusion

- **life annuity → LCA**
  - increases annuitization level for both health groups by around 12 percentage points
    - health group 1: due to both insurance feature and risk pooling
    - health group 4: mainly from insurance feature
  - less liquid wealth and higher consumption
  - more risk-averse allocation of total wealth; less risk-averse allocation of liquid wealth
  - substantially improves welfare

- **uncertain out-of-pocket HCE, in particular the end-of-life HCE**
  - reduces (increases) the demand of life annuities (LCA)
  - leads to precautionary savings and lower consumption
  - increases risky asset holding among total wealth
Implications & Future research

Implications:
- the potential of LCAs to provide at least partial protection against out-of-pocket HCE and the cost of long-term care
- design of retirement income products and public pension policy

Future research:
- incorporate monthly and new health transitions: Friedberg et al. (2014)
- household decision model
- laddered annuity purchase
- more specific institutional settings
- ... 
- optimal LTCI product design: empirical evidence from stated preference data, next year Netspar Pension Workshop?
References I


The numerical solution method

Backward induction:

- grid points for
  - after-consumption wealth $\bar{W}_t$ (endogenous grid method, Carroll, 2006)
  - annuity income $Y_t$: $x = 0\%, 1\%, ..., 100\%

\[
C_t^{*-\gamma} = \sum_{j=1}^{10} \pi_t(H_t,j) \cdot \beta \cdot \mathbb{E}_t \left\{ \frac{\partial V_{t+1}}{\partial W_{t+1}} \cdot R_{t+1}^* \right\} + \pi_t(H_t,1) \cdot \beta \cdot b^{1-\gamma} \mathbb{E}_t \left\{ W_{t+1}^{-\gamma} R_{t+1}^* \right\}
\]

where

\[
\frac{\partial V_{t+1}}{\partial W_{t+1}} = \begin{cases} 
C_{t+1}^{*-\gamma}, & \text{if } W_{t+1} + Y^A - M_{t+1} \geq W; \\
0, & \text{if } W_{t+1} + Y^A - M_{t+1} < W.
\end{cases}
\]

- Importance Sampling for out-of-pocket HCE (Rubinstein and Kroese, 2011)
- Monte Carlo method for calculating the expectation, 200,000 simulations
- linear (cubic spline) interpolation for consumption (allocation)
On optimal wealth and consumption paths

![Graph showing liquid wealth and consumption paths for different health states and annuity types at retirement for males.]

- **Health state 1 at retirement, males**
  - Liquid wealth (in $000)
  - Consumption (in $000)

- **Health state 1 at retirement, males**
  - No HCE, optimal life annuity (=66%)
  - HCE without costs of dying, optimal life annuity (=70%)
  - HCE, optimal life annuity (=57%)
  - No HCE, no annuity
  - HCE, no annuity

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On optimal asset allocations

Health state 1 at retirement, males

Allocation of total wealth to stock (in %)

- No HCE, no annuity
- HCE, no annuity
- No HCE, optimal life annuity (=66%)
- HCE without costs of dying, optimal life annuity (=70%)
- HCE, optimal life annuity (=57%)

Health state 1 at retirement, males

Allocation of after-consumption wealth to stock (in %)