Hedging long-term liabilities

"At year-end 2012, pension funds had hedged 48% of their interest rate risk. (…) a 1% fall in market interest rates would cause the average funding ratio to decline 7.8%; without interest rate hedging, the funding ratio would drop 14.9%.”

It is difficult to fit the very long end of the yield curve

Funds are sensitive to long end of term structure

source: Annual report Pensioenfonds Grafische Bedrijven 2014
**Euro swap data**
(discount rate yield curves)

**Interest rate hedging: basics**

- Duration hedging
  - Find portfolio of bonds with same duration as liabilities
  - **Liabilities longer than liquid instruments:**
    \( \Rightarrow \) leveraged position in longest liquid bonds

- Leveraged duration hedge not optimal, even risky, when yield curve shifts not parallel

- Much evidence for three yield curve factors
  - level, slope, curvature

- Time-varying factor loadings: financial crises

**Term structure models**

- Nelson-Siegel is a popular 3-factor term structure model that parsimoniously explains the shape and time variation in interest rate levels

\[
y_t(\tau) = \sum_{i=1}^{3} B_i(\tau) \tilde{f}_{it} + \epsilon_t(\tau)
\]

- A liability hedge is a portfolio with
  - same factor exposure as the liability
  - minimal residual risk

- How stable are factor loadings?
  - How did they change after 2008?

**Duration plus ...**

\[
B(\tau) = \begin{pmatrix}
1 \\
1 - e^{-\lambda \tau} \\
\frac{1 - e^{-\lambda \tau}}{\lambda \tau} - e^{-\lambda \tau}
\end{pmatrix}
\]

- First factor in Nelson-Siegel model is a parallel shift in the yield curve
  - level factor \( \Rightarrow \) corresponds to duration hedging

- Other two factors are slope and curvature
  - both governed by a single parameter \( \lambda \)
Term structure as time-varying linear combination of 3 curves

\[ y(\tau) = \sum_{i=1}^{3} B_i(\tau) \tilde{f}_i + \epsilon(\tau) \]

Econometric model

- Portfolio analysis requires (excess) returns

\[ r_{t+h}(\tau) = (p_{t+h}(\tau) - p_{t}(\tau + h)) - \left( p_{t+h}(0) - p_t(h) \right) \]

- Further scaling by maturity is like first differencing of yields

\[ \rho_{t+h}(\tau) \equiv \frac{r_{t+h}(\tau)}{\tau} = -\Delta y_{t+h}(\tau) + \frac{h}{\tau} (y_t(\tau) - y_t(h)) \]

- small \( h \): daily data
- filters out the persistence in yield levels
- gets rid of cross-sectional heteroskedasticity of returns

Transformation preserves factor structure

- Scaled excess returns

\[ \rho_{t}(\tau) = B(\tau)^{t} f_t \]

follow factor model with factors

\[ f_t = \tilde{f}_t - A \tilde{f}_{t-1} \]

- Add idiosyncratic error terms for maturities \( j = 1, \ldots, N \)

\[ \rho_{t}(\tau_j) = B(\tau_j)^{t} f_t + \epsilon_t(\tau_j) \]

that are uncorrelated across maturities.

Much time-variation in \( \lambda \) since 2008

- Least Squares rolling window estimates (100 days)
Main idea of the paper

\[ B(\tau) = \begin{pmatrix} 1 \\ \frac{1-e^{-\lambda \tau}}{\lambda \tau} \\ \frac{1-e^{-\lambda \tau}}{\lambda \tau} - e^{-\lambda \tau} \end{pmatrix} \]

- "GARCH" like dynamics for \( \lambda_t \):
  \[ \lambda_{t+1} = \phi_0 + \phi_1 \lambda_t + \phi_2 s_t \]

- The term \( s_t \) is the score of the likelihood function with respect to \( \lambda_t \):
  Dynamic Conditional Score model: see Creal, Koopman and Lucas (JAE 2013) for the general idea

The score function

\[ s_t = \frac{f_t' G_t' \Sigma_t^{-1} \epsilon_t}{(f_t' G_t' \Sigma_t^{-1} G_t f_t)^{1/2}} \]

with

\[ G_t = \left( \frac{\partial B}{\partial \lambda} \right)_t \]

and \( \Sigma_t \) the error covariance matrix

Diagonal GARCH specification: \( \Sigma_t = \sigma_t^2 I \),

\[ \sigma_t^2 = \omega^2 + \alpha \sigma_{t-1}^2 + \beta \epsilon_{t-1}^2 \epsilon_{t-1} \]

Estimation results score model

<table>
<thead>
<tr>
<th></th>
<th>DCS</th>
<th>DCS-GARCH</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi_1 )</td>
<td>0.966</td>
<td>0.961</td>
</tr>
<tr>
<td></td>
<td>(0.076)</td>
<td>(0.072)</td>
</tr>
<tr>
<td>( \phi_2 )</td>
<td>0.029</td>
<td>0.015</td>
</tr>
<tr>
<td></td>
<td>(0.008)</td>
<td>(0.005)</td>
</tr>
</tbody>
</table>

Sensitive to heteroskedasticity in residuals

Time-varying \( \lambda \) to fit curves

vertical axis: scaled returns \( \rho_t \) on Jan 29 2009
Hedging problem

- Fixed liability with **50 years** duration: \( w_0 = -1 \)
- Find portfolio with same factor exposure and minimal residual risk

\[
\min \sum_{i=1}^{N} w_i^2 \tau_i^2 \\
\sum_{i=0}^{N} w_i = 0 \\
\sum_{i=0}^{N} w_i \tau_i B_k(\tau_i) = 0 \quad k = 1, 2, 3
\]

(recall that we modelled returns scaled by maturity)

Hedge portfolio

The figure shows the portfolio allocation for hedging a liability with a 50-year maturity. The graph on the left shows Duration hedging. The right graph shows the NS factor-hedge portfolios using \( \lambda_t \) from the NS-DCS model. The shaded area is the empirical distribution of the portfolio weights over time.

Hedging results

- Hedging 50 years liability
- Model estimated using maturities up to 20 years
- Rolling window \( T = 1000 \)
- Mean Absolute Error

Conclusion

- Substantial variation in Nelson-Siegel shape parameter
  - Especially relevant since 2008
  - Interaction with residual GARCH
- Model with time-varying shape outperforms in out-of-sample hedging.