Score-Driven Nelson-Siegel: Hedging Long-Term Liabilities

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Hedging long-term liabilities

DeNederlandscheBank

EUROSYSTEEM

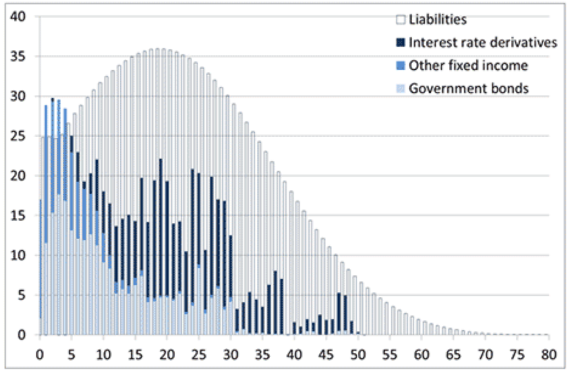


Figure 1 Cash outflows of pension benefits (liabilities) and expected cash inflows (redemptions and coupon payments) of investments in fixed-income securities (interest rate derivatives, sovereign bonds and other fixed-income securities) in EUR billion per year for the next 80 years, year-end 2012. The Figure uses the aggregate of cash inflows and outflows of pension funds.

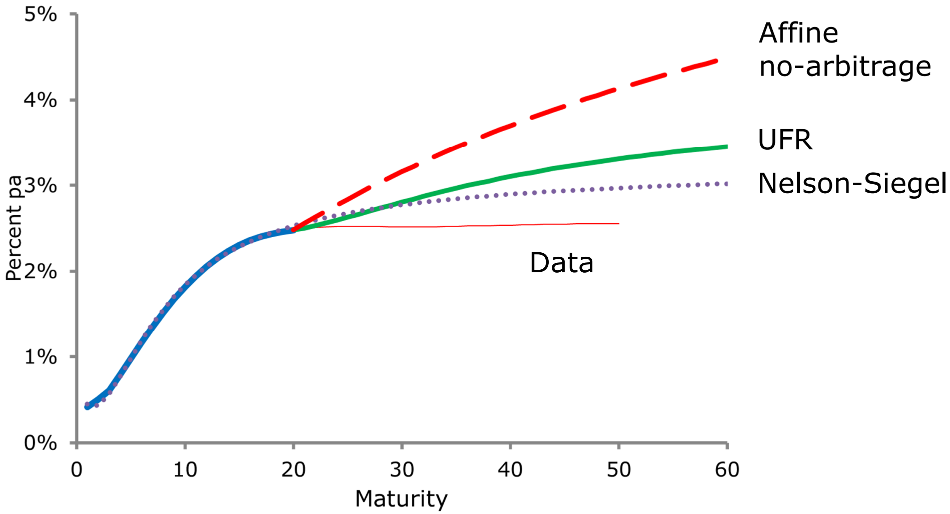
DNBulletin (Sept 2013)

"At year-end 2012, pension funds had hedged 48% of their interest rate risk.

(…)

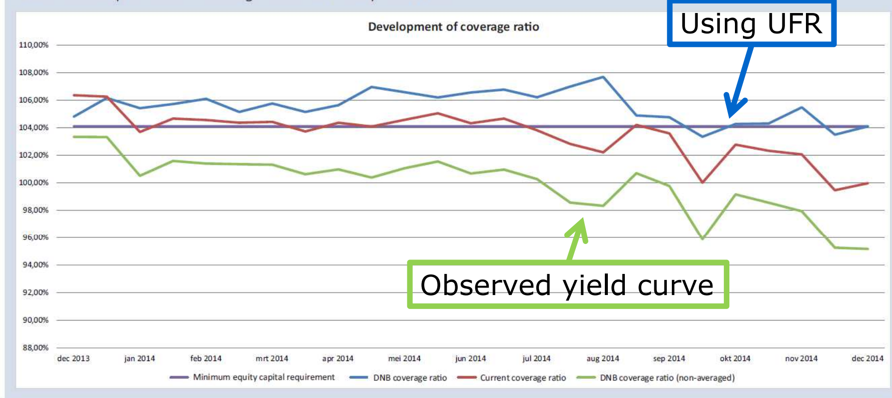
a **1%** fall in market interest rates would cause the average funding ratio to decline **7.8%**; without interest rate hedging, the funding ratio would drop **14.9%**."

It is difficult to fit the very long end of the yield curve



Funds are sensitive to long end of term structure

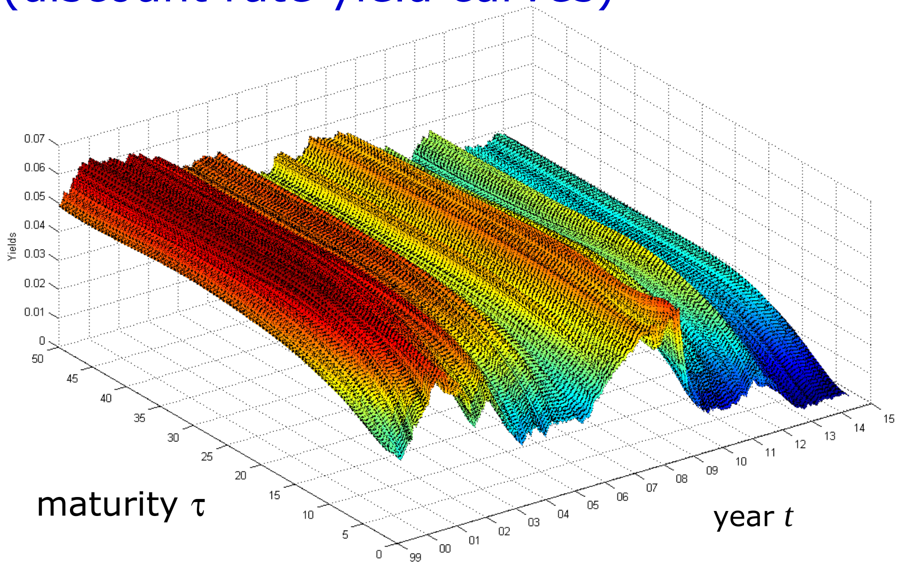
Chart 1: *Development of the coverage ratio of the PGB pension fund*



Explanation: *The interest curve as on 31 December 2014 was, just as on 31 December 2013, determined on the basis of the average of all trading days of the last three months of 2014 while applying the Ultimate Forward Rate (UFR) method. The application of the UFR means that the curve moves towards 4.2% in the long term. In the chart, the coverage ratio is shown including the UFR and averaging (DNB coverage ratio, 104.1% as on 31 December 2014), including the UFR but excluding averaging (DNB coverage ratio not averaged, 100.0% as on 31 December 2014), and excluding UFR and excluding averaging (current coverage ratio, 95.1% as on 31 December 2014).*

source: Annual report Pensioenfonds Grafische Bedrijven 2014

Euro swap data
(discount rate yield curves)



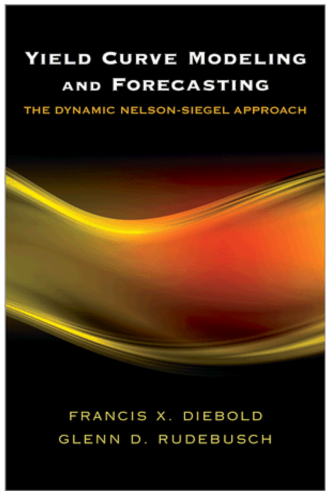
Interest rate hedging: basics

- Duration hedging
  - Find portfolio of bonds with same duration as liabilities
  - **Liabilities longer than liquid instruments**:
  - $\Rightarrow$ leveraged position in longest liquid bonds
- Leveraged duration hedge <u>not</u> optimal, even risky, when yield curve shifts not parallel
- Much evidence for three yield curve factors
  - level, slope, curvature
- Time-varying factor loadings: financial crises

Term structure models

- Nelson-Siegel is a popular 3-factor term structure model that parsimoniously explains the shape and time variation in interest rate levels

$$y_t(\tau) = \sum_{i=1}^{3} B_i(\tau)\tilde{f}_{it} + \epsilon_t(\tau)$$



- A liability hedge is a portfolio with
  - same factor exposure as the liability
  - minimal residual risk
- How stable are factor loadings?
  - How did they change after 2008?

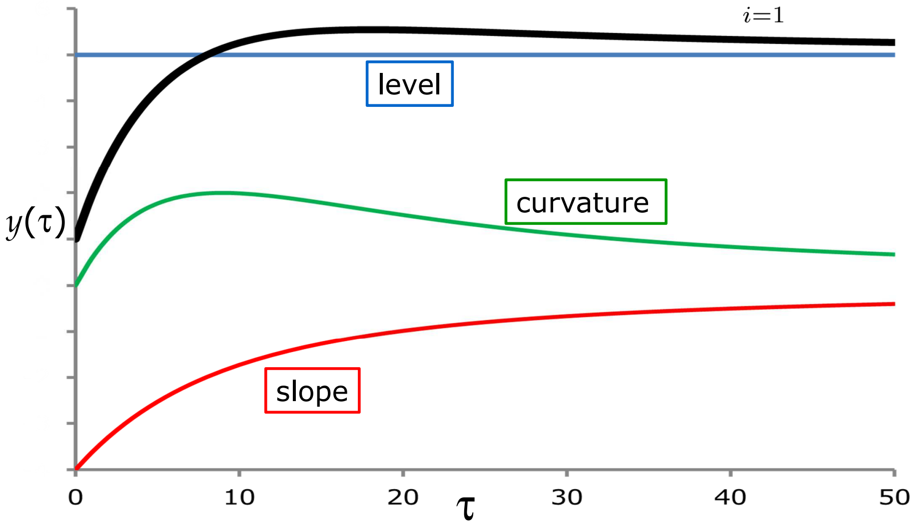
Duration plus …

$$\boxed{B(\tau) = \begin{pmatrix} 1 \\ \frac{1-\mathrm{e}^{-\lambda\tau}}{\lambda\tau} \\ \frac{1-\mathrm{e}^{-\lambda\tau}}{\lambda\tau} - \mathrm{e}^{-\lambda\tau} \end{pmatrix}}$$

- First factor in Nelson-Siegel model is a parallel shift in the yield curve
  - level factor ➔ corresponds to duration hedging
- Other two factors are slope and curvature
  - both governed by a single parameter λ

Term structure as time-varying linear combination of 3 curves

$$y(\tau) = \sum_{i=1}^{3} B_i(\tau)\tilde{f}_i + \epsilon(\tau)$$



Econometric model

$$\boxed{p_t(\tau) = \ln P_t(\tau) = -\tau y_t(\tau)}$$

- Portfolio analysis requires (excess) returns

$$r_{t+h}(\tau) = (p_{t+h}(\tau) - p_t(\tau + h)) \quad - \quad \boxed{(p_{t+h}(0) - p_t(h))}$$

- Further scaling by maturity is like first differencing of yields

$$\rho_{t+h}(\tau) \equiv \frac{r_{t+h}(\tau)}{\tau} = -\Delta y_{t+h}(\tau) + \frac{h}{\tau}(y_t(\tau) - y_t(h))$$

  - small *h*: daily data
  - filters out the persistence in yield levels
  - gets rid of cross-sectional heteroskedasticity of returns

Transformation preserves factor structure

- Scaled excess returns

$$\rho_t(\tau) = B(\tau)' f_t$$

follow factor model with factors

$$f_t = \tilde{f}_t - A\tilde{f}_{t-1}$$

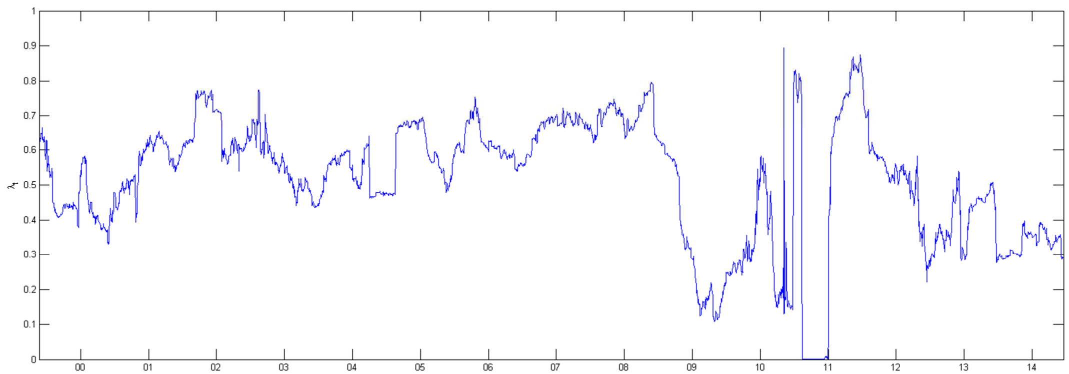
- Add idiosyncratic error terms for maturities *j* = 1, …, *N*

$$\rho_t(\tau_j) = B(\tau_j)' f_t + \epsilon_t(\tau_j)$$

that are uncorrelated across maturities.

Much time-variation in λ since 2008

- Least Squares rolling window estimates (100 days)



Main idea of the paper

$$B(\tau) = \begin{pmatrix} 1 \\ \frac{1-e^{-\lambda\tau}}{\lambda\tau} \\ \frac{1-e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau} \end{pmatrix}$$

- "GARCH" like dynamics for λ_t :

$$\boxed{\lambda_{t+1} = \phi_0 + \phi_1\lambda_t + \phi_2 s_t}$$

- The term s_t is the score of the likelihood function with respect to λ_t :
  - Dynamic Conditional Score model: see Creal, Koopman and Lucas (*JAE* 2013) for the general idea

The score function

$$s_t = \frac{f_t'\mathbf{G}_t'\Sigma_t^{-1}\epsilon_t}{\left(f_t'\mathbf{G}_t'\mathbf{\Sigma}_t^{-1}\mathbf{G}_t f_t\right)^{1/2}}$$

with

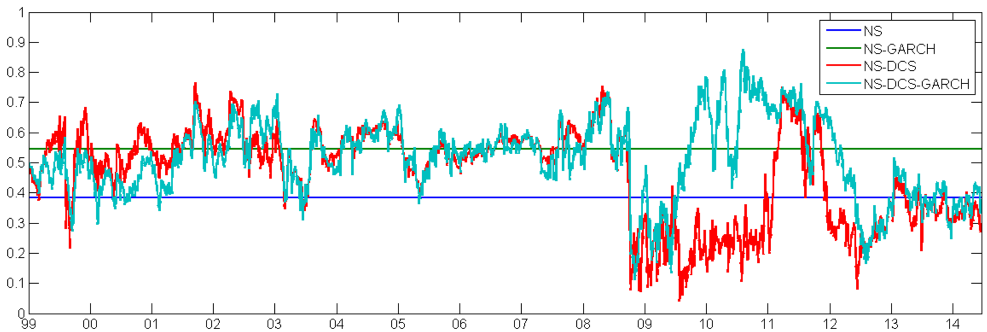
$$\mathbf{G}_t = \left(\frac{\partial \mathbf{B}}{\partial \lambda}\right)_t$$

and Σ_t the error covariance matrix

Diagonal GARCH specification: $\Sigma_t = \sigma_t^2 \boldsymbol{I}$,

$$\sigma_t^2 = \omega^2 + \alpha\sigma_{t-1}^2 + \beta\iota'\epsilon_{t-1} \odot \epsilon_{t-1}$$

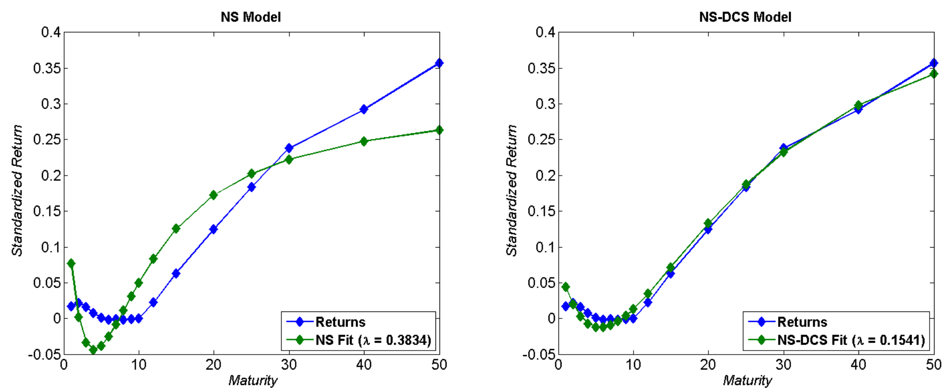
Estimation results score model



Sensitive to heteroskedasticity in residuals

	DCS	DCS-GARCH
ϕ_1	0.966 (0.076)	0.961 (0.072)
ϕ_2	0.029 (0.008)	0.015 (0.005)

Time-varying λ to fit curves



vertical axis: scaled returns ρ_t on Jan 29 2009

Hedging problem

- Fixed liability with **50 years** duration: w_0 = -1
- Find portfolio with same factor exposure and minimal residual risk

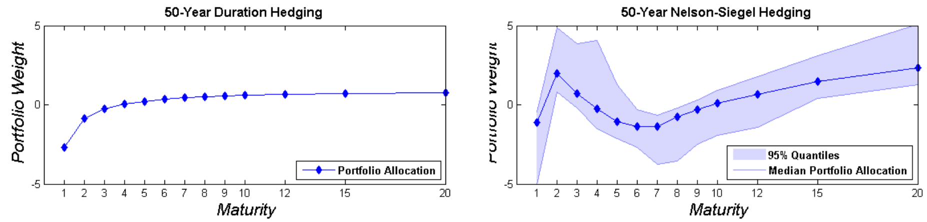
$$\min \sum_{i=1}^{N} w_i^2 \tau_i^2$$

$$\sum_{i=0}^{N} w_i = 0$$

$$\sum_{i=0}^{N} w_i \tau_i B_k(\tau_i) = 0 \qquad k = 1, 2, 3$$

(recall that we modelled returns scaled by maturity)

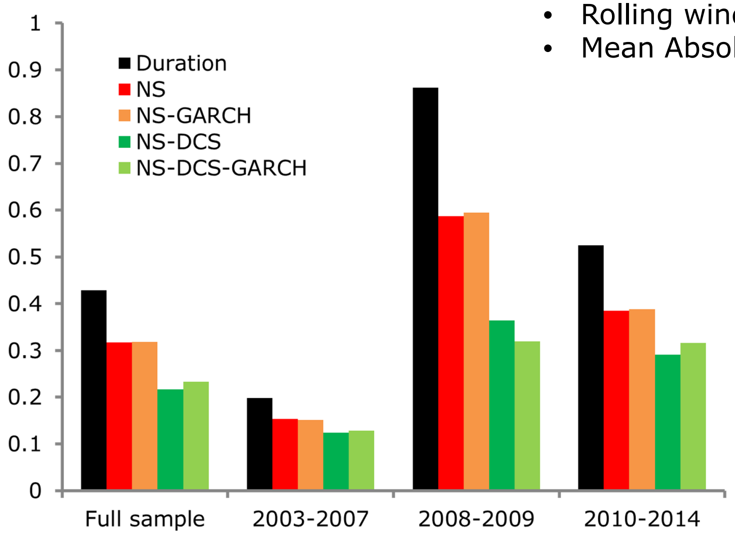
Hedge portfolio



The figure shows the portfolio allocation for hedging a liability with a 50-year maturity. The graph on the left shows Duration hedging. The right graph shows the NS factor-hedge portfolios using λ_t from the NS-DCS model. The shaded area is the empirical distribution of the portfolio weights over time.

Hedging results

- Hedging 50 years liability
- Model estimated using maturities up to 20 years
- Rolling window T = 1000
- Mean Absolute Error



Conclusion

- Substantial variation in Nelson-Siegel shape parameter
  - Especially relevant since 2008
  - Interaction with residual GARCH
- Model with time-varying shape outperforms in out-of-sample hedging.