Funding Shortfall Risk and Asset Prices

Majid Hasan

EDHEC Infrastructure Institute, Singapore
EDHEC Business School

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Motivation & Objective

- Institutions hold an increasingly larger share of assets—approximately 90% of publicly traded equity in the UK and 60% in the US (Celik and Isaksson, 2013);
- Some institutional investors, such as pensions and insurers, have contractually promised payments to their end-investors;
- These institutions often face a funding-ratio constraint, which requires them to hold sufficient wealth to be able to meet their promised payments;
- After the UK Pensions Act of 2004 stipulated that pension funds had to maintain a minimum ratio of assets to liabilities, the UK term structure went from being almost flat to downward sloping at the longer maturity end (Gromb and Vayanos (2010));
- The paper intends to explore the effects of such a constraint on asset prices, and the dynamics of promised payouts on asset prices;
- The effect of the constraint on asset prices is relevant both for understanding asset prices, as well as the optimal design of such regulation (Haldane (2014),Carney (2016)).

Funding-Ratio Constraint

- Denote the constrained institution's amount of invested invested wealth (present value of future consumption) by $F_{1,t}$;
- Denote the required amount of wealth (present value of required payouts) as $F_{1,t}^{min}$;
- The funding-ratio constraint requires

$$\underbrace{\sum_{i=t+1}^{T} E_t \left[\beta^{i-t} \frac{u_0'(c_{0,i})}{u_0'(c_{0,t})} \overbrace{c_{1,i}}^{\text{actual payout}} \right]}_{F_{1,t}} \geq \phi^{min} \underbrace{\sum_{i=t+1}^{T} E_t \left[\beta^{i-t} \frac{u_0'(c_{0,i})}{u_0'(c_{0,t})} \overbrace{c_{1,i}^{min}}^{\text{required payout}} \right]}_{F_{1,t}^{min}}$$

Real-World Relevance

- DB pension plans and insurers are constrained to maintain a certain fraction, typically between 0.8 to 1.1, of the present value of their liabilities to end-investors (Blome, Fachiner, Franzen, and Scheuenstuhl (2007))

$$F_t \geq \phi^{min} F_t^{min}; \tag{1}$$

- In Basel III (BIS (2011)), banks are required to hold equity capital of about ten percent of their total risk-weighted assets. Taking F_t^{min} as the bank's total liabilities that are owed to its depositors, the capital requirement can be written as

$$F_t \geq \left(1 + \frac{1}{1 - \phi^{capital}}\right) F_t^{min}; \tag{2}$$

- Endowments typically manage their fund according to 'capital preservation rule' (Dybvig and Qin (2016)). Assuming constant spending amounts, constant discount rates, and infinite horizon, the funding-ratio constraint can be written as

$$\frac{c^{min}}{F_t} \leq IRR, \tag{3}$$

where IRR is the one-period constant discount rate used by the fund to discount its future spending needs.

Related Literature

- The paper is closest to Grossman and Zhou (1996) and Basak (1995); Basak and Shapiro (2001);
- Grossman and Zhou (1996) considers a setting where utility is derived from terminal wealth, and the terminal wealth is required to be a certain fraction of (exogenously given) initial wealth;
- Basak (1995) considers a continuous-time setting with intertemporal consumption, where the portfolio insurer requires its wealth at a future time to be above an exogenously given threshold, and the constraint is incorporated though the utility function;
- We consider a discrete-time setting, impose the funding-ratio constraint along with budget constraint, and set the wealth threshold as the present value of an exogenously given consumption stream.

Main Intuitions

- Two regions: (1) constraint is never binding (unconstrained region), (2) constraint has a positive probability of being binding (constrained region);
- The constrained institution invests more for a given level of current payout, increasing asset prices;
- The constrained institutions at $t = 0$ moves out of (into) the constrained region upon good (bad) realisations of the state of the economy (aggregate dividend);
- Funding-shortfall risk matters more in bad states (lower aggregate dividend) of the world, making risky assets more risky, and increasing asset premia, and reducing riskfree rates;
- Given that funding-shortfall risk matters less in good states of the world, conditional cashflow covariances of risky assets with the aggregative dividend matter, affecting the cross-section of asset returns;
- Constrained institution increases its allocation to equity as the duration of minimum-payouts decreases;
- Future exogenous payouts only matter when they require the institution to increase its share of aggregate payout compared to the current level, hence increasing payouts are likely to matter more than decreasing payouts.

Main Findings

- The effect on prices is determined by the severity of the constraint, and even a few small institutions that are sufficiently constraint may have a non-negligible effect on asset prices;
- A non-linear CCAPM or a two-factor linear CCAPM is obtained, where the second factor is related to the funding-ratio;
- Lowers riskfree rate(s), increases equity premium and Sharpe ratio, lower equity return volatility, and can affect the cross-section of asset returns;
- Equity premium, and Sharpe ratio evolve in a counter cyclical manner, while volatility evolves cyclically;
- Asset return moments exhibit asymmetric dependence on innovations in the aggregate dividend;
- Time-variation in exogenous payouts affects the dynamics of asset returns, and may lead to predictability;
- Expected returns on longer maturity assets are generally higher than expected returns on shorter maturity assets (upward sloping term structure);
- The term structure of riskfree rates can be affected by the relative growth rates of aggregate dividend and institutional payouts;
- Funding-ratio constrained investors may optimally employ under-diversification and short-selling.

Policy Implications

- Higher (and countercyclical) funding-ratio requirements can decrease market volatility (conditional volatility of equity returns);
- Higher funding-ratio requirements may increase the price of risk, and, hence, the cost of capital for firms;
- Higher capital requirements for banks may increase their appetite for more-risky loans relative to the less-risky loans, and may discourage them from longer-term lending;
- A regulatory insistence on maintaining funding-ratio levels can inflate asset prices and lead to a bubble-like behaviour in the prices of more-risky assets;
- Simultaneous imposition of funding-ratio and short-sale constraints may not be optimal.

Model Setup

- Pure-exchange, discrete-time economy with one riskfree and one risky asset;
- $n = 0$ denotes the risk free asset, and $n = 1$ denotes the risky asset;
- The dividend of the risky asset, $d_{n,t}$, follows a binomial process

$$d_{1,t} = d_{1,t-1} e^{\mu - \frac{1}{2}\sigma^2 + \sigma z} \tag{4}$$

  where z can take values of -1 and 1 with an equal probability of $1/2$;
- $m = 0$ denotes a representative unconstrained institution, and $m = 1$ denotes a representative constrained institution;
- The constrained institution is required to hold a fraction, f^{min}, of the present value of an exogenous payout stream, $c_{1,t}^{min}$;
- Both institutions manage funds under identical (unconstrained) preferences

$$u(c_t) = \frac{c_t^{1-\gamma}}{1-\gamma} \tag{5}$$

  where c_t is interpreted as the institution's total payout.

Notation

- Payouts of each institution are denoted by $c_{m,t}$;
- Allocations to each asset by each institution are denoted by $\theta^n_{m,t}$;
- Prices and returns of the two assets are denoted by $P_{n,T}$ and $R_{n,t}$;
- The invested wealth of each institution is denoted by $F_{m,t} = \sum_{n=0}^{n=1} \theta^n_{m,t} P_{n,t}$;
- The total wealth of each institution is denoted by $W_{m,t} = c_{m,t} + F_{m,t}$;
- Each institution's share of aggregate endowment is denoted by $\omega_{m,t} \equiv \frac{c_{m,t}}{d_{1,t}}$;
- Unconstrained institution's share of aggregate endowment, $\omega_{0,t}$, serves as the endogenous state variable;
- Optimal quantities for the case of funding-ratio constraint and unconstrained models are denoted by $\widehat{\ }$, and $\mathring{\ }$, respectively.

Asset Allocations at time $T-1$

If c_T^{min} is independent of $d_{1,T}$, the constrained investor can maintain its funding-ratio requirement by holding a fraction, $E_{T-1}\left[\frac{c_T^{min}}{d_{1,T}}\right]$, of the aggregate wealth

$$\widehat{\theta}_{1,T-1}^{1} = \begin{cases} \omega_{1,T-1}, & \text{if } \omega_{1,T-1} > \omega_{T-1}^{min} \\ E_{T-1}\left[\omega_T^{min}\right], & \text{if } \omega_{1,T-1} \leq E_{T-1}\left[\omega_T^{min}\right]. \end{cases} \tag{6}$$

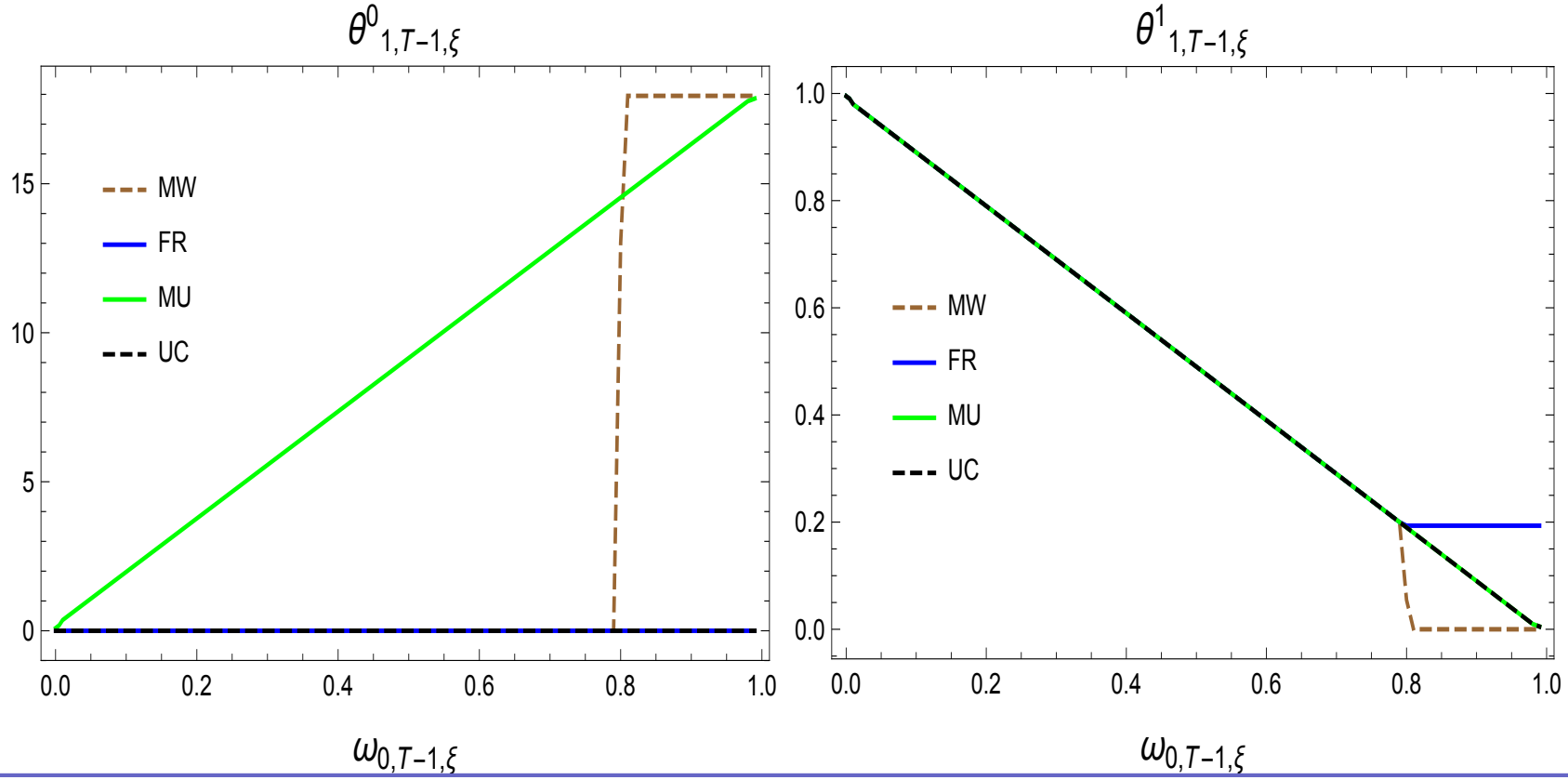
And for the unconstrained institution

$$\widehat{\theta}_{0,T-1}^{1} = \begin{cases} \omega_{0,T-1}, & \text{if } \omega_{1,T-1} > \omega_{T-1}^{min} \\ \omega_{0,T-1}\frac{1-E_{T-1}\left[\omega_T^{min}\right]}{1-\omega_{1,T-1}}, & \text{if } \widehat{\omega}_{1,T-1} \leq E_{T-1}\left[\omega_T^{min}\right], \end{cases} \tag{7}$$

where $\omega_T^{min} = \frac{c_T^{min}}{d_{1,T}}$.

Asset Allocations at time $T-1$

Figure: Bond (left panel) and equity (right panel) allocations at a single node ξ, which corresponds to the lowest aggregate dividend realisation, at time $T-1$, as a function of the unconstrained institution's share of aggregate endowment at time $T-1$. FR and UC curves always overlap in the left panel, and MU and UC curves always overlap in the right panel.



Payouts (Withdrawals) at time T

Payouts at time T are given by

$$\widehat{c}_{m,T} = \widehat{\theta}^{1}_{m,T-1} d_{1,T}, \tag{8}$$

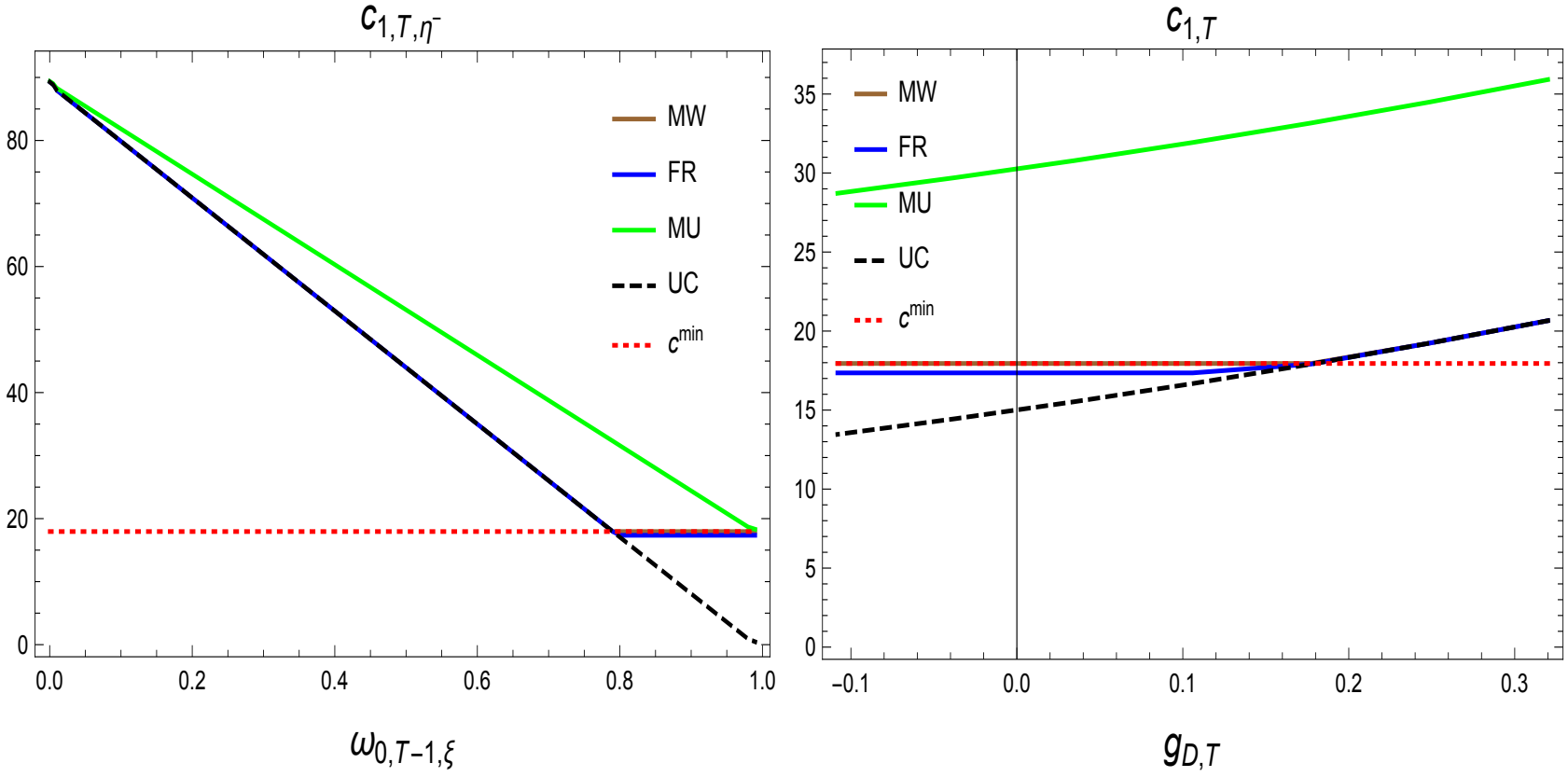
where ω^{min}_{T-1} denotes the threshold consumption share of the funding-ratio constrained investor below which the constraint becomes binding, and is given by

$$\omega^{min}_{T-1} = \phi^{min} E_{T-1}\left[\frac{c^{min}_{T}}{d_{1,T}}\right]. \tag{9}$$

We assume $\phi^{min} c^{min}_{T} < E_{T-1}[d_{1,T}]$, so that $\omega^{min}_{T-1} < 1$.

Payouts (Withdrawals) at time T

Figure: The left panels shows fund withdrawals at a single node η^-, which corresponds to a negative innovation in the aggregate dividend starting from node ξ at time $T-1$, and the right panel shows withdrawals across nodes at time T, for $\omega_{0,0} = 0.85$.



Asset Prices at time $T-1$

Asset prices at time $T-1$ are inflated due to an insurance demand.

$$\widehat{P}_{0,T-1} = \begin{cases} \beta \frac{E_{T-1}[d_{1,T}^{-\gamma}]}{d_{1,T-1}^{-\gamma}}, & \text{if } \omega_{1,T-1} > E_{T-1}[\omega_T^{min}] \\ \beta \left(\frac{1-E_{T-1}[\omega_T^{min}]}{1-\omega_{1,T-1}} \right)^{-\gamma} \frac{E_{T-1}[d_{1,T}^{-\gamma}]}{d_{1,T-1}^{-\gamma}}, & \text{if } \omega_{1,T-1} \leq E_{T-1}[\omega_T^{min}] \end{cases} \tag{10}$$

$$\widehat{P}_{1,T-1} = \begin{cases} \beta \frac{E_{T-1}[d_{1,T}^{1-\gamma}]}{d_{1,T-1}^{-\gamma}}, & \text{if } \omega_{1,T-1} > E_{T-1}[\omega_T^{min}] \\ \beta \left(\frac{1-E_{T-1}[\omega_T^{min}]}{1-\omega_{1,T-1}} \right)^{-\gamma} \frac{E_{T-1}[d_{1,T}^{1-\gamma}]}{d_{1,T-1}^{-\gamma}}, & \text{if } \omega_{1,T-1} \leq E_{T-1}[\omega_T^{min}] \end{cases}. \tag{11}$$

Asset prices can be written in terms of their unconstrained counterparts as

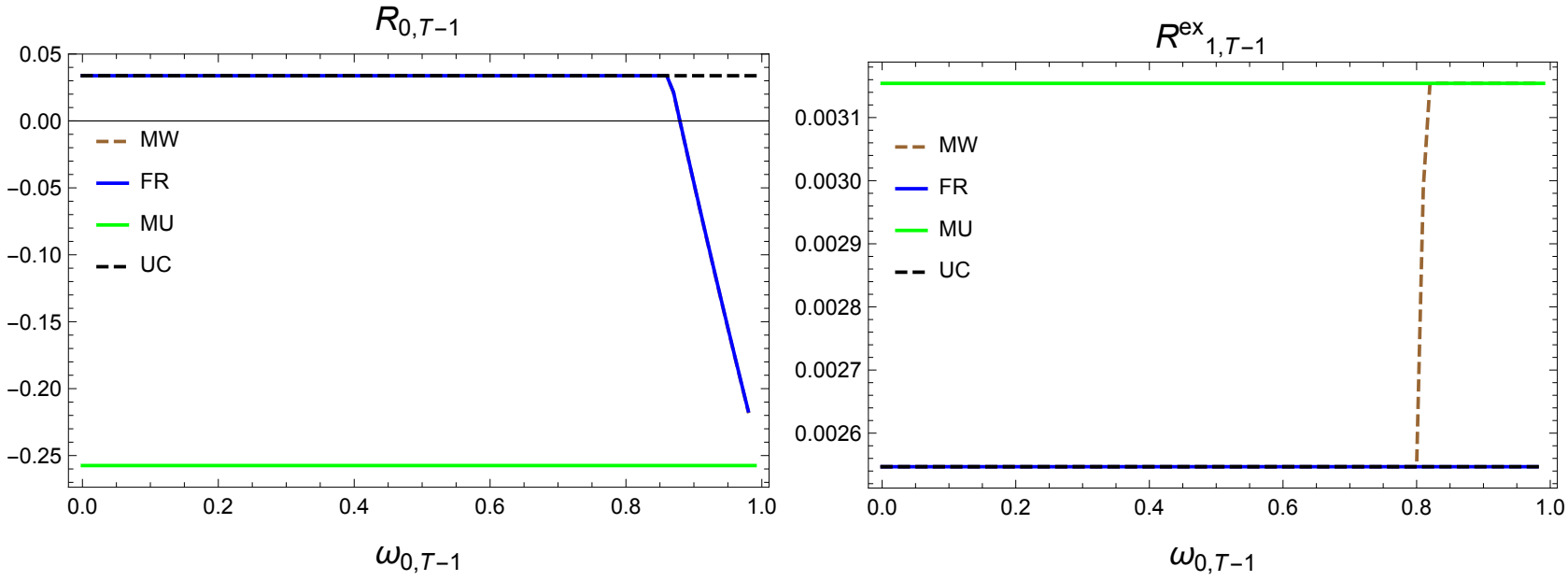
$$\widehat{P}_{n,T-1} = \mathring{P}_{n,T-1} \left(1 + \widehat{f}_{\omega_{T-1}}^{-\gamma} - 1 \right) \tag{12}$$

where

$$\widehat{f}_{\omega_{T-1}} = \min \left(1, \frac{1 - E_{T-1}[\omega_T^{min}]}{1 - \omega_{1,T-1}} \right) \tag{13}$$

Expected Bond Return and Equity Premium at Time $T-1$

Figure: Expected bond return (left panel) and equity premium (right panel) at time $T-1$.



Equilibrium at time t

With only one non-zero c^{min} at T, equilibrium prices at time t can be written as

$$\widehat{P}_{0,t} = \beta \frac{E_t[d_{1,t+1}^{-\gamma}]}{d_{1,t}^{-\gamma}} = \mathring{P}_{0,t} \tag{14}$$

$$\begin{aligned}\widehat{P}_{1,t} &= \frac{1}{d_{1,t}^{-\gamma}} E_t \left[\sum_{i=t+1}^{T-1} \beta^{i-t} d_{1,i}^{1-\gamma} + \beta^{T-t} d_{1,T}^{1-\gamma} f_{\omega_{T-1}}^{-\gamma} \right] \\ &= \underbrace{\frac{1}{d_{1,t}^{-\gamma}} E_t \left[\sum_{i=t+1}^{T} \beta^{i-t} d_{1,i}^{1-\gamma} \right]}_{\text{CCAPM Price} \equiv \mathring{P}_{1,t}} + \underbrace{\frac{1}{d_{1,t}^{-\gamma}} E_t \left[\beta^{T-t} d_{1,T}^{1-\gamma} \left(f_{\omega_{T-1}}^{-\gamma} - 1 \right) \right]}_{\text{Price Deviation due to FR Constraint} \equiv \Delta \widehat{P}_{1,t}} .\end{aligned} \tag{15}$$

Equilibrium allocations can be computed as

$$\widehat{\theta}_{m,0}^{0} = \frac{W_{m,1,u} \Delta_{u,d} W_{tot,1} - W_{tot,1,u} \Delta_{u,d} W_{m,1}}{\Delta_{u,d} W_{tot,1}} \tag{16}$$

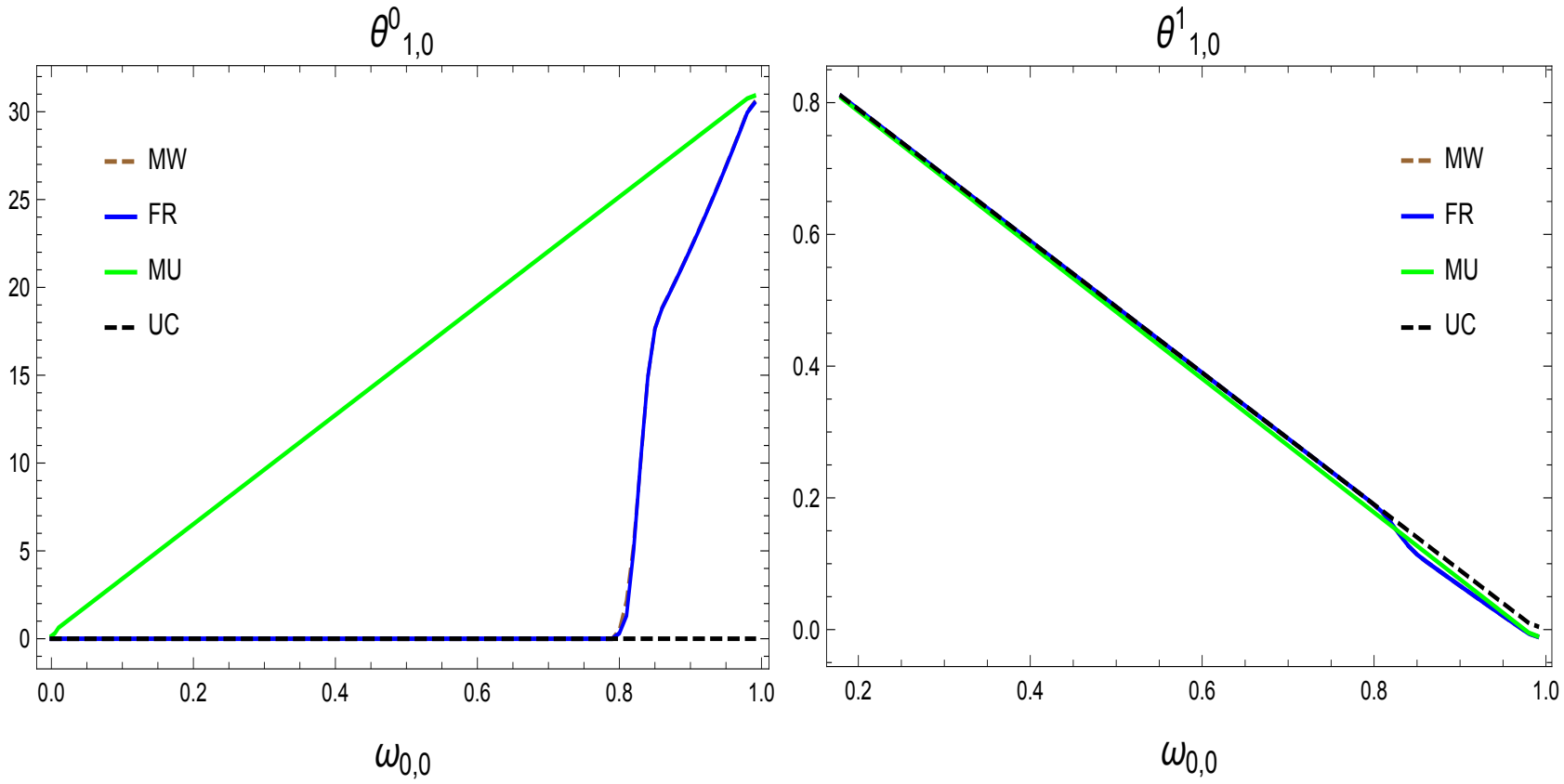
$$\widehat{\theta}_{m,0}^{1} = \frac{\Delta_{u,d} W_{m,1}}{\Delta_{u,d} W_{tot,1}}, \tag{17}$$

where

$$\Delta_{u,d} X = X_u - X_d. \tag{18}$$

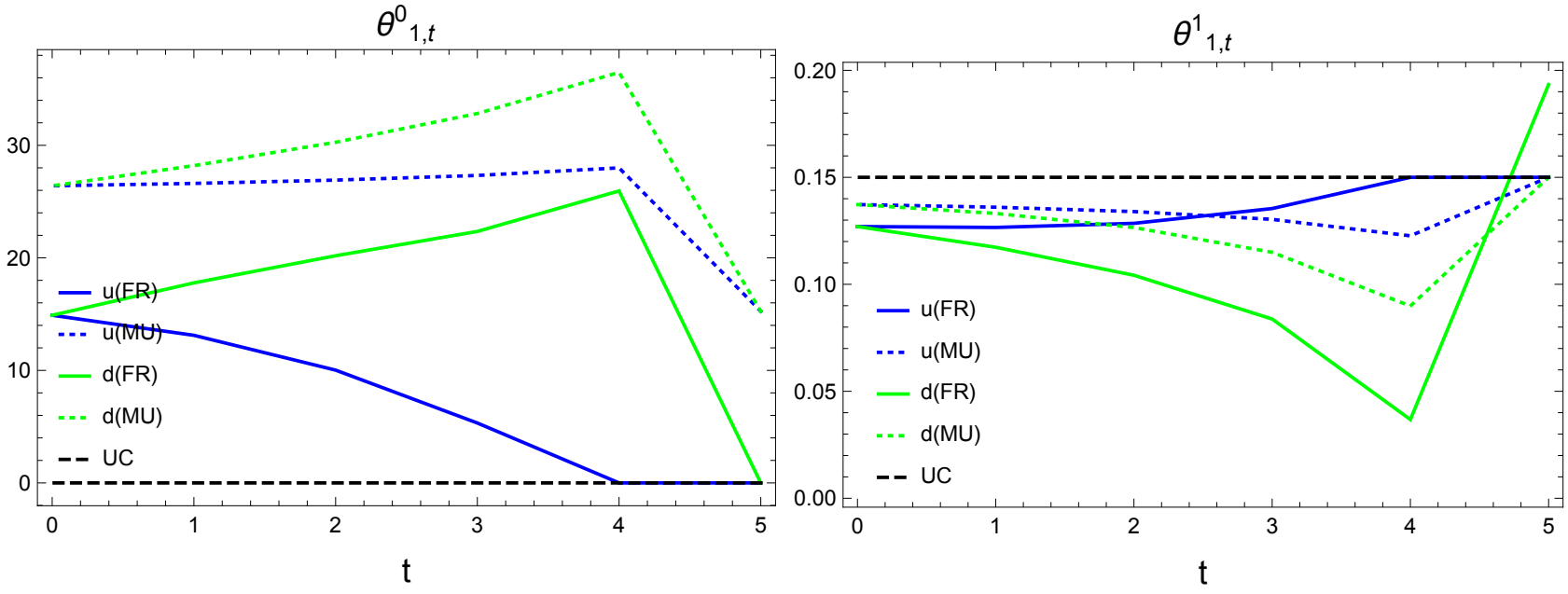
Bond and Equity Allocations at time 0

Figure: Constrained institution's bond (left panel) and equity (right panel) allocations.



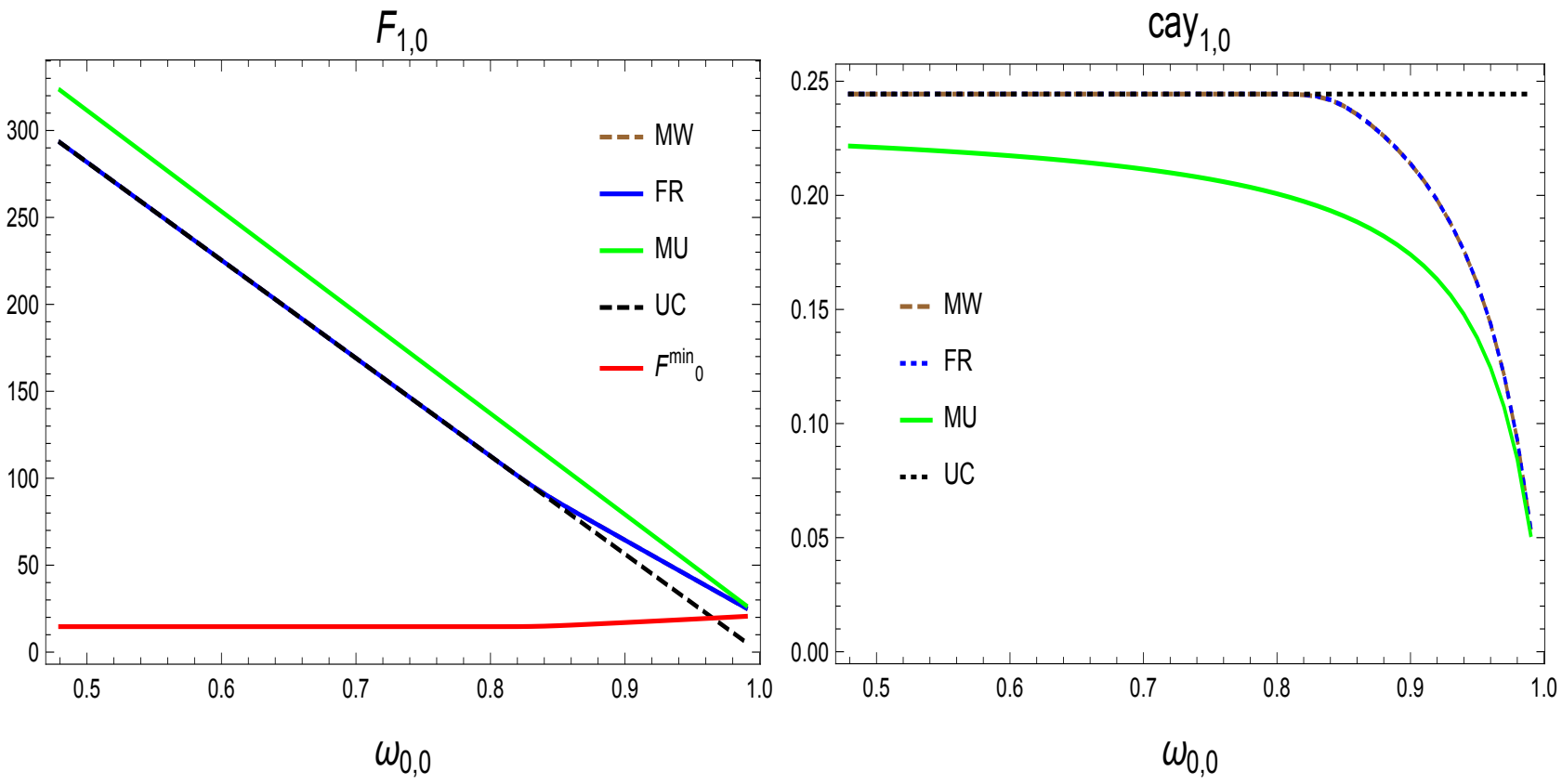
Evolution of Bond and Equity Allocations

Figure: Evolution of the constrained institution's bond (left panel) and equity (right panel) allocation.



Invested Wealth and Payout/Wealth Ratio at time 0

Figure: Constrained institution's invested wealth (left panel), and payout/invested wealth ratio (cay) at time 0.



Expected Bond Return and Equity Premium at Time t

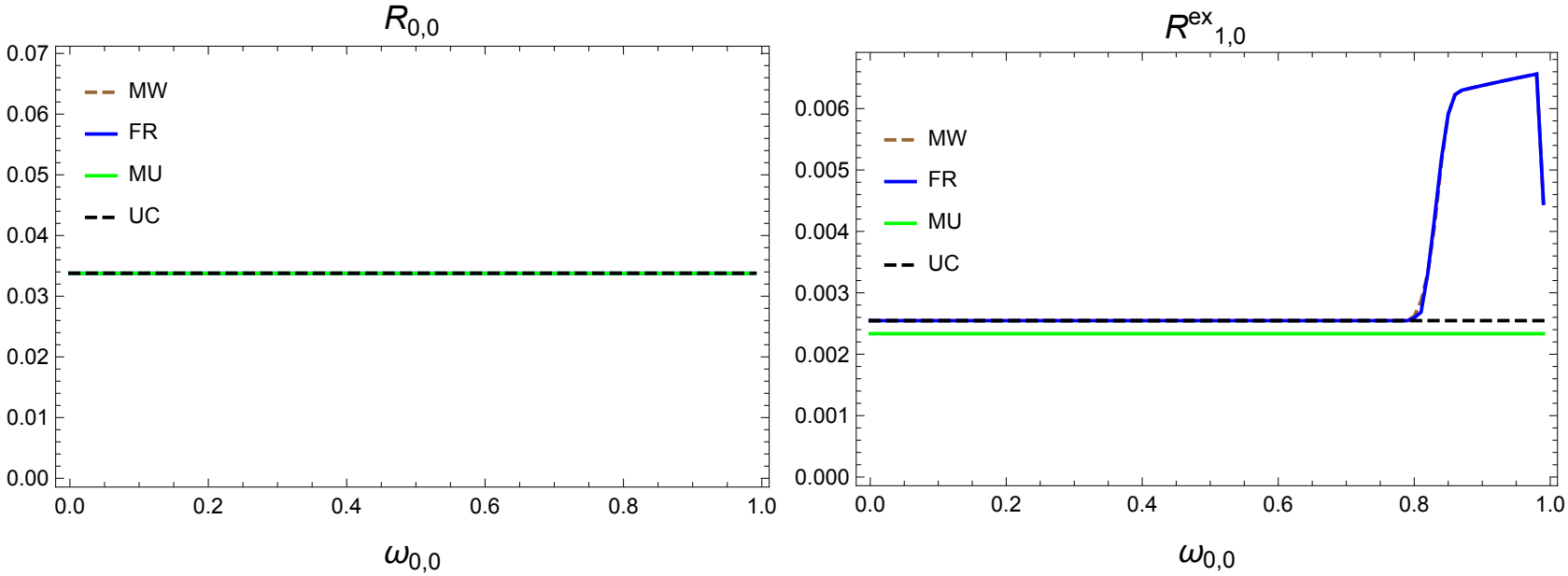
- For $t < T-1$ the expected return on bond is unchanged;
- The change in expected equity return from its unconstrained level can be approximated as

$$E_t\left[\Delta\widehat{R}_{1,t}\right] \approx \log\left(1+\frac{E_t\left[\widehat{SDF}\frac{\Delta\widehat{P}_{1,t+1}}{\mathring{P}_{1,t}}\frac{\mathring{P}_{1,t}-\mathring{P}_{1,t}(\mathcal{F}_{t+1})}{\mathring{P}_{1,t}(\mathcal{F}_{t+1})}\right]}{1+\frac{\Delta\widehat{P}_{1,t}}{\mathring{P}_{1,t}}}\right) - Var_t\left(\frac{\Delta\widehat{P}_{1,t+1}}{\mathring{P}_{1,t+1}+d_{1,t+1}}\right)$$

  where $\mathring{P}_{1,t}(\mathcal{F}_{t+1})$ denotes the time-t value of the dividend stream, $d_{1,t+1:T}$, conditional on time-$t+1$ information, $\mathcal{F}_{t+1}$;
- $\frac{\Delta\widehat{P}_{1,t}}{\mathring{P}_{1,t}}$ can be interpreted as the present value of the insurance against shortfall, relative to the aggregate unconstrained wealth;
- The expectation term can be interpreted as the covariance between loss in unconstrained equity value with discounted insurance value;
- In other words, the loss in equity value is penalised because it covaries positively with the demand for insurance
- The last term, $Var_t\left(\frac{\Delta\widehat{P}_{1,t+1}}{\mathring{P}_{1,t+1}+d_{1,t+1}}\right)$, arises due to the concavity of log returns.

Expected Bond Return and Equity Premium at Time 0

Figure: Expected bond return (left panel) and equity premium (right panel) at time 0.



Equity Premium Volatility and Sharpe Ratio at Time t

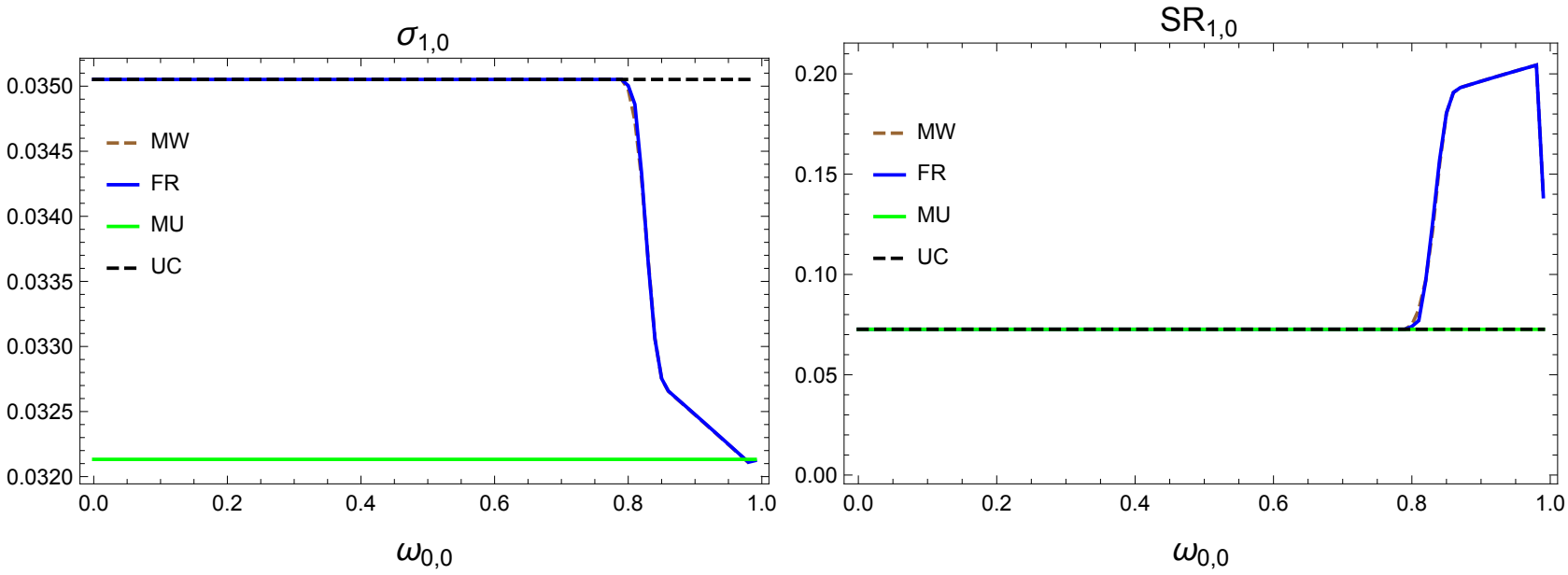
- The volatility of equity return can then be written as

$$\widehat{\sigma}_{1,t}^2 = \mathring{\sigma}_{1,t}^2 + 2\,\underbrace{Cov_t\left(\mathring{R}_{1,t}, \frac{\Delta\widehat{P}_{1,t+1}}{d_{1,t+1} + \mathring{P}_{1,t+1}} - \frac{\Delta\widehat{P}_{1,t}}{\mathring{P}_{1,t}}\right)}_{\text{Covariance} < 0}$$
$$+ Var_t\left(\frac{\Delta\widehat{P}_{1,t+1}}{d_{1,t+1} + \mathring{P}_{1,t+1}} - \frac{\Delta\widehat{P}_{1,t}}{\mathring{P}_{1,t}}\right). \tag{19}$$

- Like the expected return, the return volatility is also affected by the covariance of unconstrained equity return and value of insurance against shortfall, but in the opposite direction.

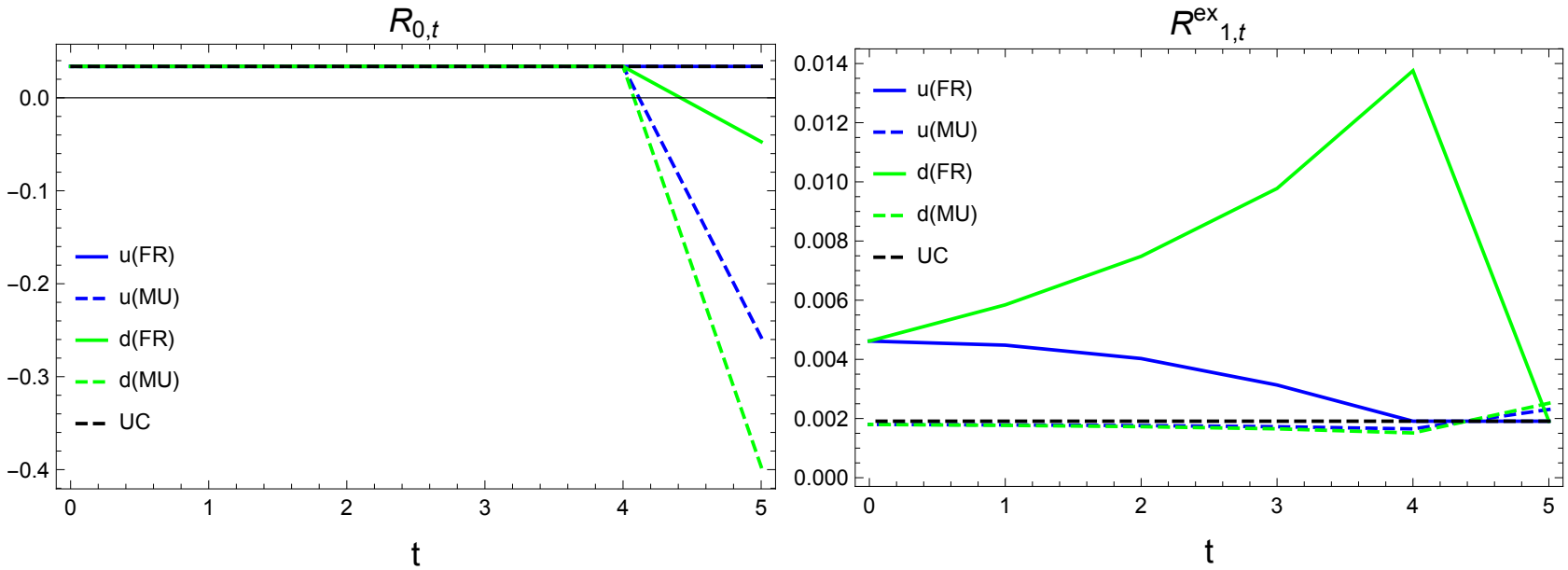
Equity Premium Volatility and Sharpe Ratio at Time 0

Figure: Equity premium volatility (left panel) and Sharpe ratio (right panel) at time 0.



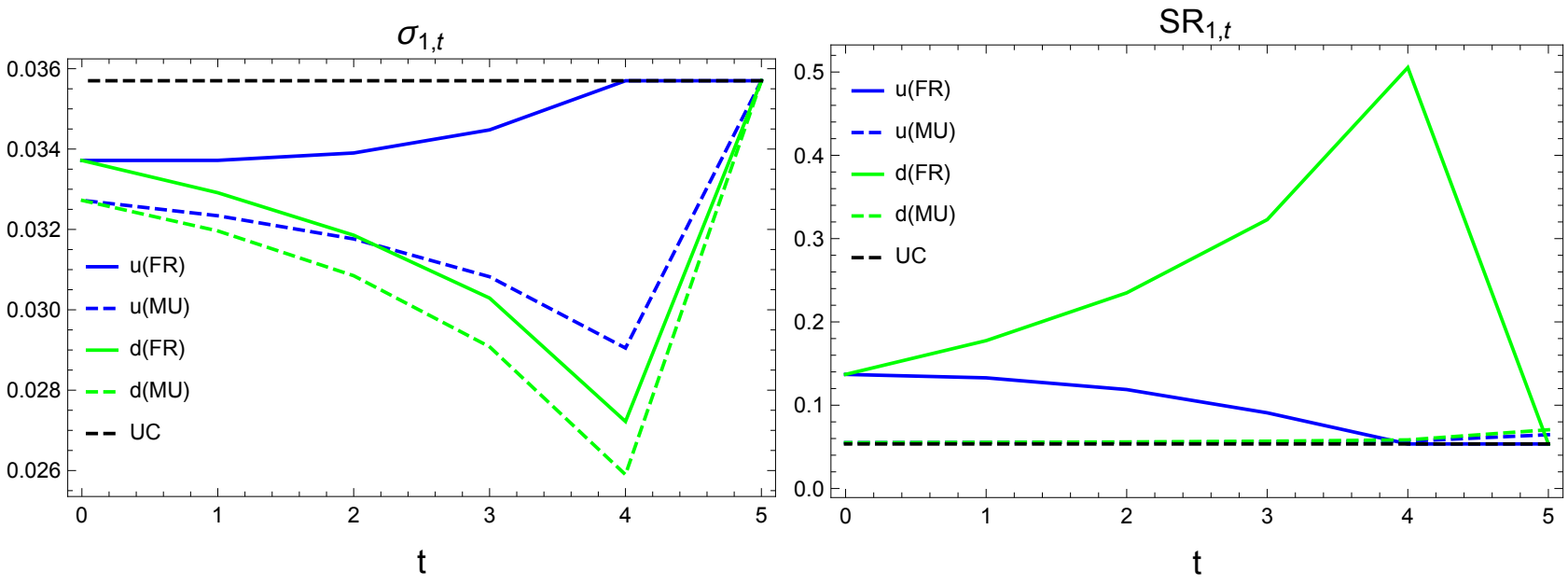
Evolution of Bond Return and Equity Premium

Figure: Evolution of bond return (left panel) and equity premium (right panel).



Evolution of Equity Premium Vol and Sharpe Ratio

Figure: Evolution of equity premium volatility (left panel) and Sharpe ratio (right panel).



Stochastic Discount Factor

The SDF between time 0 and T can be written as

$$SDF_{0,T} = \beta^T \frac{u'(c_{0,T})}{u'(c_{0,0})} = \beta^T \left(\frac{1-\omega_{1,T}}{1-\omega_{1,0}}\right)^{-\gamma} \left(\frac{d_{1,T}}{d_{1,0}}\right)^{-\gamma}, \tag{20}$$

where

$$\omega_{1,T} = \max\left(E_{T-1}\left[\omega_T^{min}\right], \omega_{1,0}\right). \tag{21}$$

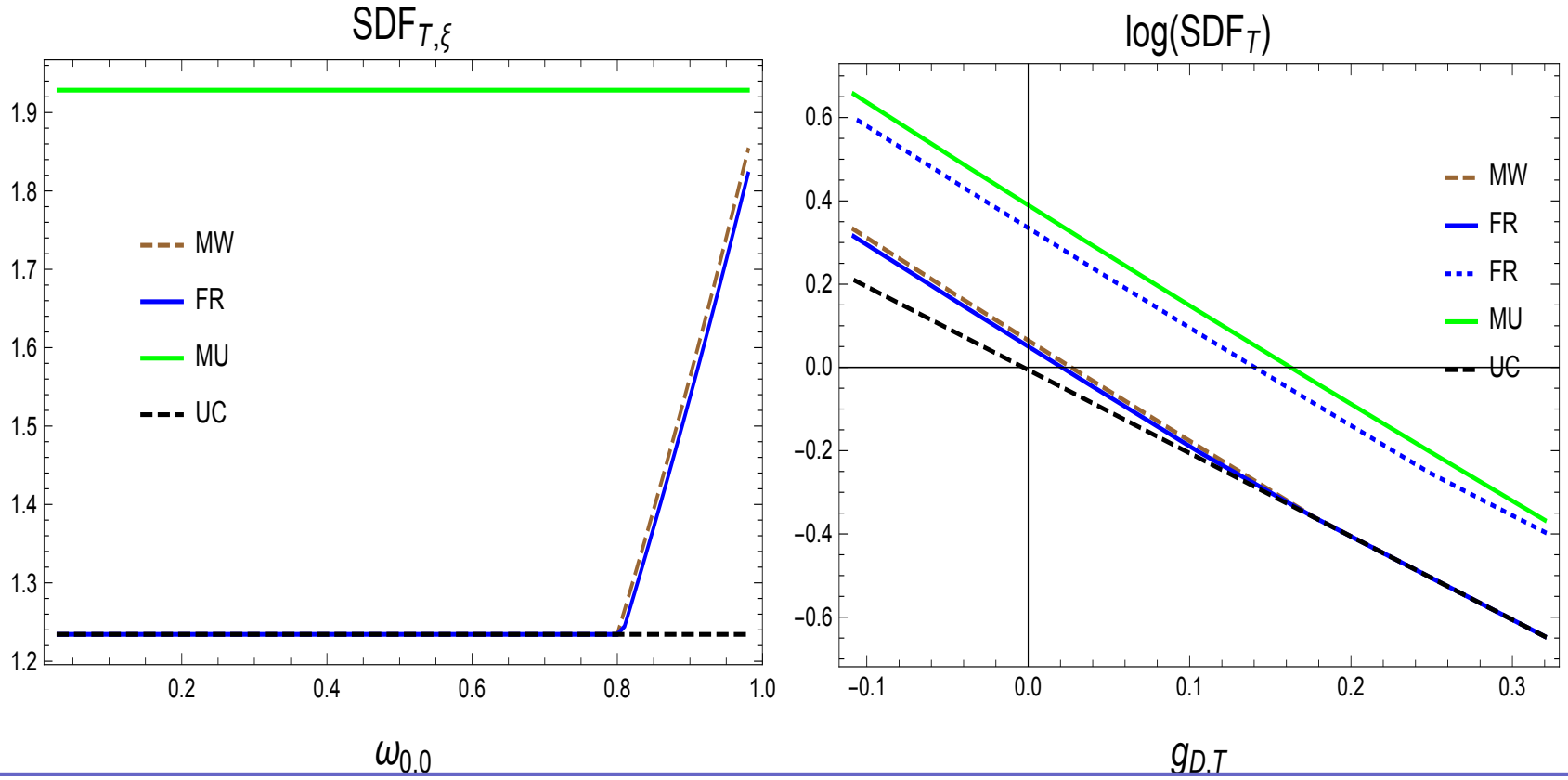
Hence, the expected excess return on any asset is determined by

$$Cov_t\left(r_{i,T}^{ex}, \log(SDF_T)\right) = -\gamma Cov_t\left(r_{i,T}^{ex}, \log\frac{d_{1,T}}{d_{1,0}}\right) - \gamma Cov_t\left(r_{i,T}^{ex}, \log\frac{1-\omega_{1,T}}{1-\omega_{1,0}}\right) \tag{22}$$

Notice that the second covariance is state-dependent and non-zero only for bad states of the world (lower realisations of the aggregate dividend).

Stochastic Discount Factor - Two Factor CCAPM

Figure: The left panel shows the SDF for the lowest aggregate dividend realization at time T for all possible values of the initial endowment share, $\omega_{0,0}$. The right Panel shows the log of SDF for different realizations of the aggregate dividend at time T, for an initial endowment share of $\omega_{0,0} = 0.85$, except for the dotted blue curve, which corresponds to an initial endowment share of 0.98.



Time-Varying Minimum-Payouts

With time-varying minimum-payouts at every date, the price of a τ-maturity bond can be written as

$$\widehat{P}^{\tau}_{0,t} = \underbrace{\frac{1}{d_{1,t}^{-\gamma}} E_t \left[\sum_{i=t+1}^{\tau} \beta^{i-t} d_{1,i}^{-\gamma} \right]}_{\text{CCAPM Price: } \mathring{P}^{\tau}_{0,t}} + \underbrace{\frac{1}{d_{1,t}^{-\gamma}} E_t \left[\sum_{i=t+1}^{\tau} \beta^{i-t} d_{1,i}^{-\gamma} \left(\prod_{j=t}^{i-1} f_{\omega_j}^{-\gamma} - 1 \right) \right]}_{\text{Price deviation due to the constraint: } \Delta\widehat{P}^{\tau}_{0,t}}, \quad (23)$$

and the price of the risky asset can be written as

$$\widehat{P}_{1,t} = \underbrace{\frac{1}{d_{1,t}^{-\gamma}} E_t \left[\sum_{i=t+1}^{T} \beta^{i-t} d_{1,t+i}^{1-\gamma} \right]}_{\text{CCAPM price: } \mathring{P}_{1,t}} + \underbrace{\frac{1}{d_{1,t}^{-\gamma}} E_t \left[\sum_{i=t+1}^{T} \beta^{i-t} d_{1,t+i}^{1-\gamma} \left(\prod_{j=t}^{i-1} f_{\omega_j}^{-\gamma} - 1 \right) \right]}_{\text{Price deviation due to the constraint: } \Delta\widehat{P}_{1,t}}. \quad (24)$$

where

$$f_{\omega_j} \equiv \min \left(1, \frac{1 - E_j \left[\omega_{j+1}^{min} \right]}{1 - \omega_{1,j}} \right). \quad (25)$$

Expected Return with Time-Varying Minimum-Payouts

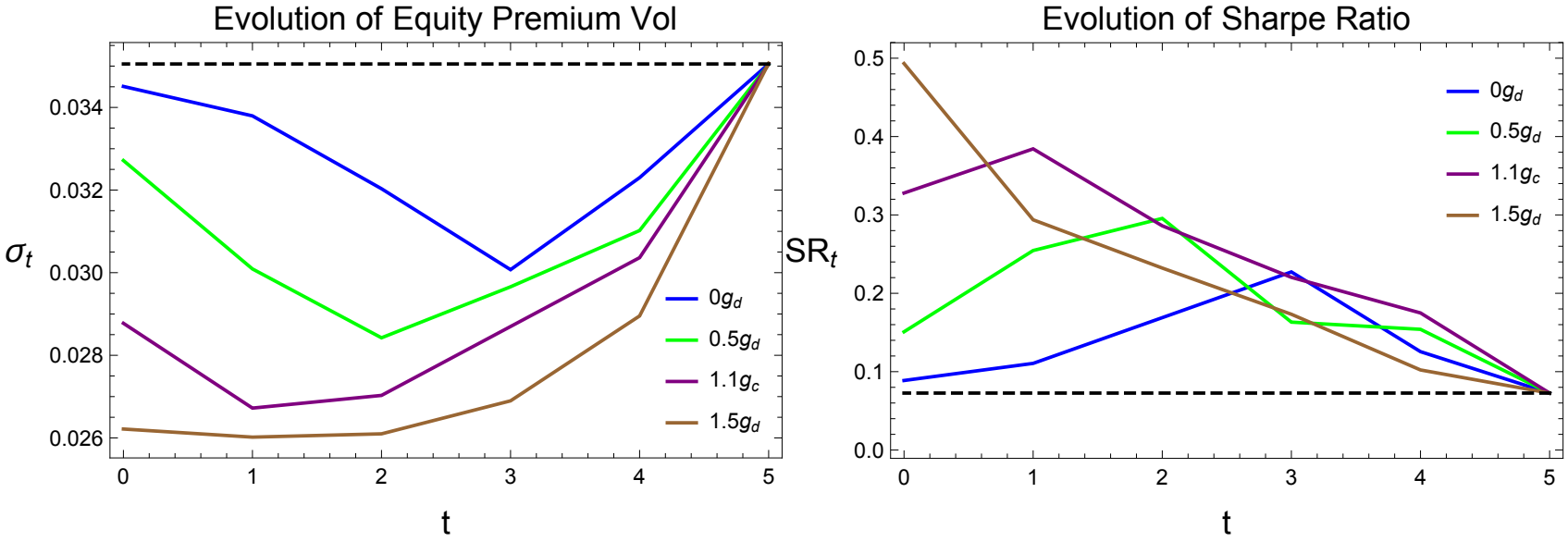
- Expected return on risky asset can be approximated as

$$E_t\left[\widehat{R}_{1,t}\right] = E_t\left[\mathring{R}_{1,t}\right] - Var_t\left(\frac{\Delta\widehat{P}_{1,t+1}}{\mathring{P}_{1,t+1} + d_{1,t+1}}\right)$$
$$+ \log\left(1 + \frac{E_t\left[\widehat{SDF}\frac{\Delta\widehat{P}_{1,t+1}}{\mathring{P}_{1,t}}\frac{\mathring{P}_{1,t}-\mathring{P}_{1,t}\left(\mathcal{F}_{t+1}\right)}{\mathring{P}_{1,t}\left(\mathcal{F}_{t+1}\right)}\right] - E_t\left[\mathring{SDF}\left(f_{\omega_t}^{-\gamma} - 1\right)\frac{\Delta\widehat{P}_{1,t+1}}{\mathring{P}_{1,t}\left(\mathcal{F}_{t+1}\right)}\right]}{1 + \frac{\Delta\widehat{P}_{1,t}}{\mathring{P}_{1,t}}}\right).$$

- This expression has an additional term that depends on $\left(f_{\omega_t}^{-\gamma} - 1\right)$, which measures the value of insurance against the immediate minimum-payout, $c_{1,t+1}^{min}$, and vanishes when the expected next period's payout already exceeds the minimum-payout, i.e. $E_t\left[\widehat{c}_{1,t+1}\right] \geq c_{1,t+1}^{min}$.
- Because $\left(f_{\omega_t}^{-\gamma} - 1\right) \geq 0$, this additional expectation is positive, and the demand for insurance against the immediate minimum-payout decreases the expected equity return.
- While the positive covariance of present value of insurance with equity losses increases the expected equity return, the demand for insurance against the immediate minimum-payout tends to decrease expected equity return.

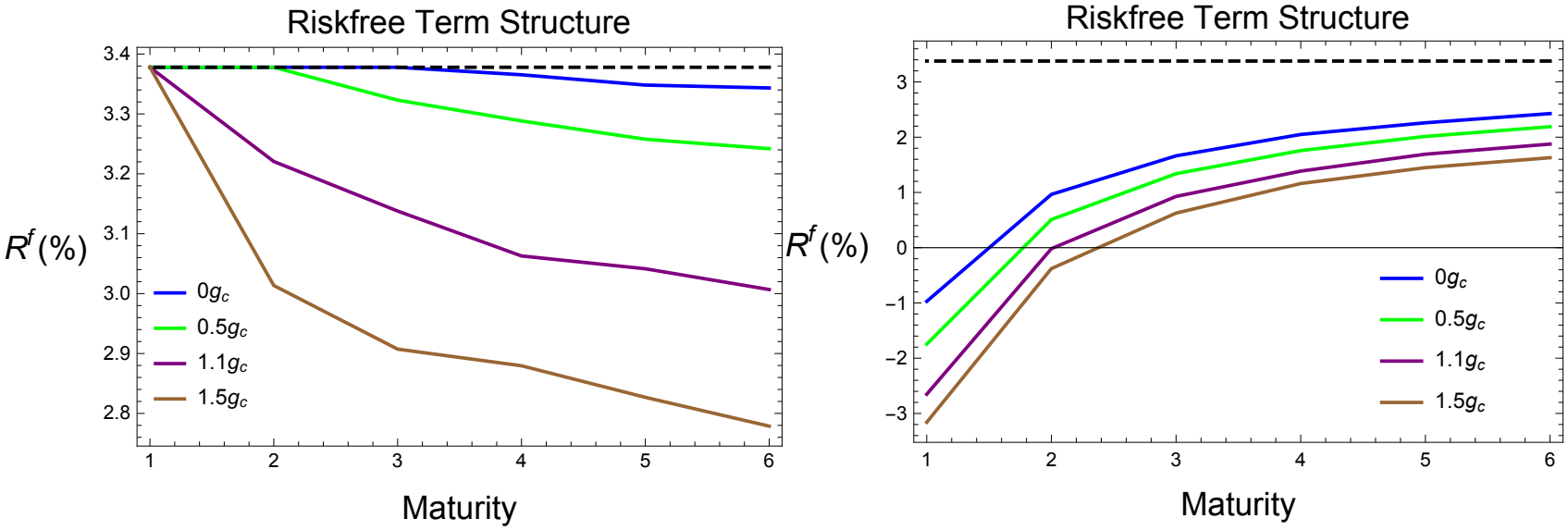
Evolution of Equity Premium Vol and Sharpe Ratio for Time-Varying Minimum Withdrawals

Figure: Evolution of equity premium volatility (left panel), and equity Sharpe ratio (right panel) along the worst possible path of the aggregate dividend, starting from an initial endowment share of 0.85 for the unconstrained institution.



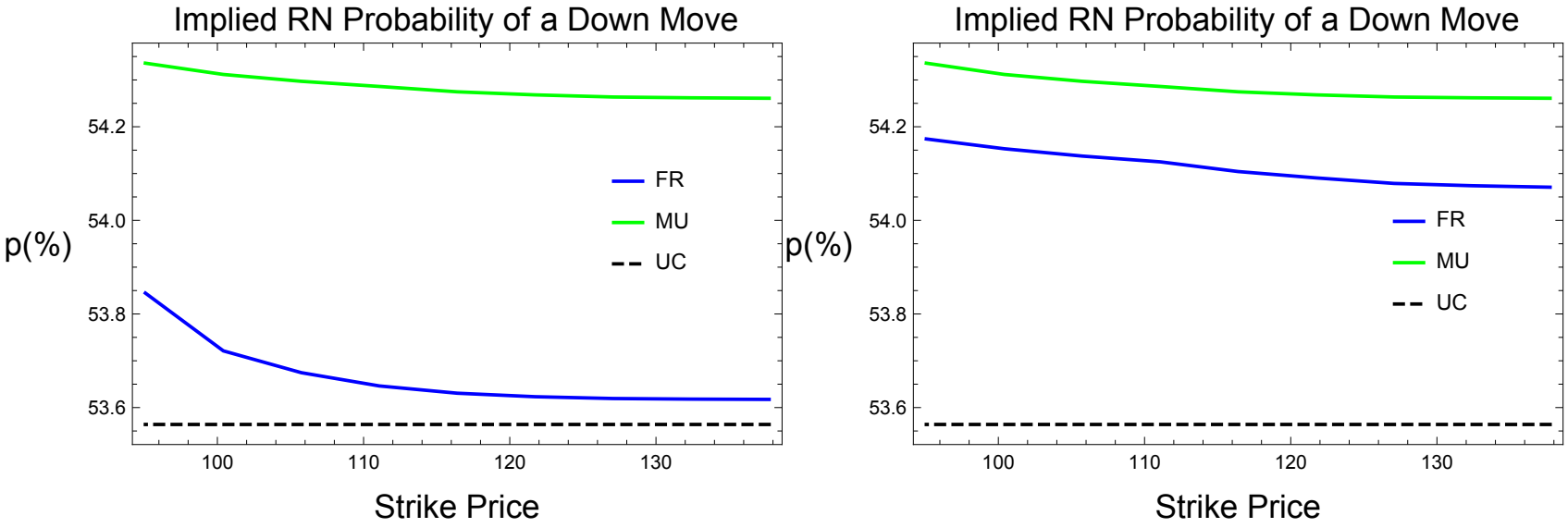
Term Structure of Riskfree Rates

Figure: Term structures of riskfree rates for two different values of the unconstrained institution's endowment share, which is 0.87 for the left panel and 0.90 for the right panel.



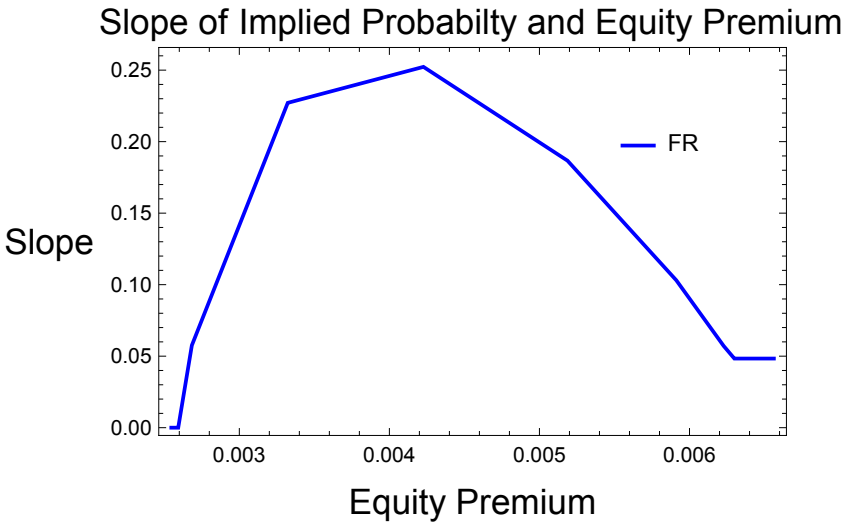
Implied Risk-Neutral Probability of a Downward Move

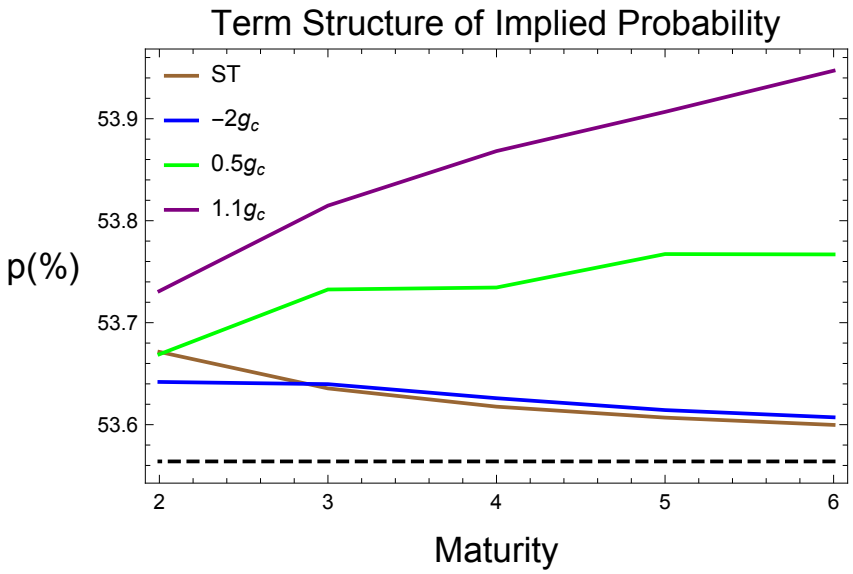
Figure: Implied risk-neutral probability of a down move as a function of strike price for two different values of the unconstrained institution's endowment share. The left panel corresponds to an initial endowment share of 0.84 and and the right panel corresponds to an initial endowment share of 0.87 for the unconstrained institution.



Implied Risk-Neutral Probability

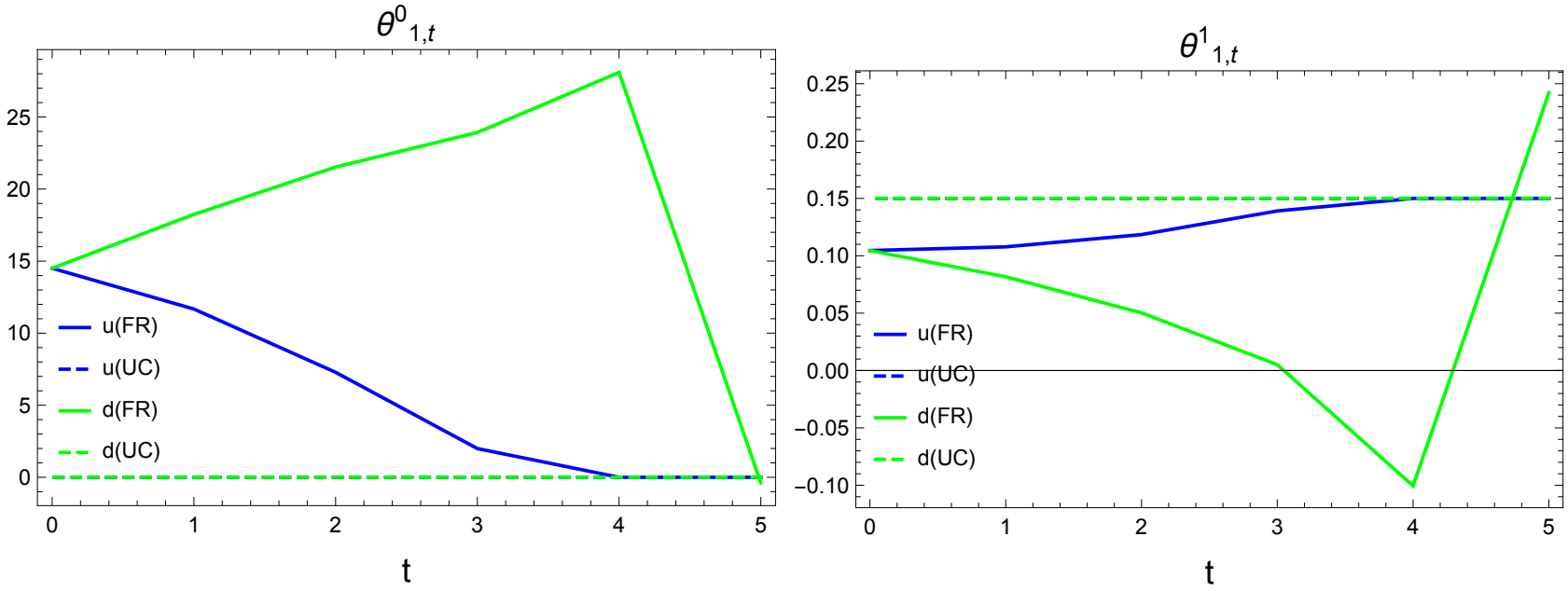
Figure: The left panel shows the slope of implied probability curve with the equity premium. The right panel shows the implied probabilities of put options with the same level of 'moneyness' as a function of maturity for different profiles of minimum-withdrawals. "ST" (short term) curve corresponds to the case when the constrained institution only has non-zero minimum-withdrawals at dates 2 and 3. In the right panel, the strike price for options at all maturities is set equal to the maximum dividend at the corresponding maturity.





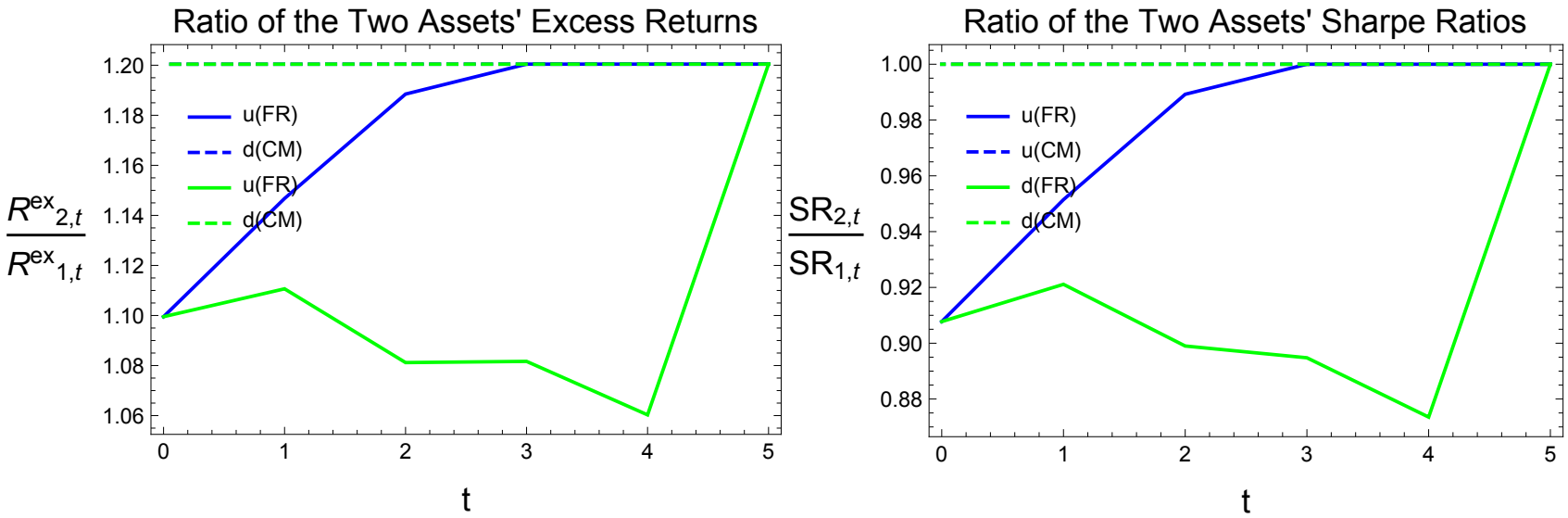
Allocations to Risky Assets

Figure: Time evolution of the constrained institution's allocations to the riskfree (left panel) and the lower-risk risky asset (right panel).



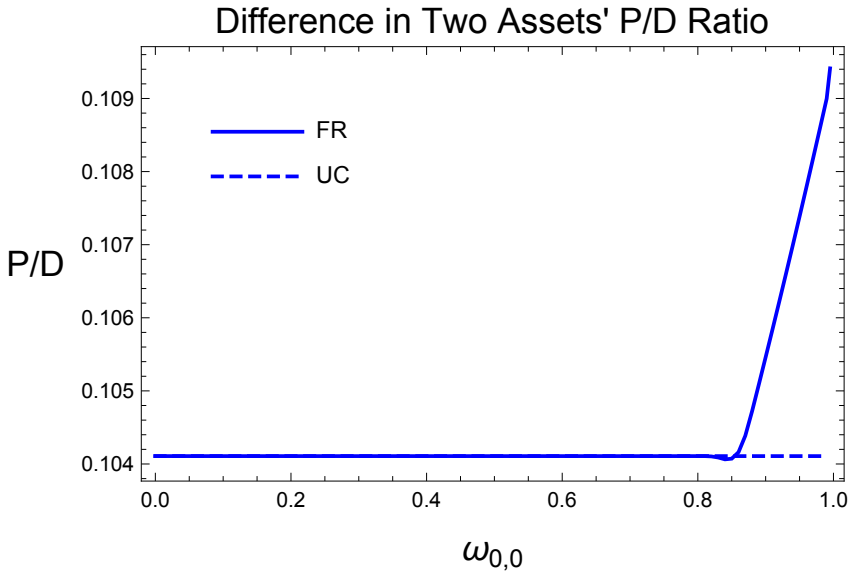
Evolution of Sharpe Ratios of the Two Risky Assets

Figure: Time evolution of the ratio of expected excess returns (left panel) and Sharpe ratios (right panel) of the two risky assets.



Price/Dividend Ratio of the Two Risky Assets

Figure: Difference between the price/dividend ratio of the two risky assets at time 0, for the funding-ratio model.



Solution with Large Number of Investors

In the case of a large number of investors, the solution can be obtained by writing f_{ω_t} as

$$f_{\omega_t} = \min\left(1, \frac{1 - \sum_{i=1}^{N_{c,t}} E_t[\omega_{i,t+1}^{min}]}{1 - \sum_{i=1}^{N_{c,t}} \omega_{i,t}}\right). \tag{26}$$

f_{ω_t} can be interpreted as a measure of the average severity of the constraint in the economy, and asset prices can will then depend on $\prod_{i=t}^{T-1} f_{\omega_i}$. Hence, asset prices at time t depend not on the consumption-share and constraint parameters of a single investor, but on average of these quantities across all constrained investors.

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