Discussion of
Score-Driven Nelson Siegel: Hedging Long-Term Liabilities

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Nelson-Siegel Dynamics

Dynamic model for $p(t, T)$, the zero-coupon bond prices at time $t$ for maturity $T \geq t$:

$$p(t, t + \tau) = \exp(-\tau B_{\lambda_t}(\tau)\tilde{f}_t) = \exp(-b_{\lambda_t}(\tau)\tilde{f}_t)$$

which are driven by a three-dimensional (column) vector process $\tilde{f}_t$. 

The Nelson Siegel functions $b_{\lambda_t}(\tau) = [\tau, 1 - e^{-\lambda\tau}, 1 - e^{-\lambda\tau} - \tau e^{-\lambda\tau}]$. 

The shape parameter $\lambda_t$ is allowed to vary over time.
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The Nelson Siegel functions

$$b_{\lambda}(\tau) = [ \tau, \frac{1}{\lambda}(1 - e^{-\lambda \tau}), \frac{1}{\lambda}(1 - e^{-\lambda \tau}) - \tau e^{-\lambda \tau} ]$$

depend on a shape parameter $\lambda$ which in the model is allowed to vary over time: $\lambda_t$. 
Dynamic Conditional Score Model

Goal is hedging: i.e. reduced sensitivity to interest rate shocks for portfolio which contains bonds and long-term liabilities.

- In usual approach the shape parameter $\lambda$ is taken constant and emphasis is on modeling of time series $\tilde{f}_t$ in

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  \[ p(t, t + \tau) = \exp(-b_{\lambda_t}(\tau)\tilde{f}_t). \]

- In this paper, estimation of $\tilde{f}_t$ is simplified and emphasis is on modeling the shape parameter $\lambda_t$:
  \[ \lambda_t - \phi_0 = \phi_1(\lambda_{t-1} - \phi_0) + u_{t-1}, \]

  with $u_{t-1}$ a deterministic function of $\lambda_{t-1}$ and other observed quantities at time $t - 1$ (scaled score function of observations wrt $\lambda_t$).
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- Idiosyncratic risks modeled by single GARCH process with equal weights for all maturities.
Results

- Cases considered
  - maturities up till 20 years or full set (i.e. also 25, 30, 40 and 50 years maturity)
  - constant or GARCH process for idiosyncratic volatility
  - constant or DCS process for shape parameter
- Key result: hedging 50-year bond using bonds with maturities of 20 or less

Table: Reduction in Variance of Portfolio Returns

<table>
<thead>
<tr>
<th></th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>First factor only</td>
<td>88.2%</td>
</tr>
<tr>
<td>NS factors, shape fixed,</td>
<td>92.6%</td>
</tr>
<tr>
<td>GARCH vol</td>
<td></td>
</tr>
<tr>
<td>NS factors, shape fixed,</td>
<td>92.7%</td>
</tr>
<tr>
<td>constant vol</td>
<td></td>
</tr>
<tr>
<td>NS factors, shape time-varying, constant vol</td>
<td>93.8%</td>
</tr>
<tr>
<td>NS factors, shape time-varying, GARCH vol</td>
<td>93.9%</td>
</tr>
</tbody>
</table>
Arbitrage

- Unadjusted form of Nelson-Siegel model with time-varying parameters, so there may be arbitrage opportunities.

- Not clear how the Christensen et al. (2011) correction term which avoids this would impact the hedging performance, since
  - that term strongly depends on the shape parameter which is no longer taken constant, and
  - examples in that paper suggest that adjustments are highest for largest maturities.
Model choice

- In-sample parameter estimation:
  - GARCH model for idiosyncratic risks gives better improvement in likelihood than changing shape parameter.
  - No information criterion values (AIC/BIC) reported.
  - May be interesting to compare hedging efficiency with other approaches that vary shape parameter:
    - Generalized Autoregressive Score approach of Creal et al. (2008) for factor model,
    - Extended Kalman Filter of Koopman et al. (2010) for piecewise constant/cubic spline model, or
    - Approximate Filter Algorithm of Hevia et al. (2015) for regime switching model.
  - or use multivariate time-varying volatility structures and heavy tails, as in Koopman, Lucas and Zamojski (2016).
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Application: hedging long-term pension liabilities

- Portfolio to hedge 50-year maturity exposures involves very large positions:
  - Median position for 20-year bond is more than 150%, and
  - 95% quantiles of -400% around maturity 7 and +500% at maturity 20.

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