Discussion of Score-Driven Nelson Siegel: Hedging Long-Term Liabilities

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Nelson-Siegel Dynamics

Dynamic model for $p(t, T)$, the zero-coupon bond prices at time t for maturity $T \geq t$:

$$p(t, t+\tau) \;=\; \exp(-\tau B_{\lambda_t}(\tau)\tilde{f}_t) \;=\; \exp(-b_{\lambda_t}(\tau)\tilde{f}_t)$$

which are driven by a three-dimensional (column) vector process $\tilde{f}_t$.

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The Nelson Siegel functions

$$b_\lambda(\tau) \;=\; [\;\tau,\; \tfrac{1}{\lambda}(1-e^{-\lambda\tau}),\; \tfrac{1}{\lambda}(1-e^{-\lambda\tau}) - \tau e^{-\lambda\tau}\;].$$

depend on a shape parameter λ which in the model is allowed to vary over time: λ_t.

Dynamic Conditional Score Model

Goal is hedging: i.e. reduced sensitivity to interest rate shocks for portfolio which contains bonds and long-term liabilities.

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- In this paper, estimation of $\tilde{f}_t$ is simplified and emphasis is on modeling the shape parameter λ_t:
$$\lambda_t - \phi_0 \ = \ \phi_1(\lambda_{t-1} - \phi_0) \ + \ u_{t-1},$$
with u_{t-1} a deterministic function of λ_{t-1} and other observed quantities at time $t-1$ (scaled score function of observations wrt λ_t).

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- Idiosyncratic risks modeled by single GARCH process with equal weights for all maturities.

Results

- Cases considered
  - maturities up till 20 years or full set (i.e. also 25, 30, 40 and 50 years maturity)
  - constant or GARCH process for idiosyncratic volatility
  - constant or DCS process for shape parameter
- Key result: hedging 50-year bond using bonds with maturities of 20 or less

Table: Reduction in Variance of Portfolio Returns

88.2%	First factor only
92.6%	NS factors, shape fixed, GARCH vol
92.7%	NS factors, shape fixed, constant vol
93.8%	NS factors, shape time-varying, constant vol
93.9%	NS factors, shape time-varying, GARCH vol

Arbitrage

- Unadjusted form of Nelson-Siegel model with time-varying parameters, so there may be arbitrage opportunities.
- Not clear how the Christensen et al. (2011) correction term which avoids this would impact the hedging performance, since
  - that term strongly depends on the shape parameter which is no longer taken constant, and
  - examples in that paper suggest that adjustments are highest for largest maturities.

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- May be interesting to compare hedging efficiency with other approaches that vary shape parameter:
  - Generalized Autoregressive Score approach of Creal et al. (2008) for factor model,
  - Extended Kalman Filter of Koopman et al. (2010) for piecewise constant/cubic spline model, or
  - Approximate Filter Algorithm of Hevia et al. (2015) for regime switching model.

or use multivariate time-varying volatility structures and heavy tails, as in Koopman, Lucas and Zamojski (2016).

Application: hedging long-term pension liabilities

- Portfolio to hedge 50-year maturity exposures involves very large positions:
    - Median position for 20-year bond is more than 150%, and
    - 95% quantiles of -400% around maturity 7 and $+500\%$ at maturity 20.

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- Perhaps more relevant to hedge movements in mid-range maturities?

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