Longevity Risk: To Bear or to Insure

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Motivation

**Longevity risk** endangers the financial security of retirees.

**Defining Characteristics**

- In contrast to mortality risk, i.e., risk of the uncertain time of death given known survival probabilities.
- **Misestimation** of future survival probabilities

**Threat to Retirement Planning**

- The entity that conventionally bears the risk (i.e., the plan sponsor) no longer does.
- **Phasing-out** of DB schemes

- Investors who accept to bear this risk **command a risk premium** (Bayraktar et al., 2009).
- **Systematic risk**

- **Scarcity of longevity-linked assets**
  - Maturation of the marketplace for longevity-linked assets is beset by challenges (Tan et al., 2015).
Longevity Risk Management

Benefits are **adjusted** according to longevity evolution.

- A mean of dealing with longevity risk **without involving investors**.
- Self-sustaining: Solvency is always **maintained**.
- **Volatility** of benefits: Subject to longevity shocks.

Benefits are **invariant to longevity evolution**, conditional on provider’s solvency.

- Longevity risk is **transferred** to contract provider.
- **Default** risk: Contract provider has **limited liability**.
- **Costlier**: Investors only accept to bear the risk, in return for some **financial reward**.
The need to generate a longevity risk premium to compensate equity holders.

Composition of Capital Buffer Entails

The need to preserve policy holders who may prefer the collective scheme if the loading is high.

Convention: Assume that the capital buffer is entirely composed of loading charged to individuals.

   e.g., Friedberg and Webb, 2007; Richter and Weber, 2011; Maurer et al., 2013; Boyle et al., 2015.
Objective: Enhance the Modeling of the Market Solution

Individuals’ Willingness to Pay to Insure

Individuals are willing to pay little to insure against longevity risk.

e.g., 0.75%\(^1\) (Weale and van de Ven, 2016) to 1%\(^2\) (Maurer et al., 2013) in contract loading.

For an annuity contract with no default risk.

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Provider’s Required Level of Buffer Capital

Substantial buffer capital may be necessary to limit default risk.

e.g., \(\approx 18\%\) of the contract’s best estimate value to limit the default rate to 1%\(^2\) (Maurer et al., 2013).

We attempt to reconcile this gap by introducing the equity holders.

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\(^1\) Immediate nominal annuity contract for a 65 year old male with \(\gamma = 2\).

\(^2\) Deferred variable annuity contract, with benefit payments that begin at age 67 to 120, for a 40 year old female with \(\gamma = 5\).
Summary of Findings (1/2)
If the annuity provider sells zero-loading contracts:

**Individuals**
- **Marginal** preference for the collective scheme.
- Certain equivalent loading of -0.2%

**Equity Holders**
- Longevity risk exposure is an inferior investment opportunity.
- No longevity risk exposure
- Marginally higher excess return
- 50% higher Sharpe ratio
- Higher standard deviation

The figures correspond to contracts for individuals with $\gamma = 5$ and the underlying portfolio is 20% invested in the risky stock index, 80% invested in the money market account.
Summary of Findings (2/2)

Consequence:
- The market-provided annuity contract would not co-exist with the collective scheme.

Outcome is robust to:
- Individuals’ risk aversion levels (e.g., $\gamma = 2, 5, \text{ and } 8$);
- Deferral period (e.g., 40 years, 20 years, and immediate);
- Stock exposure (e.g., 0%, 20%, 40%, 60%, glide path); and
- Parameter uncertainty of the longevity model time trend’s drift term.

Individuals exhibit preference for the annuity contract if:
- They are highly risk-averse (e.g., $\gamma = 10, 15, \text{ and } 20$):
  - Certainty Equivalent Loading (CEL): 0.003, 0.34 and 0.62%.
- The uncertainty surrounding life expectancies is heightened but default risk is curtailed:
  I. Doubled variance to the errors of the longevity time trend: CEL = 3.2%, zero-default-risk.
  II. Higher uncertainty of survival probability at older ages: CEL = 0.5%, zero-default-risk.

Unless otherwise stated, the figures correspond to contracts for individuals with $\gamma = 5$ and the underlying portfolio is 20% invested in the risky stock index, 80% invested in the money market account.
Related Literature

1. Group-Self-Annuitization (Pigott et al., 2005) vs. other schemes (e.g., conventional annuities):
   - **Preclude** longevity risk
     - Stamos, 2008; Donnelly et al., 2013; Milevsky and Salisbury, 2015.
   - **Disregard** the annuity provider’s **business model**
     - Stamos, 2008; Denuit et al., 2011; Qiao and Sherris, 2013; Donnelly et al., 2013; Milevsky and Salisbury, 2015; Hanewald et al., 2013; Boyle et al., 2015.
   - **Impose** the insurer’s **default risk**
     - Hanewald et al., 2013.
   - Overarching conclusion: **Preference for the collective scheme is increasing in the loading**
     - Hanewald et al., 2013; Boyle et al., 2015.

2. Longevity-indexed vs. non-indexed contracts:
   - **Omission** of insurer’s **equity holders**
     - Assume that the buffer capital is entirely composed of loading charged to individuals.
     - Richter and Weber, 2011; Maurer et al., 2013.
## Model Description

### Financial Market

- Constant interest rate, $r$
- Stochastic stock market index:
  \[ dS_t = S_t (r + \lambda_S \sigma_S) dt + S_t \sigma_S dZ_{S,t} \]

### Life Expectancy

- Lee and Carter (1992):
  \[ \ln(m_{x,t}) = a_x + b_x k_t + \varepsilon_{x,t} \]
- Time trend, $\{k_t\}_{t=t_0}^T$ follows an ARIMA(0,1,0) process.
- Omission of mortality (i.e., micro-longevity) risk.

### Individual Preference

- Choose a contract at age 25 in year $t_0$.
- Receive retirement benefits, $\Xi_t$, between ages 66 to 95, conditional on survival.
- CRRA Utility:
  \[ \int_{t_R}^T e^{-\beta t} \frac{\Xi_t^{1-\gamma}}{1-\gamma} \times_{t-t_0} p_{25} dt \]

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$t_0, t_R, T$ are the years when the individual is aged 25, 66 and 95 respectively.
$t_{t_0}p_{25}$ is the probability of someone aged 25 to be alive in $t - t_0$ year(s).
Financial Contracts for Retirement (1/2): DVA

The DVA and the GSA treat financial market risk identically (i.e., fully borne by the individuals), but differ on the longevity risk distribution.

Deferred Variable Annuity (DVA)

- Entitlements are determined using longevity forecasts on the date of contract sale, benefits received are equivalent to entitlements while provider is solvent.

- Default occurs if the DVA provider’s Value of assets < Value of liabilities
  - In default, individuals recover the residual wealth of the provider, which they use to buy a portfolio of equally-weighted bonds of maturities starting from the retirement year (or present year if retirement has begun) to the year of maximum age.

- The annuity provider can fully hedge financial market risk by adopting the reference portfolio’s investment policy.

Mathematical formalization is available in the Appendices.
Financial Contracts for Retirement (2/2): GSA

The DVA and the GSA treat **financial market risk identically** (i.e., fully borne by the individuals), but **differ** on the **longevity risk distribution**.

### Group Self-Annuitization (GSA)

- **Entitlement** calculation is **identical** to that of a DVA with zero loading.
  - Parametrized by the Assumed Interest Rate (AIR).
  - Indexed to a Reference Portfolio.

- **Entitlements are adjusted** each year by this ratio to determine the benefits paid-out.

\[
\frac{FR_t}{MFR} \begin{cases} 
\text{Benefits < Entitlement} & \text{if } FR_t < MFR \\
\text{Benefits = Entitlement} & \text{if } FR_t = MFR \\
\text{Benefits > Entitlement} & \text{if } FR_t > MFR
\end{cases}
\]

Mathematical formalization is available in the [Appendices](#).
Longevity Risk Visualized


The fan plot is based on 10,000 replications.
Boxplot of Benefits

- DVA provider’s **equity capital is 10%** of the contracts’ best estimated value (i.e., to coincide with the 90% average leverage ratio of life insurers in the U.S. between 1998 and 2011\(^1\))
- The ensuing **cumulative default rates are low**: < 0.0084%.

\[\text{Note: Annuitization capital at age 25 is normalized to 1. Financial market return is constant at 3.62%}.\]

\(^1\)A.M. Best data from Koijen and Yogo (2015)

Figures correspond to contracts for individuals with \(\gamma = 5\) and the underlying portfolio is 100% invested in the money market account.
The figure corresponds to contracts for individuals with $\gamma = 5$ and the underlying portfolio is 20% invested in the risky stock index, 80% invested in the money market account.
Key Statistics

### Individuals

**Certainty Equivalent Loading (CEL)**
- The proportional loading on the DVA contract for which the individual derives the same expected utility under the DVA and under the GSA.

<table>
<thead>
<tr>
<th>θ (%)</th>
<th>γ</th>
<th>2</th>
<th>5</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
<td>-0.350</td>
<td>-0.200</td>
<td>-0.055</td>
</tr>
<tr>
<td></td>
<td></td>
<td>[-0.362, -0.339]</td>
<td>[-0.211, -0.188]</td>
<td>[-0.067, -0.044]</td>
</tr>
<tr>
<td>20</td>
<td></td>
<td>-0.349</td>
<td>-0.200</td>
<td>-0.052</td>
</tr>
<tr>
<td></td>
<td></td>
<td>[-0.361, -0.338]</td>
<td>[-0.216, -0.184]</td>
<td>[-0.088, -0.016]</td>
</tr>
</tbody>
</table>

Values are in %.
- θ is the % invested in stocks.
- γ is the risk aversion parameter.

### Equity Holders

**Sharpe Ratio (SR)**
- The ratio of the annualized investment return in excess of the annualized return on the money market account, over its annualized standard deviation.

<table>
<thead>
<tr>
<th>Statistic</th>
<th>γ</th>
<th>2</th>
<th>5</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>R^{A_{exs}} (%)</td>
<td></td>
<td>1.44</td>
<td>1.44</td>
<td>1.44</td>
</tr>
<tr>
<td>σ^{A_{exs}} (%)</td>
<td></td>
<td>5.04</td>
<td>4.95</td>
<td>4.95</td>
</tr>
<tr>
<td>SR</td>
<td></td>
<td>0.29</td>
<td>0.29</td>
<td>0.29</td>
</tr>
</tbody>
</table>

Reference portfolio: 20% in the stock index.

\[
R^{exs} = 1.43\%  
\sigma^{exs} = 3.17\%  
SR = 0.45
\]

The values in parentheses are the 99% confidence intervals.
Sensitivity Analysis: General

Baseline Case with $\gamma = 5$: Cumulative default rate = 0.0038%; CEL = -0.2%.

Risk Aversion Level
- Individuals who are highly risk-averse prefer the DVA.
  - e.g., $\gamma = 20$; CEL = 0.62%.

Leverage Ratio
- If the DVA provider has a higher leverage ratio, then individuals prefer the GSA more.
  - e.g., Initial capital is halved to 5%.
    $\gamma = 5$: Cumulative default rate rises to 5%; CEL decreases to -12.9%.

No Material Effect

Deferral Period
- 40 years, 20 years, or an immediate annuity
  - Shorter deferral period allows for more accurate survival probabilities forecast but more imminent longevity shocks to utility.

Stock Exposure
- 0, 20, 40, 60, the optimum ($\frac{\lambda_s}{\gamma\sigma_s}$), and a glide path (90% at age 25, diminishing to 30% by age 66).
Sensitivity Analysis: Longevity Model (1/3)

Sensitivity surrounding the longevity model:

**Doubled Time Trend Errors’ Variance**
- Time trend process:  
  \[ k_t = c + k_{t-1} + \delta_t \]
- \( \delta \sim N(0, 2\sigma_\delta^2) \)

**Drift Parameter Uncertainty**
- \( k_t = c + k_{t-1} + \delta_t \)
- \( \hat{c} \) is estimated by maximum likelihood, and is distributed as \( \hat{c} \sim N(c, \sigma_c^2) \)
- For the \( l^{th} \) replication, draw a \( c_l \) from the distribution \( N(\hat{c}, \sigma_c^2) \)

**Alternate Longevity Model**
- Cairns, Blake and Dowd (2006)
- \( \text{logit}(q_{t,x}) = \kappa_t^{(1)} + \kappa_t^{(2)}(x - \bar{x}) \)
Sensitivity Analysis: Longevity Model (2/3)

Implication on individual preference and equity holders’ profitability.

**Doubled Time Trend Errors’ Variance**

- **Default rates increase** from 0.0038% to 3.41%:
  CEL = -7.7%;

- If equity capital is raised sufficiently to **eliminate default risk**:
  CEL = 3.2%;

- **Lower Sharpe ratio** with longevity risk exposure when loading is 3.2% and equity capital is raised sufficiently.

**Drift Parameter Uncertainty**

- **No material change** to the default rates, CEL, and equity holders’ investment performance.

Figures correspond to contracts for individuals with $\gamma = 5$ and the underlying portfolio is 20% invested in the risky stock index, 80% invested in the money market account.
Sensitivity Analysis: Longevity Model (3/3)

Both models are calibrated on U.S. female mortality data from 1980 to 2013, from the Human Mortality Database. The fan plot is based on 10,000 replications.

- **Higher uncertainty** on the likelihood of survival at older ages;
- Default rises to 0.48%: CEL = -0.5%;
- Absent default: CEL = 0.46%;
- **Lower Sharpe ratio** with longevity risk exposure.

Alternate Longevity Model

Cairns-Blake-Dowd

Lee-Carter
Conclusion (1/2)

- We investigate **longevity risk management** in retirement planning under two arrangements:
  - **Bearing** the risk as a collective (GSA), or;
  - **Insuring** the risk with a market-provided annuity contract (DVA).

- We model not only **individual preference** but also the **annuity provider’s business** to underscore the involvement of equity holders in enabling the market solution.

- **Individuals prefer** the arrangement that yields a **higher expected utility**.

- Equity holders’ **willingness to provide capital** depends on the **Sharpe ratio** of the investment opportunities that bear the **same financial market risks**, but are either exposed to, or not exposed to longevity risk.
Conclusion (2/2)

- We find that when the DVA is priced at its best estimate:
  - Individuals have a slight preference for the GSA;
  - Equity holders attain a lower Sharpe ratio when exposed to longevity risk.

- Market-provided annuity contracts would not co-exist with collective schemes.

- Preference for the GSA is insensitive to:
  - Risk aversion levels;
  - Contract deferral period;
  - Exposure to stock market risk;
  - Longevity time trend’s drift parameter uncertainty.

- Heightened longevity risk only enhances the appeal of a DVA to the individual if the provider restrains default risk.
  - Sharpe ratio of the equity holders remains inferior to the Sharpe ratio of the investment in the financial market only;
  - Aggravated longevity risk leads to higher variability of the equity holder payoff as well.
Appendices
Retirement Contract: Per Unit Cost

- The per unit contract cost (i.e., annuity factor) is

\[ A(h, F, t_0, x) := (1 + F) \int_{t=t_R}^{T} t - t_0 p_x^{(t_0)} \exp(-h_t \times (t - t_R)) \, dt \]

- \( t - t_0 p_x^{(t_0)} \) = conditional probability in year \( t_0 \)
  that a living individual of age \( x \)
  lives for at least \( t - t_0 \) more years

- \( h = AIR \)
- \( F = \) loading factor
- \( t_R = \) retirement year, i.e., \( t_R = t_0 + 66 - 25 \)
Retirement Contract: Entitlement

- The entitlement in period \( t, t_R \leq t \leq T \), conditional on the individual’s survival, is

\[
\Xi(h, F, t, x) := \exp\left( -h_t \times (t - t_R) \right) \times \frac{W_t^{Ref}(\theta)}{W_{t_0}^{Ref}(\theta)}
\]

\( W_t^{Ref}(\theta) = \) value of the reference portfolio at time \( t \)

\( h = AIR \)
The optimal Assumed Interest Rate (AIR) maximizes the individual’s expected utility in retirement under the DVA contract, when the reference portfolio’s investment policy is \( \theta = \{\theta_t\}_{t=t_0}^T \) invested in the stock index, and \( 1 - \theta \) invested in the money market account.

\[
h^*(t, \theta_t) := r + \frac{\beta - r}{\gamma} - \frac{1 - \gamma}{\gamma} \theta_t \sigma_S \left( \lambda_S - \frac{\gamma \theta_t \sigma_S}{2} \right)
\]

- \( h^*(t, \theta_t) \) := function definition
- \( t \) = time index, \( t, t_R \leq t \leq T \)
- \( r \) = constant short rate
- \( \beta \) = subjective discount factor
- \( \gamma \) = risk aversion parameter
- \( \theta_t \) = fraction of wealth allocated to the stock index
- \( \sigma_S \) = diffusion term of the stock index
- \( \lambda_S \) = Sharpe ratio of the stock index
Retirement Contract: DVA and GSA Entitlement

- The entitlement in period \( t, t_R \leq t \leq T \), conditional on the individual’s survival, is

**DVA**

\[
\mathbb{E}^{DVA}(h^*, F, t, x) = \mathbb{E}(h^*, F, t, x)
\]

**GSA**

\[
\mathbb{E}^{GSA}(h^*, 0, t, x) = \mathbb{E}(h^*, 0, t, x) \times \frac{FR_t}{1} \times \frac{\exp(-h^*(t, \theta_t) \times (t - t_R))}{A(h^*, 0, t_0, x)} \times \frac{W_t^{Ref}(\theta)}{W_{t_0}^{Ref}(\theta)} \times FR_t
\]

\( FR_t \) = Funding Ratio in year \( t \)
Certainty Equivalent Loading

- The Certainty Equivalent Loading (CEL) is the value such that the following holds.

\[
\mathbb{E} \left[ U \left( \frac{1}{1 + CEL} \times \mathbb{E}^{DVA}_{F=0} \right) \right] = \mathbb{E} \left[ U \left( \mathbb{E}^{GSA} \right) \right]
\]
Sensitivity Analysis: Longevity Model (1/2)

Doubled Time Trend Errors’ Variance

Mean, 5- and 95-percentile.
Sensitivity Analysis: Longevity Model (2/2)

Drift Parameter Uncertainty
Mean, 5- and 95-percentile.