TILBURG UNIVERSITY

# Longevity Risk: To Bear or to Insure

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# Motivation

**Longevity risk** endangers the financial security of retirees.

## Defining Characteristics

In contrast to mortality risk, i.e., risk of the uncertain time of death given known survival probabilities.

**Mis-estimation** of future survival probabilities

Investors who accept to bear this risk **command a risk premium** (Bayraktar et al., 2009).

**Systematic** risk

Longevity Risk

## Threat to Retirement Planning

**Phasing-out** of **DB** schemes

The entity that conventionally bears the risk (i.e., the plan sponsor) no longer does.

**Scarcity** of longevity-linked **assets**

Maturation of the marketplace for longevity-linked assets is beset by challenges (Tan et al., 2015).

# Longevity Risk Management

**Bearing** via a Collective Scheme ← **Managing Longevity Risk** → **Insuring** with an Annuity Contract

| Benefits are adjusted according to longevity evolution. | Benefits are invariant to longevity evolution, conditional on provider's solvency. |
|---|---|

Benefits are **adjusted** according to **longevity evolution**.

+ A mean of dealing with longevity risk **without involving investors**.

+ Self-sustaining: **Solvency** is always **maintained**.

− **Volatility** of benefits: Subject to **longevity shocks**.

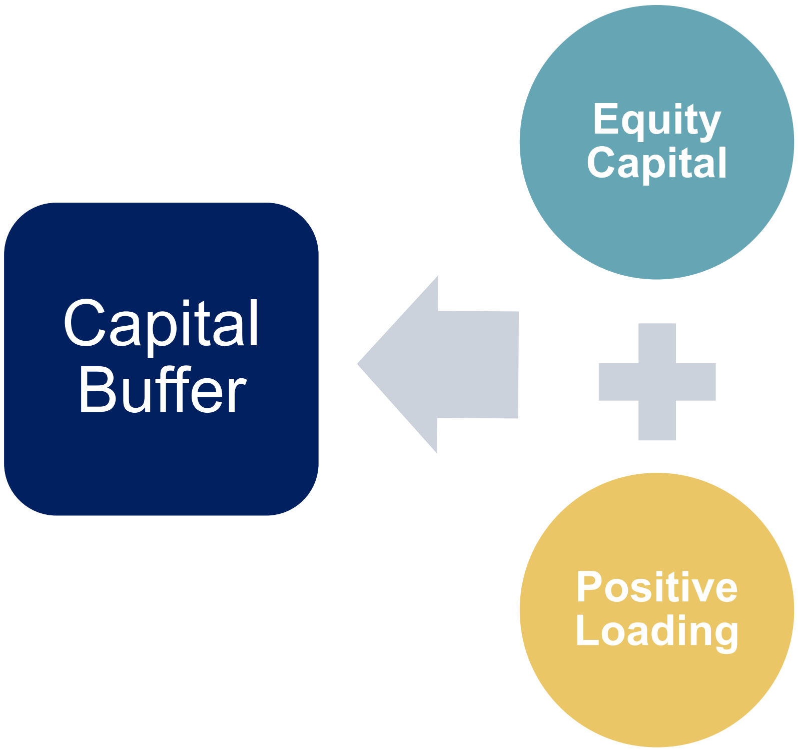
Benefits are **invariant to longevity evolution**, conditional on **provider's solvency**.

+ Longevity risk is **transferred** to contract provider.

− **Default** risk: Contract provider has **limited liability**.

− **Cost**lier: Investors only accept to bear the risk, in return for some **financial reward**.

# Contract Provider: Capital to Enable Annuity Provision




The need to generate a **longevity risk premium** to compensate equity holders.

**Composition of Capital Buffer Entails**

The need to preserve policy holders who may **prefer the collective scheme** if the loading is high.

Convention: Assume that the capital buffer is **entirely composed** of **loading charged to individuals**.

e.g., Friedberg and Webb, 2007; Richter and Weber, 2011; Maurer et al., 2013; Boyle et al., 2015.

# Objective: Enhance the Modeling of the Market Solution

**Individuals' Willingness to Pay to Insure** < **Provider's Required Level of Buffer Capital**

Individuals are willing to pay little to insure against longevity risk.

e.g., **0.75%**[1] (Weale and van de Ven, 2016) to **1%**[2] (Maurer et al., 2013) in contract loading.

For an annuity contract with **no default risk**.

**Substantial** buffer capital may be necessary to **limit default risk**.

e.g., ≈**18%** of the contract's best estimate value to **limit the default rate to 1%**[2] (Maurer et al., 2013).

We attempt to reconcile this gap by introducing the **equity holders**.

[1] Immediate nominal annuity contract for a 65 year old male with $\gamma = 2$.

[2] Deferred variable annuity contract, with benefit payments that begin at age 67 to 120, for a 40 year old female with $\gamma = 5$.

# Summary of Findings (1/2)

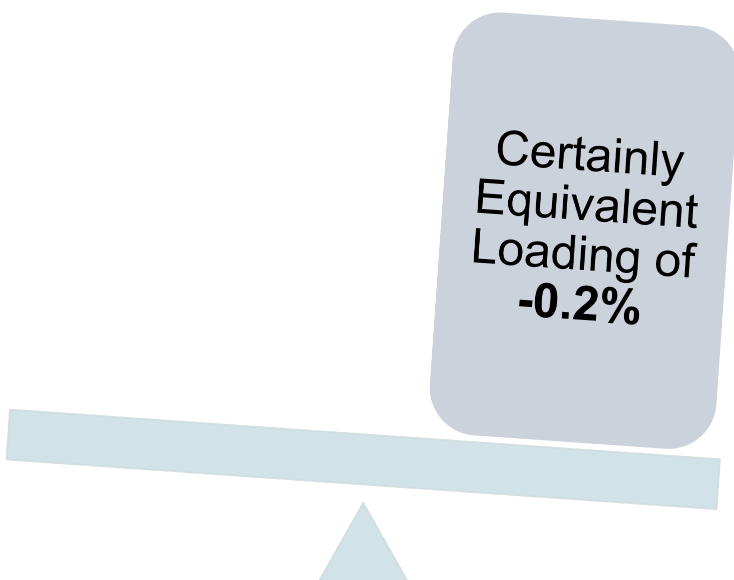
If the annuity provider sells zero-loading contracts:

## Individuals

**Marginal** preference for the **collective scheme.**

Collective Scheme

Annuity Contract

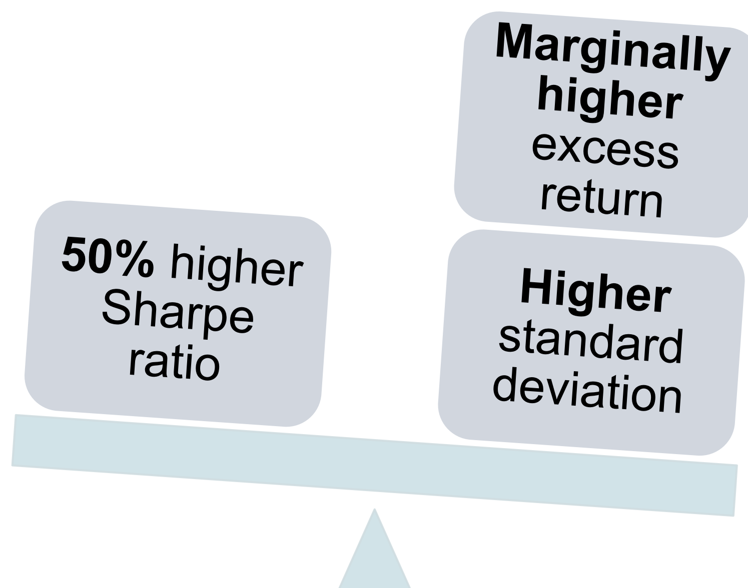



## Equity Holders

**Longevity risk exposure** is an **inferior** investment opportunity.

No Longevity Risk Exposure

Longevity Risk Exposure




The figures correspond to contracts for individuals with $\gamma = 5$ and the underlying portfolio is 20% invested in the risky stock index, 80% invested in the money market account.

# Summary of Findings (2/2)

Consequence:

- The market-provided annuity contract **would not co-exist** with the collective scheme.

Outcome is **robust** to:

- Individuals' risk aversion levels (e.g., $\gamma$ = 2, 5, and 8);
- Deferral period (e.g., 40 years, 20 years, and immediate);
- Stock exposure (e.g., 0%, 20%, 40%, 60%, glide path); and
- Parameter uncertainty of the longevity model time trend's drift term.

Individuals exhibit **preference for the annuity contract** if:

- They are **highly risk-averse** (e.g., $\gamma$ = 10, 15, and 20):
  - Certainty Equivalent Loading (CEL): 0.003, 0.34 and 0.62%.
- The **uncertainty** surrounding life expectancies is **heightened** but **default risk** is **curtailed**:
  I. Doubled variance to the errors of the longevity time trend: CEL = 3.2%, zero-default-risk.
  II. Higher uncertainty of survival probability at older ages: CEL = 0.5%, zero-default-risk.

Unless otherwise stated, the figures correspond to contracts for individuals with $\gamma = 5$ and the underlying portfolio is 20% invested in the risky stock index, 80% invested in the money market account.

# Related Literature

1. Group-Self-Annuitization (Pigott et al., 2005) vs. other schemes (e.g., conventional annuities):
   - **Preclude** longevity risk
     - Stamos, 2008; Donnelly et al., 2013; Milevsky and Salisbury, 2015.
   - **Disregard** the annuity provider's **business model**
     - Stamos, 2008; Denuit et al., 2011; Qiao and Sherris, 2013; Donnelly et al., 2013; Milevsky and Salisbury, 2015; Hanewald et al., 2013; Boyle et al., 2015.
   - **Impose** the insurer's **default risk**
     - Hanewald et al., 2013.
   - Overarching conclusion: **Preference** for the **collective scheme** is **increasing** in the **loading**
     - Hanewald et al., 2013; Boyle et al., 2015.

2. Longevity-indexed vs. non-indexed contracts:
   - **Omission** of insurer's **equity holders**
     Assume that the buffer capital is entirely composed of loading charged to individuals.
     - Richter and Weber, 2011; Maurer et al., 2013.

# Model Description

### Financial Market

- Constant interest rate, $r$
- Stochastic stock market index: $dS_t = S_t(r + \lambda_S \sigma_S)dt + S_t \sigma_S dZ_{S,t}$

### Life Expectancy

- Lee and Carter (1992): $ln\left(m_{x,t}\right) = a_x + b_x k_t + \varepsilon_{x,t}$
- Time trend, $\{k_t\}_{t=t_0}^{T}$ follows an ARIMA(0,1,0) process.
- Omission of mortality (i.e., micro-longevity) risk.

### Individual Preference

- Choose a contract at age 25 in year $t_0$.
- Receive retirement benefits, $\Xi_t$, between ages 66 to 95, conditional on survival.
- CRRA Utility: $\int_{t_R}^{T} e^{-\beta t} \frac{\Xi_t^{1-\gamma}}{1-\gamma} \times {}_{t-t_0}p_{25} dt$

$t_0, t_R, T$ are the years when the individual is aged 25, 66 and 95 respectively..

${}_{t-t_0}p_{25}$ is the probability of someone aged 25 to be alive in $t - t_0$ year(s).

# Financial Contracts for Retirement (1/2): DVA

The DVA and the GSA treat **financial market risk identically** (i.e., fully borne by the individuals), but **differ** on the **longevity risk distribution.**

**Deferred Variable Annuity (DVA)**

- Parametrized by the **Assumed Interest Rate** (AIR).
- **Indexed** to a reference portfolio.
- Similar to the variable annuity studied in the literature, e.g., Koijen et al., 2009; Maurer et al., 2013.

- **Entitlements** are determined using **longevity forecasts** on the date of contract sale, **benefits received are equivalent to entitlements** while provider is **solvent**.
- **Default** occurs if the DVA provider's $Value\ of\ assets < Value\ of\ liabilities$
  - In default, individuals **recover the residual wealth** of the provider, which they use to buy a portfolio of **equally-weighted bonds** of maturities starting from the retirement year (or present year if retirement has begun) to the year of maximum age.
- The annuity provider can **fully hedge financial market risk** by adopting the reference portfolio's investment policy.

Mathematical formalization is available in the Appendices,

# Financial Contracts for Retirement (2/2): GSA

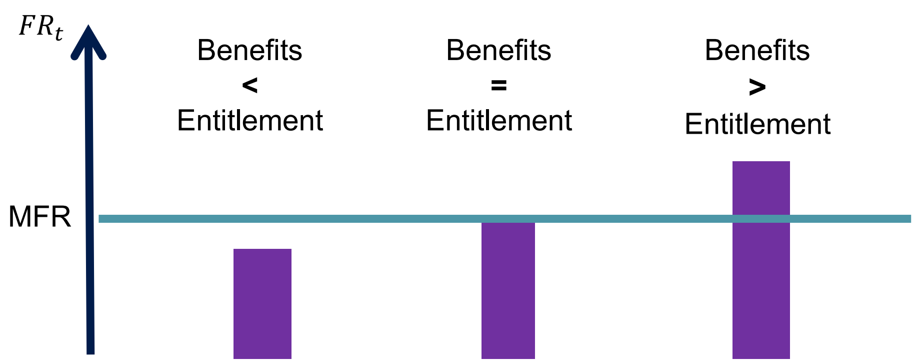
The DVA and the GSA treat **financial market risk identically** (i.e., fully borne by the individuals), but **differ** on the **longevity risk distribution.**

**Group Self-Annuitization (GSA)**

- **Entitlement** calculation is **identical** to that of a DVA with zero loading.
  - Parametrized by the Assumed Interest Rate (AIR).
  - Indexed to a Reference Portfolio.

- **Entitlements are adjusted** each year by this ratio to determine the benefits paid-out.

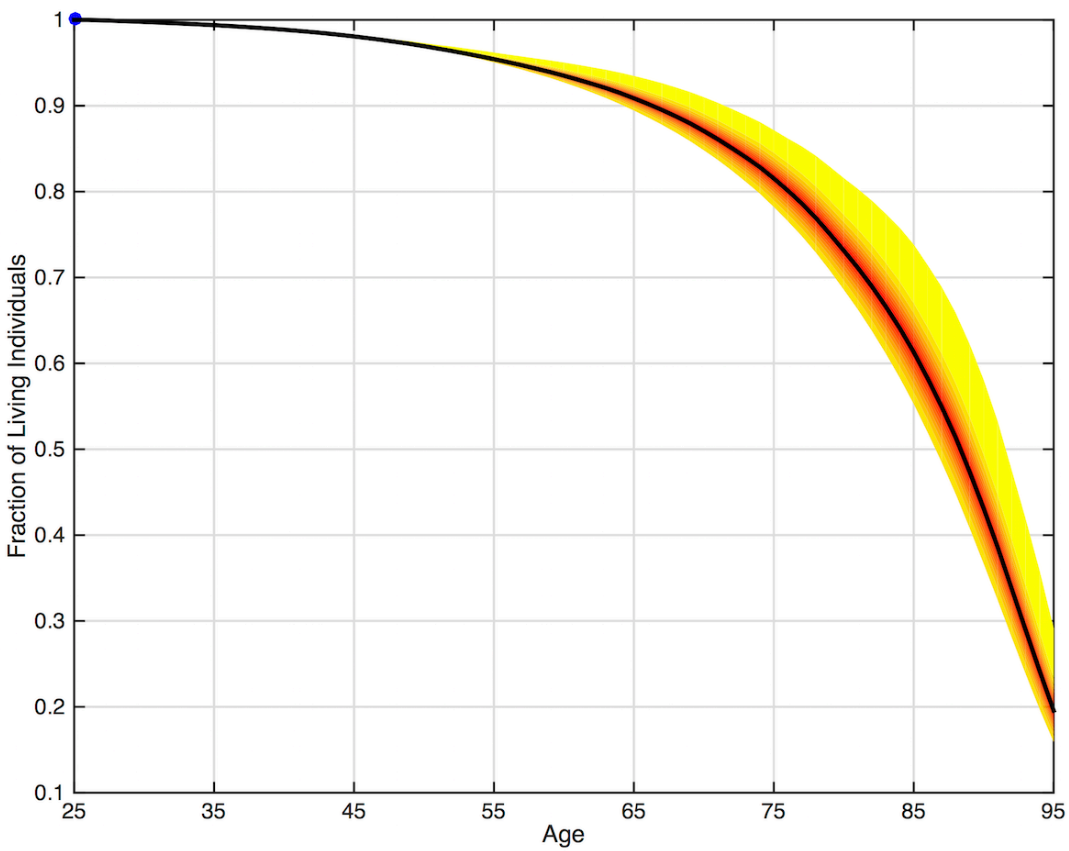
$$\frac{Funding\ Ratio\ in\ year\ t\ (FR_t)}{Minimum\ Funding\ Requirement\ (MFR)}$$




Mathematical formalization is available in the Appendices,

# Longevity Risk Visualized

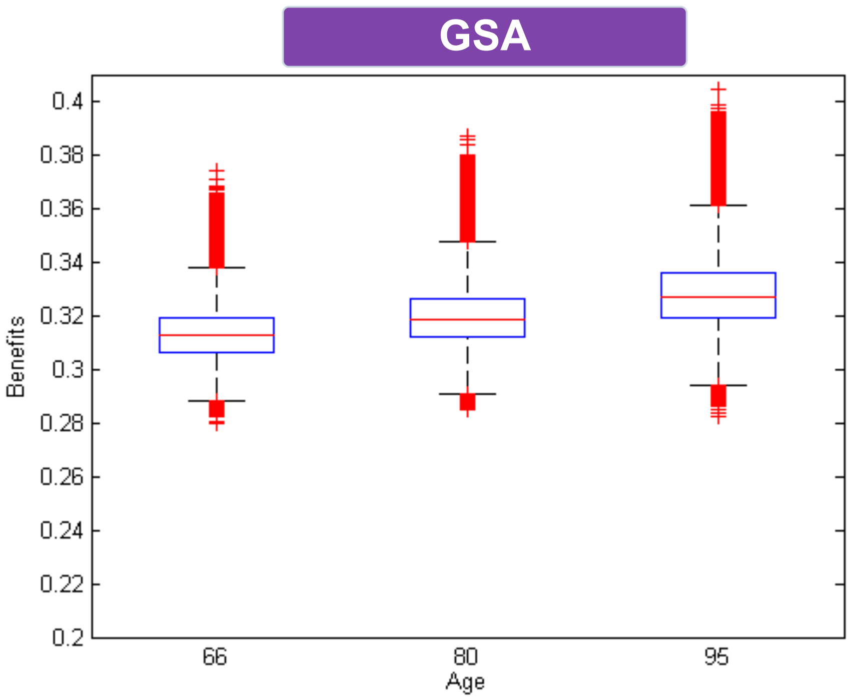
Lee and Carter (1992) model calibrated on U.S. female mortality data from 1980 to 2013, from the Human Mortality Database.

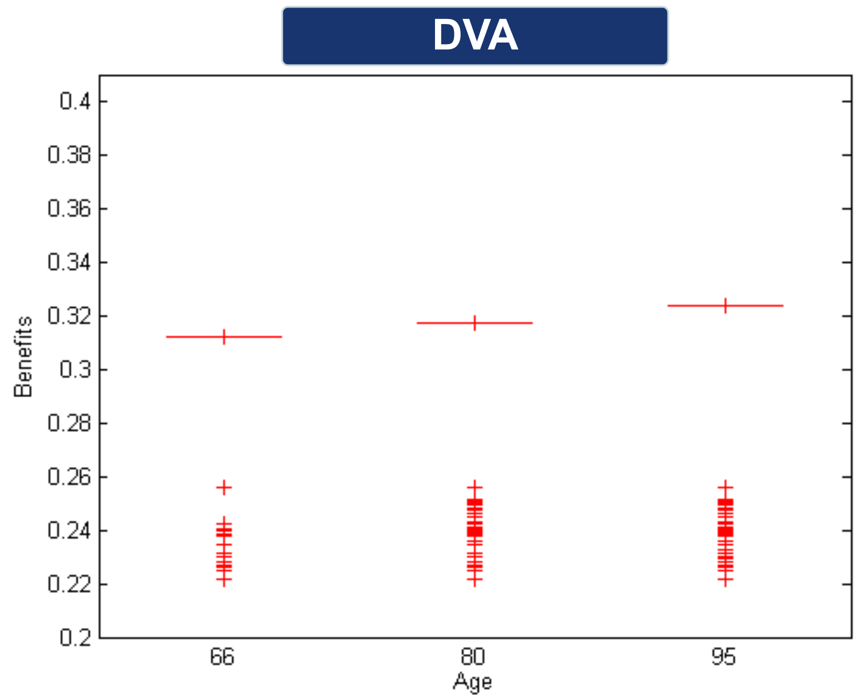



The fan plot is based on 10,000 replications.

# Boxplot of Benefits

- DVA provider's **equity capital is 10%** of the contracts' best estimated value (i.e., to coincide with the 90% average leverage ratio of life insurers in the U.S. between 1998 and 2011[1])
- The ensuing **cumulative default rates are low**: < 0.0084%.



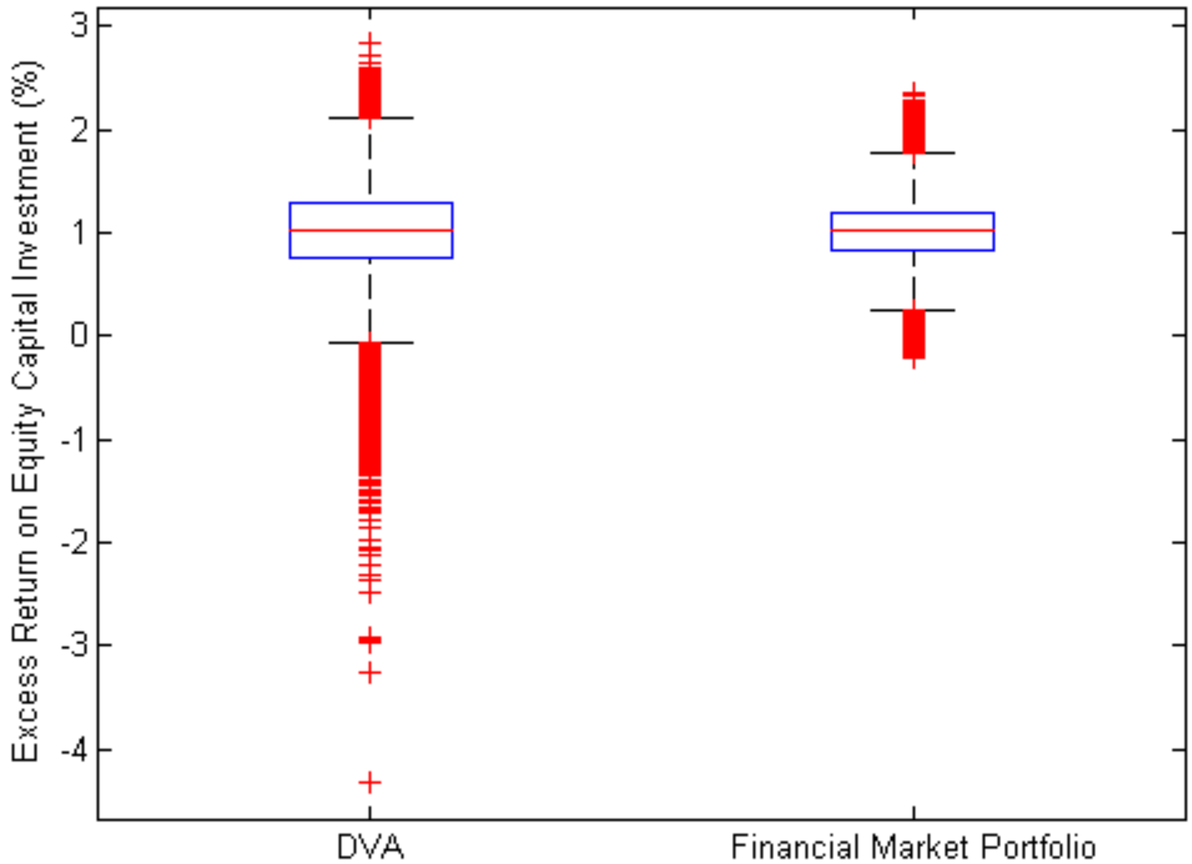




Note: Annuitization capital at age 25 is normalized to 1. Financial market return is constant at 3.62%.

[1]A.M. Best data from Koijen and Yogo (2015)

Figures correspond to contracts for individuals with $\gamma = 5$ and the underlying portfolio is 100% invested in the money market account.

# Boxplot of Equity Holders' Excess Return




Note: Annualized values.

The figure corresponds to contracts for individuals with $\gamma = 5$ and the underlying portfolio is 20% invested in the risky stock index, 80% invested in the money market account.

# Key Statistics

| Individuals | Equity Holders |
|---|---|

### Certainty Equivalent Loading (CEL)

- The **proportional loading** on the DVA contract for which the individual derives the **same expected utility** under the DVA and under the GSA.

| $\theta$ (%) | $\gamma$ 2 | 5 | 8 |
|---|---|---|---|
| 0 | -0.350 [-0.362, -0.339] | -0.200 [-0.211, -0.188] | -0.055 [-0.067, -0.044] |
| 20 | -0.349 [-0.361, -0.338] | -0.200 [-0.216, -0.184] | -0.052 [-0.088, -0.016] |

Values are in %.
$\theta$ is the % invested in stocks.
$\gamma$ is the risk aversion parameter.

### Sharpe Ratio (SR)

- The **ratio** of the **annualized** investment return in **excess** of the annualized return on the money market account, over its annualized **standard deviation**.

| Statistic | $\gamma$ 2 | 5 | 8 |
|---|---|---|---|
| $\mathbb{E}\left[R^{(A_{exs})}\right]$ (%) | 1.44 [1.44, 1.44] | 1.44 [1.44, 1.45] | 1.44 [1.44, 1.45] |
| $\sigma^{(A_{exs})}$ (%) | 5.04 [5.03, 5.06] | 4.95 [4.94, 4.96] | 4.95 [4.94, 4.96] |
| $SR$ | 0.29 [0.29, 0.29] | 0.29 [0.29, 0.29] | 0.29 [0.29, 0.29] |

Reference portfolio: 20% in the stock index.

$$R^{exs} = 1.43\%$$
$$\sigma^{exs} = 3.17\%$$
$$SR = 0.45$$

The values in parentheses are the 99% confidence intervals.

## Sensitivity Analysis: General

**Baseline Case** with $\gamma$ = 5: Cumulative default rate = 0.0038%; CEL = -0.2%.

### Risk Aversion Level

- Individuals who are **highly risk-averse** prefer the **DVA**.
  - e.g., $\gamma$ = 20; CEL = 0.62%.

### Leverage Ratio

- If the DVA provider has a **higher leverage ratio**, then individuals prefer the **GSA** more.
  - e.g., Initial capital is halved to 5%.
    $\gamma$ = 5: Cumulative default rate rises to 5%; CEL decreases to -12.9%.

<u>No Material Effect</u>

### Deferral Period

- 40 years, 20 years, or an immediate annuity
  - Shorter deferral period allows for **more accurate survival probabilities forecast** but **more imminent longevity shocks** to utility.

### Stock Exposure

- 0, 20, 40, 60, the optimum ($\frac{\lambda_S}{\gamma \sigma_S}$), and a glide path (90% at age 25, diminishing to 30% by age 66).

# Sensitivity Analysis: Longevity Model (1/3)

Sensitivity surrounding the longevity model:

## Doubled Time Trend Errors' Variance

- Time trend process: $k_t = c + k_{t-1} + \delta_t$
- $\delta \sim N(0, \mathbf{2}\widehat{\sigma_\delta^2})$

## Drift Parameter Uncertainty

- $k_t = \boldsymbol{c} + k_{t-1} + \delta_t$
- $\hat{c}$ is estimated by maximum likelihood, and is distributed as $\hat{c} \sim N(c, \sigma_c^2)$
- For the $l^{th}$ replication, **draw a $\boldsymbol{c_l}$** from the distribution $\boldsymbol{N}\left(\hat{\boldsymbol{c}}, \widehat{\boldsymbol{\sigma_c^2}}\right)$

## Alternate Longevity Model

- **Cairns, Blake and Dowd (2006)**
- $logit(q_{t,x}) = \kappa_t^{(1)} + \kappa_t^{(2)}(x - \bar{x})$

# Sensitivity Analysis: Longevity Model (2/3)

Implication on individual preference and equity holders' profitability.

## Doubled Time Trend Errors' Variance

- **Default rates increase** from 0.0038% to 3.41%:
  CEL = **-7.7%**;

- If equity capital is raised sufficiently to **eliminate default risk**:
  CEL = **3.2%**;

- **Lower Sharpe ratio** with longevity risk exposure when loading is 3.2% and equity capital is raised sufficiently.

## Drift Parameter Uncertainty

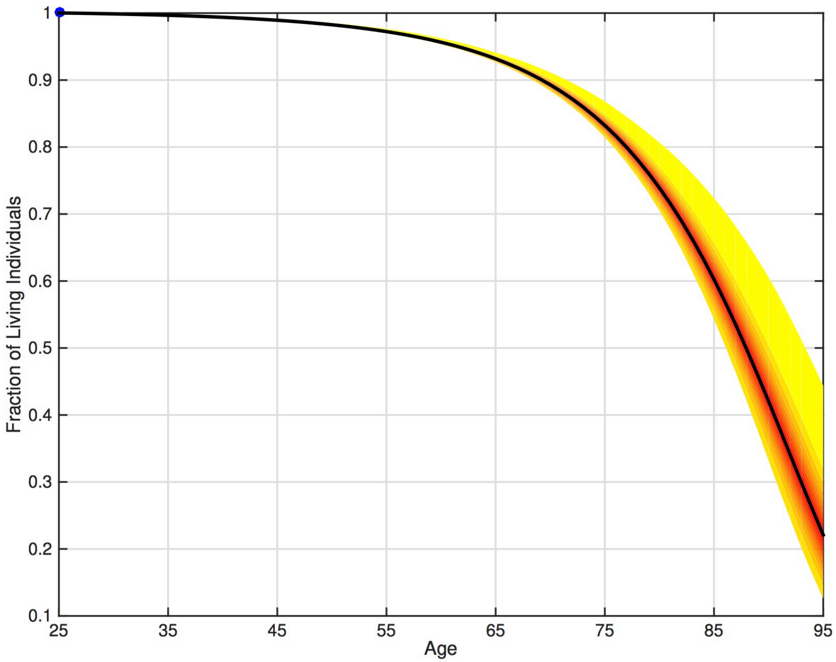
- **No material change** to the default rates, CEL, and equity holders' investment performance.

Figures correspond to contracts for individuals with $\gamma = 5$ and the underlying portfolio is 20% invested in the risky stock index, 80% invested in the money market account.

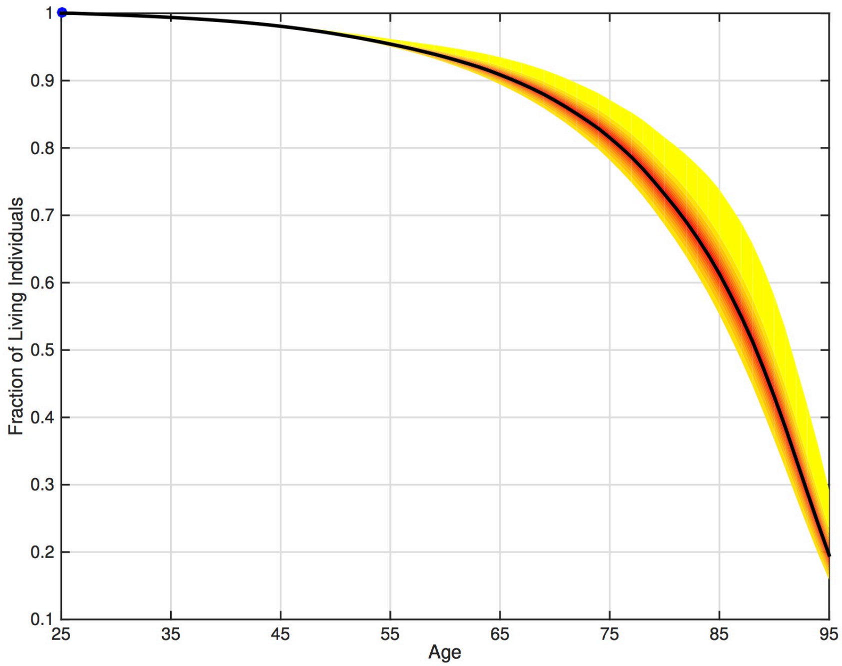
# Sensitivity Analysis: Longevity Model (3/3)

## Alternate Longevity Model

### Cairns-Blake-Dowd




### Lee-Carter




- **Higher uncertainty** on the likelihood of **survival at older ages**;
- Default rises to 0.48%: CEL = **-0.5%**;
- Absent default: CEL = **0.46%**;
- **Lower Sharpe ratio** with longevity risk exposure.

Both models are calibrated on U.S. female mortality data from 1980 to 2013, from the Human Mortality Database.

The fan plot is based on 10,000 replications.

# Conclusion (1/2)

- We investigate **longevity risk management** in retirement planning under two arrangements:
  - **Bearing** the risk as a collective (GSA), or;
  - **Insuring** the risk with a market-provided annuity contract (DVA).

- We model not only **individual preference** but also the **annuity provider's business** to underscore the involvement of equity holders in **enabling** the market solution.

- **Individuals prefer** the arrangement that yields a **higher expected utility**.

- Equity holders' **willingness to provide capital** depends on the **Sharpe ratio** of the investment opportunities that bear the **same financial market risks**, but are either **exposed to, or not exposed to longevity risk**.

# Conclusion (2/2)

- We find that when the DVA is priced at its **best estimate**:
  - Individuals have a **slight preference** for the **GSA**;
  - Equity holders attain a **lower Sharpe ratio** when **exposed to longevity risk**.

- Market-provided annuity contracts **would not co-exist** with collective schemes.

- Preference for the GSA is **insensitive** to:
  - Risk aversion levels;
  - Contract deferral period;
  - Exposure to stock market risk;
  - Longevity time trend’s drift parameter uncertainty.

- **Heightened longevity risk** only enhances the appeal of a DVA to the individual if the provider **restrains default risk**.
  - **Sharpe ratio** of the equity holders remains **inferior** to the Sharpe ratio of the investment in the financial market only;
  - Aggravated longevity risk leads to higher variability of the equity holder payoff as well.

# Appendices

# Retirement Contract: Per Unit Cost

- The per unit contract cost (i.e., annuity factor) is

$$A(h, F, t_0, x) := (1+F) \int_{t=t_R}^{T} {}_{t-t_0}p_x^{(t_0)} \exp(-h_t \times (t - t_R))\, \mathrm{d}t$$

$$\begin{aligned}
{}_{t-t_0}p_x^{(t_0)} &= \text{conditional probability in year } t_0 \\
&\quad \text{that a living individual of age } x \\
&\quad \text{lives for at least } t - t_0 \text{ more years} \\
h &= AIR \\
F &= \text{loading factor} \\
t_R &= \text{retirement year, i.e., } t_R = t_0 + 66 - 25
\end{aligned}$$

## Retirement Contract: Entitlement

- The entitlement in period $t, t_R \leq t \leq T$, conditional on the individual's survival, is

$$
\begin{aligned}
\Xi(h, F, t, x) &:= \frac{\exp\left(-h_t \times (t - t_R)\right)}{A\left(h, F, t_0, x\right)} \times \frac{W_t^{Ref}(\theta)}{W_{t_0}^{Ref}(\theta)} \\
W_t^{Ref}(\theta) &= \text{value of the reference portfolio at time } t \\
h &= AIR
\end{aligned}
$$

# Retirement Contract: Optimal AIR

- The optimal Assumed Interest Rate $(AIR)$ **maximizes** the individual's **expected utility** in retirement under the DVA contract, when the reference portfolio's investment policy is $\theta = \{\theta_t\}_{t=t_0}^{T}$ invested in the stock index, and $1 - \theta$ invested in the money market account.

$$
\begin{aligned}
h^*(t, \theta_t) &:= r + \frac{\beta - r}{\gamma} - \frac{1-\gamma}{\gamma}\theta_t \sigma_S \left(\lambda_S - \frac{\gamma \theta_t \sigma_S}{2}\right) \\
t &= \text{time index, } t,\ t_R \leq t \leq T \\
r &= \text{constant short rate} \\
\beta &= \text{subjective discount factor} \\
\gamma &= \text{risk aversion parameter} \\
\theta_t &= \text{fraction of wealth allocated to the stock index} \\
\sigma_S &= \text{diffusion term of the stock index} \\
\lambda_S &= \text{Sharpe ratio of the stock index}
\end{aligned}
$$

# Retirement Contract: DVA and GSA Entitlement

- The entitlement in period $t, t_R \leq t \leq T$, conditional on the individual's survival, is

**DVA**

$$\Xi^{DVA}(h^*, F, t, x) = \Xi(h^*, F, t, x)$$

**GSA**

$$\begin{aligned}
\Xi^{GSA}(h^*, 0, t, x) &= \Xi(h^*, 0, t, x) \times \frac{FR_t}{1} \\
&\quad \frac{\exp\left(-h^*\left(t, \theta_t\right) \times \left(t - t_R\right)\right)}{A\left(h^*, 0, t_0, x\right)} \times \frac{W_t^{Ref}\left(\theta\right)}{W_{t_0}^{Ref}\left(\theta\right)} \times FR_t \\
FR_t &= \text{Funding Ratio in year } t
\end{aligned}$$

# Certainty Equivalent Loading

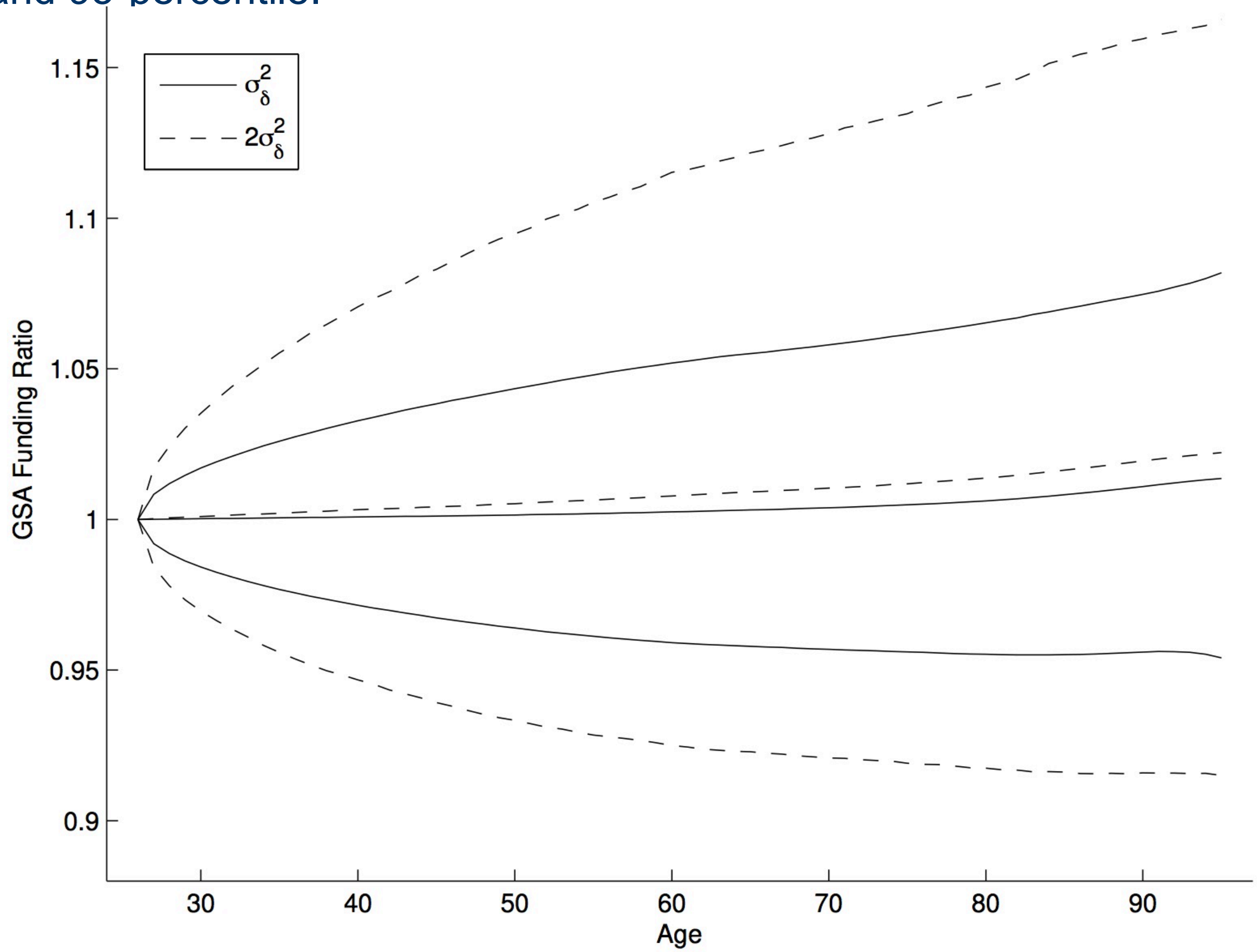
- The Certainty Equivalent Loading $(CEL)$ is the value such that the following holds.

$$\mathbb{E}\left[U\left(\frac{1}{1+CEL}\times \Xi^{DVA}|_{F=0}\right)\right] = \mathbb{E}\left[U\left(\Xi^{GSA}\right)\right]$$

# Sensitivity Analysis: Longevity Model (1/2)

## Doubled Time Trend Errors' Variance

Mean, 5- and 95-percentile.



# Sensitivity Analysis: Longevity Model (2/2)

## Drift Parameter Uncertainty

Mean, 5- and 95-percentile.