Optimal portfolio choice with health-contingent income products: The value of life care annuities *

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ABSTRACT

Whereas there is ample evidence that life-contingent income products (life annuities) have the potential to improve individual welfare, combining them with health-contingent income products (resulting in so-called life care annuities) would serve to further increase welfare for individuals who are exposed to uncertain out-of-pocket healthcare expenditure later in life. We develop a life-cycle model of annuitization, consumption, and investment decisions for a single retired individual who faces stochastic capital market returns, uncertain health status, differential mortality risks, and uncertain out-of-pocket healthcare expenditure with cost of dying. Using the calibrated model, we show that individuals who are eligible to purchase life care annuities instead of standard life annuities increase their level of annuitization by around 12 percentage points. Health status at retirement affects the extent to which the insurance feature and the pricing advantage of life care annuities contribute to this increment, with end-of-life healthcare expenditure being of particular importance. Also, life care annuities allow individuals to consume more throughout their retirement and to invest a higher proportion of their liquid wealth in the risky asset. They are willing to pay a loading up to 21% for having access to life care annuities.

Keywords: Life-cycle portfolio choice; Long-term care insurance; Retirement; Household finance.

JEL Classifications: D91, D14, G11, I13, H55, J32

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1 Introduction

Large and uncertain out-of-pocket healthcare expenditure (HCE[1]) and long-term care (LTC) costs can threaten the financial security of retirees. Individuals facing this risk can either purchase private health insurance and long-term care insurance (LTCI) or insure themselves by building up precautionary savings (De Nardi et al., 2010). However, voluntary LTCI has a small market share in countries where there is a private market, such as the United States (Brown and Finkelstein, 2011). In most countries, there is no private market for LTCI at all. While precautionary savings may provide a buffer, few individuals, especially those with significant pre-annuitized wealth, accumulate enough savings for large HCE in later life (e.g, Poterba et al., 2011; De Nardi et al., 2010; Ooijen et al., 2015).

One alternative is to integrate LTCI with old-age income provision. Ooijen et al. (2015) suggest that, in the context of the Netherlands, a lower pension that in turn provides extra income contingent on the need for long-term care might be offered to retirees in addition to the existing fixed-income pension. In the same view, Murtaugh et al. (2001) and Brown and Warshawsky (2013) propose a life care annuity (LCA), which combines a life annuity and LTCI, as a solution to the adverse selection problem in the life annuity market and the intensive underwriting in the LTCI market. Using data from a large US survey, Ameriks et al. (2011) find that the willingness to pay for life annuities with LTC coverage is much higher than for standard life annuities. Their empirical evidence motivates our normative study of optimal choices of bundled health-contingent retirement income products.

The objective of this paper is to study how optimal retirement portfolio choice and welfare are affected when individuals can choose between a health-contingent retirement income product, a conventional life annuity, bonds, and stocks. We develop a life-cycle model in which single retirees with bequest motives choose how much to annuitize at retirement and then decide on their consumption and investment paths. The annuity offered at retirement is either a conventional life annuity or a LCA. Retirees face stochastic capital market returns, uncertain health status, differential (health, age and gender specific) mortality risks, and uncertain HCE including the end-of-life HCE. Health dynamics are described by transition probabilities estimated by Brown and Warshawsky (2013). Out-of-pocket HCE is modeled as a mixture distribution with health-dependent parameters, which are allowed to vary during the last year of life. The parameters are calibrated using data from the Health and Retirement Study (HRS). Utilities in our model can be state dependent, taking into account the impact of health on the marginal utility of consumption.

Results suggest that LCAs increase the optimal annuitization level by 12 percentage points

[1] Throughout this paper, HCE refers to the out-of-pocket component of healthcare expenditure.

for individuals who are eligible to purchase them. For the those in good health, this increment is attributed in equal measure to the long-term care coverage and to the pricing advantage of LCAs. Brown and Warshawsky (2013) show that this group is able to purchase LTCI at a price below the actuarially fair premium when lapses of contracts are taken into account (which are frequent in actual practice). This paper also finds that LCAs are attractive in terms of pricing to this group. In contrast, retirees in bad health are very likely to be excluded from LTCI due to underwriting (Brown and Warshawsky, 2013). Hence this paper suggests that this may be overcome by bundling life annuities and LTCI.

Our results also show that uncertain HCE in later life reduce the demand for standard life annuities when end-of-life HCE is incorporated, in line with Sinclair and Smetters (2004), Turra and Mitchell (2008), Reichling and Smetters (2015), and Peijnenburg et al. (2015). However, when end-of-life HCE is neglected, the risk of HCE does not matter; it simply causes individuals to consume less, to hold significant precautionary savings, and to invest a lower proportion of their liquid wealth in the risky asset, confirming the results of earlier studies (e.g., De Nardi et al., 2010; Pang and Warshawsky, 2010). We also find that the presence of uncertain HCE results in a less risk-averse portfolio for total wealth which includes the annuitized wealth as well as liquid wealth.

With the support of LCAs, individuals hold less precautionary savings, allowing them to enjoy a higher level of consumption throughout their retirement phase. Moreover, they tend to increase the share of the risky asset in their liquid wealth. At a given level of annuitization, however, individuals hold a more risk-averse portfolio for their total wealth with LCAs than with standard life annuities. Consistent with Ameriks et al. (2011), we find that the welfare gain from access to a LCA market is substantial. Individuals are willing to pay as much as 10% of their wealth at retirement for this, amounting to loadings of 16% to 21% depending on their health status.

The rest of the paper is structured as follows. Section 2 briefly reviews the relevant literature. Section 3 describes the life-cycle model. Section 4 presents the results for the benchmark case and examines the welfare implication of LCAs. Sensitivity analyses are subsequently performed in Section 5. Section 6 concludes.

2 Literature review

This paper relates to three strands of literature. The first strand studies the influence of health status and HCE on portfolio choice and saving behavior. Feinstein and Lin (2006) show that poor health and sizeable HCE result in more risk-averse portfolio choices, reducing the share of risky assets. Pang and Warshawsky (2010) expand these results by including life annuities in the portfolio and allow them to be purchased in every year of retirement. Using the calibrated model, they show that the uncertain HCE may shift household portfolios from risky equity to riskless bonds, and eventually to life annuities later in life. Other studies find that HCE (Palumbo, 1999, Dynan et al., 2004, and De Nardi et al., 2010) and needs for LTC (Ameriks et al., 2011, 2015) generate a significant amount of precautionary savings and play an important part in explaining the failure of the elderly to dissave. We expand these studies by incorporating a health-contingent income product in the retirement portfolio while taking into account the end-of-life HCE.

The second strand of research focuses on the impact of HCE on annuitization decisions.[2] The existing literature is inconclusive. On the one hand, some studies argue that uncertain HCE reduces the attractiveness of annuities. Sinclair and Smetters (2004), Turra and Mitchell (2008), and Reichling and Smetters (2015) hold this view and attribute it to the correlation between mortality risk and HCE, affecting the remaining present value of life annuities. Davidoff et al. (2005) and Peijnenburg et al. (2015) emphasize that sizeable HCE can occur early in retirement, causing a need for liquidity and hence reducing demand for (illiquid) annuities. However, if HCE occurs later in retirement, it is optimal for individuals to annuitize more. This is because the increasing mortality credit at older ages creates a hedge against age-rising HCE. On the other hand, studies modeling HCE in accordance with De Nardi et al. (2010) find that uncertain HCE does not lower, or even increases, the demand for life annuities. Pashchenko (2013), examining several factors affecting annuitization decisions, reaches the conclusion that uncertain HCE does not lower annuity demand. Pang and Warshawsky (2010) build a model in which additional annuities can be bought in every year of retirement, showing that uncertain HCE makes annuities more appealing.[3] We contribute to this strand of literature by showing that the impact of HCE risk on annuitization largely depends on whether or not end-of-life HCE is taken into account. Its incorporation is found to significantly reduce the demand for life annuities.

The third strand of studies examine the supply-side potential of offering LCAs as an alternative to life annuities and LTCI purchased separately. The idea of combining life annuities with LTCI first appeared in Pauly (1990) and was extended by Murtaugh et al. (2001). Bundling the two is intended to reduce both adverse selection in the annuity market and the need for intensive underwriting in the LTCI market. Due to adverse selection, life annuities often have low money's

[2]Other rational factors argued to affect the demand for annuities include loadings (Friedman and Warshawsky, 1990; Mitchell et al., 1999), incomplete markets (Davidoff et al., 2005; Horneff et al., 2008), bequest motives (Bernheim, 1991; Lockwood, 2012; Ameriks et al., 2011; Brown, 2001), option value of delay (Blake et al., 2003; Milevsky, 1998, 2001; Milevsky and Young, 2007a, 2007b; Kingston and Thorp, 2005), pre-existing annuitization (Bütler et al., 2016; Dushi and Webb, 2004), and intra-family risk sharing (Brown and Poterba, 2000).

[3]This is because liquidity constraints in their model are less likely to bind in early retirement. Individuals are predicted to annuitize more to take advantage of the mortality credit and save significant amounts out of their annuity income to build a buffer against HCE later in life.

worth for the average individual, causing lower rate of annuitization (Mitchell et al., 1999). Intensive underwriting in LTCI excludes a significant proportion of potential buyers (Murtaugh et al., 1995), contributing to the low take up of LTCI (Brown and Finkelstein, 2011).

The theoretical rationale for bundling life annuities and LTCI is explained in Murtaugh et al. (2001). By integrating the two products, a LCA requires minimal underwriting, only excluding those who are eligible for the LTC benefit at the time of purchase, provided that the LTC segment of the LCA is attached to the life annuity as a permanent rider in return for a single premium insurance (Brown and Warshawsky, 2013).[4] In this way, LTC coverage can be made available to most of those who are currently rejected. As the currently rejected individuals have a lower life expectancy, not only will the inclusion of this group of people largely increase the potential market for LCAs but it will also make the product more affordable by reducing adverse selection.

The success of a LCA market depends on whether it can induce risk pooling of two groups of potential purchasers, the healthy and unhealthy. For the healthy, the question is whether they can purchase the bundled product at a lower premium than separately, where they can benefit from the lower periodic LTC premium that is calculated taking into account lapses. For the unhealthy, the trade-off is between receiving LTC coverage and paying a premium for the life annuity segment of LCAs in excess of the actuarially fair value. Brown and Warshawsky (2013) find that for the healthy, the bundled product and the two separate products come at roughly the same costs, while for unhealthy males a LCA costs about 7% more than the actuarially fair premium. We contribute to this strand of literature by studying whether a LCA can provide access to LTC coverage for those who do not now qualify for LTCI at present, and by quantifying the associated welfare gain, in a life-cycle framework.

3 The model

We solve a discrete-time life-cycle model which incorporates HCE risk, capital market risk, and longevity risk. The investment menu consists of a standard life annuity, a LCA, a riskless asset, and a risky asset, where either the life annuity or the LCA is available at retirement. The unit of analysis is a single individual who has just retired at age 65 and has a maximum lifespan of 100 years. Integer t symbolizes the number of years into the retirement period, starting at 0 and ending

[4] Finkelstein and McGarry (2006) find that *ex post* utilisation rates of LTC are similar between the insured and the non-insured. Brown and Warshawsky (2013) estimate actuarial cost of disability indemnity policies to be similar across health groups of consumers who are eligible for a LCA. However, they argue that defensive underwriting is necessary for LTCI with periodic premiums, making the discounted present value of premiums depend on the life expectancy of the insured. Yet periodic-premium insurance is prevalent in current LTCI markets because of its advantage in terms of liquidity and lower premiums when taking into account lapses.

at the maximum length of the retirement period $T = 35$ i.e., $t \in [0, T]$. The individual, who has a CRRA utility separable in time and bequest motives, maximizes lifetime utility by choosing how much to annuitize at retirement, to consume, and to invest in the risky asset in each period subject to the constraints of no short-selling and borrowing.

At retirement ($t = 0$), the individual chooses the fraction ($x \in [0, 1]$) of wealth W to be annuitized, a decision that can only be made once.[5] From this point onwards, the individual enters each period $t \in [0, T-1]$ with a level of liquid wealth W_t and a current health state (H_t). Hence, the level of liquid wealth immediately after annuitization is given by

$$W_0 = (1 - x)W \tag{1}$$

and H_0 is known at retirement. Conditional on survival, he or she receives annuity income Y_t and incurs medical expenses M_t, resulting in disposable wealth $W_t + Y_t - M_t$. If disposable wealth is below a wealth floor $\underline{W}$, the individual receives a government subsidy G_t giving the individual always $\underline{W}$ to consume, i.e., $G_t = \max\{0, \underline{W} - (W_t + Y_t - M_t)\}$.[6] The individual chooses consumption C_t and the proportion α_t of after-consumption wealth given by $\overline{W}_t = W_t + Y_t - M_t + G_t - C_t$ to be invested in the risky asset. End-of-life utility is derived from a bequest equal to end-of-period wealth including investment returns.

3.1 Health dynamics

Health dynamics play an important role in our analysis. We borrow the actuarial model of health transition[7] from Brown and Warshawsky (2013) who distinguish eleven health states H_t in a given period t, i.e., integer $H_t \in [1, 11]$. This classification takes into account self-reported health, history of major illness (heart problems, diabetes, lung disease and stroke), as well as whether the individual needs help with activities of daily living (ADL). Also, $H_t = 1$ indicates full health and $H_t = 11$ indicates death (see Table 1).

The individual's health evolves from period t to $t+1$ according to a Markov chain with a 11×11 transition matrix, whose $(i, j)^{th}$ element $\pi_t(i, j)$ represents the probability of being in state j in period $t + 1$ conditional on being in health state i in period t, which also depends on gender (but

[5] Peijnenburg et al. (2016) argue that annuitization is often an one-off decision due to institutional constraints, the fact that retirees make financial decisions very infrequently rather than annually, and the decline of cognitive ability to make financial decisions at old ages. This is consistent with Turra and Mitchell (2008), Peijnenburg et al. (2015) and Peijnenburg et al. (2016). However, Pang and Warshawsky (2010), Horneff et al. (2008), and Horneff et al. (2009) allow annuities to be purchased at any time during retirement.

[6] The wealth floor is equivalent to the consumption floor in Ameriks et al. (2011) and Peijnenburg et al. (2015), given that individuals cannot die with negative wealth with the specification of our bequest function in Section 3.4.

[7] The model is similar to the one developed by Robinson (1996), which is widely-used in the literature (Brown and Finkelstein, 2007, 2008) as well as by insurance companies, regulators, and government agencies.

Table 1: Classification of health states

The table summarizes the classification of health states (1 - 11). Heart problems refer to heart attack, coronary heart disease, angina, congestive heart failure, or other heart problems. Lung disease refers to chronic lung diseases like chronic bronchitis and emphysema but not asthma. Stroke refers to stroke or transient ischemic attack. Moderately disabled is defined by two to three ADL limitations or cognitive impairment with fewer than two ADL limitations. Severely disabled is defined by four to five ADL limitations or cognitive impairment with two or more ADL limitations. (Source: Table 1, Brown and Warshawsky, 2013)

Health state	History of major illness	Self-reported health	Disability status
1	None	Good to Excellent	0 ADLs, no cognitive impairment
2	None	Poor to Fair	0 ADLs, no cognitive impairment
	None	All	1 ADLs, no cognitive impairment
3	Heart problems or diabetes, but not both	All	0 ADLs, no cognitive impairment
4	Heart problems and diabetes, or Lung Disease, or Stroke	All	0 ADLs, no cognitive impairment
5	None	All	Moderately disabled
6	Heart problems or diabetes, but not both	All	Moderately disabled
7	Heart problems and diabetes, or Lung Disease, or Stroke	All	Moderately disabled
8	None	All	Severely disabled
9	Heart problems or diabetes, but not both	All	Severely disabled
10	Heart problems and diabetes, or Lung Disease, or Stroke	All	Severely disabled
11			Death

with the pertinent subscript dropped for simplicity). Thus,

$$\pi_t(i,j) = Pr(H_{t+1} = j | H_t = i), \tag{2}$$

with $\pi_t(i, 11)$ symbolizing the probability of dying from health state i in period t. Finally, we define the n-period transition by

$$\pi_t^n(i,j) = Pr(H_{t+n} = j | H_t = i). \tag{3}$$

3.2 Out-of-pocket healthcare expenditure

The cross-sectional distribution of HCE follows a mixture distribution rather than the lognormal distribution used in French and Jones (2004) and De Nardi et al. (2010).[8] The mixture distribution consists of three components, with the first representing the cluster of zero HCE observed in the data. Therefore, HCE is described by M_t,

$$M_t = \begin{cases} 0, & \text{if } I_t = 1; \\ m_t, & \text{if } I_t = 0, \end{cases}$$

where $m_t \in (0, \infty)$ represents non-zero HCE and I_t is an indicator function which is assumed to follow a Bernoulli distribution,

$$I_t \sim \text{Bernoulli}(p(\Theta_t)). \tag{4}$$

The parameter $p(\Theta_t)$ is the probability of $I_t = 1$ given Θ_t and Θ_t is the information set for the HCE distribution.

The second component of HCE models m_t with a truncated lognormal distribution, in line with French and Jones (2004). Specifically, if m_t is less than the truncation value $a(\Theta_t)$, it follows a lognormal distribution with mean parameter $\mu(\Theta_t)$ and standard deviation parameter $\sigma(\Theta_t)$.

The third component corresponds to the tail of m_t using an exponential distribution. The tail is modeled separately because a lognormal distribution does not fit it well. In particular, it is too fat at very high percentiles (e.g., beyond the 99.5^{th} percentile) compared with the empirical distribution, resulting in unrealistically high simulated HCE. However, French and Jones (2004) show that the goodness of fit in the tail is important for life-cycle models, because extremely high HCE results in a low level of wealth, while sensitivity of utility to a wealth shock is larger at a low wealth level than at a high one.

One way to deal with this issue is to calibrate the two parameters of the lognormal distribution to the mean and a large percentile (French and Jones, 2004). However, this approach can lead both to a poor fit in the left tail of the HCE distribution and to unrealistically high simulated HCE beyond the calibrated percentile, especially in the case of end-of-life HCE. An exponential distribution for the tail of non-zero HCE avoids these problems.

Specifically, if non-zero HCE m_t is greater than or equal to the truncation value $a(\Theta_t)$, $m_t - a(\Theta_t)$

[8] Modeling HCE using a lognormal distribution would either require dropping observations with zero HCE or incorporating them as small positive values. The former approach overestimates HCE while the latter approach biases the parameter estimates because the clustering of small values does not fit a lognormal distribution well.

follows an exponential distribution with mean $\lambda(\Theta_t)$. Thus,

$$\begin{cases} m_t \sim \text{Truncated LN } (\mu(\Theta_t), \sigma(\Theta_t), a(\Theta_t)), & \text{if } m_t < a(\Theta_t); \\ (m_t - a(\Theta_t)) \sim \text{Exp } (\lambda(\Theta_t)), & \text{if } m_t \geq a(\Theta_t). \end{cases}$$

The information set Θ_t includes not only the health state at time t but also at time $t+1$, i.e., $\Theta_t = \{H_t, H_{t+1}\}$. This permits one to incorporate HCE during the last year of life separately, in contradistinction with De Nardi et al. (2010) and Ameriks et al. (2011). This approach takes into account evidence suggesting that time-to-death is a strong predictor of HCE (Zweifel et al., 1999; Felder et al., 2000; Stearns and Norton, 2004). Parameter estimates presented in Section 3.6 confirm this. Also note that on average modeled HCE does increase with age, as a result of health-dependent parameters, although age is not incorporated explicitly.

The cost of dying is driven by a separate information set,

$$\Theta_t = \begin{cases} \theta_t^{i,A}, & \text{if } H_t = i \text{ and } H_{t+1} \neq 11; \\ \theta_t^{i,D}, & \text{if } H_t = i \text{ and } H_{t+1} = 11. \end{cases}$$

With this specification, $\theta_t^{i,A}$ presents where an individual in health state i at time t and is alive at time $t+1$, while $\theta_t^{i,D}$ presents if the individual is dead at time $t+1$. Hence, the parameters of the HCE model ($p(\Theta_t)$, $\mu(\Theta_t)$, $\sigma(\Theta_t)$, $a(\Theta_t)$, $\lambda(\Theta_t)$) depend not only on current health state (H_t) but also on whether the individual survives or passes away in the following period (H_{t+1}).

Using the data and calibration method explained in Section 3.6, Panel A of Table 2 exhibits seven moments of empirical HCE pertaining to individuals who die in the following year.[9] In Panel B and Panel C, the moments of simulated HCE derived from a lognormal distribution and from the 'fitted' lognormal distribution (French and Jones, 2004) differ substantially from their empirical counterparts for values beyond the 99.5$^{\text{th}}$ percentile, causing an excessive impact of HCE on both annuity demand and precautionary saving. In comparison, the mixture distribution in Panel D exhibits a good overall fit. In particular, it does not overestimate HCE of the deceased beyond the 99.5$^{\text{th}}$ percentile, with one exception. Since this exception pertains to health state 1, where HCE is low, it has limited relevance. However, the mixture model shares the tendency of the 'fitted' lognormal to underestimate standard deviation.

[9] For individuals who are alive in the following year, the three models in Table 2 have similar goodness of fit. Pooling surviving and deceased individuals results in differences in goodness of fit of the same type as shown in Table 2.

Table 2: Comparison of HCE models

The table compares seven moments of observed annual HCE (in $000 as of 2014) with the moments of three models for individuals who are dead in the following year. Using HRS data from 1998 to 2008, the 'Lognormal' model is fitted to the mean and standard deviation of the data; the 'Fitted lognormal' is fitted according to French and Jones (2004); the 'Mixture' model is fitted using the method discussed in Section 3.6;. Health states 5 to 7 and 8 to 10 are pooled in all models. An asterisk indicates that the value lies outside the 95 percent CI of the respective sample moment.

	Health state					
Moments	1	2	3	4	5 to 7	8 to 10
Panel A: Data						
Number of observations	272	180	467	723	254	376
Mean	2.8	4.5	4.2	4.1	8.7	16.9
Std	7.0	11.0	12.1	10.6	24.5	43.9
90^{th} percentile	7.9	14.1	11.0	11.9	17.9	48.9
95^{th} percentile	14.5	17.1	20.9	22.0	46.8	96.4
99.5^{th} percentile	31.7	73.6	86.0	69.0	162.8	220.5
99.9^{th} percentile	59.6	94.0	138.7	102.6	199.6	427.5
99.99^{th} percentile	65.9	98.6	139.5	113.9	206.8	504.6
Panel B: Lognormal						
Mean	2.8	4.5	4.2	4.1	8.7	16.9
Std	7.0	11.0	12.1	10.6	24.5	43.9
90^{th} percentile	6.3*	10.3*	9.4	9.3*	19.4	38.0
95^{th} percentile	10.6*	17.0	16.1	15.6*	33.3	63.9
99.5^{th} percentile	39.0	61.8	64.5	58.9	131.7	242.2
99.9^{th} percentile	80.2*	126.1*	138.9	122.5	281.6*	505.8
99.99^{th} percentile	193.7*	301.5*	354.7*	300.3*	713.3*	1244.6*
Panel C: Fitted lognormal						
Mean	2.8	4.5	4.2	4.1	8.7	16.9
Std	5.4*	14.3*	20.9*	13.6*	35.4*	38.6*
90^{th} percentile	6.4*	10.0*	8.4	9.0*	18.2	38.3
95^{th} percentile	10.0*	17.5	16.1	16.0*	33.7	62.6
99.5^{th} percentile	31.7	73.6	86.0	69.0	162.8	220.5
99.9^{th} percentile	59.9	162.9*	217.4*	154.8*	388.6*	442.1
99.99^{th} percentile	130.6*	429.9*	675.1*	416.1*	1126.1*	1035.0*
Panel D: Mixture						
Mean	2.7	4.4	4.1	4.0	8.5	16.9
Std	6.5	8.9*	10.5*	9.7*	23.8	41.0
90^{th} percentile	7.9	14.1	11.0	11.9	17.9	48.9
95^{th} percentile	15.5	22.8	23.5	23.2	49.6	97.2
99.5^{th} percentile	40.6	51.8	65.1	60.8	155.2	257.6
99.9^{th} percentile	58.2	72.0	94.1	87.2	228.9	369.7
99.99^{th} percentile	83.4*	101.0	135.7	124.8	334.5	530.0

3.3 Financial assets

The investment menu includes a standard life annuity or a LCA, a riskless bond, and a risky stock. For simplicity, individuals can purchase either a standard life annuity or a LCA at retirement as an irreversible choice.

The life annuity pays a fixed amount of money at the beginning of a period conditional upon survival, with the first payment paid immediately after purchase. To reflect adverse selection and following Brown and Warshawsky (2013), the life annuity is actuarially fairly priced according to the mortality in health state 1 at retirement.

The LCA consists of two income streams. Its basic (annuity) component pays a fixed amount of money conditional on survival as a standard life annuity. Its top-up (LTC) component pays L times as much as the basic component provided that the annuitant is in health states 5–10.[10] As in Brown and Warshawsky (2013), the LCA is priced without loadings using the pooled transition probabilities of health states 1–4 at retirement. Individuals in health states 5–10 at retirement do not qualify for this product and are therefore excluded from pricing, because they would have been immediately eligible for a top-up payment at purchase. Compared with a standard life annuity, a LCA offers a better deal for two reasons. First, it provides partial LTC coverage (to the extent that the need for LTC goes along with a transition to health states 5 to 10). Second, its pricing is more favourable due to risk pooling. This is because risk pooling substantially reduces the cost of the basic (annuity) segment, while it does not increase the top-up (LTC) component significantly provided that it is attached to the basic (annuity) segment as a permanent rider in return for a single premium (Brown and Warshawsky, 2013).

The real rate of return of the bond r_f is time-independent, thus the real gross return $R^f = 1+r_f$ accordingly. The real gross return of the stock from t to $t+1$, denoted by $\widetilde{R}_{t+1}$, follows a lognormal distribution with parameters μ_S and σ_S, with excess return given by $R^e_{t+1} = \widetilde{R}_{t+1} - R^f$. Therefore, the rate of wealth accumulation R_{t+1} pertaining to a stock-bond portfolio from t to $t+1$ is

$$R_{t+1} = R^f + \alpha_t R^e_{t+1}, \tag{5}$$

where $\alpha_t \in [0, 1]$ is the proportion invested in the stock. Note that short-selling is precluded.

[10] In Brown and Warshawsky (2013) and Murtaugh et al. (2001), the LCA has a 10-year guarantee period and two top-up income streams, one for health states 5–10 and the other for health states 8–10. For simplicity, this distinction is not made here.

3.4 Preferences

Individuals gain utility from consumption and leaving bequests. Following Koijen et al. (2016), the individual is assumed to have a time-separable but health-dependent utility function with constant relative risk aversion (CRRA) and exponential discounting parameter β. Utility from consuming C_t in health state $H_t \in [1, 10]$ is given by,

$$u(C_t, H_t) = \omega(H_t)^\gamma \frac{C_t^{1-\gamma}}{1-\gamma}, \tag{6}$$

where γ is the relative risk aversion and

$$\omega(H_t) = \begin{cases} 1, & \text{if } H_t \in [1, 4]; \\ h, & \text{if } H_t \in [5, 10]. \end{cases}$$

With this specification, the marginal utility of consumption in health states 5 to 10 differs from that in health states 1 to 4. When an individual is in health states 1 to 4, the utility function becomes standard power utility. The parameter h governs the relationship between consumption and health. If $h < 1$, the marginal utility of consumption is low when in bad health. If $h > 1$, it is high. This approach extends the utility function used in existing annuitization studies by applying the theory of health-dependent utility (Arrow, 1974), and is more realistic (Viscusi and Evans, 1990; Finkelstein et al., 2013).

Leaving a bequest of value W_t creates utility

$$v(W_t) = \frac{(bW_t)^{1-\gamma}}{1-\gamma}, \tag{7}$$

where the parameter b measures the strength of the bequest motive. Utility from bequest is also discounted by the time preference β.

3.5 The optimization problem

The individual's problem is to maximize expected lifetime utility by choosing an annuitization level at retirement, consumption, and an investment strategy for each period. The state variables are liquid wealth, health state, and annuitized income $\{W_t, H_t, Y_t\}$. The objective function is given by

$$V(W, H_0) = \max_{\{x, C_t, \alpha_t\}_{t=0}^{T}} \left\{ \mathbb{E}_0 \left[\sum_{t=0}^{T-1} \sum_{j=1}^{10} \pi_0^t(H_0, j) \cdot \beta^t \cdot [u(C_t, H_t) + \beta \cdot \pi_t(j, 11) \cdot v(W_{t+1})] \right] \right\} \tag{8}$$

where $V(\cdot,\cdot)$ is the value function at retirement. The Bellman equation for the annuitization decision is given by

$$V(W,H_0) = \max_x \mathbb{E}_0 \left\{ V_0(W_0,H_0,Y_0) \right\}, \tag{9}$$

subject to equation (1), where $V_t(\cdot,\cdot,\cdot)$ is the value function for period t. Given the solution to equation 9, the Bellman equation for each period $t \in [0, T-1]$ is given by

$$\begin{aligned} V_t(W_t,H_t,Y_t) = \max_{\{C_t,\alpha_t\}} \mathbb{E}_t \Bigg\{ & u(C_t,H_t) + \sum_{j=1}^{10} \pi_t(H_t,j) \cdot \beta \cdot V_{t+1}(W_{t+1},H_{t+1}=j,Y_{t+1}) \\ & + \pi_t(H_t,11) \cdot \beta \cdot v(W_{t+1}) \Bigg\}, \end{aligned} \tag{10}$$

subject to constraints

$$\overline{W}_t = W_t + Y_t - M_t + G_t - C_t \geq 0, \qquad \forall t \in [0, T-1]; \tag{11}$$

$$W_{t+1} = \overline{W}_t \cdot R_{t+1}, \qquad \forall t \in [0, T-1], \tag{12}$$

where the $\overline{W}_t$ is wealth level after consumption. The budget constraint (11) combined with the assumption of lognormal returns, ensures that individuals cannot die with negative wealth. At time $t = T$, the individual dies for sure and there is no annuity income and consumption. Hence,

$$V_T(W_T, H_T = 11, Y_T = 0) = v(W_T). \tag{13}$$

This optimization problem is solved numerically by backward induction, with expectations evaluated numerically using the importance sampling method. Linear interpolation is applied to consumption and cubic spline to investment strategies for points that are not on the grid (see Appendix A for details). The numerical solution provides optimal consumption and investment strategies for grid points of wealth, health state, and income. Monte Carlo simulations are financially carried out to generate 200,000 sample paths to evaluate the statistical properties of optimal decisions in terms of annuitization at retirement, as well as consumption and investment over the period of observation.

3.6 Parameter calibration

Parameters are set equal to the values often used in the literature, with a sensitivity analysis conducted in Section 5. Following Gomes and Michaelides (2005), Pang and Warshawsky (2010), and Peijnenburg et al. (2015), we set time preference[11] β equal to 0.96 and relative risk aversion γ to 5. The strength of bequest motive b is set at 0.17 (Yogo, 2009). Given this value, the wealth floor $\underline{W}$ is set equal to \$22,000, resulting in optimal minimal annual consumption between \$8,600 to \$4,400 per year depending on age and health state. This is similar to Ameriks et al. (2011) and Peijnenburg et al. (2015) with a consumption floor of \$7,000. The parameter h governs the marginal utility of consumption in health states 5–10 relative to health states 1–4. In our benchmark analysis, we set $h = 1$ so the utility function is not state-dependent.

For financial products, the real rate of return of the bond is $r^f = 3\%$ and the real rate of return of the stock is $\mu_S = 6.5\%$ with a standard deviation $\sigma_S = 16.1\%$, as in Pang and Warshawsky (2010). The standard life annuities and LCAs are discounted using r^f.[12] The amount of the top-up income stream is an attribute of LCAs and can vary. In the benchmark case, it is twice the basic payment i.e., $L = 2$, which is the optimal product design for males[13] in health state 1 as shown in Section 5, and is also roughly in line with the values in Brown and Warshawsky (2013) and Murtaugh et al. (2001). The pre-annuitization wealth level is $W = \$500,000$ (results at other wealth levels are discussed in Section 5).

Our model abstracts from the existence of annuitization prior to retirement (such as a public pension or a Defined Benefit plan paying a lifetime income stream) for three reasons. First, the form and the amount of pre-annuitized wealth varies from one country to another, while this paper seeks to avoid imposing any country-specific institutional settings. Second, the trade-off between standard life annuities and LCAs can be better studied without a significant proportion of total wealth being annuitized prior to retirement. Third, optimal income levels turn out to be largely independent of pre-retirement annuitization. In particular, retirees can always purchase additional units of annuities if their optimal income levels exceed income from pre-retirement annuitization.

The health transition probabilities $\pi_t(i, j)$ are estimated by Brown and Warshawsky (2013) using maximum likelihood method as in Kalbfleisch and Lawless (1985), based on HRS data from 1998 (Wave 4) to 2008 (Wave 9). We rely on their estimation results.[14]

The parameters of the HCE model are calibrated from HRS data for males from 1998 to 2008, with HCE during the last year of life taken from HRS exit proxy interviews. Following Brown

[11] We are aware of the high subjective discount factors found in experimental studies (e.g., Coller and Williams, 1999; Andersen et al., 2008)

[12] Pang and Warshawsky (2010) use a 3.4% discount rate for life annuities, whereas Brown and Warshawsky (2013) use a 6% discount rate for LCAs. To make the products comparable, we use r^f to discount both.

[13] We choose this value as the results in Section 4 focus on males. The optimal product design for females in health state 1 is $L = 1$. The difference in the optimal value of L is driven by the different observed HCE distribution between genders.

[14] For the goodness of fit of the estimated model, Table 3 in Brown and Warshawsky (2013) compares fitted transition probabilities with observed ones, whereas their Table 4 exhibits seven moments of mortality and disability projections. We do not repeat their work here.

and Warshawsky (2013), the HRS sample weights are adjusted to match the distribution of the U.S. population in 2010 terms of age, gender, and race. For all parameters, health states 5 to 7 (moderately disabled) and 8 to 10 (severely disabled) are pooled, because the number of observations is insufficient for parameter estimation. However, it should be noted that the transition probabilities are estimated for each state separately.

The parameter $p(\Theta_t)$ is calibrated to reflect the proportion of zero HCE,

$$p(\Theta_t) = \Pr(M_t = 0|\Theta_t), \tag{14}$$

while truncation values correspond to the 90$^{\text{th}}$ percentile. Thus

$$a(\Theta_t) = M_t^{90}|\Theta_t \tag{15}$$

where M_t^{90} is the 90$^{\text{th}}$ percentile of HCE. The parameters $\mu(\Theta_t)$ and $\sigma(\Theta_t)$ of the truncated log-normal distribution are calibrated using the following equations (Greene, 2002),

$$\mathbb{E}(\log(m_t)|m_t < a(\Theta_t), \Theta_t) = \mu(\Theta_t) - \sigma(\Theta_t)\frac{\phi(z)}{\Phi(z)} \tag{16}$$

$$\operatorname{Var}(\log(m_t)|m_t < a(\Theta_t), \Theta_t) = \sigma^2(\Theta_t)\left[1 - z\frac{\phi(z)}{\Phi(z)} - \left(\frac{\phi(z)}{\Phi(z)}\right)^2\right] \tag{17}$$

where

$$z = \frac{\log(a(\Theta_t)) - \mu(\Theta_t)}{\sigma(\Theta_t)}.$$

The mean parameter of the exponential distribution $\lambda(g, H_t, H_{t+1})$ is calibrated to the sample mean of HCE in excess of the 90$^{\text{th}}$ percentile,

$$\lambda(\Theta_t) = \mathbb{E}(m_t - a(g, H_t, H_{t+1})|m_t > a(\Theta_t), \Theta_t). \tag{18}$$

Table 3 summarizes the calibrated parameter values of the HCE model. As expected, average HCE is higher in poorer health states. Among both the survivors and the deceased individuals across all health states, there is a significant proportion of observations with zero HCE as shown by the estimates of $p(\Theta_t)$, supporting the separation of zero from non-zero HCE. For most health states, all three of truncated lognormal parameters as well as the mean parameter of the exponential distribution are higher for survivors than the deceased. This result clearly shows that by controlling for heath status, both the size of and the variation of HCE are on average greater in the last year

Table 3: Calibrated parameter values of the HCE model for males

The table reports the calibrated parameter values of the HCE model for males based on the data and method explained in Section 3.6 for the case when individuals survive in the following period (Panel A) and the case when individuals pass away (Panel B).

	Health state (H_t)					
Parameters	1	2	3	4	5 to 7	8 to 10
Panel A: Survivors ($H_{t+1} \neq 11$)						
$p(\Theta_t)$	0.117	0.159	0.064	0.095	0.194	0.283
$\mu(\Theta_t)$	6.953	7.390	7.271	7.520	7.433	7.673
$\sigma(\Theta_t)$	1.620	1.787	1.416	1.506	1.902	2.178
$a(\Theta_t)$	3405.850	4675.305	5144.762	5911.721	13651.591	21025.613
$\lambda(\Theta_t)$	4933.089	10447.075	8754.058	8527.028	26855.825	35098.905
Panel B: The deceased ($H_{t+1} = 11$)						
$p(\Theta_t)$	0.326	0.211	0.225	0.238	0.273	0.370
$\mu(\Theta_t)$	7.013	7.622	7.079	7.042	8.834	9.963
$\sigma(\Theta_t)$	2.305	2.507	2.628	2.236	3.304	3.232
$a(\Theta_t)$	7935.787	14094.379	10960.766	11874.487	17855.921	48937.307
$\lambda(\Theta_t)$	10917.782	12581.826	18057.985	16347.919	45835.725	69647.053

of life, consistent with the findings by Zweifel et al. (1999), Felder et al. (2000) and Stearns and Norton (2004), underlining the importance of incorporating the cost of dying in HCE modeling.

It is useful to examine the pattern of simulated HCE in our model, and to compare it with previous studies. Table 4 presents the moments of simulated HCE across age for individuals in health states 1 and 4 (the poorest health state for LCA eligibility) at retirement. In general, both the level and variation of HCE increase with age and poorer health. Peijnenburg et al. (2015) calculate these moments of simulated annual HCE for the model in De Nardi et al. (2010) which also assumes exponential HCE at the high end. While the age trend is similar in Table 4, there is a longer tail in early retirement in our model, shown by higher values at the 90th, 95th and 99th percentiles respectively, suggesting that our model allows large HCE to occur early in retirement, which is an important distinction and affects the demand for life annuities (Peijnenburg et al., 2015).

Table 4: Simulated annual HCE by age

The table reports six moments of the simulated annual HCE distribution (in $000 as of 2014) and the number of alive individuals at different ages for males in health state 1 (Panel A) and in health state 4 (Panel B) at retirement (age 65). Both panels are based on 200,000 simulations.

	Age							
	65	70	75	80	85	90	95	All
Panel A: Health State 1 at retirement								
Mean	1.6	2.1	2.4	2.8	3.2	3.8	4.6	2.5
Std	2.9	4.8	6.5	8.3	10.2	13.0	16.1	7.4
90^{th} percentile	3.4	4.5	5.1	5.7	6.5	7.7	8.9	5.1
95^{th} percentile	6.8	8.6	10.0	11.7	13.1	15.8	19.6	10.1
99^{th} percentile	15.0	21.5	25.9	32.5	40.6	52.3	70.3	40.6
Maximum	63.9	259.3	430.0	374.5	383.3	507.5	425.2	613.5
Number of observations	200000	187131	161430	123471	78475	37683	11105	3579031
Panel B: Health State 4 at retirement								
Mean	2.9	3.2	3.4	3.8	4.3	5.4	6.8	3.4
Std	5.7	7.5	8.8	11.0	13.1	17.8	22.4	9.0
90^{th} percentile	6.5	6.8	7.1	7.5	8.4	10.3	11.7	7.0
95^{th} percentile	9.9	11.5	12.9	14.9	18.4	24.9	34.9	12.6
99^{th} percentile	30.4	34.9	39.6	48.3	60.8	78.7	129.9	55.8
Maximum	150.2	310.1	416.0	531.9	491.1	498.4	263.4	742.5
Number of observations	200000	161560	115121	68532	31022	9171	1179	2494449

4 Simulation results

This section reports the simulation results for single males[15] with pre-annuitization retirement wealth of $500,000 using 200,000 simulated sample paths of return on stock, health dynamics, and HCE. We study optimal annuitization, wealth profiles, and consumption and investment strategies in retirement. We also quantify the welfare gain with the access to a LCA market compared to the absence of such a market. To reduce complexity, only the cases for health states 1 and 4 are shown.

4.1 Annuitization choice

4.1.1 The effect of product features and pricing of life care annuities

This subsection discusses the effect of having access to a LCA on optimal annuitization choice in the presence of HCE, disentangling the impact from product feature and pricing. Given optimal post-retirement consumption and portfolio allocation strategies, Figure 1 compares optimal annuiti-

[15] Results for females are discussed in Section 5.

zation levels in the benchmark setting between standard life annuities and LCAs. With standard life annuities available only, individuals in health state 1 at retirement achieve the optimum (in terms of the highest certainty equivalent consumption) by annuitizing 57% of their pre-annuitization retirement wealth ($500,000) (blue dashed-dotted line in the upper panel). When LCAs are available at retirement, this share increases by as much as 12 percentage points (blue solid line), resulting in a 21% increase in annuitized wealth. Compared with life annuities, LCAs have an additional insurance feature (i.e., LTC coverage) and an advantage in pricing, both of which increase the annuitization level. To quantify the effect of each driver separately, the case where LCAs are fairly priced (i.e., conditional on annuitants' own health status) is also shown (green solid line with plus sign marker). For individuals in health state 1, the life annuities are fairly priced in the benchmark setting. Therefore, half of the increase in annuitization is attributed to the insurance feature of LCAs, amounting to six percentage points, with the remainder due to the price advantage from reduced adverse selection.

Turning to health state 4, the lower panel of Figure 1 shows an equal increase in annuitization as a result of the availability of a LCA, from 41% to 53% (or a 29% increase in annuitized wealth). However, the sources of this increase are different this time. When both standard life annuities (green solid line with dot marker) and LCAs (green solid line with plus sign marker) are fairly priced, the insurance feature of LCAs is estimated to increase the optimal annuitization level by 11 percentage points, with only one percentage point due to more favorable pricing. This finding supports the claim by Brown and Warshawsky (2013) that these individuals in relatively poor health benefit from LCAs mainly because of the LTC coverage they provide.

4.1.2 The effect of uncertain out-of-pocket healthcare expenditure

In this subsection we study the effect of uncertain HCE on optimal annuitization choice by comparing the cases with and without HCE, as shown in Figure 1. According to the upper panel, it is optimal for individuals in health state 1 at retirement to annuitize 66% of their retirement wealth, given that they have access to only standard life annuities and do not face any HCE (because they have full health insurance coverage). In the situation where individuals face HCE (with no or incomplete health insurance coverage), the optimal annuitization level drops to 57%. For individuals who have access to a LCA, the level of annuitization is 64% in the absence of risky HCE, but increases to 69% when HCE is present. The lower panel of Figure 1 again refers to individuals in health state 4 at retirement. For this group, optimal levels of annuitization are generally lower than for those in health state 1, partly because they need more liquidity to meet HCE.[16] For those

[16]It is also due to that they anticipate their lower life expectancies.

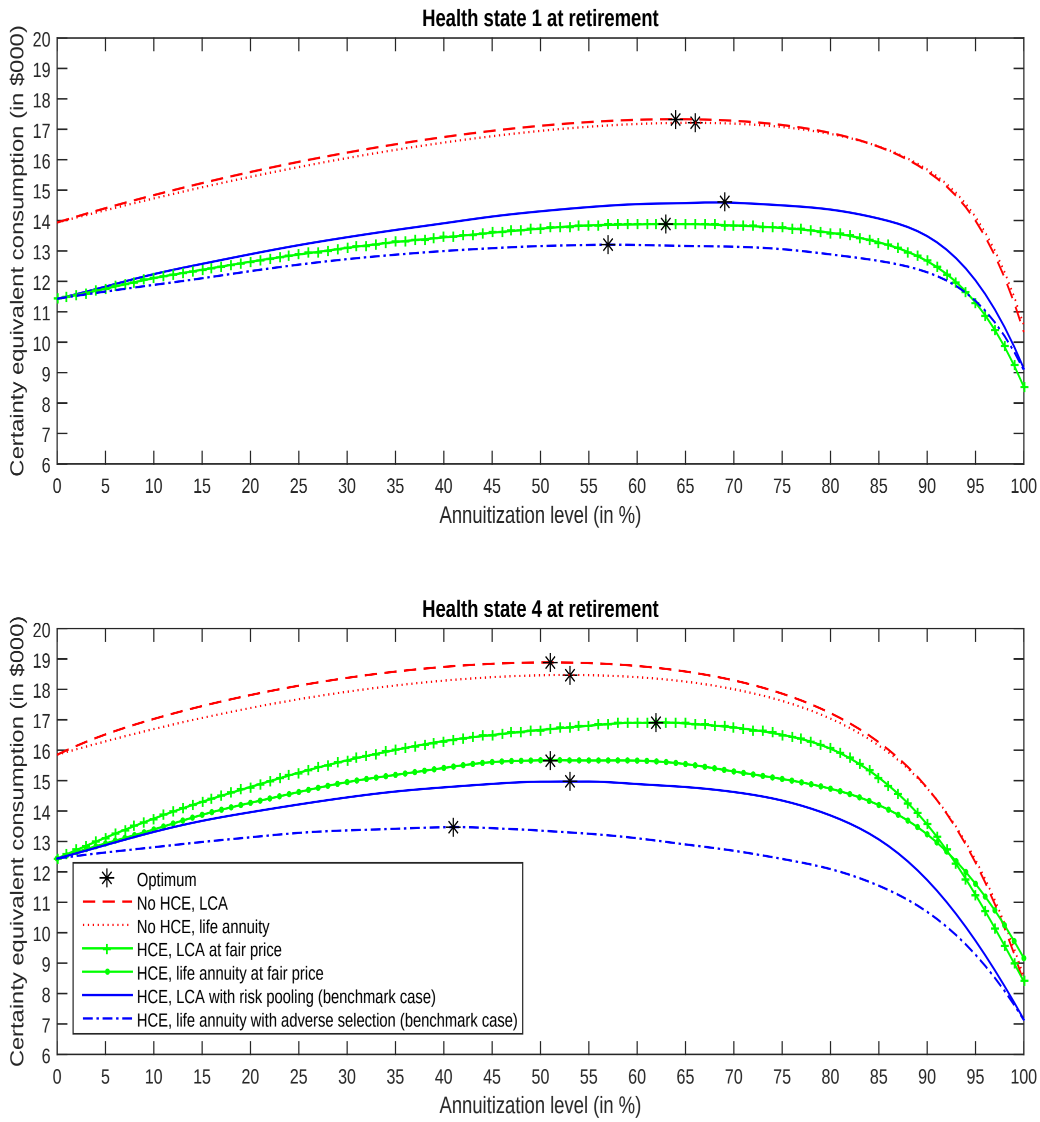

Figure 1: Optimal annuitization level. Notes: The figure compares optimal annuitization levels in terms of certainty equivalent consumption of a male retiring at age with HCE and without HCE, conditional on access to a standard life annuity or a LCA and under different pricing assumptions. The upper panel refers to health state 1 at retirement and the lower panel refers to health state 4. The cases without HCE incorporate health status uncertainty and liftime uncertainty. Fair prices are calculated using the health transition probabilities conditional on the health state of an annuitant at retirement. For individuals in health state 1 at retirement, a life annuity in the benchmark case is at the fair price for them. The results are based on the average of 200,000 simulations.

having access to standard life annuities only, the optimal level of annuitization drops to 53% (no risky HCE) and then to 41% (with risky HCE) respectively. When a LCA is available, optimal annuitization levels are 51% and 53% respectively. Therefore, the presence of uncertain HCE reduces the demand for standard life annuities, while it makes LCAs more appealing.[17]

These results are consistent with Sinclair and Smetters (2004), Turra and Mitchell (2008), Ameriks et al. (2011), Davidoff et al. (2005) and Peijnenburg et al. (2015) but contrast with Pang and Warshawsky (2010). Pang and Warshawsky (2010), using the HCE model of De Nardi et al. (2010), find that annuity demand is higher in the presence of uncertain HCE than in their absence. The reason is that the mortality credits provided by life annuities increase with age, helping individuals hedge against HCE that increases with age as well. In our model, the cost of dying is accounted for, which affects the hedging effectiveness and thus life annuity demand. For an individual who lives long (dies young) in retirement, the present value of life annuities is high (low), and the present value of the end-of-life HCE at retirement is low (high). This makes life annuities no longer an appropriate hedging instrument for HCE.

An important feature of the model is whether there is a probability of sizeable HCE early in retirement. According to Peijnenburg et al. (2015), this causes a reduction in annuity demand, because of the large loss in expected lifetime utility due to low consumption in the years following early-retirement high HCE. By way of contrast, high HCE early in retirement may occur due to the cost of dying. An individual who dies early in retirement after having occurred high end-of-life HCE obtains a low return from a life annuity, resulting in low bequeathable wealth. Therefore, sizeable end-of-life HCE early in retirement reduces the demand for life annuities.

Table 5: Optimal annuity income ($000 as of 2014)

Scenario	Standard life annuity	LCA in health state			
		1 − 4	5 − 10		
		Basic only	Basic	Top-up (L=2)	Total
Without HCE					
Health state 1 at retirement	24.342	23.021	23.021	46.042	69.063
Health state 4 at retirement	19.547	23.021	23.021	36.690	55.035
With HCE					
Health state 1 at retirement	21.023	24.820	24.820	49.639	74.459
Health state 4 at retirement	15.121	19.064	19.064	38.129	57.193

Table 5 displays the optimal annuity incomes, distinguishing between health states 1 − 4 and 5

[17]In the absence of HCE, a LCA is 'riskier' than a standard life annuity because it leads to uncertain (health-dependent) income.

– 10 in the case of a LCA. Comparing these income levels with Panel A of Table 2, males in health state 1 at retirement are able to pay their annual HCE in over 95% and 90% of cases using their top-up income from LCAs when they are in health states 5 – 7 and 8 – 10 respectively. For those in health state 4 at retirement, the numbers are slightly less than 95% and 90% due to their lower annuitization levels.

4.2 Wealth and consumption profiles

This section presents optimal wealth and consumption paths. Given pre-annuitization wealth of \$500,000, Table 6 shows the liquid wealth at retirement immediately after annuitization (W_0), juxtaposing a standard life annuity with a LCA. In the absence of HCE, an individual in health state 1 at retirement still disposes of \$170,000 and \$180,000 respectively. These values increase to \$235,000 and \$245,000 for an individual in health state 4, as a result of their lower annuitization levels. In the presence of HCE, the divergence between a life annuity and a LCA becomes more marked. Given access to a LCA, an individual in health state 1 can optimally reduce his liquid wealth to \$155,000 (rather than \$215,000 with only access to a life annuity). This \$60,000 difference appears among individuals in health state 4 at retirement as well, though they have to hold higher levels of liquid wealth than those in health state 1.

Table 6: Liquid wealth after annuitization (\$000 as of 2014)

Scenario	Standard life annuity	LCA
Without HCE		
Health state 1 at retirement	170	180
Health state 4 at retirement	235	245
With HCE		
Health state 1 at retirement	215	155
Health state 4 at retirement	295	235

Turning to the dynamics of liquid wealth in retirement, the upper panel of Figure 2 shows optimal paths of liquid wealth (W_t) for males in health state 1 at retirement.[18] When neither standard life annuities nor LCAs are available, individuals optimally decumulate their wealth from the start, however at a much slower rate in the presence of HCE (solid blue line). When a life annuity or a LCA is available, individuals start from a lower level of liquid wealth, depending on their optimal annuitization level. If protected from HCE, they decrease their wealth at a very slow rate after age 77 for both the cases of access to a life annuity and a LCA. This reflects the fact that without HCE, wealth levels depend on the bequest motive. However, in the presence of HCE,

[18] Results for health state 4 are similar and are available upon request.

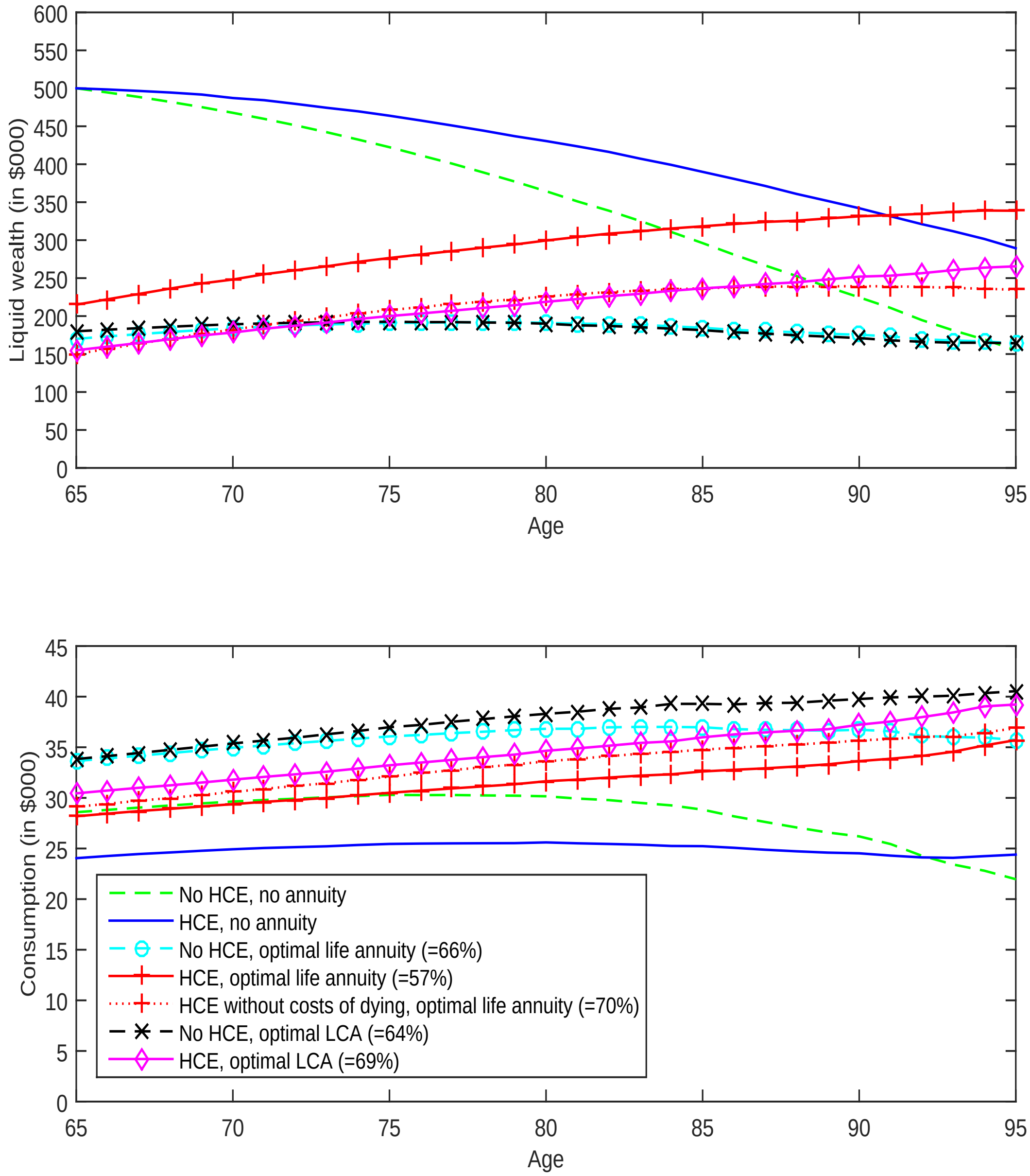

Figure 2: Optimal wealth and consumption paths in retirement. Notes: The figure displays optimal paths of liquid wealth (upper panel) and consumption (lower panel) of a male retiring at age 65 with and without HCE, conditional on availability of a standard life annuity or a LCA. The results refer to health state 1 at retirement and are based on the average of 200,000 simulations.

they need to continuously accumulate their wealth even when benefiting from the top-up payment of the LCA. More importantly, individuals consistently retain a lower level of wealth with a LCA (pink solid line with diamond marker) than with a standard life annuity (red solid line with plus sign marker), reducing their needs for precautionary savings.

The different wealth paths have their mirror images in the consumption paths (lower panel of Figure 2). Therefore, retirees have the lowest level of consumption when they face HCE and no annuity is available at all. In contrast, availability of a LCA combined with the absence of HCE leads to the highest level of consumption throughout. Last but not least, retirees facing HCE are able to consistently enjoy a higher level of consumption with the support of a LCA than with access to a life annuity.

4.3 Investment strategy

To describe an optimal investment strategy, one needs to distinguish two types of shock. One type affects the current stock of wealth, e.g., exogenous HCE to be paid out of liquid wealth. The other type refers to the investment risk, affecting the rate of return on investment (and hence terminal wealth) without changing the value of existing wealth. To understand the effect of a shock of the first type on the investment strategy, we first solve an optimal asset allocation problem in a simple one period model (see Appendix B). We find that when the uncertainty arising from this shock is small relative to the investment risk (the shock of type two), the optimal share of wealth invested in the risky asset decreases with the uncertainty of the shock. This is because an individual with CRRA utility faces a larger total risk while his or her risk appetite remains the same. However, when the uncertainty arising from the shock is large relative to the investment risk, the opposite reaction is predicted. This is due to the fact that the marginal increase in the uncertainty of terminal wealth (including investment returns) arising from the investment risk is low (since the two shocks are independent), whereas the risky asset has the benefit of a higher expected return.

Turning to the life-cycle model, the upper panel of Figure 3 shows the optimal proportion of total wealth (after-consumption wealth plus the expected present value of income from standard life annuities or LCAs) invested in the risky asset. These proportions do not vary much with age because of the property of the CRRA utility function and time-independent return parameters.[19] However, we find that individuals with access to a life annuity invest more in the risky asset in

[19] In fact, the proportion invested in the risky asset is constant across age for cases without HCE. The fluctuation for cases with HCE is due to simulation errors, given an extra source of uncertainty. Since that we model HCE using a mixture distribution which incorporates end-of-life HCE, we are not able to (numerically) evaluate expectations on HCE using the quadrature method (which is applied in ealier studies, e.g., French and Jones, 2004). Instead, these expectations are computed by Monte Carlo integration (see Appendix A), which leads to larger simulation errors compared with the quadrature method.

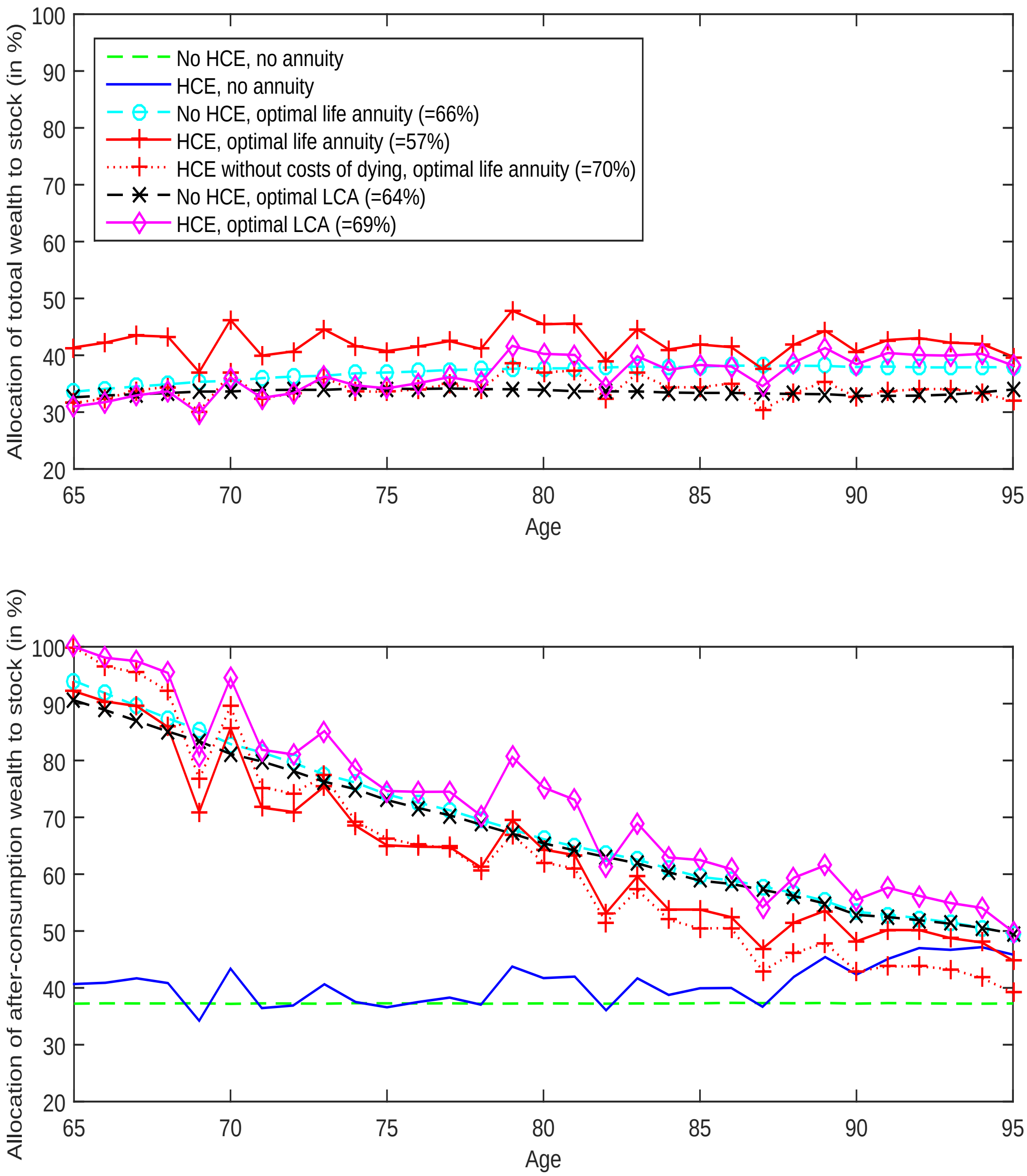

Figure 3: Optimal wealth allocations in retirement. Notes: The figure displays optimal allocation of total wealth (upper panel) and of (liquid) after-consumption wealth (lower panel) a male retiring at age 65 with and without HCE, conditional on availability of a standard life annuity or a LCA. Total wealth includes (liquid) after-consumption wealth and the annuitized wealth (i.e., expected present value of income from life annuities or LCAs). The results refer to health state 1 at retirement and are based on the average of 200,000 simulations.

the presence of HCE (incorporating end-of-life HCE) than in the absence. This is because the HCE risk is large relative to the investment risk.[20] When a LCA is available, the (partial) LTC coverage provided by LCAs reduces the uncertainty of HCE, shifting individuals to a more risk-averse portfolio than with access to only a standard life annuity.

The lower panel of Figure 3 shows the optimal proportion of (liquid) after-consumption wealth invested in the risky asset. Since this is a decision at the margin, differences are more marked. When there are no annuity products available (and hence after-consumption wealth is same as the total wealth), individuals optimally retain a stable level of investment in the risky asset. When annuity products are available, the annuitized wealth decreases as individuals receive annuity payments, and thus they generally adjust down the share of risky investment when they age. With standard life annuities, the presence of HCE deceases the share of the risky asset held by individuals. Moreover, having access to LCAs reverses this effect by allowing individuals to hold a less risk-averse portfolio than otherwise.

4.4 The effect of end-of-life healthcare expenditure

Earlier models disregarded end-of-life HCE. To see the differences this makes, the present model is solved without end-of-life HCE, using only the parameters for survivors in Table 3. We find that individuals in health state 1 (4) at retirement optimally annuitize 70% (53%) under standard life annuities, rather than 66% (53%) without HCE and 57% (41%) with end-of-life HCE in Section 4.1. Thus, the presence of HCE without the cost of dying makes standard life annuities as appealing as, if not more appealing than, the case without HCE, a result that is consistent with Pang and Warshawsky (2010) and Pashchenko (2013). However, end-of-life HCE substantially reduces the demand for standard life annuities.

Moreover, individuals without end-of-life HCE still need to hold liquid wealth and consume less than in the complete absence of HCE, but to a much lesser extent than in the case with end-of-life HCE. They also invest a smaller share of their total wealth in the risky asset not only compared to the case with end-of-life HCE, but also compared to the case in the absence of HCE (see Figure 3), which is consistent with the existing literature. These results highlight that incorporating end-of-life HCE significantly decreases both the optimal annuitization level and consumption, and increases both the precautionary savings and the share of the risky asset held by individuals.

[20]Section 4.4 discusses the optimal investment strategy without end-of-life HCE. In this case, the HCE risk is small relative to the investment risk. Therefore, individuals with the same risk appetite invest less in the risky asset in the presence of HCE than in the absence of HCE, which is consistent with Feinstein and Lin (2006) and Pang and Warshawsky (2010).

4.5 Welfare analysis: willingness to pay for life care annuities

LCAs improve individuals' welfare, indicated by the difference between the optimal consumption paths with and without LCAs in Figure 2. Alternatively, the welfare gain can be quantified by willingness to pay for access to a LCA market. This can be done in two ways. First, we calculate the amount of pre-annuitization wealth required for an individual with access to a standard life annuity only to obtain the same level of expected lifetime utility given access to a LCA at retirement and $500,000 pre-annuitization wealth. We find that males who are only able to purchase a standard life annuity are estimated to need around $550,000 ($550,000 if in health state 1 and $554,000 if in health state 4), corresponding to a welfare gain of 10% of pre-annuitization wealth at retirement.

Second, we quantify the amount of loading that an individual is willing to pay for a LCA, provided that the level of expected lifetime utility remains same as the case with only access to a standard life annuity. We find that males in health states 1 and 4 at retirement are willing to pay around 16% and 21% respectively, in excess of the actuarially fair premium for having access to a LCA. This suggests not only a substantial welfare improvement but also the margin available to insurers for covering expenses as well as profit.

5 Sensitivity analysis

In this section, we conduct sensitivity tests on whether and the extent to which our results are affected by the values of financial parameters and preference parameters.

5.1 Financial parameters

Panel A of Table 7 reports the influences of alternative values of rate of return on stocks, the health-contingent payment, the wealth floor and the pre-annuitization wealth on optimal annuitization choice. As we expected, there is a negative relationship between the rate of return on stocks and the optimal annuitization level. When the rate of return on stocks is lower (higher), individuals annuitize slightly more (less) of their pre-annuitization wealth in all cases. Yet, the demand for LCAs is consistently higher than for conventional life annuities under alternative assumptions for stock returns.

An important feature of the LCA is its top-up payment which is set as twice of the basic payment in the benchmark model ($L = 2$). As individuals in different health states may prefer different levels of top-up payment, we allow L to be varied to find the optimal product design (i.e., the value of L) for each health state and the consequent annuitization level. We find that the optimal values of L are around 2 and 3.5 for individuals in health states 1 and 4 at retirement respectively. Hence the value of L in the benchmark model is optimal for health state 1. For those in health state 4, the optimal annuitization level is only two percentage points below the benchmark model, amounting to 51% of the pre-annuitization wealth. This will lead to a top-up payment about \$60,000 per annum in the case of needing aged care, covering over 95% (90%) of annual HCE for health states 5-7 (8-10).

The existence of the wealth floor provides a form of social security for those who are not able to afford their HCE. We test whether more or less generous social security will affect the attractiveness of LCAs. When the wealth floor is set at \$15,000, LCAs remain a superior product than standard life annuities. However, when the wealth floor is set at \$30,000, more generous social security crowds out the demand for private LTC coverage for individuals in health state 1, while those in health state 4 still annuitize more with access to a LCA.

We also derive optimal annuitization choice for different levels of pre-annuitization wealth. Results show that a LCA is more appealing to a standard life annuity for individuals in the middle of the wealth distribution with pre-annuitization wealth from \$350,000 and \$1,000,000. This is because the poor (with pre-annuitization wealth of \$200,000 or lower) are unlikely to insure themselves against their HCE even with LCA and hence the wealth floor crowds out the demand for private LTC insurance. For the rich (\$2,000,000 or more), the top-up payment is very high in dollar amounts at moderate annuitization levels, leading to over insurance and reducing the attractiveness of a LCA compared with buying standard life annuity and self insuring against HCE.

It is also interesting to have a look at the demand for standard life annuity at various levels of wealth. With access to only standard life annuities, the optimal annuitization level for both health states shows a U-shape pattern as the level of pre-annuitized wealth increases. For example, individuals in health state 1 convert 68% of their wealth into life annuities when they have \$200,000, only 50% at \$750,000, but 65% at \$2,000,000. This is in contrast to Peijnenburg et al. (2015) who show that optimal annuitization level increases with wealth for individuals facing uncertain HCE. With the existence of a government subsidy and HCE risk, there are two forces that influence the annuitization level. On one hand, the need for precautionary savings to smooth consumption imposes a pressure to reduce the demand for life annuities. On the other hand, if one is almost certain that he will not be able to afford his HCE, the optimal strategy is annuitizing most of his wealth and relying on the subsidy when his HCE exceeds the annuity income.[21] Therefore, when the level of pre-annuitization wealth is low and the chance of failing to pay HCE is high, it is optimal to annuitize the majority of wealth because the second force is dominant. As the level of pre-annuitization wealth increases, the chance of failing to pay HCE becomes lower and precautionary

[21] This is better than holding liquid wealth, being impoverished by HCE and relying on the subsidy for the remaining life.

motives start to dominate, reducing the optimal annuitization level. Further increasing the level of pre-annuitization wealth leads to a recovery in the demand for life annuity to utilize the mortality credits, because a smaller proportion of wealth is needed for precautionary motives for the wealthy.

Table 7: Sensitivity analysis

The table compares optimal annuitization levels for alternative values of preference parameters (Panel A) and financial parameters (Panel B) against the benchmark model.

	Health State 1		Health State 4	
	Standard life annuity	LCA	Standard life annuity	LCA
Benchmark model	57	69	41	53
Panel A: Financial parameters				
Rate of return on stock = 5%	66	72	43	59
Rate of return on stock = 8%	56	64	40	48
Health-contingent payment: optimal L^*		69		51
Wealth floor level = \$30K	68	66	48	55
Wealth floor level = \$15K	57	70	33	46
Pre-annuitization wealth				
\$200K	68	67	52	51
\$350K	64	67	45	52
\$750K	50	69	36	61
\$1M	52	72	30	59
\$2M	65	69	57	53
Panel B: Preference parameters				
Relative risk aversion = 3	39	42	11	22
Relative risk aversion = 7	52	70	40	47
Bequest parameter = 0.32	70	74	54	62
Time preference = 0.94	56	66	38	51
Time preference = 0.98	64	70	43	54
Marginal utility in health states 5–10				
20% lower	57	67	42	53
20% higher	57	69	41	53
Female (at $L = 1$)	52	58	32	36

*The optimal L values for health state 1 and 4 at retirement are around 2 and 3.5 respectively.

5.2 Preference Parameters

We study the influence of alternative values for relative risk aversion, the bequest strength parameter, the time preference discount factor and the state-dependent utility parameter on optimal annuitization choice, as well as the results for females in the benchmark case. Panel B of Table 7 summarizes the results in this section.

We test two alternative values of relative risk aversion, 3 and 7.[22] In terms of higher relative risk aversion (7), optimal annuitization levels are only slightly different from the benchmark model. The positive effect on the optimal annuitization level of having access to LCAs is unchanged. In terms of lower relative risk aversion (3), all optimal annuitization levels are lower than the benchmark model, due to the fact that stock is more attractive in this case. However, LCAs still significantly increase the optimal annuitization level for individuals in health state 4, compared with the standard life annuity case. The effect for individuals health state 1 is smaller than the baseline model.

As the existing literature has offered inconclusive evidence on the strength of bequest motives (Hurd, 1989; De Nardi, 2004; Kopczuk and Lupton, 2007), we test different values for the strength of bequests. We change the value from 0.17 in the benchmark model to 0.32, a value that is consistent with the parameter setting for bequest motives in Gomes and Michaelides (2005). At this level of bequest strength, an individual consumes around 30% of wealth in the last period, in contrast with 20% in the benchmark case. We find that having access to LCAs still increases the optimal annuitization level, though the impact for individuals in health state 1 is smaller.

We also test our time preference discount factor. When the value of the time preference factor is lower (0.94), optimal annuitization levels are slightly smaller than those in the benchmark model. This is because individuals in the model are more impatient over future consumption and hence value both annuity products less. This is consistent with higher optimal annuitization levels when the value of the time preference factor is higher (0.98). Under both cases, the optimal annuitization levels are significantly higher when LCAs are available at retirement.

The benchmark model assumes the marginal utility of consumption is not state dependent, because there is no consensus about the parameter value for state-dependent utility in the literature and it is not even clear whether the marginal utility of consumption is higher or lower in poor health states (Viscusi and Evans, 1990; Finkelstein et al., 2009). Sims et al. (2008) find that depending on the number and combination of ADLs for which individuals need help, the utility of living in disability is between 76% and 89% of the utility of living in full health for the same number of years of life. Similarly, Finkelstein et al. (2013) find that the marginal utility of consumption is 10% – 25% lower for a one-standard deviation increase in the number of chronic diseases, relative to the marginal utility without any chronic disease. However, using data from a large survey Ameriks et al. (2015) show that the marginal utility of spending experienced by households needing long-term care is higher than those who are free of disability, justifying the conservative drawdown of wealth in retirement. Thus, we examine two state-dependent utility cases where the marginal

[22]It should be noted that we fixed $b^{(1-\gamma)}$ i.e., the ratio between marginal utility from bequest and marginal utility from consumption in the benchmark model, when we experiment different values of relative risk aversion.

utility of consumption is 20% higher and lower in health states 5–10. Results show little change in optimal annuitization levels.

As our analysis in Section 4 focuses on males, we also show the optimal annuitization choices in the benchmark case for females. The benchmark case for females has the same parameter values as for males, with an exception of the amount of the top-up payment (controlled by L). Section 3.6 sets $L = 2$ which is the optimal product design for males in health state 1. To be consistent, we set $L = 1$ in the benchmark case for females as this is found to be the optimal value for females in health state 1. We find that having access to LCAs still increases the optimal annuitization level, though the impact for females in the benchmark model is about half of that for males. A further analysis shows that the lower demand for embedded LTCI (shown by the lower optimal value of L) for females is due to the fact that the difference between the mean HCE in health states 5–7 and that in health states 8–10 is much larger for females than males.[23] As the LCA product in our model only has one level of top-up payment which does not distinguish health states 5–7 and 8–10, it suits the needs for males better than females. This makes the insurance feature of LCA less valuable for females, since one level of top-up payment cannot provide an effective hedge for their HCE with greatly different means in health states 5–7 and 8–10. Thus the optimal product design for females will require two top-up payments, of which one is paid when the insured is in health states 5–7 and the other is paid when in health states 8–10.

6 Conclusions

This paper studies how availability of a health-contingent income product (a so-called life care annuity, LCA) affects optimal decisions at and during retirement as well as individual welfare in comparison with a conventional life annuity. We develop a life-cycle model and derive optimal annuitization choice at retirement and, consumption decisions as well as portfolio allocation during retirement for single retirees with a bequest motive. The model incorporates capital market risk, differential mortality risk, uncertain health status, and uncertain out-of-pocket HCE, much of which occurs as end-of-life HCE.

We find that individuals increase their level of annuitization by around 12 percentage points when they are able to purchase a LCA, rather than being limited to a standard life annuity. For the relatively healthy retirees, this increment is attributable in equal measure to the insurance feature and to the pricing advantage of LCAs; for the relatively unhealthy ones, the insurance feature is dominant. With the support of LCAs, individuals are able to consume more throughout their

[23]When individuals are alive in the next period, the mean HCE for females in health states 8–10 is almost twice the mean HCE in health states 5–7, while it is only 31% higher for males.

retirement phase and invest a higher proportion of their liquid wealth in a risky asset. However, when differences in annuitization levels are controlled for, they tend to hold a more risk-averse portfolio for total wealth than with life annuities. Access to LCAs is estimated to afford a substantial welfare gain as well, reflected by a willingness to pay as much as 10% of their wealth at retirement, or 16% to 21% in terms of a loading on an actuarially fair LCA premium, depending on their health status.

Importantly, uncertain out-of-pocket HCE makes LCAs more appealing but reduces the demand for standard life annuities, causing retirees to generate substantial precautionary savings and hence sacrifice consumption. Uncertain end-of-life HCE in particular makes them invest a higher proportion of their total wealth (including annuitized wealth) into the risky asset.

This research has important practical implications, shedding light on the potential of LCAs to provide at least partial protection against HCE and the cost of long-term care. Combining conventional life annuities with coverage of HCE during retirement holds the promise of substantially improving the welfare of retirees while boosting the attractiveness of retirement income products.

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Appendix A The numerical solution method

The numerical solution method starts with discretizing the continuous state variables $\{W_t, Y_t\}$. Following Carroll (2006), N endogenous grid points are constructed for after-consumption wealth, resulting in $\overline{W}_t = \overline{W}_t^n$ for $n = 1, ..., N$. After some numerical experiments, we find $N = 150$ is large enough for stable and accurate results. Endogenous grid points for after-consumption wealth (rather than grid points for liquid wealth at the beginning of each period) allows us to solve for optimal consumption analytically. We discretize annuity income for annuitization levels $x = 1\%, 2\%, ..., 100\%$, leading to a grid for annuity income $Y_t = Y_t^x$. Solutions are also conditional on the discrete state variable H_t across all alive health states i.e., $H_t = i$ for $i = 1, ..., 10$. This results in wealth-health-income combinations, $\{\overline{W}_t = \overline{W}_t^n, H_t = i, Y_t = Y_t^x\}$.

The first-order condition[24] for the optimal portfolio allocation and consumption are given by

$$0 = \sum_{j=1}^{10} \pi_t(H_t, j)\mathbb{E}_t\left\{\frac{\partial V_{t+1}}{\partial W_{t+1}} R_{t+1}^e\right\} + \pi_t(H_t, 11)b^{1-\gamma}\mathbb{E}_t\left\{W_{t+1}^{-\gamma} R_{t+1}^e\right\} \tag{19}$$

$$C_t^{*-\gamma} = \sum_{j=1}^{10} \pi_t(H_t, j)\beta\mathbb{E}_t\left\{\frac{\partial V_{t+1}}{\partial W_{t+1}} R_{t+1}^*\right\} + \pi_t(H_t, 11)\beta b^{1-\gamma}\mathbb{E}_t\left\{W_{t+1}^{-\gamma} R_{t+1}^*\right\} \tag{20}$$

for $0 \le t < T - 1$, where $R_{t+1}^* = R^f + \alpha_t^* R_{t+1}^e$ and

$$\frac{\partial V_{t+1}}{\partial W_{t+1}} = \begin{cases} C_{t+1}^{*\,-\gamma}, & \text{if } W_{t+1} + Y^A - M_{t+1} \ge \underline{W}; \\ 0, & \text{if } W_{t+1} + Y^A - M_{t+1} < \underline{W}. \end{cases}$$

Equation (19) defines an implicit solution for the optimal portfolio allocation α_t^*, while equation (20) can be used to solve optimal consumption C_t^*.

Due to the property of a CRRA utility function, the portfolio allocation does not depend on consumption (but the reverse is not true). With backward induction, optimal portfolio allocation is solved first through an iterative root finding algorithm. We then use the optimal portfolio allocation to solve for optimal consumption. For each period t, optimal values are derived for all combinations $(\{\overline{W}_t = \overline{W}_t^n, H_t = i, Y_t = Y_t^x\})$ and over the distributions of random variables (portfolio returns (R_{t+1}) and HCE (M_{t+1})).

To evaluate the conditional expectations required for solving optimal asset allocation and consumption, we simulate 10,000 trajectories of portfolio returns and HCE for all periods. Portfolio returns are simulated using the Monte Carlo method. As HCE depends on the terminal health state, we generate Monte Carlo simulations of the terminal health states, conditional on the health

[24] As the value function is concave, the first-order conditions are sufficient for an optimum.

state at the beginning of each period. Next, we simulate the HCE trajectories based on the paths of health states using the importance sampling technique (Rubinstein and Kroese, 2011) to better approximate the tail of the HCE distributions. The importance distributions are uniform over $(0, M_{max})$. The value of M_{max} should be large enough to represent extremely high HCE. Conversely, unrealistically large M_{max} would reduce the effectiveness of importance sampling due to a loss of observations over a realistic domain. For each gender, M_{max} is set to twice of the observed maximum HCE after some experimentation. Increasing M_{max} further is found not to change the results. Monte Carlo integration is finally used to compute the expectations, taking into account the weights of trajectories derived from importance sampling.

Appendix B A simple model of portfolio choice with uncertain out-of-pocket healthcare expenditure

To better understand the impact of uncertain out-of-pocket HCE on portfolio choice, we study a simple one-period model. The agent starts with wealth W_0 at the beginning of the period, which becomes $W_0 R_{hc}$ after paying out-of-pocket HCE, where R_{hc} is assumed to follow a lognormal distribution with parameters $\mu_{hc} \in (0,1)$ and σ_{hc}^2. Thus, HCE can be viewed as an investment with negative and instantaneous expected rate of return.

Table 8: Optimal portfolio choice with different level of healthcare expenditure uncertainty

The table compares the optimal portfolio choices of an agent with CRRA utility and relative risk aversion equal to 5, when the standard deviation of HCE (σ_{hc}) varies from zero to five times the standard deviation of stock returns (σ_s).

σ_{hc}/σ_s	0	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5
α^*	0.373	0.371	0.369	0.367	0.364	0.362	0.363	0.371	0.387	0.413	0.448

The agent then optimally allocates $W_0 R_{hc}$ between a riskless bond and a risky stock (whose share is denoted by α), maximizing the utility from consumption C_1 at the end of the period. $R^f = 1 + r_f$ denotes real gross return on the bond and r_f is the real rate of return. The real gross return of the stock, denoted by $\widetilde{R}$, follows a lognormal distribution with parameters μ_S and σ_S, giving rise to excess return $R^e = \widetilde{R} - R^f$. The agent has a CRRA utility, consistent with the life-cycle model. For simplicity, there is no time preference and uncertainty with respect to health and survival, all of which do not affect the results in this section.

Therefore, the objective function is given by

$$\max_{\{\alpha\}} \mathbb{E}_0 \left[u(C_1)\right]$$

where $u(C) = \frac{C^{1-\gamma}}{1-\gamma}$ and γ denotes relative risk aversion, with the constraints

$$W_1 = W_0 \cdot R_{hc} \cdot (R^f + \alpha R^e),$$
$$C_1 \leq W_1.$$

It is straightforward to show that the optimal portfolio choice α^* satisfies that following equation

$$0 = \mathbb{E}_0[R_{hc}^{1-\gamma} \cdot (R^f + \alpha^* R^e)^{\gamma} \cdot R^e].$$

Table 8 summarizes the values of α^* for $\gamma = 5$, with σ_{hc} ranging from zero to $5\sigma_s$.[25] A zero standard deviation of HCE (and a zero mean) is equivalent to the optimal portfolio choice without HCE. Therefore, the α^* values show that uncertain HCE makes the agent invest in a more risk-averse portfolio compared with the absence of HCE, as long as $\sigma_{hc} < 4\sigma_s$. If $\sigma_{hc} > 4\sigma_s$, however, the agent invests in a less risk-averse portfolio.

[25] The value of μ_{hc} can be any number from 0 to 1 and does not affect the results.