Closing Down the Shop: Optimal Health and Wealth Dynamics near the End of Life*

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Abstract

The observed health decline, and increase in mortality risk exposure near the end of life coincide with less curative (e.g. hospital stay, doctor visits), and more comfort (e.g. nursing home) care, which accelerate the fall in wealth. We investigate whether these dynamics jointly result from a *closing down the shop* decision i.e. a depletion of health and wealth is optimally selected (and eventually accelerated), leading to states characterized by indifference between life, and death. Towards that aim, we expand, structurally estimate, and simulate a life cycle model of financial, and health expenses with endogenous mortality exposure (Hugonnier et al., 2013). Under economically plausible, and statistically verified conditions, we find that, unless sufficiently rich and healthy, agents will optimally select expected depletion of their health capital, and associated increase in death likelihood. Moreover, we identify a wealth and health locus below which agents *accelerate* their health depletion. Importantly, wealth is also expected to decline such that all surviving agents eventually enter the closing down phase.

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1 Introduction

Health recedes rapidly in old age.[1] Since health is a significant predictor of major health onsets,[2] and of mortality,[3] exposure to death risk also increases sharply.[4] Falling health is further accompanied by both an increase,[5] and a change in composition in health expenditures. Indeed, agents approaching the end of life substitute away from curative care (e.g. doctor visits, and short-term hospital stays) in favor of more comfort care, such as long-term and nursing care.[6] These expenses share attributes with luxury goods consumption, and are not covered by Medicare.[7] Consequently, out-of-pocket expenses increase, and wealth falls sharply in old age.[8]

The objective of this paper is to assess the optimality of these joint end-of-life health and wealth dynamics. More precisely, we look at conditions under which agents choose to *close down the shop*, i.e. health is optimally depleted towards a region associated with high mortality risk, and indifference between life and death. We further identify threshold effects whereby the depletion of the health capital is initially slowed down (but not reversed), before being *accelerated*. These conditions are associated with lower wealth, such that richer individuals delay the health depletion. However, we show that under reasonable assumptions, wealth depletion is *also* optimally selected, so that agents eventually enter the closing-down phase.

[1] For example, the percentage of people reporting declining health doubles between ages 40-70, and again between ages 70–80. See Banks et al. (2015, Fig. 5, p. 12), as well as Heiss (2011, Fig. 2, p. 124). Similar age declines in self-reported health status are reported in Smith (2007, Fig. 1, p. 740), Van Kippersluis et al. (2009, Figs, 2, 3, pp. 822–826), and Case and Deaton (2005, Fig. 6.1, p. 186), who highlight faster deterioration at lower income quartiles.

[2] Smith (2007, Tabs. 1–3, pp. 747–752) .

[3] For example, Benjamins et al. (2004, Tab. 4, p. 1303) report a doubling of mortality risk for those reporting poor compared to excellent health. See also Heiss (2011); Hurd et al. (2001); Hurd and McGarry (2002) for additional evidence on health-dependent mortality.

[4] Survivorship drops by 88.4% between ages 70, and 95, compared to a drop of 18.5% between ages 45, and 70 (Arias, 2014, Tab. B, p. 4).

[5] Health spending by all payors doubles between age 70–90, reaching $25,000 by age 90, and increasing to over $43,000 in the last year of life alone (De Nardi et al., 2015b, p. 3, 23, and Tab. 12.b, p. 24). End-of-life spending has also been estimated to represent a quarter of lifetime health expenditures (Philipson et al., 2010).

[6] See De Nardi et al. (2015b, Tab. 2, p. 7, and Figs. 3, 4, p. 22, 26) for evidence.

[7] See De Nardi et al. (2015b); Marshall et al. (2010) for evidence and discussions.

[8] Wealth falls by 50% in the last three years of life (30% in the last year alone) for those agents who die, compared to only 2% for those who don't (French et al., 2006, Fig. 1, p. 7). De Nardi et al. (2015a, p. 9) shows that median assets for individuals aged 76 in 1996 and who survived to 2006 fell from $84,000 to $44,000. Falling wealth is also correlated with occurrence of severe illness late in life, and to a lesser extent to chronic diseases (Lee and Kim, 2008). Wealth profiles remain comparatively flat in the absence of significant changes in health status and/or family composition (Poterba et al., 2015).

This paper has theoretical, and empirical contributions. First, we build upon a rich life cycle model developed in Hugonnier, Pelgrin and St-Amour (2013) to identify end-of-life dynamics. This model encompasses a dynamic health investment setup with endogenous exposure to death risk, and exogenous sickness shocks. In addition to investing in their health, agents can purchase pecuniary insurance against health shocks at actuarially fair prices, and save in risky, and risk-less assets. Agents also earn income, part of which is fixed (e.g. social security), and part which is health-dependent, reflecting their physical ability to work. Finally, preferences are characterized by subsistence consumption, and by source-dependent risk aversion with respect to mortality, morbidity, and financial risks.

We rely on the approximate closed-form solutions of that model to characterize the optimal dynamics for health, and wealth capitals. Our main theoretical results first define the conditions under which the health and wealth that result from the agent's decisions are expected to decline over time. These conditions are economically plausible and relevant for agents approaching the end of life. Indeed, they require that the agent's sickness-adjusted depreciation of health capital, and his consumption propensity is high, whereas the health-dependent component of labor income is low.

Second, we identify a U-shaped locus in the health-wealth nexus such that agents who are insufficiently rich/healthy, optimally select expected depletion of their health stock. Consequently, there exists a threshold wealth level below which all agents expect a health depletion, regardless of their health status. We can also identify an accelerating locus below which health spending falls faster than health, so that agents initially slow down – yet do not reverse – the depletion, before choosing to *accelerate* the decline in health. Importantly, agents also optimally select to decumulate wealth. Combining these element entails that health is set on a downward spiral leading to drops in available resources, further cuts in health spending, and additional depletion of the health stock. Health thus eventually falls towards low levels that are associated with very high mortality risks, and indifference between life and death. We further show that it is possible to reduce the incidence of closing-down strategies through income redistribution policies. However, the normative basis for such intervention is unclear to the extent that depletion obtains optimally, and under complete markets, and that poverty is endogenously determined.

Our second contribution is empirical. Using HRS cross-sectional data, we rely on a trivariate econometric system composed of optimal health spending, risky asset holdings, and health-dependent income to structurally estimate the model over a population of relatively old agents (75.3 years in average). This exercise allows us to estimate the model's deep parameters, and evaluate the induced parameters that are used to partition the state space. Our empirical results confirm the model's realism, as well as the economic relevance of the predicted depletion strategies.

In particular, we show that all the required conditions are met for the existence of optimal closing-down strategies. Moreover, we show that the bulk of the population is located in the health depletion region, with a subset located in the accelerating zone. A simulation exercise yields predicted life cycles that are consistent with observed end-of-life dynamics. Indeed, the simulated optimal trajectories show rapid wealth, and health depletion. Net total human and financial capital is thus exhausted by the end of the expected lifespan, at which point indifference between life and death is predicted. These results therefore are pointing towards agents jointly selecting a short lifespan, and corresponding closing-down strategies that are consistent with remaining lifetime.

The main novelty of our approach concerns the *optimality* of the joint health and wealth depletion processes near the end of life. Despite strictly preferring to live, our agents optimally close down the shop; they simultaneously act in a manner that *results* in a short terminal horizon, *and* they select a depletion strategy that is consistent with this horizon. To our knowledge, this is the first attempt to rationalize end-of-life health and wealth dynamics, rather than model them as ex-post responses to an irreversible sequence of exogenous health and/or wealth declines. Indeed most life cycle models of asset decumulation in old age rely on exogenous health status, and expenses.[9] Otherwise endogenous expenses provide direct utility flows, but have no bearing on health status (e.g. De Nardi et al., 2010; Yogo, 2009). Regardless of whether they do, endogenous mortality is almost always abstracted from.[10] Consequently, longevity is exogenously set, and cannot be altered through the agent's health decisions; in the absence of bequest motives, the optimal strategy thus fully depletes wealth reserves at death.

The rest of this paper proceeds as follows. We summarize the theoretical model in Section 2. The depletion and accelerating regions are defined, and formally characterized

[9] See De Nardi et al. (2015a, 2009), or French and Jones (2011) for examples.

[10] Exceptions with endogenous mortality include Pelgrin and St-Amour (2016); Hugonnier et al. (2013); Blau and Gilleskie (2008); Hall and Jones (2007). However, none of these papers focus on end-of-life joint dynamics for health and wealth.

in Section 3. The empirical evaluation is performed in Section 4, with main results outlined in Section 5. We close the discussion with concluding remarks in Section 6.

2 Theoretical framework

Our analysis builds upon on the theoretical framework developed in Hugonnier et al. (2013) which we extend to analyze the joint dynamics of health and wealth. The main features of this model and its approximate solution are briefly reproduced here for completeness.

2.1 Economic environment

2.1.1 Health dynamics

The agent's health level is denoted by H_t and evolves according to a stochastic version of the Grossman (1972) demand-for-health model:

$$\mathrm{d}H_t = \left((I_t/H_t)^\alpha - \delta\right) H_t \mathrm{d}t - \phi H_t \mathrm{d}Q_{st}, \quad H_0 > 0. \tag{1}$$

In this equation $I_t \geq 0$ represents the agent's flow rate of health spending;[11] $\mathrm{d}Q_{st}$ is the increment of a Poisson process with intensity λ_{s0} that captures the arrival of health shocks,[12] and $(\delta, \alpha, \phi) \in \mathbb{R}_+ \times (0,1)^2$ are constants that represent the decay rate of health in the absence of shocks, the severity of health adjustment costs and the fraction of the agent's health that is lost upon the occurence of a shock.

The agent's health level endogenously determines both the instantaneous likelihood of the agent's death and his income. Specifically, we assume that the agent's flow rate of income is given by

$$Y(H_t) = y_0 + \beta H_t \tag{2}$$

[11] The restriction to postive health investment rates is standard and simply reflects the fact that the agent cannot monetize his own health.

[12] Hugonnier et al. (2013) consider a more general model in which the arrival intensity of health shocks is a decreasing function of the agent's health level. We focus on the case of a constant arrival intensity to facilitate the presentation but our results can be extended to cover this more general case.

where $(y_0, \beta) \in \mathbb{R}_+^2$ are constants that capture health-independent elements—such as Social Security revenue—and the sensitivity of the agent's income to his health status.[13] We also assume that the agent's time of death T_m is the first jump time of a Poisson process Q_{mt} whose arrival intensity is given by

$$\lim_{h \to 0} (1/h) P_t \left[t < T_m \leq t + h \right] = \lambda_{m0} + \lambda_{m1} H_{t-}^{-\xi_m} \equiv \lambda_m(H_{t-}) \tag{3}$$

for some constants $(\xi_m, \lambda_{m0}, \lambda_{m1}) \in \mathbb{R}_+^2$ where $H_{t-} = \lim_{s \uparrow t} H_s$. In this expression the first term represents the agent's endowed exposure to mortality risk while the second term captures the fact that the agent can influence the distribution of his lifetime by investing in his health. The fact that both ξ_m and λ_{m1} are nonnegative implies that an healthier agent can naturally expect to live longer.

2.1.2 Investment opportunity set

The agnet can continuously invest in three assets: A risk-free asset with constant rate of return r, and two risky asset. The first risky asset proxies for the stock market. Its prices is denoted by S_t and evolves according to

$$\frac{dS_t}{S_t} = r\mathrm{d}t + \sigma_S \left(\mathrm{d}Z_t + \theta_t \mathrm{d}t \right)$$

where $\mathrm{d}Z_t$ is the increment of a Brownian motion that captures market risk, and (σ_S, θ) are constants which represent, respectively, the volatility of market returns and the instantaneous remuneration that investors earn for exposure to market risk.

The second risky asset that the agent can invest in is an actuarially fair health insurance contract that pays one unit of the numéraire upon the occurence of a health shock. The instantaneous return that the agent earns by investing the amount $X_t \geq 0$ in this asset over the time interval $(t, t + \mathrm{d}t]$ is given by

$$X_t \left(\mathrm{d}Q_{st} - \lambda_{s0} \mathrm{d}t \right) \tag{4}$$

[13] Old-age male participation in the labor market has increased from 26% in 1995, to 35% in 2014, 60% of which involves full time work (Bosworth et al., 2016, Figs. II.1, and 2, pp. 7, and 9). See also Bureau of Labor Statistics (2008); Toossi (2015) for further evidence of increased old age participation in the labor force.

where the first term captures the payment that the agent receives from the insurer upon the occurence of a health shock and the second term captures the instantaneous insurance premium that he pays to the insurer.

Denote by Π_t, X_t and C_t the predictable processes that track the amount that the agnet invests in the stock market, the amount he invests in the insurance and the amount he consumes per unit of time. With this notation we have that the dynamic budget constraint that governs the evolution of the agent's wealth is

$$\mathrm{d}W_t = (rW_t + Y(H_t) - C_t - I_t)\mathrm{d}t + \Pi_t \sigma_S (\mathrm{d}Z_t + \theta \mathrm{d}t) + X_t (\mathrm{d}Q_{st} - \lambda_{s0}\mathrm{d}t). \tag{5}$$

Investment in the riskless asset and the stock market is unconstrained, but we naturally assume that the agent can neither consume negative amounts nor sell insurance by imposing a nonnegativity constraint on both C_t and X_t.

2.1.3 Preferences

To close the model it remains to specify the agent's preferences. Following Hugonnier et al. (2013) we assume that the continuation utility $U_t = U_t(C)$ to an alive agent of a lifetime consumption schedule C solves a recursive integral equation of the form

$$U_t = E_t \int_t^{T_m} \left(f(C_s, U_s) - \frac{\gamma \sigma_s^2}{2U_s} - \sum_{k=m}^{s} F_k (U_s, H_s, \Delta_k U_s) \right) \mathrm{d}s \tag{6}$$

where γ is a strictly postive constant that measures the agent's local risk aversion to financial market shocks, $\sigma_t = \sigma_t(U) = \mathrm{d}\langle U, Z \rangle / \mathrm{d}t$ measures the sensitivity of the continuation utility process to these shocks, and

$$\Delta_k U_t = 1_{\{\mathrm{d}Q_{kt} \neq 0\}} (U_t - U_{t-})$$

represents the predictable jump in continuation utility triggered by the occurence of a health shock ($k = s$) or the agent's death ($k = m$). In the above equation

$$f(C, U) = \frac{\rho U}{1 - 1/\varepsilon} \left(((C - a)/U)^{1-1/\varepsilon} - 1 \right) \tag{7}$$

is the Kreps-Porteus aggregator with elasticity of intertemporal substitution $\varepsilon > 0$, time preference rate $\rho > 0$, and subsistence consumption level $a \geq 0$; and the penalty terms below the sum are given by

$$F_k(U, H, \Delta U) = \lambda_k(H) \left[\frac{\Delta U}{U} + \frac{1 - (1 + \Delta U/U)^{1-\gamma_k}}{1 - \gamma_k} \right] U$$

for some constants $\gamma_s > 0$ and $\gamma_m \in [0, 1)$. This recursive preference specification allows to disentangle the agent's behavior toward intertemporal substitution from his attitude towards risk but, as explained in Hugonnier et al. (2013), it goes one step further than Duffie and Epstein (1992) by allowing to discriminate between the the various sources of risk present in the model. Specifically, our specification implies that the agent has constant relative risk aversion $\gamma > 0$ towards financial market risk, $\gamma_s \geq 0$ towards heath risk, and $\gamma_m \in [0, 1)$ toward mortality risk. Importantly, the restriction that $\gamma_m < 1$ guarantees that, irrespective of his attitude towards the other sources of risk, the agent prefers life over death.

2.1.4 The decision problem

The agent's decision problem consists in choosing a portfolio, consumption, health insurance and health investment strategy to maximize his lifetime utility. The indirect utility associated with this problem is defined by

$$V(W_t, H_t) = \sup_{(C,\Pi,X,I)} U_t(C)$$

subject to the dynamics of the health process (1) and the budget constraint (5) where $U_t(C)$ is the continuation utility process associated with the lifetime consumption and health investment plan (C, I) through (6).

In the absence of bequests, the continuation utility process defined by (6) vanishes at death.[14] As a result, we have $\Delta_m U_t = -U_t$ and it follows that the penalty associated

[14]This assumption is imposed for tractability and can be justified by noting that while bequest motives are potentially relevant in an endogenous mortality setting, panel data evidence suggests that their role in explaining the behavior of retired agents is debatable. In particular, Hurd (2002) finds no clear evidence of a bequest motive behind savings decisions and Hurd (1987) finds no differences in the saving behavior of the elderly who have children compared to those who don't.

with mortality risk satisfies

$$\frac{F_m(U_s, H_s, \Delta_m U_s)}{\lambda_m(H_s)U_s} = \frac{\gamma_m}{1-\gamma_m} \equiv \Phi \in [1, \infty).$$

Using this observation and integrating over the conditional distribution of the agent's time of death, Hugonnier et al. (2013) show that the agent's decision problem, which features incomplete markets and a endogenous random horizon, can be conveniently recast into an equivalent infinite horizon problem with endogenous discounting and complete markets. Specifically, they show that

$$V(W_t, H_t) = \sup_{(C,\Pi,X,I)} \overline{U}_t(C)$$

where the modified continuation utility process $\overline{U}_t = \overline{U}_t(C)$ solves the infinite horizon recursive integral equation given by

$$\overline{U}_t = E_t \int_t^\infty e^{-\int_t^s \lambda_m(H_k)(1+\Phi)\mathrm{d}k} \left(f(C_s, \overline{U}_s) - \frac{\gamma\sigma_s(\overline{U})^2}{2\overline{U}_s} - F_s(\overline{U}_s, H_s, \Delta_s \overline{U}_s) \right) \mathrm{d}s. \quad (8)$$

This formulation brings to light the channels through which the agent's health status enters his decision problem. First, health can be interpreted as a durable good that generates service flows through the income rate $Y(H_t)$. Second, health determines the instantaneous probability of morbidity shocks and the rate $\lambda_m(H_t)(1+\Phi)$ at which the agent discounts future consumption and continuation utilities.

Remark 1 (Health dependent preferences) One might also reasonably object that old agents are likely to be retired and thus do earn labor income. However, this objection is inconsequential for our purposes since the agent's decision problem is iso-morphic to one with health-dependent utility, and constant base income. This results follows by effecting the change of variable $\overline{C}_t = C_t + \beta H_t$ throughout the above equations, see (Hugonnier et al., 2013, Remark 3) for details.

2.2 Optimal dynamic policies

The fact that the discount rate in (8) is endogenous implies that the agnet's decision problem does not admit a closed-form solution. To circumvent this difficulty Hugonnier et al. (2013) rely on a two step procedure. First, they show that in the exogenous mortality case where $\lambda_{m1} = 0$ the agent's decision problem admits a closed form solution. Second, they use an asymptotic expansion of the solution to the dynamic programming equation around the point $\lambda_{m1} = 0$ to compute the first order effect of endogenous mortality on the optimal policy. Adapting their results to our setting allows to derive an the following approximation to the optimal policy.

Theorem 1 (Optimal policy functions) *Assume that conditions (34), (35), and (36) of Appendix A.2 hold true, and let*

$$N_0(W, H) = W + BH + (y_0 - a)/r \tag{9}$$

where $B > 0$ is the smallest solution to (41). Up to a first order approximation the optimal policy functions are given by:

$$X^*(W, H) = \phi BH + \mathcal{X}_1 H^{-\xi_m} N_0(W, H) \tag{10}$$

$$C^*(W, H) = a + AN_0(W, H) + \mathcal{C}_1 H^{-\xi_m} N_0(W, H) \tag{11}$$

$$\Pi^*(W, H) = (\theta/(\gamma\sigma_S)) N_0(W, H) \tag{12}$$

and

$$I^*(W, H) = KBH + \mathcal{I}_1 H^{-\xi_m} N_0(W, H) \tag{13}$$

where the nonnegative constants B, $\mathcal{X}_1$, A, $\mathcal{C}_1$, K, and $\mathcal{I}_1$ are defined in Appendix A.2.

The optimal investment, consumption, portfolio and welfare are all increasing in net total wealth $N_0(W, H)$. The latter in (9) includes financial wealth W plus the present value of the human capital BH, for which B is the marginal-Q of health solving equation (41). This value can be interpreted as the capitalized value of the health-dependent capacity to generate income βH in (2), where B is an increasing function of β. In parallel, $(y_0 - a)/r$ is the net present value (NPV) of base income y_0 minus subsistence consumption a.

The first term in (13) is the order-0 investment that is proportional to health's economic value BH. The second term captures the incremental demand for health that arises from its death risk hedging capacity; that demand is increasing in the endogenous component $\lambda_{m1} H^{-\xi_m}$ of the death intensity (3). Hence the demand for health is larger when better health reduces the exposure to death risk. Importantly, health investment (13) responds to endogenous death risk. As will be seen next, the non-monotonic effects of H on $I(W, H)$ induced by the demand for death risk hedging will play a key role in the complex nonlinear dynamics for health and wealth.

The optimal rules are defined only over an *admissible* state space, i.e. the set of wealth and health levels such that net total wealth $N_0(W, H)$ is nonnegative in (9). Indeed, observe from optimal consumption (11) that admissibility is required to ensure that consumption C_t is above subsistence a. Moreover, it can be shown that the continuation utility induced by the optimal rules in Theorem 1 is given as:

$$V^*(W, H) = \Theta N_0(W, H) - \lambda_{m1} \Theta L_m H^{-\xi_m} N_0(W, H), \tag{14}$$

where the parameters are given in Appendix A.2, and which is non-negative for non-negative net total wealth. Admissibility thus ensures that the continuation utility of living V_t is positive in (14), and therefore that life is preferred to death. Otherwise, negative total wealth entails negative continuation utility, and from preferences (6), a lower utility of living ($V_t < 0$), than of dying ($V_t = 0$). To avoid this counter-intuitive outcome, positive net total wealth in equation (9) can be relied upon define the admissible region $\mathcal{A}$:

$$\mathcal{A} = \{(W, H) \in \mathbb{R} \times \mathbb{R}_+ : W \geq x(H) \equiv -(y_0 - a)/r - BH\}. \tag{15}$$

Note finally from optimal risky holdings (12) that the risky portfolio shares out of total wealth can be written as:

$$\Pi^*(W, H)/W = (\theta/(\gamma\sigma_S))(1 - w(H)/W).$$

As is well-known, portfolio shares are increasing in the financial wealth level W (e.g. Wachter and Yogo, 2010), which therefore requires that $w(H)$ be nonnegative, and consequently, that:

$$(y_0 - a)/r < 0, \tag{16}$$

i.e. base income y_0 is insufficient to cover subsistence consumption a. This restriction is also tested and confirmed empirically in Section 5, and will henceforth be imposed.

3 Optimal health and wealth dynamics

We assume from now on that the agent follows the approximate optimal rules prescribed by Theorem 1. As a result of this assumption, his health and wealth evolve according to the dynamical system formed by (1) and (5) evaluated at (10)–(13). Due to the presence of Brownian financial shocks and Poisson health shock, this dynamical system is stochastic and thus cannot be directly analyzed using standard tools such as phase portraits. To circumvent this difficulty we focus on the instantaneous expected changes in health and wealth that are implied by the optimal rules.[15]

We start by defining, and characterizing health depletion, as well as speed of depreciation in Section 3.1. We then analyze wealth depletion, as well as implications for closing-down strategies in Section 3.2. Finally, policy instruments aimed at reducing the prevalence of these depletion dynamics are presented in Section 3.3.

3.1 Optimal health depletion

The expected local change in health capital is given by:

$$E_{t-}[\mathrm{d}H] = \left[I^h(W,H)^\alpha - \tilde{\delta}\right] H \mathrm{d}t, \tag{17}$$

where we denote the conditional expectation, given filtration $\mathcal{F}_{t-}$, as $E_{t-}[\cdot] = E[\cdot \mid \mathcal{F}_{t-}]$, and let $\tilde{\delta} = \delta + \phi\lambda_{s0}$ be the sickness-adjusted expected depreciation rate. The investment-to-health ratio evaluated at the optimal investment in (13) is given by:

$$I^h(W,H) = \frac{I^*(W,H)}{H} = BK + \mathcal{I}_1 H^{-\xi_m - 1} N_0(W,H). \tag{18}$$

[15] See also Laporte and Ferguson (2007) for an analysis of expected local changes of the Grossman (1972) model with Poisson shocks.

Since our main focus concerns end-of-life decumulation, we can define the admissible region of the state space $\mathcal{D}_H$ where health *depletion* is expected:

$$\mathcal{D}_H = \{(W,H) \in \mathcal{A} : E_{t-}[\mathrm{d}H] < 0\}.$$

Interestingly, it is also possible to analyze how fast the health capital is allowed to deplete. To do so, we can define an *acceleration* subset in the health depletion region $\mathcal{AC} \subseteq \mathcal{D}_H$ whereby the investment-to-health ratio is an increasing function of health:

$$\mathcal{AC} = \left\{(W,H) \in \mathcal{D}_H : I_H^h(W,H) > 0\right\}.$$

Hence for agents with $(W,H) \in \mathcal{AC}$, a positive health gradient of the investment-to-capital ratio (18) entails that health depletion is followed by more important cuts in health investment, leading to declines in $I^h(W,H)$, and further depletion of the health capital in (17) .

The following result characterizes both the health depletion, and acceleration regions of the state space.

Theorem 2 (health depletion) *Assume that the agent follows the optimal rules in Theorem 1. Then,*

1. *The following condition:*

$$BK < \tilde{\delta}^{1/\alpha}, \tag{19}$$

is necessary and sufficient for the health depletion zone to be given by:

$$\mathcal{D}_H = \{(W,H) \in \mathcal{A} : W < y(H)\}, \tag{20}$$

where the health depletion locus is

$$y(H) = x(H) + DH^{1+\xi_m}, \tag{21}$$

$$D = \mathcal{I}_1^{-1}\left[\tilde{\delta}^{1/\alpha} - BK\right]. \tag{22}$$

2. There exists a threshold $\bar{H}_3$ given by:

$$\bar{H}_3 = \left(\frac{B}{D(1+\xi_m)}\right)^{\frac{1}{\xi_m}} > 0, \tag{23}$$

such that the accelerating region is given by:

$$\mathcal{AC} = \begin{cases} \mathcal{D}_H, & \text{if } H < \bar{H}_3; \\ \{(W, H) \in \mathcal{D}_H : W < z(H)\}, & \text{otherwise,} \end{cases} \tag{24}$$

where the acceleration locus is

$$z(H) = x(H) + \frac{BH}{1+\xi_m}. \tag{25}$$

Condition (19) states that expected health depreciation $\tilde{\delta}$ is high relative to the health capital's marginal-Q, and is appropriate for end-of-life characterization. Indeed, a high depreciation in the absence of investment (δ), or conditional upon sickness (ϕ), a high likelihood of morbidity shocks (λ_{s0}), as well as a low variable component in income (β) are all to be expected for elders. As shown in Appendix B, the expression ($\tilde{\delta}^{1/\alpha} - BK$) in (22) captures the order-0 expected depletion, i.e. in the absence of endogenous mortality. When the latter is reintroduced, it was shown earlier that optimal investment in (13) is larger, reflecting the positive incremental demand for death risk hedging provided by health capital. If condition (19) is violated, then health grows in expectation absent mortality control value; positive growth is even larger when endogenous mortality is re-introduced and no relevant health depletion region exists. Moreover, this restriction ensures that the constant $D > 0$ in equation (21), such that $y(H) > x(H)$, and the admissible health depletion subset is therefore non-empty. Finally, contrasting condition (19) with (34) in Appendix A.1 shows that the former is more restrictive, and therefore also implies that transversality is verified.

The health dynamics characterized by Theorem 2 can be analyzed through the phase diagram in (W, H) space in Figure 1. First, the admissible region $\mathcal{A}$ is bounded below by the $x(H)$ locus (15) in red, with complementary non-admissible area $\mathcal{NA}$ in shaded red region. The $W-$intercept of $x(H)$ is given by the NPV of base income deficit $-(y_0 - a)/r$ which is positive under restriction (16), whereas the $H-$ intercept is given by $\bar{H}_1 =$

$-(y_0 - a)/(rB) > 0$, consistent with requiring strictly positive wealth (resp. health) at zero health (resp. wealth) for admissibility.

Second, equation (20) states that the health depletion region $\mathcal{D}_H$ is the shaded green area located below the green $y(H)$ locus (21). Both $x(H), y(H)$ loci intersect at the same $-(y_0 - a)/r$ intercept. Under condition (19), we show in Appendix B that the $y(H)$ locus is U-shaped, and attains a unique minimum at $\bar{H}_3$ given by (23). The reasons for the non-monotonicity stem from the non-monotone effects of H on $I^h(W, H)$. Indeed, the investment-to-health ratio in (18) is monotone increasing in wealth, but not in health due to the opposing forces of net total wealth, and mortality effects. On the one hand, an increase in H raises net total wealth $N_0(W, H)$, and therefore raises I^h. Consequently, constant (and zero) expected growth is obtained by reducing W. On the other hand, an increase in H also reduces endogenous mortality risk $\mathcal{I}_1 H^{-\xi_m - 1}$, and therefore also reduces I^h. Therefore, constant zero growth requires increasing W. The analysis of the $y(H)$ locus in (21) thus reveals that the net total wealth effect is dominant at low health ($H < \bar{H}_3$), whereas the mortality risk effect dominates for healthier agents ($H > \bar{H}_3$).

Third, the accelerating locus $z(H)$ in (25) is plotted as the blue line in Figure 1; the accelerating region is the dashed blue and green subset of $\mathcal{D}_H$. Appendix B shows that this locus intersects the $x(H), y(H)$ loci at the same $-(y_0 - a)/r$ intercept, and that it intersects the $H-$axis at $\bar{H}_2 = \bar{H}_1(1 + \xi_m)/\xi_m > \bar{H}_1$; consequently, the admissible accelerating region is non-empty. Moreover, it also intersects the health depletion locus $y(H)$ at its unique minimal value $\bar{H}_3$ in (23). Consequently, all agents with $H < \bar{H}_3$ in the depletion region are also in the accelerating subset.

The local expected dynamics of health are represented by the horizontal directional arrows in Figure 1. First, only agents who are sufficiently rich (i.e. $W > y(H)$) can expect a growth in health; all others are located in the $\mathcal{D}_H$ region in which the health stock is expected to fall. In particular, there exists a threshold wealth level $\bar{W}_3 = y(\bar{H}_3)$ below which *all* agents, regardless of their health status, expect a health decline. Second, the sufficiently rich and healthy ($W > z(H)$) agents in the health depletion region $\mathcal{D}_H$ optimally slow down (but do not reverse) the depreciation of their health capital (i.e. $I^h_H < 0$). However, for $W < z(H)$, the health depletion accelerates (i.e. $I^h_H > 0$, illustrated by the thick directional vector) as falling health is accompanied by further

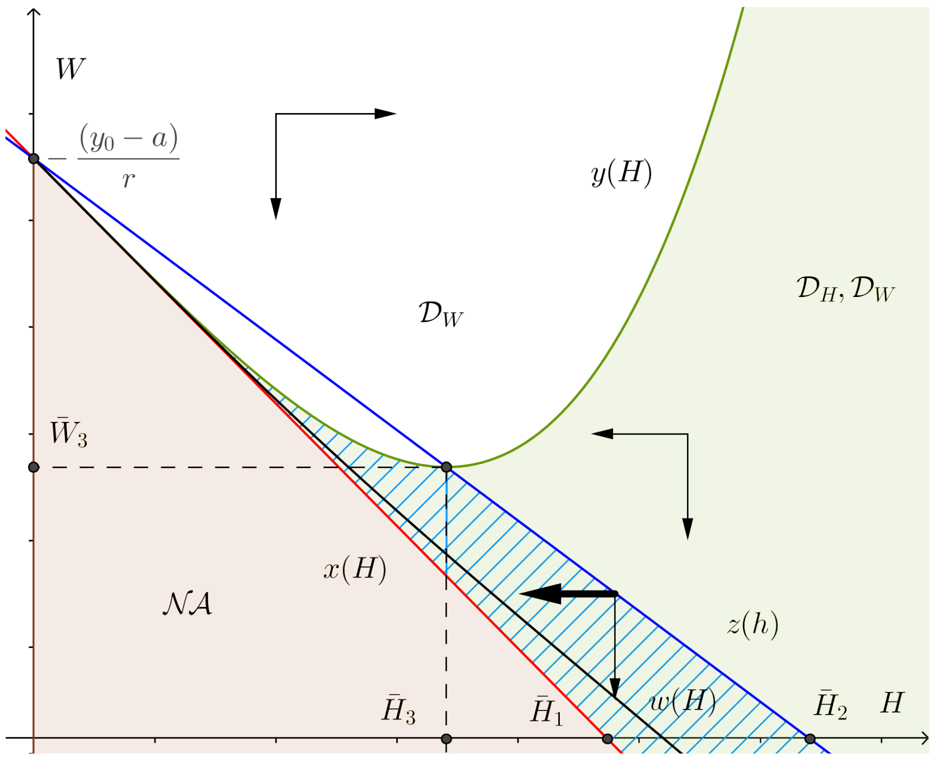

Figure 1: Joint health and wealth dynamics

Notes: Non-admissible set $\mathcal{NA}$: shaded red area under red $x(H)$ line. Health depletion set $\mathcal{D}_H$: shaded green area under green $y(H)$ green curve. Acceleration set $\mathcal{AC}$: hatched green area under blue $z(H)$ curve. Wealth depletion set $\mathcal{D}_W$: area above $w(H)$ black curve.

cuts in the investment-to-health ratio. The health dynamics thus crucially depend on the wealth levels and dynamics, an issue we now address.

3.2 Optimal wealth depletion

Since the expected net return is zero on actuarially fair insurance contracts (4), the expected changes in wealth is:

$$E_{t-}[\mathrm{d}W] = [rW + Y(H) - C^*(W,H) - I^*(W,H) + \Pi^*(W,H)\sigma_S\theta]\,\mathrm{d}t. \tag{26}$$

In parallel with the earlier analysis, the wealth depletion $\mathcal{D}_W$ region of the admissible state space can then be written as:

$$\mathcal{D}_W = \{(W,H) \in \mathcal{A} : E_{t-}[\mathrm{d}W] < 0\}\,.$$

The following result identifies realistic sufficient conditions for characterizing wealth depletion.

Theorem 3 (wealth depletion) *Assume that the elasticity of inter-temporal substitution $\varepsilon > 1$, and that the following conditions hold:*

$$\beta < Br + \tilde{\delta}^{1/\alpha}, \tag{27}$$

$$\frac{\theta^2}{\gamma} + r < A, \tag{28}$$

Then the wealth depletion zone is given by:

$$\mathcal{D}_W = \{(W, H) \in \mathcal{A} : W > w(H)\}, \tag{29}$$

where the wealth depletion locus is

$$\begin{aligned} w(H) &= \frac{x(H)[\ell(H) + r]}{\ell(H)} + \frac{k(H)}{\ell(H)}, \\ \ell(H) &= A - \frac{\theta^2}{\gamma} - r + (\mathcal{I}_1 + \mathcal{C}_1)\, H^{-\xi_m} \\ k(H) &= y_0 - a + H(\beta - KB), \end{aligned} \tag{30}$$

and where $x(H) < w(H) < y(H)$.

The conditions (27), and (28) are economically plausible and relevant for end-of-life analysis. Indeed, the restriction (27) parallels (19), and states that the health gradient of income β is low relative to expected depreciation of the health capital. Condition (28) refers to high consumption patterns. To see this, observe from (37) that the former can be rewritten as:

$$(1 + \varepsilon)\frac{\theta^2}{2\gamma} < \varepsilon(\rho - r) + (\varepsilon - 1)\frac{\lambda_{m0}}{1 - \gamma_m}.$$

Since $\gamma_m \in [0, 1)$, and assuming (as will be verified later) that the elasticity of intertemporal substitution $\varepsilon > 1$, the a high marginal propensity to consume A in (28) obtains whenever the agent is impatient, i.e. ρ is high, and/or the unconditional risk of dying λ_{m0} is high, and/or the aversion to death risk γ_m is high.

The wealth depletion locus $w(H)$ in (30) is represented as the black curve in Figure 1. Appendix B establishes that this locus has the same $H-$ intercept $-(y_0 - a)/r$, and must lie everywhere between the $x(H)$, and the $y(H)$ loci. Equation (29) states that the wealth depletion $\mathcal{D}_W$ is the area above the black curve $w(H)$, with corresponding wealth dynamics captured by the vertical directional arrows. Since $w(H)$ is located between the admissible, and the health depletion loci, most agents thus expect their wealth to fall, except at very low wealth levels in the $\mathcal{AC}$ region where rapidly receding health expenses $I(W, H)$ in (26) lead to expected increases in wealth.

These joint end-of-life health and wealth dynamics are thus consistent with a deliberate *closing down the shop* strategy. Sufficiently rich and healthy agents ($W > y(H)$) postpone health declines, e.g. by restoring levels following sickness. However, falling wealth is optimally chosen and agents eventually enter the $\mathcal{D}_H$ region where health depletion is also selected. Depreciation of the health stock accelerates once falling health and wealth push agents into the $\mathcal{AC}$ region. From endogenous death intensity (3), falling health is invariably accompanied by an increase in mortality, and a decline towards the admissible locus $x(H)$ characterized by zero net total wealth, and indifference between life and death, i.e. $V(W, H) = 0$.

Our model suggests that falling health is initially fought back, before eventually being accelerated. Although we do not distinguish between various inputs in health care, such behavior would be consistent with an end-of-life change in composition in health expenses towards more comfort, and less curative care (De Nardi et al., 2015b; Marshall et al., 2010). Importantly, regardless of whether it is accelerating or not, the optimal descent of health and increased exposure to death risk for those agents in the health depletion region obtains even when life is strictly preferred. Indeed, as discussed earlier, the non-separable preferences (6) ensure strictly positive continuation utility under life (versus zero under death), under admissible health and wealth statuses. The agents we are considering therefore have no proclivity in favor of premature death.

3.3 Reducing the prevalence of closing-down strategies

Assuming that such an objective is warranted (e.g. for public health purposes), the prevalence of closing-down strategies can be reduced through income policies. One instrument that can be used towards that aim is the base income y_0 which can be altered through Social Security, or minimal revenue policies. If we take as given the current health and wealth distribution, Figure 2 shows that an increase in base income lowers the intercept $-(y_0-a)/r$. It follows that the four loci are shifted downwards without affecting $\bar{H}_3$, leading to a lower $\bar{W}_3^1 < \bar{W}_3^0$. For given health and wealth levels, admissibility is increased, and the prevalence of the health depletion $\mathcal{D}_H$ is hence reduced.

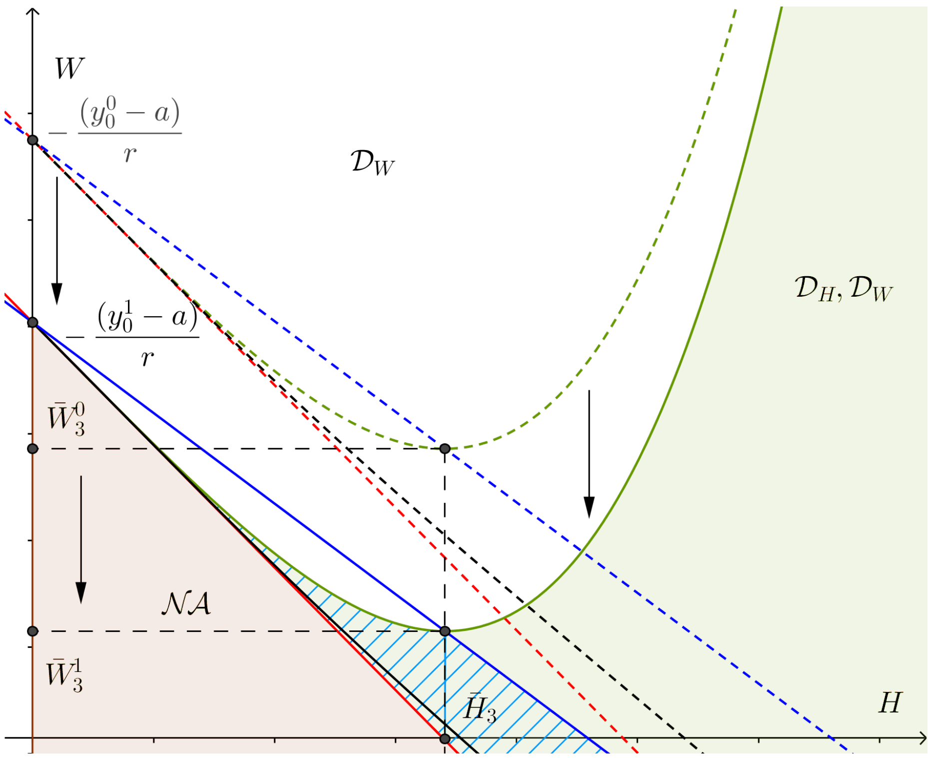

Figure 2: Reducing the prevalence of closing-down.

Notes: Effects of increase in base income y_0. Dotted lines: Initial position, solid lines: resulting position. Non-admissible set $\mathcal{NA}$: shaded red area under red $x(H)$ line. Health depletion set $\mathcal{D}_H$: shaded green area under green $y(H)$ green curve. Acceleration set $\mathcal{AC}$: hatched green area under blue $z(H)$ curve. Wealth depletion set $\mathcal{D}_W$: area above $w(H)$ black curve.

Whereas the tools for reducing the incidence of closing down are readily identified, the normative arguments in favor of intervention are less clear. Indeed, the traditional rationale of market incompleteness can hardly be invoked since closing down is obtained as an optimal dynamic strategy under a complete markets setting. Moreover, poor agents are subject to faster depreciation of their health capital and higher mortality risk. However, redistribution arguments cannot be invoked to the extent that poverty, and life expectancy are both endogenously determined as an optimal strategy. Consequently, redistributive arguments for intervening can neither be invoked.

4 Empirical evaluation

The closing-down strategy of optimal health and wealth depletion we have identified is arguably more appropriate for agents nearing death, than for younger ones. Indeed, a high sickness-augmented depreciation rate for the health capital (condition (19)), and a low ability to generate labor revenues (condition (27)) both seem legitimate for old agents in the last period of life, yet less so for younger ones. Moreover, a high marginal propensity to consume (condition (28)), as well as a base income deficit relative to subsistence consumption (condition (16)) are suitable for elders nearing end of life. Using a database of relatively old individuals, we next verify empirically whether or not these conditions are valid, and whether the admissible, depletion, and acceleration subsets have economic relevance.

Towards that objective, the structural econometric model that we rely upon to estimate the deep parameters and evaluate the induced parameters that are relevant for the various regions of the state space is based on a subset of the optimal rules in Section 2.2.

4.1 Econometric model

The tri-variate nonlinear structural econometric model that we estimate over a cross-section of agents $j = 1, 2, \ldots, n$ is the optimal investment (13), and the risky asset holdings (12), to which we append the income equation (2) :

$$I_j = KBH_j + \mathcal{I}_1 H_j^{-\xi_m} N_0(W_j, H_j) + u_j^I, \tag{31}$$

$$\Pi_j = (\theta/(\gamma\sigma_S))\, N_0(W_j, H_j) + u_j^{\Pi}, \tag{32}$$

$$Y_j = y_0 + \beta H_j + u_j^Y, \tag{33}$$

where $(u_j^I, u_j^{\Pi}, u_j^Y)$ are (potentially correlated) error terms. Data limitations discussed below explain why optimal consumption (11), and insurance (10) are omitted from the econometric model. The latter thus assumes that agents are heterogeneous only with respect to their health, and wealth statuses; the deep parameters are considered to be the same across individuals. This assumption does not appear unreasonable to the extent that we are considering a relatively homogeneous subset of old individuals, thereby ruling out potent cohort effects.

The technological, distributional, and preference parameters are estimated using the joint system (31), (32) and (33), imposing the regularity conditions outlined in Appendix A.1, and under the theoretical restrictions governing $(K, \mathcal{I}_1, B)$ that are outlined in Appendix A.2. The identification of the deep parameters is complicated by the significant non-linearities that are involved. Consequently, not all the parameters can be estimated, and we calibrate a subset. Certain calibrated parameters are set at standard values from the literature. For others however, scant information is available, and we rely on thorough robustness analysis, especially with respect to γ_m, and ϕ.[16]

The estimation approach is an iterative two-step procedure. In a first step, the convexity parameter ξ_m is fixed and a maximum likelihood approach is conducted on the remaining structural parameters. In a second step, the other structural parameters are fixed and the likelihood function is maximized with respect to ξ_m. The procedure is iterated until a fixed point is reached for both the structural parameters and the convexity parameters.

The likelihood function is written by assuming that there exist some cross-correlation between the three equations, i.e. $\mathrm{Cov}(u_j^I, u_j^\Pi, u_j^Y) \neq 0$. For the first two equations, the cross-correlation can be justified by the fact that we use an approximation of the exact solution (see Hugonnier et al., 2013, for details). Moreover, our benchmark case assumes that the three dependent variables are continuous. However, the observed risky holdings Π_j contain a significant share of zero observations. For that reason, we also experiment a mixture model specification in which the asset holdings variable is censored (Tobit) and the other two dependent variables (investment and income) are continuous, resulting in qualitatively similar results.[17]

[16]These alternative estimates, which are available upon request, are reasonably robust, with main interpretations qualitatively unaffected.

[17]Note however that our structural model neither rules out zero holdings, nor does predict a Tobit-based specification for the portfolio equation.

4.2 Data

The data base used for estimation is the 2002 wave of the Health and Retirement Study (HRS, Rand data files). This data set is the last HRS wave with detailed information on total health spending; subsequent waves only report out-of-pocket expenses. Under OOP ceilings, total health expenses I are not uniquely identified for insured agents. Also, even though the HRS contains individuals aged 51 and over, we restrict our analysis to elders (i.e. agents aged 65 and more), with nonnegative financial wealth (9,817 observations, mean age 75.3). In doing so, we avoid endogenizing the insurance choice X_t in (4) which, under near universal Medicare coverage, can be considered as exogenous. Unfortunately, this data set does not include a consumption variable, so that we omit equation (11) from the econometric model.

We construct financial wealth W_j as the sum of safe assets (checking and saving accounts, money market funds, CD's, government savings bonds and T-bills), bonds (corporate, municipal and foreign bonds and bond funds), retirement accounts (IRAs and Keoghs), and risky assets (stock and equity mutual funds) Π_j. Health status H_j is evaluated using the self-reported general health status, where we express the polytomous self-reported health variable in real values with increments of 0.75 corresponding to: 0.5 (poor), 1.25 (fair), 2.00 (good), 2.75 (very good), and 3.50 (excellent).[18]

Health investments I_j are obtained as the sum of medical expenditures (doctor visits, outpatient surgery, hospital and nursing home, home health care, prescription drugs and special facilities), and out-of-pocket medical expenses (uninsured cost over the two previous years). Finally, we resort to wage/salary income Y_j, to which we add any Social Security revenues. The estimates presented below are obtained for a scaling of \$M applied to all nominal variables (I_j, W_j, Π_j, Y_j).

Table 1 reports the median values for wealth, investment and risky asset holdings, for wealth quintiles, and self-reported health. Overall, these statistics confirm earlier findings. A first observation concerns the relative insensitivity of financial wealth to the health status.[19] Second, we find that health investment increases moderately in wealth,

[18]Self-reported health has been shown to be a valid predictor of the objective health status (Benítez-Silva and Ni, 2008; Crossley and Kennedy, 2002; Hurd and McGarry, 1995).

[19]See Hugonnier et al. (2013); Michaud and van Soest (2008); Meer et al. (2003); Adams et al. (2003) for additional evidence.

and falls sharply in health.[20] Conversely, risky holdings increase sharply in wealth, and are also higher for healthier agents.[21]

5 Results

Table 2 reports the calibrated, and estimated deep parameters (panels a–d), the induced parameters that are relevant for the various subsets (panel e), as well as the hypothesis testing for the assumptions relevant to Theorems 2, and 3 (panel f). The standard errors indicate that all the estimates are significant at the 5% level.

5.1 Deep parameters

First, the law of motion parameters in panel a are indicative of significant diminishing returns to the health production function ($\alpha = 0.69$). Moreover, depreciation is important ($\delta = 7.2\%$), and sickness is rather consequential, with additional depreciation ($\phi = 1.1\%$) suffered upon realization of the health shock.

Second, in panel b the intensity parameters indicate a high, and significant incidence of health shocks ($1 - \exp(-\lambda_{s0}) = 25\%$). The death intensity (3) parameters are realistic, with an expected lifetime of 79.0 years for an individual with an average (i.e. good) health.[22] Importantly, the null of exogenous exposure to death risk is rejected ($\lambda_{m1}, \xi_m \neq 0$), indicating that agent's health decisions are consequential for their expected life horizon. Taken together, these law of motion and risk exposure parameters compare well to estimates in Hugonnier et al. (2013), and are consistent with expectations regarding an elders' population.

[20]Similar findings with respect to wealth (e.g. Hugonnier et al., 2013; Meer et al., 2003; DiMatteo, 2003; Gilleskie and Mroz, 2004; Acemoglu et al., 2013) and health (e.g. Hugonnier et al., 2013; Smith, 1999; Gilleskie and Mroz, 2004; Yogo, 2009) have been discussed elsewhere.

[21]Similar positive effects of wealth on risky holdings have been identified in the literature (e.g. Hugonnier et al., 2013; Wachter and Yogo, 2010; Guiso et al., 1996; Carroll, 2002) whereas positive effects of health have also been highlighted (e.g. Hugonnier et al., 2013; Guiso et al., 1996; Rosen and Wu, 2004; Coile and Milligan, 2009; Berkowitz and Qiu, 2006; Goldman and Maestas, 2013; Fan and Zhao, 2009; Yogo, 2009).

[22]In particular, Hugonnier et al. (2013) show that an age-t person's remaining life expectancy can be computed using:

$$\ell(W_t, H_t) = (1/\lambda_{m0})(1 - \lambda_{m1}\kappa_0 H_t^{-\xi_m}), \quad \text{where } \kappa_0 = [\lambda_{m0} - F(-\xi_m)]^{-1} > 0.$$

The average age in our HRS sample is 75.3 years, and the expected remaining life horizon is 3.7 years for an individual with good health. The unconditional expected lifetime was 77.3 years in 2002, with 74.5 for males, and 79.9 for females (Arias, 2004).

Third, the returns parameters (μ, r, σ_S) are calibrated at standard values in panel c. The income parameters of equation (2) are both significant, and indicative of a positive health effects on income ($\beta \neq 0$), while the the base income y_0 is estimated to a value of \$8,200 (representing \$10,824 in 2016). Fourth, the preference parameters in panel d suggest a significant subsistence consumption a of \$12,700 (\$16,760 in 2016), which is larger than base income y_0. Both subsistence, and base income values are realistic.[23] Our estimate of the inter-temporal elasticity ε is larger than one, as identified by others using micro data.[24] Aversion to financial risk is realistic ($\gamma = 2.78$), whereas aversion to mortality risk is calibrated in the admissible range ($0 < \gamma_m < 1$), and close to the value set by Hugonnier et al. (2013) ($\gamma_m = 0.75$). The aversion to morbidity risk γ_s is both unidentifiable and irrelevant under the exogenous morbidity risk assumption, and in the absence of endogenous demand for insurance. Finally, the subjective discount rate is set at usual values ($\rho = 2.5\%$). Overall, we conclude that the estimated structural parameters are economically plausible.

5.2 Induced parameters

Table 2.e reports the induced parameters that are relevant for the admissible, depletion and accelerating subsets; panel f shows that the four main conditions for our theoretical results are verified. These composite parameters allow us to evaluate the position of the loci $x(H), y(H), z(H)$, and thus of the various subsets in Figure 3. The H axis also records the positions associated with Poor ($H = 0.5$), and Fair ($H = 1.25$) self-reported health statuses, where the scaling is the one used in the estimation. The W axis is reported in \$M, using the same scaling as for the estimation. The health and wealth joint distribution for the HRS data is indicated by plotting the wealth quintiles as blue points for Q_i for the poor, and fair health statuses.

First, we identify a relatively large marginal-Q of health $B = 0.1148$ in panel e, suggesting that health is very valuable.[25] Second, the large negative value for (y_0 −

[23] For example, the 2002 poverty threshold for elders above 65 was \$8,628 (source: U.S. Census Bureau).

[24] For example, Gruber (2013) finds estimates centered around 2.0, relying on CEX data. In our case, the recourse to elders' data, and the assumption of no bequest function could explain a relatively strong consumption reaction to interest rates movements.

[25] Adapting the theoretical valuation of health in Hugonnier et al. (2013, Prop. 3) reveals that an agent at the admissible locus (i.e. with $N_0(W, H) = 0$) would value a 0.10 increment in health as $w_h(0.10, W, H) = 0.10. * B * 10^6 = \$11{,}480$ (\$15,150 in 2016).

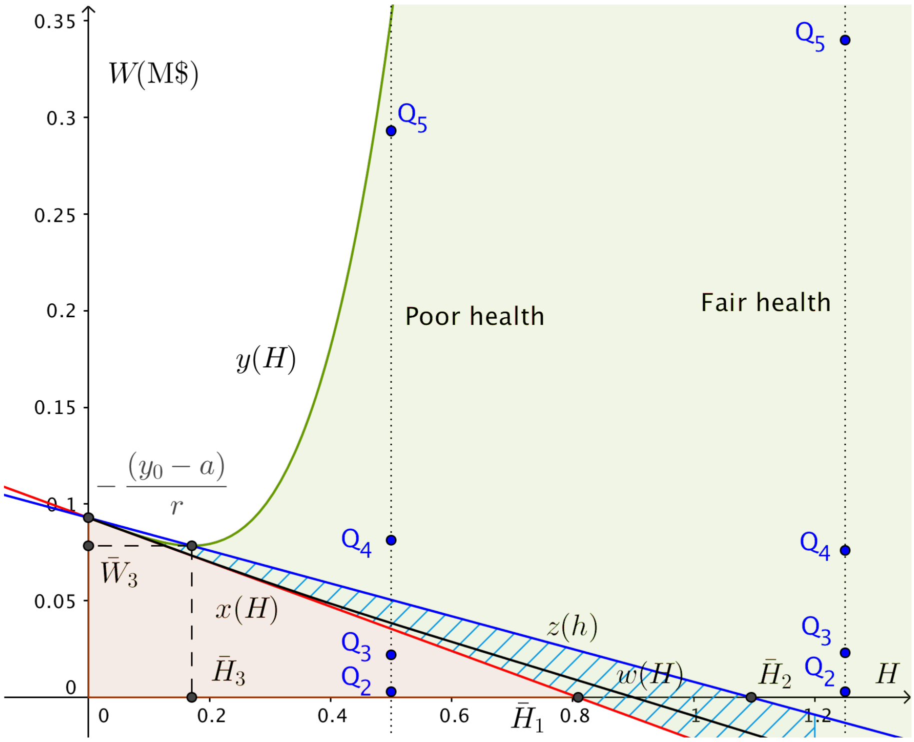

Figure 3: Estimated depletion, accelerating, and non-admissible regions

Notes: Non-admissible set $\mathcal{NA}$: shaded red area under red $x(H)$ line. Health depletion set $\mathcal{D}_H$: shaded green area under green $y(H)$ green curve. Acceleration set $\mathcal{AC}$: hatched green area under blue $z(H)$ curve. Wealth depletion set $\mathcal{D}_W$: area above $w(H)$ black curve. Position of loci, and areas evaluated at estimated parameters in Table 2. Quintile levels for wealth quintiles $Q_2, \ldots, Q_5$ ($Q_1 = 0\$$) are taken from Table 1, and are reported as blue points for health levels poor, and fair.

$a)/r$ corresponds to a capitalised base income deficit of 92,900$ (122,628$ in 2016), and confirms that condition (16) in Theorem 3 is verified.

Third, the estimated wealth depletion locus $w(H)$ is, as expected, lying between the $x(H), y(H)$ loci. It is also very low, confirming that most of the agents are in the wealth depletion region. Fourth, the value for D is significant which confirms the verification of condition (19). From the definition of $y(H)$ in (20), a large value of D also entails a very steep health depletion locus. It follows that unless very wealthy, and very unhealthy, the bulk of the population would be located in the health and wealth depletion regions.

Finally, our estimates are consistent with a narrow accelerating region $\mathcal{AC}$. Indeed, the values for $B, (y_0 - a)/r, \xi_m$ are such that intercepts $\bar{H}_1, \bar{H}_2$ are relatively low (i.e. between Fair, and Poor self-reported health), and close to one another (less than one discrete increment of 0.75). This feature of the model is reassuring since we would expect accelerating phases where agents are cutting down expenses in the face of falling health to coincide with the very last periods of the end of life.

5.3 Simulation analysis

The dynamic analysis presented thus far has focused upon local expected changes for health and wealth $E_{t-}[\mathrm{d}H], E_{t-}[\mathrm{d}W]$. In order to assess whether such small anticipated depletion translate into realistic life cycle paths for health and wealth, we conduct a Monte-Carlo simulation exercise described in further details in Appendix D. Figure 4 plots the resulting mean values for the optimal life cycles for financial wealth W_t (panel a), net total wealth $N_0(W_t, H_t)$ (panel b), health level H_t (panel c), as well as the exposure to death risk $1 - \exp[-\lambda_m(H_t)]$ (panel d).

Overall, these results confirm all our previous findings. Consistent with the data, our simulated life cycles feature a rapid end-of-life depletion of both health (Banks et al., 2015; Case and Deaton, 2005; Smith, 2007; Heiss, 2011), and wealth (De Nardi et al., 2015b; French et al., 2006) as they enter the end of life period. Indeed, recalling that expected longevity is 79.0 years, the optimal strategy is to bring down net total wealth $N_0(W_t, H_t)$ to zero (i.e. reach the lower limits of admissible set $\mathcal{A}$) at terminal age (panel b), an objective obtained by running down wealth (panel a) very rapidly (consistent with our finding of low $w(H)$ locus), and a somewhat slower decline for health (panel c).

Contrasting rich versus poor cohorts reveals that, as expected, wealth (panel a), and health (panel c) depletion is faster for poor agents, such that low-wealth individuals enter the depletion, and accelerating regions more rapidly. Moreover, exposure to death risk is higher for the poor (panel d), consistent with stylized facts,[26] except at very old age where attrition effects imply that only the very healthy poor agents remain alive, and the rich and poor exposures to mortality are converging. Put differently, our simulations indicate that agents entering the last period of life optimally select a short expected lifespan, and allocations that are consistent with optimal closing down, i.e. depletion of the health and wealth capitals during their remaining lifetime. High initial wealth thus has a moderating effect on the speed of the depletion, but not on its ultimate outcome.

[26] For example, longevity for males from a 1940 cohort in HRS are 73.3 years in the first decile of career earnings, and 84.6 if in the 10th decile (Bosworth et al., 2016, Tab. IV-4, p. 87).

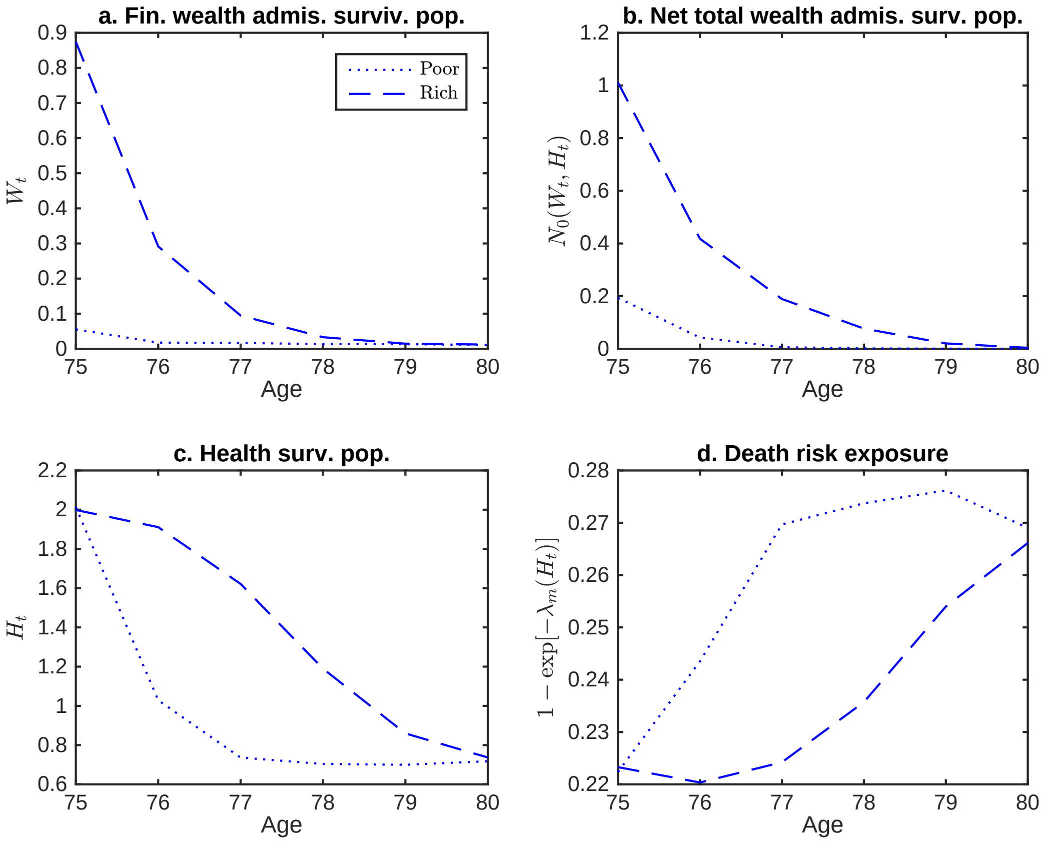

Figure 4: Simulated optimal paths

Notes: Mean values for simulated optimal life cycles taken over an initial population of 1,000 agents with 1,000 replications. Initial draw from rich (dashed lines), and poor (dotted lines) initial populations. Mean values are taken with respect to surviving, admissible agents, with positive financial wealth.

6 Conclusion

This paper identifies conditions under which agents approaching the end of life optimally select to close down the shop, i.e. run down their health, and wealth capitals, bringing them to a state where they are indifferent between life and death. We rely on closed-form solutions to a life cycle model of optimal health spending and insurance, portfolio, and consumption to characterize the end of life dynamics for health, and wealth. Our findings can be summarized as follows. First, under plausible, and empirically verified conditions, agents optimally choose an expected depletion of their health capital, unless they are sufficiently healthy and wealthy. We also identify a threshold wealth level below which health decline is independent on how healthy the agent is. Moreover, this depletion is accelerated below certain levels of health and wealth. Importantly, wealth is also expected to fall, such that all agents eventually close down the shop.

The incidence of depletion strategies can be reduced by increasing base income (e.g. through enhanced Social Security, Medicaid, or minimal revenue programs). However, whereas the positive arguments are readily obtained, the normative reasons for intervening are less clear. Indeed, continuous depletion of the health stock leading to very high death risks, and indifference between life and death is optimally selected, even in the case of agents with no predisposition for early death. Moreover, this downward spiral is obtained in a complete markets setting, such that no market failure argument for intervention can be invoked. Finally, poverty is endogenously determined such that redistributive rationales for intervening cannot be made.

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A Parametric restrictions

A.1 Regularity and transversality restrictions

Define the following elements:

$$\begin{aligned}
\chi(x) &= 1 - (1-\phi)^{-x}, \\
F(x) &= x(\alpha B)^{\frac{\alpha}{1-\alpha}} - x\delta - \lambda_{s0}\chi(-x), \\
L_m &= [(1-\gamma_m)(A - F(-\xi_m))]^{-1} > 0.
\end{aligned}$$

The theoretical model is solved under three regularity and transversality conditions that are reproduced for completeness:

$$\beta < (r + \delta + \phi\lambda_{s0})^{\frac{1}{\alpha}}, \tag{34}$$

$$\max\left(0; r - \frac{\lambda_{m0}}{1-\gamma_m} + \theta^2/\gamma\right) < A, \tag{35}$$

$$0 < A - \max\left(0, r - \frac{\lambda_{m0}}{1-\gamma_m} + \theta^2/\gamma\right) - F(-\xi_m), \tag{36}$$

where the consumption parameter A, and the price of health B, are defined in (37), and in (41).

A.2 Closed-form solutions for optimal rules parameters

The closed-form expression for the parameters in the optimal rules are obtained as follows. The insurance parameter in (10) is given as:

$$\mathcal{X}_1 = \lambda_{m1}\chi(\xi_m)\left(1/\gamma_s - 1\right) L_m.$$

The consumption parameters in (11) are:

$$A = \varepsilon\rho + (1-\varepsilon)\left(r - \frac{\lambda_{m0}}{1-\gamma_m} + \frac{\theta^2}{2\gamma}\right), \tag{37}$$

$$\mathcal{C}_1 = \lambda_{m1} A(\varepsilon - 1) L_m. \tag{38}$$

The parameters of the optimal investment in (13) are:

$$K = \alpha^{\frac{1}{1-\alpha}} B^{\frac{\alpha}{1-\alpha}}, \tag{39}$$

$$\mathcal{I}_1 = \lambda_{m1} \left(\xi_m K/(1-\alpha)\right) L_m, \tag{40}$$

where the shadow price B of health solves:

$$g(B) = \beta - (r + \delta + \phi\lambda_{s0})B - (1 - 1/\alpha)\,(\alpha B)^{\frac{1}{1-\alpha}} = 0, \tag{41}$$

subject to $g'(B) < 0$. Finally, the positive continuation utility parameter in (14) is obtained as:

$$\Theta = \rho(A/\rho)^{1/(1-\varepsilon)}.$$

B Proofs

B.1 Theorem 1

See Hugonnier et al. (2013, Thm. 2) for the general case, and evaluate the optimal policies at the restricted exogenous morbidity case $\lambda_{s1} = 0$.

B.2 Theorem 2

B.2.1 Health depletion

First, substituting the investment-to-capital ratio (18) in the expected local change for health (17) shows that:

$$E_{t-}[\mathrm{d}H] = \left\{ \left[BK + \mathcal{I}_1 H^{-\xi_m - 1} N_0(W,H) \right]^{\alpha} - \tilde{\delta} \right\} H\mathrm{d}t.$$

By admissibility, $N_0(W,H) > 0$, whereas $\mathcal{I}_1 \geq 0$ in (40). It follows that

$$E_{t-}[\mathrm{d}H]\Big|_{\lambda_{m1}=0} = \left\{ [BK]^{\alpha} - \tilde{\delta} \right\} H\mathrm{d}t < E_{t-}[\mathrm{d}H]\Big|_{\lambda_{m1}\neq 0}.$$

Since $E_{t-}[\mathrm{d}H] < 0$ by definition of health depletion, condition (19) is thus required for order-0 depletion, and therefore a necessary condition for existence of depletion zones when mortality risk is endogenous.

Second using the definition of net total wealth (9) shows that we can write:

$$E_{t-}[\mathrm{d}H] < 0 \iff W < y(H) = x(H) + DH^{1+\xi_m}, \quad D = \mathcal{I}_1^{-1}\left[\tilde{\delta}^{1/\alpha} - BK\right]$$

Condition (19) implies that $D > 0$ in (20), and is therefore sufficient for $y(H) \geq x(H), \forall H$, i.e. the locus $y(H)$ lies everywhere in the admissible zone, and consequently $\mathcal{D}_H$ is a non-empty subset of $\mathcal{A}$. Third, observe that the health depletion locus is characterized by:

$$y_H(H) = -B + (1+\xi_m)DH^{\xi_m} \begin{cases} < 0, & \text{if } H < \bar{H}_3, \\ = 0, & \text{if } H = \bar{H}_3, \\ > 0, & \text{if } H > \bar{H}_3, \end{cases} \quad \text{and}$$
$$y_{HH}(H) = \xi_m(1+\xi_m)DH^{\xi_m-1} > 0.$$

The locus $y(H)$ is therefore convex, and U-shaped and attains a unique minima at $\bar{H}_3$ in the (H, W) space, where $\bar{H}_3$ is given in (23), with corresponding wealth level $\bar{W}_3 = y(\bar{H}_3)$.

B.2.2 Acceleration

Taking the derivative of the investment-to-health ratio (18) with respect to H and rearranging shows that the accelerating region can be characterized by:

$$I_H^h(W, H) > 0 \iff W < -(y_0 - a)/r - \frac{BH\xi_m}{1+\xi_m} = x(H) + \frac{BH}{1+\xi_m} = z(H).$$

Since $B, \xi_m > 0$, $z(H) > x(H)$, i.e. this locus lies everywhere above the $x(H)$ locus, and is therefore admissible, i.e. $\mathcal{AC} \subset \mathcal{A}$. Observe furthermore that $z(0) = x(0) = y(0) = -(y_0 - a)/r$, and that:

$$z(H) - y(H) = H\left[\frac{B}{1+\xi_m} - DH^{\xi_m}\right] \begin{cases} > 0, & \text{if } H < \bar{H}_3 \\ = 0, & \text{if } H = \bar{H}_3 \\ < 0, & \text{if } H > \bar{H}_3 \end{cases}$$

again using the definition of $\bar{H}_3$ in (23). Consequently, the $z(H)$ locus is downward-sloping, has the same intercept and intersects $y(H)$ at its unique minimal value $\bar{H}_3$, and lies above (below) the $y(H)$ locus for $H < \bar{H}_3$ ($H > \bar{H}_3$). It follows that the acceleration set (i.e. the health depletion subset where $I_H^h > 0$) is the entire $\mathcal{D}_H$ for $H \in [0, \bar{H}_3]$, and otherwise the area between $y(H), z(H)$, as given in (24), and (25).

B.3 Theorem 3

Substituting the optimal investment (13), consumption (11), risky portfolio (12), and insurance (10) in the expected local change for wealth (26), and using the definition of net total wealth (9) reveals that

$$E_{t-}[\mathrm{d}W] = 0 \iff W\ell(H) = x(h)[\ell(H) + r] + k(H),$$

where

$$\begin{aligned}
\ell(H) &= \left[A - \sigma_S \theta L_0 - r + (\mathcal{I}_1 + \mathcal{C}_1)\, H^{-\xi_m}\right], \\
k(H) &= (y_0 - a) + H(\beta - BK).
\end{aligned}$$

Observe that $\varepsilon > 1$ induces $\mathcal{C}_1 > 0$ in (38), whereas $\mathcal{I}_1 > 0$ in (40). Condition (28) is therefore sufficient to guarantee that $\ell(H) > 0, \forall H$. Consequently, the wealth depletion zone $\mathcal{D}_W$ is delimited by:

$$W > \frac{x(H)[\ell(H) + r]}{\ell(H)} + \frac{k(H)}{\ell(H)} = w(H).$$

It is straightforward to show that:

$$\lim_{H \to 0} w(H) = x(0) = -(y_0 - a)/r$$

such that the $w(H)$ curve has the same intercept which is nonnegative under condition (16). We first show that the locus $w(H)$ is admissible, i.e. $w(H) > x(H)$, which simplifies to:

$$0 < x(H)r + k(H) \iff 0 < \beta - B(r + K).$$

Using the defining condition for B given by $g(B) = 0$ in (41), as well as the definition of K in (39), this condition simplifies to:

$$0 < B\tilde{\delta} - \frac{1}{\alpha}BK \iff 0 < \tilde{\delta}^{1/\alpha} - BK$$

which is induced by restriction (19). Consequently $w(H) > x(H), \forall H$, and the locus is everywhere admissible.

We next show that $w(H)$ lies everywhere below the $y(H)$ locus, i.e. $y(H) > w(H)$, which simplifies to:

$$rx(H) + k(H) < D\ell(H)H^{1+\xi_m} \iff \beta - Br - \tilde{\delta}^{1/\alpha} < DH^{\xi_m}\left[A - \sigma_S\theta L_0 - r\right]$$

As shown earlier, condition (27) implies that the left-hand side is negative. Condition (19) entails $D > 0$, whereas condition (28) ensures that the right-hand term in square bracket is positive, and is therefore sufficient for $y(H) > w(H)$, as required.

C Tables

Table 1: HRS data statistics

Variable	Wealth quintile 1	2	3	4	5
	a. Poor health ($H = 0.5$)				
Financial wealth	0.000	0.030	0.220	0.814	2.930
Investment	0.379	0.417	0.469	0.427	0.615
Risky holdings	0.005	0.079	0.216	0.485	0.800
	b. Fair health ($H = 1.25$)				
Financial wealth	0.000	0.030	0.230	0.760	3.400
Investment	0.255	0.254	0.233	0.252	0.266
Risky holdings	0.000	0.046	0.253	0.514	0.782
	c. Good health ($H = 2.0$)				
Financial wealth	0.000	0.040	0.220	0.770	3.300
Investment	0.157	0.149	0.156	0.129	0.168
Risky holdings	0.002	0.082	0.299	0.510	0.824
	d. Very good health ($H = 2.75$)				
Financial wealth	0.000	0.040	0.230	0.840	3.500
Investment	0.100	0.112	0.106	0.105	0.107
Risky holdings	0.011	0.107	0.368	0.604	0.854
	e. Excellent health ($H = 3.5$)				
Financial wealth	0.000	0.050	0.210	0.800	3.820
Investment	0.137	0.065	0.063	0.105	0.091
Risky holdings	0.010	0.131	0.350	0.520	0.861

Notes: Median (wealth), and mean values (investment, risky holdings), measured in 100'000$ (year 2002) per health status, and wealth quintiles for HRS data used in estimation.

Table 2: Estimated and calibrated parameter values

Parameter	Value	Parameter	Value	Parameter	Value
		a. Law of motion health (1)			
α	0.6940^{*}	δ	0.0723^{*}	ϕ	0.011^{c}
	(0.1873)		(0.0366)		
		b. Sickness and death intensities (3)			
λ_{s0}	0.2876^{*}	λ_{m0}	0.2356^{*}		
	(0.1419)		(0.0844)		
λ_{m1}	0.0280^{*}	ξ_m	2.8338^{*}		
	(0.0108)		(1.1257)		
		c. Income and wealth (2), (5)			
y_0	$0.0082^{*\$}$	β	0.0141^{*}		
	(0.0029)		(0.0059)		
μ	0.108^{c}	r	0.048^{c}	σ_S	0.20^{c}
		d. Preferences (6), (7)			
a	$0.0127^{*\$}$	ε	1.6738^{*}	γ	2.7832^{*}
	(0.0063)		(0.6846)		(1.3796)
ρ	0.025^{c}	γ_m	0.75^{c}	γ_s	N.I.
		e. State space subsets (41), (22), (23), (13)			
B	0.1148^{*}	$(y_0 - a)/r$	$-0.0929^{*\$}$	D	4.5088^{*}
$\bar{H}_1$	0.8093^{*}	$\bar{H}_2$	1.0460^{*}	$\bar{H}_3$	0.1743^{*}
K	0.0022^{*}	$\mathcal{I}_1$	0.0053^{*}	$\bar{W}_3$	$0.0781^{*\$}$
		f. Conditions (16), (19), (27), (28)			
$y_0 - a$	-0.0045^{*}	$BK - \tilde{\delta}^{1/\alpha}$	-0.0239^{*}		
$\beta - Br - \tilde{\delta}^{1/\alpha}$	-0.0156^{*}	$\theta^2/\gamma + r - A$	-0.5533^{*}		

Notes: *: Estimated structural and induced parameters (standard errors in parentheses), significant at 5% level; c: calibrated parameters; \$: In \$M; N.I.: non-identifiable/irrelevant under the exogenous morbidity restriction.

D Monte-Carlo simulation

The Monte-Carlo framework used to simulate the dynamic model is as follows:

1. Relying on a total population of $n = 1,000$ individuals, we initialize the health and wealth distributions at base age $t = 75$ using a common uniform distribution for health, $H_0 \sim U[0.5, 3.5]$, and two different distributions for wealth:
   - Poor: $W_0 \sim U[0.01, 0.10]$;
   - Rich: $W_0 \sim U[0.25, 1.50]$.
2. We simulate individual-specific Poisson health shocks $\mathrm{d}Q_s \sim P(\lambda_{s0})$, as well as a population-specific sequence of Brownian financial shocks $\mathrm{d}Z \sim N(0, \sigma_s^2)$ over a 10-year period $t = 75, \dots, 85$.
3. At each time period $t = 75, \dots, 85$, and using our estimated and calibrated parameters:
   (a) For each agent with health H_t, we generate the Poisson death shocks with endogenous intensities $\mathrm{d}Q_m \sim P[\lambda_m(H_t)]$, and keep only the surviving agents, with positive wealth (as imposed in the estimation) for the computation of the statistics.
   (b) We verify admissibility, for each agent with health and wealth (H_t, W_t) and keep only surviving agents in the admissible region.
   (c) We use the optimal rules $I(W_t, H_t), c(W_t, H_t), \Pi(W_t, H_t), X(H_{t-})$, as well as income function $Y(H_t)$, and the sickness and financial shocks $\mathrm{d}Q_{st}, \mathrm{d}Z_t$ in the stochastic laws of motion $\mathrm{d}H_t, \mathrm{d}W_t$.[27]
   (d) We update the health and wealth variables using the Euler approximation:

$$
\begin{aligned}
H_{t+1} &= H_t + \mathrm{d}H_t(H_t, I_t, \mathrm{d}Q_{st}) \\
W_{t+1} &= W_t + \mathrm{d}W_t\left[W_t, c(W_t, H_t), I(W_t, H_t), \Pi(W_t, H_t), X(W_t, H_t), \mathrm{d}Q_{s,t}, \mathrm{d}Z_t\right]
\end{aligned}
$$

4. We replicate the simulation 1–3 for 1,000 times.

[27]The morbidity risk aversion parameter involved in the optimal insurance $X^*(W, H)$ is calibrated at the same value of $\gamma_s = 7.40$ in Hugonnier et al. (2013).