Macroeconomic and welfare implications of different pension benefit arrangements

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Abstract

I analyze the trade-off among insurance, labor supply and savings incentives arising in the design of pay-as-you-go (PAYG) pension benefits. I consider two extreme types of pension benefits: i) a flat benefit (FL) system that pays the same pension regardless of the amount of previous contributions and ii) a notional defined contribution (NDC) system in which benefits are perfectly linked to previous contributions. The NDC system promotes higher labor supply, higher consumption inequality and generally crowds out capital more. If the level of pension contributions is low, the FL system brings a higher welfare due to higher consumption insurance. At higher levels of contributions, the NDC system leads to higher welfare due to lower labor supply distortions. General equilibrium effects tilt the welfare result in favor of FL pensions in dynamically efficient economies. The analysis suggests that pension benefit design should depend on the size of the pension system and the size of idiosyncratic risk. The welfare results can explain the contrasting reforms of pension benefits implemented in different countries, as well as the empirical correlations between pension benefit progressivity, on the one hand, and the size of pension systems and idiosyncratic risk, on the other hand.

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1 Introduction

In the past 25 years, governments tried to reform PAYG pension systems as both their efficiency and their sustainability was questioned. One popular reform was the tightening of the link between pension contributions and benefits aimed at improving the labor supply incentives of agents and reducing the deadweight losses associated with labor supply distortions. Italy, Germany, Sweden and many eastern European countries switched to a Notional Defined Contribution (NDC) pension system or a points system in which pension benefits are perfectly linked to contributions paid and, hence, to life-time earnings. However, such pension systems do not offer insurance against idiosyncratic earnings shocks. Any shock that leads to a decrease in earnings at a certain point of an agent's career such as unemployment, health problems or a temporary decrease in productivity, will be perfectly carried over to the retirement period in the form of lower pension benefits. Moreover, as I will show in the present paper, the strengthening of labor supply incentives leads to higher labor supply, but can decrease the savings rate of agents and the capital per labor ratio thus worsening welfare in dynamically efficient economies. Taking all these effects into account, I analyze under what conditions is a tightening of the link between pension contributions and pension benefits welfare enhancing.

The paper makes three contributions to the literature. First, I show that, if the level of contributions is small, a pension system with no link between contributions and benefits is preferable from a welfare point of view because of the insurance this provides against idiosyncratic earnings shocks. As the size of the pension contribution increases, the welfare gain from insurance is outweighed by the increasing welfare losses from labor supply distortions, so pension systems with a tight link between contributions and benefits, like the NDC system, become desirable. This also implies that countries in which the level of idiosyncratic earnings risk is high should not tighten the link between contributions and benefits.

Second, I highlight the relationship between the strengthening of labor supply incentives and the capital per labor ratio and the subsequent implications for welfare. Because agents work more under the NDC system, the capital per labor ratio is lower. In general equilibrium, this implies an increase in the return of capital and, hence, a higher difference between the return on capital and the return of the pension system in economies that are dynamically efficient in the sense of Cass (1972)[1]. Consequently, NDC pension systems have a more negative impact on welfare from this point of view. The general equilibrium effect highlighted in the paper is extremely important because it implies that a reform towards the NDC pension system may not be welfare enhancing even in economies with low idiosyncratic earnings risk.

Third, my analysis can explain a number of stylized facts. First, it can explain from a welfare perspective why countries with big PAYG pension systems like Italy, Sweden, Germany

[1] An economy is dynamically efficient in the sens of Cass (1972) if the return on capital is higher than the return of the pension system.

and Poland chose to switch to systems with a tight link between pension contributions and benefits, while countries with small pension systems like the UK chose to transform the earnings related part of the pension system in a flat benefit system. Second, the analysis can substantiate from a welfare perspective the empirical relationship between the size of the pension system, on the one hand, and the progressivity of the pension system[2] and the size of uninsurable idiosyncratic risk in an economy, on the other hand. Cross-country data show that there exists: i) a negative relationship between the size of the PAYG pension system and its progressivity and ii) a positive relationship between income inequality prevailing in an economy and the progressivity of pension systems. Starting with Conde-Ruiz and Profeta (2007), the literature explained these stylized facts as the outcome of voting on both the size of the pension system and its progressivity. The analysis in the present paper indicates that these stylized facts can also be substantiated from a welfare perspective considering the trade-off between insurance and labor supply incentives.

In the first part of the paper, I obtain the results presented above in a tractable general equilibrium model with two overlapping generations, idiosyncratic earnings shocks, incomplete markets, endogeneous labor supply of old agents and a PAYG pension system. In order to determine the macroeconomic and welfare implications of tightening the link between pension contributions and benefits, I compare the steady states of two economies with extreme pension system arrangements: i) a flat benefit (FL) system and ii) a NDC system. I show that old agents work more under the NDC system because the contribution is nondistortionary for labour supply. Hence, aggregate labour is higher in the NDC system. However, consumption inequality is also higher in the NDC system, because pension benefits are perfectly linked with the agents' life-time labor income. The FL system brings a higher welfare than the NDC system at low levels of pension contributions due to the insurance it provides. As the size of the pension contribution increases, the welfare losses from labor supply distortions increase outweighing the welfare gains from insurance and the NDC system eventually provides a higher welfare. The threshold contribution up to which the FL system brings a higher welfare increases with the size of the idiosyncratic earnings risk.

The impact of the pension benefit arrangement on the savings rate depends on two opposing factors. On the one hand, because old agents work more under the NDC pension system, they need to save less for their consumption. On the other hand, due to the existence of idiosyncratic earnings shocks, agents make precautionary savings. The level of precautionary savings is higher in the NDC pension system because this type of pension system does not

[2]The tightness of the link between pension contributions and benefits is the main determinant of the *progressivity* of the pension system. If the replacement rate does not vary with life-time earnings, then the pension system entails *zero progressivity*. This is the case of pure NDC and points based pension systems. NDC and points pension systems can also entail some degree of progressivity by including provisions such as minimum pensions or contributions caps. However, cross-country data shows that the degree of progressivity remains low even with such provisions. If the replacement rate decreases with life-time contributions - as is the case under the FL pension system for example - then the pension system is *progressive*.

provide insurance against shocks. Moreover, since the NDC system promotes higher labor supply, the capital per labor ratio is lower and capital crowding out is higher even if the savings rate of agents is constant. I show that the NDC pension system crowds out capital more than the FL system if labor supply is relatively elastic. If labor supply is relatively inelastic, the NDC pension system still crowds out capital accumulation more than the FL pension system as long as the level of idiosyncratic risk is not very high.

Hence, an important finding of the paper is that the strengthening of work incentives in the NDC pension system can come at the expense of lower savings rates and higher capital crowding out. This outcome impacts steady state welfare through general equilibrium effects. In economies that are dynamically efficient in the sense of Cass (1972), because the FL pension system crowds out capital formation less, the difference between the return on the investment in capital and the return on the investment in the pension system is lower than under the NDC pension system. Hence, the FL system has a less negative impact on welfare. The general equilibrium effect reinforces the welfare gains from consumption insurance of the FL pension system at low levels of contributions. As the level of the contribution increases, compared to the NDC pension system, the FL pension system brings higher welfare losses due to labor supply distortions but lower welfare losses through general equilibrium effects. Which of the two effects dominates is a quantitative question.

In the second part of the paper, I build a realistically calibrated model in order to verify the results obtained with the two generations model and to asses the importance of general equilibrium effects in determining the relative welfare of the NDC and FL economies. The full fledged model contains a realistic demographic structure with 80 overlapping generations. Agents start working when they are 20 years old and choose each period how much to save and how much to consume, whether to work or not and if they work how much labor to supply. The probability of surviving to the next age and the population growth rate are calibrated to the US economy. Each period agents are hit by persistent uninsurable earnings shocks. The tax and the pension systems are modeled consistent with the US economy. The US pension system links pension benefits with contribution, but also provides insurance against idiosyncratic shocks because the replacement rate is decreasing with life-time earnings. Hence the US pension system is in-between the NDC system and the Fl system in terms of both insurance and labor supply incentives.

With this realistically calibrated model, I confirm the macroeconomic and welfare implications of reforming a pension system towards a NDC system and a FL system, respectively, obtained with the two overlapping generations model. Compared to the FL system, the NDC system promotes higher labor supply at both intensive and extensive margin, higher consumption inequality and lower capital per labor ratio. Also, the welfare is lower under the NDC pension system. Compared with the current US system, a steady state with a NDC pension system brings a lower welfare while a steady state with a FL system implies a higher

welfare.

The present paper considers both the benefits of PAYG pension systems stemming from better insurance in economies with incomplete markets and the costs of labor supply and savings distortions. Storesletten et al. (2004) show that the US pension system reduces consumption variance with 20% due to the particular, convex shaped link between pension contributions and pension benefits. Imrohoroglu et al. (1995) obtain that, due to its consumption insurance property, a PAYG with a replacement rate of 30% is welfare improving in the US. However, these papers do not consider the labor supply distortions of PAYG pension contributions. Gruber and Wise (1998), Erosa et al. (2012), Wallenius (2013), Alonso-Ortiz (2014), Bagchi (2015) show how different pension system arrangements impact on the labor supply of agents, but do not consider the insurance properties of the pension systems.

The paper is also closely related to the literature analyzing pension reforms in economies with incomplete markets. Nishiyama and Smetters (2007) analyze the consequences of a 50% privatization of the US pension system. They conclude that this is welfare decreasing because of the lower insurance of idiosyncratic productivity shocks. However, if the privatization is accompanied by an increase in the progressivity of pension benefits and is financed by a consumption tax, then it can produce efficiency gains. Also focusing on the US pension system, Huggett and Parra (2010) determine the pension benefit function that is optimal from an ex-ante point of view. They consider benefit functions that are constant, linear or quadratic with respect to life-time income. The results show that the optimal benefit function entails more progressivity than the one currently in place. Fehr and Habermann (2008) and Fehr et al. (2013) show that reforming the current German pension benefit system towards more progressivity is welfare improving because of better insurance of idiosyncratic earnings risk.

I depart from this literature on pension reforms in a number of ways. First, I explicitly analyse the impact of changing the pension benefit arrangement on labor supply, savings and consumption inequality and explain how these macroeconomic outcomes impact on welfare. Second, I aim to explain the cross country heterogeneity in pension benefit arrangements instead of focusing on the characteristics of a single country. Finally, I highlight the fact that not only the insurance offered by pension systems with a loose link between pension contributions and benefits, but also their impact on welfare through general equilibrium effects can render them more efficient.

The rest of the paper is structured in the following way. Section 2 presents the full-fledged model with 80 overlapping generations that I calibrate in Section 4 and simulate in 5. To point out the intuition behind the results, section 3 compares the steady states of the economies with a FL and a NDC pension system using a simplified model with only two overlapping generations. I show that the results obtained with the simplified model carry on to a more realistic setting in section 5. Section 6 illustrates the policy implications of

the paper. It presents cross-country data on the relationship between pension progressivity, on the one hand, and the size of the pension system and of the after tax income inequality, on the other hand, and shows that the empirical correlations concur with the results of our theoretical analysis. Finally, section 7 concludes the paper. All the proofs are presented in the Appendix.

2 The model

The economy is comprised of T=80 overlapping generations. An agent enters the labor market at 20 years and lives until at most 100 years. The probability that an agent survives from age j to $j+1$ is s_{j+1}. The population grows at rate n. Time is discrete and each period represents a year.

2.1 Households

Agents decide how much to consume (c), save (a') and work (l) maximizing their expected utility. At the beginning of each period agents are hit by a persistent idiosyncratic earnings shock z. This is a proxy for involuntary unemployment, a change in match productivity or a health shock. The markets are incomplete, agents cannot insure against these idiosyncratic earnings shocks in any other way than saving or working more.

$$V(x) = \max_{c(x),l(x),a'(x)} u(c(x), l(x)) + \beta s_{j+1} V(x') \tag{1}$$

$$c(x)(1+\tau_c) + a'(x) = a(1 + r(1-\tau_k)) + netw(x) \tag{2}$$

$$a'(x) \geq 0 \tag{3}$$

$$l(x) \in [0, 1) \tag{4}$$

I denote by $x = (j, z, p, a)$ the state of the agent. This is comprised of the age j, the idiosyncratic earnings shock z, the pension claims built p and the stock of capital owned a. The variable $netw(x)$ represents the net earnings of the agent, τ_c is the tax on consumption, τ_k is the tax on the return on capital and r is the return on capital.

Relation (3) shows that agents face a tough constraint: they cannot borrow against future earnings.

2.2 Preferences

I consider a form for the household's utility that is consistent with a balanced growth path:

$$u(c,l) = \begin{cases} \frac{(c^{\eta}(1-l\bar{h}-\theta_P P)^{1-\eta})^{1-\sigma}}{1-\sigma} & , \text{ if } \sigma \neq 1 \\ (\eta ln(c) + (1-\eta) ln(1 - l\bar{h} - \theta_P P)) & , \text{ if } \sigma = 1 \end{cases} \tag{5}$$

Parameters σ and η give the inverse of the intertemporal elasticity of substitution (IES)= $1+\eta(\sigma-1)$ and the Frisch elasticity of labor supply: $\nu = \frac{1-l\bar{h}-\theta_P}{l\bar{h}}\frac{1-\eta(\sigma-1)}{\sigma}$. The maximum proportion of total time devoted to work in a year is represented by $\bar{h}$.

The parameter θ_P measures the disutility of participating to the labor market. This measures the fixed costs required to hold a job such as commuting time. The variable P indicates the participation to the labor market: $P=1$, if $l>0$ and $P=0$, if $l=0$.

2.3 Net labor income

If agents decide to work ($P=1$), they receive a net labor income. This is given by the average wage $w\bar{h}$, the idiosyncratic earnings shock received by the agent z, a deterministic component of the wage that depends on age k_j, the optimally chosen labor supply $l(x)$ and taxation.

$$netw(x) = w\bar{h}zk_jl(x)(1-\tau-\tau_l)+TL \tag{6}$$

The government levies a contribution to the PAYG pension system τ and a labor income tax τ_l and pays agents back a lump sum transfer TL. Due to the presence of the lump sum transfer, the labor income tax system is progressive and provides some insurance against idiosyncratic earnings shocks.

The earnings shock is persistent and lognormally distributed.

$$logz_j = \rho logz_{j-1} + \epsilon, \epsilon \sim N(0,\sigma_\epsilon^2) \tag{7}$$

2.4 Government

The government obtains revenues from taxing consumption $\tau_c C$, labor $\tau_l L$, the return on capital $\tau_k rK$ and from a 100% tax on accidental bequests Beq. These revenues are used to finance government spending G and the lump sum transfers TL. I assume that the government balances the budget in the steady state by adjusting the lump sum transfers TL.

$$\tau_c C + \tau_l L + \tau_k rK + Beq = G + TL \tag{8}$$

2.5 The PAYG pension system

The contributions to the pension system come from a proportional tax on labor income τ. I analyze the macroeconomic and welfare implications of transiting towards two types of pension benefit arrangements.

The first one is the Notional Defined Contribution (NDC) system. This pension system perfectly links life-time contributions with pension benefits. Agents pay contributions proportional to their labor earnings. These contributions are used to build up the agent's claims for the pension benefit that will be paid to him once he retires (relation (9)). Once the agent

reaches the official retirement age, he is entitled to receive the pension benefit computed as his pension claims divided by the expected survival period at retirement s_{ret} measured in years (relation (10)).

$$p(j) = p(j-1)(1+r^P) + \tau w\bar{h}zk_j l(x) \tag{9}$$

$$b(j,p) = \frac{p}{s_{ret}} \tag{10}$$

In line with the way that the NDC system is arranged in practice, the annual accrual rate of the agent's pension claims r^P is given by the growth rate of the aggregate wage bill. This ensures that the budget of the pension system is balanced in the stationary competitive equilibrium. The growth rate of the aggregate wage bill is equal to the gross population growth rate in the stationary equilibrium:

$$1 + r^P = 1 + n \tag{11}$$

The second arrangement that I analyze is the flat benefit system (FL). Under the FL system, the benefit received from the pension system does not depend on the previous contributions of the agent and is constant across agents. Since there is no link between pension contributions and pension benefits, agents do not build pension claims during their working life.

$$p(j) = 0 \tag{12}$$

$$b(j,p) = b \tag{13}$$

I compare the NDC and FL systems, starting from the model calibrated on the US pension system. Contributions are levied proportional to labor income. Benefits in the US system are linked to life-time pension contributions to some extent. More specifically, the pension claims under the US system are based on the average annual earnings in the 35 highest earning years. Since this definition in not tractable in my model, I will compute pension claims based on average annual life-time earnings. This overestimates slightly the tightness of the link between pension contributions and benefits and understates the insurance provided by the pension system.

$$p(j) = \begin{cases} \frac{jp(j-1)+w\bar{h}zk_j l(x)}{j+1}, \text{ if } w\bar{h}zk_j l(x) < c \\ \frac{jp(j-1)+c}{j+1}, \text{ if } w\bar{h}zk_j l(x) \geq c \end{cases} \tag{14}$$

where c is a cap set on pension claims.

Another characteristic of the US pension system is the fact that the ratio between benefits

and pension claims is decreasing with the size of the pension claim (equation (15)). This implies that the return of the pension system is higher for low earning agents who build up smaller pension claims so the US pension system offers some insurance against idiosyncratic earnings shocks. Hence, the US system is in between the FL and the NDC system: it links pension benefits to contributions but also offers insurance through declining replacement rates.

$$b(j,p) = \begin{cases} 0.9p \text{ for } p < b_1 \\ 0.9b_1 + 0.32(p - b_1) \text{ for } p \in (b_1, b_2) \\ 0.9b_1 + 0.32(b_2 - b_1) + 0.15(p - b_2) \text{ for } p > b_2 \end{cases} \tag{15}$$

2.6 Firms

Firms operate a Cobb-Douglas technology $F(A, K, L) = AK^{\alpha}L^{1-\alpha}$. Profit maximization yields the following conditions:

$$r + \delta = \frac{\partial F(A, K, L)}{\partial K} \tag{16}$$

$$w = \frac{\partial F(A, K, L)}{\partial L} \tag{17}$$

where A is the total factor productivity, K is the capital stock, δ is the depreciation rate of capital.

The stationary competitive equilibrium of the economy described in sections 2.1-2.6 is presented in Definition 1.

Definition 1. *Given the exogeneous survival probabilities $\{s_j\}_{j=1,T}$, the population growth rate $\{n\}$ and the government policies $\{G, \tau_c, \tau_l, b\}$, a stationary competitive equilibrium of the model is represented by a set of time-invariant allocations $\{c(x), l(x), a'(x)\}$ for every state x, prices $\{w, r\}$, policies $\{\tau, TL\}$, accidental bequests $\{Beq\}$ and a distribution of agents across states $\mu(x)$ such that:*

1. *Agents take optimal decisions given prices and policies;*

2. *Firms take the optimal decision given prices;*

3. *Bequests are equal to the assets of the deceased:*

$$Beq = \sum_x (1 - s_j)a(x)\mu(x)$$

4. The budget of the pension system balances:

$$w\bar{h}\tau \sum_x l(x)zk_j\mu(x) = \sum_x b(x)\mu(x)$$

5. The government budget balances:

$$\tau_c C + \tau_l L + \tau_k rK + Beq = G + TL$$

6. Capital market clears:

$$K = \sum_x a(x)\mu(x)$$

7. Labour market clears:

$$L = \sum_x l(x)zk_j\mu(x)$$

8. The distribution of agents across states is stationary.

Before I calibrate the model and run the policy experiments, I compare the macroeconomic and welfare implications of having a FL or a NDC pension system in a small scale model in Section 3. Using a very tractable model helps us to point out the intuition of the main results of the paper. Later on, i will show that the results obtained with the small model carry on to a large scale model.

3 A two-overlapping generations model

In this section I consider a small scale version of the model described in Section 2. In this simplified version of the model it is easier to point out the trade-offs implied by different pension benefit arrangements.

I consider that the economy has only T=2 overlapping generations: young (y) and old (o). Population grows at rate $n = 0$ and I abstract from stochastic survival from young age to old age.

Young agents can be thought of as the group of prime-aged workers (25-55 years) who decide how much to consume (c^y) and save (a). To simplify the analysis, I consider that they supply labor inelastically and are not affected by idiosyncratic earnings shocks. Hence all young agents have the same earnings $w\bar{z}$ when young. With the assumption of inelastic labor supply the preferences of young agents will be given only by their consumption $u(c^y)$.

Old agents can be considered as the group of old workers (55-85 years old). At the beginning of their old age, agents are hit by the idiosyncratic earnings shock z with mean $\bar{z}$ and variance σ^2. They decide how much to consume (c^o) and work (l^o), but they do not save.

In a two period model, agents can choose how much labor they supply only at the intensive margin. Hence the preferences of old agents are given by (5), but with no disutility from participating to the labor market $\theta_P = 0$.

I focus on the stationary steady state as described in Definition 1.

The setup of the model presented in this section is related to Harenberg and Ludwig (2015). In contrast to them, I consider the labor supply decision of old agents, but abstract from aggregate shocks. However, the most important difference is that I also consider a NDC pension system and study its macroeconomic and welfare implications in comparison with the FL pension system.

3.1 Labor supply distortions imposed by the pension system

The intratemporal consumption-leisure choice faced by an old agent in the two overlapping generations model is described by the following first order condition:

$$\frac{-u_l(c^o(z), l^o(z))}{u_c(c^o(z), l^o(z))} = \left(wz(1-\tau) + \frac{\partial b(z)}{\partial l^o(z)} \right) \tag{18}$$

If pension benefits are linked to pension contributions, a higher labor supply when old increases pension benefits, i.e. $\frac{\partial b(z)}{\partial l^o(z)} > 0$. Relation (18) shows that such a pension system lowers the labor supply distortion.

Particularizing relation (18) to the FL and the NDC pension system, I obtain:

$$\text{FL:} \frac{-u_l(c^o(z), l^o(z))}{u_c(c^o(z), l^o(z))} = wz(1-\tau) \tag{19}$$

$$\text{NDC:} \frac{-u_l(c^o(z), l^o(z))}{u_c(c^o(z), l^o(z))} = wz \tag{20}$$

Hence, the NDC pension system leaves the labor supply decision of old agents undistorted, while the FL system imposes labor supply distortions. In a model with more than 2 overlapping generations, the NDC pension system distorts the labor supply decision of agents at every age until the official retirement age, but distortions are lower than under the FL system. Although the NDC system perfectly links pension contributions to benefits, it still offers a different rate of return (usually lower) than capital and this distorts the supply of labor.

3.2 A closed form solution

I impose a number of assumptions on the parameters of the model in order to obtain an analytical solution.

Assumption 1. *Capital depreciates fully ($\delta = 1$) and there is no technological progress ($A_t = A_{t+1} = A$). Preferences of agents are represented by the life-time utility described in*

relation (5) *with* $\sigma = 1$ *and* $\theta_P = 0$.

With the above simplifying assumptions I obtain a closed form solution for the model. The resulting aggregate labor and capital per labor ratio are presented in Proposition 1.

Proposition 1. *Under Assumption 1, aggregate labor (l) and capital to labor ratio (k) in the stationary steady state of the economies with a FL pension system and a NDC pension system, respectively, are equal to:*

$$l^{FL} = \frac{(1+\eta)(1-\alpha)(1-\tau)\bar{z}}{1-\alpha\eta-(1-\alpha)\eta\tau} \tag{21}$$

$$k^{FL} = \left(\frac{A\bar{z}(1-\alpha)(1-\tau)s^{FL}}{l^{FL}}\right)^{\frac{1}{1-\alpha}} \tag{22}$$

$$l^{NDC} = \frac{(1+\eta-\tau(1-\eta))(1-\alpha)\bar{z}}{1-\alpha\eta} \tag{23}$$

$$k^{NDC} = \left(\frac{A\bar{z}(1-\alpha)(1-\tau)s^{NDC}}{l^{NDC}}\right)^{\frac{1}{1-\alpha}} \tag{24}$$

where

$$s^{FL} = \frac{\beta\Gamma^{FL}}{\eta+\beta\Gamma^{FL}} \tag{25}$$

$$\Gamma^{FL} = \frac{\alpha}{1-\alpha}l^{FL}E\left[\frac{1}{\frac{\alpha}{1-\alpha}l^{FL}+\tau l^{FL}+z(1-\tau)}\right] \tag{26}$$

$$s^{FL} = \frac{\beta\Gamma^{NDC}}{\eta+\beta\Gamma^{NDC}} \tag{27}$$

$$\Gamma^{NDC} = \frac{\alpha}{1-\alpha}l^{NDC}E\left[\frac{1}{\frac{\alpha}{1-\alpha}l^{NDC}+z+\bar{z}\tau}\right] \tag{28}$$

Labor supply is decreasing in the pension contribution rate τ under both types of pension systems. Equations (19) and (20) show that the pension contribution distorts the intratemporal choice between leisure and consumption in the FL pension system, but not in the NDC pension system. However, the pension contribution lowers the accumulation of savings under both types of pension systems. The smaller amount of capital leads, in equilibrium, to a lower demand for labor also in the case of the NDC pension system.

The pension contribution impacts the capital to labor ratio through three channels: i) the distortion of the intertemporal savings-consumption decision - reflected by the term $1-\tau$ -, ii) the savings rate s and iii) the labor supply l. An increase in the pension contribution lowers the capital to labour ratio through the distortion of intertemporal choices and through the lower savings rate. However, an increase in the pension contribution also lowers labour supply and this triggers an increase in the capital per labor ratio.

Proposition 2 presents the impact of the pension benefit arrangement on aggregate labor and capital to labor ratio.

Proposition 2. *Under Assumption 1:*

1. *Aggregate labour is higher in the economy with a NDC pension system, i.e.* $l^{NDC} > l^{FL}$*;*
2. *In an economy without idiosyncratic earnings risk, the NDC pension system crowds out capital formation more than the FL pension system, i.e.* $k^{NDC}_{\sigma=0} < k^{FL}_{\sigma=0}$*;*
3. *For each* (α, τ)*, there exists* $\eta^*(\alpha, \tau) \in (0, 1]$ *such that:*
   - *if* $\eta < \eta^*(\alpha, \tau)$ *(labor supply is relatively elastic), the capital to labor ratio increases with respect to the size of the idiosyncratic risk more under the FL pension system than under the NDC pension system, i.e.* $\frac{\partial k^{FL}}{\partial \sigma^2} > \frac{\partial k^{NDC}}{\partial \sigma^2}$*. Consequently, the NDC pension system crowds out capital formation more than the FL pension system at any level of idiosyncratic earnings risk, i.e.* $k^{NDC} < k^{FL}, \forall\ \sigma$*;*
   - *if* $\eta > \eta^*(\alpha, \tau)$ *(labor supply is relatively inelastic), the capital to labor ratio increases with respect to the size of the idiosyncratic risk more under the NDC pension system than under the FL pension system, i.e.* $\frac{\partial k^{FL}}{\partial \sigma^2} < \frac{\partial k^{NDC}}{\partial \sigma^2}$*. In this case the FL pension system can crowd out capital formation more than the NDC pension system, i.e.* $k^{NDC} > k^{FL}$*, when the size of idiosyncratic earnings risk is high enough.*

The fact that aggregate labor is higher under the NDC pension system is intuitive, since this type of pension system does not distort the consumption-leisure decision.

The capital prevailing under the two pension systems is determined by two opposing effects. On the one hand, since the NDC pension system eliminates labor supply distortions, people work more when they are old and aggregate labor is higher. Hence, if the savings rate is the same in the two pension systems, the capital per labor ratio in the NDC pension system is automatically lower (equation (23)). However, the proof in the Appendix shows that also the savings rate in the NDC pension system is lower in the absence of idiosyncratic shocks. Because old agents work more in the NDC system, they can insure the same old-age consumption level by making lower savings when young compared to the FL pension system. Considering these effects, the result that capital per labor ratio is lower in the NDC system in the absence of shocks, i.e. $k^{NDC}_{\sigma=0} < k^{FL}_{\sigma=0}$, is intuitive.

On the other hand, as the proof in the Appendix shows, capital is increasing in the size of the idiosyncratic risk under both types of pension systems. In the face of a higher level of idiosyncratic earnings risk, agents make more precautionary savings when young, hence capital increases.

Faced with more volatile earnings, agents also adjust labor supply when old: high earning agents work more, while low earning agents work less. If labor supply is relatively elastic ($\eta < \eta^*$), agents adjust their labor supply easier and hence increase their savings for precautionary reasons to a lesser extent. Consequently, when $\eta < \eta^*$ the NDC pension system economy crowds out capital formation more than the FL pension system at any level of idiosyncratic productivity risk.

If labor supply is relatively inelastic ($\eta > \eta^*$), labor supply is more difficult to adjust and agents rely more on increasing precautionary savings. This effect is more pronounced in an economy with an NDC pension system because this offers lower consumption insurance (Proposition 3). Hence, capital increases with the level of idiosyncratic risk to a larger extent in the NDC pension economy than in the FL economy. It is unclear however, how high the level of risk must be to insure that the precautionary motive dominates and the NDC system crowds out capital formation less than the FL system.

Next, I investigate the insurance provided by the two pension systems.

Proposition 3. *Under Assumption 1, consumption inequality is lower in the FL pension system economy than in the NDC pension system economy. Both types of pension systems lower consumption inequality compared to an economy with no pension system.*

The FL pension system lowers consumption inequality because it provides a return that decreases with agents' earnings. This comes from the fact that contributions are proportional to earnings , but benefits are flat. The NDC pension system, however, does not provide direct consumption insurance, because pension benefits are perfectly linked to contributions and, hence, to earnings. Compared to an economy without a pension system, the reduction in consumption inequality in the NDC pension system is brought about by the fact that due to capital crowding out firms also reduce demand for labor. Hence, all types of agents retire earlier in the economy with a NDC pension system, lowering their exposure to idiosyncratic earnings shocks.

3.3 Welfare

I turn to assessing the impact that the different macroeconomic outcomes have on the ex-ante utility of agents living under the two types of pension systems. I consider the implication of increasing the contribution to the pension system for the utility that agents have at birth:

$$SWF = U(c^y) + \beta EU(c^o, l^o)$$

The following proposition presents the trade-off among consumption insurance, labor supply and capital accumulation that defines the welfare in the two economies.

Proposition 4. *Under Assumption 1, the change in welfare caused by a marginal increase in the contribution to the pension system is composed of the following three terms:*

1. *the welfare loss from lower consumption due to the higher contribution to the pension system (ω_1');*
2. *the welfare gain from higher consumption due to the higher pension benefit (ω_1'');*
3. *the impact of capital crowding out (general equilibrium effects - ω_2).*

	FL	*NDC*
ω_1'	$-\frac{\eta}{1-\tau} - \frac{\beta\eta}{1-\tau}$	$-\frac{\eta}{1-\tau} - \frac{\beta\Gamma^{NDC}}{1-\tau}$
ω_1''	$\beta\frac{1-\alpha}{\alpha}\Gamma^{FL}$	$\beta\frac{1-\alpha}{\alpha}\bar{z}\frac{\Gamma^{NDC}}{l^{NDC}}$
ω_2	$\frac{\partial ln(k^{FL})}{\partial\tau}\Phi^{FL} + \frac{\partial ln(s^{FL})}{\partial\tau}\frac{\beta\tau\Gamma^{FL}(1-\alpha)}{\alpha}$	$\frac{\partial ln(k^{NDC})}{\partial\tau}\Phi^{NDC}$
Φ	$\eta\alpha(\beta+1) - \beta(\alpha+\tau(1-\alpha))\frac{1-\alpha}{\alpha}\Gamma^{FL}$	$\eta\alpha(\beta+1) - \beta(1-\alpha)\Gamma^{NDC}$

$$\omega_1 = \omega_1' + \omega_1'' \tag{29}$$

$$\frac{\partial SWF}{\partial\tau} = \omega_1 + \omega_2 \tag{30}$$

An increase in the contribution to the pension system produces a reallocation of the income sources for consumption: net wages decrease while pension benefits increase. The welfare loss due to lower consumption from net wages is measured by the term ω_1', while the welfare gain of consumption from higher pension benefits is given by ω_1''. The term ω_1'' increases with Γ and, hence, with the size of the idiosyncratic risk and of consumption inequality in the economy (see proof of Proposition 2 for the positive relationship between Γ and σ). Intuitively, the higher is consumption inequality in the economy, the more valuable is the insurance provided by the pension system and the higher is the present value of the additional pension benefit agents will receive. I call $\omega_1 = \omega_1' + \omega_1''$ *the consumption reallocation effect.*

A higher contribution reduces savings and, hence, crowds out capital formation: $\frac{\partial ln(k)}{\partial\tau} < 0$. The lower level of capital to labor ratio impacts on welfare in general equilibrium through factor prices: the return on capital increases (positive welfare effect measured by $\beta(\alpha + \tau(1-\alpha))\Gamma^{FL}$ and $\beta(1-\alpha)\Gamma^{NDC}$, respectively) and wages decrease (negative welfare effect measured by $\eta\alpha(\beta+1)$). The net effect is captured by the term Φ whose sign is closely related to the dynamic efficiency of the economy. More precisely, the proof in Appendix 2 shows that $\Phi_{\sigma=0,\tau=0} > 0$ if and only if the economy is dynamically efficient in the absence of idiosyncratic shocks and the pension system. In this case, the introduction of the PAYG pension system has a negative impact on welfare through *general equilibrium effects*, i.e. $\omega_2 < 0$.

Proposition 5 shows that in the absence of idiosyncratic shocks and based only on the consumption reallocation effect ω_1, the NDC pension system dominates from a welfare perspective the FL system at any level of pension contribution. This is because it promotes lower labor supply distortions. However, the general equilibrium effects captured by ω_2 favor the FL system, if the economy is dynamically efficient.

Proposition 5. *Under Assumption 1, in the absence of idiosyncratic earnings risk* ($\sigma = 0$)*, a marginal increase in the contribution to the pension system has a lower negative impact on welfare through the consumption reallocation effect under the NDC system than under the FL system, i.e.* $\omega_1^{NDC}|_{\sigma=0} \geq \omega_1^{FL}|_{\sigma=0}$*. At the marginal introduction of the pension system, the impact on welfare of the two pension systems is the same, i.e.* $\omega_1^{NDC}|_{\sigma=0,\tau=0} = \omega_1^{FL}|_{\sigma=0,\tau=0}$*.*

If the economy is dynamically efficient ($\Phi_{\sigma=0,\tau=0} > 0$)*, a marginal increase in the contribution to the pension system has a more negative impact on welfare through general equilibrium effects in the case of the NDC pension system* ($\omega_2^{NDC}|_{\sigma=0} < \omega_2^{FL}|_{\sigma=0}$)*.*

Two effects determine the general equilibrium result. First, the NDC system crowds out capital formation more than the FL system in the absence of idiosyncratic risk (Proposition 2), i.e. $\frac{\partial ln(k^{NDC})}{\partial \tau} < \frac{\partial ln(k^{FL})}{\partial \tau} < 0$. Second, the impact of the FL system on welfare through capital crowding out is lower, i.e. $\Phi_{\sigma=0}^{FL} < \Phi_{\sigma=0}^{NDC}$.

In the absence of idiosyncratic shocks, at the marginal introduction of the pension system ($\tau = 0$), the impact on welfare through the consumption reallocation effect is the same in the case of the two pension systems ($\omega_1^{FL}|_{\sigma=0,\tau=0} = \omega_1^{NDC}|_{\sigma=0,\tau=0}$). However the general equilibrium effect brings a higher welfare in the case of the FL system ($\omega_2^{FL}|_{\sigma=0} > \omega_2^{NDC}|_{\sigma=0}$). In conclusion, in the absence of idiosyncratic shocks, at the marginal introduction of the pension system, the FL system dominates the NDC system from a welfare perspective.

However, as τ increases past the marginal introduction of the pension system, the consumption reallocation effect ω_1 has a less negative impact on welfare in the case of the NDC system. This is due to the lower labor supply distortions promoted by this system. Hence, when τ increases, the consumption reallocation effect and the general equilibrium effect ω_2 act in opposite directions favoring the NDC system and the FL system, respectively. The overall impact on welfare is not straightforward. I will answer this question through quantitative exercises performed with the small scale model in Section 3.4 and with the big model in Section 5.

Proposition 6 shows how ω_1 changes as the level of idiosyncratic risk increases.

Proposition 6. *A higher level of idiosyncratic risk increases the welfare gains (or decreases the welfare losses) from the consumption reallocation effect under the FL system more than under the NDC system, i.e.* $\frac{\partial(\omega_1^{FL} - \omega_1^{NDC})}{\partial \sigma^2} > 0$*.*

The consumption reallocation effect ω_1 increases with the size of idiosyncratic risk σ, because the value of the pension system as an insurance device captured by ω_1'' increases. Hence, welfare losses brought by the pension systems decrease or welfare gains increase. The consumption reallocation effect ω_1 increases more in the case of the FL system due to the better insurance provided by this pension system. Hence, starting from $\sigma = 0$, increasing the level of idiosyncratic risk will lead to $\omega_1^{FL} > \omega_1^{NDC}$ for low levels of τ.

In the absence of idiosyncratic risk, the welfare losses brought by general equilibrium effects ω_2 are higher under the NDC system. In the presence of idiosyncratic risk, the difference

between the NDC and the FL pension system is expected to decrease, but still persist at least for low values of σ. This happens for two reasons.

First, the FL pension system crowds out capital formation less than the NDC system for low levels of σ and the difference between the capital per labor ratio under the two systems increases with the size of τ (see proof of Proposition 2). Second, compared to the NDC system, the impact of the FL pension system on welfare through general equilibrium effects contains the additional term $\Omega = \beta\tau\Gamma^{FL}\frac{\eta(1-\alpha)^2}{\alpha(1-\alpha\eta-\tau\eta(1-\alpha))}$ (see relations (41) and (42) in Appendix 1) that is always positive and increases with the level of pension contributions.

In conclusion, the analysis performed with the small scale model indicates that the FL pension system brings a higher welfare than the NDC pension system at low levels of pension contributions due to both consumption insurance and general equilibrium effects. The NDC pension system eliminates the deadweight loss stemming from labor supply distortions, but it is a quantitative question whether this effect dominates the insurance and general equilibrium effects at high levels of pension contributions. I will assess the relative importance of insurance, labor supply distortions and general equilibrium effects in a calibrated version of the small scale model in the next subsection and in a calibrated version of the full fledged model in Section 5.

3.4 A numerical example

In order to illustrate better the implications of the FL and the NDC pension system on welfare, I present in this subsection a calibrated version of the small scale model. I consider a lognormal distribution for the idiosyncratic productivity shock $z = e^y, y \sim N(\mu, \sigma^2)$ and assume that a period of the model represents 30 years. Following the relevant literature (Storesletten et al. (2004), French (2005), Kaplan (2012)), I set the locational parameters of z by assuming the following values at yearly frequency: $\mu_y = 0$ and $\sigma_y^2 = 0.014$. The value for the variance of the earnings shock is consistent with estimates for the persistent part of the earnings process. Since we considered young agents to be homogeneous, we abstract from the permanent earnings shocks that affect agents when entering the labor market. We calibrate $\beta = 0.99^{30}$ and $\alpha = 0.3$ in line with Song (2011), Gonzalez-Eiras and Niepelt (2008), Harenberg and Ludwig (2015). The leisure share is set to $1 - \eta = 0.65$. For this value of η, at the contribution rate of 10.6% currently in place in the US, old agents choose to work on average 6 years under the FL system and 7 years under the NDC system. This implies a retirement age of 61 and 62 years, respectively.

For the set of parameters I chose, the capital to labor ratio is below the first best level in the absence of the pension system (Figure 1). Hence, the economy is dynamically efficient. The NDC pension system crowds out capital more than the FL pension system at every level of contribution to the pension system (τ). In Proposition 2 we pointed out that if labor supply is not very elastic, the NDC pension system could crowd out capital less than the FL

pension system if the size of the idiosyncratic risk is very high. We find that this does not happen in the small scale model for a realistically calibrated value of σ.

Figure 1: Capital to labor ratio

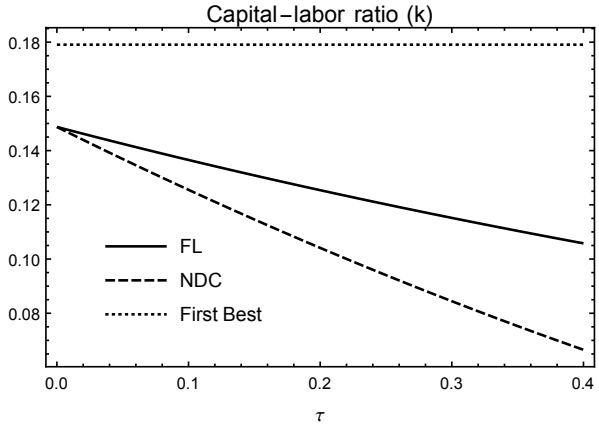


Figure 2 presents the difference in ex-ante utility between the FL and the NDC pension economies at different levels of pension contribution. The welfare change caused by a marginal increase in the contribution to the pension system is shown in Figure 3.

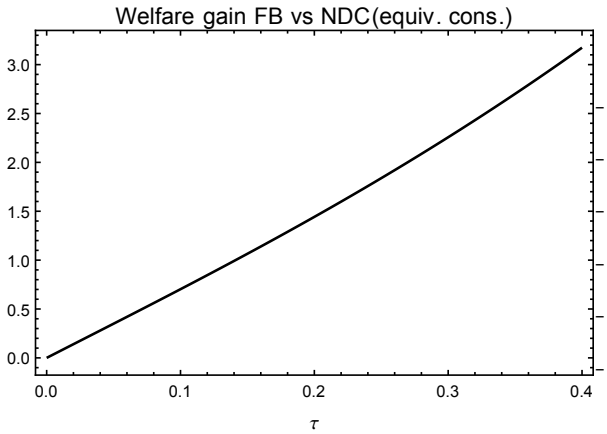


Figure 2: Difference in welfare

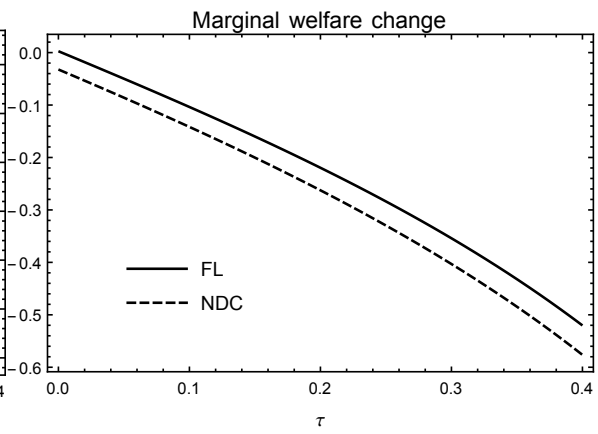


Figure 3: Marginal welfare change

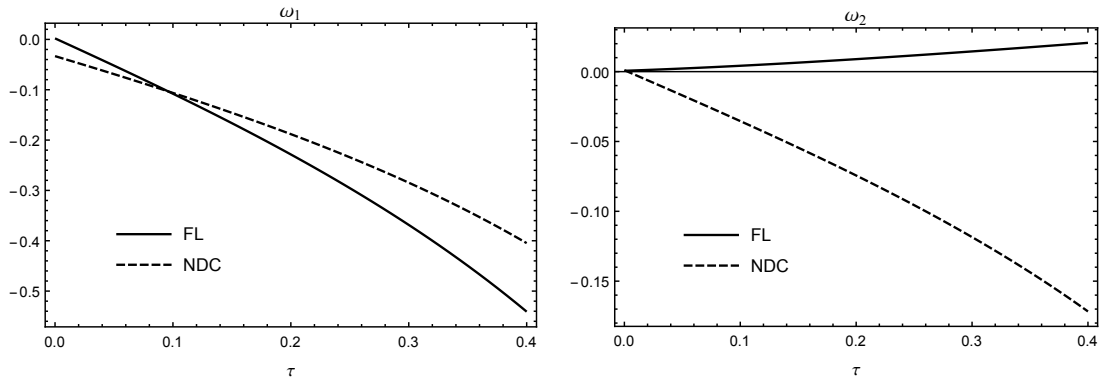
The FL pension system brings a higher welfare than the NDC pension system at all levels of pension contribution ($SWF^{FL} - SWF^{NDC} > 0$). To see why this happens, we analyse the two components of the marginal welfare change identified in Proposition 4. These are illustrated in Figure 4, panels (a) and (b).

First, as anticipated by Proposition 5, the FL pension system has a lower negative impact on welfare through consumption reallocation ($\omega_1^{FL} > \omega_1^{NDC}$) if the level of contribution is small (Figure 4, panel (a)). A marginal increase of the FL pension system leads to lower welfare losses if the contribution is less than 10%. This is due to the insurance that the

FL pension provides. As the level of pension contribution increases, the welfare loss from the higher labor supply distortions of the FL pension system increases and the welfare gain from insurance decreases. Consequently, for high levels of pension contributions, the marginal welfare loss from consumption reallocation becomes higher in the FL system ($\omega_1^{FL} < \omega_1^{NDC}$).

Figure 4: The components of marginal welfare change

(a) Consumption reallocation effect (ω_1) (b) General equilibrium effect (ω_2)



Second, as Figure 4 panel (b) shows, the FL pension system has a less negative impact on welfare through general equilibrium effects at any level of pension contribution ($\omega_2^{FL} > \omega_2^{NDC}$). This result was anticipated by Proposition 6 and is due to the fact that the NDC pension system crowds out capital formation more than the FL pension. Moreover, as the level of pension contribution τ increases, the negative impact on welfare through general equilibrium effects amplifies under the NDC pension system, but decreases under the FL pension system. Two effects contribute to this result: i) the difference in capital to labor ratio between the two pension systems increases with the level of pension contribution (Figure 1 and Proposition 2) and ii) the impact of the FL pension system on welfare through general equilibrium effects contains the additional term Ω (relation (42)) that is always positive and increases at all levels of pension contributions.

Overall, general equilibrium effects ensure that the FL pension system brings a higher welfare also at high levels of pension contributions.

4 Calibration

This section lays down the calibration of the model presented in Section 2. The parameters of the model are calibrated to match key variables of the US economy (table 1). The annual population growth rate is equal to 1.2%, the average for the US over the past 40 years. The survival probabilities s_j are taken from Bell and Miller (2005).

Table 1: Calibration

Parameter	Description	Value	
n	Population growth rate	0.012	
α	Capital share in output	0.35	
ρ	Autoregressivity of earnings process	0.97	
σ_ϵ^2	Variance of earnings process	0.02	
G/Y	Government expenditure to GDP	0.18	
τ_c	Consumption tax	0.05	
τ_k	Return on capital tax	0.36	
τ_l	Labor income tax	0.17	
Parameter	Description	Value	Target
δ	Depreciation rate	0.06	I/Y=0.22
β	Time preference	0.998	K/Y=3.0
σ	Preference parameter	3	$IES = 0.5$
η	Consumption share	0.55	Hours worked across the life-cycle
$\theta_P(j)$	Disutility of labor force participation	$0.1 + 0.2j^2$	Participation rate across the life-cycle
$\bar{h}$	Maximum hours worked	0.45	
τ	Contribution to PAYG pensions	0.11	Balanced budget
TL/Y	Lump sum transfers to GDP	0.05	Balanced budget

The characteristics of the earnings process are taken from Kitao (2014): the autoregressivity $\rho = 0.97$, the variance $\sigma_\epsilon^2 = 0.02$, the deterministic part of the earnings process that depends on age k_j.

The government expenditure is equal to 18% of GDP, the average of the variable for the US economy. The depreciation rate is set to $\delta = 0.06$ by targeting the value of 22% for the ratio of investment to GDP. The time preference $\beta = 0.998$ is obtained by targeting a capital to output ratio of 3.0. The capital share in output is $\alpha = 0.35$.

The consumption tax and return on capital tax are taken from the US tax code: $\tau_c = 0.05$ and $\tau_k = 0.36$. The contribution to the pension system is obtained from the condition of a balanced budget. The resulting figure is $\tau = 0.108$, close to the actual contribution of 0.106 currently in place in the US. The pension benefit is given by formula (15) where the benefit intervals are the ones corresponding to the year 2002: $b_1 = 7104$ USD, $b_2 = 42804$ USD. The pension system insures a replacement rate of 42%, slightly higher than the value of 39% estimated by OECD (2005).

For the labor income tax, I take the value of 0.28 used by Trabandt and Uhlig (2011) and deduct the equilibrium social security contributions of 0.11. This implies $\tau_l = 0.17$.

In the simulations, agents are allowed to take up social security benefits starting with the early retirement age currently set in the US at 62 years. Although agents face penalties in the form of permanent benefit reductions and an earnings test if they draw social social security

benefits before the full retirement age of 66, the data shows that around 50% of agents retire at the early retirement age.

To calibrate the preferences of individuals, I set the intertemporal elasticity of substitution to 0.5, in line with the literature. For the consumption share η and the disutility of labor force participation $\theta_P(j)$, I use two targets from the data: i) the number of hours worked across the life-cycle and ii) the labor force participation rate across the life-cycle. To compute the number of hours worked and the labor force participation across the life-cycle, I use data for men from the 1962-2016 March CPS. Figure 5 shows the calibrated labor force participation rate versus the actual data, while figure 6 presents the calibrated hours worked across the life-cycle versus the actual data.

Figure 5: Labor force participation rate

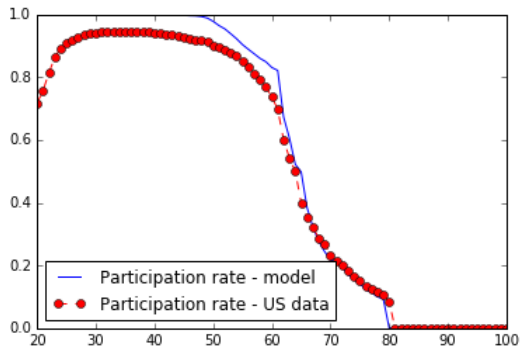
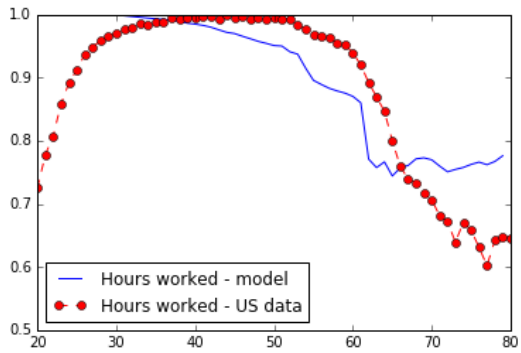


Figure 6: Hours worked



The calibration overestimates the labor force participation rate until the official retirement age. This is because the labor force participation across the life-cycle is influenced by factors that are not in the model such as education decisions (especially for agents between 20 and 30 years), disability and discouragement.

5 Simulations

With the model calibrated as described in Section 4, I perform a number of simulations. To compare the welfare under the three types of pension benefit arrangements, I use the concept of *ex-ante* utility or utility of agents at birth.

$$SWF = E_0 \sum_{j=1}^{T} s_{j+1} \beta^i u(c_j, l_j) \tag{31}$$

I investigate the macroeconomic and welfare consequences of replacing the US benefit arrangement, in turn, with a NDC system and a FL system. I compare the stationary equilibrium of the model calibrated for the US economy with the steady states of the same econ-

omy but with the two alternative benefit arrangements. Table 2 presents the macroeconomic aggregates under the three types of pension systems.

Table 2: Macroeconomic implications of changing the benefit function

	Benchmark (US system)	NDC	FL
Interest rate	5.7%	5.6%	5.5%
Capital per labor	-	0.9%	1.2%
Labor	-	0.4%	-1.3%
Welfare - consumption eq.	-	-0.9%	1.4%

The results obtained with the simplified model in Section 3 are confirmed in the full fledged model. The NDC system promotes higher labor supply than both the US system and the FL pension system because it lowers labor supply distortions. Figure 7 and 8 show that the NDC system strengthens work incentives at the intensive and the extensive margin: agents work more hours and stay longer in the labor force under this pension system.

The development of consumption inequality across the life-cycle under the three types of pension arrangements is in line with the results obtained in the small model (Figure 9). Consumption inequality is the highest under the NDC pension system because this does not provide insurance against idiosyncratic shocks and the lowest under the FL pension system. The US pension system is in-between the NDC and the FL system in terms of insurance against idiosyncratic shocks.

The NDC system crowds out capital formation more than the FL pension system but less than the US system. Using the two overlapping generations model of section 3, we obtained that the NDC pension system crowds out capital formation more than the FL system as long as the level of idiosyncratic risk is not too high.

Figure 7: Labor force participation rate

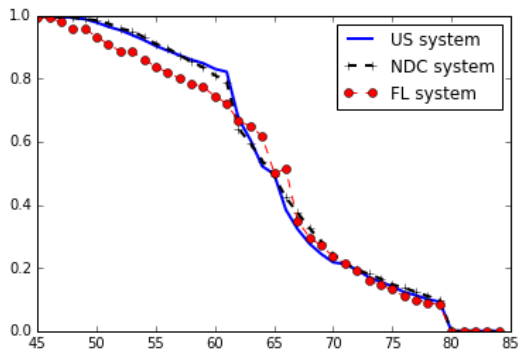


Figure 8: Hours worked

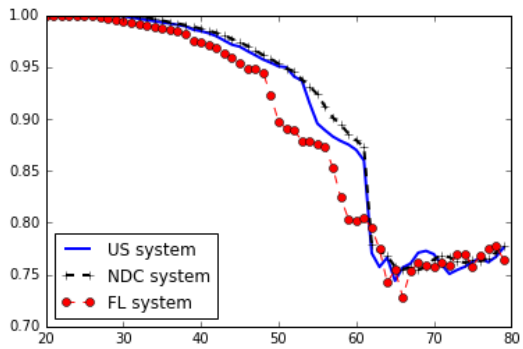
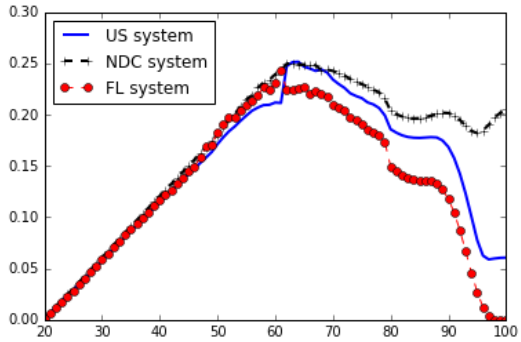


Figure 9: Consumption inequality



Overall, individuals are worse off in the steady state of the NDC economy than under the current US pension system: agents need a compensation of 0.9% of annual consumption in order to achieve the same welfare as in the initial steady state. Although the NDC system strengthens work incentives at both the intensive and extensive margin and crowds out capital formation less than the US system, it provides no insurance against idiosyncratic shocks and welfare is lower. The converse is true under the FL system: individuals have a higher ex-ante utility than under the US system. The FL system provides more insurance against idiosyncratic shocks and crowds out capital formation less than the US system.

6 Policy implications

This section points out the policy implications of the analysis performed in this paper. A first important conclusion is that one can substantiate from a welfare point of view the contrasting reforms implemented in various countries in the past 25 years. Countries with large PAYG pension systems like Italy, Sweden, Germany and eastern European countries switched to NDC or points pension systems in order to improve labor supply incentives. Countries with small PAYG systems like the UK switched to a flat benefit system preferring insurance to the strengthening of labor supply incentives.

As most of pension progressivity stems from the tightness of the link between pension contributions and benefits, our paper also has a policy implication regarding the cross country heterogeneity in pension progressivity. The literature starting with Conde-Ruiz and Profeta (2007) pointed out a negative relationship between the progressivity of pension systems and the size of the pension system and a positive relationship between the progressivity of the pension system and the size of income inequality.

I also illustrate the empirical relationships using cross-country data for the following countries: Sweden, France, Germany, Netherlands, Austria, Belgium, Finland, Italy, Spain, UK, US, Canada, Australia, New Zealand, Japan. As a proxy for the size of the idiosyncratic

earnings risk, I consider the ratio between the 90th percentile and the 10th percentile of disposable income (income after taxes and transfers) for the population aged 18-64 (working age). The data set is obtained from the OECD database and I compute the average of the data recorded for 1974-2012. The pension progressivity index is taken from OECD Pensions at a Glance (2005)[3]. The size of the pension system is proxied by the gross replacement rate of the individual with an average income provided by OECD Pensions at a Glance (2005).

Figure 10 presents the relationship between the progressivity and the size of the pension system. The figure documents a considerable heterogeneity in the progressivity of pension systems and it indicates a strong negative correlation between pension progressivity and the size of the pension system. The present paper substantiates this negative relationship from a welfare point of view: small pension systems are very progressive because the welfare gains from insurance dominate the welfare losses from labor supply distortions, while large pension system entail little progressivity in order to alleviate labor supply distortions.

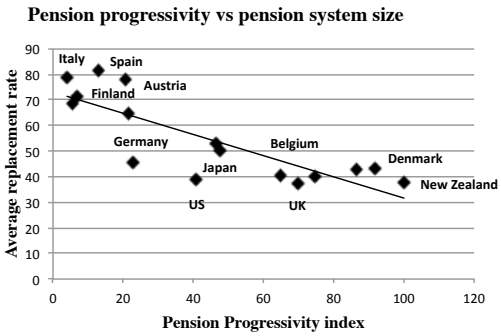


Figure 10

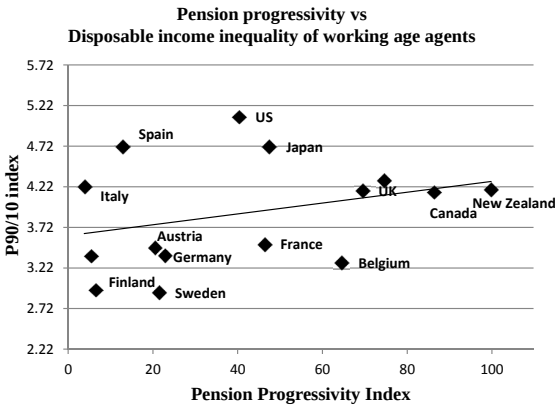


Figure 11

Figure 11 shows the relation between disposable income inequality of working age agents and pension progressivity. The relationship between the two variables is positive, indicating that countries with a higher income inequality have more progressive pension arrangements. The present paper also substantiates this positive relation from a welfare perspective, indicating that countries that have a higher level of uninsurable idiosyncratic risk should implement more progressive pension systems.

7 Conclusion

In this paper I analyzed the macroeconomic and welfare implications of two extreme types of pension benefits arrangements: the FL and NDC pension systems. I found that the FL

[3]In the paper I analyze only the implications of pension benefit progressivity. However, the OECD pension progressivity index also considers the progressivity of pension contributions.

system leads to lower aggregate labor, but also to lower consumption inequality. As long as the level of idiosyncratic risk is not very high, the FL system also crowds out capital formation less than the NDC system.

Comparing the welfare under the two pension systems, if I abstract from general equilibrium effects, the FL pension system brings a higher welfare at low levels of pension contributions. As the level of pension contribution becomes higher, the NDC pension system brings a higher welfare due to the lower labor supply distortions. This result can explain why countries with a high size of the PAYG pension system like Germany, Italy, France and Poland switched to a perfect link between pension contributions and pension benefits. Countries with a small size of the pension system like the UK prefer a FL pension system because of the insurance it provides. The impact of the pension systems on welfare through general equilibrium effects tilt the balance in favor of the FL pension system in dynamically efficient economies.

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Appendix 1

Proof of Proposition 1

The optimization problem of the household in the model with only 2 overlapping generations is the following:

$$\max_{c^y, c^o(z), l^o(z), a_t} u(c^y) + \beta E u(c^o(z), l^o(z))$$
$$c^y + a = w\bar{z}(1-\tau)$$
$$c^o(z) = a(1+r) + wzl^o(z)(1-\tau) + b(z)$$
$$l^o \in [0, 1)$$

The first order conditions of the household's problem are:

$$\frac{-u_l(c^o(z), l^o(z))}{u_c(c^o(z), l^o(z))} = \left(wz(1-\tau) + \frac{\partial b(z)}{\partial l^o(z)} \right) \tag{32}$$

$$u_c(c^y) = \beta(1+r)E[u_c(c^o(z), l^o(z))] \tag{33}$$

With the preferences considered in Assumption 1, using (32) and the budget constraints of the agents and government, we find closed form solutions for consumption and labor as a function of savings and prices:

FL	NDC
$c^y = w\bar{z}(1-\tau) - a$	$c^y = w\bar{z}(1-\tau) - a$
$c^o(z) = \eta(a(1+r) + wz(1-\tau) + w\tau l)$	$c^o(z) = \eta(a(1+r) + wz + w\bar{z}\tau)$
$l^o(z) = \eta - (1-\eta)\frac{a(1+r)+w\tau l}{wz(1-\tau)}$	$l^o(z) = \eta - (1-\eta)\frac{a(1+r)+w\bar{z}\tau}{wz}$

Dividing the first order conditions of the firm's problem (16) and (17), we get a relation for the return on capital that we substitute in the formulas for consumption and labor supply:

$$1 + r = \frac{\alpha}{1-\alpha}\frac{l}{a}w$$

We obtain:

FL	NDC
$c^y = w\bar{z}(1-\tau) - a$	$c^y = w\bar{z}(1-\tau) - a$
$c^o(z) = \eta w(\frac{\alpha}{1-\alpha}l + z(1-\tau) + \tau l)$	$c^o(z) = \eta w(\frac{\alpha}{1-\alpha}l + z + \bar{z}\tau)$
$l^o(z) = \eta - (1-\eta)\frac{\frac{\alpha}{1-\alpha}l+\tau l}{z(1-\tau)}$	$l^o(z) = \eta - (1-\eta)\frac{\frac{\alpha}{1-\alpha}l+\bar{z}\tau}{z}$

The expressions for consumption are substituted in (33). We solve for the level of assets:

$$a = \frac{\beta\Gamma}{\beta\Gamma + \eta}\bar{z}(1-\tau)(1-\alpha)\left(\frac{a}{l}\right)^{\alpha} \Rightarrow k = \left(\frac{s\bar{z}(1-\tau)(1-\alpha)}{l}\right)^{1-\alpha} \tag{34}$$

where $\Gamma^{FL} = \frac{\alpha}{1-\alpha} lE\left[\frac{1}{\frac{\alpha}{1-\alpha}l+z(1-\tau)+\tau l}\right]$, $\Gamma^{NDC} = \frac{\alpha}{1-\alpha} lE\left[\frac{1}{\frac{\alpha}{1-\alpha}l+z+\bar{z}\tau}\right]$ and $s = \frac{\beta\Gamma}{\beta\Gamma+\eta}$.

We aggregate individual labor supply:

$$l = \bar{z} + \int_i z_i l_i dF(z_i) \Rightarrow$$

$$l^{FL} = \frac{(1+\eta)(1-\alpha)(1-\tau)\bar{z}}{1-\alpha\eta-\eta\tau(1-\alpha)} \tag{35}$$

$$l^{NDC} = \frac{(1+\eta-(1-\eta)\tau)(1-\alpha)\bar{z}}{1-\alpha\eta} \tag{36}$$

Proof of Proposition 2

Using (21) and (23), we get:

$$l^{NDC} - l^{FL} = \frac{(1-\alpha)(1-\alpha(1-\tau)-\tau)\tau\eta(1-\eta)}{(1-\alpha\eta)(1-\alpha\eta(1-\tau)-\tau\eta)} > 0$$

Using (22) and (24), we show that capital accumulation is lower in the NDC pension system in absence of idiosyncratic productivity shocks ($\sigma = 0$) at all levels of pension contributions:

$$sgn\left(k^{NDC}_{\sigma=0} - k^{FL}_{\sigma=0}\right) = sgn\left(\left(\frac{k^{NDC}_{\sigma=0}}{k^{FL}_{\sigma=0}}\right)^{1-\alpha} - 1\right) =$$

$$= sgn\left(-\frac{\tau\eta(1+\alpha\beta)(1-\eta)(1+\alpha+\tau(1-\alpha)))}{(1-\alpha\eta-\tau\eta(1-\alpha))(\eta(1+\alpha)+\alpha\beta(1+\eta)+\tau(\eta(1-\alpha)-\alpha\beta(1-\eta)))}\right) = -$$

Using the logarithm version of (22) and (24) , we show that, in absence of idiosyncratic productivity shocks ($\sigma = 0$), an increase in the contribution to the pension system reduces capital under the NDC pension system more than under the FL pension system:

$$\frac{\partial(ln(k^{NDC}_{\sigma=0}) - ln(k^{FL}_{\sigma=0}))}{\partial\tau} =$$

$$-\frac{\eta(1+\alpha\beta)(1-\eta)[(1-\alpha)(\eta+1)(1-\tau)\alpha\beta\tau + (\eta(1+\alpha)+\alpha\beta(\eta+1)+\tau\eta(1-\alpha))(1+\alpha(1-\tau)+\tau))]}{(1-\alpha)(1-\tau)(\eta(1+\alpha)+\alpha\beta(\eta+1)+\tau\eta(1-\alpha))(\eta(1+\alpha)+\alpha\beta(\eta+1)+\tau(\eta(1-\alpha)-\alpha\beta(1-\eta)))(1-\alpha\eta-\tau\eta(1-\alpha))} < 0$$

We compute the change of the capital stock with respect to the variance of the idiosyncratic risk:

$$\frac{\partial k^{1-\alpha}}{\partial\sigma^2} = \frac{A(1-\alpha)(1-\tau)}{l}\frac{\partial s}{\partial\sigma^2} = \frac{A(1-\alpha)(1-\tau)\eta\beta}{l(\eta+\beta\Gamma)^2}\frac{\partial\Gamma}{\partial\sigma^2}$$

The relationship above holds for both the FL and the NDC economy.

We determine the sensitivity of Γ with respect to the size of the idiosyncratic shock by taking a second order Taylor expansion around $(E(z) = 1, \tau)$ and differentiating with respect

to σ^2:

$$\frac{\partial \Gamma^{FL}}{\partial \sigma^2} \approx \frac{\partial^2 \Gamma^{FL}}{\partial z^2}\bigg|_{\sigma=0} = \frac{\alpha}{1-\alpha} l^{FL} \frac{(1-\tau)^2}{\left(\frac{\alpha}{1-\alpha} l^{FL} + \tau l^{FL} + 1 - \tau\right)^3} > 0 \tag{37}$$

$$\frac{\partial \Gamma^{NDC}}{\partial \sigma^2} \approx \frac{\partial^2 \Gamma^{NDC}}{\partial z^2}\bigg|_{\sigma=0} = \frac{\alpha}{1-\alpha} l^{NDC} \frac{1}{\left(\frac{\alpha}{1-\alpha} l^{NDC} + 1 + \tau\right)^3} > 0 \tag{38}$$

$$\frac{1}{l^{FL}} \frac{\partial \Gamma^{FL}}{\partial \sigma^2} - \frac{1}{l^{NDC}} \frac{\partial \Gamma^{NDC}}{\partial \sigma^2} \approx \frac{\alpha \tau P(\eta)}{(1-\alpha)(1-\tau)(1+\tau+\alpha(1-\tau))^3} \tag{39}$$

where

$$P(\eta) = 1 - 3\eta + 3\eta^2(1 - (1-\tau)(1-\alpha)^2) + \eta^3(-3\alpha(1-\tau)(\alpha(1-\tau)+\tau) - \tau^2 + \alpha^3(2 - 3\tau + \tau^2))$$

From relations (37) and (38), we obtain that capital accumulation increases with the level of idiosyncratic risk in both economies.

It is straightforward to show that:

$$P(0) = 1 > 0, P(1) = -(1-\alpha)^3(2-\tau)(1-\tau) < 0$$
$$P(\eta) = 0 \text{ has discriminant } \Delta = -27(1-\alpha)^6(1-\tau)^2\tau^2 < 0$$

Consequently $P(\eta) = 0$ has one real solution $\eta^*(\alpha, \tau)$ and $\eta^*(\alpha, \tau) \in (0, 1]$.

We distinguish 2 cases:

- $\eta < \eta^*(\alpha, \tau)$: $P(\eta) > 0$ and capital increases faster in the FL economy than in the NDC economy when the size of idiosyncratic risk increases. Hence for these values of labor supply elasticity, the NDC pension system crowds out capital formation more than the FL pension system at any level of idiosyncratic productivity risk.

- $\eta > \eta^*(\alpha, \tau)$: $P(\eta) < 0$ and capital in an NDC pension system economy increases faster than in an FL pension system economy. Hence, although $k^{NDC}(0, \tau) < k^{FL}(0, \tau)$, a sufficiently high increase in the level of idiosyncratic risk may lead to a lower capital to labor ratio in the case of the FL pension system.

Proof of Proposition 3

We consider as a measure of consumption inequality the ratio of two quantiles of the consumption distribution:

$$c_{inequality} = \frac{c_H}{c_L}$$

where $z_H - z_L = 2\sigma$.

Using the closed for solutions for c^H and c^L obtained in the proof of Proposition 2, we obtain the following relations:

$$c^{FL}_{inequality} - c^{NDC}_{inequality} = -\frac{2(1-\alpha)\sigma\tau\eta(\tau+1+\alpha(1-\tau))}{(1+\tau+\alpha(1-\tau)-\sigma(1-\alpha\eta))(1+\tau+\alpha(1-\tau)(\sigma\eta+1)+\eta-\sigma(1-\tau))} < 0$$

$$c^{FL}_{inequality} - c^{FL}_{inequality}|_{\sigma=0} = -\frac{2(1-\alpha)\sigma\tau(1+\eta)}{(1+\alpha-\sigma+\alpha\sigma\eta)(1-\sigma+\tau\sigma\eta+\alpha(1-\tau)(1+\sigma\eta))} < 0$$

$$c^{NDC}_{inequality} - c^{NDC}_{inequality}|_{\sigma=0} = -\frac{2(1-\alpha)\sigma\tau(1-\alpha\eta)}{(1+\alpha-\sigma+\alpha\sigma\eta)(1+\alpha-\sigma+\tau-\alpha\tau+\alpha\sigma\eta)} < 0$$

Proof of Proposition 4

We write all the macroeconomic variables in terms of the τ, k and s:

$$a = sw\bar{z}(1-\tau) = s\bar{z}(1-\tau)(1-\alpha)k^\alpha \Rightarrow 1 = \frac{s\bar{z}(1-\tau)(1-\alpha)k^{\alpha-1}}{l}$$

$$c^y = \bar{z}(1-s)(1-\tau)(1-\alpha)k^\alpha$$

FL	NDC
$c^o(z) = \eta(1-\tau)(1-\alpha)k^\alpha(s\bar{z}\alpha k^{\alpha-1} + z + \tau s\bar{z}(1-\alpha)k^{\alpha-1})$	$c^o(z) = \eta(1-\alpha)k^\alpha(s\bar{z}\alpha k^{\alpha-1}(1-\tau) + z + \bar{z}\tau)$
$l^o(z) = \eta - (1-\eta)s\bar{z}k^{\alpha-1}\frac{\alpha+\tau(1-\alpha)}{z}$	$l^o(z) = \eta - (1-\eta)\frac{s\bar{z}(1-\tau)\alpha k^{\alpha-1}+\bar{z}\tau}{z}$

The ex ante utility of an agent is:

$$SWF(\tau, k, s) = U(c^y) + \beta E(U(c^o(z), l^o(z)))$$

The change in ex ante utility following a marginal introduction of a pension system is:

$$\begin{aligned}\frac{\partial SWF}{\partial \tau} = & \frac{\partial U(c^y)}{\partial c^y}\left(\frac{\partial c^y}{\partial \tau} + \frac{\partial c^y}{\partial k}\frac{\partial k}{\partial \tau} + \frac{\partial c^y}{\partial s}\frac{\partial s}{\partial \tau}\right) + \\ & + \beta E\left[\frac{\partial U(c^o(z), l^o(z))}{\partial c^o(z)}\left(\frac{\partial c^o(z)}{\partial \tau} + \frac{\partial c^o(z)}{\partial k}\frac{\partial k}{\partial \tau} + \frac{\partial c^o(z)}{\partial s}\frac{\partial s}{\partial \tau}\right)\right] + \\ & + \beta E\left[\frac{\partial U(c^o(z), l^o(z))}{\partial l^o(z)}\left(\frac{\partial l^o(z)}{\partial \tau} + \frac{\partial l^o(z)}{\partial k}\frac{\partial k}{\partial \tau} + \frac{\partial l^o(z)}{\partial s}\frac{\partial s}{\partial \tau}\right)\right]\end{aligned}$$

We plug in the formulas for $c^y, c^o(z), l^o(z)$. We group the terms of the derivative in order

to obtain the term ω_1 of Proposition 4:

$$\omega_1 = \frac{\partial U(c^y)}{\partial c^y}\frac{\partial c^y}{\partial \tau} + \beta E\left[\frac{\partial U(c^o(z), l^o(z))}{\partial c^o(z)}\frac{\partial c^o(z)}{\partial \tau} + \frac{\partial U(c^o(z), l^o(z))}{\partial l^o(z)}\frac{\partial l^o(z)}{\partial \tau}\right]$$
$$\omega_1^{FL} = -\frac{\eta}{1-\tau} - \frac{\beta\eta}{1-\tau} + \beta\frac{1-\alpha}{\alpha}\Gamma^{NDC}$$
$$\omega_1^{NDC} = -\frac{\eta}{1-\tau} - \frac{\beta\Gamma^{NDC}}{1-\tau} + \beta\frac{1-\alpha}{\alpha}\frac{\Gamma^{NDC}}{l^{NDC}}$$

The formulas for the term ω_2 are:

$$\omega_2 = \frac{\partial k}{\partial \tau}\left(\frac{\partial U(c^y)}{\partial c^y}\frac{\partial c^y}{\partial k} + \beta E\left[\frac{\partial U(c^o(z), l^o(z))}{\partial c^o(z)}\frac{\partial c^o(z)}{\partial k} + \frac{\partial U(c^o(z), l^o(z))}{\partial l^o(z)}\frac{\partial l^o(z)}{\partial k}\right]\right) +$$
$$+ \frac{\partial s}{\partial \tau}\left(\frac{\partial U(c^y, l^y)}{\partial c^y}\frac{\partial c^y}{\partial s} + \beta E\left[\frac{\partial U(c^o(z), l^o(z))}{\partial c^o(z)}\frac{\partial c^o(z)}{\partial s} + \frac{\partial U(c^o(z), l^o(z))}{\partial l^o(z)}\frac{\partial l^o(z)}{\partial s}\right]\right)$$
$$\omega_2^{FL} = \frac{\partial ln(k^{FL})}{\partial \tau}\left(\eta\alpha(1+\beta) - \beta(\alpha + \tau(1-\alpha))\frac{1-\alpha}{\alpha}\Gamma^{FL}\right) + \frac{\partial ln(s^{FL})}{\partial \tau}\frac{\beta\Gamma^{FL}\tau(1-\alpha)}{\alpha}$$
$$\frac{\partial ln(s^{FL})}{\partial \tau} = (1-\alpha)\frac{\partial ln(k^{FL})}{\partial \tau} + \frac{1}{1-\tau} + \frac{\partial ln(l^{FL})}{\partial \tau}$$
$$\omega_2^{NDC} = \frac{\partial ln(k^{NDC})}{\partial \tau}\left(\eta\alpha(1+\beta) - \beta(1-\alpha)\Gamma^{NDC}\right)$$

Proof of Proposition 5

1. We compute for $\sigma = 0$ the difference between the impact on welfare through consumption reallocation triggered by a marginal increase in the FL and NDC pension system, respectively.

$$(\omega_1^{FL} - \omega_1^{NDC})|_{\sigma=0} = \frac{2\beta\tau\eta P(\tau)}{(1-\tau)^2(1+\alpha(1-\tau)+\tau)(1+\tau+\alpha)} \leq 0 \tag{40}$$

   where $P(\tau) = \tau^2(1-\alpha)^2 + 2\tau\alpha(1-\alpha) - (1-\alpha^2)$. The result follows from the fact that $P(\tau) < 0$ for $\tau \in [-\frac{1+\alpha}{1-\alpha}, 1]$.

2. We compute the difference between the impact on welfare through general equilibrium effects triggered by a marginal increase in the FL and NDC pension system, respectively.

$$\omega_2^{FL} - \omega_2^{NDC} = \frac{\partial ln(k^{FL})}{\partial \tau}\left(\eta\alpha(\beta+1) - \beta(1-\alpha)\Gamma^{FL}\right) -$$
$$\frac{\partial ln(k^{NDC})}{\partial \tau}\left(\eta\alpha(\beta+1) - \beta(1-\alpha)\Gamma^{NDC}\right) + \beta\tau\Gamma^{FL}\frac{\eta(1-\alpha)^2}{\alpha(1-\alpha\eta-\tau\eta(1-\alpha))} \tag{41}$$

   The term:

$$\Omega = \beta\tau\Gamma^{FL}\frac{\eta(1-\alpha)^2}{\alpha(1-\alpha\eta-\tau\eta(1-\alpha))} \tag{42}$$

plays no role in determining the relative welfare of FL and NDC pensions at the marginal introduction of the pension system $\tau = 0$. However, past the marginal introduction of the pension system, this term is positive and increasing in τ, favoring the FL system from the point of view of general equilibrium effects.

From Proposition 2, we know that:

$$\begin{aligned}
&\Gamma^{NDC}_{\sigma=0} \leq \Gamma^{FL}_{\sigma=0} \leq \Gamma_{\sigma=0,\tau=0} \Rightarrow \\
&\Phi^{NDC}_{\sigma=0} \geq (\eta\alpha(\beta+1) - \beta(1-\alpha)\Gamma^{FL})_{\sigma=0} \geq \Phi_{\sigma=0,\tau=0} > 0
\end{aligned} \tag{43}$$

Also from Proposition 2, we know that the NDC pension system crowds out capital formation strictly more than the FL pension system when $\sigma = 0$:

$$0 > \frac{\partial ln(k^{FL})}{\partial \tau} > \frac{\partial ln(k^{NDC})}{\partial \tau} \tag{44}$$

From relations (41), (43), (44) it follows that $0 > \omega_2^{FL}|_{\sigma=0} > \omega_2^{NDC}|_{\sigma=0}$.

Proof of Proposition 6

We use the same Taylor expansion as in the proof of proposition 2. Specifically, we employ relations (37) and (38) to compute the sensitivity of ω_1^{FL} and ω_1^{NDC} with respect to the variance of the idiosyncratic earnings risk σ^2. We obtain the following:

$$\begin{aligned}
\frac{\partial \omega_1^{FL}}{\partial \sigma^2} =& \beta \frac{1-\alpha}{\alpha} \frac{\partial \Gamma^{FL}}{\partial \sigma^2} \approx \left.\frac{\partial^2 \Gamma^{FL}}{\partial z^2}\right|_{\sigma=0} = \beta l^{FL} \frac{(1-\tau)^2}{\left(\frac{\alpha}{1-\alpha} l^{FL} + \tau l^{FL} + 1 - \tau\right)^3} \\
\frac{\partial \omega_1^{NDC}}{\partial \sigma^2} =& \beta \frac{\partial \Gamma^{NDC}}{\partial \sigma^2} \left(-\frac{1}{1-\tau} + \frac{1-\alpha}{\alpha} \frac{\bar{z}}{l^{NDC}} \right) \approx \\
=& \beta \left(-\frac{1}{1-\tau} + \frac{1-\alpha}{\alpha} \frac{1}{l^{NDC}} \right) \frac{\alpha}{1-\alpha} l^{NDC} \frac{1}{\left(\frac{\alpha}{1-\alpha} l^{NDC} + 1 + \tau\right)^3}
\end{aligned}$$

We expand the expressions above:

$$\begin{aligned}
\frac{\partial \omega_1^{FL}}{\partial \sigma^2} =& \beta \frac{(1-\alpha)(1+\eta)(1-\alpha\eta-(1-\alpha)\eta\tau))^2}{(1+\alpha+\tau-\alpha\tau)^3} > 0 \\
\frac{\partial \omega_1^{NDC}}{\partial \sigma^2} =& \beta \frac{((1-\alpha)(1-\tau)-2\alpha\eta)(1-\alpha\eta)^2}{(1-\tau)(1+\alpha+\tau-\alpha\tau)^3} > 0
\end{aligned}$$

We compute the difference between the two sensitivities:

$$\frac{\partial\omega_1^{FL}}{\partial\sigma^2}-\frac{\partial\omega_1^{NDC}}{\partial\sigma^2}=\frac{\beta\left[(1-\tau)(1-\alpha)(1+\eta)(1-\alpha\eta-(1-\alpha)\eta\tau))^2-((1-\alpha)(1-\tau)-2\alpha\eta)(1-\alpha\eta)^2\right]}{(1-\tau)(1+\tau+\alpha(1-\tau))^3}=$$
$$=\frac{\beta\left[(1-\tau)(1-\alpha)\eta\left[(1-\alpha\eta)^2+(1-\alpha)^2\tau^2\eta(1+\eta)-2(1-\alpha\eta)\tau(1+\eta)(1-\alpha)\right]+2\alpha\eta(1-\alpha\eta)^2\right]}{(1-\tau)(1+\tau+\alpha(1-\tau))^3}=$$
$$=\frac{\beta\left[(1-\tau)(1-\alpha)\eta P(\tau)+2\alpha\eta(1-\alpha\eta)^2\right]}{(1-\tau)(1+\tau+\alpha(1-\tau))^3}$$

The minimum of the numerator is given by the minimum of $P(\tau)$ that is achieved for $\tau_{min}=\frac{1-\alpha\eta}{(1-\alpha)\eta}$. The value of the numerator in τ_{min} is $(1-\alpha\eta)^2\left(2\alpha\eta+\frac{1-\eta}{\eta}\right)>0$. Hence the difference between the two sensitivities is positive.

Appendix 2

Appendix 3

In this Appendix we consider the welfare of the FL and NDC pension systems in an economy that is dynamically inefficient in the absence of idiosyncratic shocks ($\Phi_{\sigma=0,\tau=0}<0$). Under Assumption 1, in an economy with no idiosyncratic risk ($\sigma=0$) a marginal increase in the contribution to the pension system has a higher positive impact on welfare through general equilibrium effects in the case of the NDC pension system, ($\omega_2^{NDC}>\omega_2^{FL}$) if the level of pension contribution τ is low.

For this proof we consider first the case $\tau=0$. Equation (41) becomes:

$$(\omega_2^{FL}-\omega_2^{NDC})_{\sigma=0,\tau=0}=\left(\frac{\partial ln(k^{FL})}{\partial\tau}-\frac{\partial ln(k^{NDC})}{\partial\tau}\right)\Phi_{\sigma=0,\tau=0}<0 \tag{45}$$

Increasing τ further has the following effects:

- the crowding out effect under the NDC pension system worsens faster than under the FL pension system (see Proof of Proposition 2):

$$0>\frac{\partial ln(k^{FL})}{\partial\tau}>\frac{\partial ln(k^{NDC})}{\partial\tau} \tag{46}$$

- however, the impact of the general equilibrium effect on welfare also shrinks faster in the NDC pension system:

$$\Gamma_{\sigma=0}^{NDC}\leq\Gamma_{\sigma=0}^{FL}\leq\Gamma_{\sigma=0,\tau=0}\Rightarrow 0>\Phi_{\sigma=0}^{NDC}\geq\Phi_{\sigma=0}^{FL}\geq\Gamma_{\sigma=0,\tau=0} \tag{47}$$

- ω_2^{FL} has an additional component $\beta\tau\Gamma^{FL}\frac{\eta(1-\alpha)^2}{\alpha(1-\alpha\eta-\tau\eta(1-\alpha))}$ that is positive and increasing with the level of pension contributions τ.

From the above we conclude that, when $\Phi_{\sigma=0} < 0$, we can have a higher welfare gain from general equilibrium effects in the NDC system, but only for low levels of τ.