Annual VaR From High Frequency Data

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Motivation

- Banks and Insurance Risk Management
- Basel and Solvency II
- Security Capital Requirement

Literature on Scaling Law

- Square root of T rule
  - Valid if returns are iid normal
  - how good is it under different circumstances?

- *Danielsson and Zigrand (2006)* effect of jumps on VaR
- *Wang et al. (2011)* failure of $\sqrt{T}$ due to serial dependence

- *Christoffersen et al. (1998)* point out that this rule performs well at medium level of volatility
- Aggregation problems: availability of data

Temporal Aggregation

- If $\mathrm{r_t}|\sigma_t^2 \sim \mathrm{N}(0, \sigma_t^2)$

  then $\mathrm{R_{t+T}}|\mathrm{S}_{t+T}^2 \sim \mathrm{N}(0, \mathrm{S}_{t+T}^2)$

  with

$$\mathrm{R_{t+T}} = \sum_{i=1}^{T} \mathrm{r_{t+i}}$$

$$\mathrm{S}_{t+T}^2 = \sum_{i=1}^{T} \sigma_{t+i}^2$$

- *Andersen et al. (2003)* shows data supports conditional normality

Mixture distribution

- Returns are fat-tailed conditional on lagged information

$$p(\mathrm{R_{t+T}}|\mathrm{I_t}) = \int p(\mathrm{R_{t+T}}|\mathrm{S^2_{t+T}})p(\mathrm{S^2_{t+T}}|\mathrm{I_t})d\mathrm{S^2_{t+T}}$$

- The final step is a model for σ^2_{t}
  - $h_t = \ln \sigma^2_t$
  - HAR vs Fractional
  - Implied properties of $\mathrm{S^2_{t+T}}$

Data

- Sample from November 1995 to November 2015
- SPY and Dow Jones Constituents
- Compute RV from 5-minutes returns

Data

- Returns distribution

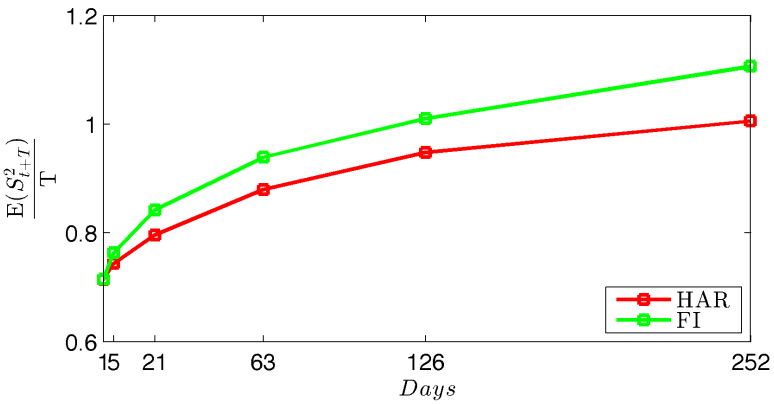
	$r_{i,t}$				$r_{i,t}/\sigma_{i,t}$			
Stock	Mean	St.dev.	Skew.	Kurt.	Mean	St.dev.	Skew.	Kurt.
Min	-0.069	1.142	-0.202	5.747	-0.016	0.873	0.006	2.696
0.50	0.010	1.561	0.126	8.728	0.020	0.927	0.060	2.842
Max	0.080	2.367	0.588	16.171	0.084	0.970	0.154	3.060
SPY	-0.005	1.021	-0.104	10.535	0.053	0.961	0.003	2.789

- Variance distribution

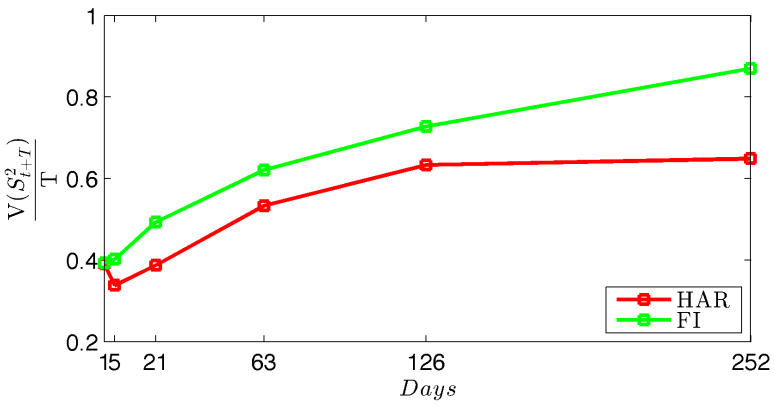
	$\sigma^2_{i,t}$				$\ln \sigma^2_{\mathrm{i,t}}$			
Stock	Mean	St.dev.	Skew.	Kurt.	Mean	St.dev.	Skew.	Kurt.
Min	1.585	2.354	6.165	75.501	-0.070	0.832	-0.063	2.250
0.50	2.806	4.102	10.557	215.180	0.477	0.952	0.347	2.975
Max	7.173	15.194	58.820	3905.738	1.297	1.203	0.514	3.854
SPY	1.148	2.174	9.715	161.728	-0.473	1.046	0.259	3.189

Cumulative Variance Moments

Mean



Vol

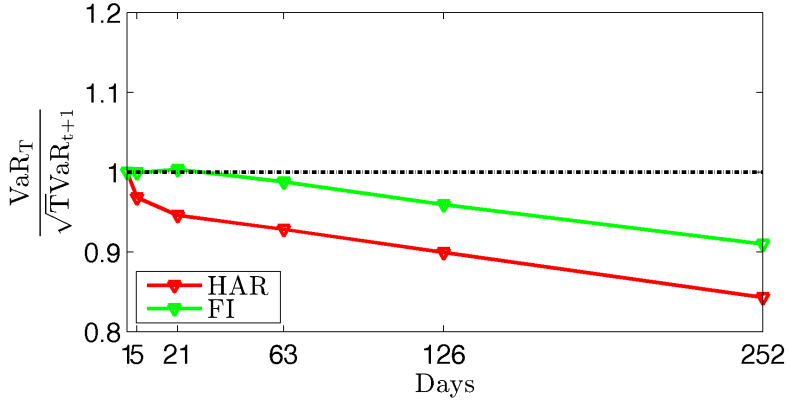


- Conditional on σ_t^2 at medium level in sample
- Both moments grow faster than T
- Longer memory $\Rightarrow$ stronger increase
- Implications:
  1. Growth in scaled mean $\Rightarrow$ deviation from $\sqrt{T}$-rule
  2. Growth in scaled volatility $\Rightarrow$ more fat tailed

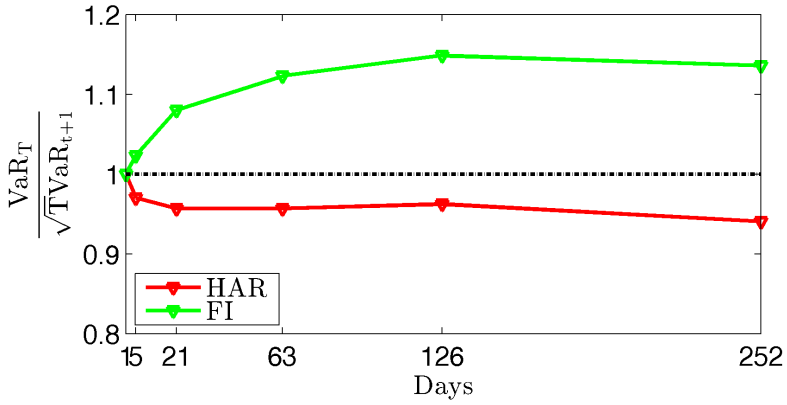
Persistence and Long Term VaR

Heterogeneity in persistence among stocks

Low



High



- Scaling affected by persistence
- HAR lower ratio than Fractional

General Pattern in Results

- Heterogeneity in persistence
- Deviations from $\sqrt{T}$-rule

$$x_{i,T} = \beta_0 + \beta_1 \ln T + \beta_2 \lambda_i + \beta_3 \lambda_i \ln T + u_{i,T}$$

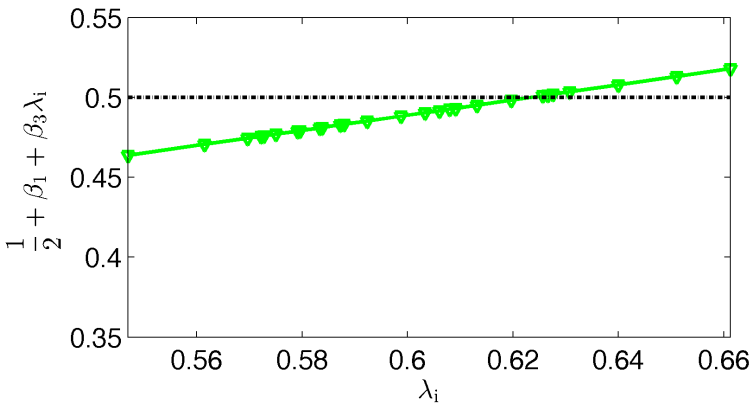
$$x_{i,T} = \ln \mathrm{VaR}_{i,T} - \frac{1}{2} \ln T - \ln \mathrm{VaR}_{i,t+1}$$

Total Effect

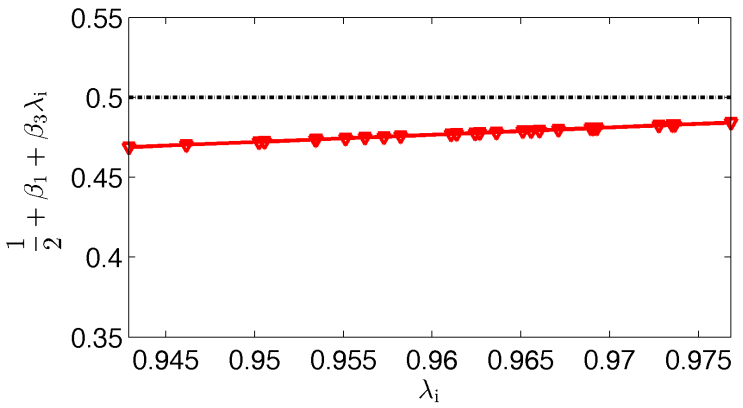
VaR scaling with T

$$\frac{\partial \ln \mathrm{VaR}_{i,T}}{\partial \ln T} = \frac{1}{2} + \beta_1 + \beta_3 \lambda_i$$

Fractional



HAR



Conclusion

- We propose a method to compute VaR for any horizon T
- We show that time series characteristics are relevant in computing VaR
- We show that $\sqrt{\mathrm{T}}$-rule performs on average well when taking into account fat tails for next day VaR

HAR model

$$h_{t+1} = \mu + a_1(h_t - \mu) + a_2(h_t^w - \mu) + a_3(h_t^m - \mu) + \omega\eta_{t+1}$$

$$\begin{aligned}
Y_{t+1} &= FY_t + GE_{t+1} \\
\mathbb{E}[Y_{t+T}] &= F^T Y_t \\
\mathbb{V}[Y_{t+T}] &= \omega^2 \sum_{j=0}^{T-1} F^j GG'(F^j)'
\end{aligned}$$

Fractional model

$$(1-L)^d(h_t-\mu)=\omega\eta_{t+1}$$

Define $y_t = h_t - \mu$.

$$y_{t+T}=\sum_{s=1}^{T} b_s y_{t+T-s}+\omega\eta_{t+T}$$

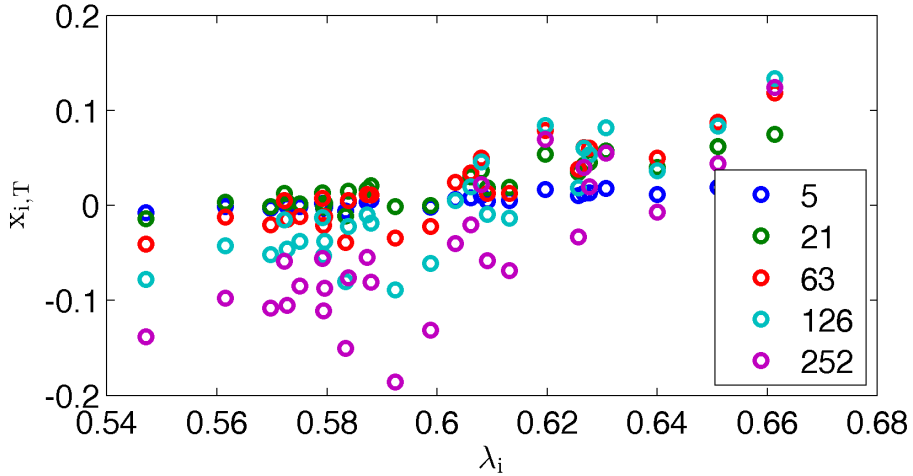
where

$$b_{s+1}=b_s\frac{s-d}{s+1}$$

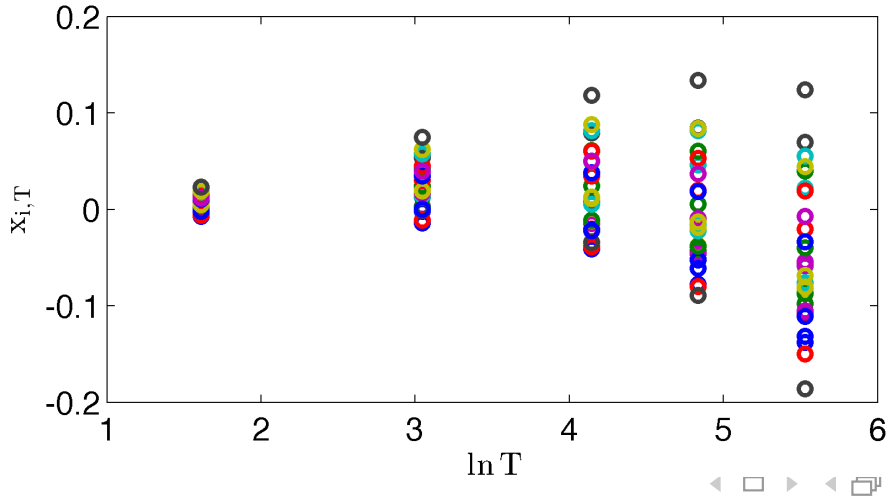
with $b_0 = 1$.

Fractional

Persistence

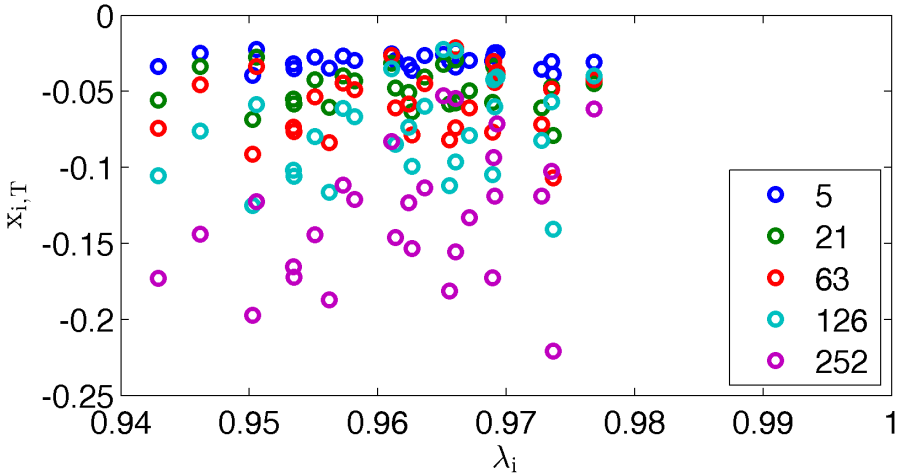


Time

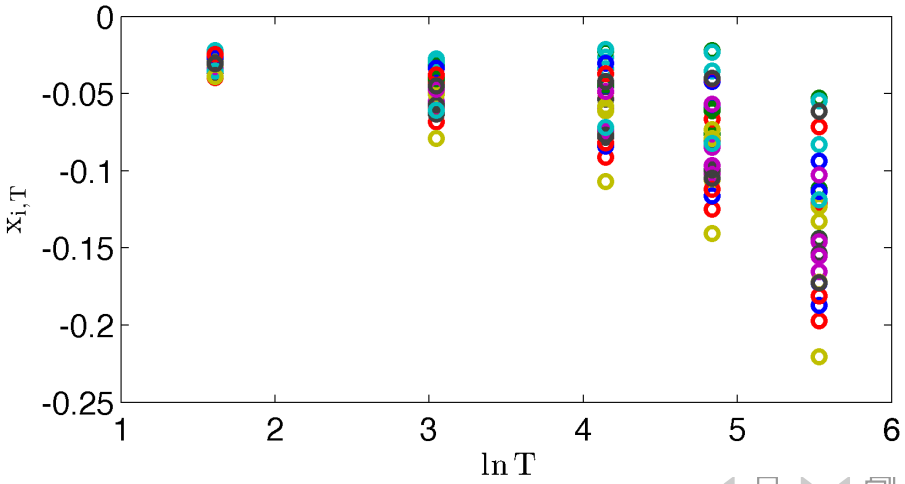


HAR

Persistence



Time



Data Appendix

Stock	Skew.	Kurt.
Min	0.237	4.187
0.10	0.346	4.388
0.25	0.367	4.506
0.50	0.442	5.049
0.75	0.523	5.610
0.90	0.606	6.322
Max	0.770	7.573
SPY	0.397	4.873