Annual VaR From High Frequency Data

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Motivation

- Banks and Insurance Risk Management
- Basel and Solvency II
- Security Capital Requirement
Literature on Scaling Law

- Square root of $T$ rule
  - Valid if returns are iid normal
  - how good is it under different circumstances?

- Danielsson and Zigrand (2006) effect of jumps on VaR
- Wang et al. (2011) failure of $\sqrt{T}$ due to serial dependence

- Christoffersen et al. (1998) point out that this rule performs well at medium level of volatility
- Aggregation problems: availability of data
Temporal Aggregation

- If \( r_t | \sigma_t^2 \sim N(0, \sigma_t^2) \) then \( R_{t+T} | S_{t+T}^2 \sim N(0, S_{t+T}^2) \)

with

\[
R_{t+T} = \sum_{i=1}^{T} r_{t+i}
\]
\[
S_{t+T}^2 = \sum_{i=1}^{T} \sigma_{t+i}^2
\]

- Andersen et al. (2003) shows data supports conditional normality
Mixture distribution

- Returns are fat-tailed conditional on lagged information

\[ p(R_{t+T}|I_t) = \int p(R_{t+T}|S_{t+T}^2)p(S_{t+T}^2|I_t)dS_{t+T}^2 \]

- The final step is a model for \( \sigma_t^2 \)
  - \( h_t = \ln \sigma_t^2 \)
  - HAR vs Fractional
  - Implied properties of \( S_{t+T}^2 \)
Data

- Sample from November 1995 to November 2015
- SPY and Dow Jones Constituents
- Compute RV from 5-minutes returns
## Data

- **Returns distribution**

<table>
<thead>
<tr>
<th>Stock</th>
<th>Mean</th>
<th>St.dev.</th>
<th>Skew.</th>
<th>Kurt.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Min</td>
<td>-0.069</td>
<td>1.142</td>
<td>-0.202</td>
<td>5.747</td>
</tr>
<tr>
<td>0.50</td>
<td>0.010</td>
<td>1.561</td>
<td>0.126</td>
<td>8.728</td>
</tr>
<tr>
<td>Max</td>
<td>0.080</td>
<td>2.367</td>
<td>0.588</td>
<td>16.171</td>
</tr>
<tr>
<td>SPY</td>
<td>-0.005</td>
<td>1.021</td>
<td>-0.104</td>
<td>10.535</td>
</tr>
</tbody>
</table>

- **Variance distribution**

<table>
<thead>
<tr>
<th>Stock</th>
<th>Mean</th>
<th>St.dev.</th>
<th>Skew.</th>
<th>Kurt.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Min</td>
<td>1.585</td>
<td>2.354</td>
<td>6.165</td>
<td>75.501</td>
</tr>
<tr>
<td>0.50</td>
<td>2.806</td>
<td>4.102</td>
<td>10.557</td>
<td>215.180</td>
</tr>
<tr>
<td>Max</td>
<td>7.173</td>
<td>15.194</td>
<td>58.820</td>
<td>3905.738</td>
</tr>
<tr>
<td>SPY</td>
<td>1.148</td>
<td>2.174</td>
<td>9.715</td>
<td>161.728</td>
</tr>
</tbody>
</table>
Cumulative Variance Moments

- Conditional on $\sigma_t^2$ at medium level in sample
- Both moments grow faster than $T$
- Longer memory $\Rightarrow$ stronger increase
- Implications:
  1. Growth in scaled mean $\Rightarrow$ deviation from $\sqrt{T}$-rule
  2. Growth in scaled volatility $\Rightarrow$ more fat tailed
Persistence and Long Term VaR

Heterogeneity in persistence among stocks

- Scaling affected by persistence
- HAR lower ratio than Fractional
General Pattern in Results

- Heterogeneity in persistence
- Deviations from $\sqrt{T}$-rule

\[
    x_{i,T} = \beta_0 + \beta_1 \ln T + \beta_2 \lambda_i + \beta_3 \lambda_i \ln T + u_{i,T}
\]

\[
    x_{i,T} = \ln \text{VaR}_{i,T} - \frac{1}{2} \ln T - \ln \text{VaR}_{i,t+1}
\]
Total Effect

VaR scaling with $T$

$$\frac{\partial \ln \text{VaR}_{i,T}}{\partial \ln T} = \frac{1}{2} + \beta_1 + \beta_3 \lambda_i$$

Fractional

HAR
Conclusion

- We propose a method to compute VaR for any horizon $T$
- We show that time series characteristics are relevant in computing VaR
- We show that $\sqrt{T}$-rule performs on average well when taking into account fat tails for next day VaR
HAR model

\[ h_{t+1} = \mu + a_1 (h_t - \mu) + a_2 (h^w_t - \mu) + a_3 (h^m_t - \mu) + \omega \eta_{t+1} \]

\[ Y_{t+1} = F Y_t + G E_{t+1} \]
\[ \mathbb{E}[Y_{t+T}] = F^T Y_t \]
\[ \mathbb{V}[Y_{t+T}] = \omega^2 \sum_{j=0}^{T-1} F^j G G' (F^j)' \]
Fractional model

\[(1 - L)^d(h_t - \mu) = \omega \eta_{t+1}\]

Define \(y_t = h_t - \mu\).

\[y_{t+T} = \sum_{s=1}^{T} b_s y_{t+T-s} + \omega \eta_{t+T}\]

where

\[b_{s+1} = b_s \frac{s - d}{s + 1}\]

with \(b_0 = 1\).
Fractional Persistence

Time

\[
x_{i,T}
\]

\[
\lambda_i
\]

\[
\ln T
\]
HAR

Persistence

![Persistence Diagram]

Time

![Time Diagram]
## Data Appendix

<table>
<thead>
<tr>
<th>Stock</th>
<th>Skew.</th>
<th>Kurt.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Min</td>
<td>0.237</td>
<td>4.187</td>
</tr>
<tr>
<td>0.10</td>
<td>0.346</td>
<td>4.388</td>
</tr>
<tr>
<td>0.25</td>
<td>0.367</td>
<td>4.506</td>
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<tr>
<td>0.50</td>
<td>0.442</td>
<td>5.049</td>
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<tr>
<td>0.75</td>
<td>0.523</td>
<td>5.610</td>
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<tr>
<td>0.90</td>
<td>0.606</td>
<td>6.322</td>
</tr>
<tr>
<td>Max</td>
<td>0.770</td>
<td>7.573</td>
</tr>
<tr>
<td>SPY</td>
<td>0.397</td>
<td>4.873</td>
</tr>
</tbody>
</table>