Pension Fund Restoration Policy in General Equilibrium

Netspar Pension Day 2016

Pim Kastelein

October 14, 2016

Supervised by R.M.W.J. Beetsma and W.E. Romp
Overview

1 Motivation

2 Model

3 Results

4 Conclusions
Research question

What are the business cycle effects and distributional consequences of pension fund restoration policy after the economy has been hit by a financial shock?
Motivation

Pension funds suffered large financial losses in 2008 ...

<table>
<thead>
<tr>
<th>Country</th>
<th>2008</th>
</tr>
</thead>
<tbody>
<tr>
<td>Turkey</td>
<td>19.00</td>
</tr>
<tr>
<td>Korea</td>
<td>4.09</td>
</tr>
<tr>
<td>Germany</td>
<td>1.60</td>
</tr>
<tr>
<td>Czech Republic</td>
<td>0.32</td>
</tr>
<tr>
<td>Greece</td>
<td>-0.89</td>
</tr>
<tr>
<td>Mexico</td>
<td>-2.03</td>
</tr>
<tr>
<td>Slovak Republic</td>
<td>-2.08</td>
</tr>
<tr>
<td>Italy</td>
<td>-6.30</td>
</tr>
<tr>
<td>Spain</td>
<td>-8.00</td>
</tr>
<tr>
<td>Norway</td>
<td>-8.70</td>
</tr>
<tr>
<td><strong>Simple average</strong></td>
<td><strong>-10.83</strong></td>
</tr>
<tr>
<td>Switzerland</td>
<td>-11.30</td>
</tr>
<tr>
<td>Austria</td>
<td>-12.94</td>
</tr>
<tr>
<td>Poland</td>
<td>-14.28</td>
</tr>
<tr>
<td>Luxembourg</td>
<td>-14.39</td>
</tr>
<tr>
<td>Chile</td>
<td>-14.58</td>
</tr>
<tr>
<td>Portugal</td>
<td>-14.66</td>
</tr>
<tr>
<td>Finland</td>
<td>-15.00</td>
</tr>
<tr>
<td>Netherlands</td>
<td>-15.70</td>
</tr>
<tr>
<td>Hungary</td>
<td>-17.64</td>
</tr>
<tr>
<td>Belgium</td>
<td>-19.89</td>
</tr>
<tr>
<td>Australia</td>
<td>-20.60</td>
</tr>
<tr>
<td><strong>Weighted average</strong></td>
<td><strong>-20.93</strong></td>
</tr>
<tr>
<td>United States</td>
<td>-24.00</td>
</tr>
<tr>
<td>Ireland</td>
<td>-35.00</td>
</tr>
</tbody>
</table>

Source: OECD (2011)

**Figure**: Pension funds’ real investment rate of return in selected OECD countries in 2008 (Natali, 2011)
Motivation

... and as a result were heavily underfunded

![Estimated median percentage surplus or deficit of 2,100 exchange-listed companies’ aggregate defined benefit obligations](image)

(*) Companies are grouped by country of domicile. Therefore, all data represent pension plans’ administered by headquarteried companies and not the pension plans of the country of domicile.

Source: Thomson Reuters Datastream.

**Figure:** Median pension fund deficits of 2100 OECD companies in 2007, 2008, and 2009 (Laboul, 2010)
Motivation

How do pension funds typically function?

- Workers pay pension fund contributions (typically as a *share* of labour income)
- In return, workers accumulate pension benefits to be received upon retirement
- Pension fund invests paid contributions by workers

What does underfunded mean?

- Assets: value of managed assets (i.e. holding of capital stock)
- Liabilities: PDV of existing pension promises to fund participants
- No action entails pension fund exhausts assets
Motivation

Pension fund regulations prescribe speedy restoration of funding adequacy

- In e.g. Denmark, Finland, Germany, Iceland, Norway, Sweden (Pugh and Yermo, 2008)
- In The Netherlands through Financieel Toetsings Kader

However, undertaken measures differ widely (OECD, 2013)

- Less indexation of pension benefits
- Increased contribution payments
- Writing down of accumulated pension benefits (last resort)
Motivation

Theoretically, not just a matter of bring assets closer to liabilities:

- Δ distributional consequences
- Δ implications for macroeconomic aggregates

Writing down accumulated pension benefits:

- Liabilities ↓
- Mostly hurts retirees
- Retirees have higher MPCW → aggregate consumption drops

Increasing contribution payments:

- Assets ↑
- Mostly hurts workers
- Distorts labour supply and in turn aggregate supply
Candidate model: Gertler (1999)

- Life-cycle behaviour in model calibrated at business cycle frequency
- Economy is populated by two groups of agents: workers and retirees
- Workers retire, retirees decease
- Agents take into account finiteness of life when optimising
- Government policy is non-Ricardian (and so is our pension fund)
Our model:


Innovation:

- Introduce pension fund framework of Romp (2013) to flexibly embed various types of pension funds (DB, DC, etc.)
- Highlight how pension fund policy influences agent’s incentives
Model

Discuss model elements with focus on own innovations:

1. Retiree and worker decision problem
2. Pension fund

Skip:

1. Production
2. Government
Retiree and worker decision problem

- Each period, agents choose consumption $c_t^i$, labour supply $l_t^i$, and real balances $m_t^i$, $i = \{r, w\}$
- Maximise expected lifetime utility with RINCE preferences (see e.g. Weil (1989) and Farmer (1990))
- Risk neutrality, but nontrivial preference for intertemporal consumption smoothing
Retiree and worker decision problem

How is the pension fund embedded in the decision problem of agents?

- Agents pay contribution 'tax' $\tau$ on labour income
- Agents accumulate $\nu$ share of labour income as additional per-period pension benefits to be received upon retirement (annuity)
- Pension fund can mark up or write down stock of per-period pension benefits with indexation instrument $\mu$
- Pension fund announces $\tau$, $\nu$, and $\mu$ at the start of each period
Distorted labour supply decision

Effective retiree tax rate:

\[ \tau^r_t = \tau_t - (R^r_t - 1) \nu_t \]

\[ R^r_t = 1 + \mu_{t+1} \frac{\gamma}{1 + r_{t+1}} R^r_{t+1} \]

Effective worker tax rate:

\[ \tau^w_t = \tau_t - R^w_t \nu_t \]

\[ R^w_t = \frac{\mu_{t+1}}{1 + r_{t+1}} \left( \frac{\omega}{\Omega_{t+1}} R^w_{t+1} + (1 - \frac{\omega}{\Omega_{t+1}}) R^r_{t+1} \right) \]

\[ \Omega_{t+1} = \omega + (1 - \omega) \left( \frac{1 - \tau^w_{t+1}}{1 - \tau^r_{t+1} \xi} \right)^{v_2} (\epsilon_{t+1})^{\frac{1}{1-\sigma}} \]
Funding gap policy rule:

$$K_{t+1}^f - L_{t+1}^f = \nu(K_t^f - L_t^f)$$

- $K_t^f$ = value of managed assets
- $L_t^f$ = value of extended pension promises to current fund participants
- $\nu \in [0, 1)$ denotes closure speed
- if $K_t^f < L_t^f \rightarrow$ either $\mu_t < 1$, or $\nu_t \downarrow$, or $\tau_t \uparrow$
- $\nu_{\mu}$ denotes share of gap to be closed through indexation instrument $\mu$
- fix $\nu_t = \nu$
Production and government

Production:

- Perfectly competitive final goods sector
- Imperfectly competitive intermediate goods sector with Calvo (1983) pricing
- Perfectly competitive capital goods sector subject to capital adjustment costs

Government:

- Central bank follows standard Taylor rule with interest rate smoothing
Results

1. Pension fund calibration
2. Restoration policy after unexpected capital stock shock
3. Impulse response functions
4. Welfare implications
## Table: Pension fund parameters and targeted pension fund variables

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Formula</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Accrual rate</td>
<td>$\nu$</td>
<td>0.0055</td>
</tr>
<tr>
<td>Implied contribution rate</td>
<td>$\tau$</td>
<td>0.022</td>
</tr>
<tr>
<td>Implied pension fund capital to output ratio</td>
<td>$\frac{K_f}{y}$</td>
<td>0.89</td>
</tr>
<tr>
<td>Implied retiree pension transfers to output ratio</td>
<td>$\frac{Pr_r}{f}$</td>
<td>0.049</td>
</tr>
<tr>
<td>Implied pension fund capital to aggregate capital ratio</td>
<td>$\frac{K_f}{k}$</td>
<td>0.274</td>
</tr>
</tbody>
</table>
As in Shimer (2012), suppose unexpected capital stock shock evaporates 10% of capital.
Pension fund now has $K_t^f < L_t^f \rightarrow$ conduct restoration policy.
(a) Defined Contribution

(b) Indexation
Pension fund balance sheet

Labour income tax rates

Indexation

(c) Hybrid

(d) Defined Benefit
Impulse response diagrams I

Output

Capital

Retiree labour supply

Worker labour supply

-4.00%
-3.50%
-3.00%
-2.50%
-2.00%
-1.50%
-1.00%
-0.50%

0 5 10 15 20 30

-10.50%
-9.50%
-8.50%
-7.50%
-6.50%
-5.50%
-4.50%
-3.50%

0 5 10 15 20

Laissez-faire  Defined Contribution  Indexation  Hybrid  Defined Benefit

Laissez-faire  Defined Contribution  Indexation  Hybrid  Defined Benefit
Figure: Impulse response diagrams for various variables after a 10% capital stock shock, compared over different pension systems. Values are in percentual deviation from the steady state.
Welfare implications

Table: First period equivalent variations (as a percentage of GDP) compared to Defined Contribution scenario after a 10% capital stock shock across different pension arrangements.

<table>
<thead>
<tr>
<th></th>
<th>Retirees EV</th>
<th>Workers EV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Indexation</td>
<td>0.19%</td>
<td>−0.42%</td>
</tr>
<tr>
<td>Hybrid</td>
<td>0.49%</td>
<td>−1.68%</td>
</tr>
<tr>
<td>Defined Benefit</td>
<td>0.78%</td>
<td>−3.04%</td>
</tr>
</tbody>
</table>

- Only compute welfare of current groups of workers and retirees (income effects in labour supply)
- When pension funding gap is closed over time, part of welfare gains come at expense of future generations
Sensitivity analyses

- Low retiree productivity $\rightarrow$ welfare improving scope of DB fund $\uparrow$
- Large DB pension fund $\rightarrow$ shelter retirees from shocks $\uparrow$
- Increased life-expectancy $\rightarrow$ retirees better equipped against shocks
- Trade-off between slow and fast recovery
Concluding remarks

Major take-aways:

- Economies with DC-type pension funds behave similarly to economies without pension funds
- Significant deviations when pension funds use $\tau$ to fill funding gaps
- DB pension funds distort labour supply, but can shelter retirees from shocks
- Retiree self-sufficiency, size of pension funds, and speed of recovery are key determinants


Retiree decision problem

\[ V_{t}^{r,i} \left( \frac{1 + r_t}{\gamma} a_{t-1}^{r,i}, \mu_t P_{t}^{r,i} \right) = \max_{c_t^{r,i}, a_t^{r,i}, l_t^{r,i}, m_t^{r,i}} \left[ \left( c_t^{r,i} \right)^{v_1} (1 - l_t^{r,i})^{v_2} (m_t^{r,i})^{v_3} \right]^{\frac{1}{\rho}} + \beta \gamma \left[ V_{t+1}^{r,i} \left( \frac{1 + r_{t+1}}{\gamma} a_{t+1}^{r,i}, \mu_{t+1} P_{t+1}^{r,i} \right) \right]^{\frac{1}{\rho}} \]

subject to:

\[ c_t^{r,i} + a_t^{r,i} + \frac{i_t}{1 + i_t} m_t^{r,i} = \frac{1 + r_t}{\gamma} a_{t-1}^{r,i} + (1 - \tau_t) \xi w_t l_t^{r,i} + \mu_t P_{t}^{r,i} - \tau_t^g \]

\[ P_{t+1}^{r,i} = \mu_t P_{t}^{r,i} + \nu_t \xi w_t l_t^{r,i} \]
Retiree decision problem

Special attention to labour supply decision:

\[ 1 - l_t^{r,i} = \frac{v_2}{v_1} \frac{c_t^{r,i}}{(1 - \tau_t^r)\xi w_t}, \]

where \( \tau_t^r = \tau_t - (R_t^r - 1)\nu_t \) and \( R_t^r = 1 + \mu_{t+1} \frac{\gamma}{1+r_{t+1}} R_{t+1}^r \)

Optimisation gives:

- Retiree consumes fraction \( \epsilon_t \pi_t \) of total lifetime wealth in period \( t \)
- Total lifetime wealth: PDV of disposable income
Retiree decision problem

Optimisation gives:

\[ c_{t+1}^{r,i} = \left[ \beta (1 + r_{t+1}) \frac{(1 - \tau_t^r)w_t}{(1 - \tau_{t+1}^r)w_{t+1}} \right]^{\sigma} \left( \frac{1 + i_{t+1}}{i_{t+1}} \frac{i_t}{1 + i_t} \right)^{v_3 \rho} c_t^{r,i} \]

\[ 1 - l_t^{r,i} = \frac{v_2 \frac{c_t^{r,i}}{v_1 (1 - \tau_t^r)\xi w_t}}{v_1 (1 - \tau_t^r)\xi w_t} \]

\[ m_t^{r,i} = \frac{v_3}{v_1} \frac{1 + i_t}{i_t} c_t^{r,i} \]

where \( \tau_t^r = \tau_t - (R_t^r - 1) \nu_t \) and \( R_t^r = 1 + \mu_{t+1} \frac{\gamma}{1 + r_{t+1}} R_{t+1}^r \)
Worker decision problem

\[ V_{t,j}^w (1 + r_t a_{t-1}^w j, \mu_t P_t^w j) = \max_{c_t^w j, a_t^w j, l_t^w j, m_t^w j} \left[ (c_t^w j)^{v_1} (1 - l_t^w j)^{v_2} (m_t^w j)^{v_3} \right]^p + \beta \left[ \omega V_{t+1,j}^w (1 + r_{t+1} a_t^w j, \mu_{t+1} P_{t+1}^w j) + (1 - \omega) V_{t+1}^{r,j} ((1 + r_{t+1}) a_t^{r,j} \mu_{t+1} P_{t+1}^{r,j}) \right]^{\frac{1}{p}} \]

subject to the constraints that become operative once he retires and:

\[ c_t^w j + a_t^w j + \frac{i_t}{1 + i_t} m_t^w j = (1 + r_t) a_{t-1}^w j + (1 - \tau_t) w_t l_t^w j + f_t - \tau_t^g \]

\[ P_{t+1}^w j = \mu_t P_t^w j + \nu_t w_t l_t^w j \]
Worker decision problem

\[ \omega c_{t+1}^{w,j} + (1 - \omega)c_{t+1}^{r,j} \Lambda_{t+1} = c_t^{w,j} \left[ \beta (1 + r_{t+1}) \Omega_{t+1} \left( \frac{(1 - \tau_t^w)w_t}{(1 - \tau_{t+1}^w)w_{t+1}} \right)^{v_2\rho} \left( \frac{1 + i_{t+1}}{i_{t+1}} \frac{i_t}{1 + i_t} \right)^{v_3\rho} \right]^\sigma \]

\[ 1 - l_t^{w,j} = \frac{v_2}{v_1} \frac{c_t^{w,j}}{(1 - \tau_t^w)w_t} \]

\[ m_t^{r,j} = \frac{v_3}{v_1} \frac{1 + i_t}{i_t} c_t^{w,j} , \]

with:

\[ \Lambda_{t+1} = (\epsilon_{t+1})^{\frac{\sigma}{1 - \sigma}} \]

\[ \chi_{t+1} = \left( \frac{1 - \tau_{t+1}^w 1}{1 - \tau_{t+1}^r \xi} \right)^{v_2} \]

\[ \Omega_{t+1} = \omega + (1 - \omega)\chi_{t+1}(\epsilon_{t+1})^{\frac{1}{1 - \sigma}} \]
Aggregation

Aggregation is straight-forward due to:

- Linearity of $l_t^z$ and $m_t^z$ in $c_t^z$
- $\pi_t$ and $\epsilon_t \pi_t$ the same for all workers and retirees, respectively

Example:

$$l_t^r = \sum_i (1 - \frac{v_2}{v_1} \frac{c_t^{r,i}}{(1 - \tau_t^r) \xi w_t}) = N^r - \frac{v_2}{v_1} \frac{c_t^r}{(1 - \tau_t^r) \xi w_t}$$

$$c_t^r = \sum_i \left( \epsilon_t \pi_t \left( \frac{(1 + r_t)}{\gamma} a_t^{r,i} + h_t^{r,i} \right) \right) = \epsilon_t \pi_t \left( (1 + r_t) a_{t-1}^r + h_t^r \right)$$
Aggregation

\[ l_t^r = \sum_i \left( 1 - \frac{v_2}{v_1} \frac{c_{t,i}^r}{(1 - \tau_t^r)\xi w_t} \right) = \mathcal{N}^r - \frac{v_2}{v_1} \frac{c_t^r}{(1 - \tau_t^r)\xi w_t} \]

\[ l_t^w = \sum_j \left( 1 - \frac{v_2}{v_1} \frac{c_{t,j}^w}{(1 - \tau_t^w)w_t} \right) = \mathcal{N}^w - \frac{v_2}{v_1} \frac{c_t^w}{(1 - \tau_t^w)w_t} \]

\[ l_t = l_t^w + \xi l_t^r, \]

\[ d_t^r = \sum_i \left( (1 - \tau_t)\xi w_t l_t^{r,i} + \mu_t P_t^{r,i} - \tau_t^g \right) = (1 - \tau_t)\xi w_t l_t^r + \mu_t P_t^{r,f} - \tau_t^g \mathcal{N}^r \]

\[ d_t^w = \sum_j \left( (1 - \tau_t)w_t l_t^{w,j} + f_t - \tau_t^g \right) = (1 - \tau_t)w_t l_t^w + f_t \mathcal{N}^w - \tau_t^g \mathcal{N}^w, \]
\[ h^r_t = \sum^i (d^r_{t,i} + \frac{\gamma}{1 + r_{t+1}} h^r_{t+1,i}) = d^r_t + \frac{\gamma}{1 + r_{t+1}} h^r_{t+1} \]

\[ h^w_t = \sum^j \left( d^w_{t,j} + \frac{1}{1 + r_{t+1}} \left( \frac{\omega}{\Omega_{t+1}} h^w_{t+1,j} + (1 - \frac{\omega}{\Omega_{t+1}}) h^r_{t+1,j} \right) \right) \]

\[ = d^w_t + \frac{1}{1 + r_{t+1}} \left( \frac{\omega}{\Omega_{t+1}} h^w_{t+1} + (1 - \frac{\omega}{\Omega_{t+1}}) \frac{1}{\psi} h^r_{t+1} \right), \]

\[ a^r_t = (1 + r_t) a^r_{t-1} + d^r_t - c^r_t - \frac{i_t}{1 + i_t} m^r_t + \]

\[ (1 - \omega) \left( (1 + r_t) a^w_{t-1} + d^w_t - c^w_t - \frac{i_t}{1 + i_t} m^w_t \right) \]

\[ a^w_t = \omega \left( (1 + r_t) a^w_{t-1} + d^w_t - c^w_t - \frac{i_t}{1 + i_t} m^w_t \right) \]

\[ a_t = a^w_t + a^r_t, \]
Aggregation III

\[ c_t^r = \sum_i \left( \epsilon_t \pi_t \left( \frac{(1 + r_t)}{\gamma} a_{t-1}^{r,i} + h_t^{r,i} \right) \right) = \epsilon_t \pi_t \left( (1 + r_t) a_{t-1}^r + h_t^r \right) \]

\[ c_t^w = \sum_j \left( \pi_t \left( (1 + r_t) a_{t-1}^{w,j} + h_t^{w,j} \right) \right) = \pi_t \left( (1 + r_t) a_{t-1}^w + h_t^w \right) \]

\[ c_t = c_t^r + c_t^w, \]

\[ m_t^r = \sum_i \left( \frac{v_2}{v_1} \frac{1 + i_t}{i_t} c_t^{r,i} \right) = \frac{v_2}{v_1} \frac{1 + i_t}{i_t} c_t^r \]

\[ m_t^w = \sum_j \left( \frac{v_2}{v_1} \frac{1 + i_t}{i_t} c_t^{w,j} \right) = \frac{v_2}{v_1} \frac{1 + i_t}{i_t} c_t^w \]

\[ m_t = m_t^w + m_t^r \]
Liabilities:

\[ L_t^f = R_{t,f}^r P_{t,f}^r + R_{t,f}^w P_{t,f}^w \]

Counterparts of \( P_{t,f}^r \) and \( P_{t,f}^w \):

- \( P_{t,f}^r \) = aggregate stock of per-period pension benefits of currently retired
- \( P_{t,f}^w \) = aggregate stock of per-period pension benefits of currently working

Counterparts of \( R_{t}^r \) and \( R_{t}^w \):

\[ R_{t,f}^r = 1 + \frac{\gamma}{1 + r_{t+1}} R_{t+1}^r \]

\[ R_{t,f}^w = \frac{1}{1 + r_{t+1}} (\omega R_{t+1}^w + (1 - \omega) R_{t+1}^r) \]
Back to (??)

\[ y_t = \left[ \int_0^1 (y_{z,t})^{\frac{\theta-1}{\theta}} \, dz \right]^{\frac{\theta}{\theta-1}} \]

\[ y_{z,t} = y_t \left[ \frac{P_{z,t}}{P_t} \right]^{-\theta} \]

\[ P_t = \left[ \int_0^1 (P_{z,t})^{1-\theta} \, dz \right]^{\frac{1}{1-\theta}} \]

\[ y_{z,t} = (A_t^{lap} l_{z,t})^\alpha (k_{z,t})^{1-\alpha} \]
\[ w_t = mc_t\left[\alpha \left(\frac{k_{z,t}}{A_{t}^{lap} l_{z,t}}\right)^{1-\alpha}\right] A_{t}^{lap} \rightarrow mc_t = \frac{w_t l_{z,t}}{\alpha y_{z,t}} \]

\[ r_t^k = mc_t[(1-\alpha)\left(\frac{A_{t}^{lap} l_{z,t}}{k_{z,t}}\right)^{\alpha}] \rightarrow mc_t = \frac{r_t^k k_{z,t}}{(1 - \alpha) y_{z,t}} \]

\[ f_{z,t} = \frac{P_{z,t}}{P_t} y_{z,t} - w_t l_{z,t} - r_t^k k_{z,t} \]

\[ = y_{z,t} \left(\frac{P_{z,t}}{P_t} - mc_t\right) \]

\[ \frac{k_{z,t}}{l_{z,t}} = \frac{1 - \alpha \ w_t}{\alpha \ r_t^k} \]

\[ mc_t = \left(\frac{w_t}{\alpha A_t^{lap}}\right)^{\alpha} \left(\frac{r_t^k}{1 - \alpha}\right)^{1-\alpha} \]
Production III

\[
\frac{P_t^*}{P_t} = \frac{\theta}{\theta - 1} \frac{\sum_{i=0}^{\infty} (\zeta \beta)^i \Delta t + i \left( \frac{1}{P_{t+i}} \right)^{1-\theta} y_{t+i} mc_{t+i} \frac{P_{t+i}}{P_t}}{\sum_{i=0}^{\infty} (\zeta \beta)^i \Delta t + i \left( \frac{1}{P_{t+i}} \right)^{1-\theta} y_{t+i}}
\]

\[
P_t = \left[ \zeta (P_{t-1})^{1-\theta} + (1 - \zeta) (P_t^*)^{1-\theta} \right]^{\frac{1}{1-\theta}}
\]

\[
k_t = (1 - \delta) k_{t-1} + (1 - S \left[ \frac{i^k_t}{i^k_{t-1}} \right]) i^k_t
\]

\[
1 + r_t = \frac{P^k_t (1 - \delta) + r^k_t}{P^k_{t-1}}
\]

\[
1 = P^k_t \left( 1 - S \left[ \frac{i^k_t}{i^k_{t-1}} \right] - S' \left[ \frac{i^k_t}{i^k_{t-1}} \right] \frac{i^k_t}{i^k_{t-1}} \right) + \frac{P^k_{t+1}}{1 + r_{t+1}} S' \left[ \frac{i^k_{t+1}}{i^k_t} \right] (\frac{i^k_t}{i^k_t})^2
\]
\[ l_t = \int_0^1 l_{z,t} \, dz \]

\[ k_{t-1} = \int_0^1 k_{z,t} \, dz \]

\[ y_t = \left[ \int_0^1 (y_{z,t})^{\frac{\theta-1}{\theta}} \, dz \right]^{\frac{\theta}{\theta-1}}, \text{ with } y_{z,t} = (A_{t}^{lap} l_{z,t})^{\alpha} (k_{z,t})^{1-\alpha} \]

\[ f_t N^w = \int_0^1 f_{z,t} \, dz = \int_0^1 \left( \frac{P_{z,t}}{P_t} - mc_t \right) y_{z,t} \, dz \text{, with } y_{z,t} = (A_{t}^{lap} l_{z,t})^{\alpha} (k_{z,t})^{1-\alpha} \]

\[ a_t + K_f^t + \tau_t w_t l_t - \mu_t P_{t,r} = P_t^k k_t + \frac{m_t}{1 + i_t} \]

\[ y_t = c_t + i_t^k \]

\[ i_t = \eta i_{t-1} + (1 - \eta) [r_{t+1} + \gamma_{y} \pi^{P}_{t} + \gamma_{y} \tilde{y}_t] \]
\[ \tau_t^g = m_{t-1} \frac{P_{t-1}}{P_t} - m_t \]