Optimal Long-Term Allocation for a Pension Fund

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Pension Funds

- Defined Benefits (DB)
  - Target: Pension should be about 60% of last wage
  - Risk is with employer
- Defined Contributions (DC)
  - Pension is a fraction of accumulated saving
  - Risk is with employee
- Pension funds very generous in the past, demographics changed
- Financial crisis destroyed wealth
- Shift from DB to DC (focus on DC)

Research Question

Concern about the financing of pensions in a low interest rate environment

- low interest rate $\rightarrow$ low return on assets
- low discount rate $\rightarrow$ high liabilities

If interest rates raise

- losses on bond holdings
- decrease in value of future liabilities

Question: How is the optimal allocation of pension funds affected by the low interest rate environment?

Summary of our Research

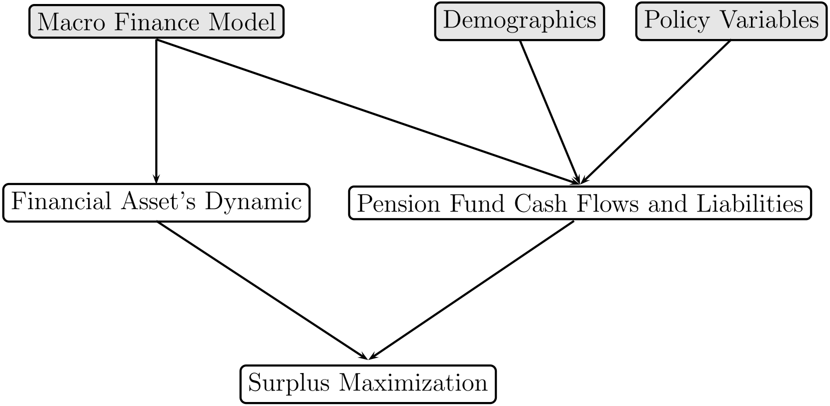
- We construct a working macro-finance model
  - Calibrated quarterly over 1985:Q1 - 2013:Q2
  - for the USA, the Euro Area, and Switzerland
- What impact of low interest rate?
  - Increase uncertainty on liabilities in the short run
  - Increase liabilities duration
- How much do legal restrictions matter?
  - Short sale constraints reduce performance
- Pension funds often manage assets as if there were no liabilities?
  - It is important to take liabilities into account

The Road We Go

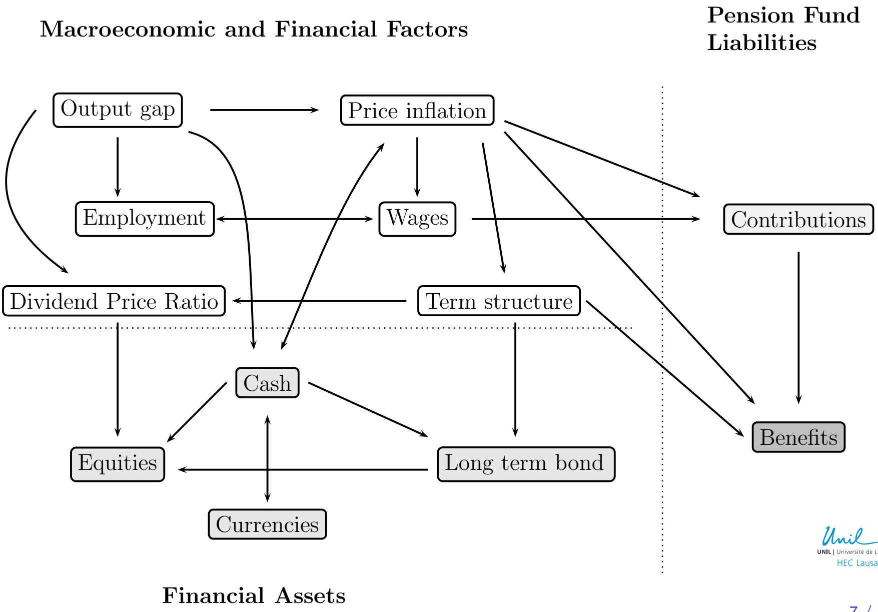
- 3-country macro-finance model
- Various asset classes: Stocks/Bonds/FX, Real Estate, Commodities
- Calibrate a pension fund as to represent 10% of Swiss population
- Generate future return on assets and return on liabilities
- Seek portfolio allocation that maximizes surplus

Macro-Factors and Asset-Liability Correlation

The Logic



Macro-Finance Model



Macro-Finance Model

- Earlier work: Dynamic Stochastic General Equilibrium Model
  - Calés, Jondeau, and Rockinger (2012), Long-Term Portfolio Management with a Structural Macroeconomic Model.
- Small number of structural parameters
- But estimation is slow, forecasting is time consuming
- And flexibility is limited

Macrofinance Model

Our Approach

- Restricted Vector Error Correcting Model

$$A_0 \; \Delta X_{t+1} = \tilde{\mu} + A_1 \; \Delta X_t + A_2 \; X_t + A_3 \; \varepsilon_{t+1}$$

- ΔX_t contains stationary variables, ε_{t+1} is the error term
- Matrices A_0, A_1, and A_2 are restricted by economic theory
- Matrix A_3 captures contemporaneous correlation among variables
- The model is a stationary, parsimonious, restricted VAR(1) model

$$Y_{t+1} = \Phi_0 + \Phi_1 \; Y_t + \Phi_2 \; \eta_{t+1}$$

with $Y_{t+1} = \left[\begin{array}{c} \Delta X_{t+1} \\ X_{t+1} \end{array} \right]$

Main asset classes: Typical dynamics

- 3-month interest rate: Taylor rule

$$r_{t+1}^{(3m)} = \mu_{r,0} + \mu_{r,1}\ \pi_{t+1} + \mu_{r,2}\ og_{t+1} + \varepsilon_{r,t+1}$$

$$\Delta r_{t+1}^{(3m)} = \phi_{r,0} + \phi_{r,1}\ \hat{\varepsilon}_{r,t} + \phi_{r,2}\ \Delta\pi_{t+1} + \phi_{r,3}\ \Delta og_{t+1} + u_{r,t+1}$$

- 10-year government bond rate: Term premium

$$y_{t+1}^{(10)} = \mu_{y,0} + \mu_{y,1}\ r_{t+1}^{(3m)} + \mu_{y,2}\ \pi_{t+1} + \mu_{y,3}\ og_{t+1} + \varepsilon_{y,t+1}$$

$$r_{t+1}^{(10)} = D_t^{(10)}\ y_t^{(10)} - D_{t+1}^{(9)}\ y_{t+1}^{(9)}$$

- Real stock return: Dividend discount model

$$dp_{t+1} = \mu_{dp,0} + \mu_{dp,1}(y_{t+1}^{(10)} - \pi_{t+1}) + \varepsilon_{d,t+1}$$

$$\rho_{s,t+1} = \phi_{s,0} + \phi_{s,1}\ \hat{\varepsilon}_{d,t} + \phi_{s,2}\ \Delta(r_{t+1}^{(3m)} - \pi_{t+1})$$

$$\phi_{s,3}\ \Delta(r_{t+1}^{(10)} - \pi_{t+1}) + \cdots + u_{s,t+1}$$

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Term structure to discount pension fund cash flows

- Economic Approach
  - Model 3M T-Bills, 2 year, and 10 year government bond rates
  - Nelson-Siegel approach to recover entire structure forecasts
  - Discount using $R^{(i)}_{t+T} = R^{(i),Gov}_{t+T} + \pi$ with $\pi = 1\%$ or 2%
- Regulatory Approach
  - 'Technical rate': average of long-term government bond rate and return of a risky portfolio, smoothed over a long period of time
  - Lags behind markets

Macrofinance Model

- Excess log-returns are linear in macroeconomic and financial factors

$$x_{t+1} = (x_{1,t+1}, \cdots, x_{N,t+1})' = M\ Y_{t+1}$$

  with $x_{i,t+1} = r_{i,t+1} - r_{t+1}^{(3m)}$

- Depending on M, returns can be real or nominal, hedged or not against currency risk
- Forecasts over long horizons ($x_{i,t:t+T}$) are easy to obtain
- The covariance matrix is estimated in closed form

Liability Side: Population Dynamics

A Nightmare for a Finance person

- We assume a DC pension fund with 1/10th of Swiss population
- Demographics is calibrated using Swiss data
- Pedagogical Model with Active employees (A)/Retired (R)/Dead (M)
- Initial population: $P_0 = (P'_{A,0}, P'_{R,0}, P'_{M,0})'$
- $P_{t+1} = \Pi' \ P_t$, where Π is a transition matrix

$$\Pi = \begin{pmatrix} \Pi_{AA} & \Pi_{AR} & \Pi_{AM} \\ 0 & \Pi_{RR} & \Pi_{RM} \\ 0 & 0 & I \end{pmatrix}$$

- Open pension fund: Active members who leave the plan are replaced by new actives

Liability Side: Cash Flows

- L_{t+T} is the present value of expected future cash flows CF_{t+T+i}

$$L_{t+T} = E_t \sum_{i=1}^{\infty} \frac{1}{(1+R_{t+T}^{(i)})^i} \left[\sum_{x=65}^{105} Pens_{R,t+T+i}^{(x)} - \sum_{x=25}^{64} Cont_{A,t+T+i}^{(x)} \right]$$

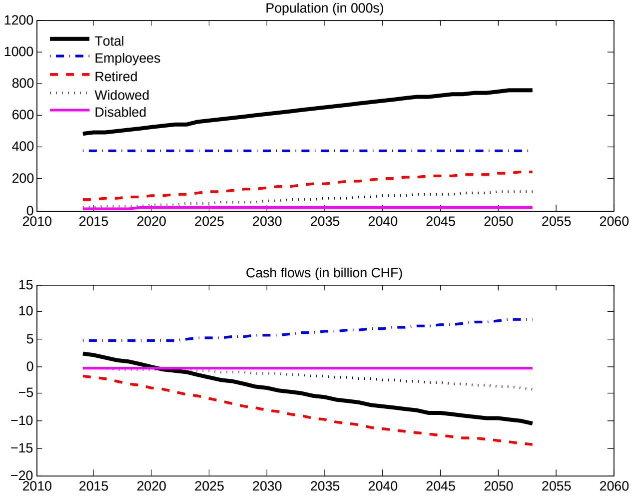
- Cash flows for Employees and Retirees of Age $x+1$

$$\begin{aligned} Cont_{A,t+1}^{(x+1)} &= \Pi_{AA}^{(x,x+1)} \; P_{A,t}^{(x)} \; C_A^{(x)} \; S_{A,t}^{(x)} \\ Pens_{R,t+1}^{(x+1)} &= \Pi_{RR}^{(x,x+1)} \; P_{R,t}^{(x)} \; C_R \; B_t^{(x)} \end{aligned}$$

where

- $C_A^{(x)}$: contribution rate and C_R: conversion rate
- Salary: $S_{A,t+1}^{(x+1)} = S_{A,t}^{(x)} \; (1+\pi_{t+1}^{w}) \; (1+ms^{(x,x+1)})$
- Employee wealth: $\Delta B_{t+1}^{(x+1)} = R_{t+1}^{e} B_t^{(x)} + C_A^{(x)} S_{A,t}^{(x)}$
- Retiree wealth: $\Delta B_{t+1}^{(x+1)} = R_{t+1}^{p} B_t^{(x)}$

Population and Cash-Flow Dynamics



- CF prospective is not looking very good

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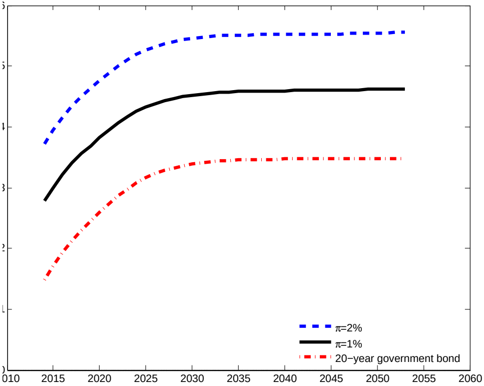
Future discount rate

- $\bar{R}_{t+T}$ future yield to maturity (IRR)

$$E_t \sum_{i=1}^{\infty} \frac{CF_{t+T+i}}{(1+\bar{R}_{t+T})^i} = E_t \sum_{i=1}^{\infty} \frac{CF_{t+T+i}}{(1+R_{t+T}^{(i)})^i}.$$



- Discount rates go up in the future

Low Interest Rate Environment

- Three facets
  - Level of liabilities / Dynamics of liabilities / Duration of liabilities
- If $CF_t = 10$ bln, then $L_t = CF/R_t$ and $r_{L,t+1} = \log(R_t) - \log(R_{t+1})$
- The level of liabilities is very sensitive to changes in interest rate:
  - $R_t = 1\% \rightarrow L_t = 1000$ bln $R_t = 2\% \rightarrow L_t = 500$ bln
  - $R_t = 3\% \rightarrow L_t = 333$ bln $R_t = 4\% \rightarrow L_t = 250$ bln
- An increase in interest rate strongly decreases liabilities:
  - if R_t up from 2% to 3%: L_t down by 33% $(r_{L,t+1} = -40\%)$
  - if R_t up from 3% to 4%: L_t down by 25% $(r_{L,t+1} = -29\%)$
- Conclusion: Huge uncertainty in the short run

  Expect large negative liability returns in short run

Low Interest Rate Environment: Duration

$$D_t = \frac{1}{L_t} E_t \sum_{i=1}^{\infty} i \frac{CF_{t+i}}{(1+R_t^{(i)})^i} \quad \text{where} \quad L_t = E_t \sum_{i=1}^{\infty} \frac{CF_{t+i}}{(1+R_t^{(i)})^i}.$$

Premium	Duration
$\pi = 1\%$	70.1
$\pi = 2\%$	52.9

- Conclusion: 10 year bonds do not have the right duration
  Need longer-term instruments

Optimal portfolio choice

- Funded Ratio:

$$F_{t+k} = \frac{A_{t+k}}{L_{t+k}} = \frac{A_t \exp(r_{A,t:t+k})}{L_t \exp(r_{L,t:t+k})} \quad \Rightarrow \quad r_{F,t:t+k} = r_{A,t:t+k} - r_{L,t:t+k}$$

- Pension fund optimization problem:

$$\max_{allocation} E_t[r_{F,t:t+k}] - \frac{\lambda}{2} V_t[r_{F,t:t+k}]$$

- Optimal portfolio has 3 components:
  - 3M T-Bill loading
  - Performance seeking portfolio (minimum global variance)
  - Liabilities hedging portfolio (independent from asset returns)

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Hedging Properties of Asset Classes

	Expected return	Volatility	Correlation inflation	Correlation with liab. $\pi = 1\%$	$\pi = 2\%$
Assets			(nominal)	(real)	(real)
Cash	0.3	3.1	79.5	-40.2	-51.9
U.S. bond	1.3	4.2	44.4	50.5	58.5
U.S. equity	7.9	9.4	28.8	29.9	34.5
E.A. bond	0.7	5.1	25.3	54.4	61.5
E.A. equity	6.5	11.1	10.7	23.2	22.9
Swiss bond	0.5	4.2	26.2	58.3	70.3
Swiss equity	9.4	11.7	11.8	19.5	21.8
Commodities	6.6	23.1	64.8	-23.4	-22.1
Swiss real est.	3.3	6.1	25.4	27.9	37.0
Liabilities					
$\pi = 1\%$	-2.3	14.5	-18.5		
$\pi = 2\%$	0.2	10.8	-24.6		

Optimal Assets-Only and Assets-Liabilities Portfolios

$\pi = 1\%$ (in %)	Assets-Only GMVP	$\lambda = 50$	$\lambda = 20$	Assets-Liabilities LHP	$\lambda = 50$	$\lambda = 20$
No weight restriction						
Cash	53.0	19.5	-30.7	-129.0	-158.8	-203.5
U.S. bond	-17.9	10.3	52.6	55.3	82.1	122.2
U.S. equity	11.1	18.3	29.0	9.7	16.9	27.7
E.A. bond	14.9	6.5	-6.0	44.2	35.3	21.9
E.A. equity	-2.0	-1.1	0.2	3.0	3.8	4.9
Swiss bond	36.4	14.3	-18.8	133.0	108.9	72.9
Swiss equity	-3.3	9.8	29.3	8.3	21.1	40.3
Commodities	2.7	7.2	14.0	-0.2	4.4	11.3
Swiss real estate	5.0	15.2	30.3	-24.3	-13.6	2.5
Positivity restrictions						
Cash	43.4	18.9	0.0	0.0	0.0	0.0
U.S. bond	0.0	11.1	12.0	0.0	0.0	6.3
U.S. equity	6.5	17.5	33.5	9.0	23.9	44.8
E.A. bond	10.1	5.9	0.0	46.7	38.9	23.3
E.A. equity	0.0	0.0	0.0	0.0	0.0	0.0
Swiss bond	31.3	14.5	0.0	44.3	27.6	0.0
Swiss equity	0.0	9.4	25.8	0.0	9.5	25.6
Commodities	3.7	7.3	11.6	0.0	0.0	0.0
Swiss real estate	5.0	15.4	17.2	0.0	0.0	0.0

Optimal Assets-Only and Assets-Liabilities Portfolios

$\pi = 1\%$	Assets-Only			Assets-Liabilities		
(in %)	GMVP	$\lambda = 50$	$\lambda = 20$	LHP	$\lambda = 50$	$\lambda = 20$
No weight restriction						
Expected surplus μ_S	3.4	6.2	10.2	4.3	7.0	11.1
Risk σ_S	13.6	13.6	14.4	11.2	11.4	12.7
Cost of A-O allocation	–	–	–	29.2	14.3	5.5
Cost of positivity restrictions	0.2	0.0	0.1	13.2	7.7	4.2
Positivity restrictions						
Expected surplus μ_S	3.7	6.2	9.3	3.6	5.6	8.6
Risk σ_S	13.5	13.6	14.3	12.3	12.5	13.3
Cost of A-O allocation	–	–	–	14.9	6.5	2.1

- A-O allocation is harmful for Pension funds
- Positivity restrictions are harmful for A-L funds

Conclusions

- Pension funds have (very) long duration
- Allow for cash borrowing to improve liability hedging
- Asset-Only is a bad idea: prevents liability hedging
- Short-term horizon is not good: at least 10 to 20 years