Stability of Participation in Collective Pension Schemes

An Option Pricing Approach

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Introduction

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- Worldwide examples from mandatory to voluntary pension schemes:

  The U.K. (April 2015): withdraw pension assets at the age of 55

  Canada (May 2015): Minister of Finance: "*consider allowing additional voluntary contributions to the country's mandatory Canada Pension Plan*"

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- An option-pricing approach: participants get an irreversible "exit option"
- **Goal**: providing policy recommendations for a voluntary collective pension scheme such that participants are inclined to stay
- Main contribution of this paper is to model participation decisions as an American option to explore sustainability of the pension scheme

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- Source of risk: returns on investments (geometric Brownian motion)
- Regulation policy to adjust financial shocks in the funding ratio (F):

$$E_0^{\mathbb{Q}}\left(F_t - \bar{F}\right) = \alpha^t \left(F_0 - \bar{F}\right), \ \forall t \geq 0, \ \alpha \in (0,1)$$

Low α: immediate recovery

High α: smoothing risks by allowing funding ratio fluctuations

Classification of Collective Pension Schemes

- The period t total correction Ω_t follows from regulation:

$$\Omega_t = \underbrace{\int_{I_t^w} \pi_{s,t} ds}_{=(1-\omega)\Omega_t} + \underbrace{(1-\gamma_t)\left(\Lambda_t + \int_{I_t^r} B_{s,t} ds\right)}_{=\omega\Omega_t}$$

$(1-\omega)\,\Omega_t$: part of correction achieved through recovery contributions

$\omega\Omega_t$: part of correction achieved through adjusting indexation

Parameter:	$\omega = 0$	$\omega \in (0,1)$	$\omega = 1$
Pension scheme:	DB ($\gamma = 1$)	hybrid	collective DC ($\pi = 0$)

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- American option: exercise at each date until retirement (applying approximation methods)
- Decision is based on full participation of current and future cohorts (optimistic beliefs)

Benchmark Parameters

Description	Symbol	Value
Entry age	t_0	0
Retirement age	t_R	40
Age of death	t_D	60
Target funding ratio	$\bar{F}$	1
Risk smoothing	α	0.5
Interest rate (risk free)	r	0.02
Portfolio return volatility	σ	0.15
Wage	w	1
Accrual rate	ψ	$70\%/t_R$

Age-Dependent Contribution Pure Defined-Benefit

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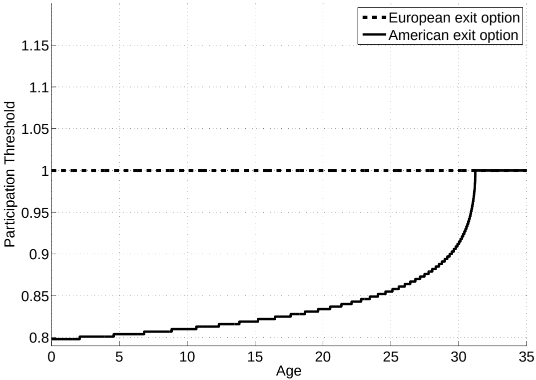
- For the DB pension scheme: 1 state variable $\rightarrow$ assets of the pension fund is the underlying
- Finite Difference (FD) method to approximate the option value (technical details are in the paper)
- European (**American**) option: threshold = 100% (**79.8%**)

Participation Thresholds



Dashed line: European option: 100% at all ages
Solid line: American option: 79.8% at entry

Age-Dependent Contribution

Hybrid Pension Scheme

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- Not only the pension fund's assets, but also the pension entitlements of other participants affect the participation decision
- Least Squares Monte Carlo (LSMC) method to approximate the option value based on 10^5 simulation runs (technical details in the paper)

$$Part_{s,t} = X_t \beta_{t-s} + \varepsilon$$

$$X_t = \left(1, F_t, F_t^2, \max\left(1 - F_t, 0\right), L_t, L_t^2, \underbrace{F_t L_t}_{=A_t}, B_{t-10,t}, B_{t-20,t}, \ldots, B_{t-50,t}\right)$$

Example of a Hybrid Pension Scheme

	$\omega = 0.5$ (hybrid)				
Age	0	10	20	30	40
β_1	1.332	-0.903	0.180	1.803	-4.423
β_F	-1.356	0.548	-3.081	-4.612	3.411
β_{F^2}	0.631	-0.341	1.416	0.768	0.328
$\beta_{\max(1-F,0)}$	-0.444	1.750	3.343	7.371	11.213
β_L	-0.016	-0.029	-0.052	-0.082	-0.058
$\beta_{L^2} * 10^6$	1.679	-0.855	4.897	16.511	-0.004
β_A	0.006	0.009	0.011	0.024	0.004
$\beta_{B_{10}}$	1.910	9.424	2.958	3.449	5.567
$\beta_{B_{20}}$	-0.041	2.598	12.531	3.886	5.998
$\beta_{B_{30}}$	1.099	2.251	6.042	15.791	7.741
$\beta_{B_{40}}$	1.740	2.626	4.480	7.770	8.066
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Uniform Contribution Policies

Contribution Rate

- Age-dependent (old): $c_{t-\nu,t} = \bar{c}_\nu + \frac{1}{t_R}(1-\omega)\,\Omega_t$
- Uniform (new): $c_{t-\nu,t} = \bar{c} + \frac{1}{t_R}(1-\omega)\,\Omega_t$
- $\bar{c}_\nu = \psi R_\nu$ is increasing with age
  $\bar{c} = \frac{1}{t_R}\int_0^{t_R}\bar{c}_\nu d\nu$ is uniform across ages

Motivation

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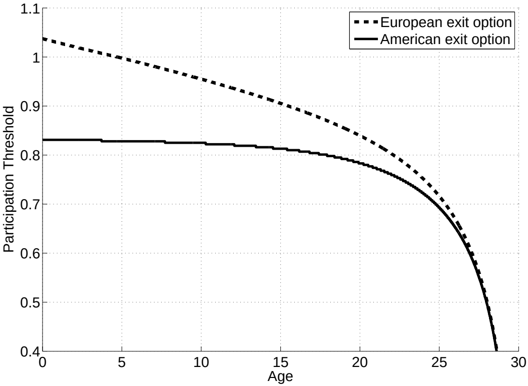
- Common type of contributions in many collective public sector pension plans (e.g. ABP, PfZW, USS)
- Discussions on abolishing "doorsneesystematiek" in the Dutch occupational pension scheme: DNB Position Paper (January 2015), Kamerbrief SZW (July 2014), CPB Policy Letter (January 2014), AFM Position Paper (January 2015), Pensioenfederatie (December 2013)

Participation Thresholds Pure DB



Dashed line: European option: 103.7% at entry
Solid line: American option: 83.1% at entry

Combining Uniform and Age-Dependent

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- Contribution policy:

$$
\begin{aligned}
c_{t-\nu,t} &= (1-\zeta)\left[\bar{c} + \frac{1}{t_R}(1-\omega)\,\Omega_t\right] + \ldots \\
\ldots &\quad \zeta\left[\bar{c}_\nu + (1-\omega)\,\Omega_t \frac{\left(\bar{c}_{t_R} - \bar{c}_\nu\right)^\theta}{\int_0^{t_R}\left(\bar{c}_{t_R} - \bar{c}_u\right)^\theta du}\right]
\end{aligned}
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- Uniform: $\zeta = 0$; age-dependent: $\zeta = 1$; compromise: $\zeta \in (0,1)$

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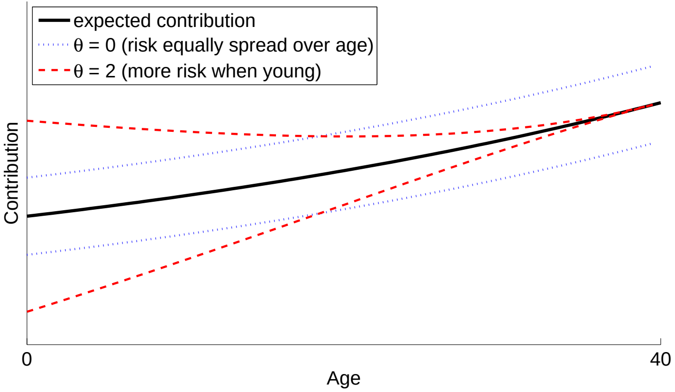
- Uniform: $\zeta = 0$; age-dependent: $\zeta = 1$; compromise: $\zeta \in (0,1)$

- Risk equally spread over working cohorts: $\theta = 0$ more risk when young: $\theta > 0$ (see next slide)

Graphical Illustration



Likelihood that no cohort exits

		$\omega = 0$ (DB)			$\omega = 0.5$ (hybrid)			$\omega = 1$ (DC)		
	ζ	0	0.5	1	0	0.5	1	0	0.5	1
$\sigma = 0\%$	$\alpha \in (0,1)$	0	0	**100**	0	0	**100**	0	0	**100**
	$\alpha = 0.25$	92.9	**95.5**	48.6	44.2	36.9	36.2	**36.6**	27.3	26.2
$\sigma = 10\%$	$\alpha = 0.5$	89.8	93.1	48.0	45.1	38.1	37.5	36.5	26.3	25.5
	$\alpha = 0.75$	83.9	88.3	46.7	**45.9**	41.5	41.0	36.6	25.3	24.7
	$\alpha = 0.25$	94.5	**95.7**	47.8	43.1	33.3	33.0	**29.3**	20.4	19.9
$\sigma = 15\%$	$\alpha = 0.5$	91.9	93.3	46.8	**43.7**	34.4	34.2	28.6	19.4	19.0
	$\alpha = 0.75$	86.4	87.3	45.0	43.7	37.8	37.6	27.2	18.1	18.1
	$\alpha = 0.25$	**95.5**	**95.5**	46.9	41.1	30.6	30.3	**24.3**	17.7	17.4
$\sigma = 20\%$	$\alpha = 0.5$	92.7	92.7	45.6	**41.2**	31.7	31.6	23.8	17.4	17.2
	$\alpha = 0.75$	86.8	85.2	43.1	39.0	33.9	33.9	21.7	13.3	15.2

Conclusion

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- Uniform contributions can act stabilizing, since old workers are subsidized by the young, while the young benefit more from the American option
- **This paper provides policy recommendations for a voluntary collective pension scheme such that participants are inclined to stay**

Additional Slides

Related Literature

- **Benefits from collective funded pension plans**: Teulings and De Vries (2006); Ball and Mankiw (2007); Gollier (2008); Cui et al. (2011); Chen et al. (2014)
- **Conditions on voluntary participation in funded pension schemes**: Hemert (2005); Bommel (2007); Siegmann (2011); Beetsma et al. (2013); Beetsma and Romp (2013)
- **Applying option pricing theory to pensions**: Fung and Chan (1995); Blake (1998); Kocken (2006); Timmermans et al. (2011); Broeders et al. (2013)
- **American option of early retirement**: Friedman and Shen (2002); Chevalier (2006)
- **Least Squares Monte Carlo (LSMC) approach to valuate pension and life insurance products**: Pelsser et al. (2007); Bernard and Lemieux (2008); Cathcart and Morrison (2009); Boyer and Stentoft (2013); Chen (2015)

Value of Participation

- Value of participation for generation s with age t_M:

$$Part_{s,s+t_M} = \underbrace{\exp\left[-r\left(t_R - t_M\right)\right]\Pi_{s+t_R}\left(B_{s,s+t_R}\right)}_{\text{Discounted future retirement benefits}}$$

$$- \underbrace{E^{\mathbb{Q}}_{s+t_M}\left[\int_{t_M}^{t_R} c_{s+u}\exp\left[-r\left(u - t_M\right)\right]du\right]}_{\text{Discounted expected contribution payments}}$$

$$- \underbrace{\min\left(1, F_{s+t_M}\right)\Pi_{s+t_M}\left(B_{s,s+t_M}\right)}_{\text{Value obtained from exiting}}$$

- We can algebraically derive that the participation threshold is at exactly 100%

Participation Decision at Retirement (DB European)

- Value of participation at the age of t_R, i.e. the retirement age:

$$Part_{s,s+t_R} = \underbrace{\Pi_{s+t_R}\left(B_{s,s+t_R}\right)}_{\text{PV retirement benefits}} - \underbrace{0}_{\text{Contributions}} - \underbrace{\min\left(1, F_{s+t_R}\right)\Pi_{s+t_R}\left(B_{s,s+t_R}\right)}_{\text{Value obtained from exiting}}$$

$$\Longleftrightarrow \quad Part_{s,s+t_R} = \max\left(0, 1 - F_{s+t_R}\right)\Pi_{s+t_R}\left(B_{s,s+t_R}\right)$$

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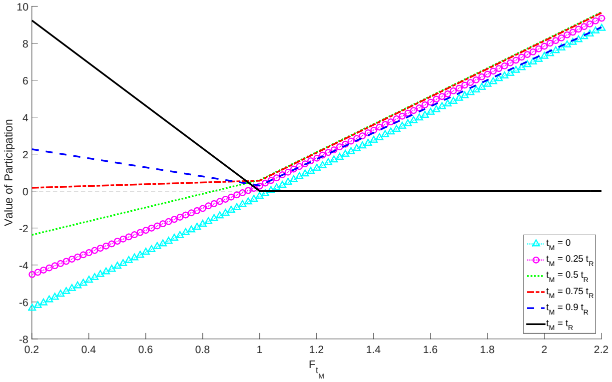
$$\iff Part_{s,s+t_R} = \max\left(0, 1 - F_{s+t_R}\right)\Pi_{s+t_R}\left(B_{s,s+t_R}\right)$$

- Overfunding: $Part_{s,s+t_R} = 0.$

  Underfunding: $Part_{s,s+t_R} = (1 - F_{s+t_R})\,\Pi_{s+t_R}\left(B_{s,s+t_R}\right) > 0.$

Participation Decisions (DB European)



Voluntary participation for funding ratio $F_0 \geq 103.7\%$ at entry

Example of a Hybrid Pension Scheme

	$\omega = 0.5$ (hybrid)				
Age	0	10	20	30	40
β_1	1.162	-1.397	0.757	-0.228	-4.423
β_F	-2.562	1.122	-4.158	-2.874	3.411
β_{F^2}	1.347	-0.256	2.104	0.314	0.328
$\beta_{\max(1-F,0)}$	0.659	2.144	3.055	7.601	11.213
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Exit Probabilities

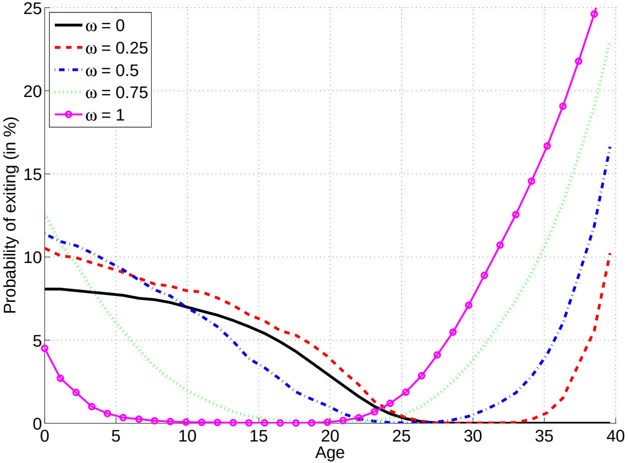
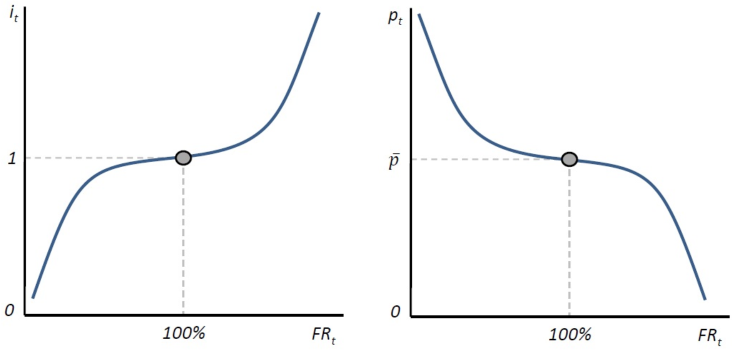
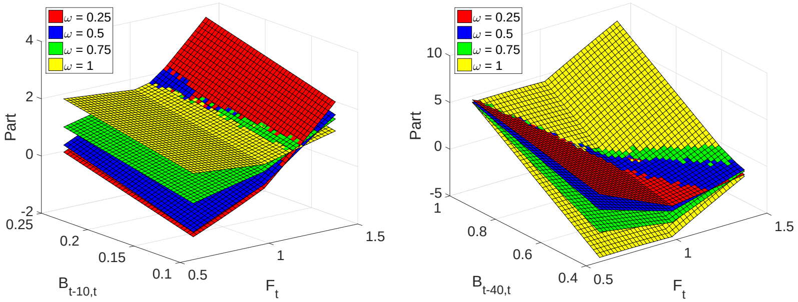


Illustration Contribution Rate



(i = *indexation*, p = *contribution*, FR = *funding ratio*)

Graphical Presentation (Hybrid American)



B: pension entitlements

F: funding ratio

Part: value of participation

Key Findings (Hybrid American)

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- At old ages and underfunding: the value of participation decreases due to reduced indexation
- Vice versa, at young ages:
  - A higher indexation rate benefits the older cohorts at the expense of the young workers, implying less room to reduce the contribution rate
- **DB pension scheme**: exiting never optimal at the retirement age
  Hybrid and DC pension scheme: exiting optimal for low funding rates. Also when the funding ratio is around 100% and the own pension entitlements are particularly low

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- For the hybrid pension scheme, the old bear part of the risk though indexations, such that the young can bear more risks by setting θ large

- Under the DB pension scheme, a short smoothing period is optimal, while under the hybrid pension scheme some risk smoothing might prevent old generations from exiting close before retirement

Least Squares Monte Carlo Approximation Method

1. Run $Q = 10^5$ simulation runs over a "burn-in" period of 100 years.

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7. This way, we have an approximation for the value of participation at each age ν given some current state X_t:

$$\hat{Part}_{t-\nu,t} = X_t \hat{\beta}_\nu - \min\left(1, \hat{F}_t\right) \Pi_t \left(B_{t-\nu,t}\right).$$

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