Non-annuitization in the Annuity Market

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1 Introduction

Lifetime uncertainty affects an individual's choice of consumption, savings and bequests. Facing the risk of lifetime uncertainty, individuals have to decumulate slowly in case that they outlive their assets. A life annuity, which provides a smooth income stream once purchased, is a perfect financial instrument to hedge this risk. In previous literature, a life annuity has been shown to significantly improve individual welfare. Yaari (1965) has shown that an actuarially-fair annuity dominates normal assets for individuals without bequest motive. Davidoff et al. (2005) relaxes the conditions proposed in Yaari (1965) and requires only that annuities pay a rate of return that is greater than the return on normal assets. They also find that positive annuitization remains optimal with market incompleteness and bequest motive. Mitchell et al. (1999) has estimated in an empirical study that individuals are willing to give up around one quarter of their wealth to obtain a life annuity.

Despite the large welfare gains shown in life cycle models, few retirees voluntarily purchase private annuities. The stark contrast between the theoretical predictions and the empirical evidence is called the 'Annuitization Puzzle'. The weak demand for annuities has been largely attributed to three factors: adverse selection in the private annuity market, the wealth preannuitized in social security system, and bequest motives. However, any one of them fails to explain the lack of interest in annuities. This prompt a search for explainations outside of rational choice framwork from behavioral considerations (Brown et al., 2008; Benartzi et al., 2011).

The first contribution of this paper is to combine the three factors and explain in a rational choice framwork why so many individuals, while holding a positive portfolio of assets, do not invest in annuities. Davidoff et al. (2005) has shown that if individuals have bequest motives, all assets that they do not wish to bequeath should be annuitized, which is also implicit in Yaari (1965). In practice, this is not observed. This obvious yet counterfactural prediction relies heavily on an extreme assumption that the annuity market is actuarially fair. If we relax this condition and combine bequest motives with imperfect information in the annuity market, we are able to show that the low-health (high-mortality) types of individuals are not willing to annuitize. This non-annuitization is further aggravated by adverse selection in the annuity market. As the low health types quit the annuity market, the insurers have to set a higher price of annuities to compensate for the decrease in the average mortality rate of insurees. This, in turn, causes more individuals with low-health types to quit the annuity market.

A social security system aggravates the non-annuitization in the private annuity market. We show that a pay-as-you-go social security, whether its return higher or lower than the return of private annuities, increases the threshold health type that is willing to annuitize and decreases the return from the pooling annuity market. A mandatory social security system is certainly not a perfect substitute for private annuities, but it can significantly reduce the demand for private annuities. In an recent empirical study, Hosseini (2015) supports our findings.

A second contribution is to show that other generic imperfections (marketing cost, administration cost, oligopoly profits, etc.) lowers the real return from the pooling annuity market and makes non-annuitization possible. We use a load factor λ, the percentage by which the premium cost exceeds the expected return, to describe the imperfections in the annuity market. Contrary to Mitchell et al. (1999) and Lockwood (2012), we don't include adverse selection into the load factor. Actually, adverse selection does not explain the differential between the premium cost and the expected payouts. We also show that bequest motives affect individuals' tolerance towards imperfections in the annuity market. With a strong bequest motive, a slight imperfection in the annuity market may inhibit the healthiest individuals from purchasing annuities.

The paper is organised as follows. Section 2 presents the two-period model with bequest motives and lifetime uncertainty. Section 3 describes the role of adverse selection in the annutiy market. Section 4 introduces the pay-as you-go social security system. Section 5 discusses other generic imperfections in the annuity market. Section 6 concludes.

2 Model with Bequest Motives

Our model is a variation of Abel (1986). Consider an individual who has a bequest motive and lives for a maximum of two periods. He faces lifetime uncertainty at the beginning of the second period. If he dies young (with probability $1-\mu$), he leaves accidental bequests to his children; if he survives (with probability μ), he splits his wealth into old-age consumption and intentional bequests. During the first period ('youth'), he supplies inelastically one unit of labor, receives wage rate w and chooses a portfolio of riskless bonds S and annuities A. In the second period ('old age'), he is retired and lives on his savings and annuities. We assume that the individual receives a bequest transfer Z upon birth. He gives birth to his children at the beginning of the second period before the lifetime uncertainty is resolved. His budget constraint during youth is given by:

$$C_1 + S + A = w + Z, \tag{1}$$

where C_1 is youth consumption. If he dies young, the annuity firm pays nothing. Only the return from the riskless bonds goes to his heir as accidental bequests:

$$B^A = (1+r)S. \tag{2}$$

If he survives, he splits his wealth into old-age consumption and intentional bequests at the beginning of the second period:

$$C_2 + B^I = (1+r)S + (1+r^A)A, \tag{3}$$

where C_2 is old-age consumption and B^I is intentional bequests. We assume that bequests are non-negative, $B^i \geq 0, i = I, A$. To obtain his lifetime budget constraint, we first combine his youth budget constraint (1) with his old-age budget constraint (3):

$$C_2 + B^I = (1+r^A)(w + Z - C_1) - (r^A - r)S, \tag{4}$$

then substitute (2) into (4) to obtain his lifetime budget constraint:

$$C_2 + B^I = (1+r^A)(w + Z - C_1) - \frac{r^A - r}{1+r} B^A. \tag{5}$$

The individual derives utility from both his consumption and the bequests he leaves to his heir (Dynan et al., 2002). Hence, the individual's lifetime expected utility is given by:

$$\mathbb{E}\Lambda \equiv U(C_1) + \frac{1-\mu}{1+\rho} V(B^A) + \frac{\mu}{1+\rho} \left(U(C_2) + V(B^I) \right), \tag{6}$$

where ρ is the rate of time preference.

The individual maximizes his expected lifetime utility (6) subject to his life time budget constraint (5) and the non-negativity of bequests by choosing C_1, C_2 and B^I. The first order conditions that determine the optimal consumption and bequests are given by:

$$U'(C_1) - (1+r^A)\frac{\mu}{1+\rho}U'(C_2) = 0, \tag{7}$$

$$U'(C_2) - V'(B^I) = 0, \tag{8}$$

$$V'(B^A) - \frac{r^A - r}{1+r}\frac{\mu}{1-\mu}V'(B^I) = 0. \tag{9}$$

Assume that the individual purchases actuarially-fair annuities. The rate of return from the annuity is based on his survival rate:

$$1 + r^A = \frac{1+r}{\mu}, \quad 0 < \mu < 1. \tag{10}$$

Substituting (10) into (9), we obtain:

$$V'(B^A) = V'(B^I), \tag{11}$$

Hence $B^A = B^I$. Combining with (2) and (3) we obtain $B^A = B^I = (1+r)S$, $C_2 = (1+r^A)A$. With actuarially fair annuities, the bequests are insured. No matter the individual survives or not, the return from riskless bonds will be given to his heir as bequests. If he survives, his old age consumption will be solely financed by his annuity payments.

To obtain analytical solutions, we assume that $U(\cdot)$ is isoelastic:

$$U(C) = \begin{cases} \frac{C^{1-1/\sigma}-1}{1-1/\sigma} & \text{if} \quad \sigma > 0, \sigma \neq 1, \\ \ln C & \text{if} \quad \sigma = 1, \end{cases} \tag{12}$$

where σ is the intertemporal elasticity of substitution. $V(\cdot)$ takes the following functional form:

$$V(B) = \begin{cases} \eta\frac{(\theta+B)^{1-1/\sigma}-1}{1-1/\sigma} & \text{if} \quad \sigma > 0, \sigma \neq 1, \\ \eta\ln(\theta + B) & \text{if} \quad \sigma = 1, \end{cases} \tag{13}$$

where η describes the strength of bequest motive, θ determines the threshold wealth below which consumers leave no bequests to their children. We assume that θ is positive. This functional form is widely used in bequest literature (see Table 1). Lockwood (2012) refers to $V(B)$ as *Threshold Bequest Motives*. With this bequest motive, bequests are luxury goods, and the marginal utility of bequests is diminishing. To better understand the property of threshold bequest motives, consider a simple case where the strength of bequest motive η equals

Table 1: Functional forms of bequest motives in various studies.

	Bequest form	Strength of bequest motive	Threshold parameter
Pashchenko (2013)	$\eta\frac{(\phi+B)^{1-\sigma}}{1-\sigma}$	$\eta = 2,360$	$\phi = 273,000$
Lockwood (2012)	$a\frac{(y_h+B)^{1-\sigma}}{1-\sigma}$	$a \in [0,2]$	$y_h = y_{pre}$ [a]
Ameriks et al. (2011)	$\varpi\frac{\left(\phi+\frac{B}{\varpi}\right)^{1-\gamma}}{1-\gamma}$	$\varpi = 47.6$	$\phi = 7,280$
De Nardi et al. (2010)	$\theta\frac{(k+B)^{1-\nu}}{1-\nu}$	$\theta = 2,360$	$k = 273,000$

Note. Notation is the same as used by the authors, except for B which denotes bequest. These functional forms are all special cases of the functional form defined in equation (13).

[a] y_{pre} is defined as the individual's pre-existing annuity income.

to 1, and individuals survive into the second period with wealth W. If the second period wealth W is less than threshold wealth θ, individuals leave no bequests to their children; if W is greater than θ, individuals distribute the wealth beyond θ equally between old age consumption and bequests. Hence, the larger the θ, the higher the degree to which bequests are luxury goods. If we remove the threshold parameter ($\theta = 0$), $V(B)$ degenerates to the *Homothetic Bequest Motives*, as in Abel (1986). With this bequest motive, the rich and the poor leave the same proportion of their wealth as bequests to their children, which is inconsistent with the empirical evidence.

Combining (7)-(13), the individual chooses his optimal consumption and bequest plans:

$$C_1 = \phi(r)\left[w + Z + \frac{\theta}{1+r}\right], \tag{14}$$

$$C_2 = \left(\frac{1+r}{1+\rho}\right)^{\sigma}\phi(r)\left[w + Z + \frac{\theta}{1+r}\right], \tag{15}$$

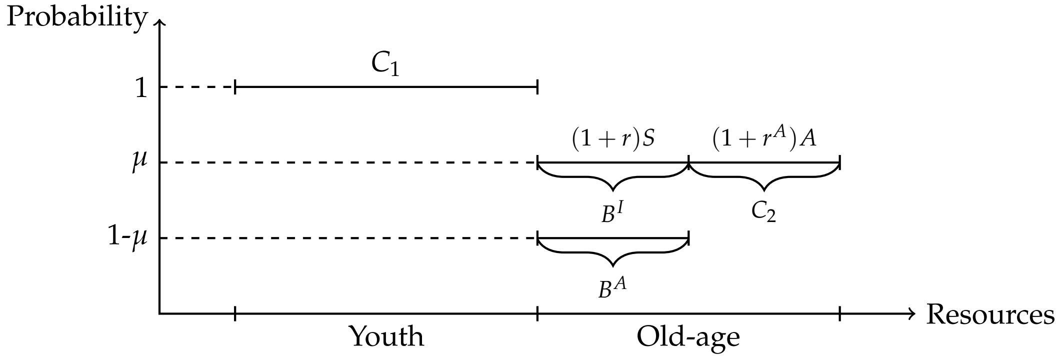
$$B^I = B^A = \eta^{\sigma}\left(\frac{1+r}{1+\rho}\right)^{\sigma}\phi(r)\left[w + Z + \frac{\theta}{1+r}\right] - \theta. \tag{16}$$

where $\phi(r)$ is given by:

$$\phi(r) = \left[1 + \frac{\mu+\eta^{\sigma}}{1+r}\left(\frac{1+r}{1+\rho}\right)^{\sigma}\right]^{-1}. \tag{17}$$

The individual's intertemporal choices are illustrated in Figure 1.

Figure 1: Intertemporal Choices



3 Adverse Selection in the Annuity Market

One explanation for the non-annuitization is the adverse selection in the annuity market. Suppose that individuals are differentiated by their health types μ, which is uniformly distributed between μ_L and μ_H. Apart from the health types, they have the same preferences and utility functions. The expected lifetime utility of an individual with health type μ is given by:

$$U(C_1(\mu)) + \frac{1-\mu}{1+\rho}V(B^A(\mu)) + \frac{\mu}{1+\rho}\left[U(C_2(\mu)) + V(B^I(\mu))\right], \tag{18}$$

and his lifetime budget constraint is given by:

$$C_2(\mu) + B^I(\mu) = (1+r^P)(w + Z - C_1(\mu)) - \frac{r^P - r}{1+r}B^A(\mu), \tag{19}$$

where r^P is the return from the annuity market. A type μ individual maximizes his lifetime utility (18) subject to the budget constraint (19), the first order conditions that determine the optimal consumption and bequests are given by:

$$U'(C_1(\mu)) - (1+r^P)\frac{\mu}{1+\rho}U'(C_2(\mu)) = 0, \tag{20}$$

$$U'(C_2(\mu)) - V'(B^I(\mu)) = 0, \tag{21}$$

$$V'(B^A(\mu)) - \frac{r^P - r}{1+r}\frac{\mu}{1-\mu}V'(B^I(\mu)) \geq 0, \quad (= 0 \text{ if } A(\mu) > 0). \tag{22}$$

Assume that the individual purchases positive amount of annuities, $A(\mu) > 0$. Combining (12)-(13) with (20)-(22), we solve for the individual's optimal consumption, annuity and bequest plans:

$$C_1(\mu) = \left[(1+r^P)\frac{\mu}{1+\rho}\right]^{-\sigma} \phi(\mu, r^P)(1+r^P)\left[w + Z + \frac{\theta}{1+r}\right], \tag{23}$$

$$C_2(\mu) = \phi(\mu, r^P)(1+r^P)\left[w + Z + \frac{\theta}{1+r}\right], \tag{24}$$

$$B^I(\mu) = \eta^\sigma \phi(\mu, r^P)(1+r^P)\left[w + Z + \frac{\theta}{1+r}\right] - \theta, \tag{25}$$

$$B^A(\mu) = \left[\frac{r^P - r}{1+r}\frac{\mu}{1-\mu}\right]^{-\sigma} \eta^\sigma \phi(\mu, r^P)(1+r^P)\left[w + Z + \frac{\theta}{1+r}\right] - \theta, \tag{26}$$

$$A(\mu) = \left[1 + \eta^\sigma - \left[\frac{r^P - r}{1+r}\frac{\mu}{1-\mu}\right]^{-\sigma} \eta^\sigma\right] \phi(\mu, r^P)\left[w + Z + \frac{\theta}{1+r}\right], \tag{27}$$

where $\phi(\mu, r^P)$ is given by:

$$\phi(\mu, r^P) = \left[(1+r^P)^{1-\sigma}\left(\frac{\mu}{1+\rho}\right)^{-\sigma} + \left(\frac{r^P - r}{1+r}\right)^{1-\sigma}\left(\frac{\mu}{1-\mu}\right)^{-\sigma}\eta^\sigma + 1 + \eta^\sigma\right]^{-1}. \tag{28}$$

We assume that there is perfect competition among the annuity firms. Each individual knows his health type μ, but the annuity firms cannot observe their μ. Without exact information of an individual's health status, annuities firms have to offer a single price to all annuitants. There will be a pooling equilibrium in the annuity market.

The low-health types, however, may find the pooling equilibrium price unattractive. Abel (1986) restricts his analysis to a group of individuals purchasing positive annuities by assuming a lower bound for the low health types. We extend his analysis by removing the restrictions on the low health types. There is no guarantee that individuals will have positive demand for annuities. Instead, we assume that there exists a cut-off health type μ_C, such that when individuals have a lower health type $\mu \leq \mu_C$, they stop purchasing annuities $A(\mu) = 0$. From equation (27), for the cut-off health type μ_C we have:

$$1 + \eta^\sigma - \left[\frac{r^P - r}{1+r}\frac{\mu_C}{1-\mu_C}\right]^{-\sigma} \eta^\sigma = 0. \tag{29}$$

In a perfectly competitive annuity market, zero-profit implies that all revenues are paid to the surviving annuitants in their old age:

$$(1+r)\int_{\mu_C}^{\mu_H} A(\mu)h(\mu)d\mu = (1+r^p)\int_{\mu_C}^{\mu_H} A(\mu)h(\mu)\mu d\mu, \tag{30}$$

the return from the annuity market is given by:

$$1 + r^p = (1+r)\frac{\int_{\mu_C}^{\mu_H} A(\mu)h(\mu)d\mu}{\int_{\mu_C}^{\mu_H} A(\mu)h(\mu)\mu d\mu}. \tag{31}$$

The cut-off μ_C and pooling rate of return r^P can be jointly determined by combining (29) with (31).

To show intuitively how the low healthy types of individuals are crowded out of the annuity market by pooling equilibrium and adverse selection, we simulate the model by using parameter values that are widely-accepted in the annuity literature (see, among many others, Caliendo et al., 2014; Pashchenko, 2013; Lockwood, 2012). The interest rate and the rate of time preference are three percent per year. The length of each period is set to be 40 years. It follows that for each period $r = (1 + r_a)^{40} - 1$, $\rho = (1 + \rho_a)^{40} - 1$. Assume a cobb-douglas production function $Y = AK^{\alpha}L^{1-\alpha}$ with capital efficiency parameter $\alpha = 0.3$ and capital labor ratio $\frac{K}{L} = 2.5$, then we have the wage rate $w = \frac{1-\alpha}{\alpha}\frac{K}{L}r = 13.18$. For simplicity we assume that the bequest transfer $Z = 0$. The intertemporal elasticity of substitution in the benchmark case is $\sigma = 1$ (i.e. log-utility). The strength of the bequest motive is chosen such that individuals derive half of the utility from leaving bequests compared to their own consumption, $\eta = 0.5$.[1] Following Lockwood (2012), we assume that the threshold wealth θ equals the fraction of wealth that is already annuitized in the public and employer pensions. From empirical research around twenty percent of income are invested in the public pension system in OECD countries, $\theta = 0.2w$ (Pension at a Glance 2013). The lowest and highest survival probabilities (health types) are 0.1 and 0.9, respectively. The structural parameters are summarized in Table 2.

Given these parameters, we solve for the cut-off health type $\mu_C = 0.5387$ and pooling rate of return $r^P = 3.1933$. It follows that the annual rate of return from private annuities is $r_a^P = (1 + r^P)^{\frac{1}{40}} - 1 = 0.036$. Individuals with health type μ lying between μ_L and μ_C are crowded out of the private annuity market by pooling equilibrium and adverse selection. We decompose the crowding-out effect into two parts and illustrate these effects in Figure 2.

First, we investigate the size of the crowding-out effect of pooling equilibrium. Suppose that there is no adverse selection in the annuity market, the return for annuities is based on the average survival probability:

$$1 + \bar{r} = \frac{1 + r}{\bar{\mu}}. \tag{32}$$

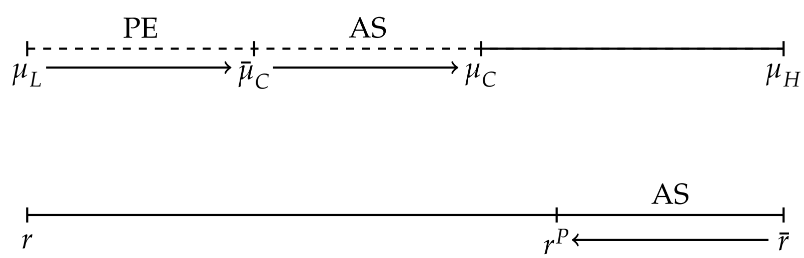
Assume that facing the average mortality based return for annuities, the cut-off low health

[1] We test the robustness of the intertemporal elasticity of substitution and the strength of bequest motive in Appendix A.

Table 2: Structural parameters

r_a	annual interest rate	0.03
r	interest rate per period	2.26
ρ_a	annual time preference parameter	0.03
ρ	time preference parameter per period	2.26
w	wage rate	13.18
Z	bequest transfer	0
θ	threshold wealth parameter	$0.2w$
α	capital efficiency parameter	0.3
σ	intertemporal elasticity of substitution	1
η	strength of bequest motive	0.5
μ_L	lowest survival probability	0.1
μ_H	highest survival probability	0.9

Figure 2: Non-annuitization Explained by Pooling Equilibrium and Adverse Selection



Note. PE stands for Pooling Equilibrium. AS stands for adverse selection.

type that is unwilling to annuitize is $\bar{\mu}_C$. Replacing r^P with $\bar{r}$ in (29), we have the following condition for the cut-off health type $\bar{\mu}_C$:

$$1 + \eta^\sigma - \left[\frac{\bar{r} - r}{1 + r} \frac{\bar{\mu}_C}{1 - \bar{\mu}_C}\right]^{-\sigma} \eta^\sigma = 0. \tag{33}$$

Solving equation (33), we obtain $\bar{\mu}_C = 0.25$. About 19 percent of individuals with their health types lying between μ_L and $\bar{\mu}_C$ are crowded out of the private annuity market by pooling equilibrium.

Since $\bar{\mu}_C < \mu_C$, the non-annuitization of the next 36 percent of individuals with their health types lying bewteen $\bar{\mu}_C$ and μ_C are attributed to adverse selection. As the low health types drop out of the annuity market, the insurers have to raise the price of annuities to compensate for the increased average survival probability of insurees. In turn, more low health types of individuals are crowded out of the annuity market. As shown in Figure 2, adverse selection also decreases the rate of return for annuities in the pooling equilibrium.

4 Social Security and the Annuity Market

In this section we show that a **pay-as-you-go** social security system aggravates the non-annuitization in the private annuity market. With a mandatory social security system individuals are compelled to pay a tax T in youth. If they survive into old age, they receive a social security $(1+r^S)T$. Assume that the population is constant and normalized to one; there is no aggregate uncertainty, a fraction μ of type μ individuals will survive. The tax revenues collected from youth are transfered to the old age:

$$\int_{\mu_L}^{\mu_H} Th(\mu)d\mu = \int_{\mu_L}^{\mu_H} (1+r^S)T\mu h(\mu)d\mu, \tag{34}$$

the internal return for the social security tax if the individual survives is given by:

$$1+r^S = \frac{1}{\bar{\mu}}. \tag{35}$$

The return from the social security may be higher or lower than the return from the pooling annuity market. The lifetime budget constraint of a type μ individual is given by:

$$C_2(\mu) + B^I(\mu) = (1+r^P)(w+Z-C_1(\mu)) - \frac{r^P - r}{1+r}B^A(\mu) + (r^S - r^P)T. \tag{36}$$

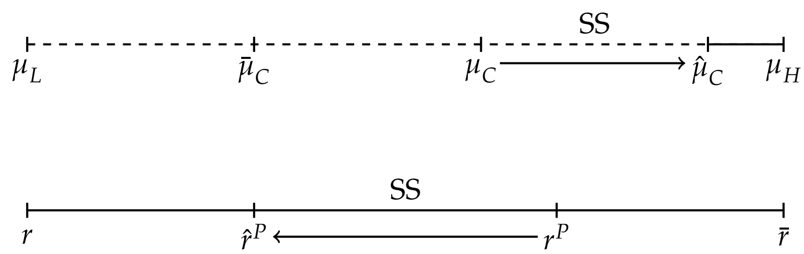
Comparing the lifetime budget constraint (36) with (19), the augmented term on the right hand side reflects the increased (decreased) lifetime resources due to the higher (lower) return from the social security system. When a type μ individual maximizes his expected lifetime utility function (18) subject to his budget constraint (36), the amount of private annuities that he is going to purchase is given by:

$$A(\mu) = \left[1+\eta^\sigma - \left[\frac{r^P - r}{1+r}\frac{\mu}{1-\mu}\right]^{-\sigma}\eta^\sigma\right]\phi(\mu, r^P)\left[w+Z+\frac{r^S - r^P}{1+r^P}T + \frac{\theta}{1+r}\right] - \frac{1+r^S}{1+r^P}T. \tag{37}$$

For the cut-off health type μ_C that purchase zero annuities, we have:

$$A(\mu_C) = \left[1+\eta^\sigma - \left[\frac{r^P - r}{1+r}\frac{\mu_C}{1-\mu_C}\right]^{-\sigma}\eta^\sigma\right]\phi(\mu, r^P)\left[w+Z+\frac{r^S - r^P}{1+r^P}T + \frac{\theta}{1+r}\right] - \frac{1+r^S}{1+r^P}T$$
$$= 0. \tag{38}$$

Figure 3: Non-annuitization Explained by Pay-As-You-Go Social Security



Note. SS stands for Pay-As-You-Go Social Security.

As long as the tax rate is positive, i.e. $T > 0$, the cut-off health type individuals must satisfy:

$$1 + \eta^{\sigma} - \left[\frac{r^P - r}{1 + r} \frac{\mu_C}{1 - \mu_C} \right]^{-\sigma} \eta^{\sigma} > 0. \tag{39}$$

Comparing the cut-off non-annuitization health type condition (39) with (29), for the same return in the pooling annuity market r^P, the cut-off health type μ_C increases with the introduction of the pay-as-you-go social security system. Since the return from the pooling annuity market is given by:

$$1 + r^p = (1 + r) \frac{\int_{\mu_C}^{\mu_H} A(\mu) h(\mu) d\mu}{\int_{\mu_C}^{\mu_H} A(\mu) h(\mu) \mu d\mu}, \tag{40}$$

the increase in the cut-off μ_C in turn increases the average survival rate of annuitants and decreases the rate of return from the pooling annuity market. The equilibrium μ_C and r^P after the introduction of the social security system are jointly determined by (38) and (40).

To get an explicit example of how a pay-as-you-go social security system aggravates the adverse selection and lowers the return of private annuities, we simulate the model by assuming a social security tax rate of twenty percent, $T = 0.2w$. Other structural parameters are the same as listed in Table 2. We solve for the cut-off health type $\hat{\mu}_C = 0.8579$ and the pooling rate of return $\hat{r}^P = 2.6822$, which are illustrated in Figure 3. Another 40 perfect of individuals lying between $\mu_C$and $\hat{\mu}_C$ who do not purchase annuities are explained by the pay-as-you-go social security system. Meanwhile, the annual rate of return from private annuities is reduced to $\hat{r}_a^P = (1 + \hat{r}^P)^{\frac{1}{40}} - 1 = 0.033$.

5 Generic Imperfections in the Annuity Market

Up to now we have assumed that there is no 'transaction cost' in the annuity market. In fact, the differential between the premium costs and the expected return must cover marketing costs, administration costs and the oligopoly profits (Lockwood, 2012; Heijdra and Mierau, 2012; Mitchell et al., 1999). We assume that the real return from the pooling annuity market is given by $(1 + r^R) = (1 - \lambda)(1 + \hat{r}^P)$, where λ is the *load factor*, the percentage by which the premium cost exceeds the expected return.[1] The higher the λ, the higher the imperfections in the annuity market. Thus parameter λ is also the *imperfection indicator*.

For health type μ individuals, we assume that there exists a cut-off return $r^C(\mu)$ such that they purchase no annuities if the real return is lower than the cut-off return:

$$A(\mu) = 0 \quad \text{if} \quad r^R < r^C(\mu), \tag{41}$$

where $r^C(\mu)$ is determined by the condition that the marginal benefit of purchasing the *first unit* of annuities equals the marginal cost:

$$V'(B^A(\mu)) - \frac{r^C(\mu) - r}{1 + r} \frac{\mu}{1 - \mu} V'(B^I(\mu)) = 0, \tag{42}$$

with the non-annuitization initial condition, we have $B^A(\mu) = C_2(\mu) + B^I(\mu)$. The marginal benefit of the second period wealth must be equal, $U'(C_2(\mu)) = V'(B^I(\mu))$. Combining with the functional forms (12) and (13), we obtain:

$$\frac{V'(B^A(\mu))}{V'(B^I(\mu))} = \frac{\eta\left[(1 + \eta^\sigma)C_2(\mu)\right]^{-1/\sigma}}{\eta\left[\eta^\sigma C_2(\mu)\right]^{-1/\sigma}} = \left(\frac{\eta^\sigma}{1 + \eta^\sigma}\right)^{1/\sigma}, \tag{43}$$

then we can solve for the cut-off return for health type μ individuals:

$$r^C(\mu) = r + (1 + r)\frac{1 - \mu}{\mu}\left(\frac{\eta^\sigma}{1 + \eta^\sigma}\right)^{1/\sigma}. \tag{44}$$

For individuals lying between $\hat{\mu}_C$ and μ_H, they will drop out of the annuity market if the real return is lower than the cut-off return due to the imperfections in the annuity market:

$$1 + r^R = (1 - \lambda)(1 + \hat{r}^P) < 1 + r^C(\mu), \tag{45}$$

Substituting (44) into (45), we obtain the following proposition:

[1] The load factor does not explain the effects of *adverse selection* or *social security*, contrary to Mitchell et al. (1999).

Proposition 1. *The higher the imperfection in the annuity market, the less annuities consumers purchase. Above the threshold imperfection $\bar{\lambda}(\mu)$, type μ individuals stop purchasing annuities:*

$$A(\mu) = 0 \quad \textit{if} \quad \lambda > \bar{\lambda}(\mu),$$
$$\bar{\lambda}(\mu) \equiv 1 - \frac{1 + r^C(\mu)}{1 + \hat{r}^P} = 1 - \frac{1+r}{1+\hat{r}^P}\left[1 + \frac{1-\mu}{\mu}\left(\frac{\eta^\sigma}{1+\eta^\sigma}\right)^{1/\sigma}\right]. \tag{46}$$

Given the structural parameters in Table 2, and the pooling rate of return $\hat{r}^P = 2.6822$, we can calculate the cut-off load factor (imperfection) for the healthiest type of individuals:

$$\bar{\lambda}(\mu_H) = 1 - \frac{1+r}{1+\hat{r}^P}\left[1 + \frac{1-\mu_H}{\mu_H}\left(\frac{\eta^\sigma}{1+\eta^\sigma}\right)^{1/\sigma}\right] = 0.081. \tag{47}$$

If the load factor is higher than 8.1 percent, even the healthiest type of individuals will drop out of the private annuity market. Note that the strength of bequest motive also affects the maximum imperfection of the annuity market that individuals will tolerate:

Corollary 1. *The stronger the bequest motive, the lower the threshold imperfection in the annuity market above which consumers stop purchasing annuities:*

$$\frac{\partial \bar{\lambda}(\eta)}{\partial \eta} < 0. \tag{48}$$

Intuitively, since there is a trade-off between bequests and annuities, when individuals have a stronger bequest motive, they tend to be less tolerant towards the imperfections in the annuity market. For instance, when the strength of bequest motive η increases from 0.5 to 1, the cut-off load factor for the healthiest type of individuals is reduced to:

$$\bar{\lambda}(\mu_H)|_{\eta=1} = 0.065. \tag{49}$$

As long as the load factor exceeds 6.5 percent, the healthiest type of individuals will drop out of the private annuity market.

6 Conclusion

In this paper we have built on the two-period model by Abel (1986), and extend the model to include a threshold bequest motive and heterogeneous individuals without lower bounds on their survival probabilities. We combine bequest motives, adverse selection and a mandatory social security system to explain the non-annuitization phenomenon in the private annuity market. Assuming asymmetric information in the private annuity market, our numerical simulation shows that pooling equilibrium and adverse selection account for 19 and 36 percent of non-annuitization, respectively. Another 40 percent of non-annuitization is attributed to a pay-as-you-go social security with a tax rate of twenty percent. Meanwhile, adverse selection reduces the return from the pooling annuity market. A mandatory pay-as-you-go social security system, whether its return higher or lower than the return of private annuities, aggravates the adverse selection and reduces the return from the pooling annuity market even further.

By introducing 'transaction cost' into our model, we can show that even the healthiest type of individuals may not be willing to invest in private annuities while holding a positive portfolio of assets. The transaction cost is denoted by a load factor λ, which describes the imperfection of the annuity market. We also show that a stronger bequest motive decreases individuals' tolerance towards imperfections in the annuity market. With a strong bequest motive, individuals may stop purchasing annuities facing a slight imperfection in the private annuity market. Our numerical simulation shows that a less than ten percent load factor can drive the healthiest type of individuals out of the annuity market, and the threshold load factor (imperfection) decreases with the strength of bequest motive.

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