Stability of Participation in Collective Pension Schemes: An Option Pricing Approach[a]

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Abstract

This paper contributes to the discussion about mandatory participation in collective funded pension schemes. It explores under what circumstances individual participants exercise the option to exit such scheme if participation is voluntary. First, we show how the willingness to participate increases when there are more future exercise dates. Then, we show how the pension fund's set of policy instruments can be designed to minimise the likelihood that cohorts exit the pension scheme. The instruments consist of contribution and indexation policies. Recovery of the funding ratio (ratio of assets over liabilities) to its regulatory target level may be based on uniform contributions or age-dependent contributions which are actuarially fair in expectation. Specifically, while the value of the exit-option deters younger workers from exiting the pension fund, a uniform contribution policy encourages older workers to stay in the pension scheme.

Keywords: defined-benefit and collective defined-contribution pension funds; participation decision; actuarially fair, uniform and age-dependent contribution; European, Bermuda and American option; Least Squares Monte Carlo simulations; stability.

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1 Introduction

The key contribution of this paper is the application of option pricing theory to explore participation decisions in voluntary collective funded pension schemes. This approach can be used to design contribution and benefit policy instruments of voluntary collective pension schemes in such a way that participants have an incentive to stay in the scheme, which can serve as an alternative for making participation in collective funded pension schemes mandatory.

Participation in collective pension schemes can be either mandatory or voluntary. Many countries feature pension arrangements with mandatory participation. Examples are the sub-national civil servants' pension schemes in the U.S. and most occupational pension arrangements in the Netherlands and Denmark. The funded pension schemes in Australia, Chile, Iceland, Norway, Sweden and Switzerland, among others, are mandatory for all employees or even all wage earners (OECD, 2013). Mandatory participation may be beneficial for several reasons. First, and most important, individuals are protected against the consequences of their own myopia, which deters them from saving enough for their retirement. Second, it allows for intergenerational risk-sharing. This is *ex-ante* welfare enhancing as it allows shocks to be divided over a large group of subsequent generations. Consequently, shocks have less impact on the disposable income of participants in a collective pension scheme compared to participants in individual schemes. (Gordon and Varian, 1988; Shiller, 1999; Ball and Mankiw, 2007; Gollier, 2008; Cui et al., 2011; Chen et al., 2014). Finally, participants in a collective scheme avoid the need to take complex investment decisions.

Despite these advantages, mandatory participation is under pressure. Increasing labour market mobility and self employment require more flexible pension arrangements (Chen and Beetsma, 2014). Furthermore the potential benefit of intergenerational risk sharing may become smaller due to ageing of society. Also the quest for more individual freedom of choice has increased. We analyse this flexibility by studying pension schemes with voluntary participation. The question is what this additional flexibility implies for the stability of participation in collective pension schemes. This stability is important for the remaining participants to continue to be able to reap the benefits from participation. It may also be systemically important, because a run on the assets of a large pension fund may have profound consequences on the financial markets in which it has invested.

This paper applies option pricing techniques to analyse the decision to participate in a voluntary collective funded pension scheme. Under voluntary participation a participant has an option to (continue to) participate in, or oppositely, to exit the pension scheme. Specifically, we investigate how a pension fund can deploy its policy instruments to reduce the likelihood that a cohort wants to leave the pension fund. Hence, the analysis in this paper provides leads for meeting the quest for more individual freedom of choice (Bovenberg et al., 2007; Westerhout, 2011; Beetsma et al., 2012; Beetsma and Romp, 2013), while not unnecessarily endangering the stability of pension schemes.

With mandatory participation under increasing pressure, it is important for the policy authorities, such as pension fund supervisors, to understand the consequences of offering pension fund participants an exit-option. In our analysis, we consider different design features of the exit-option, ranging from a "European" option with a single pre-specified exit date to an "American" option that allows for the possibility to exit at any moment until the option expires. An example of the first type is when (only) at the moment of retirement the participant can choose between taking out his accumulated balance or receiving an annuity payment until death. This is the case for Australia, Chile, Denmark, Sweden and Switzerland.[1] By contrast, in the U.K. participants have the option to withdraw their entire balance at any moment after the age of 55, while in the U.S. this option exists during the entire working career. We capture these variants by modelling options with a fixed number of pre-specified exercise dates or a continuum of exercise dates. An example of an option with multiple exercise moments (a "Bermuda" option) concerns the recent introduction in the U.K. of the obligation of employers to automatically enrol employees every three years into an occupational pension scheme. Participants can withdraw their contributions within a month after enrolment. Thereafter, contributions are locked in the pension scheme until the age of 55. Depending on the pension scheme one might be able to reduce or increase the level of contributions. In particular, the non-profit "NEST" pension scheme, which was set up as part of the government's workplace pension reforms, allows for a "contribution holiday". The participant can keep his retirement pot and start contributing again at a later date.

We set up a model with multiple overlapping generations, in which participants have the option to stay in their pension fund or to once-and-for-all exit it. Upon exit, his pension entitlements are converted into an individual defined-contribution retirement account. This choice might be optimal when the funding ratio, the value of the fund's assets over its liabilities, is low. By exiting the participant does not share in the future recovery burden. Investment risks affect the financial position of the pension fund, which can deploy two instruments, the contribution and the indexation rate, to restore its financial position. This recovery is required by regulation in our model and can be spread out over a longer or shorter period. The types of pension contracts we consider range from collective defined-benefit (DB), in which all the adjustment takes place through the contributions, to a collective defined-contribution (DC) scheme, in which all adjustment occurs through indexation. We also analyse hybrid contracts, with adjustments along both dimensions. In all contract specifications the accrual and indexation rates are uniform for all participants. For the

[1] In Chile, the latter possibility only exists if the annuity exceeds some mandatory minimum. In Sweden, the participant may choose between an annuity until death or an annuity with a fixed maturity of at least five years.

contribution, on the other hand, we consider three cases: an actuarially fair contribution, a uniform contribution (as is common in many collective pension arrangements throughout the world) and an age-dependent contribution that puts a relatively large share of the adjustment (in the case of underfunding as well as overfunding) on young cohorts. To obtain our results, we apply the Least Squares Monte Carlo (LSMC) approach as proposed by Longstaff and Schwartz (2001). By now, several other studies have applied the LSMC approach to pensions and life insurance products, e.g. Pelsser et al. (2007); Bernard and Lemieux (2008); Cathcart and Morrison (2009); Boyer and Stentoft (2013).

Our key findings are the following. For realistic assumptions the likelihood of participants exiting the voluntary scheme is fairly high. It increases for a longer recovery period. Ceteris paribus, young participants are more inclined to exit a DB scheme or a hybrid scheme relying more on contribution adjustments for recovery. Older participants are more inclined to exit a collective DC scheme or a hybrid scheme in which recovery relies more on indexation adjustments. Further, participants are more inclined to continue participation if the number of moments that the exit-option can be exercised is high. Counterintuitively, more freedom of choice to exit actually improves participation. Applying a uniform contribution in the presence of an American exit-option helps to keep both young and old participants in the pension fund and, as such, has an important stabilizing influence on the pension fund. The American exit-option is relatively valuable to the young cohorts as it gives them many potential exercise opportunities. The uniform contribution is relatively valuable to the old cohorts. They are implicitly subsidized by the young through the uniform contributions: as a result of the shorter period over which they are discounted, the newly accrued pension entitlements associated with an extra year's work are more valuable for older than for younger working cohorts. This *pay-as-you-go effect* is present in many collective funded public sector pension plans, such as those in Australia, Canada, Germany, the Netherlands, Norway, Switzerland, the U.K. and the sub-national civil servants' plans in the U.S. (Ponds et al., 2011).

Closest to the current paper is Chen (2015). This paper extends Chen (2015) into a number of directions. In contrast to Chen (2015) we consider hybrid pension schemes and we introduce the indexation rate as an additional instrument to restore the pension fund's financial position. In addition, we allow for the contribution policy to be age-dependent. This way we can explore what policy instrument settings are conducive to all cohorts continuing their participation in the pension fund. This paper differs from other papers studying the decision to participate in a collective pension fund by applying an option pricing approach based on risk-neutral valuation, rather than a utility-based framework. Siegmann (2011) analyses funding ratio thresholds at which an individual would voluntarily participate in a DB pension fund. Molenaar et al. (2011) analyse whether a low funding ratio creates incentives for participants to exit a pension plan. In line with our results, they find that particularly the young and the old working cohorts are likely to exit the pension fund. The exit incentive of the young is driven by the pay-as-you-go effect, while the exit incentive of the old is driven by the fact that indexation reductions affect their relatively large accumulated pension benefits most. Other articles studying participation in collective pension funds are Van Hemert (2005), Van Bommel (2007), Beetsma et al. (2012) and Beetsma and Romp (2013). Except for applying a utility-based approach, these papers cast their analysis in a context with two overlapping generations, while we allow for a more realistic setting with a continuum of overlapping generations and a potential continuum of exercise moments.

This paper also connects to the literature on the stability of pension schemes and actuarially fairness. Dufresne (1989) investigates to what extent fluctuations in contributions and funding ratios can be reduced, by analysing the mean and variance of the pension fund variables. Hassler and Lindbeck (1997) show that a notional DC pension scheme is stable, when actuarially fairness and a balanced budget are necessary conditions for stability in a pay-as-you-go pension system. From this perspective the Swedish public pension pillar, which is based on a pay-as-you-go notional DC scheme, looks attractive. However, our paper considers funded pension schemes. Chen and Romp (2015) propose a method to model the behaviour of funded pension schemes by distributing the required recovery by regulation over the policy instruments, such that the pension system is globally stable, as it is expected to converge to a unique steady state. This feature holds regardless of the extent of risk-sharing and the type of financing (DB, DC or hybrid). We apply this method to ensure non-exploding simulation paths, while we focus on improving stability in terms of participation. Kleinow and Schumacher (2015) show that actuarial fairness is not straightforward, when risk-sharing is implemented through conditional indexation. They compute a recursive formula in the context of a model with two overlapping generations, such that contributions are actuarially fair for entry generations. We are not concerned with actuarial fairness per se, but we focus on the setting of the instruments that protects the pension fund's stability in terms of participation to the maximum extent possible.

The remainder of this paper is structured as follows. Section 2 presents the model, while Section 3 presents the benchmark parameter settings. As a stepping stone for the ensuing analysis, Section 4 explores the exit-option under a DB scheme when there is a fixed exercise date, while Section 5 turns to the American exit-option. Section 6 explores how stability in terms of participation can be enhanced by deploying a uniform contribution policy, which is no longer actuarially fair. Finally, in Section 7 we conclude the main text of this paper. Technical details are found in the Appendix.

2 The Model

This section presents the model. Section 2.1 describes the underlying economy and the individuals inhabiting the economy. Section 2.2 discusses the valuation of random cash-flows, while Section 2.3 explains the various pension schemes.

2.1 The Economy and its Agents

All processes in the model are specified under the risk neutral measure $\mathbb{Q}$. We assume that the market is complete and that the only source of risk is investment return risk. The value P_t of the pension fund's investment portfolio follows a geometric Brownian motion

$$dP_t = rP_t dt + \sigma P_t dW_{P,t}, \tag{1}$$

where the drift exactly equals the instantaneous risk-free rate r under the risk neutral measure and where σ is the instantaneous volatility of the portfolio return.

A single period in the model corresponds to one year. An individual works from age $t_0 = 0$ until his retirement age t_R, while he is retired from age t_R until the age at which he dies, t_D. The parameters t_0, t_R and t_D are all constant. Moreover, the size of the new cohort entering the workforce each period is constant over time and normalized to unity. Hence, we abstract from demographic risks. We also abstract from unemployment risk and inflation risk. In fact, we assume that the inflation rate is zero.[2] The cohort entering the labour market at time $t = s$ is referred to as "cohort s". Furthermore, we assume that the wage profile is constant over an individual's working life. We normalize the annual wage rate to unity. Hence,

$$w_{s,t} = \begin{cases} 1, & \text{for } t - s \in [0, t_R], \\ 0, & \text{otherwise}, \end{cases}$$

where $w_{s,t}$ is the wage of a participant aged $\nu = t - s$. At date t cohort s contributes a fraction $c_{s,t}$ of its wage to the pension fund.

2.2 Valuation Method

Key to the analysis will be the participant's option to exit a collective pension fund. At time t, the price of any security or contract with random pay-off X_u at $u \geq t$ is given by $\Pi_t(X_u)$. According to the martingale representation theory, we can price securities with respect to

[2] Allowing for non-zero inflation would complicate the algebra, without affecting the main results.

the expectation under the risk-neutral measure $\mathbb{Q}$. Hence, we obtain

$$\Pi_t\left(X_u\right)=\exp\left[-r\left(u-t\right)\right]E_t^{\mathbb{Q}}\left(X_u\right),$$

where $E_t^{\mathbb{Q}}$ is the expectation under the risk-neutral measure $\mathbb{Q}$ conditional on the information available at time t. We assume that the market is complete and, therefore, the risk-free interest rate can be used as the unique numéraire.

2.3 The Pension Schemes

In this subsection we define the different pension schemes. We distinguish between individual DC schemes and collective schemes. Within the collective schemes we define a continuum of pension schemes ranging from DB to collective DC. We end this section with a description of the collective-to-individual pension option, i.c., the exit-option.

2.3.1 The Individual Defined-Contribution Pension Scheme

In the individual DC pension scheme a participant accumulates assets by paying contributions and earning investment returns. At retirement the accumulated assets are used to buy an annuity. The accumulated pension assets of cohort s at time t are

$$A_{s,t}^{DC}=\int_s^t c^{DC}\frac{P_t}{P_u}du,\quad \text{for } t-s\in\left[0,t_R\right],$$

where c^{DC} is the constant contribution. Assets at retirement, $A_{s,s+t_R}^{DC}$, are used to buy an annuity that yields a constant benefit B^{DC} until death. This benefit is easily calculated as (see Appendix A.1)

$$B^{DC}=rA_{s,s+t_R}^{DC}/\left\{1-\exp\left[-r\left(t_D-t_R\right)\right]\right\}.$$

The individual DC scheme is actuarially fair by construction. By applying the valuation method, we simply obtain

$$\Pi_t\left(A_{s,t}^{DC}\right)=A_{s,t}^{DC}.$$

2.3.2 The Collective Pension Scheme

The collective schemes are more complex. We run through several steps in this section to model them. First, we define the participation setting in the collective scheme. Second, we present the asset dynamics and the valuation of the liabilities. Third, we turn to the various policy instruments available to the collective scheme. Fourth, we address the regulation of the pension scheme and define the equilibrium targets. Fifth, we parametrise the various collective pension schemes. We finish this subsection with a description of the recovery contribution policy.

Participation Setting Denote I_t as the set of participating cohorts in the collective pension scheme at time t. These cohorts must have entered the labour market at time $s \in [t - t_D, t]$. Under "full participation" all cohorts currently alive participate in the pension fund. Hence, in this case, $I_t = \{s : t - s \in [0, t_D]\}, \forall t$, where $t - s$ is the age of cohort s. Under full participation the set of working cohorts in the collective pension scheme at time t is

$$I_t^w = \{s : t - s \in [0, t_R]\} \cap I_t,$$

while the set of retired cohorts participating at time t is

$$I_t^r = \{s : t - s \in [t_R, t_D]\} \cap I_t.$$

Asset Dynamics The pension fund's assets A_t evolve as

$$dA_t = \frac{dP_t}{P_t} A_t + \left(C_t - B_t^{TOT}\right) dt.$$

Hence, the pension fund's assets grow according to the stochastic portfolio return (dP_t/P_t) as defined in equation (1) plus the total volume of contributions (C_t) minus the total volume of benefit payments $\left(B_t^{TOT}\right)$.

Valuation of the Liabilities The actuarially-fair price of the pension entitlements at time t of cohort s is

$$\Pi_t (B_{s,t}) = R_{t-s} B_{s,t},$$

where R_{t-s} is the discount factor of pension entitlements and $B_{s,t}$ are the accumulated pension entitlements of cohort s at time t. To derive the discount factor we distinguish between the pre- and the post-retirement period. For a cohort of age $\nu = t - s$ the discount factor is given by

$$\begin{aligned} R_\nu &= \begin{cases} \exp\left[-r\left(t_R - \nu\right)\right] \int_{t_R}^{t_D} \exp\left[-r\left(u - t_R\right)\right] du, & \text{for } \nu \in [0, t_R], \\ \int_\nu^{t_D} \exp\left[-r\left(u - \nu\right)\right] du, & \text{for } \nu \in (t_R, t_D), \end{cases} \\ &= \frac{1}{r} \exp\left\{-r\left[t_R - \min\left(t_R, \nu\right)\right]\right\} \left(1 - \exp\left\{-r\left[t_D - \max\left(t_R, \nu\right)\right]\right\}\right). \end{aligned}$$

The pension fund's liabilities equal the discounted pension entitlements integrated over all participating cohorts[3]

$$L_t = \int_{I_t} R_{t-s} B_{s,t} ds.$$

The Policy Instruments The collective pension scheme has two policy instruments available to respond to financial shocks: the rate at which accrued pension entitlements are indexed and the contribution. We refer to the policies associated with these instruments as the "indexation policy" and the "contribution policy".

The indexation policy is specified as follows. Pension entitlements of cohort s at time t evolve as

$$dB_{s,t} = [\psi w_{s,t} + (\gamma_t - 1) B_{s,t}] dt, \text{ for } t - s \in (0, t_D),$$

where ψ is the accrual rate as a constant fraction of the wage rate and γ_t is the (gross) indexation rate. Note that the wage rate, and, hence, also accrual, is zero during retirement, i.e. $w_{s,t} = 0$ for $t - s \notin [0, t_R]$. Because entitlements are zero at the moment of entry into the labour force, we have $B_{s,s} = 0$. The indexation rate allows the pension fund to respond to the current funding ratio, which in turn is affected by the pension fund's investment returns. Hence, indexation policy is best understood as a correction to the pension entitlements as a result of the pension fund's investment returns. Because inflation is zero, we calibrate the indexation policy such that the indexation rate is $\gamma_t = 1$ when the funding ratio is at its long-run target level (see below). Hence, in that case, retirement benefits grow at rate zero. Integrating over all retired cohorts, the pension fund's period-t aggregate benefit payments are

$$B_t^{TOT} = \gamma_t \int_{I_t^r} B_{s,t} ds.$$

The second policy instrument concerns the contribution. The contribution by an individual from cohort s at time t is the sum of an actuarially-fair contribution $(\bar{c}_{t-s})$ and a recovery contribution $(\pi_{s,t})$. Aggregate contributions over all working cohorts are given

[3]Note that the valuation of the liabilities does not include the value of a potential exit-option. The liabilities are calculated under the implicit assumption that none of the current participants will exit the pension fund. This is in line with how liabilities are calculated in practice. Including the participation decision in the valuation of the liabilities would complicate the model so substantially that it is beyond the scope of the current paper. The specific way of calculating the liabilities does not interfere with the optimality of an individual's participation decision given the policy followed by the pension fund. However, the pension fund's policy itself is based on the funding ratio and is therefore affected by the way the liabilities are calculated.

by

$$C_t = \int_{I_t^w} c_{s,t} ds = \int_{I_t^w} \left(\bar{c}_{t-s} + \pi_{s,t}\right) ds,$$

where the actuarially-fair component of the individual contribution is

$$\bar{c}_{t-s} = \psi w_{s,t} R_\nu = \begin{cases} \psi R_{t-s}, & \text{for } t-s \in [0, t_R], \\ 0, & \text{otherwise.} \end{cases}$$

This actuarially fair component ignores the potential value of the exit-option that an individual may hold. The recovery contribution can be positive or negative, depending on the pension fund's financial position.

Regulation The collective pension schemes are regulated, for example by a supervisor appointed by the government. The key input is the funding ratio

$$F_t = A_t / L_t.$$

The regulator requires pension funds to target a funding ratio of $\bar{F}$. Moreover, when the actual funding ratio F_t deviates from the target funding ratio, it requires the pension fund to close the gap between the two at a sufficiently high speed according to the following equation

$$E_t^{\mathbb{Q}} \left(F_{t+dt} - \bar{F}\right) = \alpha^{dt} \left(F_t - \bar{F}\right), \quad 0 < \alpha < 1, \tag{2}$$

where α denotes a regulatory smoothing parameter. This smoothing parameter allows for a gradual adjustment to financial shocks. The smoothing parameter is crucial in determining the distribution of the adjustment burden across cohorts. For example, when the parameter is high, recovery from a low funding ratio will be smoothed out over a long horizon, implying that the oldest cohorts will have died before the full adjustment has taken place. In the sequel, we will refer to $F_t - \bar{F}$ as the "funding gap". Appendix A.2 shows that the rule above can be rewritten as

$$E_t^{\mathbb{Q}} \left(F_{t+u} - \bar{F}\right) = \alpha^u \left(F_t - \bar{F}\right), \quad \forall u \geq 0.$$

This implies that the funding ratio is expected to converge to $\bar{F}$ in the long run

$$\lim_{u \to \infty} E_t^{\mathbb{Q}} \left(F_{t+u}\right) = \bar{F}. \tag{3}$$

Equilibrium Targets We define the "equilibrium" value of the liabilities as their value under full participation, while the funding ratio is at its regulatory target, i.e. $F_t = \bar{F}$. Appendix A.4 derives the equilibrium liabilities $\left(\bar{L}\right)$ and target funding ratio as

$$\begin{aligned}\bar{L} =& \frac{\psi}{r}\left(t_R\left(t_D - t_R\right) - \frac{1}{r^2}\left\{1 - \exp\left[r\left(t_R - t_D\right)\right]\right\}\left[1 - \exp\left(-rt_R\right)\right]\right),\\ \bar{F} =& 1.\end{aligned}$$

Classification of Collective Pension Arrangements The pension fund thus uses its two policy instruments, the recovery contribution $(\pi_{s,t})$ and the indexation rate (γ_t), to manage a funding gap. These instruments have a different impact on the pension fund's participants. The recovery contribution is paid by the active members (the workers), while the indexation rate affects both the active and retired participants. Its incidence differs by the relative amount of accumulated pension entitlements.

We now can define the "total correction" Ω_t. It captures the part in period t resulting from regulatory policy that has to be covered by additional contributions and deviations of the gross indexation rate from unity. Appendix A.5 shows that

$$\int_{I_t^w} \pi_{s,t} ds + (1 - \gamma_t)\left(\Lambda_t + \int_{I_t^r} B_{s,t} ds\right) = \Omega_t.$$

The pension fund's board can now easily apply its policy instruments to navigate the funding ratio. We define ω as the constant fraction of the correction Ω_t that is to be achieved through adjusting indexation. By definition, the remainder $(1 - \omega)$ of the correction is to be achieved through contribution adjustments. Hence,

$$\omega\Omega_t = (1 - \gamma_t)\left(\Lambda_t + \int_{I_t^r} B_{s,t} ds\right), \tag{4}$$

$$(1 - \omega)\,\Omega_t = \int_{I_t^w} \pi_{s,t} ds. \tag{5}$$

This setting allows us to classify collective pension schemes according to their value for ω. On the one end, if $\omega = 0$, the scheme is of the collective DB type. There is no uncertainty about the benefits, as $\gamma_t = 1, \forall t$. Instead, all investment risk is absorbed through changes in current and future contributions. On the other hand, if $\omega = 1$, the pension scheme is of the collective DC type. None of the correction takes place through contributions. In this case, all investment risk is allocated directly to the active participants through an adjustment of their entitlements and to the retirees through an adjustment of their benefits. For $0 < \omega < 1$, we have a hybrid pension scheme. A hybrid scheme uses both benefit and contribution adjustments to allocate investment risk to the participants. Table 1 summarizes

Table 1: Classification of specific collective pension schemes

Parameter:	$\omega = 0$	$\omega \in (0,1)$	$\omega = 1$
Pension scheme:	DB	hybrid	collective DC

these cases. In the sequel we refer to ω as the "hybridity" parameter.

The Recovery Contribution Policy On top of the actuarially-fair contribution, participants contribute to the pension fund's recovery. To allow for flexibility in the recovery contribution policies, we weigh the part of the recovery contribution covered by a worker of age ν by the factor $\left(\bar{c}_{t_R} - \bar{c}_\nu\right)^\theta$, $\theta \geq 0$, which for $\theta > 0$ is positive and decreasing in age, since $\frac{\partial\left(\bar{c}_{t_R} - \bar{c}_\nu\right)}{\partial \nu} < 0$. Appendix A.6 shows that the time t age-dependent recovery contribution paid by a participant of age ν is

$$\pi_{t-\nu,t} = (1-\omega)\,\Omega_t \frac{\left(\bar{c}_{t_R} - \bar{c}_\nu\right)^\theta}{\int_{I_t^w} \left(\bar{c}_{t_R} - \bar{c}_{t-s}\right)^\theta ds}.$$

Now we can distinguish between different recovery policies. For $\theta = 0$, the recovery burden through contributions is spread equally over all working cohorts. We refer to this as the "uniform policy". For $\theta > 0$ a relatively larger part is absorbed by a worker when he is younger. Figure 1 illustrates different age-dependent contributions for the case of a DB pension scheme ($\omega = 0$). The solid line represents the actuarially-fair contribution ($\bar{c}_\nu$), while the other lines illustrate how recovery is spread over the working cohorts given some θ. The lines above (below) the solid line are based on a funding ratio of 99% (101%). Appendix A.6 also shows that $\frac{\partial c_{t-\nu,t}}{\partial \Omega_t} \geq 0$, hence the age-dependent recovery contribution is increasing in the correction factor Ω_t, and that $\frac{\partial^2 c_{t-\nu,t}}{\partial \Omega_t \partial \nu} \leq 0$ for $\theta > 0$, implying that an increase in the recovery burden has to be absorbed by the younger workers in particular.

2.3.3 The Option to Exit the Collective Scheme

We have now described the individual DC scheme and the full continuum of collective schemes, ranging from DB to collective DC. In the DB scheme financial shocks are fully absorbed by adjusting contributions. In the collective DC scheme shocks are fully absorbed by adjusting indexation. Now, as a final step to complete our model, we introduce the option for a participant to exit the collective scheme. Doing so means that he switches from the collective scheme to the individual DC scheme.

For the sequel, we assume the following. By default an individual enters the labour market as a member of a collective pension scheme. The individual holds an option, to be specified

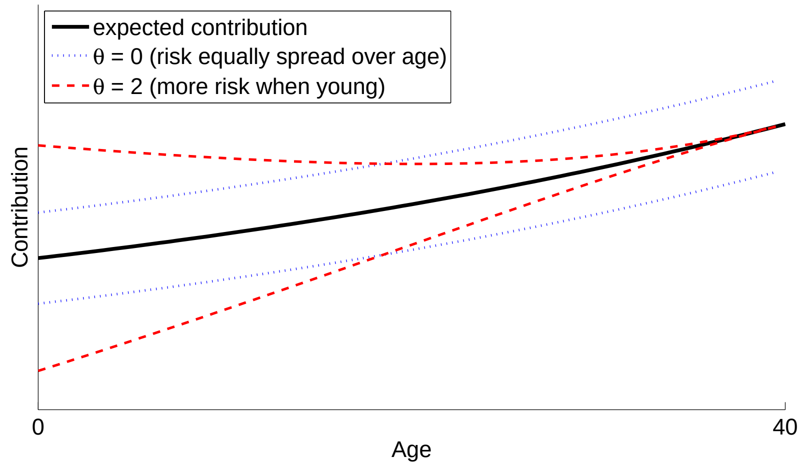


Figure 1: Graphical illustration of the age-dependent contribution.

below for different cases, to exit the collective scheme.[4] The option cannot be exercised after retirement date. Hence, if it has not been exercised before or at the retirement date, then at time t a participant of age $\nu \in (t_R, t_D)$ receives retirement benefits $B_{t-\nu,t}$. If the exit-option is exercised at some age $\nu \in [0, t_R]$ during the accrual phase, then the accumulated pension rights $B_{t-\nu,t}$ are converted into personal assets $A^{DC}_{t-\nu,t}$ according to this rule

$$A^{DC}_{t-\nu,t} = \min\left(1, F_t\right) \Pi_t \left(B_{t-\nu,t}\right).$$

This rule implies that in case of underfunding, $F_t < 1$, the amount of personal assets he receives is the actuarial value of his entitlements reduced by a factor equal to the fraction by which the current funding ratio falls below 100 percent. In the case of overfunding, the individual simply receives the actuarial value of his entitlements. If he would receive more, he would exit at the retirement age and buy an annuity that pays higher benefits than the benefits he receives as a participant of the pension fund. Effectively, in the case of overfunding the pension fund's participants encounter a penalty when leaving the pension fund. This penalty resembles a written put option on the pension fund's assets.

After exiting the pension fund at age ν, the individual transfers his personal assets into an individual DC account of which the value during the remaining part of his working life evolves as

$$A^{DC}_{t-\nu,\tau} = A^{DC}_{t-\nu,t} \frac{P_\tau}{P_t} + \int_t^\tau c^{DC} \frac{P_\tau}{P_u} du, \quad \text{for } \tau - t + \nu \in [\nu, t_R].$$

[4] A thorough analysis of default options for pension plans is provided by Madrian and Shea (2001). They find that only a small fraction of participants decides to opt out.

Table 2: Benchmark parameter values

Description	Symbol	Value
Entry age	t_0	0
Retirement age	t_R	40
Age of death	t_D	60
Target funding ratio	$\bar{F}$	1
Regulatory smoothing parameter	α	0.5
Risk-free interest rate	r	0.02
Portfolio return volatility	σ	0.15
Wage	w	1
Accrual rate	ψ	$0.7/t_R$

3 Parametrization and Simulation Setup

Now we turn to the numerical part of the analysis. Table 2 reports the choice of the benchmark parameter values. As a robustness check, we will later also explore other parameter settings. We assume a regulatory target funding ratio of 100% and we assume that the pension scheme operates under full participation. Hence, we set $\bar{F} = 1$ and $I_t = \{s : t - s \in [0, t_D]\}, \forall t$. This means that all cohorts who are alive at time t, i.e. cohorts $s \in [t - t_D, t]$, participate in the collective pension scheme. Such a situation could be the result of participation having been mandatory so far or of a good investment performance of the pension fund so far.

Our analysis is based on $Q = 10^5$ simulation runs, each with a "burn-in" period of 100 years, after which we start evaluating the simulation results.[5] This way, we do not obtain the results around an equilibrium state, but around a more realistic setting in which the various cohorts have been confronted differently to the risks. Time steps in our simulations need to be small to approximate continuous time. We set the time steps at $\delta = 0.1$, implying 10 possible dates per annum to exercise the exit-option when it is of the American type. For convenience, the benchmark calculations are based on a cohort that starts working at time $t_0 = 0$. Hence, for this cohort $s = 0$ time equals age.

4 The "European" Exit-Option under a DB Pension Scheme

We start our numerical analysis with recovery contributions that are equally spread over all working cohorts by setting $\theta = 0$. Later we consider other values for θ as well. We

[5]This burn-in period is intended to converge to the long-run distribution of our state variables. Given the initial values under our benchmark parameter setting, the distribution of the variables converges in 30 to 40 years.

now turn to the analysis of the exit-option. Exercising the option means a switch to the individual DC scheme and not exercising the option means the participant remains in the collective scheme. In this section we assume some simplifications that will be relaxed in later sections. In particular, we allow for only one working age at which the participant can exercise the option, i.e. the European exit-option. In the next section the option can be exercised continuously. We also assume a contribution policy that is actuarially fair when all variables are at their equilibrium values, an assumption that we will relax in Section 6. Hence, under full participation and $\theta = 0$ we can write

$$c_{t-\nu,t} = \bar{c}_{\nu} + \pi_{t-\nu,t} = \bar{c}_{\nu} + \frac{(1-\omega)}{t_R}\Omega_t.$$

Finally, we confine ourselves to a DB pension scheme. In the following sections we explore the exit-option also under the other schemes.

Under the DB scheme benefits are fully guaranteed, while all investment risks are absorbed through contribution adjustments. Hence, $\omega = 0$ and $\gamma_t = 1, \forall t$, so that pension entitlements grow uniformly over an individual's working career

$$B_{s,t} = \begin{cases} \psi\,(t-s)\,, & \text{for } (t-s) \in [0, t_R]\,, \\ B_{s,s+t_R}, & \text{for } (t-s) \in (t_R, t_D)\,, \\ 0, & \text{otherwise.} \end{cases}$$
$$= \begin{cases} \psi \min\left(t-s, t_R\right), & \text{for } (t-s) \in [0, t_D)\,, \\ 0, & \text{otherwise.} \end{cases}$$

Put alternatively, a retiree's pension is simply the accrual rate multiplied by the years of service (ψt_R). The constant benefit during retirement and the full participation imply that liabilities are always equal to their equilibrium value, i.e. $dL_t = 0$. The only policy instrument available to the pension fund is the recovery contribution, which is derived in Appendix A.7 as

$$\pi_{t-\nu,t} = \frac{[r - (\log\alpha)]\left(\bar{L} - A_t\right)}{t_R}. \tag{6}$$

This means that the recovery contribution is increasing in the shortfall of the pension fund's assets from its liabilities, decreasing in the number t_R of working cohorts that need to close this shortfall and increasing in the difference between the risk-free interest rate, r, and the logarithm of the smoothing parameter. Note that $-\log\alpha > 0$. An increase in the smoothing parameter implies that the funding gap needs to be closed less fast. Hence, the recovery contribution can be reduced, while the recovery itself is stretched out over a longer horizon. However, Appendix A.7 shows that the long-run expected recovery contribution equals zero.

Now we turn to the decision by the individual participant. There is only one age at which he can decide to exit the collective scheme and this decision is irreversible. A cohort s worker of age t_M decides to continue his participation in the pension scheme when the so-called "value of participation" is positive. This is the discounted value of his future pension benefits, $\exp\left[-r\left(t_R - t_M\right)\right]\Pi_{s+t_R}\left(B_{s,s+t_R}\right)$, minus the expected discounted sum of the contributions to be paid from now until retirement, $E^{\mathbb{Q}}_{s+t_M}\left[\int_{t_M}^{t_R} c_{s+u}\exp\left[-r\left(u-t_M\right)\right]du\right]$, minus the payout received upon exiting, $\min\left(1, F_{s+t_M}\right)\Pi_{s+t_M}\left(B_{s,s+t_M}\right)$. Hence, the value of participation is given by

$$\begin{aligned} Part_{s,s+t_M} = & \exp\left[-r\left(t_R - t_M\right)\right]\Pi_{s+t_R}\left(B_{s,s+t_R}\right) \\ & -E^{\mathbb{Q}}_{s+t_M}\left[\int_{t_M}^{t_R} c_{s+u}\exp\left[-r\left(u-t_M\right)\right]du\right] \\ & -\min\left(1, F_{s+t_M}\right)\Pi_{s+t_M}\left(B_{s,s+t_M}\right). \end{aligned}$$

Appendix A.9 shows that for $F_{s+t_M} = 1$, we have that $Part_{s,s+t_M} = 0$ and $\partial Part_{s,s+t_M}/\partial F_{s+t_M} > 0$. Hence, the value of participation is strictly positive for funding ratios above one and strictly negative for funding ratios below one. This way, due to the actuarially-fair contribution, the participation threshold funding ratio is exactly 100% for all ages when there is only a single age at which the exit-option can be exercised.

5 The "American" Exit-Option

Now we turn to the analysis of the exit-option given that the participant has the possibility to exit at any moment until retirement. Because of the increased number of exercise dates, this American exit-option is more valuable than the European exit-option considered in the previous section. This pushes the value of participation further upwards. To approximate the value of participation under the American option, we choose a finite partition $t \in \{t_0, t_0+\delta, t_0+2\delta, \ldots, t_R\}$ for the exercise dates. An analytical expression for the value of participation would be cumbersome to derive algebraically, if this is at all possible. Therefore, we use the so-called Least Squares Monte Carlo (LSMC) approximation method to determine the option value. We explain this method in Appendix B. In the next sections we apply this method to analyse different pension schemes.

5.1 A Defined-Benefit Pension Scheme

We first consider the DB pension scheme with guaranteed benefits. Therefore, the only variable relevant for the participation decision is the funding ratio. We approximate the value of participation for cohort s using the following regression model

$$Part_{s,t} = \left[\begin{array}{cccc} 1_Q & F_t & F_t^2 & \max(1 - F_t, 0) \end{array} \right] \beta_{t-s} + \varepsilon, \tag{7}$$

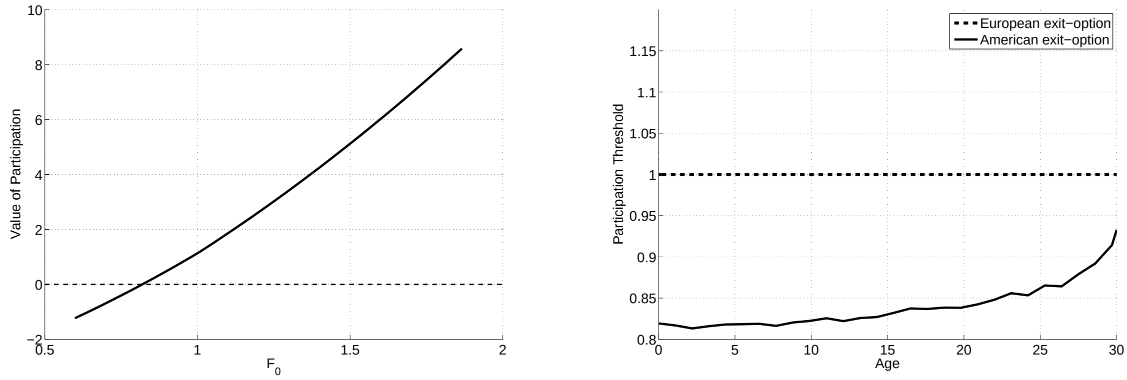
This model fits the value of participation as a function of the funding ratio. In particular, besides a linear term, we include a quadratic function of the funding ratio (F_t) and the extent of underfunding, i.e., $\max(1 - F_t, 0)$.

Figure 2(a) depicts the value of participation at age $t_0 = 0$ as a function of the funding ratio at entry F_0. The value of participation is increasing in F_0 and positive for large enough values of F_0. Due to the right to exit at any age up to retirement, the participation threshold has fallen to 81.93%, which is substantially below the original threshold of 100% for the single exercise age. If the funding ratio increases the participant benefits from a reduction in contributions, while if the funding ratio decreases the participant has the opportunity to exit the pension fund and forego recovery contributions. The disadvantage of exercising the exit option is that he loses a fraction $(1 - F_t)$ of his pension entitlements. However, this loss is small when he is young, because he has barely accumulated any pension entitlements. In fact, at entry participants have a call option on the assets of the pension fund, because they obtain a zero pay-off when exiting at entry, while the value of participation is positive when the funding ratio exceeds 81.93%. Hence, when underfunding at entry is only limited, the benefit of staying in the pension fund outweighs the cost associated with restoring the funding ratio. Further, an increase in α implies more smoothing and dampens the restoration contribution (equation (6)) towards zero for two reasons: more future cohorts contribute to the recovery, while, in addition, even without the inflow of new cohorts a lengthening of the restoration period would dampen the recovery contribution of all existing working cohorts and would enhance the future recovery contribution of existing young cohorts. Notice that in the most extreme case, in which α would be equal to unity (formally not possible under the model), the restoration contribution would be the interest payment on a net debt (the difference between liabilities and assets) that is permanently rolled over.

The solid line in Figure 2(b) shows the participation threshold as a function of age in the case of the American exit-option. Because the value of the option is larger when there are more future exercise dates, the threshold is lower at young ages.

5.2 Collective DC and Hybrid Pension Schemes

Now, we turn to collective DC and hybrid pension schemes. Under the former, the only policy instrument is the indexation rate. Under the latter, the pension fund uses both recovery contributions and the indexation rate as instruments. Given the target funding ratio of 100% and full participation, Appendix A.5 shows that the instruments are determ-



(a) Value of participation at entry under the American exit-option

(b) Participation thresholds at different ages

Figure 2: Value of participation and the corresponding exercise thresholds in the DB pension scheme for the European and American exit-option.

ined through the following two expressions

$$(\gamma_t - 1)\left(\frac{1-\omega}{\omega}\Lambda_t + \frac{1}{\omega}\int_{t_R}^{t_D} B_{t-s,t} ds\right) = A_t \left[r - (\log\alpha)\right] - F_t E_t^{\mathbb{Q}}\left(\frac{dL_t}{dt}\right) + (\log\alpha) L_t + \dots$$

$$\dots \int_0^{t_R} \bar{c}_s ds - \int_{t_R}^{t_D} B_{t-s,t} ds,$$

$$\int_0^{t_R} \pi_{t-s,t} ds = \frac{1-\omega}{\omega}(1-\gamma_t)\left(\Lambda_t + \int_{t_R}^{t_D} B_{t-s,t} ds\right),$$

where, now,

$$\exp(-rdt)\,\Lambda_t = L_t - \int_{t_D - dt}^{t_D} R_s B_{t-s,t} ds - \frac{1-\exp(-rdt)}{r}\int_{t_R-dt}^{t_D-dt} B_{t-s,t} ds.$$

The retirement benefits are uncertain, because the indexation rate is uncertain. While in the case of the DB pension scheme the participation decision only depends on the funding ratio, it now depends on a wider set of variables. To see this, suppose, for example, that we fix the funding ratio at 100% and vary the pension entitlements of the very old. If these entitlements are relatively low, then the pension fund's liabilities can be expected to increase, because the average level of the entitlements will rise once the low entitlements of the very old have dropped out. An increase in the (average) liabilities will lead to reduced future indexation. Hence, it becomes relatively more attractive for active cohorts to exit. The opposite is the case when the entitlements of the very old are relatively high.

We approximate the value of participation for cohort s by expanding the regression in equation (7) as follows

$$Part_{s,t} = X_t \beta_{t-s} + \varepsilon, \tag{8}$$

$$\text{with } X_t = \left[\begin{array}{cccccccccc} 1_Q & F_t & F_t^2 & \max(1-F_t,0) & L_t & L_t^2 & \underbrace{F_t L_t}_{=A_t} & B_{t-10,t} & B_{t-20,t} & \ldots & B_{t-50,t} \end{array} \right]. \tag{9}$$

Note that the regression coefficient vector β_{t-s} is age dependent. Hence, we expand the previous regression model with linear and quadratic terms in the total liabilities (L_t), the assets (A_t) and the entitlements of a limited number of cohorts. We do not include the entitlements of all the cohorts, as the entitlements of cohorts close in age would be close, possibly leading to multi-collinearity. In addition, we exclude $B_{t,t}$, since the pension entitlements are zero at entry, while we also exclude the pension entitlements of the deceased generation at time t, $B_{t-t_D,t}$. We denote the vector of estimated regression coefficients by $\hat{\beta}_{t-s}$.

Table 3 reports the coefficient estimates of equation (8) for different cohort ages and different values of the hybridity parameter ω. The estimates yield several interesting insights. First, the coefficient on the degree of underfunding max $(1-F_t, 0)$ increases with age. In case of underfunding more pension entitlements are lost when the exit-option is exercised at a higher age, because older participants have accumulated more pension entitlements. Second, the coefficient on liabilities L_t is negative and is generally decreasing with age, while the coefficient on assets A_t is positive and increasing with age. Hence, the value of participation decreases with the liabilities and increases with the assets of the pension fund. These effects become stronger at higher ages, because the participants have accumulated more pension entitlements, so that the effects of lower indexation (when liabilities increase, ceteris paribus) and higher indexation (when assets increase, ceteris paribus) become larger. Third, at young ages, the coefficients on liabilities and assets become smaller in absolute value as the pension scheme relies more on recovery through indexation adjustments (i.c., ω increases). The consequences of restoration are smaller for the young, because they have accumulated fewer pension entitlements. The preceding effect, that the indexation policy affects the participation decision more strongly when pension entitlements are large, dominates at higher ages. Furthermore, the variation in the liabilities and assets increases with the hybridity parameter ω, as the recovery through indexation adjustments has a persistent effect on the aggregate benefit payments. Fourth, the coefficient on the pension entitlements of the cohort closest in age to cohort s has the largest value, in line with the intuition that the value of participation for an individual is affected most by his own pension entitlements.

Table 3: Coefficient estimates of equation (8) for the approximation of the value of participation under the American exit-option.

	$\omega = 0.25$ (hybrid)					$\omega = 0.5$ (hybrid)				
Age	0	10	20	30	40	0	10	20	30	40
β_1	0.606	-0.402	0.822	4.034	-4.864	1.332	-0.903	0.180	1.803	-4.423
β_F	-1.649	-1.139	-2.958	-3.503	3.691	-1.356	0.548	-3.081	-4.612	3.411
β_{F^2}	0.941	0.592	1.567	0.136	0.373	0.631	-0.341	1.416	0.768	0.328
$\beta_{\max(1-F,0)}$	0.357	2.347	3.368	7.479	11.323	-0.444	1.750	3.343	7.371	11.213
β_L	0.011	-0.030	-0.106	-0.153	-0.028	-0.016	-0.029	-0.052	-0.082	-0.058
$\beta_{L^2} * 10^6$	0.150	-2.291	16.921	48.788	-1.201	1.679	-0.855	4.897	16.511	-0.004
β_A	0.013	0.015	0.015	0.029	-0.006	0.006	0.009	0.011	0.024	0.004
$\beta_{B_{10}}$	-0.042	6.444	6.759	9.714	3.429	1.910	9.424	2.958	3.449	5.567
$\beta_{B_{20}}$	-3.558	1.193	18.050	8.153	4.004	-0.041	2.598	12.531	3.886	5.998
$\beta_{B_{30}}$	-1.902	2.422	10.173	22.371	4.835	1.099	2.251	6.042	15.791	7.741
$\beta_{B_{40}}$	-3.501	2.820	11.460	13.933	5.528	1.740	2.626	4.480	7.770	8.066
$\beta_{B_{50}}$	-2.335	1.785	8.722	9.433	3.827	0.723	2.318	4.295	4.117	6.144
	$\omega = 0.75$ (hybrid)					$\omega = 1$ (Collective DC)				
Age	0	10	20	30	40	0	10	20	30	40
β_1	0.614	-1.260	-0.026	1.439	-4.719	0.619	-1.340	-0.016	1.569	-5.827
β_F	0.218	1.078	-3.228	-5.755	4.270	0.335	1.148	-3.579	-6.455	6.368
β_{F^2}	-0.111	-0.248	1.611	1.402	-0.186	-0.125	-0.142	1.941	1.744	-1.172
$\beta_{\max(1-F,0)}$	-0.154	1.985	3.373	7.156	11.267	-0.107	2.120	3.268	7.036	11.546
β_L	-0.004	-0.011	-0.037	-0.065	-0.103	-0.002	-0.010	-0.039	-0.069	-0.156
$\beta_{L^2} * 10^6$	0.221	0.122	1.706	5.343	0.685	-0.082	-0.260	0.465	0.987	1.150
β_A	0.002	0.004	0.008	0.023	0.014	0.000	0.002	0.008	0.026	0.023
$\beta_{B_{10}}$	1.393	7.646	2.092	0.894	9.780	2.333	7.230	2.698	-0.610	15.573
$\beta_{B_{20}}$	-0.552	1.996	10.916	3.233	9.277	-0.696	3.144	10.202	4.481	13.247
$\beta_{B_{30}}$	0.129	0.182	5.699	14.073	13.001	0.156	-0.129	7.611	12.898	19.868
$\beta_{B_{40}}$	0.425	0.915	2.648	7.812	12.268	0.481	1.098	2.526	10.831	17.218
$\beta_{B_{50}}$	0.132	0.851	3.241	2.724	10.240	0.220	0.933	3.542	2.653	15.463

We can gain additional insights from Figure 3, which presents the estimated value of participation for different combinations of the independent variables in equation (8) and different values of the hybridity parameter ω. We always set the other explanatory variables at their equilibrium values. For example, panel (a) of Figure 3 depicts the value of participation at age 10 as a function of the funding ratio and the pension entitlements of the cohort of age 10, while we set $L_t = \bar{L}$, $B_{t-20,t} = 20\psi$, $B_{t-30,t} = 30\psi$ and $B_{t-40,t} = B_{t-50,t} = t_R\psi$. First, we observe that the value of participation typically increases in the funding ratio[6] and in the own entitlements. Second, we see that at a high age - see panel (d) for the age of 40 - the more the pension fund relies on indexation (i.e., the higher is ω), the more the

[6] The full effect of the funding ratio is calculated as the effect of an increase in F_t on the term $\beta_F F_t + \beta_{F^2} F_t^2 + \beta_{\max(1-F,0)} \max(1 - F_t, 0) + \beta_A \bar{L} F_t$.

figure surface rotates anti-clockwise for given own entitlements. For given above-target funding ratio, the larger is ω, the higher is the value of participation, because the older participant has a larger claim on extra indexation in the future. For given below-target funding ratio, the value of participation is decreasing in ω. The reason is that a larger fraction of the underfunding has to be worked away through reduced indexation when ω is higher.[7] At low ages the value of participation rotates clockwise for given own entitlements when ω increases. Since young generations have low pension entitlements, the higher indexation rate in the case of overfunding benefits the older cohorts at the expense of young workers, because it leaves less room to reduce the contribution, which would benefit the young. Now, for given below-target funding ratio, the value of participation is increasing in ω. The higher degree of indexation allows for a smaller restoration contribution, because it shifts a larger part of the restoration burden to the old cohorts who have relatively high entitlements. Third, for the case of the DB pension scheme, we saw that exiting at the retirement age would never be optimal. However, for the case of the DC pension scheme and the hybrid pension schemes, it could be optimal to exit at the retirement age, as we observe negative values of participation in panel (d). This particularly holds for combinations of low funding ratios and low own pension entitlements. In the case of underfunding, the old participant can be expected to make a large contribution to restoring the pension fund's financial health through reduced indexation. This effect is compensated when his own entitlements are high, but not when they are low – remember that the lump-sum that he gets when he exits is linked to both his own entitlements and the funding ratio.

5.3 Exit Distribution under the Collective Pension Scheme

So far we focused on valuation of the exit-option. In this section we explore the likelihood of participants from different cohorts to exercise their exit-option. Figure 4(a) presents these probabilities as a function of age for different values of the hybridity parameter ω, with the regulatory smoothing parameter α and the investment risk parameter σ at their benchmark values. In each simulation run we checked for each cohort age after the burn-

[7] More formally, suppose that $dL_t = 0$, then equation (12) from Appendix A.5 yields

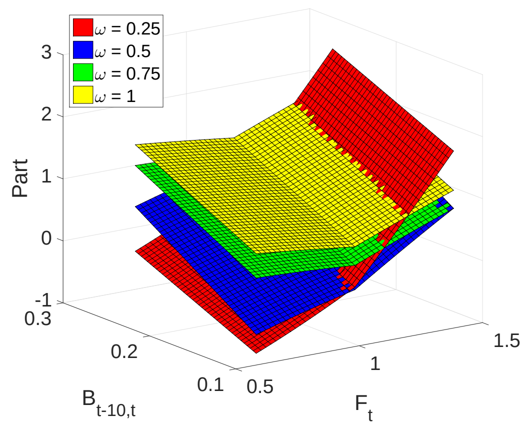
$$(1-\gamma_t)\int_{I_t^r} B_{s,t}ds = -A_t\left[r-(\log\alpha)\right]-(\log\alpha)L_t-\int_{I_t^w}\bar{c}ds+\int_{I_t^r}B_{s,t}ds-\int_{I_t^w}\pi_{s,t}ds$$

$$\Rightarrow\begin{cases}\frac{\partial\gamma_t}{\partial A_t} = & A_t\frac{[r-(\log\alpha)]}{\int_{I_t^r}B_{s,t}ds}>0\\ \frac{\partial\gamma_t}{\partial L_t} = & \frac{(\log\alpha)}{\int_{I_t^r}B_{s,t}ds}<0\end{cases}$$

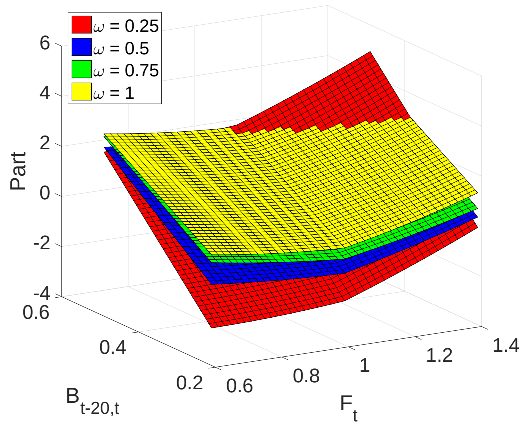
Hence, the indexation rate is positively affected by the funding ratio

$$\frac{\partial\gamma_t}{\partial F_t}>0.$$

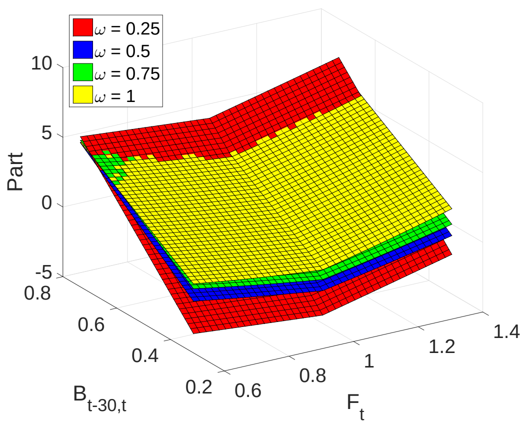
When γ_t is low, then dL_t is typically negative, which amplifies this effect. However, when dL_t is positive, the relation between γ_t and F_t becomes ambiguous.



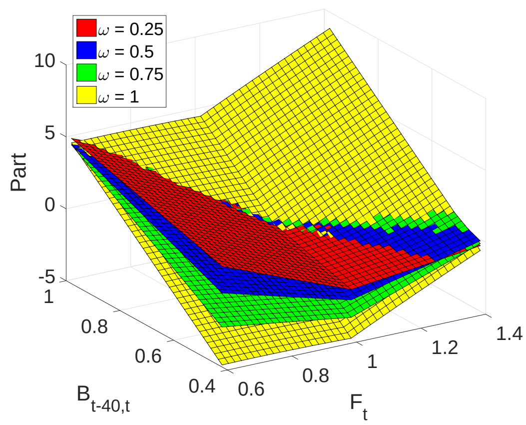
(a) Value of participation at age 10.



(b) Value of participation at age 20



(c) Value of participation at age 30.



(d) Value of participation at age 40.

Figure 3: Approximated value of participation at various ages under the American exit-option for different combinations of the explanatory variables in equation (8) and the hybridity parameter ω.

in period whether the value of participation was negative. A negative participation value triggers the execution of the exit option. The figure thus exhibits the frequency of negative participation values at each age. The exit probabilities under the DB pension scheme ($\omega = 0$), represented by the solid black lines, exhibit a peak around the age of 30. For young participants, the value of the exit-option is relatively high due to the large number of remaining exit possibilities. Hence, the very young are inclined not to exit. However, the option loses value and, hence, the inclination to exit becomes stronger as one grows older. However, beyond a certain point, the likelihood of exiting starts to decrease with age, because the remaining period over which recovery contributions may have to be paid shrinks. As we already saw, under the DB scheme exiting is never optimal at the retirement age. For the hybrid pension schemes, the likelihood to exit is increasing with age, because the contribution to recovery through indexation increases with the level of the entitlements.

In panels (b)-(e) of Figure 4 we vary the regulatory smoothing parameter α and the investment risk parameter σ. The exit probabilities rise, ceteris paribus, with the smoothing parameter. This is the net result of two opposite effects. On the one hand, a higher value of the smoothing parameter implies that the funding ratio on average deviates more from its target level. This implies a higher chance that it becomes so low that at least some cohorts want to leave the scheme. On the other hand, a higher smoothing parameter means that future entrants face a larger share of the recovery burden, which reduces the incentive to exit the pension fund.

When the investment portfolio becomes more risky young workers are more likely to stay in the pension fund, while older workers are more likely to exit ceteris paribus. The intuition is as follows. The pension contract of the young resembles a call option on the pension fund's assets. Higher return volatility raises both the expected degree of underfunding in case underfunding happens and the expected degree of overfunding when overfunding takes place. Given the young's limited pension entitlements, and similar to a standard call option, the expected loss from the former is limited, while the expected benefit from the latter rises. The pension contract of older workers, however, resembles a written put option on the assets of the pension fund: they gain relatively little from overfunding, but they can lose significantly in the case of underfunding by exiting and giving up a fraction of the pension entitlements. The value of a written put option also decreases if the volatility of the underlying asset increases. Ceteris paribus, in the case of a DB scheme or a scheme that is close to DB (ω is low), the incentive to exit shortly before retirement is weak. Only for very small degrees of underfunding the participant would want to exit. When the degree of underfunding is more severe, the loss from given up entitlements by exiting dominates the restoration contribution that will be incurred over the short period until retirement. When ω is high, in the case of severe underfunding the almost-retired worker will suffer from low indexation during his retirement period and has a stronger incentive to exit, ceteris paribus. Hence, close to retirement, the probability of exiting increases with the volatility of the investment returns, σ. This particularly holds when more adjustment occurs through indexation (ω is high), because the pension benefits will be downgraded more during retirement in the case of underfunding.

6 Uniform Contribution Policies

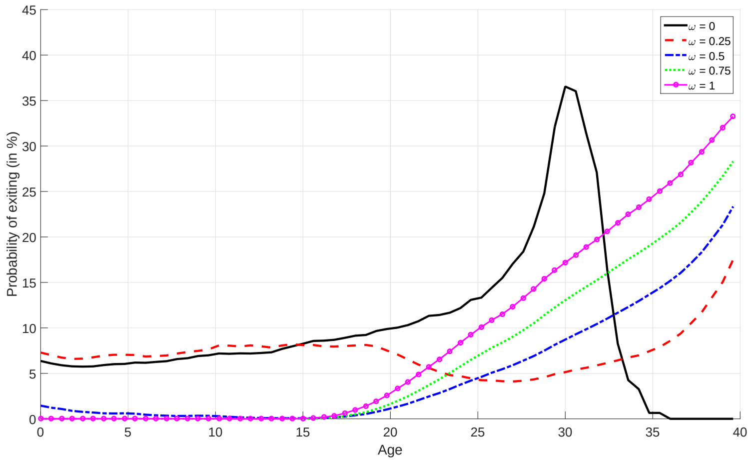
So far, we have considered contributions that are actuarially fair when the funding ratio is at its long-run target. However, in reality collective pension schemes usually rely on uniform contributions, i.e. contributions that are independent of age in particular. These contributions are not actuarially fair. Typically, the young pay relatively too much for their pension accrual and the old pay too less. In this section we explore the stability of the pension fund by varying the contribution policy between being actuarially fair (in equilibrium) and uniform. Intuitively, a uniform contribution policy benefits older workers at the expense of younger workers and, therefore, discourages the former group from exiting the pension fund. However, the exit-option of the younger workers is more valuable than that of the older workers, because the value of the American option is larger when there are more future exercise dates. Therefore, younger workers are also discouraged from exiting.

Appendix A.12 derives the uniform contribution for a general setting. However, as before, we assume a target funding ratio of 100% and full participation. Appendix A.12 shows that under these assumptions we can write the uniform contribution as

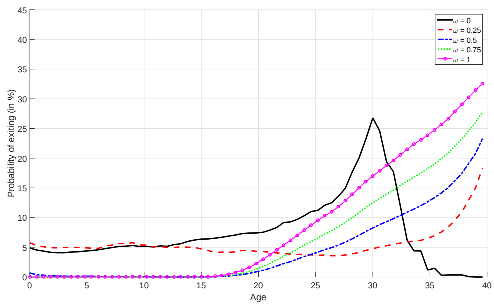
$$\begin{aligned} c_{s,t} =& \bar{c} + \pi_t^{unif}, \\ \text{where } \bar{c}t_R =& \psi \int_0^{t_R} R_u du \text{ and } \pi_t^{unif} = \frac{(1-\omega)}{\int_{I_t^w} 1 ds}\Omega_t. \end{aligned} \tag{10}$$

Hence, the total equilibrium contributions ($\bar{c}t_R$) are equal to the market price of the total pension accrual.

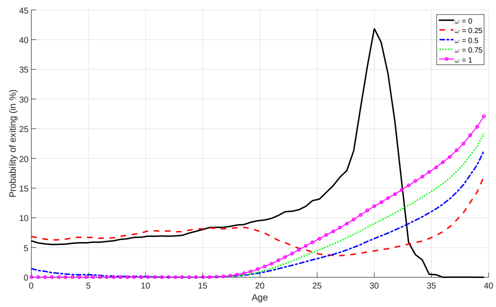
Table 4 reports the actuarially fair and uniform contributions for a pension fund that operates under full participation. The remainder of this section applies the uniform contribution to first the DB scheme and second the collective DC and hybrid schemes. Finally, we study a compromise between the uniform and actuarially-fair contribution policies, thereby analysing what policy settings may be able to raise the probability that no participant wants to exit the pension scheme.



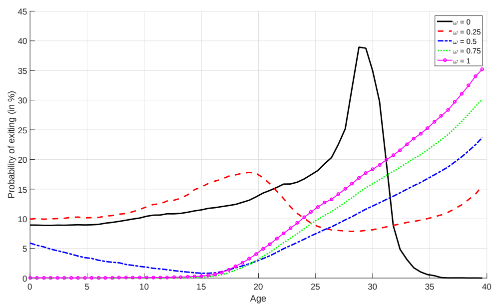
(a) Benchmark: $\alpha = 0.5$ and $\sigma = 15\%$



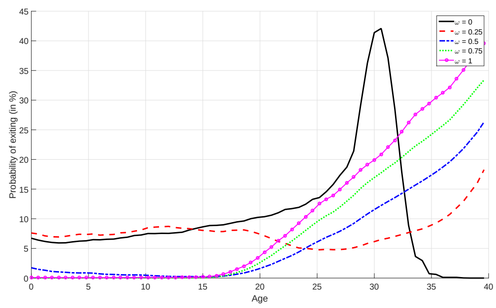
(b) $\alpha = 0.25$ and $\sigma = 15\%$



(c) $\alpha = 0.5$ and $\sigma = 10\%$



(d) $\alpha = 0.75$ and $\sigma = 15\%$



(e) $\alpha = 0.5$ and $\sigma = 20\%$

Figure 4: Probability of exercising the exit-option as a function of age for different values of the hybridity parameter (ω), the smoothing parameter (α) and the investment risk level (σ).

Table 4: Contribution at time t for an agent of age ν under a target funding ratio of 100% and full participation.

Policy	Contribution $(c_{t-\nu,t})$
Actuarially fair:	$\bar{c}_\nu + (1-\omega)\,\Omega_t \frac{\left(\bar{c}_{t_R}-\bar{c}_\nu\right)^\theta}{\int_0^{t_R}\left(\bar{c}_{t_R}-\bar{c}_u\right)^\theta du}$
Uniform:	$\bar{c} + \frac{1}{t_R}(1-\omega)\,\Omega_t$

6.1 The Defined-Benefit Pension Scheme

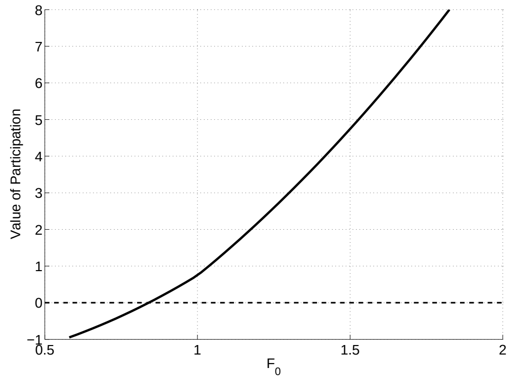
We start with the DB scheme, setting $\omega = 0$ in equation (10), while the indexation policy becomes $\gamma_t = 1, \forall t$, so that pension entitlements grow uniformly over an individual's working career:

$$B_{s,t} = \begin{cases} \psi \min\left(t-s, t_R\right), & \text{for } (t-s) \in [0, t_D)\,. \\ 0, & \text{otherwise.} \end{cases}$$

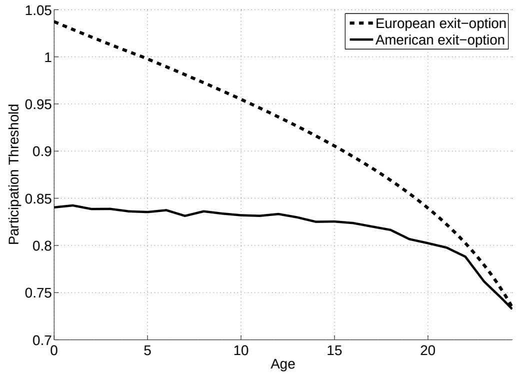
Figure 5(a) depicts, as a function of the funding ratio, the values of participation under the American option at entry into the labour force. The participation threshold corresponds to the point where the curved line crosses the dashed line, i.e. the horizontal axis. Appendix A.13 analytically derives the participation thresholds as a function of age for the European exit-option. These are depicted by the dashed line in panel (b). These thresholds are no longer equal to 100% (recall Figure2(b)), because the uniform contribution benefits older workers at the expense of younger workers. Instead, they decrease with age and exceed 100% for young workers and fall below 100% for older workers. For the American option, we again use our approximation method. The solid line in panel (b) represents the participation thresholds under this option. Because the value of the American option is larger when there are more future exercise dates, the difference between the dashed line (a threshold funding ratio of 103.7%) and the solid line (a threshold funding ratio of 84.2%) is largest at entry.

6.2 The Collective DC and Hybrid Pension Schemes

Next we turn to the other pension arrangements. Table 5 reports the coefficients from the estimation of equation (8) for different ages and values of the hybridity parameter. The estimates yield several interesting insights. First, the impact of underfunding increases with age, because older participants have accumulated more pension entitlements. Second, the effect of higher liabilities is negative and decreasing with age, while the effect of higher assets is positive and increasing with age. Third, at low ages, the coefficients of the assets



(a) Value of participation at entry under the American exit-option



(b) Participation thresholds at different ages

Figure 5: Value of participation and the corresponding exercise thresholds for the European and American exit-options under the DB pension scheme with a uniform contribution.

and the liabilities tend to zero when adjustment relies more on indexation policy, i.c., ω increases. Young participants have fewer pension entitlements and, therefore, the consequences of recovery through indexation are smaller for the young. Fourth, the value of participation is mostly affected by the pension entitlements of the cohort closest in age to the participant under consideration. These results are all in line with those discussed earlier on the basis of Table 3.

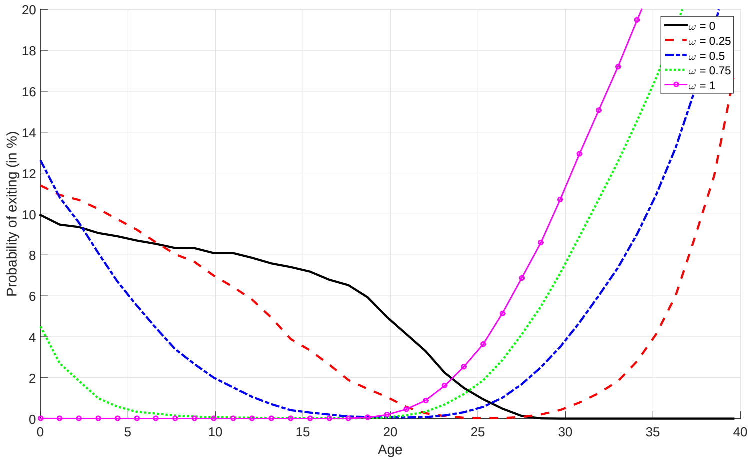
Table 5: Coefficient estimates of equation (8) under a uniform contribution policy for the approximation of the value of participation under the American exit-option.

	$\omega = 0.25$ (hybrid)					$\omega = 0.5$ (hybrid)				
Age	0	10	20	30	40	0	10	20	30	40
β_1	1.269	-0.734	1.628	0.288	-4.864	1.162	-1.397	0.757	-0.228	-4.423
β_F	-3.104	-0.917	-4.951	-1.417	3.691	-2.562	1.122	-4.158	-2.874	3.411
β_{F^2}	1.642	0.692	2.611	-0.212	0.373	1.347	-0.256	2.104	0.314	0.328
$\beta_{\max(1-F,0)}$	1.298	2.733	2.788	7.547	11.323	0.659	2.144	3.055	7.601	11.213
β_L	-0.007	-0.054	-0.159	-0.172	-0.028	-0.001	-0.044	-0.076	-0.097	-0.058
$\beta_{L^2} * 10^6$	1.557	-1.175	14.542	31.085	-1.201	0.886	0.104	4.501	10.800	-0.004
β_A	0.017	0.018	0.019	0.030	-0.006	0.011	0.014	0.016	0.029	0.004
$\beta_{B_{10}}$	1.397	9.667	13.145	12.665	3.429	0.312	10.239	4.877	5.554	5.567
$\beta_{B_{20}}$	-2.180	3.833	22.927	11.869	4.004	-1.952	3.882	15.170	5.235	5.998
$\beta_{B_{30}}$	-0.158	5.390	18.015	26.843	4.835	-0.931	3.698	8.224	18.492	7.741
$\beta_{B_{40}}$	-1.625	5.336	18.542	18.788	5.528	-1.200	4.090	7.643	9.944	8.066
$\beta_{B_{50}}$	-0.800	4.164	14.022	12.538	3.827	-1.113	3.376	6.111	5.669	6.144
	$\omega = 0.75$ (hybrid)					$\omega = 1$ (DC)				
Age	0	10	20	30	40	0	10	20	30	40
β_1	1.115	-1.131	0.487	0.156	-4.719	0.625	-1.216	-0.018	0.906	-5.827
β_F	-1.638	1.100	-3.860	-4.326	4.270	-0.301	1.133	-3.174	-5.738	6.368
β_{F^2}	0.854	-0.245	2.041	1.021	-0.186	0.152	-0.072	1.891	1.669	-1.172
$\beta_{\max(1-F,0)}$	-0.373	1.991	3.117	7.444	11.267	-0.302	2.172	3.371	7.213	11.546
β_L	-0.004	-0.012	-0.053	-0.074	-0.103	0.000	-0.008	-0.038	-0.068	-0.156
$\beta_{L^2} * 10^6$	1.095	0.307	1.622	3.626	0.685	0.010	-0.053	0.404	0.651	1.150
β_A	0.005	0.009	0.012	0.027	0.014	0.000	0.002	0.008	0.025	0.023
$\beta_{B_{10}}$	1.446	7.598	3.264	2.511	9.780	2.071	7.137	3.022	0.300	15.573
$\beta_{B_{20}}$	-1.035	1.443	12.643	3.936	9.277	-0.759	2.840	10.271	4.743	13.247
$\beta_{B_{30}}$	-0.323	-0.121	7.086	15.417	13.001	-0.204	-0.171	7.498	13.066	19.868
$\beta_{B_{40}}$	-0.091	0.085	4.476	8.928	12.268	0.222	0.707	2.505	10.756	17.218
$\beta_{B_{50}}$	-0.355	0.440	4.434	3.524	10.240	-0.013	0.754	3.528	2.737	15.463

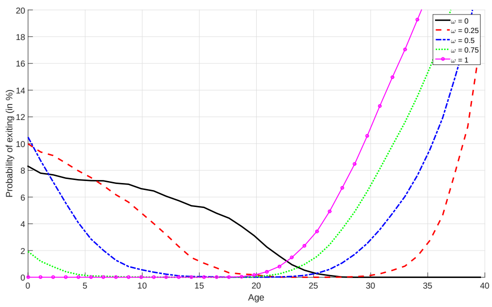
6.3 Exit Distribution under the Collective Pension Scheme

It is also interesting to analyse the probabilities of exercising the exit-option under a uniform contribution policy. Analogous to Figure 4, Figure 6 depicts the likelihood that participants exercise their exit-option.

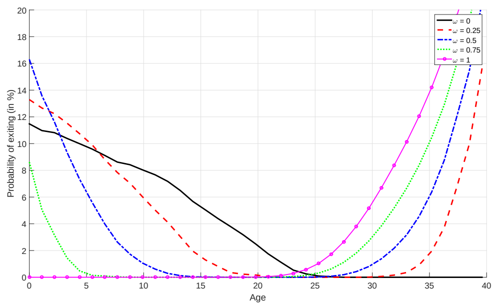
In contrast to the case of the actuarially fair contribution policy, now the exit probabilities are always decreasing with age under DB until they reach zero at a working age between 25 and 30, and after which they remain zero. This shows the attractiveness of the uniform contribution policy for older workers as they are being subsidized by young workers. For the hybrid schemes the exit likelihood is decreasing in age for relatively young workers and increasing for relatively older workers. For younger workers the increasing benefit



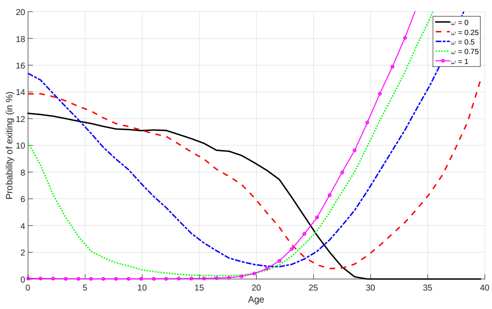
(a) Benchmark: $\alpha = 0.5$ and $\sigma = 0.15$.



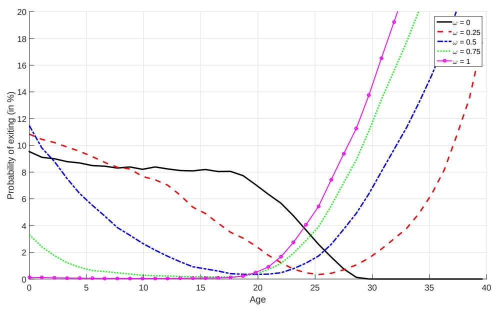
(b) $\alpha = 0.25$ and $\sigma = 0.15$



(c) $\alpha = 0.5$ and $\sigma = 0.10$



(d) $\alpha = 0.75$ and $\sigma = 0.15$



(e) $\alpha = 0.5$ and $\sigma = 0.20$

Figure 6: Probability of exercising the exit-option under a uniform contribution policy as a function of age for different values of the hybridity parameter (ω), the smoothing parameter (α) and the investment risk level (σ).

of the uniform contribution dominates the falling option value in combination with the increasing burden of recovery through indexation when there is underfunding, while for older workers it is the opposite. In fact, for the collective DC scheme, the exit likelihood remains zero until a certain point after which it starts to increase sharply with age. As before, exit probabilities of older workers increase with investment risk (σ), while they also have a general tendency to increase with the degree of regulatory smoothing (α). Finally, compared to the case of an actuarially-fair contribution policy, depicted in Figure 4, exit probabilities are generally higher for young workers, because they pay higher contributions under the uniform contribution policy.

6.4 Combining the Actuarially Fair and Uniform Contribution Policies

The American exit-option is relatively valuable for young workers due to the many exit-opportunities, thereby discouraging them from exiting the pension scheme. By contrast, a uniform contribution policy has the opposite effect on the young's willingness to stay due to the the fact they are subsidizing the old participants. Using a contribution policy that forms a "compromise" between the uniform contribution policy and the actuarially fair (in equilibrium) contribution policy would provide the pension fund additional flexibility in reducing the chances that any cohort wants to exit the pension fund. To explore this in detail we define a general contribution policy. Under full participation and a target funding ratio $\bar{F} = 1$ this more general contribution policy is formulated as a weighted average of the above alternatives

$$c_{t-\nu,t} = (1-\zeta)\left[\bar{c} + \frac{1}{t_R}(1-\omega)\,\Omega_t\right] + \zeta\left[\bar{c}_\nu + (1-\omega)\,\Omega_t \frac{\left(\bar{c}_{t_R} - \bar{c}_\nu\right)^\theta}{\int_0^{t_R}\left(\bar{c}_{t_R} - \bar{c}_u\right)^\theta du}\right]. \qquad (11)$$

We refer to ζ as the "actuarial fairness parameter". If $\zeta = 1$, the contribution policy is actuarially fair in equilibrium, while if $\zeta = 0$, it is uniform. For $\zeta \in (0,1)$, the pension fund applies a compromise between these two extremes. The remainder of this section explores the stability of a DB pension scheme, a hybrid pension scheme (with $\omega = 0.5$) and a collective DC pension scheme for different values of the actuarial fairness parameter (ζ), the reallocation of contribution risk across cohorts (θ), the extent of risk smoothing (α) and the level of investment risk (σ). Note that the parameter θ is irrelevant under the collective DC scheme ($\omega = 1$) and under the uniform contribution policy ($\zeta = 0$).

Table 6 reports the probability that no participant wants to exit (after the burn-in period). The table yields several interesting insights. First, in the absence of investment risk, i.e. $\sigma = 0\%$, it is never optimal to exit the pension scheme under an, in equilibrium, actuarially-fair contribution policy. However, for values of ζ lower than one, the younger cohorts that are subsidizing the older cohorts through the pay-as-you-go element in the overall contribution always want to exit, because in this case the exit-option has no value. Second, (higher) investment risk generates a positive probability that the funding ratio becomes so low that certain cohorts want to exit. At the same time, the exit-option becomes (more) valuable, especially for the younger cohorts. The highest likelihood that all cohorts want to stay in the pension fund is no longer attained under an actuarially-fair contribution policy, but by introducing some degree of uniformity, i.e. by setting $\zeta < 1$. Third, the likelihood that no cohort exits under the DB pension scheme is higher when contribution risks are spread equally across generations ($\theta = 0$), while this likelihood under the hybrid pension scheme is typically larger when young workers bear more risk ($\theta = 2$). The reason is that the indexation rate is mainly born by the older workers, who have the largest pension entitlements. Shifting more of the contribution risks to the young better balances the risks among the cohorts, making it less likely that any one of them wants to exit. Fourth, as it reduces the volatility of the funding ratio, a short smoothing period raises the likelihood that no cohort exits under the DB scheme. However, under the hybrid schemes, the larger indexation response to a given degree of underfunding raises the likelihood that cohorts close to retirement exit. Overall, the likelihood that no cohort exits under a hybrid scheme is highest for the combination of a short smoothing period and a high level of investment risk. Finally, comparing the various pension schemes, we find that the likelihood that no cohort exits is highest under the DB pension scheme, where only the young workers have an incentive to exit when the funding ratio falls below a certain threshold.

Table 6: Likelihood that no cohort exists

			$\omega=0$ (DB)					$\omega=0.5$ (hybrid)					$\omega=1$ (DC)				
		ζ	0	0.25	0.5	0.75	1	0	0.25	0.5	0.75	1	0	0.25	0.5	0.75	1
$\theta=0$	$\sigma=0\%$	$\alpha\in(0,1)$	0	0	0	0	**100**	0	0	0	0	**100**	0	0	0	0	**100**
	$\sigma=5\%$	$\alpha=0.1$	84.7	87.7	90.9	**93.6**	48.4	40.3	40.5	40.8	40.2	39.4	45.0	36.5	35.7	34.6	33.5
		$\alpha=0.25$	84.0	87.4	90.5	93.0	48.5	40.0	41.0	41.5	40.9	40.2	45.6	36.6	36.0	35.0	34.0
		$\alpha=0.5$	82.2	85.5	88.8	91.4	48.0	40.1	42.2	42.8	42.4	41.6	**46.6**	37.7	37.1	36.2	35.3
		$\alpha=0.75$	79.3	81.5	85.6	88.4	47.7	38.7	44.8	**45.8**	45.5	44.9	39.6	39.8	39.3	38.6	37.7
	$\sigma=10\%$	$\alpha=0.1$	90.2	91.9	**93.5**	92.5	47.3	43.5	36.2	36.3	36.0	35.6	36.4	27.7	27.2	26.6	26.1
		$\alpha=0.25$	89.5	91.5	92.7	91.2	47.3	44.2	36.6	36.9	36.6	36.2	**36.6**	27.8	27.3	26.8	26.2
		$\alpha=0.5$	88.4	90.0	91.3	89.6	46.8	45.1	37.6	38.1	37.9	37.5	36.5	26.7	26.3	25.9	25.5
		$\alpha=0.75$	86.3	87.0	88.3	86.5	45.7	**45.9**	40.5	41.5	41.3	41.0	36.6	25.5	25.3	25.0	24.7
	$\sigma=15\%$	$\alpha=0.1$	91.8	**93.2**	93.1	90.2	46.3	42.5	32.7	32.8	32.7	32.5	**29.4**	21.5	21.1	20.8	20.5
		$\alpha=0.25$	91.4	92.3	92.0	88.1	46.4	43.1	33.1	33.3	33.2	33.0	29.3	20.7	20.4	20.2	19.9
		$\alpha=0.5$	89.9	90.9	90.3	84.3	45.6	**43.7**	34.0	34.4	34.4	34.2	28.6	19.4	19.4	19.2	19.0
		$\alpha=0.75$	87.5	87.8	86.7	84.9	44.2	43.7	36.8	37.8	7.7	37.6	27.2	17.5	18.1	18.2	18.1
	$\sigma=20\%$	$\alpha=0.1$	92.6	**93.2**	92.2	87.7	45.1	40.9	30.2	30.1	30.0	29.9	24.3	18.1	17.8	17.7	17.5
		$\alpha=0.25$	91.8	91.9	90.7	85.3	45.6	41.1	30.5	30.6	30.4	30.3	**24.3**	17.7	17.7	17.5	17.4
		$\alpha=0.5$	90.3	90.1	89.2	81.2	44.7	**41.2**	31.4	31.7	31.7	31.6	23.8	16.7	17.4	17.3	17.2
		$\alpha=0.75$	87.3	86.3	85.2	81.2	42.3	39.0	32.9	33.9	33.9	33.9	21.7	10.8	13.3	14.8	15.2
$\theta=2$	$\sigma=0\%$	$\alpha\in(0,1)$	0	0	0	0	**100**	0	0	0	0	**100**	0	0	0	0	**100**
	$\sigma=5\%$	$\alpha=0.1$	84.7	89.8	91.9	**92.2**	47.9	40.3	40.8	41.6	41.5	41.1	45.0	36.5	35.7	34.6	33.5
		$\alpha=0.25$	84.0	89.0	90.7	90.7	48.2	40.0	41.3	42.3	42.2	41.7	45.6	36.6	36.0	35.0	34.0
		$\alpha=0.5$	82.2	87.2	89.6	89.0	48.1	40.1	42.4	43.6	43.5	43.0	**46.6**	37.7	37.1	36.2	35.3
		$\alpha=0.75$	79.3	83.6	86.9	86.3	48.0	38.7	44.9	**46.3**	46.1	45.6	39.6	39.8	39.3	38.6	37.7
	$\sigma=10\%$	$\alpha=0.1$	90.2	92.4	**92.5**	89.3	47.4	43.5	36.5	37.2	37.4	37.6	36.4	27.7	27.2	26.6	26.1
		$\alpha=0.25$	89.5	91.4	91.4	88.3	47.4	44.2	36.9	37.7	37.9	38.1	**36.6**	27.8	27.3	26.8	26.2
		$\alpha=0.5$	88.4	90.0	89.6	86.8	47.1	45.1	37.9	39.0	39.1	39.2	36.5	26.7	26.3	25.9	25.5
		$\alpha=0.75$	86.3	87.2	86.8	83.6	46.2	**45.9**	40.6	42.0	42.0	41.8	36.6	25.5	25.3	25.0	24.7
	$\sigma=15\%$	$\alpha=0.1$	91.8	**92.6**	90.8	88.3	46.8	42.5	33.1	33.7	34.1	34.6	**29.4**	21.5	21.1	20.8	20.5
		$\alpha=0.25$	91.4	91.8	89.7	87.1	46.6	43.1	33.4	34.3	34.7	35.0	29.3	20.7	20.4	20.2	19.9
		$\alpha=0.5$	89.9	89.9	88.4	84.7	46.1	**43.7**	34.3	35.4	35.7	36.0	28.6	19.4	19.4	19.2	19.0
		$\alpha=0.75$	87.5	87.0	84.8	80.9	44.4	43.7	36.8	38.4	38.6	38.6	27.2	17.5	18.1	18.2	18.1
	$\sigma=20\%$	$\alpha=0.1$	**92.6**	92.1	90.3	86.4	45.9	40.9	30.5	31.0	31.4	32.0	24.3	18.1	17.8	17.7	17.5
		$\alpha=0.25$	91.8	90.6	88.8	85.4	45.8	41.1	30.8	31.4	31.9	32.5	**24.3**	17.7	17.7	17.5	17.4
		$\alpha=0.5$	90.3	89.3	86.4	83.1	45.0	**41.2**	31.6	32.5	33.0	33.4	23.8	16.7	17.4	17.3	17.2
		$\alpha=0.75$	87.3	85.2	82.7	77.6	42.7	39.0	33.0	34.5	34.8	35.1	21.7	10.8	13.3	14.8	15.2

7 Conclusion

Popular resistance against mandatory participation in collective pension schemes seems to be increasing, a development that has been fuelled by deteriorating funding ratios as a result of the recent crisis and rising life expectancy, increasing demand for freedom of choice and enhanced labour market mobility. This paper has explored the implications of giving pension fund participants the option to exit the pension fund. We have considered a DB pension fund, a collective DC pension fund and a hybrid pension fund. The value of the option is highest for the youngest workers, because the number of remaining decision moments is highest.

Our main findings are the following. First, young participants are, ceteris paribus, more inclined to exit a DB scheme or a hybrid scheme relying more on contribution adjustments for recovery. Old participants, on the other hand, are more inclined to exit a collective DC scheme or a hybrid scheme in which recovery relies more on indexation adjustments. Second, participants are more inclined to continue participation if the number of moments that the exit-option can be exercised is high. Counterintuitively, more freedom of choice to exit actually improves participation. Third, the likelihood of participants exiting the voluntary scheme is fairly high for realistic assumptions. Fourth, higher investment risk favours young cohorts through the value of their option, but at the expense of older workers. Hence, reducing equity risk exposure might stimulate young generations to exit. Fifth, a longer smoothing period to bring the funding ratio back to its long-term target raises the likelihood that the pension fund becomes financially distressed, leading to participants exercise the exit-option. Sixth, the DB pension scheme features the lowest likelihood that any cohort wants to exit, because no cohort has an incentive to leave close to retirement. Finally, when investment risk is large, the value of the exit-option is large for young workers. In this case, a uniform contribution policy can act as a stabilising force: young workers are reluctant to exit, because they benefit from the value of the option, while the older workers are reluctant to exit, because they are subsidized by the contributions of the young. However, when investment risk is low, an age-dependent contribution policy enhances participation stability.

The analysis in this paper can be extended into several directions. One extension would be to consider more "refined" options, such as a partial withdrawal of accumulated assets from a pension fund or the possibility to withdraw resources only at the cost of a fine. The latter is quite common in the U.S. A second extension would be to include additional sources of risk into the model, such as demographic, interest rate and wage risks. Using the LSMC approach this may not be too complicated. However, not all types of these risks are hedgeable, which contradicts the complete market assumption. This could be solved by replacing the risk-neutral pricing approach by the utility indifference pricing approach.

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A Derivations

A.1 Derivation of the Annuity under Individual DC

The annuityB^{DC} under the individual DC scheme is calculated from

$$\begin{aligned} A^{DC}_{s,s+t_R} =& E^{\mathbb{Q}}_{s+t_R}\left\{\int_{s+t_R}^{s+t_D} \exp\left[-r\left(u-s-t_R\right)\right] B^{DC} du\right\} = B^{DC} E^{\mathbb{Q}}_{s+t_R}\left[\int_0^{t_D-t_R} \exp\left(-ru\right) du\right] \\ \iff B^{DC} =& r A^{DC}_{s,s+t_R} / \left\{1-\exp\left[-r\left(t_D-t_R\right)\right]\right\}. \end{aligned}$$

A.2 Regulation Policy

Suppose $n \in \mathbb{N}$. Then, we can use equation (2) and the law of iterated expectations to derive

$$\begin{aligned} E^{\mathbb{Q}}_t\left(F_{t+ndt}-\bar{F}\right) =& E^{\mathbb{Q}}_t\left[E^{\mathbb{Q}}_{t+(n-1)dt}\left(F_{t+ndt}-\bar{F}\right)\right] \\ =& \alpha^{dt} E^{\mathbb{Q}}_t\left(F_{t+(n-1)dt}-\bar{F}\right) \\ =& \alpha^{dt} E^{\mathbb{Q}}_t\left[E^{\mathbb{Q}}_{t+(n-2)dt}\left(F_{t+(n-1)dt}-\bar{F}\right)\right] \\ =& \alpha^{2dt} E^{\mathbb{Q}}_t\left(F_{t+(n-2)dt}-\bar{F}\right) \\ =& \ldots \\ =& \alpha^{ndt} E^{\mathbb{Q}}_t\left(F_t-\bar{F}\right) \\ =& \alpha^{ndt}\left(F_t-\bar{F}\right) \\ \Rightarrow E^{\mathbb{Q}}_t\left(F_{t+s}-\bar{F}\right) =& \alpha^{s}\left(F_t-\bar{F}\right). \end{aligned}$$

A.3 Dynamics of the Liabilities

In order to determine $E_t^{\mathbb{Q}}(dL_t) = E_t^{\mathbb{Q}}(L_{t+dt}) - L_t$, we first derive

$$\begin{aligned} B_{s,t+dt} =& B_{s,t} + dB_{s,t} \\ =& B_{s,t} + \left[\psi w_{t-s} + (\gamma_t - 1) B_{s,t}\right] dt \\ =& \psi w_{t-s} dt + B_{s,t}\left[1 + (\gamma_t - 1) dt\right] \end{aligned}$$

and

$$R_{\nu+dt} = \begin{cases} \int_{\nu+dt}^{t_D} \exp\left[-r(u-\nu-dt)\right] du, & \text{for } \nu + dt \geq t_R, \\ \exp\left[-r(t_R - \nu - dt)\right] \int_{t_R}^{t_D} \exp\left[-r(u - t_R)\right] du, & \text{for } \nu + dt < t_R, \end{cases}$$

$$\iff R_{\nu+dt} = \exp(rdt) R_\nu + \frac{1}{r}\left[1 - \exp(rdt)\right] \mathbb{I}_{\nu+dt>t_R},$$

where the indicator function $\mathbb{I}_{\nu+dt>t_R}$ is one if the condition in its subscript is fulfilled and zero, otherwise. Then, the liabilities at $t + dt$ can be written as

$$\begin{aligned} L_{t+dt} =& \int_{I_{t+dt}} R_{t+dt-s} B_{s,t+dt} ds \\ =& \psi dt \int_{I_{t+dt}^w} R_{t+dt-s} w_{t-s} ds + \left[1 + (\gamma_t - 1) dt\right] \int_{I_{t+dt}} R_{t+dt-s} B_{s,t} ds \\ =& \psi dt \int_{I_{t+dt}^w} R_{t+dt-s} ds + \left[1 + (\gamma_t - 1) dt\right] * \ldots \\ \ldots& \int_{I_{t+dt}} \left\{ \exp(rdt) R_{t-s} + \frac{1}{r}\left[1 - \exp(rdt)\right] \mathbb{I}_{t-s+dt>t_R} \right\} B_{s,t} ds \\ =& \psi dt \int_{I_{t+dt}^w} R_{t+dt-s} ds + \exp(rdt)\left[1 + (\gamma_t - 1) dt\right] * \ldots \\ \ldots& \left\{ \int_{I_{t+dt}} R_{t-s} B_{s,t} ds - \frac{1 - \exp(-rdt)}{r} \int_{I_{t+dt}^r} B_{s,t} ds \right\} \\ =& \psi dt \int_{I_{t+dt}^w} R_{t+dt-s} ds + \Lambda_t \left[1 + (\gamma_t - 1) dt\right], \end{aligned}$$

where we denote Λ_t as

$$\begin{aligned} \Lambda_t =& \exp(rdt) \int_{I_{t+dt}} R_{t-s} B_{s,t} ds - \frac{\exp(rdt) - 1}{r} \int_{I_{t+dt}^r} B_{s,t} ds \\ =& \exp(rdt) \int_{I_{t+dt}} R_{t-s} B_{s,t} ds - \int_0^{dt} \exp(ru)\, du \int_{I_{t+dt}^r} B_{s,t} ds, \end{aligned}$$

which is the compounded value of period t's liabilities of the period $t + dt$'s participants, minus the compounded value of period t's pension payouts to period $t + dt$'s retirees.

A.4 Equilibrium Liabilities and Funding Ratio

In equilibrium, we have $F_t = \bar{F}$, $A_t = \bar{A} = \bar{F}\bar{L}$, $\gamma_t = 1$ and full participation $\forall t$. Hence, the equilibrium pension entitlements at time t are

$$\bar{B}_{s,t} = \begin{cases} (t-s)\,\psi, & \text{for } (t-s) \in [0, t_R]\,, \\ t_R\psi, & \text{for } (t-s) \in (t_R, t_D)\,, \end{cases}$$
$$= \min\left(t-s, t_R\right)\psi, \text{ for } \nu \in [0, t_D)\,.$$

The equilibrium liabilities are

$$\begin{aligned}\bar{L} &= \int_0^{t_D} R_u \bar{B}_{t-u,t} du \\ &= \psi \int_0^{t_D} R_u \min\left(u, t_R\right) du \\ &= \psi \left[\int_0^{t_R} u R_u du + t_R \int_{t_R}^{t_D} R_u du\right],\end{aligned}$$

which can be further rewritten as follows

$$\begin{aligned}\bar{L} =& \psi \left[\int_0^{t_R} u R_u du + t_R \int_{t_R}^{t_D} R_u du\right] \\ =& \frac{\psi}{r}\left(\left\{1 - \exp\left[r\left(t_R - t_D\right)\right]\right\} \int_0^{t_R} u \exp\left[r\left(u - t_R\right)\right] du + t_R \int_{t_R}^{t_D} \left\{1 - \exp\left[r\left(u - t_D\right)\right]\right\} du\right) \\ =& \frac{\psi}{r}\left\{1 - \exp\left[r\left(t_R - t_D\right)\right]\right\} \exp\left(-r t_R\right) \frac{1}{r^2}\left[1 + \exp\left(r t_R\right)\left(r t_R - 1\right)\right] + \ldots \\ & \ldots \frac{\psi}{r} t_R \left(t_D - t_R + \frac{1}{r}\left\{\exp\left[r\left(t_R - t_D\right)\right] - 1\right\}\right) \\ =& \frac{\psi}{r}\left\{1 - \exp\left[r\left(t_R - t_D\right)\right]\right\} \frac{1}{r^2}\left[\exp\left(-r t_R\right) + r t_R - 1\right] + \ldots \\ & \ldots \frac{\psi}{r} t_R \left(t_D - t_R + \frac{1}{r}\left\{\exp\left[r\left(t_R - t_D\right)\right] - 1\right\}\right) \\ =& \frac{\psi}{r}\left(t_R\left(t_D - t_R\right) - \frac{1}{r^2}\left\{1 - \exp\left[r\left(t_R - t_D\right)\right]\right\}\left[1 - \exp\left(-r t_R\right)\right]\right),\end{aligned}$$

where we used

$$\int_0^{t_R} s \exp\left(r s\right) ds = \frac{1}{r^2}\left[1 + \left(r t_R - 1\right)\exp\left(r t_R\right)\right].$$

There is no change in assets in equilibrium

$$0 = d\bar{A} = r\bar{A}dt + \left(\bar{C} - \bar{B}^{TOT}\right) dt.$$

Then, we can derive an expression for the funding ratio target as follows

$$
\begin{aligned}
0 =& r\bar{A} + \bar{C} - \bar{B}^{TOT} \\
=& r\bar{F}\bar{L} + \int_0^{t_R} \psi R_u du - \int_{t_R}^{t_D} \psi t_R du \\
=& r\bar{F}\bar{L} + \frac{\psi}{r^2} \left\{1 - \exp\left[-r\left(t_D - t_R\right)\right]\right\}\left[1 - \exp\left(-r t_R\right)\right] - \left(t_D - t_R\right)\psi t_R \\
\iff \bar{F}\bar{L} =& \frac{\psi}{r}\left(\left(t_D - t_R\right)t_R - \frac{1}{r^2}\left\{1 - \exp\left[-r\left(t_D - t_R\right)\right]\right\}\left[1 - \exp\left(-r t_R\right)\right]\right) \\
\iff \bar{F} =& 1.
\end{aligned}
$$

A.5 Policy Instruments

For the assets, we obtain

$$
\begin{aligned}
A_{t+dt} =& A_t + dA_t \\
=& \left(1 + \frac{dP_t}{P_t}\right)A_t + \left(C_t - B_t^{TOT}\right)dt \\
=& \left(1 + \frac{dP_t}{P_t}\right)A_t + \left(\int_{I_t^w} c_{s,t} ds - \gamma_t \int_{I_t^r} B_{s,t} ds\right)dt \\
E_t^{\mathbb{Q}}\left(A_{t+dt}\right) =& \left(1 + r dt\right)A_t + \left(\int_{I_t^w} c_{s,t} ds - \gamma_t \int_{I_t^r} B_{s,t} ds\right)dt.
\end{aligned}
$$

Hence, we can write an expression for the expected change in the funding gap

$$
\begin{aligned}
E_t^{\mathbb{Q}}\left(\frac{A_{t+dt} - L_{t+dt}}{dt}\right) =& \frac{1 + r dt}{dt} A_t + \int_{I_t^w} c_{s,t} ds - \gamma_t \int_{I_t^r} B_{s,t} ds - \ldots \\
\ldots \psi \int_{I_{t+dt}^w} R_{t+dt-s} ds - \Lambda_t \left(\frac{1}{dt} + \gamma_t - 1\right) \\
\iff \Omega_t =& \int_{I_t^w} \pi_{s,t} ds + \left(1 - \gamma_t\right)\left(\Lambda_t + \int_{I_t^r} B_{s,t} ds\right),
\end{aligned}
$$

where

$$
\begin{aligned}
\Omega_t \equiv& \frac{E_t^{\mathbb{Q}}\left(A_{t+dt} - L_{t+dt}\right) - \left(1 + r dt\right)A_t + \Lambda_t}{dt} + \psi \int_{I_{t+dt}^w} R_{t+dt-s} ds - \int_{I_t^w} \bar{c}_{t-s} ds + \int_{I_t^r} B_{s,t} ds \\
=& E_t^{\mathbb{Q}}\left(\frac{A_{t+dt} - L_{t+dt}}{dt}\right) + \underbrace{\frac{1}{dt}\left(\Lambda_t + \psi dt \int_{I_{t+dt}^w} R_{t+dt-s} ds\right)}_{=\frac{L_{t+dt}}{dt} + (1-\gamma_t)\Lambda_t} - \underbrace{\frac{1}{dt}\left[\left(1 + r dt\right)A_t + dt \int_{I_t^w} \bar{c}_{t-s} ds - dt \int_{I_t^r} B_{s,t} ds\right]}_{=E_t^{\mathbb{Q}}\left(\frac{A_{t+dt}}{dt}\right) - \int_{I_t^w} \pi_{s,t} ds - (1-\gamma_t)\int_{I_t^r} B_{s,t} ds}.
\end{aligned}
$$

The first term on the right-hand side of the first line of the expression for Ω_t equals the expected funding gap, $E_t^{\mathbb{Q}}\left(A_{t+dt} - L_{t+dt}\right)$, minus the compounded assets, $\left(1 + r dt\right)A_t$, plus

Λ_t, divided by the time step size dt. The second term is the period $t+dt$ market price of the aggregate pension accrual of period $t+dt$'s working cohorts. The third term represents the aggregate equilibrium contributions of period t's working cohorts, while the last term denotes the aggregate pension entitlements of period t's retirees. Further, Λ_t is the compounded value to period $t+dt$ of period t's liabilities to the period $t+dt$ participants, minus the aggregate compounded pension payouts to period $t+dt$'s retirees between t and $t+dt$.

To adhere to the regulatory policy, the pension fund can vary the amount of contributions (C_t) and the gross indexation rate (γ_t). Using the policy imposed by the regulator, equation (2), we derive

$$\begin{aligned}
E_t^{\mathbb{Q}}\left(\frac{A_t+dA_t}{L_t+dL_t}\right)-\bar{F} &= \alpha^{dt}\left(\frac{A_t}{L_t}-\bar{F}\right)\\
\Longleftrightarrow \quad \frac{A_t+E_t^{\mathbb{Q}}(dA_t)}{L_t+E_t^{\mathbb{Q}}(dL_t)} &= \alpha^{dt}\left(\frac{A_t}{L_t}-\bar{F}\right)+\bar{F}\\
\Longleftrightarrow \quad \left(rA_t+C_t-B_t^{TOT}\right)dt &= \left(\alpha^{dt}-1\right)\left(A_t-\bar{F}L_t\right)+\left[\alpha^{dt}\left(F_t-\bar{F}\right)+\bar{F}\right]E_t^{\mathbb{Q}}(dL_t)\\
\Longleftrightarrow \quad rA_t+C_t-B_t^{TOT} &= \left(\frac{\alpha^{dt}-1}{dt}\right)\left(A_t-\bar{F}L_t\right)+\left[\alpha^{dt}\left(F_t-\bar{F}\right)+\bar{F}\right]E_t^{\mathbb{Q}}\left(\frac{dL_t}{dt}\right)\\
\Longleftrightarrow \quad rA_t+C_t-B_t^{TOT} &= (\log\alpha)\left(A_t-\bar{F}L_t\right)+F_tE_t^{\mathbb{Q}}\left(\frac{dL_t}{dt}\right)\\
\Longleftrightarrow \quad C_t-B_t^{TOT} &= F_tE_t^{\mathbb{Q}}\left(\frac{dL_t}{dt}\right)-\left[r-(\log\alpha)\right]A_t-(\log\alpha)\bar{F}L_t\\
\Longleftrightarrow \quad \int_{I_t^w}\pi_{s,t}ds+(1-\gamma_t)\int_{I_t^r}B_{s,t}ds &= F_tE_t^{\mathbb{Q}}\left(\frac{dL_t}{dt}\right)-A_t\left[r-(\log\alpha)\right]-(\log\alpha)\bar{F}L_t-\int_{I_t^w}\bar{c}_{t-s}ds+\int_{I_t^r}B_{s,t}ds
\end{aligned} \tag{12}$$

where, going from the first to the second line, we have used the independence of the change in assets from the change in liabilities conditional on period t information, and, going from the fourth to the fifth line, we have used that $dt\downarrow 0$ as well as l'Hôpital's rule to get $\frac{\alpha^{dt}-1}{dt}=\lim_{x\downarrow 0}\frac{\alpha^x-1}{x}=\log\alpha$. Note that $\log\alpha<0$, since $\alpha\in(0,1)$. Hence, we can write the policies as

$$\begin{aligned}
(\gamma_t-1)\left(\frac{1-\omega}{\omega}\Lambda_t+\frac{1}{\omega}\int_{I_t^r}B_{s,t}ds\right) &= A_t\left[r-(\log\alpha)\right]-F_tE_t^{\mathbb{Q}}\left(\frac{dL_t}{dt}\right)+(\log\alpha)\bar{F}L_t+\int_{I_t^w}\bar{c}_{t-s}ds-\int_{I_t^r}B_{s,t}ds,\\
\int_{I_t^w}\pi_{s,t}ds &= \frac{1-\omega}{\omega}(1-\gamma_t)\left(\Lambda_t+\int_{I_t^r}B_{s,t}ds\right),
\end{aligned}$$

where the last expression follows directly by combining equations (4) and (5), while the next-to-last line follows from substituting the last line back into equation (12).

A.6 Age-Dependent Recovery Contribution

The recovery contributions are weighed by the agent's actuarially fair contribution level, $(\bar{c}_{t_R} - \bar{c}_\nu)$, which decreases with age, since $\frac{\partial(\bar{c}_{t_R} - \bar{c}_\nu)}{\partial \nu} < 0$. Then, using equation (5) we can derive

$$\begin{aligned}(1-\omega)\,\Omega_t &= \int_{I_t^w} \pi_{s,t} ds = \int_{I_t^w} \left(\bar{c}_{t_R} - \bar{c}_{t-s}\right)^\theta \Theta_t ds \\ \iff \Theta_t &= \frac{(1-\omega)\,\Omega_t}{\int_{I_t^w} \left(\bar{c}_{t_R} - \bar{c}_{t-s}\right)^\theta ds} \\ \Rightarrow \pi_{t-\nu,t} &= (1-\omega)\,\Omega_t \frac{\left(\bar{c}_{t_R} - \bar{c}_\nu\right)^\theta}{\int_{I_t^w} \left(\bar{c}_{t_R} - \bar{c}_{t-s}\right)^\theta ds}.\end{aligned}$$

Hence, the age-dependent contribution at time t for a participant with age ν is

$$c_{t-\nu,t} = \bar{c}_\nu + (1-\omega)\,\Omega_t \frac{\left(\bar{c}_{t_R} - \bar{c}_\nu\right)^\theta}{\int_{I_t^w} \left(\bar{c}_{t_R} - \bar{c}_{t-s}\right)^\theta ds}.$$

If we assume that the pension fund operates under full participation, i.e. $I_t = \{s : t-s \in [0, t_D]\}$, $\forall t$, then the following holds

$$c_{t-\nu,t} = \bar{c}_\nu + (1-\omega)\,\Omega_t \frac{\left(\bar{c}_{t_R} - \bar{c}_\nu\right)^\theta}{\int_0^{t_R} \left(\bar{c}_{t_R} - \bar{c}_u\right)^\theta du}.$$

We can derive the following comparative statics

$$\begin{aligned}\frac{\partial c_{t-\nu,t}}{\partial \Omega_t} &= (1-\omega)\, \frac{\left(\bar{c}_{t_R} - \bar{c}_\nu\right)^\theta}{\int_0^{t_R} \left(\bar{c}_{t_R} - \bar{c}_u\right)^\theta du} \geq 0 \\ \Rightarrow \frac{\partial^2 c_{t-\nu,t}}{\partial \Omega_t \partial \nu} &= \frac{-(1-\omega)\,\theta \left(\bar{c}_{t_R} - \bar{c}_\nu\right)^{-\theta}}{\int_0^{t_R} \left(\bar{c}_{t_R} - \bar{c}_u\right)^\theta du} \frac{\partial \bar{c}_\nu}{\partial \nu} \leq 0, \text{ for } \theta \geq 0.\end{aligned}$$

Hence, the recovery contribution increases with the restoration parameter Ω_t, since $\frac{\partial c_{t-\nu,t}}{\partial \Omega_t} \geq 0$. However, the restoration particularly relies on the young workers when $\theta > 0$, since the required restoration part falls with age $\left(\frac{\partial^2 c_{t-\nu,t}}{\partial \Omega_t \partial \nu} < 0\right)$.

A.7 Recovery Contribution under the DB Pension Scheme

Under the DB pension scheme we have that $\omega = 0$, so from equations (4) and (5) we obtain $\gamma_t = 1$ and $\Omega_t = \int_{I_t^w} \pi_{s,t} ds$. Hence, we can write equation (12) as

$$\Omega_t = F_t E_t^{\mathbb{Q}} \left(\frac{dL_t}{dt}\right) - A_t \left[r - (\log \alpha)\right] - (\log \alpha)\, \bar{F} L_t - \int_{I_t^w} \bar{c}_{t-s} ds + \int_{I_t^r} B_{s,t} ds.$$

Under full participation we have that $dL_t = 0$ and $L_t = \bar{L}$, so we can write the above equation as

$$\begin{aligned}
\Omega_t =& - A_t \left[r - (\log \alpha)\right] - (\log \alpha)\, \bar{L} - \int_0^{t_R} \bar{c}_u du + \int_{t_R}^{t_D} B_{s,s+u} du \\
=& - A_t \left[r - (\log \alpha)\right] - (\log \alpha)\, \bar{L} - \int_0^{t_R} \psi R_u du + \psi t_R \left(t_D - t_R\right) \\
=& - A_t \left[r - (\log \alpha)\right] - (\log \alpha)\, \bar{L} - \psi \left(t_R \left(t_D - t_R\right) - \frac{1}{r^2} \left\{1 - \exp\left[r\left(t_R - t_D\right)\right]\right\} \left[1 - \exp\left(-r t_R\right)\right] \right) \\
=& \left[r - (\log \alpha)\right] \left(\bar{L} - A_t\right).
\end{aligned}$$

Hence, the recovery contribution becomes

$$\pi_{t-\nu,t} = \left[r - (\log \alpha)\right] \left(\bar{L} - A_t\right) \frac{\left(\bar{c}_{t_R} - \bar{c}_\nu\right)^\theta}{\int_0^{t_R} \left(\bar{c}_{t_R} - \bar{c}_u\right)^\theta du}.$$

Using equation (3), we can show that the long-run expected recovery contribution equals zero:

$$\begin{aligned}
& \lim_{u\to\infty} E_t^{\mathbb{Q}} \left(F_{t+u}\right) = \bar{F} = 1 \\
\Longleftrightarrow\ & \lim_{u\to\infty} E_t^{\mathbb{Q}} \left(A_{t+u}\right) = \bar{L}, \\
\Longleftrightarrow\ & \lim_{u\to\infty} E_t^{\mathbb{Q}} \left(\pi_{t+u-\nu,t+u}\right) = \lim_{u\to\infty} \left[r - (\log \alpha)\right] E_t^{\mathbb{Q}} \left(\bar{L} - A_{t+u}\right) \frac{\left(\bar{c}_{t_R} - \bar{c}_\nu\right)^\theta}{\int_0^{t_R} \left(\bar{c}_{t_R} - \bar{c}_s\right)^\theta ds} = 0.
\end{aligned}$$

Note that

$$\frac{\partial \pi_{t-\nu,t}}{\partial \alpha} = \frac{\partial (\log \alpha)}{\partial \alpha} \frac{\partial \pi_{t-\nu,t}}{\partial (\log \alpha)} = \frac{A_t - \bar{L}}{\alpha} \frac{\left(\bar{c}_{t_R} - \bar{c}_\nu\right)^\theta}{\int_0^{t_R} \left(\bar{c}_{t_R} - \bar{c}_u\right)^\theta du}.$$

This is negative in the case of underfunding, i.e. $\bar{L} > A_t$. A higher value of $\alpha \in (0,1)$ implies more smoothing and, hence, the effect on the recovery contribution is mitigated if there is underfunding.

A.8 Discounted Funding Ratios

We can use the result from Appendix A.2 to derive

$$
\begin{aligned}
& E_t^{\mathbb{Q}}\left[\int_t^T F_u \exp(-ru)\, du\right] \\
=& E_t^{\mathbb{Q}}\left[\int_t^T \left(F_u - \bar{F} + \bar{F}\right) \exp(-ru)\, du\right] \\
=& \int_t^T \left[E_t^{\mathbb{Q}}\left(F_u - \bar{F}\right) \exp(-ru) + \bar{F} \exp(-ru)\right] du \\
=& \int_t^T \left[\alpha^{u-t}\left(F_u - \bar{F}\right) \exp(-ru) + \bar{F} \exp(-ru)\right] du \\
=& \exp(-rt) \int_0^{T-t} \left[\alpha^u\left(F_t - \bar{F}\right) + \bar{F}\right] \exp(-ru)\, du \\
=& \exp(-rt) \left\{\left(F_t - \bar{F}\right) \int_0^{T-t} \alpha^u \exp(-ru)\, du + \bar{F} \int_0^{T-t} \exp(-ru)\, du\right\} \\
=& \exp(-rt) \left[\left(F_t - \bar{F}\right) \int_0^{T-t} \exp\{-u\left[r - (\log\alpha)\right]\}\, du + \bar{F} \int_0^{T-t} \exp(-ru)\, du\right] \\
=& \left(F_t - \bar{F}\right) \frac{\exp(-rt)}{r - (\log\alpha)} \left(1 - \exp\{(t-T)\left[r - (\log\alpha)\right]\}\right) + \frac{\bar{F}}{r}\left[\exp(-rt) - \exp(-rT)\right].
\end{aligned}
$$

Hence, $E_t^{\mathbb{Q}}\left[\int_t^T F_u \exp(-ru)\, du\right]$ satisfies the Markov property, as its value depends only on time t's funding ratio F_t.

A.9 Participation Under the DB Pension Scheme

Under full participation we have that $dL_t = 0$ and $L_t = \bar{L}$. Hence, we can write

$$
\begin{aligned}
& E^{\mathbb{Q}}_{s+t_M}\left[\int_{t_M}^{t_R} c_{s,s+u}\,\text{exp}\left[-r\left(u-t_M\right)\right]du\right] \\
&= \int_{t_M}^{t_R}\left\{\bar{c}_u + \left[r-\left(\log\alpha\right)\right]E^{\mathbb{Q}}_{s+t_M}\left(\bar{L}-A_{s+u}\right)\frac{\left(\bar{c}_{t_R}-\bar{c}_u\right)^{\theta}}{\int_0^{t_R}\left(\bar{c}_{t_R}-\bar{c}_s\right)^{\theta}ds}\right\}\text{exp}\left[-r\left(u-t_M\right)\right]du \\
&= \int_{t_M}^{t_R}\left\{\psi R_u + \left[r-\left(\log\alpha\right)\right]E^{\mathbb{Q}}_{s+t_M}\left(\bar{L}-A_{s+u}\right)\frac{\left(\bar{c}_{t_R}-\bar{c}_u\right)^{\theta}}{\int_0^{t_R}\left(\bar{c}_{t_R}-\bar{c}_s\right)^{\theta}ds}\right\}\text{exp}\left[-r\left(u-t_M\right)\right]du \\
&= \int_{t_M}^{t_R}\psi R_u\,\text{exp}\left[-r\left(u-t_M\right)\right]du + \dots \\
&\dots\frac{r-\left(\log\alpha\right)}{\int_0^{t_R}\left(\bar{c}_{t_R}-\bar{c}_s\right)^{\theta}ds}E^{\mathbb{Q}}_{s+t_M}\left\{\int_{t_M}^{t_R}\left(\bar{L}-A_{s+u}\right)\left(\bar{c}_{t_R}-\bar{c}_u\right)^{\theta}\text{exp}\left[-r\left(u-t_M\right)\right]du\right\} \\
&= \int_{t_M}^{t_R}\frac{\psi}{r}\,\text{exp}\left[-r\left(t_R-t_M\right)\right]\left\{1-\text{exp}\left[-r\left(t_D-t_R\right)\right]\right\}du + \dots \\
&\dots\frac{\left[r-\left(\log\alpha\right)\right]\bar{L}}{\int_0^{t_R}\left(\bar{c}_{t_R}-\bar{c}_s\right)^{\theta}ds}\int_{t_M}^{t_R}E^{\mathbb{Q}}_{s+t_M}\left(1-F_{s+u}\right)\left(\bar{c}_{t_R}-\bar{c}_u\right)^{\theta}\text{exp}\left[-r\left(u-t_M\right)\right]du \\
&= \frac{\psi}{r}\left(t_R-t_M\right)\text{exp}\left[-r\left(t_R-t_M\right)\right]\left\{1-\text{exp}\left[-r\left(t_D-t_R\right)\right]\right\} + \dots \\
&\dots\left(1-F_{s+t_M}\right)\frac{\left[r-\left(\log\alpha\right)\right]\bar{L}}{\int_0^{t_R}\left(\bar{c}_{t_R}-\bar{c}_s\right)^{\theta}ds}\int_0^{t_R-t_M}\alpha^u\left(\bar{c}_{t_R}-\bar{c}_{t_M+u}\right)^{\theta}\text{exp}\left(-ru\right)du,
\end{aligned}
$$

where we used the last equation of Appendix A.2 to go from the next-to-last line to the last line.

A rational participant of arbitrary age t_M who has started working at time s decides to stay in the pension scheme when the value of participation is positive. When the participation decision is irreversible, then the latter is given by the discounted value of his future pension benefits during retirement minus the payout obtained from exiting minus the expected discounted sum of the contributions to be paid from now until retirement:

$$
\begin{aligned}
Part_{s,s+t_M} &= \text{exp}\left[-r\left(t_R-t_M\right)\right]\Pi_{s+t_R}\left(B_{s,s+t_R}\right) - \min\left(1,F_{s+t_M}\right)\Pi_{s+t_M}\left(B_{s,s+t_M}\right) - \dots \\
&\dots E^{\mathbb{Q}}_{s+t_M}\left[\int_{t_M}^{t_R} c_{s+u}\,\text{exp}\left[-r\left(u-t_M\right)\right]du\right] \\
&= \text{exp}\left[-r\left(t_R-t_M\right)\right]\Pi_{s+t_R}\left(B_{s,s+t_R}\right) - \min\left(1,F_{s+t_M}\right)\Pi_{s+t_M}\left(B_{s,s+t_M}\right) - \\
&\dots\frac{\psi}{r}\left(t_R-t_M\right)\text{exp}\left[-r\left(t_R-t_M\right)\right]\left\{1-\text{exp}\left[-r\left(t_D-t_R\right)\right]\right\} + \dots \\
&\dots\left(F_{s+t_M}-1\right)\frac{\left[r-\left(\log\alpha\right)\right]\bar{L}}{\int_0^{t_R}\left(\bar{c}_{t_R}-\bar{c}_u\right)^{\theta}du}\int_0^{t_R-t_M}\alpha^u\left(\bar{c}_{t_R}-\bar{c}_{t_M+u}\right)^{\theta}\text{exp}\left(-ru\right)du.
\end{aligned}
$$

Then, we have that

$$\frac{\partial Part_{s,s+t_M}}{\partial F_{s+t_M}} = \begin{cases} \frac{[r-(\log\alpha)]\bar{L}}{\int_0^{t_R}\left(\bar{c}_{t_R}-\bar{c}_u\right)^\theta du}\int_0^{t_R-t_M}\alpha^u\left(\bar{c}_{t_R}-\bar{c}_{t_M+u}\right)^\theta \exp\left(-ru\right)du > 0, & \text{for } F_{s+t_M}\geq 1, \\ \frac{[r-(\log\alpha)]\bar{L}}{\int_0^{t_R}\left(\bar{c}_{t_R}-\bar{c}_u\right)^\theta du}\int_0^{t_R-t_M}\alpha^u\left(\bar{c}_{t_R}-\bar{c}_{t_M+u}\right)^\theta \exp\left(-ru\right)du - \Pi_{s+t_M}\left(B_{s,s+t_M}\right), & \text{for } F_{s+t_M}< 1. \end{cases}$$

Furthermore, suppose that $F_{s+t_M} = 1$, then

$$\begin{aligned} Part_{s,s+t_M} =& \exp\left[-r\left(t_R-t_M\right)\right]\Pi_{s+t_R}\left(B_{s,s+t_R}\right) - \Pi_{s+t_M}\left(B_{s,s+t_M}\right) - \dots \\ & \dots \frac{\psi}{r}\left(t_R-t_M\right)\exp\left[-r\left(t_R-t_M\right)\right]\left\{1-\exp\left[-r\left(t_D-t_R\right)\right]\right\} \\ =& \exp\left[-r\left(t_R-t_M\right)\right]R_{t_R}\psi t_R - R_{t_M}\psi t_M - \dots \\ & \dots \frac{\psi}{r}\left(t_R-t_M\right)\exp\left[-r\left(t_R-t_M\right)\right]\left\{1-\exp\left[-r\left(t_D-t_R\right)\right]\right\} \\ =& \left(t_R-t_M-t_R+t_M\right)\exp\left[-r\left(t_R-t_M\right)\right]\frac{\psi}{r}\left\{1-\exp\left[-r\left(t_D-t_R\right)\right]\right\} \\ =& 0. \end{aligned}$$

Hence, the participation threshold is exactly at a funding ratio of 100% when the contribution policy is actuarially fair in equilibrium and the participation decision is irreversible. The value of participation at the retirement age is obtained by $t_M = t_R$

$$\begin{aligned} Part_{s,s+t_R} =& \Pi_{s+t_R}\left(B_{s,s+t_R}\right) - \min\left(1, F_{s+t_R}\right)\Pi_{s+t_M}\left(B_{s,s+t_R}\right) \\ =& \max\left(0, 1-F_{s+t_R}\right)\Pi_{s+t_M}\left(B_{s,s+t_R}\right) \geq 0. \end{aligned}$$

Hence, the value of participation at the retirement age is always positive.

A.10 Indexation Rate under Full Participation

Under full participation, we obtain

$$E_t^{\mathbb{Q}}\left(dL_t\right) = L_{t+dt} - L_t = \psi dt\int_0^{t_R} R_s ds + \Lambda_t\left[1+\left(\gamma_t-1\right)dt\right] - L_t,$$

which can be used to derive an expression for $(\gamma_t - 1)$ under full participation and $\bar{F} = 1$:

$$
\begin{aligned}
&(\gamma_t - 1)\left(\frac{1-\omega}{\omega}\Lambda_t + \frac{1}{\omega}\int_{t_R}^{t_D} B_{t-s,t}ds\right) = \dots \\
&\quad \dots A_t\left[r - (\log\alpha)\right] - F_t E_t^{\mathbb{Q}}\left(\frac{dL_t}{dt}\right) + (\log\alpha)\, L_t + \int_0^{t_R} \bar{c}_u du - \int_{t_R}^{t_D} B_{t-s,t}ds \\
\iff &(\gamma_t - 1)\left[\Lambda_t\left(\frac{1-\omega}{\omega} + F_t\right) + \frac{1}{\omega}\int_{t_R}^{t_D} B_{t-s,t}ds\right] = \dots \\
&\quad \dots \int_0^{t_R} \bar{c}_u du - F_t\psi\int_0^{t_R} R_s ds - \int_{t_R}^{t_D} B_{t-s,t}ds + L_t(\log\alpha) + A_t\left[\frac{1}{dt} + r - (\log\alpha)\right] - \frac{F_t\Lambda_t}{dt} \\
\iff &(\gamma_t - 1)\left[\Lambda_t\left(\frac{1-\omega}{\omega} + F_t\right) + \frac{1}{\omega}\int_{t_R}^{t_D} B_{t-s,t}ds\right] = \dots \\
&\quad \dots \psi\left(1 - F_t\right)\int_0^{t_R} R_s ds - \int_{t_R}^{t_D} B_{t-s,t}ds + L_t(\log\alpha) + A_t\left[\frac{1}{dt} + r - (\log\alpha)\right] - \frac{F_t\Lambda_t}{dt} \\
\iff &(\gamma_t - 1) = \omega\left\{\psi\left(1 - F_t\right)\int_0^{t_R} R_s ds - \int_{t_R}^{t_D} B_{t-s,t}ds + L_t(\log\alpha) + A_t\left[\frac{1}{dt} + r - (\log\alpha)\right] - \frac{F_t\Lambda_t}{dt}\right\} / \dots \\
&\quad \dots \left[\Lambda_t\left(1 - \omega + \omega F_t\right) + \int_{t_R}^{t_D} B_{t-s,t}ds\right],
\end{aligned}
$$

where going from the first to the second expression we have used the above expression for $E_t^{\mathbb{Q}}(dL_t)$ and going from the second to the third expression we have used $\bar{c}_s = \psi R_s$.

A.11 Condition for Participation at Retirement under Full Participation

During retirement, the pension entitlements evolve as

$$dB_{s,t} = (\gamma_t - 1)\, B_{s,t}dt,$$

hence

$$
\begin{aligned}
B_{s,t+dt} =& B_{s,t} + (\gamma_t - 1)\, B_{s,t}dt \\
=& B_{s,t}\left[1 + (\gamma_t - 1)\, dt\right] \\
B_{s,t+2dt} =& B_{s,t+dt}\left[1 + (\gamma_{t+dt} - 1)\, dt\right] \\
=& B_{s,t}\left[1 + (\gamma_t - 1)\, dt\right]\left[1 + (\gamma_{t+dt} - 1)\, dt\right] \\
&\vdots \\
B_{s,t+ndt} =& B_{s,t}\prod_{i=0}^{n-1}\left[1 + (\gamma_{t+idt} - 1)\, dt\right].
\end{aligned}
$$

At retirement date, an individual decides to stay in the pension fund if and only if

$$
\begin{aligned}
& \min\left(1, F_{s+t_R}\right) \Pi_{s+t_R}\left(B_{s,s+t_R}\right) \\
& \leq \int_{t_R}^{t_D} \exp\left[-r\left(u-t_R\right)\right] E_{s+t_R}^{\mathbb{Q}}\left(B_{s,s+u}\right) du \\
& = \int_{0}^{t_D-t_R} \exp\left(-ru\right) E_{s+t_R}^{\mathbb{Q}}\left(B_{s,s+t_R+u}\right) du \\
& = B_{s,s+t_R} \int_{0}^{t_D-t_R} E_{s+t_R}^{\mathbb{Q}}\left\{\prod_{i=0}^{(u/dt)-1}\left[1+\left(\gamma_{s+t_R+idt}-1\right) dt\right]\right\} \exp\left(-ru\right) du \\
\iff & \int_{0}^{t_D-t_R} E_{s+t_R}^{\mathbb{Q}}\left\{\prod_{i=0}^{(u/dt)-1}\left[1+\left(\gamma_{s+t_R+idt}-1\right) dt\right]\right\} \exp\left(-ru\right) du \\
& \geq \min\left(1, F_{s+t_R}\right) R_{t_R} = \frac{1-\exp\left[-r\left(t_D-t_R\right)\right]}{r} \min\left(1, F_{s+t_R}\right),
\end{aligned}
$$

where the indexation rate follows from the last equation in Appendix A.10.

A.12 Uniform Contribution Policy

Under the uniform contribution policy, the contribution is independent of age

$$
c_{s,t} = \bar{c} + \pi_t^{unif},
$$

where $\bar{c}$ is the equilibrium contribution and π_t^{unif} is the time t recovery contribution. There is no change in assets in equilibrium

$$
0 = d\bar{A} = r\bar{A}dt + \left(\bar{C} - \bar{B}^{TOT}\right) dt.
$$

Then, we can derive an expression for the equilibrium contribution as follows

$$
\begin{aligned}
0 & = r\bar{A} + \bar{C} - \bar{B}^{TOT} \\
& = r\bar{F}\bar{L} + \int_{0}^{t_R} \bar{c} du - \int_{t_R}^{t_D} \psi t_R du \\
& = r\bar{F}\bar{L} + t_R\bar{c} - \left(t_D - t_R\right) \psi t_R \\
\iff \bar{c} & = \psi\left(t_D - t_R\right) - \frac{r\bar{F}\bar{L}}{t_R} \\
& = \psi\left\{\left(1-\bar{F}\right)\left(t_D-t_R\right) + \frac{\bar{F}}{t_R r^2}\left\{1-\exp\left[-r\left(t_D-t_R\right)\right]\right\}\left[1-\exp\left(-rt_R\right)\right]\right\} \\
& = \psi\left[\left(1-\bar{F}\right)\left(t_D-t_R\right) + \frac{\bar{F}}{t_R}\int_{0}^{t_R} R_u du\right].
\end{aligned}
$$

Hence, for $\bar{F} = 1$, the aggregate equilibrium contributions are equal to the market price for the aggregate pension accrual

$$\bar{c}t_R = \psi \int_0^{t_R} R_u du.$$

Using equation (5), the uniform recovery policy can be written as

$$\begin{aligned}(1-\omega)\,\Omega_t =& \int_{I_t^w} \pi_t^{unif} ds \\ \iff \pi_t^{unif} =& \frac{(1-\omega)}{\int_{I_t^w} 1 ds}\Omega_t.\end{aligned}$$

A.13 Irreversible Participation Decision with the DB Pension Scheme and Uniform Contributions

Under the DB pension scheme we have that $\omega = 0$, so from equations (4) and (5) we obtain $\gamma_t = 1$ and $\Omega_t = \int_{I_t^w} \pi_{s,t} ds$. Hence, we can write equation (12) as

$$\Omega_t = F_t E_t^{\mathbb{Q}}\left(\frac{dL_t}{dt}\right) - A_t\left[r - (\log\alpha)\right] - (\log\alpha)\,\bar{F}L_t - \int_{I_t^w} \bar{c} ds + \int_{I_t^r} B_{s,t} ds.$$

Under full participation and $\bar{F} = 1$ we have that $dL_t = 0$ and $L_t = \bar{L}$, so we can write the above equation as

$$\begin{aligned}\Omega_t =& -A_t\left[r-(\log\alpha)\right] - (\log\alpha)\,\bar{L} - \int_0^{t_R} \bar{c} ds + \int_{t_R}^{t_D} B_{s,s+u} du \\ =& A_t\left[r-(\log\alpha)\right] - (\log\alpha)\,\bar{L} - t_R\bar{c} + \psi t_R\left(t_D - t_R\right) \\ =& \left[r-(\log\alpha)\right]\left(\bar{L} - A_t\right).\end{aligned}$$

This way, we can write

$$\begin{aligned}& E_{s+t_M}^{\mathbb{Q}}\left\{\int_{t_M}^{t_R} c_{s,s+u} \exp\left[-r\left(u-t_M\right)\right] du\right\} \\ =& E_{s+t_M}^{\mathbb{Q}}\left\{\int_{t_M}^{t_R} \left(\bar{c} + \frac{1}{t_R}\Omega_{s+u}\right) \exp\left[-r\left(u-t_M\right)\right] du\right\} \\ =& E_{s+t_M}^{\mathbb{Q}}\left(\int_{t_M}^{t_R} \left\{\psi\left(t_D-t_R\right) - \frac{\left[r-(\log\alpha)\right]A_{s+u} + (\log\alpha)\,\bar{L}}{t_R}\right\} \exp\left[-r\left(u-t_M\right)\right] du\right) \\ =& \left[\psi\left(t_D-t_R\right) - \frac{(\log\alpha)\,\bar{L}}{t_R}\right]\int_{t_M}^{t_R} \exp\left[-r\left(u-t_M\right)\right] du - \frac{\left[r-(\log\alpha)\right]\bar{L}}{t_R} E_{s+t_M}^{\mathbb{Q}}\left[\int_{t_M}^{t_R} \frac{A_{s+u}}{\bar{L}} \exp\left[-r\left(u-t_M\right)\right] du\right] \\ =& \frac{1-\exp\left[-r\left(t_R-t_M\right)\right]}{r}\left[\psi\left(t_D-t_R\right) - \frac{(\log\alpha)\,\bar{L}}{t_R}\right] - \frac{\left[r-(\log\alpha)\right]\bar{L}}{t_R} E_{s+t_M}^{\mathbb{Q}}\left[\int_{t_M}^{t_R} F_{s+u} \exp\left[-r\left(u-t_M\right)\right] du\right].\end{aligned}$$

From Appendix A.8 and $\bar{F}=1$ we obtain

$$
\begin{aligned}
&E^{\mathbb{Q}}_{s+t_M}\left[\int_{t_M}^{t_R} F_{s+u}\exp\left[-r\left(u-t_M\right)\right]du\right]\\
=&E^{\mathbb{Q}}_{s+t_M}\left[\int_{s+t_M}^{s+t_R} F_{u}\exp\left[-r\left(u-s-t_M\right)\right]du\right]\\
=&\exp\left[r\left(s+t_M\right)\right]E^{\mathbb{Q}}_{s+t_M}\left[\int_{s+t_M}^{s+t_R} F_{u}\exp\left(-ru\right)du\right]\\
=&\left(F_{s+t_M}-1\right)\frac{1-\exp\left\{\left(t_M-t_R\right)\left[r-\left(\log\alpha\right)\right]\right\}}{r-\left(\log\alpha\right)}+\frac{1-\exp\left[-r\left(t_R-t_M\right)\right]}{r}.
\end{aligned}
$$

A rational participant of arbitrary age t_M who has started working at time s decides to stay in the pension scheme when the value of participation is positive. The latter is given by the discounted value of his future pension benefits during retirement, minus the expected discounted sum of the contributions to be paid from now until retirement, minus the payout obtained from exiting

$$
\begin{aligned}
Part_{s,s+t_M}=&\exp\left[-r\left(t_R-t_M\right)\right]\Pi_{s+t_R}\left(B_{s,s+t_R}\right)-E^{\mathbb{Q}}_{s+t_M}\left[\int_{t_M}^{t_R} c_{s+u}\exp\left[-r\left(u-t_M\right)\right]du\right]-\dots\\
&\dots\min\left(1,F_{s+t_M}\right)\Pi_{s+t_M}\left(B_{s,s+t_M}\right).
\end{aligned}
$$

Using the above derivations, we can write the latter equation as follows

$$
\begin{aligned}
Part_{s,s+t_M}=&\exp\left[-r\left(t_R-t_M\right)\right]\Pi_{s+t_R}\left(B_{s,s+t_R}\right)-E^{\mathbb{Q}}_{s+t_M}\left[\int_{t_M}^{t_R} c_{s+u}\exp\left[-r\left(u-t_M\right)\right]du\right]-\dots\\
&\dots\min\left(1,F_{s+t_M}\right)\Pi_{s+t_M}\left(B_{s,s+t_M}\right)\\
&=\exp\left[-r\left(t_R-t_M\right)\right]\Pi_{s+t_R}\left(B_{s,s+t_R}\right)-\frac{1-\exp\left[-r\left(t_R-t_M\right)\right]}{r}\left[\psi\left(t_D-t_R\right)-\frac{\left(\log\alpha\right)\bar{L}}{t_R}\right]+\dots\\
&\dots\frac{\left[r-\left(\log\alpha\right)\right]\bar{L}}{t_R}E^{\mathbb{Q}}_{s+t_M}\left[\int_{t_M}^{t_R} F_{s+u}\exp\left[-r\left(u-t_M\right)\right]du\right]-\min\left(1,F_{s+t_M}\right)\Pi_{s+t_M}\left(B_{s,s+t_M}\right)\\
&=\exp\left[-r\left(t_R-t_M\right)\right]\Pi_{s+t_R}\left(B_{s,s+t_R}\right)-\frac{1-\exp\left[-r\left(t_R-t_M\right)\right]}{r}\left[\psi\left(t_D-t_R\right)-\frac{r\bar{L}}{t_R}\right]+\dots\\
&\dots\left(F_{s+t_M}-1\right)\frac{\bar{L}}{t_R}\left(1-\exp\left\{\left(t_M-t_R\right)\left[r-\left(\log\alpha\right)\right]\right\}\right)-\min\left(1,F_{s+t_M}\right)\Pi_{s+t_M}\left(B_{s,s+t_M}\right)\\
&=\frac{\bar{L}}{t_R}\left(F_{s+t_M}-1\right)\left(1-\exp\left\{\left(t_M-t_R\right)\left[r-\left(\log\alpha\right)\right]\right\}\right)-\min\left(1,F_{s+t_M}\right)\Pi_{s+t_M}\left(B_{s,s+t_M}\right)-\dots\\
&\dots\frac{1-\exp\left[-r\left(t_R-t_M\right)\right]}{r}\left[\psi\left(t_D-t_R\right)-\frac{r\bar{L}}{t_R}\right]+\exp\left[-r\left(t_R-t_M\right)\right]\Pi_{s+t_R}\left(B_{s,s+t_R}\right).
\end{aligned}
$$

Table 7: Summary statistics of the funding ratio

Percentile or statistic	min	5%	10%	50%	mean	90%	95%	max
F_t	0.60	0.81	0.84	0.99	1.00	1.17	1.23	1.69

A.14 Compromise Between Actuarially Fair and Uniform Contribution Policy

Now, the age-dependent contribution at time t for a participant with age ν is

$$c_{t-\nu,t} = (1-\zeta)\left[\bar{c} + \frac{(1-\omega)}{\int_{I_t^w} 1ds}\Omega_t\right] + \zeta\left[\bar{c}_\nu + (1-\omega)\,\Omega_t \frac{(\bar{c}_{t_R} - \bar{c}_\nu)^\theta}{\int_{I_t^w} (\bar{c}_{t_R} - \bar{c}_{t-s})^\theta ds}\right],$$

where the parameter ζ determines whether the contribution policy is actuarially fair in equilibrium ($\zeta = 1$), uniform ($\zeta = 0$) or a compromise ($\zeta \in (0,1)$). If we assume a target funding ratio of 100% and we assume that the pension fund operates under full participation, i.e. $\bar{F} = 1$ and $I_t = \{s : t - s \in [0, t_D]\}$, $\forall t$, then the contribution becomes

$$c_{t-\nu,t} = (1-\zeta)\left[\bar{c} + \frac{(1-\omega)}{t_R}\Omega_t\right] + \zeta\left[\bar{c}_\nu + (1-\omega)\,\Omega_t \frac{(\bar{c}_{t_R} - \bar{c}_\nu)^\theta}{\int_0^{t_R} (\bar{c}_{t_R} - \bar{c}_s)^\theta ds}\right].$$

We can derive the following comparative statics

$$\begin{aligned}
\frac{\partial c_{t-\nu,t}}{\partial \Omega_t} &= \frac{(1-\zeta)(1-\omega)}{t_R} + \zeta(1-\omega)\frac{(\bar{c}_{t_R} - \bar{c}_\nu)^\theta}{\int_0^{t_R} (\bar{c}_{t_R} - \bar{c}_s)^\theta ds} \geq 0 \\
\Rightarrow \frac{\partial^2 c_{t-\nu,t}}{\partial \Omega_t \partial \nu} &= \frac{-\zeta\theta(1-\omega)(\bar{c}_{t_R} - \bar{c}_\nu)^{\theta-1}}{\int_0^{t_R} (\bar{c}_{t_R} - \bar{c}_s)^\theta ds}\frac{\partial \bar{c}_\nu}{\partial \nu} \leq 0, \text{ for } \theta \geq 0.
\end{aligned}$$

Hence, the contribution increases with the restoration parameter Ω_t, since $\frac{\partial c_{t-\nu,t}}{\partial \Omega_t} \geq 0$. However, the restoration particularly relies on the young workers, since the required restoration part falls with age $\left(\frac{\partial^2 c_{t-\nu,t}}{\partial \Omega_t \partial \nu} \leq 0\right)$.

A.15 Summary Statistics of the Funding Ratio

An overview of the statistics of F_t is shown in Table 7. Hence, 90% of the funding ratio simulations are within 81% and 123%.

B Approximation Method American Exit-Option

We use the Least Squares Monte Carlo (LSMC) approximation method to solve for the values of participation at each exercise date using backward recursion. The American exit-option is approximated by choosing a fine partition $\nu \in \{t_0, t_0 + \delta, t_0 + 2\delta, \ldots, t_R\}$ for the set of exercise ages. Here we describe the approximation method.

Step 1: run burn-in period We evaluate Q simulations after a "burn-in" period of 100 years. This way, we do not obtain the results around an equilibrium state, but around a more realistic setting whereby generations have been confronted differently to the risks. Depending on the initial values under our benchmark parameter setting, the distribution of the variables converges in 30 to 40 years.

Step 2: run an individual's working period After the burn-in simulations, we run another t_R years, which resembles the working period of an individual entering the labour market at time $t = 100$. During this working period, we register the values of X_t, which is the matrix with control variables at time t as given by equation (9). Hence, we do this for the following dates: $t = 100$, $t = 100 + \delta$, $t = 100 + 2\delta$, ..., $t = 100 + t_R$.

Step 3: run an individual's retirement period Similarly, we run the retirement period of the individual and register the value of continuation at the retirement age. The value of continuation for simulation run i for generation $s = 100$ at time $t = 100 + t_R$ is obtained by

$$Cont_{s,t,i} = \int_0^{t_D - t_R} \exp\left(-ru\right) B_{s,t+u,i} du.$$

Step 4: approximate value of participation at retirement We model the value of participation at retirement by the following regression model

$$Cont_{t-t_R,t} = X_t \beta_{t_R} + \varepsilon.$$

Using the estimated regression coefficient vector $\hat{\beta}_{t_R}$, we calculate the regression fit for the value of continuation as

$$\hat{Cont}_{t-t_R,t} = X_t \hat{\beta}_{t_R}.$$

Step 5: define values at maturity We need to determine the optimal exercise decisions using backward recursion. We start at the maturity date for the option to exit, which is the retirement age t_R, i.e. at time $t = 100 + t_R$, and register the corresponding payout.

Then, we run Q simulation paths and set up a $[Q \times 1]$ −vector $ExAge$, where each element equals t_R. Similarly, we define a $[Q \times 1]$ −pay out vector $Payout$. We initialize by setting it equal to max $\left(Stop_{t-t_R,t,i}, \hat{Cont}_{t-t_R,t,i}\right)$, where $Stop_{t-t_R,t}$ is the $[Q \times 1]$ −vector of values of stopping and $\hat{Cont}_{t-t_R,t}$ is the $[Q \times 1]$ −vector of values of continuation at time t for the generation of age t_R, and i denotes the i'th element (run) in these vectors. Initially, we also set $\nu_{old} = t_R$.

Step 6: move one step back in time with step size δ We now consider the simulation values at age $\nu = \nu_{old} - \delta$ and apply following regression model

$$Cont_{t-\nu,t} = X_t \beta_\nu + \varepsilon,$$

where the left-hand side is the vector of continuation values and X_t is the matrix with control variables at time t, as given by equation (9).

The value of continuation for a cohort of age ν at time t for simulation run i is given by

$$Cont_{t-\nu,t,i} = Payout_i \exp\left[-r\left(ExAge_i - \nu\right)\right] - \int_\nu^{ExAge_i} c_{t-s,t+s-\nu,i} \exp\left[-r\left(s-\nu\right)\right] ds,$$

i.e., the discounted payout at the exercise age, minus the discounted future contribution payments along this simulated path. Using the estimated regression coefficient vector $\hat{\beta}_\nu$, we calculate the regression fit for the value of continuation as

$$\hat{Cont}_{t-\nu,t} = X_t \hat{\beta}_\nu.$$

Step 7: update vectors when stopping is optimal The value of stopping for a cohort of age ν at time t is obtained by

$$Stop_{t-\nu,t} = \min\left(1, F_t\right) \Pi_t\left(B_{t-\nu,t}\right).$$

Then, for each simulation path i at time t, we can determine whether the value of stopping is larger than the approximated value of continuation, i.e. we check whether $Stop_{t-\nu,t,i} > \hat{Cont}_{t-\nu,t,i}$. For each simulation run where stopping is optimal, the corresponding elements in $ExAge$ are updated to ν and the corresponding elements in $Payout$ are updated to $Stop_{t-\nu,t}$. For example, if $Stop_{t-\nu,t,i} > \hat{Cont}_{t-\nu,t,i}$, we update the elements for simulation run i by setting $ExAge_i := \nu$ and $Payout_i := Stop_{t-\nu,t,i}$.

Step 8: backward recursion If $\nu > t_0 = 0$, then we set $\nu_{old} := \nu$ and go back to Step 6. If $\nu = t_0 = 0$, then we go to Step 9.

Step 9: determine the value of participation Given the current state X_t we have the approximated value of participation at age ν at time t, which is given by

$$\begin{aligned} \hat{Part}_{t-\nu,t} =& \hat{Cont}_{t-\nu,t} - Stop_{t-\nu,t} \\ =& X_t \hat{\beta}_\nu - \min\left(1, \hat{F}_t\right) \Pi_t \left(B_{t-\nu,t}\right). \end{aligned}$$