

Optimal Long-Term Allocation with Pension Fund Liabilities

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Abstract

We build a macroeconomic model for Switzerland, the Euro Area, and the USA that drives the dynamics of several asset classes and the liabilities of a representative Swiss (defined-contribution) pension fund. This encompassing approach allows us to generate correlations between returns on assets and liabilities. We calibrate the economy using quarterly data between 1985:Q1 and 2013:Q2. Using a certainty equivalent approach, we demonstrate that a liabilities-hedging portfolio outperforms an assets-only strategy by between 5% and 15% per year. The main reason for such a large improvement is that the optimal assets-only portfolio is typically long in cash, whereas hedging liabilities require the pension fund to be short in cash. It follows that imposing positivity restrictions in the construction of the portfolio also results in a large cost, between 4% and 8% per year. This estimate suggests that allowing pension funds to hedge their liabilities through borrowing cash and investing in a diversified bond portfolio helps to enhance the global portfolio return.

Keywords: Asset Liability Management, Defined Contributions, Surplus maximization.

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Evaluation of a life company is a mixture of art and science.

I.C. Smart, 1977

1 Introduction

Over the last five to ten years, the majority of developed countries have experienced a low-return environment with particularly low short-term interest rates. For instance, the Swiss three-month interbank rate has been below 0.5% since the first quarter of 2009. This new environment raises new issues for pension fund management. Some pension funds have been rather generous with their retirees and have made large promises to contributing employees. With increasing life expectancy and a low-return environment, it is now more difficult to keep those commitments and promises. For these reasons, several pension funds have already changed their type of pension plan, for instance from defined benefits to defined contributions; in addition, the technical and conversion rates, which define the commitment of pension funds to insured beneficiaries, have been regularly decreased over the recent period.

Although difficulty in generating returns is not specific to pension funds, this industry is particularly prone to it. Interest-rate risk being the main source of risk for pension liabilities, standard hedging techniques suggest holding a large amount of long-term debt, which currently generates low returns. Because stocks have suffered during the crisis, the funded ratio of most pension funds has deteriorated.

As standard cash-flow matching and immunization techniques perform poorly in a low-return environment, pension funds have recently considered alternative portfolio allocation approaches, such as surplus optimization and liabilities-driven investment (LDI), to reconcile liabilities hedging and performance enhancing. The main objective of this paper is to evaluate the ability of mixed approaches to generate sufficient return in the current environment of low interest rates.

To evaluate these approaches, several issues need to be addressed. First, we need to describe the dynamics of the return on liabilities. Typically, changes in the dynamics of the pension fund population or changes in price and wage inflation may require changes in investment strategies. In addition, pension funds with a growing or declining population may have different optimal investment policies. Therefore, the approach sometimes found in the literature, consisting of proxying return on liabilities with the long-term government bond rate, should be improved. Instead, we describe the dynamics of liabilities using a model for the pension fund.

Second, the academic research on surplus maximization and LDI generally assumes no restriction on portfolio weights. This is particularly important because these approaches are designed to form a hedge portfolio with a priori long and short positions. In practice, pension funds have many regulatory restrictions and may not be able to invest in portfolios with short positions. It is therefore important to analyze the performance of these strategies when portfolio weight restrictions are taken into account.

Finally, it is common practice to discount pension fund liabilities using either past interest rates smoothed over some period of time (actuarial valuation) or term-structure forecasts (economic valuation). In the current context, in which interest rates are historically low, this may have important consequences for the optimal portfolio allocation because smoothed discount rates will first overestimate and then underestimate actual interest rates. As we are interested in the optimal financial choice of the pension fund, we consider its actual economic situation. As a consequence, we adopt a fully economic point of view, in which the discounting is based on term-structure forecasts. We also view the plan population as an open group and describe its dynamics and renewal over time.

In the empirical part of the paper, we investigate the optimal long-term investment process of a representative defined-contribution Swiss pension fund.¹ The majority of Swiss pension funds are managed under the defined-contribution principle, in which ben-

¹The specific aspects of the Swiss “three-pillar” system have been described in Bütler and Ruesch (2007) and Von Ah (2010).

efits are defined as a fraction of the accumulated savings and therefore depend on the ability of pension-fund managers to generate a sufficient return on assets. In principle, in a defined-contribution plan, the risk is not borne by the sponsor but by the insured people. Thus, for the pension fund, managing assets in relation to liabilities may not be a priority. In fact, during the financial crisis, the low performance of pension funds in terms of asset management has worsened their global situation: As of 2011, 30% of Swiss pension funds have a surplus ratio below 100%. Unfunded liabilities amount to 7.3 billion CHF for private funds and 35 billion for public funds (OFS, 2013). Designing an investment strategy that is able to – at least – match liabilities is nowadays an important objective.

We estimate an international macrofinance model, which allows us to forecast the returns and risks of Swiss and international asset classes. The model is a stationary restricted vector error-correction model generating predictions of macroeconomic and financial factors. As returns on assets and liabilities are related through the factors generated by the macrofinance model, liabilities risk can be partly hedged by investing in an appropriate combination of assets. This allows us to investigate various important questions in a controlled environment: (1) What is the cost of allocating assets without considering their relation to liability, i.e., ignoring liability hedging. (2) What is the cost of restrictions on portfolio weights?² (3) How does the evolution of the pension-fund population affect the optimal allocation?

We evaluate the cost of suboptimal allocation strategies using a measure of certainty equivalent. We provide evidence that neglecting liabilities-hedging portfolios results in a cost ranging between 5% and 15% per year depending on the risk aversion. The main reason for such a large cost is that the optimal assets-only portfolio is typically long in cash, whereas the pension fund should be short in cash to hedge liabilities. It follows that imposing positivity restrictions in the construction of the portfolio also results in a large cost, between 4% and 8% per year. These estimates suggest that allowing pension

²We consider only positivity constraints. Actual pension funds may be subjected to more stringent conditions limiting the amounts invested abroad or the amounts allocated to some asset classes.

funds to hedge their liabilities through borrowing cash and investing in a diversified bond portfolio help to enhance the global portfolio return. A limit to this approach is the reduced availability of long-term government bonds in Switzerland.

The rest of the paper is organized as follows. In Section 2, we describe the different blocks of the model: the macrofinance model, the modeling of the assets and liabilities, and the optimal allocation strategy. In Section 3, we present the data and a preliminary investigation of the liabilities and the hedging properties of the assets. In Section 4, we discuss results regarding the optimal allocation of the pension fund under different scenarios. Section 5 concludes.

2 Model

In **Figure 1**, we display the various blocks of our global model. First, a macrofinance model describes the dynamics of the important macroeconomic and financial factors (output growth, inflation, wages, term structure, etc.). These factors serve as inputs to the financial assets forecast model to generate the expected returns, volatilities, and correlations of the various asset classes over long-term horizons.

As displayed on the right-hand-side of the figure, the pension-fund model combines the macroeconomic and financial factors with demographic and regulatory parameters (mortality rates, contribution rate, conversion rate, etc.) to generate forecasts of population and cash flows. By discounting future cash flows (contributions and pensions) with term-structure forecasts, our model generates forecasts of liabilities and eventually of returns on liabilities.

As the figure also illustrates, both financial forecasts and returns on liabilities are brought together in the allocation module to construct assets-only and assets-liabilities optimal portfolios. Dependence between returns on assets and returns on liabilities, which determines the hedging properties of the financial assets, is estimated by combining the macrofinance model and the pension-fund model through Monte Carlo simulations.

We then investigate the economic consequences of imposing or relaxing portfolio weight restrictions. Once the optimal portfolio allocation is computed, we can evaluate the pension fund performance in the last module. The rest of this section describes the main aspects of the various components.

2.1 Macrofinance Model

Our macrofinance model considers the USA, the Euro Area, and Switzerland, as Swiss pension funds are likely to concentrate their allocation in these regions.³ We need a closed model to perform long-term forecasting and simulation. The main variables described by the model are: output gap, GDP deflator inflation, consumption price inflation, wage inflation, employment growth, unemployment rate, three-month T-bills rate, 10-year government bond rate, log price-dividend ratio, currencies, and commodity price inflation. The estimation is performed at quarterly frequency over the period 1985:Q1 – 2013:Q2. The model could be re-estimated each quarter, as new observations become available.

The model is based on the error-correction principle put forward by Engle and Granger (1987). As all variables in level are nonstationary, their interactions are modeled through cointegration relations. Given the number of variables involved in the model (10 for each of the three regions), we cannot estimate an unrestricted VAR(1) because the resulting number of unknown parameters would be too large (30×30 for the lag parameter matrix and $30 \times 31/2$ for the covariance matrix). Instead, we estimate a restricted version, in which explanatory variables are selected based on economic theory and statistical significance. In the resulting model, all parameter estimates have the sign and magnitude predicted by theory and are highly significant. Residuals of the cointegration relations are stationary and represent driving forces toward the equilibrium levels described in the cointegration relations. The short-run dynamics of the model for the variables in difference is also driven by macroeconomic and finance theory.⁴ The error-correction

³Our setting could be easily adapted to other countries.

⁴A natural alternative representation of this (reduced-form) macrofinance model would be a dynamic stochastic equilibrium (DSGE) model. Calés, Jondeau, and Rockinger (2013) have investigated such a

model is written as:

$$A_0 \Delta X_{t+1} = \tilde{\mu} + A_1 \Delta X_t + A_2 X_t + A_3 \varepsilon_{t+1}, \quad (1)$$

where ΔX_t contains n stationary variables of the system and the innovations vector, ε_{t+1} , has zero mean and identity covariance matrix. Matrices A_0 , A_1 , and A_2 have restrictions imposed by economic theory and matrix A_3 captures residual contemporaneous correlation among the variables. Closing the model in a VAR form yields:

$$\begin{bmatrix} A_0 & 0_n \\ -I_n & I_n \end{bmatrix} \begin{bmatrix} \Delta X_{t+1} \\ X_{t+1} \end{bmatrix} = \begin{bmatrix} \tilde{\mu} \\ 0_n \end{bmatrix} + \begin{bmatrix} A_1 & A_2 \\ 0_n & I_n \end{bmatrix} \begin{bmatrix} \Delta X_t \\ X_t \end{bmatrix} + \begin{bmatrix} A_3 & 0_n \\ 0_n & 0_n \end{bmatrix} \begin{bmatrix} \varepsilon_{t+1} \\ 0_n \end{bmatrix}, \quad (2)$$

or with obvious notations:

$$B_0 Y_{t+1} = \mu + B_1 Y_t + B_2 \eta_{t+1}. \quad (3)$$

Solving the model indicates that the final model is a stationary, parsimonious, restricted VAR(1) model:

$$Y_{t+1} = \Phi_0 + \Phi_1 Y_t + \Phi_2 \eta_{t+1}, \quad (4)$$

where $\Phi_0 = B_0^{-1}\mu$, $\Phi_1 = B_0^{-1}B_1$, and $\Phi_2 = B_0^{-1}B_2$. Matrix Φ_2 captures contemporaneous correlation among the variables and $\Sigma = \Phi_2\Phi_2'$ is the covariance matrix of the error term.

With this model, we are able to forecast macroeconomic and financial factors, which in turn drive the dynamics of returns on assets and liabilities. A general description of the relations in the macrofinance model is given in **Figure 2**. It indicates, for a given country, how macroeconomic and financial factors interact to predict the dynamics of financial assets and liabilities. Arrows indicate causal links from one variable to the model for long-term asset allocation. Although explicitly modeling economic and financial theoretical relations in a complete model is very appealing, it also raises several empirical issues that render an international extension out of reach. In this paper, we rely on a simplified, reduced-form version of this model.

others. For instance, output gap helps predict inflation, employment, short-term rate, and the dividend-price ratio.

This macrofinance model also generates the correlation between returns on assets and liabilities, which are then used for the assets-liabilities allocation. Using Monte Carlo simulations of the model, we can infer confidence intervals for expected returns and portfolio weights. We can also deduce risk measures, such as Value-at-Risk and expected shortfall of the pension-fund surplus.

2.2 Modeling Assets

The most common assets are cash, bonds, and equities within each country. We relate the dynamics of asset returns to the macrofinance model. The monetary policy reaction function typically describes the response of the short-term interest rates ($r_{t+1}^{(3m)}$) to output gap (og_{t+1}) and inflation (π_{t+1}) pressures (Taylor, 1993). The equilibrium relation is written as:

$$r_{t+1}^{(3m)} = \mu_{r,0} + \mu_{r,1} \pi_{t+1} + \mu_{r,2} og_{t+1} + \varepsilon_{r,t+1}. \quad (5)$$

The short-run dynamics describes the evolution of the short-term rate toward this long-run equilibrium level:

$$\Delta r_{t+1}^{(3m)} = \mu_{\Delta r,0} + \mu_{\Delta r,1} \hat{\varepsilon}_{r,t} + \mu_{\Delta r,2} \Delta \pi_{t+1} + \mu_{\Delta r,3} \Delta og_{t+1} + \varepsilon_{\Delta r,t+1}, \quad (6)$$

where $\hat{\varepsilon}_{r,t}$ denotes the estimated residual of the equilibrium relation (Equation (5)).

As suggested by the expectations hypothesis of the term structure and the Fisher relation, the 10-year government bond rate ($y_{t+1}^{(10)}$) is driven by the short-term rate, with a term premium that varies with inflation and growth expectations:

$$y_{t+1}^{(10)} = \mu_{y,0} + \mu_{y,1} r_{t+1}^{(3m)} + \mu_{y,2} \pi_{t+1} + \mu_{y,3} og_{t+1} + \varepsilon_{y,t+1}. \quad (7)$$

The holding period return is obtained as:

$$r_{t+1}^{(10)} = D_t^{(10)} y_t^{(10)} - D_{t+1}^{(9)} y_{t+1}^{(9)}, \quad (8)$$

where $D_t^{(k)} = (1 - \exp(-ky_t^{(k)}))/(1 - \exp(-y_t^{(k)}))$ is Macaulay's duration of a bond of maturity k years.

The real stock return dynamics depend on the evolution of the log dividend price ratio (dpr) (Campbell and Shiller, 1988 and 2001, Barberis, 2000). The dpr is in fact nonstationary and needs to be described by a cointegration relation. The main determinant of dpr is found to be the real long-term interest rate:

$$dpr_{t+1} = \mu_{dpr,0} + \mu_{dpr,1} (y_{t+1}^{(10)} - \pi_{t+1}) + \varepsilon_{dpr,t+1}, \quad (9)$$

and the short-run dynamics of the real stock return is given by:

$$\rho_{s,t+1} = \mu_{s,0} + \mu_{s,1} \hat{\varepsilon}_{dpr,t} + \mu_{s,2} \Delta(r_{t+1}^{(3m)} - \pi_{t+1}) + \mu_{s,3} \Delta(r_{t+1}^{(10)} - \pi_{t+1}) + \varepsilon_{s,t+1}. \quad (10)$$

These various relations provide a rather good description of US asset returns. For the Euro Area and Switzerland, we need to account for international arbitrage. It is clear that long-term rates and stock returns in Europe are also driven to some extent by US factors. For this reason, we also introduce US variables as potential drivers of financial returns in the Euro Area. Similarly, US and European variables are potential drivers of Swiss financial returns. By proceeding in this way, we capture the well-known contemporaneous correlation between these variables, while retaining an economic interpretation of the estimated relations.

Finally, currency hedging may be an important issue for pension funds, in particular in a context in which large currency fluctuations may partly offset asset returns. For instance, over the recent period, the Swiss franc has experienced large fluctuations against the euro and US dollar. For this reason, we assume that all investments in US

and European assets are fully hedged. Even with full currency hedging, exchange rates have to be modeled because they may be important drivers of monetary policy and/or international arbitrage. For instance, the evolution of the Swiss short-term rate has been partly driven by the evolution of the euro-Swiss franc exchange rate over the last years.

In addition to the classical assets described above, we also added commodities and real estate as investment vehicles for pension funds. These asset classes are known to be somewhat uncorrelated to classical assets and may therefore provide good diversification opportunities to pension funds in terms of diversification. Modeling commodities and real estate returns is a difficult task in a simplified macroeconomic framework. Both asset returns are found to be related to economic growth: global (US and European) growth for commodities and regional (European and Swiss) growth for Swiss real estate.⁵

We denote the nominal log-return of asset i between t and $t + 1$ by $r_{i,t+1}$ and the real log-return by $\rho_{i,t+1} = r_{i,t+1} - \pi_{t+1}$. Even if the three-month T-bills rate is risky in the long run, we maintain the usual distinction between the three-month rate, $r_{t+1}^{(3m)}$, and the returns on the other risky assets, grouped in $r_{t+1} = (r_{1,t+1}, \dots, r_{N,t+1})'$. We also define the excess log-returns relative to the three-month rate as $x_{i,t+1} = r_{i,t+1} - r_{t+1}^{(3m)}$, $i = 1, \dots, N$. The vector x_{t+1} is linear in macroeconomic and financial factors (Y_{t+1}) through the relation: $x_{t+1} = M Y_{t+1}$, where M is a selection matrix.⁶

The expected excess log-returns and associated covariance matrix at $t + k$, conditional on the information set at date t , are given by:

$$\begin{aligned} \mu_{x,t+k} &= E_t[x_{t+k}] = M [(I_n + \Phi_1 + \dots + \Phi_1^{k-1})\Phi_0 + \Phi_1^k Y_t], \\ \Sigma_{xx,k} &= V_t[x_{t+k}] = M(I_n + \Phi_1 + \dots + \Phi_1^{k-1}) \Sigma (I_n + \Phi_1 + \dots + \Phi_1^{k-1})' M'. \end{aligned}$$

⁵We use the composite S&P Goldman Sachs Commodity index and the IAZI Investment Real Estate index. As pension funds are often investing in direct real estate projects, we consider a representative index of the Swiss real estate. The IAZI index is based on a pool of approximately 50% of all transactions at actual market conditions. The issue of illiquidity of real-estate investment is mitigated by the long investment horizon we consider.

⁶As Y_t includes inflation and exchange rates, this relation can be used to define nominal or real returns and returns hedged or unhedged against currency risk. One simply needs to adjust matrix M accordingly.

Finally, if we define annualized cumulative log-returns as $x_{t:t+k}^{\{k\}} = \frac{1}{k} \sum_{i=1}^k x_{t+i}$, we obtain that expected excess log-returns and the covariance matrix between t and $t+k$, conditional on the information set at date t , are given by:

$$\begin{aligned}\mu_{x,t:t+k}^{\{k\}} &= \frac{1}{k} M[(I_n - \Phi_1)^{-1}(kI_n - \Phi_1 - \dots - \Phi_1^k)\Phi_0 + (\Phi_1 + \dots + \Phi_1^k)Y_t], \\ \Sigma_{xx}^{\{k\}} &= \frac{1}{k} [M\Sigma M' + M(I_n + \Phi_1)\Sigma(I_n + \Phi_1)'M' + \dots \\ &\quad + M(I_n + \Phi_1 + \dots + \Phi_1^{k-1})\Sigma(I_n + \Phi_1 + \dots + \Phi_1^{k-1})'M'] .\end{aligned}$$

These two elements will be used later to determine the optimal mean-variance assets-only allocation.

2.3 Modeling Liabilities

To analyze the real effect of macroeconomic factors on a pension fund, we need a description of the liabilities of the fund. However, modeling liabilities is clearly a difficult task because the evolution of pension funds is country specific and even pension-fund specific. Different systems coexist across countries, such as “pay as you go” versus “funded pension plan” or “defined-contribution” versus “defined-benefit” plans. In addition, liabilities may be computed differently according to the organization of the fund. The main components of the liabilities of a fund are the dynamics of the population of insured employees and pensioners and the dynamics of the financial cash flows (contributions and benefits).

We adopt the point of view of a Swiss pension fund. The Swiss social security system is based on three pillars. The first pillar (state pension) is designed to cover basic needs. The second pillar (fully-funded occupational pension funds) covers all salaried employees with a minimum annual income. The third pillar is private saving. See **Appendix 1** for a more detailed description of the Swiss social security system.

Pension funds can be of different types. They can be regulated by public or private law; they can be single funds or collective foundations; they can be autonomous pension

providers or can reinsure part of their risks; they can be based on defined-contribution plans or defined-benefit plans. As most pension funds are private funds with defined-contribution plans (97% of funds, 90% of active members in 2011), we focus on defined-contribution regulation.⁷

As the liability of a pension fund is the present value of its expected future cash flows, we need to forecast the future evolution of the fund. In doing so, our objective is to be as close as possible to the most likely evolution of the fund for the population, the cash flows, and the discounting. As discussed in Winklevoss (1993) and Impavido (2011), there exist many ways to generate the future evolution of a pension fund. Regarding the dynamics of the pension-fund population, there are two main approaches. In an open pension fund, current plan members will eventually leave, retire, or die, and new individuals will join the plan (*open-group valuation*). In a closed pension fund, members are limited to the current ones (*closed-group valuation*). As we are interested in long-run investment strategies, we consider an open pension fund, in which the evolution of the plan's population will be for the most part controlled by the assumed replacement rate.⁸

Berkelaar and Kouvenberg (2010) consider the case of a closed-group valuation in which only pension entitlements that have been earned by employees up to the valuation date are considered. As a consequence, they find a relatively low duration of the liabilities, approximately 15 years. In a going concern valuation, the duration is much longer, in particular in a low interest-rate environment, in which cash flows far in the future contribute in a non-negligible way to liabilities. In our estimates, the duration would be below 25 years in a closed-group valuation but above 50 years in an open group with a

⁷In 2011, there are 2191 registered pension funds in Switzerland, with 3.787 million active members and 1.041 million beneficiaries. 2099 pension funds are under private regulation (3.161 million active members), whereas 92 pension funds are under public regulation (0.6 million active members). Total liabilities amount to 625 billion CHF. Total benefits paid amount to 44.2 billion CHF and total contributions to 47.1 billion CHF (20 billion paid by employees and 27.1 paid by employers). See Bundesamt für Statistik (2013). Barenco (2012) provides additional details on the Swiss pension fund industry.

⁸More formally, we adopt a *going concern valuation*, so that liabilities today are not only based on accrued benefits as of today but also on future entitlements that will be earned by plan members in the future. In a *plan termination valuation*, one would assume that the plan is closed at the day of the valuation, so that it does not take future income and benefits into account.

replacement rate equal to 1. As we will see below, this has important implications for the investment strategy of the pension fund.

We now turn to the two main ingredients required to generate scenarios of the liability side of a pension fund, namely, the demographics and the financial cash flows.

2.3.1 Population Dynamics

The dynamics of the population and cash flows of a closed pension fund can be described as Markov processes (see Janssen, 1966, Janssen and Manca, 1967, Wolthuis, 1994, and Mettler, 2005, for Switzerland). Assume that the initial population is partitioned as follows, according to the status and age of the beneficiaries:⁹

- Active employees (A) with initial population $P_{A,0} = (P_{A,0}^{(25)}, \dots, P_{A,0}^{(64)})'$,
- Retired (R) with initial population $P_{R,0} = (P_{R,0}^{(65)}, \dots, P_{R,0}^{(105)})'$,
- Dead (M) with initial population $P_{M,0} = (P_{M,0}^{(25)}, \dots, P_{M,0}^{(105)})'$.

The initial population of the pension fund is grouped in vector $P_0 = (P'_{A,0}, P'_{R,0}, P'_{M,0})'$.

In a closed pension fund, with time independent transition probabilities, the population dynamics is simply described by $P_{t+1} = \Pi' P_t$, where Π is a time-homogenous transition matrix:

$$\Pi = \begin{pmatrix} \Pi_{AA} & \Pi_{AR} & \Pi_{AM} \\ 0 & \Pi_{RR} & \Pi_{RM} \\ 0 & 0 & I \end{pmatrix}, \quad (11)$$

where Π_{XY} is the transition probability from state X to state Y and I is a conformable identity matrix. Matrix Π is parameterized using mortality tables. As time goes by, active members retire and eventually die.

⁹We assume that young active employees start contributing to their pension plan at age 25 and the legal retirement age is 65. We also assume that all insured members (or their surviving spouses) die at the maximal age 105.

In practice, the majority of pension funds are open, so that some active members leave the plan before retirement, whereas some new active members join.¹⁰ In this setting, it suffices to increase/decrease the positions in $P_{A,t}$ where exits or new entries occur. Assume that the active members who leave the plan between date t and $t+1$ are replaced with a ratio ψ . For instance, a ratio $\psi = 1$ means that each year, all active members leaving the plan will be replaced by a new cohort. We call ψ the replacement rate. In addition, assume that the age structure of the newly hired employees is denoted by $\Theta_A = (\Theta_A^{(25)}, \dots, \Theta_A^{(64)})'$, where $\Theta_A^{(i)}$ is the proportion of new employees with age i . Then the number of new employees who have to be recruited at date $t+1$ to compensate leaving employees and satisfy the assumed replacement rate is: $[\psi e' - e' \Pi'_{AA}] P_{A,t}$, where e is a vector of ones. Finally, given the age structure of the newly hired employees, the new active population at $t+1$ will be:¹¹

$$P_{A,t+1} = \Pi'_{AA} P_{A,t} + \Theta_A [\psi e' - e' \Pi'_{AA}] P_{A,t}. \quad (12)$$

2.3.2 Financial Cash Flows

Future cash flows include contributions (based on earnings) paid by active members and benefits (based on accumulated savings) paid to pensioners.

Salary received during the period t to $t+1$ has two components: (1) the wage inflation, which is the increase in wage that affects all employees (π_{t+1}^w); and (2) the merit scale ($ms^{(x,x+1)}$), which depends on the working age of the employees. For active employees of age x at date t , the next year salary is given by:

$$S_{A,t+1}^{(x+1)} = S_{A,t}^{(x)} (1 + \pi_{t+1}^w) (1 + ms^{(x,x+1)}) \equiv S_{A,t}^{(x)} (1 + g_{t+1}^{(x,x+1)}), \quad (13)$$

¹⁰In Switzerland, plan members who leave the plan before retirement are entitled to vested benefits. As they join a new pension fund, the former one must transfer all vested benefits to the new one. In counterpart, new plan members will bring their vested benefits to the pension fund. Therefore, in such a situation, we reduce pension-fund liability by the value of vested benefits paid to the former employee and increase its liability by the value of vested benefits brought by the new one.

¹¹A similar expression may be found in Mettler (2005).

where $g_{t+1}^{(x,x+1)}$ denotes the total salary growth for employees of age x .

Accumulated savings has two components: (1) the initial savings accumulated before the employee joins the pension plan; (2) the annual contributions paid by employees to the plan. Both components will generate interest income. For ease of exposition, we assume that the pension fund starts at date 0 with active employees and pensioners having some initial savings.

Initial savings, denoted by $B_0^{exo} = (B_{A,0}^{exo'}, B_{R,0}^{exo'}, 0)'$, is exogenous from the pension fund perspective.¹² It generates interest income at rate R_{t+1}^e for employees and at rate R_{t+1}^p for pensioners:¹³

$$\begin{aligned} B_{A,t+1}^{(x+1)exo} &= B_{A,t}^{(x)exo} (1 + R_{t+1}^e), & x + 1 &= 26, \dots, 64, \\ B_{R,t+1}^{(65)exo} &= B_{A,t}^{(64)exo} (1 + R_{t+1}^e), \\ B_{R,t+1}^{(x+1)exo} &= B_{R,t}^{(x)exo} (1 + R_{t+1}^p), & x + 1 &= 66, \dots, 105. \end{aligned}$$

The annual contribution paid by employees to the plan is a fraction of current salary and depends on the age of the employees. Once paid, the contribution also earns interest (at rate R_{t+1}^e). This component is endogenous from the pension fund perspective, as the fund may decide to change the contribution rate. Starting with $B_{A,0}^{(x)end} = B_{R,0}^{(x)end} = 0$, the dynamics of endogenous saving is given by:

$$\begin{aligned} B_{A,t+1}^{(x+1)end} &= B_{A,t}^{(x)end} (1 + R_{t+1}^e) + C_A^{(x)} S_{A,t}^{(x)}, & x + 1 &= 26, \dots, 64, \\ B_{R,t+1}^{(65)end} &= B_{A,t}^{(64)end} (1 + R_{t+1}^e) + C_A^{(64)} S_{A,t}^{(64)}, \\ B_{R,t+1}^{(x+1)end} &= B_{R,t}^{(x)end} (1 + R_{t+1}^p), & x + 1 &= 66, \dots, 105. \end{aligned}$$

¹²In practice, we reconstruct this initial saving by retropolating past salaries, past contributions, and therefore past savings.

¹³In Switzerland, two different rates are used to remunerate employees and pensioners savings: R_{t+1}^e is close to the LPP/BVG-minimum interest rate and R_{t+1}^p is close to the technical rate. The reference base for the LPP/BVG-minimum interest rate is the yield on federal bonds in addition to equity, bond and real estate trends. It was set equal to 1.5% in 2013 by the Federal Council. The technical rate is determined by pension funds based on their balance-sheet situation and financial markets prospects. The recommendation of the Swiss Chamber of Pension Actuaries for 2013 was 3%. In general, we have $R_{t+1}^e < R_{t+1}^p$. We use capital R for simple return and lower case r for log-return.

Total saving is then given by: $B_{A,t+1}^{(x+1)} = B_{A,t+1}^{(x+1)exo} + B_{A,t+1}^{(x+1)end}$ for employees and $B_{R,t+1}^{(x+1)} = B_{R,t+1}^{(x+1)exo} + B_{R,t+1}^{(x+1)end}$ for pensioners.

Finally, in a defined-contribution plan, financial cash flows received and paid by the pension fund are defined as follows. Contributions paid by employees are a fraction of annual salary. Benefits received by retirees are a fraction of the accumulated savings at time of retirement. Therefore, cash flows combine the evolution of the insured population, salaries, and accumulated savings:

$$\begin{aligned} CF_{A,t+1}^{(x+1)} &= \Pi_{AA}^{(x,x+1)} P_{A,t}^{(x)} C_A^{(x)} S_{A,t}^{(x)}, & x+1 &= 26, \dots, 64 \\ CF_{R,t+1}^{(65)} &= \Pi_{AR}^{(64,65)} P_{A,t}^{(64)} C_R B_{A,t}^{(64)}, \\ CF_{R,t+1}^{(x+1)} &= \Pi_{RR}^{(x,x+1)} P_{R,t}^{(x)} C_R B_{R,t}^{(x)}, & x+1 &= 66, \dots, 105, \end{aligned}$$

where $C_A^{(x)}$ is the contribution rate (in proportion to salary) paid by employee of age x and C_R is the conversion rate (in proportion to accumulated saving).¹⁴

It is clear from these formulas that computing liabilities for active members requires salary-growth forecasts to accrue additional benefits before retirement. These projections are provided by the macrofinance model. We also use Swiss-level statistics on the salary scale across ages, to estimate as precisely as possible total salary growth, $g_{t+1}^{(x,x+1)}$.

For the purpose of exposition, we have presented a very simplified version of the actual dynamics of the cash flows of the pension fund.¹⁵ We have voluntarily omitted disabled employees, surviving spouses and orphans, exiting employees, and vesting, and we did not differentiate male and female populations. The description of the complete model would exceed the usual format of a paper. In the empirical section, we use the full-fledged model, although from an economic point of view, differences with the simplified model are of second order.

¹⁴Contribution rates and the conversion rate are subject to regular changes in Switzerland. The minimum contribution rates, set by the Law on occupational pension schemes (BVG / LPP), are currently equal to 7%, 10%, 15%, and 18% of insured salary for ages [25-34], [35-44], [45-54], [55-65], respectively. The conversion rate is equal to 6.85% for men and 6.8% for women for total accumulated savings.

¹⁵A more complete presentation can be found in Mettler (2005).

2.3.3 Liabilities

For an asset-liability management exercise, we need to estimate the current and future liabilities to determine the expected return on liabilities. We turn to this issue now. Current time is t and we consider a time horizon T , for which we determine the liabilities. We denote by L_{t+T} the future liabilities at time $t + T$. Once future cash flows have been determined, L_{t+T} is the present value of expected future cash flows (CF_{t+T+i}) discounted with the discount rate of the maturity i ($R_{t+T}^{(i)}$):

$$L_{t+T} = E_t \sum_{i=1}^{\infty} \frac{CF_{t+T+i}}{(1 + R_{t+T}^{(i)})^i}, \quad (14)$$

where $CF_t = \sum_{x=65}^{105} CF_{R,t}^{(x)} - \sum_{x=25}^{64} CF_{A,t}^{(x)}$, i.e., the net cash flow remaining after subtracting contributions paid by employees from the benefits paid to pensioners.

An important question in Equation (14) is the choice of the discount rate. In *actuarial practice*, pension funds use the so-called technical rate to discount cash flows. The technical rate is typically based on an average of the long-term government bond rate and the expected return of a risky portfolio, smoothed over a long period of time. See the technical guidelines FRP4/DTA4 of the Swiss Chamber of Pension Actuaries (2010). This definition has important consequences: First, technical rates may be different from one pension fund to the other, so that the resulting actuarial balance sheets are not always comparable. Second, the technical rate is a backward-looking rate and may not reflect the expected evolution of future interest rates and financial returns. This is particularly relevant after the subprime crisis, because the current low level of interest rates has never been seen before.

To avoid these possible biases, we consider an *economic approach*, in which cash flows are discounted using term-structure forecasts. To do so, we use a model describing and forecasting the term structure of bonds issued by the Swiss Confederation. Our approach is based on Nelson and Siegel (1987). We model the dynamics of interest rates for three maturities (3 months, 2 years, and 10 years). With their forecast at date $t+T$, we back out

Nelson-Siegel parameters, which describe the level, slope, and curvature of a given term structure. Then, we determine the complete term structure of the Swiss Confederation, $R_{t+T}^{(i)SC}$, at date $t + T$ using the Nelson-Siegel formula. See **Appendix 2** for details. This approach ensures that the curve goes through the interest rate forecasts for the three maturities. As pension-fund liability is likely to be riskier than Swiss Confederation debt, we add a premium π to the forecasted rate and use $R_{t+T}^{(i)} = R_{t+T}^{(i)SC} + \pi$ to discount future cash flow CF_{t+T+i} .¹⁶

It should be mentioned that it is common in the academic literature on pension funds to proxy the return on liabilities with a long-term government rate, with a maturity typically between 10 and 20 years (Hoevenaars et al., 2008, Van Binsbergen and Brandt, 2008). Such an approach is much simpler to implement, as it does not require modeling the liability side of the pension-fund balance sheet. However, it also neglects some potentially important fund-specific risk factors such as the dynamics of the insured population or the evolution of salaries. More importantly, it dramatically underestimates the actual duration of liabilities. As a consequence, it generally overestimates the hedging properties of government bonds with 10-year to 20-year maturities. Finally, as we will see below, it also over-estimates the actual return on liabilities. For all these reasons, it will lead to an inadequate asset allocation.

Eventually, we define the log-return on liabilities as $r_{L,t+1} = \log(L_{t+1}/L_t)$ and the excess log-return as $x_{L,t+1} = r_{L,t+1} - r_{t+1}^{(3m)}$. We also define the expected excess log-return as $\mu_{L,t+k} = E_t[x_{L,t+k}]$ and the variance of the return on liabilities as $\sigma_{L,k}^2 = V_t[x_{L,t+k}]$. Given the way liabilities are constructed, the return on liabilities is clearly a nonlinear function of the macrofinance factors. The nonlinearity for the most part comes from capitalizing interest income and discounting cash flows. Consequently, we cannot use the standard formula described in Section 2.2 for a VAR model to compute the covariances/correlations between assets and liabilities. Instead, we rely on Monte

¹⁶Our approach is different from the one described by Diebold and Li (2006), who forecast Nelson-Siegel parameters. As we are estimating a macrofinance model, it is more natural to model the reference interest rates directly and then to recover the Nelson-Siegel parameters.

Carlo simulations and proceed as follows. We simulate a large number of samples of the macrofinance model and deduce returns on assets and returns on liabilities. Then, for a given investment horizon, we compute the resulting covariance/correlation matrix, which in turn is used for the allocation process.

2.4 Portfolio Allocation

We now turn to the optimal portfolio allocation in presence of liabilities. A first intuitive approach consists in matching liability cash flows with asset cash flows of the same maturity. This *cash-flow matching* approach is essentially done using fixed-income instruments such as zero-coupon bonds or inflation-indexed zero-coupon bonds. When such instruments do not exist, a less precise approach relies on *immunization*, which allows an imperfect matching based on equalizing assets and liabilities durations. A limitation of these approaches is their low expected return, as most of the assets held are fixed-income instruments.

A second approach, called *surplus maximization*, drops the requirement of an asset portfolio perfectly correlated with liabilities, and seeks the maximal expected return on surplus, defined as the difference between the expected returns on assets and liabilities (Sharpe and Tint, 1990, and Ezra, 1991). By allowing for a mix of asset classes, this approach is expected to produce higher expected returns. In addition, it may hedge noninterest risks borne by pension-fund liabilities. For instance, in the context of portfolio choice, Benzoni, Collin-Dufresne, and Goldstein (2007) indicate that labor income risk can be hedged by investing more in stocks because wages and stocks are highly correlated in the long run. In our context, a pension-fund future stream of cash flows depends on wages. Therefore, the same intuition as for portfolio choice carries over to surplus maximization.

For a formal discussion of surplus maximization, we assume that the pension fund maximizes the funded ratio over the next k periods. It is defined as $F_{t+k} = A_{t+k}/L_{t+k}$, with $A_{t+k} = A_t \exp(r_{A,t:t+k})$ and $L_{t+k} = L_t \exp(r_{L,t:t+k})$, where $r_{x,t:t+k}$ denotes the cu-

mulative log return between t and $t + k$. In logarithm, the funded ratio is written as $\log(F_{t+k}) = \log(A_{t+k}) - \log(L_{t+k})$, so that maximizing F_{t+k} is equivalent to maximizing the surplus log-return $r_{F,t:t+k} = r_{A,t:t+k} - r_{L,t:t+k}$. We will use the exponent $\{k\}$ to denote the annualized cumulated return over k periods. For instance, we have $r_{F,t:t+k}^{\{k\}} = \frac{1}{k} \sum_{i=1}^k r_{F,t+i} = \frac{1}{k} r_{F,t:t+k}$. We note that, as surplus log-return is defined as the difference, using nominal or real returns will not make any difference. The presentation below is made with nominal returns, but the same formulas would apply for real returns.

Assuming that the return on liabilities is given, the optimal allocation for horizon k rests on selecting the weights of the risky assets, which we denote by $\alpha_t^{\{k\}} = \{\alpha_{1,t}^{\{k\}}, \dots, \alpha_{N,t}^{\{k\}}\}$ and the weight of the one-period bond $\alpha_{0,t}^{\{k\}} = 1 - \alpha_t^{\{k\}'} e$, where e is the vector of 1. We do not assume any restrictions on portfolio weights for the moment, but later we will also consider the case of positivity constraints on all assets, $0 \leq \alpha_{i,t}^{\{k\}} \leq 1$, $i = 0, \dots, N$. Campbell and Viceira (1999, 2001, 2002) indicate that, after log-linearization, the asset portfolio excess log-return can be written as:

$$x_{A,t:t+k}^{\{k\}} \equiv r_{A,t:t+k}^{\{k\}} - r_{t:t+k}^{\{3m\}\{k\}} = \alpha_t^{\{k\}'} x_{t:t+k}^{\{k\}} + \frac{1}{2} \alpha_t^{\{k\}'} (\sigma_x^{\{k\}2} - \Sigma_{xx}^{\{k\}} \alpha_t^{\{k\}}), \quad (15)$$

where $\Sigma_{xx}^{\{k\}} = V_t[x_{t:t+k}^{\{k\}}]$ and $\sigma_x^{\{k\}2} = \text{diag}(\Sigma_{xx}^{\{k\}})$, as described in Section 2.2. As a consequence, the surplus annualized log-return is given by:

$$r_{F,t:t+k}^{\{k\}} = \alpha_t^{\{k\}'} x_{t:t+k}^{\{k\}} - x_{L,t:t+k}^{\{k\}} + \frac{1}{2} \alpha_t^{\{k\}'} (\sigma_x^{\{k\}2} - \Sigma_{xx}^{\{k\}} \alpha_t^{\{k\}}), \quad (16)$$

with expected return and variance given by:

$$\begin{aligned} E_t[r_{F,t:t+k}^{\{k\}}] &= \alpha_t^{\{k\}'} \mu_{t:t+k}^{\{k\}} - \mu_{L,t:t+k}^{\{k\}} + \frac{1}{2} \alpha_t^{\{k\}'} (\sigma_x^{\{k\}2} - \Sigma_{xx}^{\{k\}} \alpha_t^{\{k\}}), \\ V_t[r_{F,t:t+k}^{\{k\}}] &= \alpha_t^{\{k\}'} \Sigma_{xx}^{\{k\}} \alpha_t^{\{k\}} + \sigma_L^{\{k\}2} - 2\alpha_t^{\{k\}'} \sigma_{xL}^{\{k\}}, \end{aligned}$$

where $\mu_{x,t:t+k}^{\{k\}}$ denotes the vector of expected excess log-returns on assets, $\sigma_L^{\{k\}2} = V_t[x_{L,t:t+k}^{\{k\}}]$ and $\sigma_{xL}^{\{k\}} = \text{cov}_t[x_{t:t+k}^{\{k\}}, x_{L,t:t+k}^{\{k\}}]$.

The pension fund maximizes the mean-variance criterion for the surplus:

$$\begin{aligned} \max_{\{\alpha\}} q_t^{\{k\}}(\alpha) &= \left[E_t[r_{F,t:t+k}^{\{k\}}] + \frac{1}{2}V_t[r_{F,t:t+k}^{\{k\}}] \right] - \frac{\lambda}{2}V_t[r_{F,t:t+k}^{\{k\}}] \\ &= \alpha' \mu_{x,t:t+k}^{\{k\}} - \mu_{L,t:t+k}^{\{k\}} + \frac{1}{2}\alpha' \sigma_x^{\{k\}2} - \frac{\lambda}{2}\alpha' \Sigma_{xx}^{\{k\}} \alpha + \frac{1-\lambda}{2}(\sigma_L^{\{k\}2} - 2\alpha' \sigma_{xL}^{\{k\}}), \end{aligned} \quad (17)$$

where λ denotes the risk aversion parameter. For pension funds, λ is typically much larger than 1. In the absence of portfolio weight restrictions, we find the optimal weights in risky assets:

$$\alpha_{AL,t}^{\{k\}} = \frac{1}{\lambda}(\Sigma_{xx}^{\{k\}})^{-1} \left(\mu_{x,t:t+k}^{\{k\}} + \frac{1}{2}\sigma_x^{\{k\}2} + (\lambda - 1)\sigma_{xL}^{\{k\}} \right). \quad (18)$$

For comparison purposes, the unrestricted optimal weights in risky assets in an assets-only allocation are given by the following expression (Campbell and Viceira, 2004):

$$\alpha_{AO,t}^{\{k\}} = \frac{1}{\lambda}(\Sigma_{xx}^{\{k\}})^{-1} \left(\mu_{x,t:t+k}^{\{k\}} + \frac{1}{2}\sigma_x^{\{k\}2} + (\lambda - 1)(-\sigma_{x0}^{\{k\}}) \right), \quad (19)$$

where $\sigma_{x0}^{\{k\}} = cov_t[x_{t:t+k}^{\{k\}}, r_{t:t+k}^{(3m)\{k\}}]$ is the covariance between asset excess returns and the T-bills return.

Expression (18) demonstrates that the optimal portfolio is composed of three sub-portfolios:

- The liabilities-hedging portfolio (LHP):

$$\alpha_{LHP,t}^{\{k\}} = \left(1 - \frac{1}{\lambda} \right) (\Sigma_{xx}^{\{k\}})^{-1} \sigma_{xL}^{\{k\}}. \quad (20)$$

This expression is reminiscent of the OLS regression when the return on liabilities is projected on the return on assets. It is therefore the risky portfolio, the return of which is the most correlated with the return on liabilities. It is in this sense that this portfolio can be named LHP. This portfolio is independent from expected

returns and corresponds to the global minimum-variance portfolio in an assets-only allocation.

- The performance-seeking portfolio (PSP):

$$\alpha_{PSP,t}^{\{k\}} = \frac{1}{\lambda} (\Sigma_{xx}^{\{k\}})^{-1} \left(\mu_{x,t:t+k}^{\{k\}} + \frac{1}{2} \sigma_x^{\{k\}2} \right). \quad (21)$$

This portfolio is independent from the characteristics of the pension fund and corresponds to the standard mean-variance portfolio in the context of an assets-only allocation.

- The 3-month T-bills component: $\alpha_{0,t}^{\{k\}} = 1 - \alpha_{AL,t}^{\{k\}'}$.

The sum of LHP weights is not a priori equal to 0, but weights can be positive as well as negative. Imposing positivity restrictions would clearly affect the properties of this portfolio. In particular, it is likely that hedging liabilities risk requires large weights in bonds, so that the amount of wealth devoted to the PSP is likely to be limited and the resulting portfolio is likely to generate poor performance. In addition, with positivity restrictions, there is no closed-form solution to the optimal portfolio, and portfolio weights must be computed numerically. In the empirical application, we consider the case with no weight restrictions and the case with positivity restrictions on all asset classes.¹⁷

3 Preliminary Analysis

The main objective of the paper is to investigate the investment performance of a representative defined-contribution pension fund and several configurations regarding the

¹⁷It should be mentioned that an additional, related, approach, called *liabilities driven investment* (LDI), also relies on the construction of separate portfolios (Martellini, 2006): A first portfolio is explicitly constructed to hedge pension-fund liabilities (liability-matching portfolio); the second portfolio aims at generating performance, following standard asset allocation approaches. Typically, the hedging portfolio would be based on a short position in cash and a long position in long-term bonds or swaps. The consideration of the two types of portfolios constituting the eventual optimal portfolio appears relevant because it sheds insights on the objectives a fund manager has: performance seeking versus liability-hedging. Obviously, if regulation limits the asset classes or the amounts invested in them, the optimal hedging portfolio may not be feasible.

population dynamics, the discount rate, and the weight restrictions. We start with the investigation of some properties of returns on assets and liabilities. We comment on different steps of the construction of the return on liabilities. Then we move to the hedging properties of the various assets against inflation and real liabilities risks.

3.1 Liabilities

Regarding the liabilities model, we use aggregate Swiss data for 2013 to reflect average population structure, salary scale, etc. We consider a pension fund that would have a given fraction ($1/10^{th}$) of resident population for both active members and pensioners. Assumptions regarding population dynamics, salaries, and savings correspond to Swiss aggregate data. Assumptions on contribution and conversion rates correspond to the state pension plan (Law on occupational pension schemes, LPP/BVG).

The first two ingredients of liabilities are the dynamics of population and cash flows, which determine the numerator of the liabilities in equation (14). Both dynamics are affected by the replacement rate. **Figures 3 to 5** display how the population and cash flows of the representative pension fund are expected to evolve over time. Those figures depend on the current structure of the Swiss population and some assumptions for the replacement rate as well as structural assumptions. In Figure 3, the replacement rate is $\psi = 1$, i.e., the number of employees does not vary through time (380,000). Employees leaving the plan before retirement are all replaced by new employees, with the same age structure. The number of pensioners (retired, surviving spouses, and disabled) increases over time as employees retire or die, from 101,000 in 2014 to 384,000 in 2054. The number of disabled is small and does not increase over time, so that the disabled do not contribute significantly to the population dynamics. The ratio of beneficiaries-over-employees increases from 26% in 2014 to 101% in 2054.

Figure 3 also displays the resulting evolution of cash flows paid/received by the various groups over time. Contributions from employees are expected to increase slowly because of the trend in price and salary (from 4.7 billion CHF per year in 2014 to 8.8 billion in

2054). In contrast, benefits paid to retired and surviving spouses would increase much faster due to the increase in the number of pensioners and the effect of inflation on future benefits (from -2.4 billion CHF in 2014 to -19.2 billion in 2054). Given that the figure represents signed cash flows, cash flows paid to beneficiaries are negative and decreasing. The net cash flow of the fund is first positive and then negative from 2020 forward (from 2.3 billion CHF per year in 2014 to -10.4 billion in 2054).

Figure 4 illustrates the case of a mature pension fund, with a replacement rate of $\psi = 0.8$, i.e., only 80% of employees leaving the plan are replaced by new employees. Thus, the number of employees decreases by approximately 1.15% per year. At the end of 2054, the number of employees decreases to 233,000, while the number of pensioners has increased to 318,000. In 2054, the total number of insured people is 551,000 and the ratio of pensioners-over-employees is as high as 136%. The contributions paid by employees remain at the same level over time (from 4.7 to 5.1 billion CHF per year), as the decrease in population is compensated by salary growth. Benefits increase over time, although less than for $\psi = 1$ (-16 billion CHF in 2054), so that net cash flows are equal to -10.9 billion in 2054. This amount is slightly larger in absolute value than for $\psi = 1$.

Figure 5 corresponds to the case of a growing pension fund, with a replacement rate of $\psi = 1.2$, so that the number of employees increases by approximately 1.15% per year. The number of employees increases to 593,000 in 2054, while the number of beneficiaries also increases to 466,000 for a ratio of pensioners-over-employees of 78.6%. As a consequence, the increase in benefits paid to pensioners (-23 billion CHF in 2054) is more than compensated for by the increase in contributions paid by employees (14.4 billion CHF), so that net cash flows are equal to -8.6 billion CHF in 2054, i.e., less than in the case $\psi = 1$.

These numbers confirm that it is more difficult to manage a pension fund with a mature population (a high pensioners-to-employees ratio) than a pension fund with a growing population (a low pensioners-to-employees ratio). Due to the dynamics of the

population, the pension fund will need to generate higher expected returns from the portfolio to fulfill its liability obligations.

To compute liabilities, we discount future expected cash flows using the forecasted term structure of the government bond rate plus a risk premium.¹⁸ We consider two risk premia ($\pi = 1\%$ and 2%) to evaluate the effect of the discount rate on the optimal allocation in an assets-liabilities framework. In our macrofinance model, interest rates are expected to increase after a minimum level reached in 2013. A premium of 2% approximately corresponds to the average discount rate used by pension funds in 2013 (an average technical rate of 3%). However, as long-term government bond rates are expected to increase in the future to a level close to 3.5% , adding a premium of 2% would exceed the maximum level of 4.5% recommended by the regulator. We therefore also consider the case $\pi = 1\%$, which would correspond to a discount rate of approximately 4.5% in the long term. To illustrate the dynamics of the discount rate over time, we compute the internal rate of return \bar{R}_{t+T} corresponding to the liability at date $t + T$ by solving the equality:

$$E_t \sum_{i=1}^{\infty} \frac{CF_{t+T+i}}{(1 + \bar{R}_{t+T})^i} = E_t \sum_{i=1}^{\infty} \frac{CF_{t+T+i}}{(1 + R_{t+T}^{(i)})^i}. \quad (22)$$

This measure is displayed in **Figure 6**. In the current low interest-rate environment, the long-term government bond rate is expected to start increasing in 2014 to approximately 3.5% in the year 2025. The discount rate would reach 4.5% at that time with a premium of $\pi = 1\%$ and 5.5% with a premium of $\pi = 2\%$.

Figure 7 displays the evolution of liabilities of the representative pension fund. It illustrates that the replacement rate and the discount rate may have a huge impact on the present value of future cash flows. A low risk premium ($\pi = 1\%$) would generate

¹⁸In Switzerland, the risk of pension funds is not only borne by the firm sponsor. There is also a guarantee fund, which operates at the country level. It subsidizes institutions with an unfavorable age structure and also guarantees statutory benefits owed by insolvent pension funds. However, the guarantee is limited and covers benefits up to a certain upper limit. The guarantee fund is financed by the contributions of all pension funds.

extreme variations of liabilities in the short run, while a high premium ($\pi = 2\%$) would be accompanied by a relatively stable dynamics of liabilities. Sometime around 2030, the discount rates stabilize. Liabilities stabilize around 2025 before increasing afterwards. The expected value of liabilities is approximately 200 billion CHF for a low premium ($\pi = 1\%$), whereas it is only 150 billion for a higher premium ($\pi = 2\%$), reflecting the lower value of the discounted cash flows. Changes in the replacement rate also play a role, although less significantly.

The dynamics of liabilities is driven by a combination of two effects: (1) the large increase in the discount rate; (2) the increase in expected cash flows. To understand why liabilities decrease to such a large degree in the short run with a low premium, assume constant cash flows ($CF_t = CF$) and a flat term structure ($R_t^{(i)} = R_t$). In this case, liabilities at date t would be given by the perpetuity formula, $L_t = CF/R_t$. Now, if the discount rate increases from 2.5% to 4%, one obtains a decrease in liabilities by 37.5%. In the case in which cash flows are growing with a growth rate of $g = 1\%$, Gordon's formula, $L_t = CF/(R_t - g)$, indicates that this time liabilities decrease by 50%. This is the order of magnitude of the decrease we observe in the figure.

In fact, the main reason for the extreme sensitivity of liabilities to a low discount rate is the fact that the pension fund is not expected to close (open-group approach). This implies that cash flows far in the future still contribute to the liabilities. This effect can be measured by the duration of the liabilities of the fund. The duration depends on the replacement rate and the risk premium, as displayed in **Table 1**. A pension fund with a high replacement rate and low premium ($\psi = 1.2$ and $\pi = 1\%$) has a duration longer than 90 years, while the duration decreases to 51 years for a low replacement rate ($\psi = 0.8$) and decreases to 42 years when in addition we assume a high premium ($\pi = 2\%$).

3.2 Expected Returns

In the case of a perpetuity, the return on liabilities is given by: $r_{L,t+1} = \log(L_{t+1}/L_t) = \log(R_t) - \log(R_{t+1})$. It varies like the negative of the change in the discount rate. Thus, if

the term structure is shifted upwards, the liabilities will drop and the return on liabilities would fall.

Figure 8 displays the term structure of expected returns on liabilities. As interest rates are expected to increase over the coming years, the expected return on liabilities is negative in the short run and increases as the horizon increases. With a low premium, the return on liabilities is very low for short horizons. It is as low as -25% for a one-year horizon with a replacement rate of $\psi = 1.2$ (-6% with $\psi = 0.8$). In contrast, a high premium would generate a higher, although still negative, return on liabilities (from -11% with $\psi = 1.2$ to -2.5% with $\psi = 0.8$). At a horizon of 20 years, we observe some convergence of the average return on liabilities. It lies in the range $[-4.3\%; -0.9\%]$ for $\pi = 1\%$ and $[-0.5\%; 0.5\%]$ for $\pi = 2\%$. These measures have important implications from the pension-fund management perspective. The recent financial crisis has worsened the financial situation of the majority of the pension funds, in particular because of its low-return environment. Now, the prospects are much more favorable, because the expected increase in interest rates will mechanically reduce the expected return on liabilities, although fixed-income instruments will suffer from the rise in interest rates.

It should be mentioned that this evolution does not only reflect the (past and future) evolution of the representative interest rate but also captures the expected evolution of future cash flows. The figure also displays the 20-year government bond rate, often used in the literature as a proxy for the return on liabilities. As we can see, this return severely overestimates the actual return on liabilities, for two main reasons: First, it does not incorporate the effect of the cash flow dynamics. The increase in cash flows contributes positively to the value of liabilities but negatively to the return on liabilities.¹⁹ Second, the duration of the 20-year bond is much smaller than the duration of liabilities, and therefore its return is much less sensitive to a translation in the term structure.

Table 2 displays the expected return and risk of the various asset classes considered in our paper. The numbers presented in the table are real annualized log-returns computed

¹⁹In the case of a perpetuity with cash flows growing at rate g , we have $r_{L,t+1} = -\Delta \log(R_{t+1} - g)$. It is easy to verify that the derivative of $r_{L,t+1}$ with respect to g is negative.

over a 20-year horizon. As expected, cash has the lowest expected return in the long run. Its volatility is far from 0, however, and is actually only slightly below the volatility of long-term bonds (3% versus 4.2%). Next, in an international comparison, US bonds are expected to generate higher returns than the European and Swiss counterparts. As foreign investments are fully hedged, the difference between expected bond returns for the most part is from higher expected US short-term rates. US stock markets are expected to grow at 8% per year which compares to approximately 6.5% in Europe and 9.3% in Switzerland. Because of stock-return predictability and negative correlation between innovations in the dividend-price ratio and equity return, stock volatility is smaller in the long run than in the short run (approximately 20% for a 1-year horizon) (see Barberis, 2000). The commodities expected return is approximately 6.5% but with a large risk close to 25%. Swiss real estate promises a return of approximately 3.3% per year with a relatively low long-run volatility.

For a 20-year horizon, the expected return on liabilities is equal to -2.3% in our benchmark case ($\psi = 1$ and $\pi = 1\%$), meaning that the liabilities of the pension funds are expected to decrease due to the projected increase in interest rates. In a growing plan, the expected return on liabilities will be even lower. In contrast, with a premium of 2%, the expected return would be positive (for $\psi = 1$ and 0.8), although at a relatively low level. Investing in Swiss bonds would just match the expected return objective. It is worth emphasizing that the optimal asset allocation will not be fundamentally affected by the level of expected return on liabilities. What will really matter for long-term investment decisions are the expected returns on assets and their hedging properties, i.e., the correlation properties between the return on liabilities and the return on the various asset classes.

We also note that the volatilities of the return on liabilities are relatively high, between 8.5% and 19.4%, which would classify liabilities as risky according to the levels usually observed for assets. The main reason for this relatively high risk is again that liabilities are more sensitive to changes in interest rates in the context of long duration. Therefore,

the uncertainty about future interest rates induces a higher volatility of liabilities. For instance, for $\pi = 1\%$, increasing the replacement rate from 0.8 to 1.2 implies an increase in duration from 50 to 92 years and an increase in volatility from 10% to 19.4%. A higher discount premium would render liabilities less risky, as they would be less affected by changes in interest rates.

3.3 Hedging Properties

We now investigate the hedging properties of the various asset classes. We decompose hedging properties into two components: inflation hedging and real-liabilities hedging properties. Inflation hedging is a fundamental property of asset classes, as it is independent from the liabilities of a given pension fund. In contrast, real-liabilities hedging will be pension-fund specific. As we consider a representative pension fund, we are for the most part concerned by the hedging properties against real-liabilities risk as measured in our different scenarios.

Table 3 presents the correlations between nominal return and inflation on the one hand and between real return on assets and real return on liabilities on the other hand. As already emphasized by Campbell and Viceira (2005) and Hoevenaars et al. (2008), a short-term bond is the best inflation hedge at the long horizon. As the main objective of monetary policy is to maintain inflation close to the target level, both series are intrinsically related. For Switzerland, the correlation between the two series is as high as 80%. In contrast, long-term bonds only provide a weak hedge against inflation, with a low 26% correlation. Swiss stocks prove out to be an even worse hedge, with a correlation of only 12%. This limited hedging capability was already mentioned by Fama and Schwert (1977) and Campbell and Shiller (1988): High inflation increases interest rates, which lowers stock prices, but it also tends to increase dividends, leading to higher stock prices. Both effects partly compensate each other and the net effect is small.

The second-best hedges against inflation are commodities, with a correlation of 65%. This property is expected, given that commodities prices tend to anticipate future changes

in consumption prices (Gorton and Rouwenhorst, 2006). In contrast, Swiss real estate does not provide a good hedge against inflation, probably because house prices in Switzerland have grown relatively fast over the last decade, while otherwise inflationary pressures were under control. We also observe that all of our measures of return on liabilities are negatively correlated to inflation, in the range $[-26.6\%; -16.1\%]$. This suggests that the main effect of inflation is through the discount rate. Cash flows are also affected but in a more ambiguous manner, because in the long run, inflation increases both contributions and benefits.

The table also displays the hedging properties of the asset classes for real-liabilities risk. Although the ranking of assets in terms of hedging capability is not altered by changing the replacement rate and the premium, we observe that the correlations change significantly from one case to the other. The best hedges are Swiss long-term bonds, with a correlation ranging from 50% (for $\psi = 1.2$ and $\pi = 1\%$) to 77% (for $\psi = 0.8$ and $\pi = 2\%$). This observation demonstrates that, although the correlation of bonds and liabilities is strong, the relation is far from being perfect and reveals the importance of explicitly modeling the liabilities of a pension fund. There are in fact two main reasons for this result: First, the 10-year government bond is a poor proxy for liabilities because the liabilities duration is much longer than the duration of this bond. Second, there are other sources of risk included in the return on liabilities, which are not captured by long-term bond returns, such as the dynamics of the pension fund population and the dynamics of salaries. Intuitively, one may have anticipated those findings. It is only a full model similar to the one considered here that allows us to quantify the actual amounts. The next-best hedges are European and US bonds, with rather large correlations (all above 45%).

Equities have intermediate hedging properties. In this class, US equities are better hedges than European and Swiss equities. For low replacement rate and high premium (low duration), the correlations with real liabilities are the highest (36% for US equities and 24% for Swiss equities). For a high replacement rate and low premium (long du-

ration), the correlations are relatively low (30% for US equities and only 8% for Swiss equities).

Regarding the remaining asset classes, we note that Swiss real estate is a relatively good hedge for a low replacement rate and low premium (correlation of 37%) but is much less correlated with liabilities for a high replacement rate (18%). Cash and commodities have the worst hedging properties in our asset universe (with correlations in the range $[-55\%; -35\%]$ and $[-30\%; -16\%]$, respectively).²⁰

In sum, a high replacement rate and low premium imply a higher liabilities duration and, in general, lower correlation with all asset classes. This result suggests that it will be more difficult to find good hedges against real liabilities risk in case of longer duration. The main reason is the lack of bonds with very long maturities.

4 Strategic ALM

In this section, we analyze the implementation of assets-liabilities strategies by pension funds in a low interest-rate environment. We measure the cost of several suboptimal strategies. In particular, we investigate the case of assets-only allocation or positivity restrictions. All the allocations we discuss in this section are based on real returns over a 20-year horizon.²¹

Equations (18) and (19) give the optimal unrestricted portfolio weights for an assets-liabilities and an assets-only allocation, respectively. The corresponding liabilities-hedging

²⁰Having negative correlation with liabilities does not necessarily exclude these assets from the optimal portfolio. It simply suggests that they will not be part of the liabilities-hedging portfolio, if negative weights are precluded. However, these assets could be part of the performance-seeking portfolio, provided their expected returns are sufficiently large or their correlations with other assets provide sufficient diversification.

²¹As can be seen in the assets-liabilities optimization program (17), using nominal or real returns will not affect the optimal portfolio significantly, as the criterion is based on the expected return on the asset portfolio in excess of the return on liabilities. The only difference would derive from the definition of the covariance matrix for nominal versus real returns. For this reason, to save space, we only report our results in real terms, so that the optimal portfolio is constructed based on asset returns in excess of expected inflation.

portfolio (LHP) and optimal global minimum-variance portfolio (GMVP) are defined by:

$$\begin{aligned}\alpha_{LHP,t}^{\{k\}} &= (\Sigma_{xx}^{\{k\}})^{-1} \sigma_{xL}^{\{k\}}, & \text{and} & & \alpha_{0,LHP,t}^{\{k\}} &= 1 - (\alpha_{LHP,t}^{\{k\}})' e, \\ \alpha_{GMVP,t}^{\{k\}} &= -(\Sigma_{xx}^{\{k\}})^{-1} \sigma_{x0}^{\{k\}}, & \text{and} & & \alpha_{0,GMVP,t}^{\{k\}} &= 1 - (\alpha_{GMVP,t}^{\{k\}})' e.\end{aligned}$$

These portfolios may be out of reach because of the negativity of some of the weights.

Table 4 indicates the optimal allocation and portfolio performance for the assets-only (AO) and assets-liabilities (AL) portfolios for our benchmark case ($\psi = 1$ and $\pi = 1\%$). We consider three different portfolios. The first portfolios are the global minimum variance portfolios (denoted by GMVP for assets-only and LHP for assets-liabilities). Differences between the GMVP and LHP are important because they are unrelated to expected returns, which are known to be difficult to forecast, and only consider hedging properties. We also consider mean-variance portfolios with two different levels of risk aversion, $\lambda = 50$ and 20 , as described by Equation (18) for assets-liabilities allocation and Equation (19) for assets-only allocation. As pension funds are expected to adopt rather safe strategies, we consider the case $\lambda = 50$ as our benchmark. It corresponds to an annualized volatility of the asset portfolio of approximately 3–4%, a level often adopted by insurance companies. The case $\lambda = 20$ corresponds to a volatility in the range 5–6%, which is often chosen by pension funds or endowment funds.

4.1 Unrestricted Optimal Portfolios

We start with unrestricted optimal portfolios (Panel A) and with assets-only portfolios. If we consider the unrestricted GMVP, it is essentially long in cash and Swiss bonds (53% and 36%, respectively) and short in US bonds (−18%). The large weight in cash is not surprising as cash has low volatility and provides a very good hedge against inflation. If we now consider the mean-variance portfolio with $\lambda = 50$ and 20 , there is a reallocation in favor of riskier assets. The weights of equities, US bonds, commodities, and real estate increase, whereas the weights of cash and Swiss and European bonds decrease. For

$\lambda = 20$, the optimal assets-only portfolio is short in cash and Swiss bonds and long in US bonds and equities, Swiss equities, and real estate.

We now turn to assets-liabilities portfolios, which take the correlation of assets with liabilities into account. The unrestricted LHP is very different from the unrestricted GMVP. The investor for the most part invests in Swiss, US, and European bonds (133%, 55%, and 44%, respectively) and short in cash and real estate (-129% and -24%). We clearly see the effect of the correlation matrix used for the GMVP and LHP and the role of hedging liabilities risk. On the one hand, the GMVP tries to minimize the absolute portfolio risk, by combining short-term and long-term bonds. On the other hand, the LHP tries to minimize the variance of the difference between the return on the asset portfolio and the return on liabilities. This is done by borrowing cash and investing the portfolio in all types of long-term bonds (for a total of 233% of initial wealth).

In the optimal mean-variance portfolio with $\lambda = 50$ and 20, the LHP is combined with a performance-seeking portfolio. It therefore reduces the weight devoted to bonds and increases the weight to equities. We note that the role played by US bonds is twofold: It contributes to the LHP, but its weight also increases when the pension fund put more emphasis on performance.

The table also reports the expected return and risk (μ_A, σ_A) of the asset portfolio and the expected return and risk (μ_S, σ_S) of the surplus. Several results are worth mentioning. First, in all cases, the optimal allocations are expected to generate a positive surplus return: close to 4% for the lowest risk portfolios and close to 10% for the riskier portfolios. Second, the risk of the surplus is rather high (above 10% for all portfolios). This reflects the high risk of the return on liabilities for a low premium. Third, in all cases, the assets-only portfolios generate lower expected return and higher risk for the surplus than the corresponding assets-liabilities portfolios.

To evaluate the gain for an investor from being allowed to switch from a suboptimal portfolio to the optimal assets-liabilities portfolio, we use the certainty equivalent. It is defined as the difference between the utility of the optimal assets-liabilities portfolio

$q_t^{\{k\}}(\alpha_{AL,t}^{\{k\}})$ and the utility of the suboptimal portfolio $q_t^{\{k\}}(\alpha_{sub,t}^{\{k\}})$:

$$\begin{aligned} CE_t^{\{k\}} &= q_t^{\{k\}}(\alpha_{AL,t}^{\{k\}}) - q_t^{\{k\}}(\alpha_{sub,t}^{\{k\}}) \\ &= (\alpha_{AL,t}^{\{k\}} - \alpha_{sub,t}^{\{k\}})'(\mu_{t:t+k}^{\{k\}} + \frac{1}{2}\sigma_x^{\{k\}2}) - \frac{1-\lambda}{2}(\alpha_{AL,t}^{\{k\}} - \alpha_{sub,t}^{\{k\}})'\sigma_L^{\{k\}2} \\ &\quad - \frac{1-\lambda}{2}(\alpha_{AL,t}^{\{k\}'}\Sigma_{xx}^{\{k\}}\alpha_{AL,t}^{\{k\}} - \alpha_{sub,t}^{\{k\}'}\Sigma_{xx}^{\{k\}}\alpha_{sub,t}^{\{k\}}). \end{aligned}$$

As the formula indicates, the certainty equivalent allows us to convert differences in portfolio variances in units of the portfolio expected returns. In the present context, we consider the premium that the investor is willing to pay to switch from the assets-only portfolio to the assets-liabilities portfolio. There is a huge gain for pension funds to invest in the assets-liabilities portfolio: The premium is equal to 14% in the benchmark case $\lambda = 50$, which reflects the fact that mitigating risk matters greatly for pension funds. It decreases to 5.5% for lower risk aversion ($\lambda = 20$). For $\lambda = 50$, this premium is due to a combination of two effects: the decrease in surplus volatility (from 13.6% to 11.4%) and the increase in the surplus expected return (from 4.1% to 5%).

4.2 Optimal Portfolio with Positivity Restrictions

We have discussed the optimal portfolio allocation when the pension fund is allowed to sell some asset classes short. This is not the case in practice. We therefore consider now the case with positivity restrictions. In this case, the optimal portfolios for the surplus maximization and the asset-only cases are obtained by solving the following programs:

$$\min_{\{\alpha\}} V[r_{F,t:t+k}^{\{k\}}] \quad \text{and} \quad \min_{\{\alpha\}} V[r_{A,t:t+k}^{\{k\}}],$$

where positivity restrictions are imposed by setting $0 \leq \alpha_i \leq 1$, $i = 1, \dots, N$, and $1 - \alpha'e \geq 0$ during the numerical optimization. Positivity restrictions will clearly play a key role in the assets-liabilities allocation, as the LHP is naturally constructed on shorting cash.

If we start with the assets-only portfolio, we note that there are only limited differences with the unrestricted portfolios discussed previously (Panel B). In the GMVP, the portfolio is for the most part invested in cash and Swiss bonds (43% and 31%, respectively) with zero weight in US bonds. In the mean-variance portfolio with $\lambda = 50$, we have essentially the same portfolio as previously. In fact, the premium the investor would be willing to pay to switch from the restricted assets-only portfolio to the unrestricted one is negligible.

In contrast, the possibility for pension funds to borrow cash for investing in the liabilities-hedging portfolio plays an important role. The reason is that the optimal, unrestricted, LHP is short in cash. When borrowing cash is not allowed, the optimal LHP has a larger weight in Swiss and European bonds and US equities (44%, 47%, and 9%, respectively). This evidence suggests that allowing pension funds to hedge their liabilities through borrowing cash and investing in a diversified bond portfolio would help enhance the global portfolio performance. We observe similar patterns in the mean-variance portfolios with $\lambda = 50$ and 20. For all assets-liabilities portfolios, imposing positivity restrictions results in a smaller expected return and a higher risk for the surplus.

The premium the investor would be willing to pay to be allowed to invest in the unrestricted mean-variance portfolio is 7.7% for $\lambda = 50$ and 4.2% for $\lambda = 20$. Such large premiums are due to the benefits for the investor to borrow cash for investing in long-term bonds. We also observe that, when cash borrowing is not allowed, the premium to switch from assets-only to assets-liabilities allocation is decreased: from 14% to 6.5% for $\lambda = 50$ and from 5.5% to 2.1% for $\lambda = 20$. This result is explained by the fact that the pension fund has less degree of freedom when not allowed to borrow cash for hedging its liabilities.

It should be noticed that, even with positivity restrictions, the gain of assets-liabilities management is quite substantial compared to standard assets-only allocation. For $\lambda = 50$, the surplus expected return is slightly decreased (from 4.1% to 3.5%), and the surplus volatility is decreased (from 13.6% to 12.5%). As we can see from this decomposition, the

main implication of investing in the assets-only portfolio is that it does not put sufficient emphasis on hedging liabilities risk and therefore does not invest sufficiently in bonds.

4.3 Alternative Specifications

In **Table 5**, we investigate the effect of changing the replacement rate on the optimal allocation. The first three columns of the table are devoted to the case $\psi = 0.8$ (smaller duration), and the last three columns are devoted to the case $\psi = 1.2$ (longer duration).

We start again with unrestricted allocations. We note that the main effect of the replacement rate is to change the size of the investment in cash and bonds, which we could interpret as a change in leverage. With $\psi = 0.8$, the weights of Swiss, US, and European bonds are 112%, 43%, and 41%, respectively, while the pension fund should borrow 98% of cash. With $\psi = 1.2$, the weights increase to 186%, 67%, and 31%, respectively, while the pension fund should borrow 162% of cash. In doing so, the investor tries to increase the duration of the LHP. As the optimal allocation of a pension fund with a growing population is very short in cash, it is not surprising that the cost of the assets-only allocation is higher than in the case of a mature plan. It is clearly apparent that the lack of longer-term bonds is a strong limitation in the construction of an effective LHP.

If we now consider the portfolios with positivity restrictions, we observe that the optimal weights barely change compared with the case $\psi = 1$. The reason is again that, as cash borrowing is not allowed, the pension fund cannot leverage the portfolio to increase its duration. The table also reveals that for $\psi = 1.2$ the search for long-duration assets points in favor of US equities. For the LHP, their weight is as high as 20%, whereas it was only 9% for $\psi = 1$. In the case of a growing plan ($\psi = 1.2$), the premium to pay for switching to the unrestricted portfolio is higher: for $\lambda = 50$, the cost of assets-only allocation is 22% per year and the cost of positive weights restrictions is 13%. This is approximately twice the costs that we found for $\psi = 1$. Conversely, for a mature plan, the opportunity costs of the assets-only allocation and positive weights restrictions are lower, although they remain substantial.

Finally, we consider in **Table 6** the optimal assets-liabilities allocation in the case of a higher risk premium, $\pi = 2\%$. As discussed before, this case corresponds to liabilities with higher expected returns and lower volatilities. In all cases, the unrestricted LHP is long in bonds and short in cash (for instance, 205% in bonds and -111% in cash for $\psi = 1$). However, the short position in cash is clearly reduced compared to the low premium case (by approximately 20 to 30%). This can be explained by the fact that the duration is smaller with higher premium and therefore, increasing the asset portfolio duration by shorting cash is less necessary.

In LHP, we observe that the expected return on surplus is rather low (below 1% for $\psi = 0.8$ with or without weights restrictions), with a relatively high risk (a minimum of 5%). This suggests that there may not be sufficient room for maneuver to generate positive surplus over the long term.

5 Conclusion

The literature on how to manage optimal defined-contribution plans is relatively scarce. This is regrettable, because at the time this paper is written, the majority of pension plans have moved to defined contributions. The reason for this is the increase in life expectancy, which puts pressure on the sponsoring companies of many pension funds facing difficulties in honoring past promises. This problem is exacerbated because of significant losses during the 2000/03 recession and the subprime crisis, which is associated with the current phase of low interest rates.

In this paper, we demonstrate how to perform an ALM study from a financial prospective for defined-contribution plans. Given international interest in the way the Swiss pension fund system operates, we adopt the Swiss framework in our investigation. This framework tells us how to calibrate the model for our empirical investigation.

To perform this ALM study, we start with an economic model that drives both the asset and liability sides of the pension-fund balance sheet. The economic model provides

forecasts on the macroeconomic and financial factors that drive the expected returns and risks of assets and liabilities. On the asset side, we generate scenarios for the USA, the Euro Area, and Switzerland, which allow us to forecast the expected returns and risks of several asset classes diversified internationally. On the liability side, we adopt an economic approach in the way liabilities are constructed, so that future financial cash flows are discounted using term-structure forecasts. This approach allows us to estimate the correlation between assets and liabilities, which is the key ingredient in constructing a liabilities hedging portfolio.

We calibrate the economy using quarterly data between 1985:Q1 and 2013:Q2. Using a certainty equivalent approach, we demonstrate that failing to construct a liabilities-hedging portfolio results in a cost between 5% and 15% per year. The main reason for such a large cost is that the optimal assets-only portfolio is typically long in cash, whereas hedging liabilities requires the pension fund to be short in cash. It follows that imposing positivity restrictions in the construction of the portfolio also results in a large cost, between 4% and 8% per year. This estimate suggests that allowing pension funds to hedge their liabilities through borrowing cash and investing in a diversified bond portfolio would help enhancing the global portfolio return. One caveat of this conclusion regarding the optimal investment strategy is that government bonds with long maturities (longer than 10 years) are scarcely available in Switzerland. As of 2013, bonds issued by the Swiss Confederation with a remaining maturity above 20 years account for less than 2% of GDP.

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Appendix

Appendix 1: Swiss Social Security System

The Swiss pension system is based on three pillars: (1) a state pension (first pillar); (2) a fully-funded occupational pension fund (second pillar); and (3) a private pension (third pillar).²²

The state pension system (AHV-IV / AVS-AI, i.e., old-age, survivors, and disability insurance) is a pay-as-you-go system, introduced in 1948. It is designed to provide a basic subsistence level to all retired residents in Switzerland. Current workers have to finance the pensions of currently retired people through a tax on labor income.

As the first pillar only covers basic needs, a complementary insurance is required to reach a normal living standard. The occupational pension fund system is designed for complementing the first pillar. To account for the unfavorable demographic evolution (increase in life expectancy and decrease in birth rate), which is responsible for the rise in the old-age dependency ratio, it became mandatory in 1985 (BVG / LPP, i.e., occupational retirement, survivors and disability pension plans). It covers all salaried employees with a minimum annual income. Together, the first two insurance systems should ensure that retired people maintain their former standard of living, i.e., they should jointly provide approximately 60% of the last salary.

The second pillar works as follows: Contributions paid by employees are credited on a personal account. The accumulated capital generates an annual interest, which should be at least equal to the minimum interest rate defined by the federal government (the minimum BVG / LPP rate). All pension funds have to cover the mandatory occupational benefit insurance. However, the majority of them also offer extra-benefit insurance. Pension benefits are strictly proportional to the accumulated savings. At retirement, the annual pension is defined as the accumulated savings times a conversion factor. The legal

²²Details on the Swiss pension system can be found in Bütler and Ruesch (2007) and Barenco (2012).

conversion factor is fixed by the federal government. Pension funds can use a different factor, provided the annual pension is at least equal to the legal minimum pension.

The majority of occupational pension plans were initially defined as defined-benefit plans. However, to ensure the portability of retirement savings, most plans have now switched to defined-contribution plans.

The third pillar is voluntary privately-financed saving. It is designed to cover the income gap during old age. A first pillar 3a is supported by the federal government through tax exemption and is typically invested in bank and insurance products. Pillar 3b is free private saving, so that it includes all types of savings.

Appendix 2: Forecasting the Term Structure of Interest Rates

A common approach to forecasting the term structure of interest rates is based on Nelson and Siegel (1987). Diebold and Li (2006) are the first to use the Nelson-Siegel approach for forecasting. Their idea is to estimate the three Nelson-Siegel β parameters, which describe the level, slope, and curvature of the term structure. Then, they use a VAR model to predict the future β parameters. Forecasts of β parameters are in turn used to forecast the term structure. We experimented with this approach and found that it does not perform well, even within sample. The main reason is that the β parameters are extremely persistent and their changes are difficult to predict with past values. As a consequence, the approach fails at predicting changes in the term-structure patterns.

Our approach is also related to Nelson and Siegel (1987), although in a different way. As we have a model for predicting the main macroeconomic and financial factors, we use our model to obtain reference points along the term structure. In fact, in our VECM approach, we describe the dynamics of the three-month T-bills rate and the 2-year and 10-year government bond rates. Using these points, we then infer the entire term structure.

Formally, let us consider the forecast of the term structure for the next quarter, from t to $t + 1$. The macrofinance model gives us predictions for the three bond rates, which we

denote by $\hat{r}_{t+1}^{(3m)}$, $\hat{y}_{t+1}^{(2)}$, and $\hat{y}_{t+1}^{(10)}$. The specification proposed by Nelson and Siegel (1987) is that the (zero-coupon) yield of maturity m should be driven by the relation:

$$NS(\beta, m) = \beta_1 + \beta_2 \frac{1 - e^{-m/\tau}}{m/\tau} + \beta_3 \left(\frac{1 - e^{-m/\tau}}{m/\tau} - e^{-m/\tau} \right). \quad (23)$$

Limit maturities indicate that the overnight rate should be equal to $NS(\beta, 0) = \beta_1 + \beta_2$, whereas the rate of a perpetuity should be equal to $NS(\beta, \infty) = \beta_1$. These limit cases allow us to interpret β_1 and β_2 as the level and the slope of the term structure. Parameter τ corresponds to the maximum of the curvature of the term structure. Diebold and Li (2006) and Diebold, Rudebusch, and Aruoba (2006) have concluded that τ should be set such that the maximum of the curvature is reached for a maturity of approximately 2 years. In this context, β_3 can be interpreted as the curvature of the term structure.

Our approach consists in using our interest rate forecasts to back out the three β parameters: $(\hat{r}_{t+1}^{(3m)}, \hat{y}_{t+1}^{(2)}, \hat{y}_{t+1}^{(10)}) \rightarrow (\hat{\beta}_1, \hat{\beta}_2, \hat{\beta}_3)$ from Equation (23). The β parameters then result from the resolution of a 3-by-3 linear system for which explicit solutions are straightforward to obtain. Then, given these parameters $\hat{\beta} = (\hat{\beta}_1, \hat{\beta}_2, \hat{\beta}_3)$, we can reconstruct the complete term structure using $NS(\hat{\beta}, m)$ for annual maturities.

The main advantage of our approach is that it ensures that the forecasted term structure passes through the forecasts produced by the macrofinance model for the three-month T-bills rate, as well as the 2-year, and 10-year government bond rates. The forecasted term structure is therefore consistent with the macrofinance model.

Table 1: Duration of Liabilities (in years)

Premium	Replacement rate		
	$\psi = 0.8$	$\psi = 1.0$	$\psi = 1.2$
$\pi = 1\%$	50.9	70.1	92.4
$\pi = 2\%$	41.6	52.9	66.8

Note: The table reports the duration of liabilities, for various values of the replacement rate ($\psi = 0.8, 1, \text{ and } 1.2$) and the risk premium ($\pi = 1\%$ and 2%). Duration of liabilities at date t is computed as:

$$D_t = \frac{1}{L_t} E_t \sum_{i=1}^{\infty} i \frac{CF_{t+i}}{(1 + R_t^{(i)})^i} \quad \text{where} \quad L_t = E_t \sum_{i=1}^{\infty} \frac{CF_{t+i}}{(1 + R_t^{(i)})^i}.$$

Table 2: Real Expected Return and Volatility of Assets and Liabilities

	Expected return	Volatility
Assets		
Cash	0.34	3.05
US bond	1.31	4.19
US equity	7.87	9.43
E.A. bond	0.67	5.06
E.A. equity	6.50	11.08
Swiss bond	0.49	4.23
Swiss equity	9.35	11.70
Commodities	6.61	23.12
Swiss real estate	3.31	6.11
Liabilities		
$\psi = 1.0; \pi = 1\%$	-2.28	14.47
$\psi = 1.0; \pi = 2\%$	0.19	10.75
$\psi = 0.8; \pi = 1\%$	-0.93	10.20
$\psi = 0.8; \pi = 2\%$	0.46	8.46
$\psi = 1.2; \pi = 1\%$	-4.27	19.38
$\psi = 1.2; \pi = 2\%$	-0.48	13.61

Note: The table reports the expected return and volatility of the asset classes and liabilities we consider in the paper. The measures correspond to annualized log-returns over 20 years in real terms.

Table 3: Hedging Properties of Asset Classes

(in %)	Correlation with inflation	Correlation with liabilities					
		$\psi = 1.0$ $\pi = 1\%$	$\psi = 1.0$ $\pi = 2\%$	$\psi = 0.8$ $\pi = 1\%$	$\psi = 0.8$ $\pi = 2\%$	$\psi = 1.2$ $\pi = 1\%$	$\psi = 1.2$ $\pi = 2\%$
Assets	(nominal)	(real)	(real)	(real)	(real)	(real)	(real)
Cash	79.5	-40.2	-51.9	-46.6	-55.1	-35.5	-40.5
US bond	44.4	50.5	58.5	60.4	60.0	46.8	52.7
US equity	28.8	29.9	34.5	34.2	36.0	29.6	33.6
E.A. bond	25.3	54.4	61.5	67.2	69.0	45.6	55.9
E.A. equity	10.7	23.2	22.9	27.4	27.9	15.9	21.8
Swiss bond	26.2	58.3	70.3	71.9	77.1	50.3	58.1
Swiss equity	11.8	19.5	21.8	22.6	24.0	7.7	16.2
Commodities	64.8	-23.4	-22.1	-29.2	-28.4	-16.0	-16.4
Swiss real estate	25.4	27.9	37.0	37.3	36.3	18.2	29.4
		(nom.)	(nom.)	(nom.)	(nom.)	(nom.)	(nom.)
Inflation	–	-18.5	-24.6	-26.2	-26.6	-16.1	-19.8

Note: The first column of the table reports the correlation between nominal returns on assets and liabilities and inflation. Subsequent columns report the correlation between real return on assets and real return on liabilities. The last row reports the correlation between inflation and the various definitions of (nominal) liabilities. Inflation is measured by the growth rate of the consumer price index. We consider a 20-year horizon.

Table 4: Optimal Assets-Only and Assets-Liabilities Portfolios ($\psi = 1$ and $\pi = 1\%$)

(in %)	Assets-Only			Assets-Liabilities		
	GMVP	$\lambda = 50$	$\lambda = 20$	LHP	$\lambda = 50$	$\lambda = 20$
Panel A: No weight restriction						
Cash	53.0	19.5	-30.7	-129.0	-158.8	-203.5
US bond	-17.9	10.3	52.6	55.3	82.1	122.2
US equity	11.1	18.3	29.0	9.7	16.9	27.7
E.A. bond	14.9	6.5	-6.0	44.2	35.3	21.9
E.A. equity	-2.0	-1.1	0.2	3.0	3.8	4.9
Swiss bond	36.4	14.3	-18.8	133.0	108.9	72.9
Swiss equity	-3.3	9.8	29.3	8.3	21.1	40.3
Commodities	2.7	7.2	14.0	-0.2	4.4	11.3
Swiss real estate	5.0	15.2	30.3	-24.3	-13.6	2.5
Expected return and risk:						
μ_A	1.1	3.9	8.0	2.0	4.8	8.8
σ_A	2.1	3.2	6.3	7.9	8.4	10.2
μ_S	3.4	6.2	10.2	4.3	7.0	11.1
σ_S	13.6	13.6	14.4	11.2	11.4	12.7
Cost of Assets-Only allocation	–	–	–	29.2	14.3	5.5
Cost of positivity restrictions	0.2	0.0	0.1	13.2	7.7	4.2
Panel B: Positivity restrictions						
Cash	43.4	18.9	0.0	0.0	0.0	0.0
US bond	0.0	11.1	12.0	0.0	0.0	6.3
US equity	6.5	17.5	33.5	9.0	23.9	44.8
E.A. bond	10.1	5.9	0.0	46.7	38.9	23.3
E.A. equity	0.0	0.0	0.0	0.0	0.0	0.0
Swiss bond	31.3	14.5	0.0	44.3	27.6	0.0
Swiss equity	0.0	9.4	25.8	0.0	9.5	25.6
Commodities	3.7	7.3	11.6	0.0	0.0	0.0
Swiss real estate	5.0	15.4	17.2	0.0	0.0	0.0
Expected return and risk:						
μ_A	1.4	3.9	7.0	1.3	3.3	6.3
σ_A	2.2	3.2	5.6	3.4	3.9	6.0
μ_S	3.7	6.2	9.3	3.6	5.6	8.6
σ_S	13.5	13.6	14.3	12.3	12.5	13.3
Cost of Assets-Only allocation	–	–	–	14.9	6.5	2.1

Note: The table reports the optimal portfolio weights corresponding to the assets-only portfolio and assets-liabilities portfolio, for the global minimum variance portfolio (GMVP and LHP, respectively) and for mean-variance portfolios with risk aversion $\lambda = 50$ and 20 . Log-returns are in real terms. (μ_A, σ_A) and (μ_S, σ_S) denote the expected return and volatility of the assets portfolio and the surplus, respectively. We consider a 20-year horizon.

Table 5: Optimal Assets-Liabilities Portfolios ($\psi = 0.8$ and 1.2)

(in %)	$(\psi = 0.8; \pi = 1\%)$			$(\psi = 1.2; \pi = 1\%)$		
	LHP	$\lambda = 50$	$\lambda = 20$	LHP	$\lambda = 50$	$\lambda = 20$
Panel A: No weight restriction						
Cash	-97.6	-128.1	-173.8	-161.8	-191.0	-234.7
US bond	43.2	70.2	110.7	66.9	93.5	133.3
US equity	4.5	11.8	22.7	34.9	41.5	51.5
E.A. bond	41.1	32.2	18.9	31.4	22.7	9.7
E.A. equity	4.0	4.7	5.8	-2.6	-1.7	-0.4
Swiss bond	112.0	88.4	53.0	185.8	160.7	123.1
Swiss equity	4.2	17.1	36.4	-5.9	7.2	26.8
Commodities	-0.3	4.3	11.2	0.3	4.9	11.7
Swiss real estate	-11.0	-0.5	15.1	-49.0	-37.8	-21.0
Expected return and risk:						
μ_A	1.6	4.4	8.4	1.5	4.3	8.4
σ_A	6.7	7.1	9.1	9.7	10.0	11.3
μ_S	2.5	5.3	9.3	5.8	8.6	12.6
σ_S	6.5	6.9	8.8	15.9	16.1	17.0
Cost of Assets-Only allocation	20.0	9.8	3.8	44.8	22.0	8.5
Cost of positivity restrictions	7.2	4.6	2.9	26.0	13.1	6.3
Panel B: Positivity restrictions						
Cash	0.0	0.0	0.0	0.0	0.0	0.0
US bond	0.0	0.0	10.8	0.0	0.3	6.3
US equity	5.3	20.3	40.8	19.5	38.6	64.6
E.A. bond	44.8	37.0	21.4	34.3	24.5	8.6
E.A. equity	0.0	0.0	0.0	0.0	0.0	0.0
Swiss bond	50.0	33.2	1.0	46.2	36.6	15.8
Swiss equity	0.0	9.5	26.0	0.0	0.0	4.8
Commodities	0.0	0.0	0.0	0.0	0.0	0.0
Swiss real estate	0.0	0.0	0.0	0.0	0.0	0.0
Expected return and risk:						
μ_A	1.0	3.0	6.1	2.1	3.5	5.9
σ_A	3.4	3.8	5.8	3.4	4.1	6.0
μ_S	1.9	4.0	7.1	6.3	7.8	10.1
σ_S	7.5	7.8	9.1	17.5	17.6	18.1
Cost of Assets-Only allocation	12.0	5.2	1.6	18.1	8.8	3.1

Note: The table reports the optimal portfolio weights corresponding to the assets-liabilities portfolio, for the global minimum variance portfolio (LHP) and for mean-variance portfolios with risk aversion $\lambda = 50$ and 20 . We consider the cases $(\psi = 0.8; \pi = 1\%)$ and $(\psi = 1.2; \pi = 1\%)$, respectively. Log-returns are in real terms. (μ_A, σ_A) and (μ_S, σ_S) denote the expected return and volatility of the assets portfolio and the surplus, respectively. We consider a 20-year horizon.

Table 6: Optimal Assets-Liabilities Portfolios ($\pi = 2\%$)

(in %)	$\psi = 1.0$		$\psi = 0.8$		$\psi = 1.2$	
	LHP	$\lambda = 50$	LHP	$\lambda = 50$	LHP	$\lambda = 50$
Panel A: No weight restriction						
Cash	-111.0	-141.2	-67.5	-98.5	-132.6	-162.4
US bond	50.4	77.3	20.1	47.5	61.2	87.9
US equity	8.3	15.5	8.2	15.4	13.1	20.2
E.A. bond	25.7	17.1	34.1	25.3	49.9	40.8
E.A. equity	-1.1	-0.3	0.7	1.5	1.6	2.4
Swiss bond	129.5	105.6	119.8	96.1	115.1	91.4
Swiss equity	6.8	19.7	5.2	18.1	3.6	16.5
Commodities	2.3	6.9	0.1	4.7	4.1	8.6
Swiss real estate	-11.0	-0.5	-20.7	-10.0	-15.9	-5.4
Expected return and risk:						
μ_A	2.1	4.8	1.3	4.1	2.5	5.2
σ_A	6.7	7.3	5.7	6.2	7.4	8.1
μ_S	1.9	4.6	0.9	3.6	2.9	5.7
σ_S	7.2	7.6	4.9	5.5	10.4	10.7
Cost of Assets-Only allocation	20.3	9.9	14.1	6.9	25.4	12.4
Cost of positivity restrictions	8.0	5.3	4.1	2.7	11.1	7.0
Panel B: Positivity restrictions						
Cash	0.0	0.0	0.0	0.0	0.0	0.0
US bond	0.0	0.0	0.0	0.0	0.0	0.0
US equity	8.7	24.2	5.8	20.7	14.1	31.7
E.A. bond	25.1	17.4	31.0	23.6	50.3	41.0
E.A. equity	0.0	0.0	0.0	0.0	0.0	0.0
Swiss bond	66.3	50.2	63.2	46.0	35.7	23.8
Swiss equity	0.0	8.2	0.0	9.8	0.0	3.5
Commodities	0.0	0.0	0.0	0.0	0.0	0.0
Swiss real estate	0.0	0.0	0.0	0.0	0.0	0.0
Expected return and risk:						
μ_A	1.2	3.2	1.0	3.0	1.7	3.3
σ_A	3.3	3.7	3.3	3.7	3.5	4.0
μ_S	1.0	3.0	0.6	2.6	2.2	3.8
σ_S	8.2	8.5	5.7	6.1	11.4	11.6
Cost of Assets-Only allocation	11.4	4.6	9.5	4.1	13.4	5.5

Note: The table reports the optimal portfolio weights corresponding to the assets-liabilities portfolio, for the LHP and for mean-variance portfolios with risk aversion $\lambda = 50$ and 20. Log-returns are in real terms. (μ_A, σ_A) and (μ_S, σ_S) denote the expected return and volatility of the assets portfolio and the surplus, respectively. We consider a 20-year horizon.

Figure 1: Structure of the Global Model

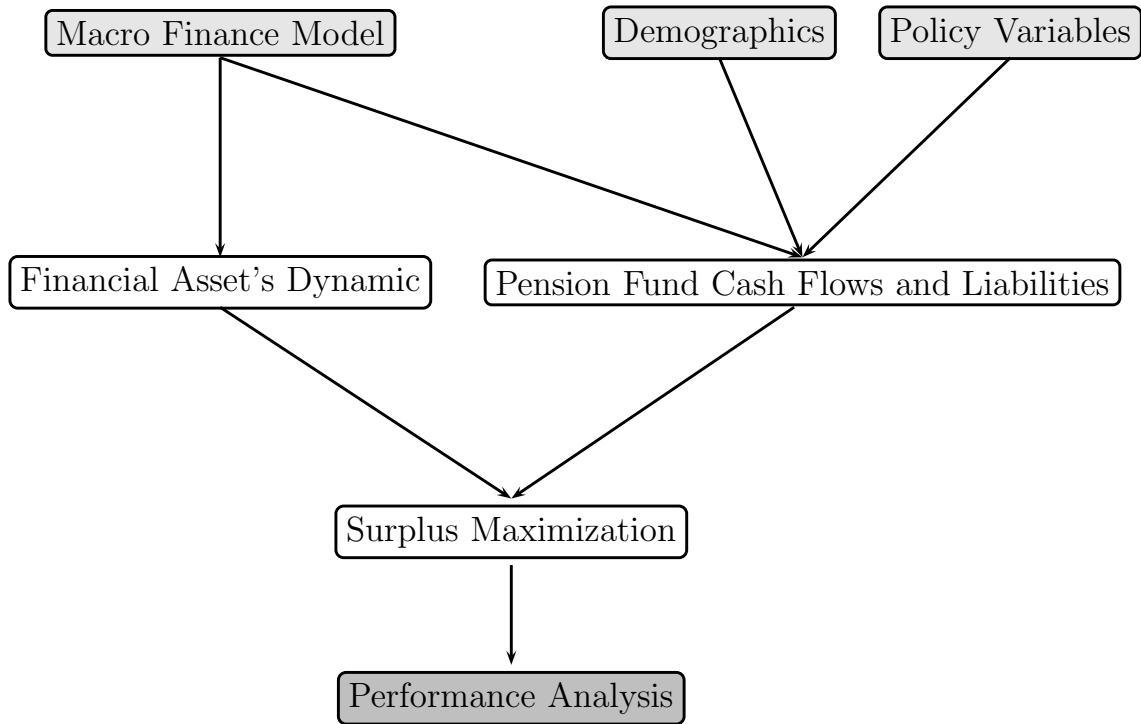


Figure 2: Structure of the Macro-finance Model

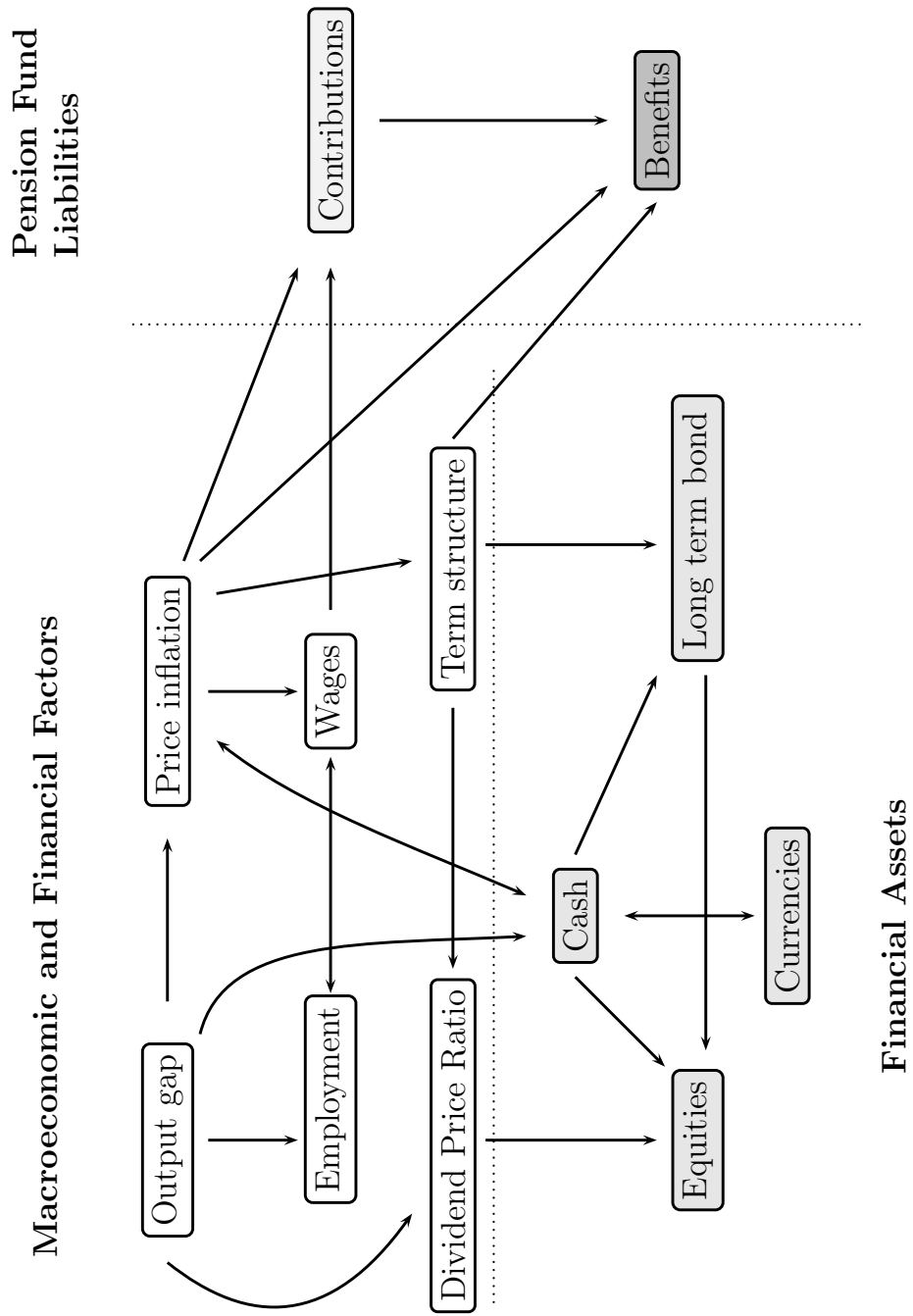
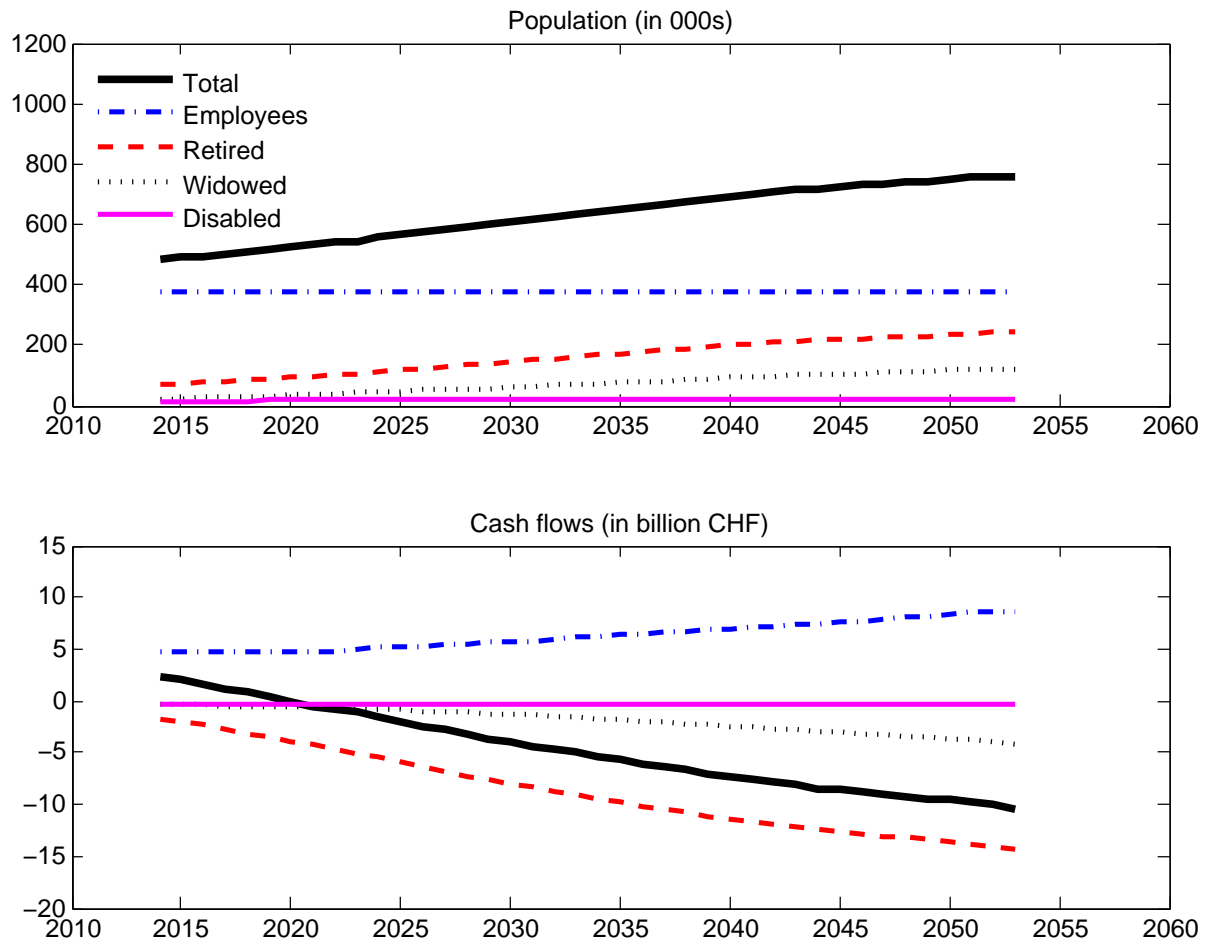
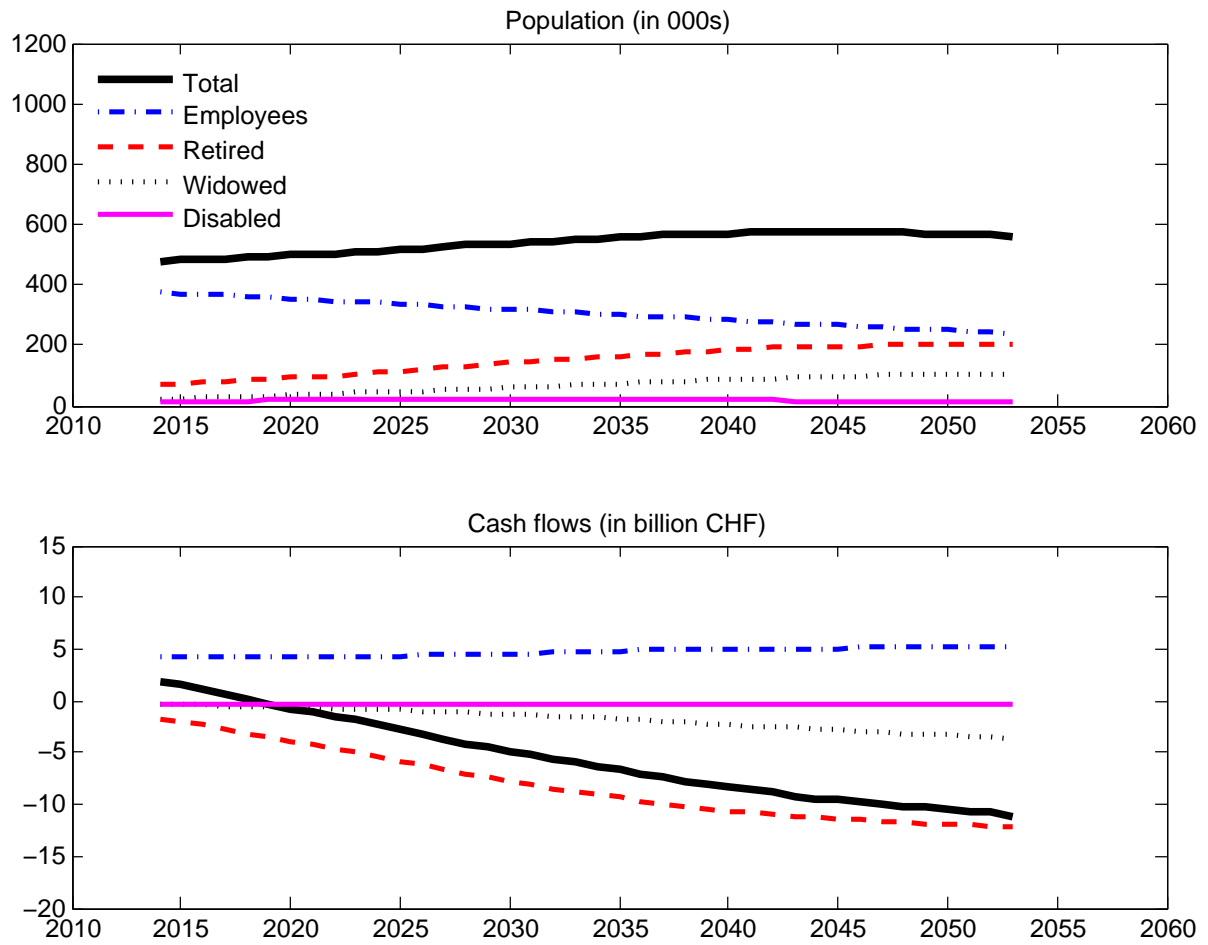


Figure 3: Population and Cash Flows of the Pension Fund ($\psi = 1$)



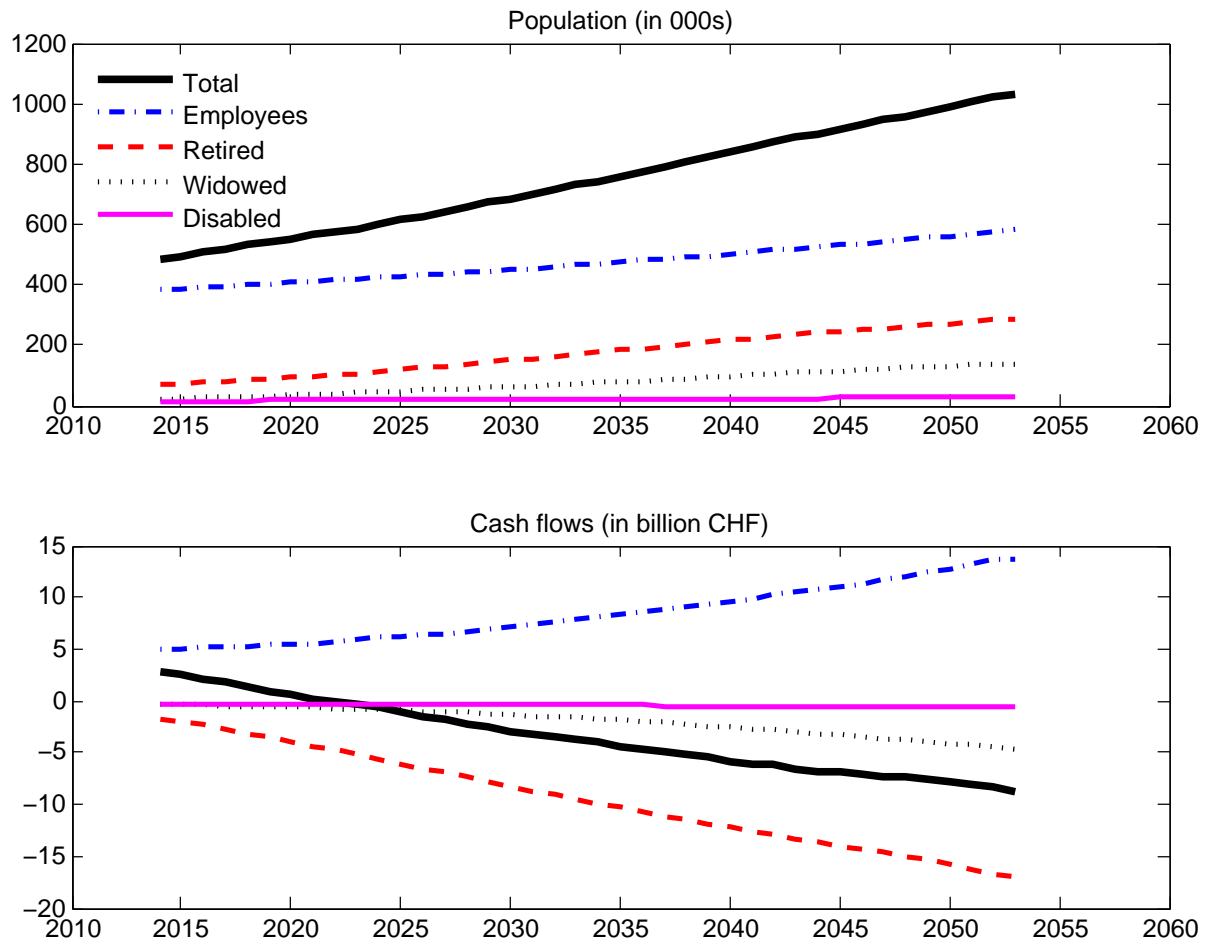
Note: This figure displays the expected evolution of insured population for the next 40 years starting from 2014 on. Population includes employees, retired, surviving spouses, and disabled.

Figure 4: Population and Cash Flows of the Pension Fund ($\psi = 0.8$)



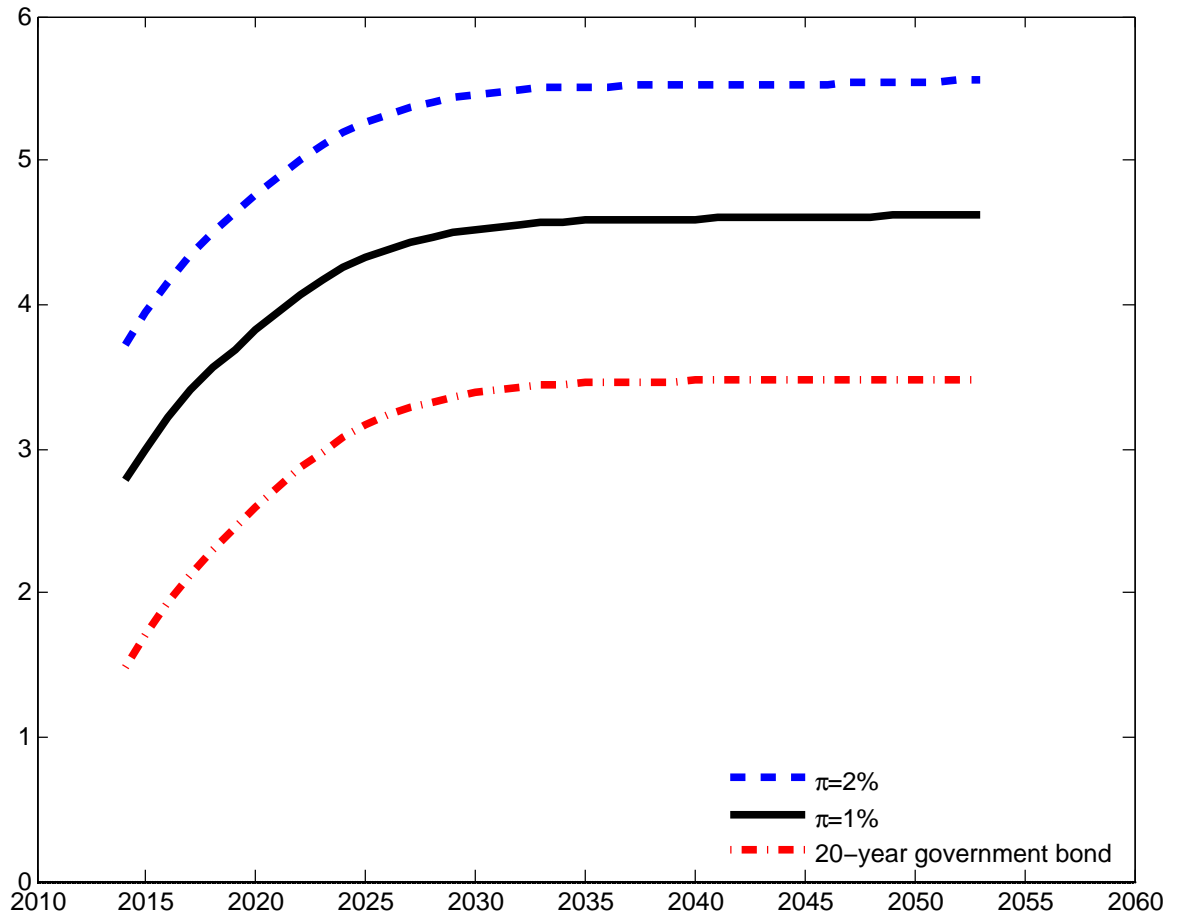
Note: This figure displays the expected evolution of insured population for the next 40 years starting from 2014 on. Population includes employees, retired, surviving spouses, and disabled.

Figure 5: Population and Cash Flows of the Pension Fund ($\psi = 1.2$)



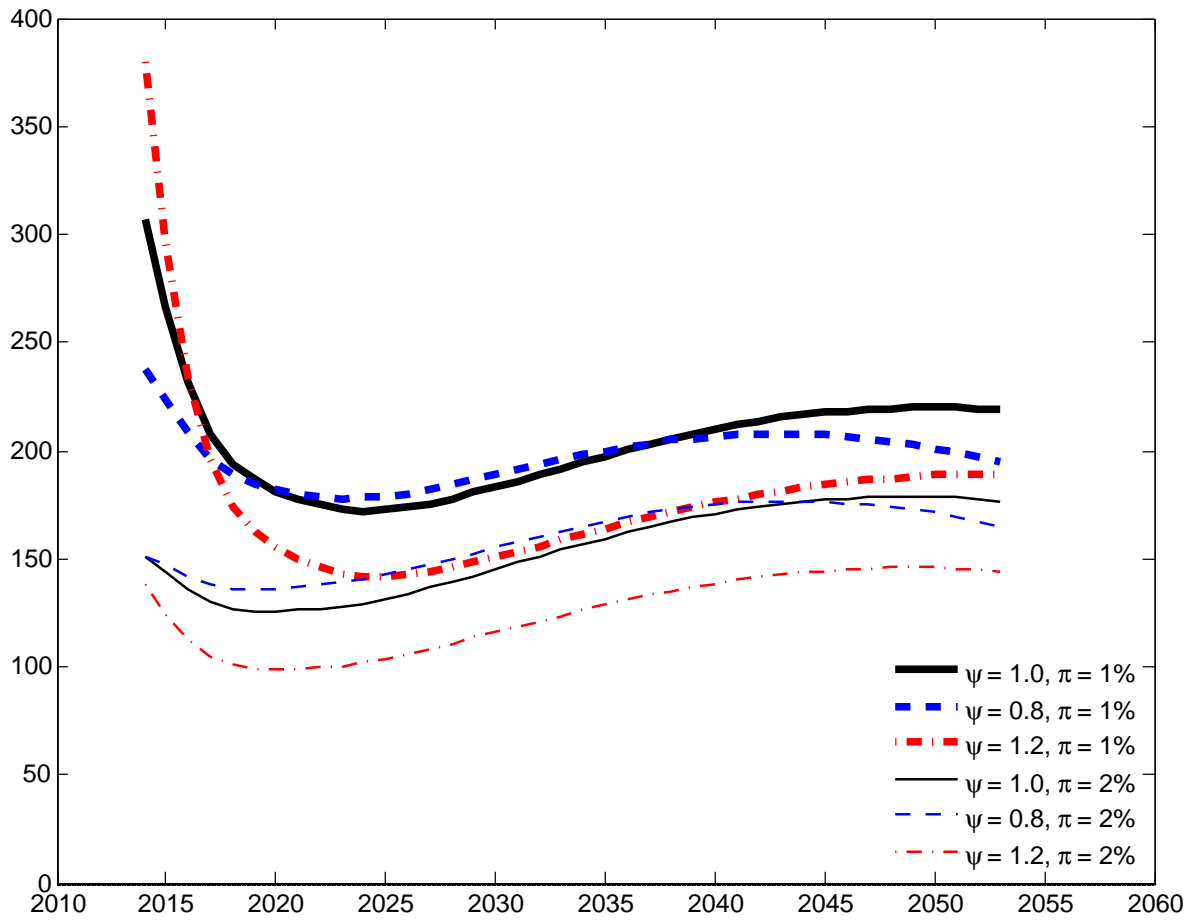
Note: This figure displays the expected evolution of insured population for the next 40 years starting from 2014 on. Population includes employees, retired, surviving spouses, and disabled.

Figure 6: Discount Rate (in %)



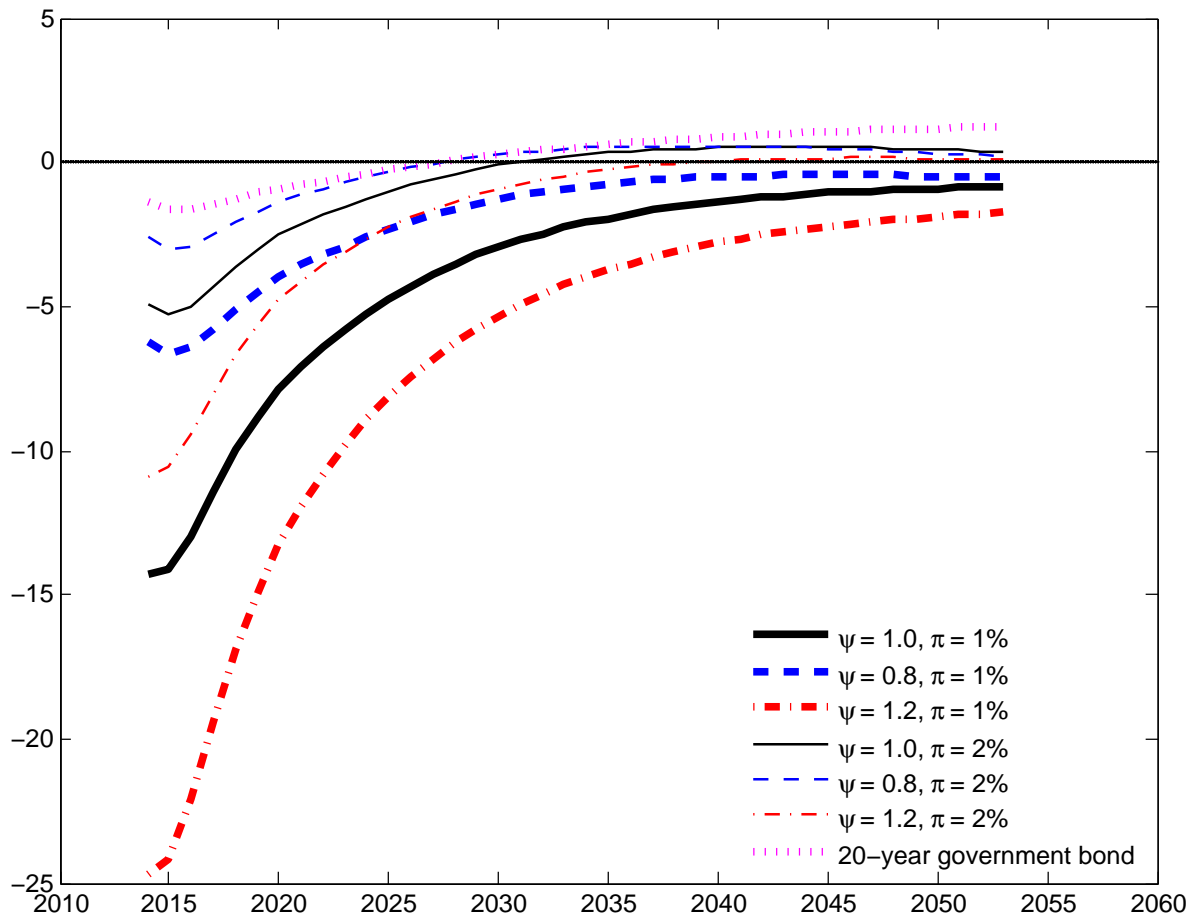
Note: This figure displays the expected evolution of the discount rate for the next 40 years starting from 2014 on. The figure also displays the 20-year government bond rate.

Figure 7: Liabilities (in billion CHF)



Note: This figure displays the expected evolution of the liabilities for the next 40 years starting from 2014 on.

Figure 8: Return on Liabilities (in %)



Note: This figure displays the expected evolution of the return on liabilities for the next 40 years starting from 2014 on. The figure also displays the return on a 20-year government bond.