Liquidity Risk Premium for Long-Term Investors
—Dynamic Portfolio Choice with Time-Varying Trading Costs

Joost Driessen * Ran Xing *

*Finance Department, Tilburg University

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Liquidity Level Premium

A large amount of empirical findings of **liquidity premium** (6%-7%),

Liquidity level of assets is priced

- Amihud and Mendelson (1986), bid-ask spread
- Brennan and Subrahmanyam (1996), a measure of market depth
- Brennan, Chordia, and Subrahmanyam (1998), trading volume
- Datar, Naik, and Radcliffe (1998), turnover rate
- Amihud (2002), ILLIQ

Liquidity Risk Premium

Since 2003, researchers document large **liquidity risk premium** (1%-7%).

Market liquidity risk (time variation of market liquidity) is priced

- Pastor and Stambaugh (2003), shocks on PS measure (price impact)
- Acharya and Pedersen (2005), shocks on market ILLIQ

Obstacle: It is hard to identify liquidity risk premium from other risk premiums, behavioral anomalies, and liquidity level premium empirically, since

- *Measures of liquidity risk are highly correlated with other risk factors and investment sentiment*;
- *Usually an asset with low liquidity also has a large exposure to market liquidity risk.*

Existing Theoretical Papers

Most theoretical papers assume **time constant** trading costs, and the liquidity premiums found are small ($< 1\%$ per year).

- Constantinides (1986):

  Investors accommodate large trading costs by reducing the frequency and volume of trade. (25 bps per year)

- Jang, Koo, Liu and Loewenstein (2007):

  Time-varying investment opportunity set. (46 bps per year)

Research Question

- How much can **time variation** of trading costs explain the large liquidity risk premium found empirically?

- What are the sources of the large liquidity premium found empirically?

  Liquidity level premium, liquidity risk premium or other risk premiums and behavioral anomalies?

Our Paper

- We solve a dynamic portfolio choice problem with **time-varying trading costs** (systematic liquidity risk)
  - Liquidity risk premium calibrated under realistic assumptions
- We model trading cost as **price impact of trades**
  - Primary source of liquidity risk
- We analyze the problem of **large institutional investors**
  - Face large price impact; Marginal investor

Main Findings

- The **liquidity risk premium** required as a compensation to the time variation of trading costs **is negligible**, **less than 3 bps** per year.
- Even if there is extremely large forced selling during market downturn, the liquidity risk premium increases to 20 bps per year at most.
- The relative importance of liquidity risk premium (4% to 28%) is smaller than the liquidity level premium.

Related Papers

- Acharya and Pedersen (2005)
  Liquidity-adjusted CAPM;
  One-period model with exogenous trading amount.
- Lynch and Tan (2011)
  Portfolio choice problem of individual investors;
  Shocks on labor income as background risk.
- Garleanu and Pedersen (2013)
  Portfolio choice problem with price-impact cost;
  Special assumptions on utility function and return dynamic.

Our paper solves a dynamic portfolio choice problem with realistic price-impact costs and utility function for large institutional investors, and calibrates a reasonable size for liquidity risk premium.

Return Dynamics

- Two assets, one risky r_{t+1} and one risk-free r_f,

$$r_{t+1} = \mu_t + \sigma_r u_{t+1}, \qquad u_{t+1} \sim N(0,1) \tag{1}$$

- One driving factor for the states of the world, AR(1)

$$F_{t+1} = \rho F_t + v_{t+1}, \quad v_{t+1} \sim N(0,1), \quad Corr_t(u_{t+1}, v_{t+1}) = \theta \tag{2}$$

- Time-varying expected returns

$$\mu_t = \mu_0 + aF_t \tag{3}$$

Trading Cost (Price Impact of Trade)

- Price impact of trade (in percentage)

$$PI_t(V_t) = V_t \sigma_r^2 \lambda_t \tag{4}$$

V_t: \$ trading volume; λ_t: trading cost parameter.

- Quadratic form of trading cost (price-impact cost)

$$TC_t(V_t) = \frac{1}{2} PI_t(V_t) * V_t = \frac{1}{2} V_t^2 \sigma_r^2 \lambda_t \tag{5}$$

- Time-varying trading-cost rates

$$ln\lambda_t = ln\lambda_0 + bF_t \tag{6}$$

Dynamic Portfolio Choice Problem

- Objective Function: CRRA utility of the final wealth

$$\max_{\alpha_0, \alpha_1, \ldots, \alpha_{T-1}} E_0\left(\frac{W_T^{1-\gamma} - 1}{1-\gamma}\right) \tag{7}$$

Decision Variable: α_t, weight in risky asset at time step t; $\alpha_t(F_t)$

- Value function J and the Bellman equation

$$J_t = \max_{\alpha_t, \ldots, \alpha_{T-1}} E_t\left(\frac{W_T^{1-\gamma} - 1}{1-\gamma}\right), \qquad J_t = \max_{\alpha_t} E_t(J_{t+1}) \tag{8}$$

The problem is solved using backward induction. We grid search the optimal weight from the last step to the first.

Parameter Values of Benchmark Case

- $r_f = 0.02$, risk-free rate, 2%;
- $\rho = 0.7$, time persistency of the state variable F_t;
- $\gamma = 2.5$, risk aversion level of the investor;
- $T = 10$, 10 time steps for 10 years;
- $\mu_c = 0.04$, long-run mean of the expected return, 4%;
- $a = 0.01$, time variation of the expected return, sd=1%;
- $\sigma_r^2 = 0.01$, time constant return volatility, sd=10%;
- $\theta = 0$, 0 correlation between shocks on F_t and r_t;

Calibration of Trading Cost Parameters

- Bikker, Spierdijk and van der Sluis (2007) from ABP shows that on average, a buy (sell) of 1.5 (1.6) million Euro leads to a price impact of 40 (60) basis points;

- $PI_t(V_t) = V_t \sigma_r^2 \lambda_c$=0.40%, Price impact of trade;
- $V_t = 1.5$ *million Euro*, Trading volume: 0.015*100 million;
- $\lambda_c = 26.88$, Trading cost parameter;
- $W_0 = 1$, Initial level of wealth, 100 million Euro;
- $b = 0.315$, Time variation of trading cost parameter;
  Calibrated using annual ILLIQ;
  95% Confidence Interval: $[0.3 * \lambda_c,\ 3.5 * \lambda_c]$;

Optimal Weights without Trading Costs

- Myopic aim portfolio weight (when $Corr_t(u_{t+1}, v_{t+1}) = 0$)

$$\alpha_t^{Myopic} = \frac{\mu_t - r_f + \frac{\sigma_r^2}{2}}{\gamma \sigma_r^2} \tag{9}$$

- Long-run aim portfolio weight (when $Corr_t(u_{t+1}, v_{t+1}) = 0$)

$$\alpha^{LongRun} = \frac{\mu_c - r_f + \frac{\sigma_r^2}{2}}{\gamma \sigma_r^2} = 100\% \tag{10}$$

Optimal Trading Strategy with Trading Costs

Consistent with Garleanu and Pedersen (2013)

- Trade partially towards the myopic aim, α_t^{Myopic};
- "Aim in front of the target", chase long-run aim, $\alpha^{LongRun}$.

Liquidity Level Premium and Liquidity Risk Premium

- Liquidity Level Premium

  The **decrease** in the expected return μ_t of the trading-cost-free risky asset that the investor requires to be **indifferent** between having access to the risky asset **without** rather than **with trading costs**.

- Liquidity Risk Premium

  $\cdots$ **without** rather than **with time variation in trading costs**.

Setting 1 (Benchmark):
trading motive: time varying expected returns μ_t

Setting 2 (Acharya and Pedersen 2005):
additional trading motive: releasing and rebuilding the portfolio per year

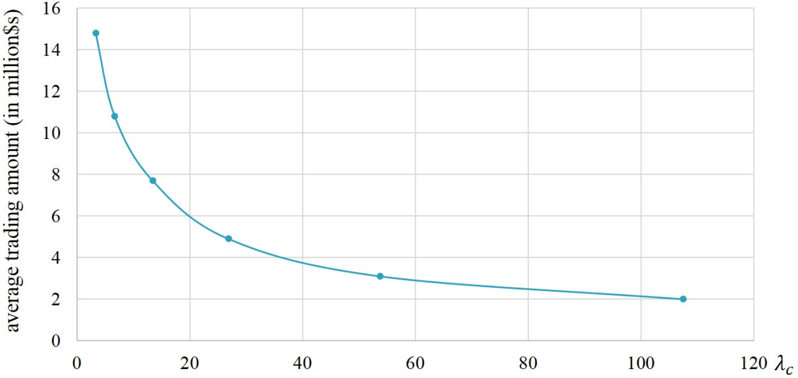
Setting 3 (with Forced Selling):
additional trading motive: forced selling during market downturn

Table: Liquidity Risk Premium for Benchmark Setting (with time varying expected returns)

in bps	$Corr(u_t, v_t)$			
	0	***-0.2***	***-0.4***	***-0.6***
Liquidity Level Premium (Total)	16.43	15.86	16.48	15.68
Liquidity Risk Premium (Total)	**-0.623**	**-0.582**	**-0.353**	**-0.104**
Total Liquidity Premium	15.80	15.27	16.12	15.58
liquidity risk premium for $Cov(\lambda_t, r_t)$	***0.000***	***0.042***	***0.270***	***0.519***
premium for $Cov(\lambda_t, r_t)$ *as % of total premium*	*0.0%*	*0.3%*	*1.7%*	*3.3%*

Endogenous Trading Amount

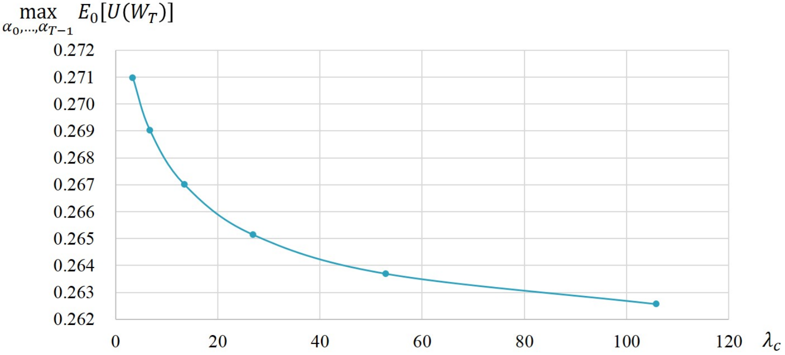
Figure: Trading Amount for Different Levels of Trading Cost



- Investor can trade more when trading cost λ_c is low and trade less when it is high.

Benefit from the Time Variation of λ_t

Figure: Expected Utilities for Different Levels of Trading Cost



- Since the expected utility $E_0[U(W_T)]$ is convex in trading cost parameter λ_c, time variation in λ_c leads to a higher expected utility.

Robustness Check: Liquidity Risk Premium across λ_c (with time varying expected returns)

in bps, $Corr(u_t, v_t) = -0.3$	Average value of trading-cost parameter λ_c					
	3.36	***6.72***	***13.44***	***26.88***	***53.76***	***107.52***
Liquidity Risk Premium (Total)	**-0.167**	**-0.316**	**-0.393**	**-0.494**	**-0.370**	**-0.297**
Total Liquidity Premium	7.54	10.44	13.51	16.34	18.77	20.63
avg. trading amount (million$s)	*14.8*	*10.8*	*7.7*	*4.9*	*3.1*	*2.0*

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Table: Liquidity Risk Premium (with fixed frequency of releasing and rebuilding)

in bps	$Corr(u_t, v_t)$	Frequency of rebuilding (per X years)			
		1	***2***	***5***	***10***
Liquidity Risk Premium	0	**1.509**	**0.907**	**0.847**	**0.861**
	-0.3	**1.645**	**1.163**	**1.925**	**1.439**
	-0.6	**1.859**	**1.447**	**2.659**	**1.846**
Total Liquidity Premium	0	194.23	183.47	146.92	94.98
	-0.3	205.01	194.19	155.61	102.72
	-0.6	212.15	201.45	163.50	108.27
avg. trading amount (million $s)	*0*	*9.49*	*8.75*	*11.39*	*12.43*
	-0.3	*9.26*	*8.83*	*11.46*	*13.06*
	-0.6	*9.18*	*8.87*	*11.68*	*13.39*

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additional trading motive: forced selling during market downturn

Table: Liquidity Risk Premium (with forced selling)

in bps	$Corr(u_t, v_t)$		
	0	***-0.3***	***-0.6***
Liquidity Risk Premium (Total)	**2.22**	**11.53**	**20.83**
Total Liquidity Premium	51.61	64.19	75.44
liquidity risk premium as % of total liquidity premium	*4%*	*18%*	*28%*
liquidity risk premium for $Cov_t(V_{t+1}^2, r_{t+1})$	*1.17*	*3.80*	*6.03*
liquidity risk premium for $Cov_t(\lambda_{t+1}, r_{t+1})$	*0.00*	*0.75*	*0.93*
liquidity risk premium for $Cov_t(\lambda_{t+1} V_{t+1}^2, r_{t+1})$	*0.00*	*9.31*	*18.61*

Main Findings

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