Risk Measures with Volatility Risk

A. Pollastri P. Schotman S. Straetmans

Maastricht University

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Motivation

In a parametric approach VaR is obtained as follows:

$$\mathrm{VaR}^{\alpha}_{\mathrm{t}+1} = \sigma_{\mathrm{t}+1} \mathrm{Q}(\alpha) \tag{1}$$

$\sigma_{\mathrm{t}+1}$ is unknown and it needs to be forecasted:

$$\widehat{\mathrm{VaR}^{\alpha}}_{\mathrm{t}+1} = \widehat{\sigma}_{\mathrm{t}+1} \mathrm{Q}(\alpha) \tag{2}$$

We want to investigate the effect of distributional assumptions for volatility model on tail risk measures.

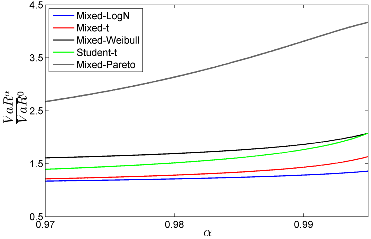
1. Model
2. Risk Measures
3. Dependence

In order to make inference about RMs we want to obtain the unconditional density of returns:

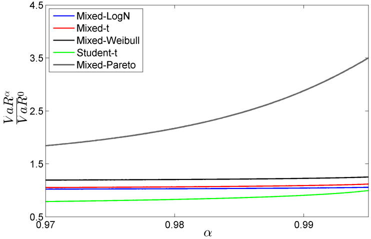
$$\mathrm{p}(\mathrm{r}_{\mathrm{t}+1}) = \int \mathrm{p}(\mathrm{r}_{\mathrm{t}+1}|\sigma^2_{\mathrm{t}+1})\mathrm{p}(\sigma^2_{\mathrm{t}+1})\mathrm{d}\sigma^2_{\mathrm{t}+1} \tag{3}$$

We also assume a SV model for the variance dynamics given by:

$$\ln \sigma^2_{\mathrm{t}+1} = \mu + \omega \mathrm{z}_{\mathrm{t}+1} \tag{4}$$



(a) ω high



(b) ω low

In the literature tail dependence is measured as follows:

$$\tau = \lim_{\alpha \to 1} \tau(\alpha) \tag{5}$$

$$\tau(\alpha) = \mathbb{P}\left(\mathrm{r}^1 > \mathrm{VaR}^1(\alpha) \middle| \mathrm{r}^2 > \mathrm{VaR}^2(\alpha)\right) \tag{6}$$

We study the effect of VoV on this coefficient.

In the bivariate case, the (co-)variance process is a matrix composed by 3 elements. A VAR(1) with autoregressive component:

$$\boldsymbol{\zeta}_{\mathrm{t}} = \begin{pmatrix} \ln \sigma_{11,\mathrm{t}} \\ \ln \sigma_{22,\mathrm{t}} \\ \ln \frac{1+\rho_{\mathrm{t}}}{1-\rho_{\mathrm{t}}} \end{pmatrix}$$

$$\boldsymbol{\zeta}_{\mathrm{t}+1} = \boldsymbol{\mu} + \mathrm{A}\Big(\boldsymbol{\zeta}_{\mathrm{t}} - \boldsymbol{\mu}\Big) + \boldsymbol{\omega}\mathrm{Z}_{\mathrm{t}+1}$$

with $\boldsymbol{\omega}\boldsymbol{\omega}' = \Omega$ is the covariance matrix of $\mathrm{Z}_{\mathrm{t}+1}$.

In order to obtain the probability given by (6), we adopt the following hierarchical model:

$$\begin{aligned} \mathrm{r}_{t+1} \Big| \mathbf{\Sigma}_{t+1} &\sim \mathcal{N}(0, \mathbf{\Sigma}_{t+1}) \\ \mathbf{\Sigma}_{t+1} &= \mathrm{f}(\boldsymbol{\mu}, \mathbf{\Omega}) \end{aligned}$$

$$\mathbf{\Omega} = \begin{pmatrix} \omega_{11} & \omega_{12} & \omega_{13} \\ \omega_{21} & \omega_{22} & \omega_{23} \\ \omega_{31} & \omega_{32} & \omega_{33} \end{pmatrix}$$

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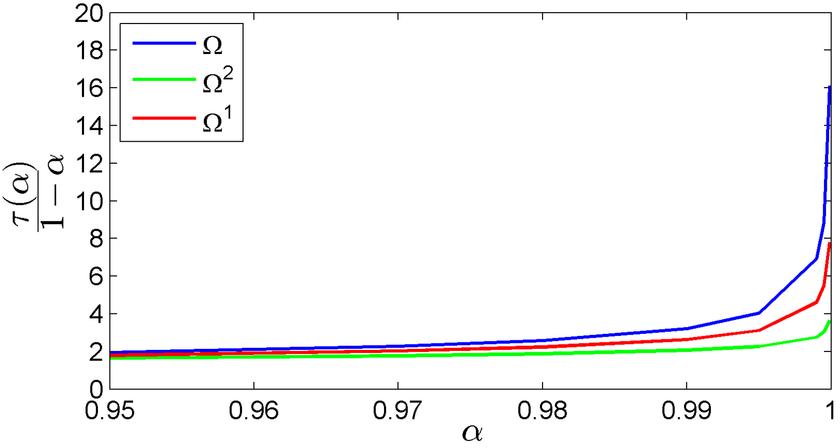
$$\boldsymbol{\Omega}^1 = \begin{pmatrix} \omega_{11} & \omega_{12} & 0 \\ \omega_{21} & \omega_{22} & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

In order to obtain the probability given by (6), we adopt the following hierarchical model:

$$
\begin{aligned}
\mathrm{r}_{t+1} \Big| \boldsymbol{\Sigma}_{t+1} &\sim \mathcal{N}(0, \boldsymbol{\Sigma}_{t+1}) \\
\boldsymbol{\Sigma}_{t+1} &= \mathrm{f}(\boldsymbol{\mu}, \boldsymbol{\Omega})
\end{aligned}
$$

$$
\boldsymbol{\Omega}^2 = \begin{pmatrix} \omega_{11} & 0 & 0 \\ 0 & \omega_{22} & 0 \\ 0 & 0 & 0 \end{pmatrix}
$$



20
18
16
14
12
10
8
6
4
2
0
$\frac{\tau(\alpha)}{1-\alpha}$
Ω
Ω^2
Ω^1
0.95
0.96
0.97
0.98
0.99
1
α

Conclusions and possible extensions

- In the univariate framework we have shown that the distributional assumptions of volatility innovations have a significant effect on RMs
- In the dependence study we have shown that different structures of $\mathbf{\Omega}$ lead to distinct values for the coefficient of dependence.

We would like to:

- Extend the current framework to the case where we have a portfolio of assets
- Study the effect of parameter uncertainty
- Consider forecasting horizons longer than one period

Thank you for your attention!