

# Optimal Design of Funded Pension Schemes under Financial Fairness

with Applications to the Dutch Pension Reform

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Netspar Pension Day 2015

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# Introduction.

- ▶ Collective risk sharing w.r.t. investment risk can be organized in many ways
- ▶ In a CDC system, the existing framework mainly uses indexation rules as a way of CRS, e.g. nFTK
- ▶ Not so many theoretical supports on how the indexation rules are determined
- ▶ We try to give a systematic approach in which the risk sharing rules are calculated from principles

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# Main Innovations

## The Mohopeff Approach

We work with the *Mohopeff* approach in the decumulation phase:  
Moving-horizon Pareto Efficiency and Financial Fairness.



# Main Innovations

## The Mohopeff Approach

- ▶ Moving horizon: a dynamic approach.  
The risk sharing rule is calculated in a projected pension system (*P-system*) at each time point.
- ▶ Pareto efficiency: a utility basis.  
Utility functions are used to determine the optimal degree of risk sharing.
- ▶ Financial fairness: a value basis.  
Specify the market values of the benefit first which shall help determining the risk sharing rules.

Maintain the value profile (market value,  $\mathbb{Q}$ ) and optimize the risk-return profile (utility functions, Pareto efficiency)

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# Main Results

- ▶ Dynamic situation-dependent risk sharing rules.
- ▶ As we shall see, linear risk sharing rules can be optimal under specific settings. But not necessarily...
- ▶ The approach can be calibrated by taking into consideration of the discontinuity problem.

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# Agenda

1. Description of the underlying CDC system
2. Introduction to the Mohopeff approach for calculating the distribution rules
3. Illustration from Numerical results
4. Short discussion on discontinuity problem

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# The CDC Pension System.

# The CDC Pension System

## Basic Settings

- ▶ Focus on allocation decisions → fix investment decisions..
- ▶ OLG setting. Realistic demography profiles can be assumed → mortality risk...
- ▶ Consider the decumulation phase only: generations make lump-sum contributions at entry.
- ▶ The lump-sum contribution is converted into a variable nominal annuity at the time of entry.

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# The CDC Pension System

## The Key Problem

- ▶ At each time we consider the *aggregate benefit*: the sum of all the to-be-paid-out benefits at that time.
- ▶ One has to decide how to distribute between

$$A_\tau = B_\tau + F_\tau$$

- ▶  $B_\tau$  is the current aggregate benefit,
- ▶  $F_\tau$  is the buffer(fund) for future liabilities.

We propose to solve this inter-temporal risk sharing problem using the Mohopeff approach.

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# The Mohoeff Approach.

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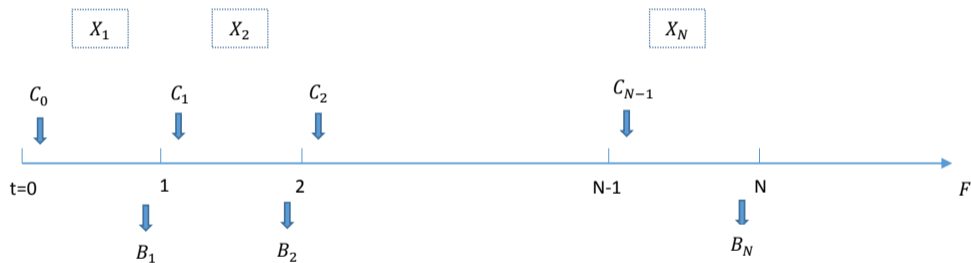
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# Multi-period PEFF

## Basic Environment



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# Multi-period PEFF

## Basic Environment

- ▶  $C_t$ : contribution paid in at time  $t$ . Deterministic.
- ▶  $B_t$ : benefit paid out at time  $t$ . Decision variable.
- ▶  $F_t$ : buffer (fund) size at time  $t$ .
- ▶  $X_t$ : stochastic gross asset return at time  $t$ : investment risk. Distribution is known at time 0 under  $\mathbb{P}$  and  $\mathbb{Q}$ .

Budget constraint:

$$B_t + F_t = (C_{t-1} + F_{t-1})X_t := A_t.$$

# Multi-period PEFF

## Pareto Efficiency

- ▶ Each benefit payment is considered as a separate agent...
- ▶ Specify utility functions  $(u_1, u_2, \dots, u_N, u_p)$  for  $(B_1, B_2, \dots, B_N, F_N)$  respectively.
- ▶ A risk sharing rule  $(B_1, B_2, \dots, B_N, F_N)$  is *Pareto efficient* if there does not exist any other risk sharing rule  $(\tilde{B}_1, \tilde{B}_2, \dots, \tilde{B}_N, \tilde{F}_N)$  such that

$$\left( \mathbb{E}^{\mathbb{P}} u_1(\tilde{B}_1), \dots, \mathbb{E}^{\mathbb{P}} u_N(\tilde{B}_N), \mathbb{E}^{\mathbb{P}} u_p(\tilde{F}_N) \right) \succeq \left( \mathbb{E}^{\mathbb{P}} u_1(B_1), \dots, \mathbb{E}^{\mathbb{P}} u_N(B_N), \mathbb{E}^{\mathbb{P}} u_p(F_N) \right).$$



# Multi-period PEFF

## Financial Fairness

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*Financial fairness* indicates that the undiscounted market values of the benefits should match some given vector  $v$ :

$$\left( \mathbb{E}^{\mathbb{Q}} B_1, \dots, \mathbb{E}^{\mathbb{Q}} B_N, \mathbb{E}^{\mathbb{Q}} F_N \right) = (v_1, \dots, v_N, v_F).$$

The vector  $(\mathbb{E}^{\mathbb{Q}} B_1, \dots, \mathbb{E}^{\mathbb{Q}} B_N, \mathbb{E}^{\mathbb{Q}} F_N)$  shall be called a *value profile*.

# Multi-period PEFF

## Main Results

- ▶ There always exists a unique risk sharing rule that is both PE and FF. It has the following form

$$\begin{aligned}B_t &= f_t(A_t), \\F_t &= A_t - f_t(A_t), \\A_{t+1} &= (C_t + F_t)X_{t+1}.\end{aligned}$$

- ▶ A numerical algorithm tells how to compute the  $f_t$ 's.

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# The Mohopeff Approach

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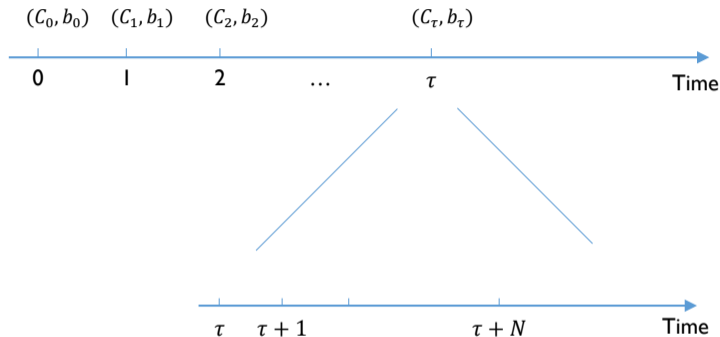
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- ▶ The Mohopeff approach is a moving-horizon version of the PEFF approach.
- ▶ At each time point of the *underlying system* (U-system), one calculates the distribution rule in a *projected pension system* (P-system) by applying the PEFF approach.
- ▶ Note that each aggregate benefit shall be regarded as separate agents.

# U-system and P-system



U-system: real-world scenarios

P-system: projected system

$$(C_\tau, \tilde{C}_{\tau;\tau+1}, \dots, \tilde{C}_{\tau;\tau+(N-1)})$$

$$(E_\tau^Q B_{\tau;\tau+1}, \dots, E_\tau^Q B_{\tau;\tau+N}, E_\tau^Q F_{\tau;\tau+N})$$

# Inputs for a P-system

- ▶ the estimated deterministic stream of aggregate contributions → synthesize the expected inflation and demography profiles.
- ▶ the utility functions → *utility knob*.
- ▶ the value profile → the *value adjustment* process.
- ▶ the distributions of the investment risks.

# Utility Knob

- ▶ front-protection: the aggregate benefits are assumed to be more risk averse than the end buffer, thus the end buffer will absorb more volatility.
- ▶ back-protection: the aggregate benefits are assumed to be less risk averse than the end buffer, thus the end buffer will absorb less volatility.
- ▶ balanced-protection: assume equal risk aversion levels to both the aggregate benefits and the end buffer.

# Value Adjustment

Within the P-system at time  $\tau$  one does the following.

1. First let the market value of the projected end-phase buffer equal the projected future liabilities at that time.
2. Any surplus or deficits between the current funding status and the projected funding status shall be smoothed proportionally within the  $N$ -year horizon in terms of market value.

This determines to what extent the current pension payments contribute to the funding status recovery in terms of market value  $\rightarrow$  the idea of  $N$ -year recovery period...

The investment returns have little impact over the *holistic* funding status, which may change due to changes in demography and economic environment.



# Output of the P-system

- ▶ The MP-PEFF in the P-system tells

$$B_{\tau+1} = f_{\tau;\tau+1}(A_{\tau+1})$$

i.e. the pension payment is a contingent claim over the value of the asset mix.

- ▶ When we move on to the next year, another new P-system is set up to utilize the most updated information and we have

$$B_{\tau+2} = f_{\tau+1;\tau+2}(A_{\tau+2})$$

# Output of the P-system

## Implications from Explicit Solution

- ▶ The benefit approximately equals its market value plus a proportion from the excess return from the total asset.
- ▶ The proportion summarizes the information within the  $N$ -year horizon, including the expected return.

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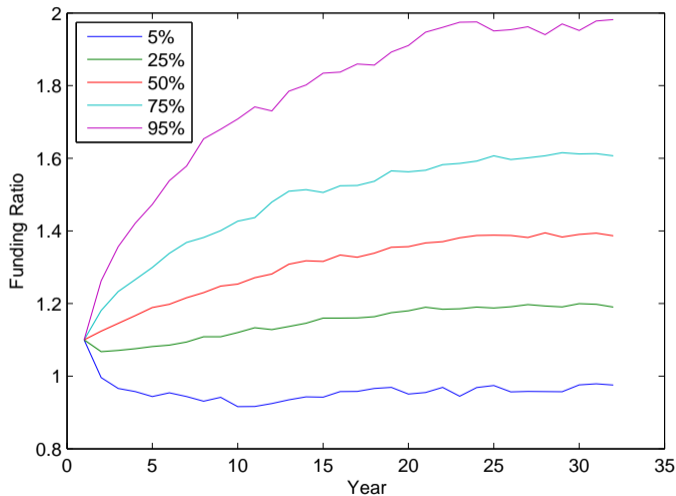
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## An ALM Study: Numerical Results.

- ▶ The Mohopeff approach is tested over the CPB model (See Draper 2014).
- ▶ No demography changes: one representative agent for each cohort who shall stay in the system for 20 years.
- ▶ Horizon of the P-system is chosen to be 10 years.

# Numerical Results

Exponential Utility, Starting BFR = 110%



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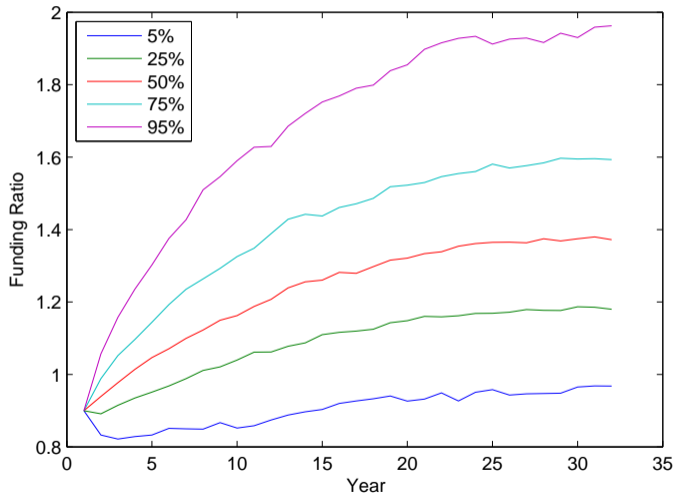
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# Numerical Results

Recovery from Low Funding Status, Starting BFR = 90%



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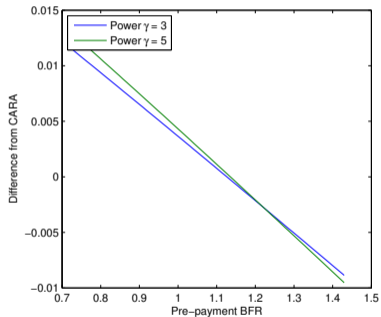
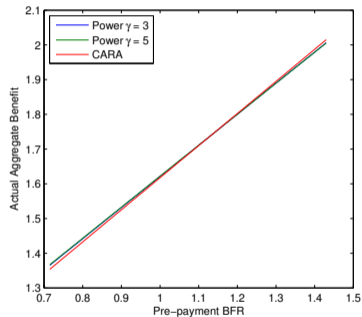
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# Numerical Results

## Risk Sharing Rule: Exponential vs. Power, Balanced-Protection

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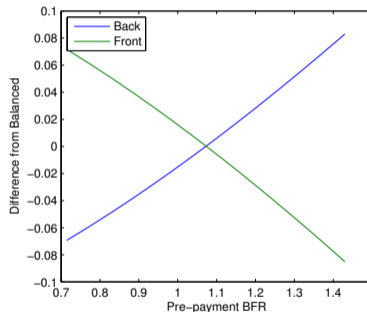
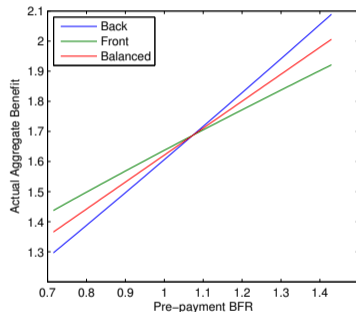
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# Numerical Results

## Risk Sharing Rule: Effect of Utility Knobs, Power Utility

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## The Concept of Tolerance Band and the Discontinuity Problem.

# Discussion on Discontinuity Problem

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- ▶ The system above didn't take into consideration the discontinuity problem...
- ▶ The new entrants may immediately face a benefit cut at entry → less attractive
- ▶ The buffer size may be so large that the existing cohorts want to immediately terminate the system

# Tolerance Band

- ▶ If the buffer stays within the tolerance band, the new entrants will feel welfare-improving to step into the system, and the existing cohorts also feel welfare-improving not to terminate the fund.
- ▶ The *floor* of the TB is calculated by comparing the aggregate utility of the new entrants between stepping into the system and going for individual contracts from the open market.
- ▶ The *cap* of the TB is calculated by comparing the aggregate utility of the existing cohorts between stepping into the system and terminating the system and going for individual contracts from the open market.
- ▶ TB shall be used to properly calibrate the model.
- ▶ Work in progress...

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**Thanks!**  
***Bedankt!***