# The Liquidity Risk Premium for Long-Term Investors

## Dynamic Portfolio Choice with Time-Varying Trading Costs *

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March 2015

**Abstract**

Recent empirical papers (e.g. Pastor and Stambaugh 2003, Acharya and Pedersen 2005) document large liquidity risk premiums in stock market. To investigate whether time variation of trading costs has the potential to generate large liquidity risk premiums, we solve a dynamic portfolio choice problem with time-varying trading costs, CRRA utility and a time-varying investment opportunity set. We find that, even under the assumptions of extremely high trading-cost rates and large trading motives, the premium generated by the time variation of trading-cost rates (liquidity risk premium) is negligible, less than 3 bps per year. Larger trading amounts and higher trading frequency increase the premium for the level of trading costs (liquidity level premium) only but not the liquidity risk premium. Assuming forced selling during market downturn enlarges the liquidity risk premium to about 10-20 bps per year, and this premium is generated by the interaction of large trading amounts and high trading-cost rates during the market downturn, rather than the high trading-cost rate itself.

*We thank Frank de Jong, Alberto Manconi, Lieven Baele, and seminar participants at Tilburg, and Netspar's Liquidity Risk Workshop for valuable comments. Driessen and Xing are at Tilburg University. Emails: J.J.A.G.Driessen@uvt.nl, r.xing_1@uvt.nl.

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## Dynamic Portfolio Choice with Time-Varying Trading Costs

Recent empirical papers (e.g. Pastor and Stambaugh 2003, Acharya and Pedersen 2005) document large liquidity risk premiums in stock market. To investigate whether time variation of trading costs has the potential to generate large liquidity risk premiums, we solve a dynamic portfolio choice problem with time-varying trading costs, CRRA utility and a time-varying investment opportunity set. We find that, even under the assumptions of extremely high trading-cost rates and large trading motives, the premium generated by the time variation of trading-cost rates (liquidity risk premium) is negligible, less than 3 bps per year. Larger trading amounts and higher trading frequency increase the premium for the level of trading costs (liquidity level premium) only but not the liquidity risk premium. Assuming forced selling during market downturn enlarges the liquidity risk premium to about 10-20 bps per year, and this premium is generated by the interaction of large trading amounts and high trading-cost rates during the market downturn, rather than the high trading-cost rate itself.

*JEL classification:*

*Keywords:* Liquidity premium; Liquidity risk; Dynamic portfolio choice; Trading Costs; Price impact.

# 1 Introduction

Nowadays, the fact that asset liquidity (the ease to trade) affects asset price has been well accepted in both academia and industry. In the recent decade, how and how much the liquidity risk (the time variation of the ease to trade) affects the price of an asset attract large amounts of attentions. Many papers document large liquidity risk premium in stock returns. For example, Pastor and Stambaugh (2003) document 7% return difference per annum between the highest and lowest deciles sorted by exposures to their liquidity risk factor; and Acharya and Pedersen (2005) find a liquidity risk premium of 1.1% per annum calibrated using their liquidity-adjusted CAPM. However, the estimation of liquidity risk premium empirically is inaccurate, mainly for two reasons. First, the currently available measures of liquidity risk (e.g. time variations of Amihud's ILLIQ measure, Kyle's lambda and measure of price impact in Pastor and Stambaugh 2003 etc.) are highly correlated with other risk factors, such as the market factor in CAPM, size factor in Fama-French 3 factors model and other unidentified risk factors, and it is impossible for these risk factors to be fully controlled. As a consequence, the liquidity risk premiums documented always include other risk premiums. Second, measures of stock liquidity levels and exposures to liquidity risk are highly correlated across stocks, because of the collinearity problem[1], it is hard to disentangle liquidity risk premium and liquidity level premium. In this paper, we quantify the magnitude of liquidity risk premium using our knowledge of liquidity risk, rather than solely relying on the econometric techniques. We find that the largest possible premium can be generated by the time variation of trading costs is substantially smaller than the liquidity risk premium found empirically. Therefore, those large liquidity risk premiums documented empirically capture liquidity level premiums and other risk premiums as well.

Theoretically, we solve a dynamic portfolio choice problem with time-varying trading costs, and calibrate the largest possible liquidity risk premium using partial equilibrium approach. Most existing papers of portfolio choice problem with trading costs assume time constant trading-cost rate (e.g. Constantinides 1986; Liu 2004; Lo, Mamaysky and Wang 2004; Jang, Koo, Liu and Loewenstein 2007 etc.). Different from them, we assume

[1] Refer to Acharya and Pedersen (2005)

time-varying trading-cost rates[2] (liquidity risk) to study the magnitudes of liquidity risk premiums. Under this setting, all sources of liquidity level premiums and liquidity risk premiums are tractable and can be accurately identified. For the exogenous trading motives, we first allow the stock expected returns to vary over time. Later in this paper, we also analyze the cases with the rebuilding of the portfolio at a fixed time frequency and the forced selling during the market downturn to further explore the possible sources of liquidity risk premium.

Following Garleanu and Pederson (2013), we use quadratic form of trading costs, which models the costs induced by the price impact of trades. Previous literature documents that the price impact costs of trades plays a crucial role in financial market, and it is the type of trading costs that varies significantly with the market condition. Since this paper focuses on the liquidity risk premium generated by the time variation of trading costs, we choose to use the quadratic trading costs. In addition, we study the portfolio choice problem of large institutional investors, to whom the price impact of trades is one of their main concerns. We focus on large institutional investors because large institutional investors are more sensitive to the market condition, expose more to liquidity risk and thus they request higher liquidity risk premium as a compensation. Since the price impact of trades for large investors can be extremely large during market downturn, and they are likely to be forced to trade at that time because of redemption or regulations, their losses of utility caused by liquidity risk should be large. Besides, they are more sophisticated than retail investors, therefore they are more likely to be the marginal investors who decide the prices.

Our analysis shows that the liquidity risk premium generated by the correlation between trading-cost rates and stock returns, which is documented as the main source of liquidity risk (e.g. Amihud 2002, Pastor and Stambaugh 2003 and Acharya and Pedersen 2005 etc.), is negligible, less than 3 bps, under both the benchmark setting with time varying expected returns and the setting with fixed frequency of rebuilding the portfolio. Once we

[2]Liquidity risk is defined in many different ways in previous literature. For example, Huang (2003) defines it as the randomly arrived liquidity shocks; in Vayanos (2004), it refers to the time variation of needs to liquidate; Ang, Papanikolaou and Westerfield (2014) uses it for the uncertainty of the length of non-trading interval. In this paper, we follow Acharya and Pedersen (2005) and define liquidity risk as the time variation of trading costs which is more consistent with the reality in stock market.

add exogenous liquidity shocks during the market downturn into the setting, the liquidity risk premium become economically significant, about 10 to 20 bps, and accounts for a large fraction, 4% to 28%, of the total liquidity premium. Forced selling and high trading-cost rate during the market downturn together hurt the investor hardly and thus generate large liquidity risk premium. But the liquidity risk premium generated by any of these two itself is still negligible, which indicates that large liquidity risk premiums documented in previous empirical papers, such as Pastor and Stambaugh (2003) and Acharya and Pedersen (2005), compensate for the forced selling and high trading-cost rate during market downturn together, rather than simply the high trading-cost rate itself. Consistently, we show that the market turnover actually increases during market downturn in market data.

Besides, we find that in addition to the actual trading costs they are going to pay, liquidity level premium also compensates for the loss of utility caused by the suboptimal decision made by investor to avoid high trading costs. Galeanue and Pedersen (2013) find that investor chooses the optimal trading amount to balance the trading costs and the utility loss of trading less. Following this thought, the fraction of liquidity level premium compensating for the actual trading costs should depend on both the level of trading-cost rates and the elasticity of the trading motive. Moreover, the total liquidity premium is limited by the utility loss of not trading (or not investing when trading is forced) at all. Since if expected trading costs are too high, investor can always choose to not trade (or not invest) at all to avoid the trading costs.

The paper is organized as follows. Section 2 discusses related literature and contributions. Section 3 describes the dynamic portfolio choice problem with quadratic and time-varying trading costs. Section 4 solves the problem numerically. In Section 5, we calculate the implied liquidity level premiums and liquidity risk premiums under the benchmark setting, setting with fixed frequency of rebuilding the portfolio and setting with forced selling during market downturn separately. Section 6 compare the correlation between market returns and turnovers indicated by our model and that in market data, and followed by conclusions in Section 7.

## 2 Related Literature and Contributions

In the previous decade, a large amount of papers investigate the magnitudes of liquidity[3] and liquidity risk premiums in stock returns, both theoretically and empirically. And this paper is one of those trying to combine theories with empirics.

One major thread of theoretical literature is the analysis of portfolio choice with trading costs. Most papers in this thread assume time constant trading-cost rate, which might be true for explicit costs (e.g. brokerage commissions) but is definitely not true for implicit trading costs (e.g. bid-ask spreads and price impact costs). As a starting point of this thread, Constantinides (1986) shows that for realistic proportional costs, the per-annum liquidity premium that must be offered to induce a constant relative risk aversion (CRRA) investor to hold the illiquid asset instead of an otherwise identical liquid asset is an order of magnitude smaller than the trading-cost rate itself. Other papers following this thread include Liu (2004), Jang, Koo, Liu and Loewenstein (2007), Lynch and Tan (2011), Collin-Dufresne, Daniel, Moallemi and Saglam (2012), Garleanu and Pedersen (2013) etc. Different from them, we let trading-cost rates vary over time to study the magnitude of liquidity risk premium, and we include forced selling during market downturn to further explore how it interacts with the time varying trading-cost rates and inflates the liquidity risk premium.

Empirically, our paper provides a tractable framework to identify different sources of liquidity level and liquidity risk premiums, and estimate their magnitudes. A number of empirical papers (e.g. Amihud and Mendelson 1986, Amihud 2002, Pastor and Stambaugh 2003 and Acharya and Pedersen 2005) find substantial differences in expected returns across portfolios sorted on liquidity measures, with a magnitude ranges from 4% to 7% per annum. Some recognize it as the premium for the level of illiquidity (liquidity level premium), while others understand it as the premium for liquidity risk or both. In general, existing theories can hardly explain the large liquidity premiums found empirically. Our paper helps to

[3] In the previous research of asset illiquidity there are many different definitions. For example, the existence of non-trading interval in Diamond (1982), Ang, Papanikolaou and Westerfield (2014); the limitation on trading quantities (e.g. Longstaff (2001)); or trading at deterministic times (e.g. Kahl, Liu, and Longstaff (2003); Koren and Szeidl (2003); Schwartz and Tebaldi (2006); Longstaff (2009) etc.). The type of illiquidity we study in this paper is the trading costs, the most common one investigated in both liquidity pricing and portfolio choice literature (e.g. Constantinides (1986); Grossman and Laroque (1990); Vayanos (1998); Pastor and Stambaugh (2003); Lo, Mamaysky and Wang (2004); Acharya and Pedersen (2005) etc.).

understand the possible sources of these liquidity premiums and evaluate the validity of their magnitudes based on our knowledge.

Different from other models, liquidity-adjusted CAPM proposed by Acharya and Pedersen (2005) assumes time varying trading-cost rates. It provides a unified framework for understanding the various channels through which liquidity risk may affect asset prices. The primary limitation of liquidity-adjusted CAPM is that it is a one-period model. Trading frequency and trading amount are determined exogenously. In reality, both of them should be determined endogenously by investors, and these decisions should affect liquidity level and liquidity risk premiums in equilibrium. To make the trading frequency and trading volume endogenous, a multi-period model is required. Furthermore, as in Constantinides (1986), investor might coordinate his/her trading behaviors over time to reduce the total trading costs and thus require lower liquidity premiums, and such coordination is only possible under dynamic setting. Since our model is solved under dynamic setting with time varying trading-cost rates, the equilibrium liquidity premiums calibrated are more realistic.

To our best knowledge, Lynch and Tan (2011) and Garleanu and Pedersen (2013) are the only two dynamic portfolio choice papers assuming time-varying trading-cost rates.

Lynch and Tan (2011) shows that permanent shocks on labor income and return predictability produce additional trading motive and thus first-order liquidity premium. Their numerical results also show that time-varying trading-cost rates further inflate the premium since under their setting trading-cost rate is high when agent trades the most. Different from Lynch and Tan (2011), we study the portfolio choice problem of large institutional investors instead of individual investors. Institutional investors are more likely to be the marginal investors in financial market who decide the market price than individual investors. They are afraid of the price impact of trades, and since the price impact varies a lot with market condition, it has the potential to generate a large liquidity risk premium. Because of this, we use time-varying quadratic trading costs (costs generated by the price impact of trades), instead of the percentage trading costs as Lynch and Tan (2011) do. In addition, institutional investors care more about regulations and redemption than labor income, therefore we assume exogenous liquidity shocks rather than labor income shocks. And we show that the forced selling of institutional investors during market downturn actually interacts with the time variation of trading costs and enlarges the liquidity risk premium.

Garleanu and Pedersen (2013) defines the portfolio choice problem as a standard form of linear quadratic control, which ensures the existence and tractability of analytical solution. Since the linear quadratic control requires special assumptions on objective function and return dynamic, they model the changes of stock price instead of stock returns, and a nonstandard objective function instead of the CRRA or CARA most people use. As a consequence, reasonable liquidity premium cannot be implied by their model. Different from their paper, to obtain realistic liquidity premiums in equilibrium, we give up the analytical solution and choose to use more widely-accepted assumptions: CRRA utility, dynamics of relative returns and time-varying expected return.

Besides, based on the finding in Garleanu and Pedersen (2013) that investor balance between trading costs and loss of utility caused by investing sub-optimally, we separate the liquidity level premium to two parts, the part compensating for trading costs and the part compensating for utility loss of investing sub-optimally. We find that liquidity premium required by investor depends not only on the trading-cost rate, but also the elasticity of investor's trading motive.

This paper is also one of many theoretical papers which try to bridge the gap between the small liquidity premium implied by existing theories and the large liquidity spreads found empirically. In general, there are three branches within this thread.

1. Use different forms of trading costs (illiquidity):

   Liu (2004) and Lo, Mamaysky and Wang (2004) use the fixed trading costs. Realistic fixed trading costs can still hardly explain the large magnitude of liquidity premium. Longstaff (2001) limits the maximum amount of each transaction; and Garleanu (2008) models the illiquidity as the delay of trades. Though large liquidity premium can be derived under these two settings, those assumptions are not realistic. Stock market rarely limits the maximum amount of each transaction and delay of trades almost never happened there.

2. Add more trading motives other than portfolio rebalancing:

The influence of trading costs largely relies on the trading frequency and the trading amounts. More trades indicates a larger liquidity premium. The most popular way within this branch is to add the time-varying investment opportunity set (return predictability or time-varying volatility). With this setting, many papers, such as Jang, Koo, Liu and Loewenstein (2007) and Lynch and Tan (2011), derive more trades and relatively larger liquidity premium at the equilibrium (or optimal policy). But the premium is still not big enough.

Another choice is to create more trading motives with background risk. For example, Lynch and Tan (2011) induces the wealth shocks (shocks to labor incomes) in their model; Lo, Mamaysky and Wang (2004) and Garleanu (2008) both assume time-varying endowments in each period; and Huang (2003) assumes that all investors face liquidity shocks and have to release their positions at some time point.

3. Assume time-varying trading-cost rates:

   If and only if the trading-cost rates are assumed to be time-varying, the liquidity risk and the corresponding premium exist. To our best knowledge, Lynch and Tan (2011) is the only paper which uses time-varying trading-cost rates in the setting of portfolio choice as one of the crucial assumptions to explain the large liquidity premium. However, the addition premium generated is still very small.

Our paper belongs to the third branch and mainly contributes to the relation between time-varying trading-cost rates (liquidity risk) and the large total liquidity risk premium found empirically. Taking the fruit of the second branch, we assume time-varying investment opportunity set in our benchmark model and allow frequent rebuilding of the portfolio and forced selling later. We find that even under the boldest assumptions on trading-cost rate and trading motives, the liquid risk premium calibrated are still substantially smaller than those documented empirically. It makes us believe that the liquidity risk are overestimated in previous literature.

Finally, this paper provides useful implications to the trading cost management of long-term investors. First, the optimal strategy of our dynamic portfolio choice problem indicates that long-term investors should trade partially each time and gradually towards the aim to reduce the price impact of trades. Second, this paper helps them to understand how much do high trading costs and time variation of trading-cost rates matter to them. They should do a self-assessment of their trading frequency, average trading amount, elasticity of their trading motives, and their trading motives during the market downturn: the more they are expected to trade and the less elastic their trading motives are, the more they should care about trading costs; and if they are expected to trade a lot during the market downturn, they should also care liquidity risk more. Third and last, they can use the liquidity level and liquidity risk premiums calibrated in this paper as a reference when making the investment decision.

# 3 Model

Large institutional investors trade risk off against return and the benefit of rebalancing against trading costs throughout their investment process. To grab these main figures, we consider a dynamic portfolio choice problem with time varying trading costs. For simplicity, we consider an economy with one risky asset and one riskless asset. The log returns are used. The risk free rate $r_f$ is constant over time, and the return dynamic for the risky asset is as follow:

$$r_{t+1} = \mu_t + \sigma_r u_{t+1} \tag{1}$$

$\mu_t$ is the conditional mean return. $\sigma_r$ is the volatility parameter. Return shock $u_{t+1}$ is normally distributed with mean 0 and standard deviation of 1. We assume that $\mu_t$ depends on a common driving factor $F_t$ which follows an AR(1) process,

$$F_{t+1} = \rho F_t + v_{t+1} \tag{2}$$

$$\mu_t = \mu_0 + aF_t \tag{3}$$

In our model, $F_t$ is the only state variable which drives the time variations of both expected return $\mu_t$ and the trading costs parameter $\lambda_t$, which will be introduced later. $v_{t+1}$ is the shock on $F_{t+1}$. For convenience, we also assume it follows standard normal distribution, and we define the correlation between the shocks of returns and state variable $F_t$ as $Corr(u_t, v_t) = Corr$. The time persistency parameter of $F_t$ is $\rho$, and the long-run mean value of $F_t$ is zero. $a$ decides the magnitude of the time variation of $\mu_t$. The expected return is constant over time if $a = 0$. By substituting $F_t$ into the expression of $\mu_t$, we can easily find that $\mu_t$ is also an AR(1) processes, and $\mu_0$ is the long-run mean of the expected return.

Trading is costly in our setting. Following Garleanu and Pedersen (2013), we use quadratic trading costs, which is associated with the dollar amount of the trade $V_t$ and can be written as

$$TC_t = \frac{1}{2} V_t^2 \sigma_r^2 \lambda_t \tag{4}$$

Quadratic trading costs depend on $V_t^2$, rather than $V_t$ as percentage trading costs do. It can be thought as the costs caused by the price impact of trades. Trading $V_t$ moves the price by

$$PI_t = V_t \sigma_r^2 \lambda_t \tag{5}$$

and the effective proportional trading cost in this case $c_t$ equals to the average price move $\frac{1}{2} PI_t$,

$$c_t = \frac{1}{2} PI_t = \frac{1}{2} V_t \sigma_r^2 \lambda_t \tag{6}$$

This results to a total trading costs of $V_t$ times the average price move $\frac{1}{2} PI_t$, which gives the expression of $TC_t$. $\frac{1}{2} V_t^2 \sigma_r^2 \lambda_t$ is a natural choice of trading costs. To understand this, suppose that a dealer takes the other side of the trade, holds this position for a period of time and releases it at the end of the period. Then the dealer's risk is $V_t^2 \sigma_r^2$ and the trading cost is the dealer's compensation for risk, depending on the dealer's risk aversion reflected by $\lambda_t$. We assume the log value of the trading costs parameter $\lambda_t$ also depends on state variable $F_t$ and follows an AR(1) process.

$$\ln \lambda_t = \ln \lambda_0 + bF_t \tag{7}$$

$b$ is the sensitivity of $\ln \lambda_t$ to common shocks in $F_t$, and $\ln \lambda_0$ is the long-run mean of $\ln \lambda_t$. Trading-cost rate is constant over time if $b = 0$.

The investor is endowed with strictly positive initial wealth $W_0$ and has a finite investment horizon $T$. To simplify the problem, we assume that the investor only cares about the level of wealth at time $T$, $W_T$. In particular, the investor maximizes the expected CRRA utility of the final wealth $W_T$,

$$E_0(\frac{W_T^{1-\gamma} - 1}{1-\gamma}) \tag{8}$$

Under our setting, the weight in risky asset at each time step, $\alpha_t, t = 1, 2, , T-1$, serves as the control variable. Investor's objective is to choose the dynamic trading strategy $(\alpha_0, \alpha_1, \ldots, \alpha_{T-1})$ to maximize the expected utility of the final wealth at time $T$,

$$\max_{\alpha_0, \alpha_1, \ldots, \alpha_{T-1}} E_0(\frac{W_T^{1-\gamma} - 1}{1-\gamma}) \tag{9}$$

We define weight in risky asset for three different time points in each time step as below,

- $\alpha_{t-}$, weight in risky asset before rebalancing
- $\alpha_t$, weight in risky asset after rebalancing but before trading costs are paid
- $\alpha_{t+}$, weight in risky asset after rebalancing and trading costs are paid

At each time step $t$, the investor should first choose the optimal weight in risky asset before the trading costs are paid, $\alpha_t$, and $\alpha_{t+}$ is defined as the weight in risky asset after the trading costs are paid. Since the weight in risky asset will change after the trading costs are paid, $\alpha_t$ cannot be used directly for the calculation of trading costs. The amount of trading costs can be expressed as below:

$$TC_t(W_t, \alpha_t, \alpha_{t-}, \lambda_t) = \frac{1}{2}((\alpha_t - \alpha_{t-})W_t)^2 \sigma_{rc}^2 \lambda_t \tag{10}$$

$\alpha_{t-}$ is the weight in risky asset before rebalancing. $(\alpha_t - \alpha_{t-})W_t$ is the dollar amount of the trade at time step $t$. We assume that all trading costs are paid with the money in the risky asset. The weight in risky asset after trading costs are paid can be expressed as the wealth in risky asset after trading costs over the total wealth after trading costs,

$$\alpha_{t+} = \frac{\alpha_t W_t - TC_t}{W_t - TC_t} = \frac{\alpha_t W_t - \frac{1}{2}((\alpha_t - \alpha_{t-})W_t)^2 \sigma_{rc}^2 \lambda_t}{W_t - \frac{1}{2}((\alpha_t - \alpha_{t-})W_t)^2 \sigma_{rc}^2 \lambda_t} \tag{11}$$

Each weight in risky asset before trading costs are paid, $\alpha_t$ has a corresponding weight in risky asset after trading costs are paid, $\alpha_{t+}$. Using $\alpha_t$ as the control variable is exactly the same as using $\alpha_{t+}$. For convenience, we use $\alpha_t$ for the expression of wealth dynamic. The level of wealth in next time step, $W_{t+1}$, is

$$W_{t+1} = (1 - \alpha_t)W_t exp?(r_f) + (\alpha_t W_t - \frac{1}{2}((\alpha_t - \alpha_{t-})W_t)^2 \sigma_{rc}^2 \lambda_t) exp?(r_{t+1}) \tag{12}$$

and the weight of risky asset before rebalancing in next time step, $\alpha_{(t+1)-}$, is

$$\alpha_{(t+1)-} = \frac{(\alpha_t W_t - \frac{1}{2}((\alpha_t - \alpha_{t-})W_t)^2 \sigma_{rc}^2 \lambda_t) exp?(r_{t+1})}{W_{t+1}} \tag{13}$$

the value function $J$ at each time step $t$ can be expressed as

$$J(W_t, \alpha_{t-}, F_t, t) = \max_{\alpha_t} E_t(\frac{W_T^{1-\gamma} - 1}{1 - \gamma}) \tag{14}$$

and the Bellman equation for this dynamic portfolio choice problem is

$$J(W_t, \alpha_{t-}, F_t, t) = \max_{\alpha_t} E_t[J(W_{t+1}, \alpha_{(t+1)-}, F_{t+1}, t+1)] \tag{15}$$

The problem is solved using backward induction. We search numerically for the weight in risky asset $\alpha_t$ which maximizes the expected utility of the final wealth from the last period to the first.

# 4 Numerical Solution

In this section, we solve this dynamic portfolio choice problem numerically to understand how an investor should choose the weight in risky asset to benefit from the high returns and avoid the trading costs at the same time.

In this paper, we use the setting with time-varying expected returns as the benchmark setting. People may think it is more reasonable to use the setting with time-constant expected return as the benchmark and treat the setting with time-varying expected returns as a deviation. Since the target of this paper is to see how large a liquidity risk premium can be generated under a realistic setting, we need trading motives to generate it. When the expected return is time-constant, the only trading motive is to rebalance the portfolio. Such trading motive is usually very small, and it does not even exist under the usual condition of market clearing: both the initial weight and the optimal weight in risky asset are 100% in our setting. Therefore, we choose to use the setting with time-varying expected returns as the benchmark. With time-varying expected returns, the myopic optimal weight in risky asset varies over time, and such variation introduces exogenous trading motives into the problem.

## 4.1 Parameter Values

We assume that the long-term average of the expected annual returns of the risky asset is 4% ($\mu_0 = 0.04$) and the standard deviation of return shocks is 10% ($\sigma_r^2 = 0.01$). Risk free rate is 2% ($r_f = 0.02$), and the risk aversion level of our representative investor is 2.5 ($\gamma = 2.5$). Then if there is no trading cost and time variation in expected returns ($\mu_t = \mu_0 = 0.04$), the analytical solution of the optimal weight is

$$\alpha^{LongRun} = \frac{\mu_0 - r_f + \sigma_r^2/2}{\gamma\sigma_r^2} = 100\% \tag{16}$$

for all $t$. It means it is optimal for the representative investor to invest all his wealth in risky assets. To add the time variation of the expected returns to our setting, we set the standard deviation of the shocks on expect returns as 1% ($a = 0.01$)[4].

Since we want to document the largest possible liquidity risk premium be generated by the time variation of trading costs, we assume high trading-cost rates with the largest possible time variation in it. For the level of trading costs parameter $\lambda_c$, we take the estimates in Bikker, Spierdijk and van der Sluis (2007) as a reference and assume that the price impact of a 1.5 million dollar trade is 40 basis points, $\lambda_c = 26.88$[5], which indicates a trading costs of 20 basis points (half the price impact). This assumption is also consistent with the numbers found in most papers of price impact (for example, Chan and Lakonishok 1997 find a price impact about 54 bps, and Keim and Madhavan 1997 find a price impact about 30 bps to 65 bps). In addition, we allow the trading-cost rate to be 3 times higher in the robustness check. We set the parameter for time variation of the trading-cost rates to 0.3149 (b = 0.3149), which is calibrated using the ln value of the annual ILLIQ measure proposed in Amihud (2002)[6]. ILLIQ, as $\lambda$ in our model, is a measure of price impact calculated as the absolute value of the daily return divided by the daily dollar trading volume. Under this setting, the 95% confidence interval of $\lambda_t$ is $[0.18 * \lambda_c, 5.64 * \lambda_c]$, which means a 2 standard deviation positive (negative) shock on $\lambda_t$ makes it more than five times (less than one fifth) its long-term level.

We set the time persistency of the state variable $F_t$ as 0.7 ($\rho = 0.7$), which is calibrated using the monthly data of ILLIQ, and we set the initial wealth as 100 million dollars, ($W_0 = 1$), which is about the median size of U.S. hedge funds. Since hedge funds are more likely to be the marginal traders in the financial market and more likely to experience liquidity shocks than large mutual funds and pension funds do, we choose to use the median size of hedge funds in our benchmark setting. Considering there is only 1 asset in our economy, 100 million dollars holdings of one single asset is large enough to generate a significant price impact of trades. To further make sure we will not underestimate the liquidity risk premium, we also solve the problem with higher level of wealth, which equivalents to higher

[4]Assuming dividend yield as the only predictor, we use the dividend yield data from 1952 to 2010 for the calibration of parameter $a$. The calibration using monthly data indicates an annual standard deviation of 0.92%, and using annual data, it increases to 1.86%.

[5]It is calibrated using equation (5).

[6]The annual ILLIQ values from 1952 to 2010 are used for the calibration.

trading-cost rate under our setting[7]. We solve the portfolio choice problem for 10 years, 1 step each year. For the calculation of liquidity risk premium, we solve the problem for different values of the correlation between shocks on realized returns and trading-cost rates, $Corr(v_t, u_t) = 0, -0.2, -0.4, -0.6$.

To sum up, all assumptions we make try to push the liquidity level and liquidity risk premiums up. We assume high level of trading-cost rate, large time variation in it (from one fifth to five time the mean level), large trading motives (time varying expected returns, fixed frequency of rebuilding the portfolio and exogenous liquidity shocks later), single risky asset (no spreading of trades over stock to reduce the price impact of trades), large institutional investors (high price impact of trades and high exposure to liquidity shocks), and 1 year per step (no spreading of trades within a year to reduce the price impact of trades).

## 4.2 Numerical Results

The dynamic portfolio choice problem is solved by backward recursion. Gaussian Quadrature is used to deliver the joint distribution of shocks on state variable $F_t$ and return shocks $(v_t, u_t)$. 4 points are used for each shock. Figure 1 shows the weights in risky asset both before and after rebalancing under the benchmark case with time constant trading-cost rate. Here we assume zero correlation between the shocks of investment opportunity set and the returns shocks thus there is no hedging demand.

[Insert Figure 1 about here]

In Figure 1, we denote the weight before rebalancing, $\alpha_{t-}$, with black circle, the weight after rebalancing and trading costs, $\alpha_{t+}$, with red star, the myopic optimal weight if there is no trading costs, $\alpha_t^{Myopic}$, with pink cross, and optimal weight in long run if there is no trading costs, $\alpha^{LongRun}$, with green dash line. The expression of $\alpha_t^{Myopic}$ is

$$\alpha_t^{Myopic} = \frac{\mu_t - r_f + \sigma_r^2/2}{\gamma \sigma_r^2} \tag{17}$$

[7] Since the wealth level and the trading amount are the only 2 assumptions based on absolute value, and the trading-cost parameter $\lambda_c$ is calibrated based on the trading amount, an increase in trading-cost parameter $\lambda_c$ equivalents to an increase in wealth level in our model.

which varies over time with the conditional expected return $\mu_t$.

We can see from Figure 1, in each time step, the investor trades partially towards the myopic optimal weight, $\alpha_t^{Myopic}$. If investor trades the entire way from $\alpha_{t-}$ to $\alpha_t^{Myopic}$, he needs to pay a large amount of trading costs; but if he does not trade at all and keep the weight at $\alpha_{t-}$, he loses too much utility by deviating from the $\alpha_t^{Myopic}$. Therefore, it is optimal to trade partially towards the aim. The optimal amount to trade is decided by the trade-off between the marginal utility gain of getting closer to the aim and the marginal trading costs incurred. We will show later that both the loss of utility caused by deviation from $\alpha_t^{Myopic}$ and the actual trading costs incurred should be compensated in the form of a higher expected return (liquidity premium). Besides, the investor resists to trade further away from $\alpha^{LongRun}$ since it will generate more trading costs in the future. These results are consistent with the main findings in Garleanu and Pedersen (2013), when trading is costly, the investor should trade partially towards the current aim, and also aim in front of the target (consider the long-run optimal weight, $\alpha^{LongRun}$).

# 5 Liquidity Level Premium and Liquidity Risk Premium

In the previous section, we have shown that trading costs make investor deviate from the optimal solution in the frictionless market. In a competitive market, investor should require a liquidity premium to compensate for the loss of utility caused by trading costs. Therefore, in this section, we compute both the liquidity level premium, the premium compensates for the level of trading costs, and the liquidity risk premium, the premium compensates for the time variation of trading costs.

In this paper, the liquidity level premium is defined to be the decrease in the long-term level of the expected return, $\mu_0$, on the trading-cost-free risky asset that the investor requires to be indifferent between having access to the risky asset without rather than with trading costs. The liquidity risk premium is defined to be the decrease in the long-term level of the expected return, $\mu_0$, on the trading-cost-free risky asset that the investor requires to be indifferent between having access to the risky asset without rather than with *time variation* in trading costs. Hence these premiums are calculated under a partial equilibrium setting.

If the premium is positive, it means that the investor should be compensated for the loss caused by trading costs or time variation of trading costs, and if it is negative, investor even benefits from it.

First, we discuss intuitively the sources of liquidity level premium and liquidity risk premium. As we mentioned in previous section, investors should be compensated for both the actual trading costs and the loss of utility caused by the deviation from the optimal weight. Liquidity level premium measures such compensations in term of a higher expected return. It is worth noting that the part of liquidity level premium compensating for the actual trading costs depends on the total amount of trading costs which is the product of cost rate and total trading amount, rather than the cost rate itself. The larger the trading amount, the higher the liquidity level premium.

The liquidity risk premium measures the compensation for the loss of utility caused by the time variation of trading-cost rates, also in term of the increase in expected return. The time variation of trading-cost rates has three different effects on the utility of the investor, thus it also enters the liquidity risk premium through three different channels: the variance of the cost rates (*Variance Effect*), the covariance between trading costs and stock returns (*Covariance Effect*), and the additional freedom to choose the weight in risky asset introduced by the time variation of cost rates (*Choice Effect*).

1. *Variance Effect:* Since the representative investor is risk averse, the time variation of trading costs should be penalized. A positive premium should be required as a compensation.

2. *Covariance Effect:* Investor hates to pay large amount of trading costs during the market downturn, hence negative covariance between the trading costs and stock returns $Cov(c_t, r_t)$ should be penalized, and a positive premium should be required as a compensation.

3. *Choice Effect:* Under dynamic setting, investor responses actively to the time variation of cost rates. Investor trades more when cost rate is low and trades less when it is high. In this case, investor benefits from the time variation of cost rates, and if this

effect dominates, a negative liquidity risk premium should be found. We show later that it is actually negative under some settings.

To sum up, whenever we try to explain the magnitude of the liquidity risk premium, we should bear in mind that it is an aggregated result of these three different effects instead of just a single one.

## 5.1 Benchmark Setting

Under the benchmark setting, the only trading motive is the time-varying myopic aim introduced by the time-varying expected returns. To calculate the liquidity level premium and liquidity risk premium, we solve four different cases of the portfolio choice problem:

*Case 1:* with constant expected return and *no* trading costs
($a = 0, b = 0, \lambda_t = \lambda_c = 0$)
*Case 2:* with time-varying expected returns and *no* trading costs
($a = 0.01, b = 0, \lambda_t = \lambda_c = 0$)
*Case 3:* with time-varying expected returns and *constant* trading-cost rate
($a = 0.01, b = 0, \lambda_t = \lambda_c = 26.88$)
*Case 4:* with time-varying expected returns and *time-varying* trading-cost rates
($a = 0.01, b = 0.315, \lambda_c = 26.88$)

We calculate the expected utility for each case. The initial position in risky asset is assumed as 100%, the long-term optimal weight in risky asset. *Case 1*, the case with constant expected return and no trading costs is used as the benchmark case for the calculation of liquidity level and liquidity risk premiums. Specifically, for each of *Case 2,3,4*, we find the corresponding level of expected return in the benchmark case (*Case 1*) which makes the investor indifferent between holding the risky asset in this case and the one in the benchmark case (it means investor has the same expected utility when holding any of these two risky assets). Then we compare the corresponding levels of expected returns across different cases. The difference of corresponding expected returns between *Case 2* and *Case 3* is recorded as liquidity level premium, and the difference of the corresponding expected returns between *Case 3* and *Case 4* is recorded as liquidity risk premium. The liquidity risk premium for $Cov(\lambda_t, r_t)$ is calculated as the liquidity risk premium additional to the liquidity risk premium for the case with zero correlation between return shocks and shocks on trading-cost rate, $Cov(\lambda_t, r_t) = 0$.

[Insert Table 1 about here]

Table 1 reports the liquidity level premium and liquidity risk premium under the benchmark setting, for different values of the correlation between the return shocks and shocks on trading-cost rate (shocks on state variable $F_t$), 0, -0.2, -0.4, -0.6. Firstly, we could see the magnitudes of liquidity risk premium are extremely small, ranging from -0.623 to -0.104 bps, significantly smaller than the liquidity level premium, which ranges from 16.43 to 15.68 bps. The effect of time variation of cost rates on investors utility is substantially smaller than the effect of the level of trading costs. Although the premium for $Cov(c_t, r_t)$, which large documented in previous literature as the main component of liquidity risk premium such as Pastor and Stambaugh (2003) and Acharya and Pedersen (2005), indeed increases as the correlation between returns and cost rates becomes more negative. It is always smaller than 1 bps and accounts for 3.3% of the total liquidity premium at most.

You may wonder why liquidity risk premiums are negative in all 4 settings. As we have mentioned, it means the *Choice Effect* dominates the *Variance Effect* and *Covariance Effect*, investor is willing to have time variation in trading-cost rates since he/she can react according to the realized cost rate and it increases the expected utility. To better explain how the *Choice Effect* generates a negative liquidity risk premium, we first have a look at the expected utility of the investor. The solid blue curve in Figure 2 plots the expected utility of the final wealth as a function of the trading-cost parameter $\lambda_c$ under the optimal strategy, for the benchmark case with time constant trading-cost rate and zero correlation between return shocks and shocks on $F_t$. Since trading costs limit investor's rebalancing of the portfolio, the expected utility decreases with the increase of trading-cost parameter $\lambda_c$. When $\lambda_c$ becomes larger, investor will choose to trade less, then the increase of $\lambda_c$ will have a smaller effect on investors expected utility. Therefore, the expected utility is strictly convex in the trading-cost parameter $\lambda_c$, which means the linear combination of any two points on the blue curve is above the blue curve. Then if we assume $\lambda_c$ is stochastic at time 0, instead of deterministic, the expect utility of the investor should always be higher than the expect utility indicated by the blue curve. The uncertainty in trading-costs parameter $\lambda_c$ always increases the expected utility of investor. Because of this, when we introduce the time variation into the trading-cost parameter $\lambda$, the *Choice Effect* makes the liquidity risk premium in the benchmark setting negative.

Secondly, it is also worth noting that liquidity level premium, ranging from 15.68 to 16.48 bps, is significantly smaller than the 65 bps [8] price impact costs of an average trade under the benchmark setting. This result is consistent with Constantinides (1986), which shows the liquidity premium is an order of magnitude smaller than the trading-cost rate itself. The reason is that the liquidity level premium compensates for the actual trading costs generated and the loss of utility caused by the existence of trading costs, rather than simply the rate of trading costs. When the loss of utility of trading less is small, investor will choose to trade less to avoid some trading costs. Under the benchmark setting, the only trading motive is to trade towards the time varying myopic optimal weight introduced by the time varying expected return $\mu_t$. If there is no trading cost, the utility gain of doing that is about 22 bps in term of a higher expected return $\mu_0$. It means the upper limit of the liquidity premium is 22 bps, since if trading-cost rate is too high, investor will choose to not trade at all and bear all the utility loss of not trading. As Table 1 shows, the total liquidity premium is about 15.27-16.12 bps, slightly smaller than the 22 bps. It indicates that it is optimal for investor to trade slightly towards the myopic optimal weight to reduce the utility loss of investing sub-optimally. Later in Table 2, we show that a small faction of liquidity level premium compensates for actual trading costs, about 4.18 bps (out of 16.34 bps), and a large fraction compensates for the utility loss of deviating from the myopic optimal weight, about 12.65 bps (out of 16.34 bps). This result is in accordance with the conjecture in Galeanue and Pedersen (2013) that investor balances between the trading costs and the loss of utility caused by deviating of the myopic optimal weight to maximize the expected utility.

[8] The average trading amount for the benchmark setting is 4.9 million dollars corresponding to a price impact costs of 1/2*20bps* (4.9m/1.5$m$) = 65.3 bps.

To further investigate how the level of trading-cost rate affects the magnitudes of liquidity risk premium as well as liquidity level premium, we solve the dynamic problem for different values of trading-cost parameter, $\lambda_c$=3.36 (1/8*26.88), 6.72, 13.44, 26.88, 53.76 and 107.52 (4*26.88). In addition, to see how investor adjusts the trading amount according to the level of cost rate, and how it affects the composition of liquidity level premium, we decompose the liquidity level premium into the premium compensating for trading costs (liquidity level premium compensating for TC), and the premium compensating for the loss of utility caused by deviating from the myopic optimal weight if there is no trading cost (liquidity level premium compensating for the deviation from optimal weight). The liquidity level premium compensating for TC is calculated by simulations. We calculate the trading costs generated as a percentage of total wealth for each time step of each simulation, and use the average value across all 10,000 simulations and all 10 steps each as a measure of liquidity level premium compensating for the actual trading costs. The liquidity level premium compensating for the deviation from optimal weight is calculated by deducting the liquidity level premium compensating for TC from the total liquidity level premium. The average trading amount per time step across all 10,000 simulations and all 10 steps each is also reported for each level of the cost rate.

[Insert Table 2 about here]

Table 2 reports liquidity level premium, for the parts compensating for TC and compensating for the deviation from optimal weight separately, liquidity risk premium and average trading amount across different trading-cost rates, from 3.36 (1/8*26.88) to 107.52 (4*26.88). The correlation between return shocks and shocks on $F_t$, $Corr(u_t, v_t)$ is set to -0.3 in all cases. We could see that the liquidity risk premium changes only slightly from -0.494 bps to -0.297 bps when the cost rate becomes 4 times as large as before. Considering a cost rate 4 times large indicates a 1.6% price impact from a 1.5 million $ trade, the liquidity risk premium of -0.297 bps is negligible. The magnitude of cost rate does not have a significant effect on the liquidity risk premium under our benchmark setting. It is worth noting that since the wealth level and the trading amount are the only 2 assumptions based on dollar value in our model, and the trading-cost parameter $\lambda_c$ is calibrated based on the trading amount, an increase in trading-cost parameter $\lambda_c$ equivalents to an increase in wealth level in our model. It means the liquidity premiums calculated for the setting with initial level of wealth as 100 million dollars and $\lambda_c$=4*26.88 are the same as the setting with initial level of wealth as 400 million dollars and $\lambda_c$=26.88. Therefore, the results shown in Table 2 also indicate that the liquidity risk premium is small also for higher levels of wealth.

[Insert Figure 3 about here]

Consistent with the Constantinides (1986) and Galeanue and Pedersen (2013), we find that investor to trade less when the trading-cost rate is higher. Table 2 and Figure 3 both show that the average trading amount per year decreases from 4.9 million dollars to 2.0 million dollars when the cost rate becomes 4 time the level before, and it increases to 14.8 million dollars when the cost rate becomes one eighth the level before. In addition, Figure 3 shows that the optimal trading amount is decreasing and convex in the cost rate. Trading amount is more sensitive to the cost rate when it is low. Also, as the model in Galeanue and Pedersen (2013) predicts, investor balances between the trading costs and the loss of utility caused by deviating from the optimal weight when there is no trading cost to maximize the expected utility. The relative importance of liquidity level premium compensating for TC decreases monotonically with the trading-cost rate (from 64% for $\lambda_c = 1/8 * 26.88$ to 14% for $\lambda_c = 4 * 26.88$); and the premium compensating for the deviation from optimal weight decreases with the cost rate (from 2.88 bps for $\lambda_c = 1/8 * 26.88$ to 18.05 bps for $\lambda_c = 4 * 26.88$). More interestingly, different from the implications of models in Amihud and Mendelson (1986) and Acharya and Pedersen (2005) etc., Figure 4 shows that rather than being proportional to trading-cost rate, the equilibrium liquidity premium is increasing and concave in the trading-cost rate, and there is an upper limit on the liquidity premium, which is about 22 bps for our benchmark setting. It is because in their models, both the trading amount and the trading frequency are exogenous, total trading costs, as the product of cost rate and total trading amount, increases linearly with the cost rate. While in our benchmark setting, the trading amount is decided endogenously by the tradeoff between trading costs and utility gain of trading more. As we have mentioned, when the trading-cost rate is too high, investor will choose not to trade at all and bear all the utility loss of not trading, which makes the upper limit of the liquidity premium. Therefore, under our setting, the liquidity premium does not only depend on the trading-cost rate, trading amount and trading frequency, but also on the sensitivity of investors expected utility on the trading behavior.

[Insert Figure 4 about here]

### 5.2 Setting with Fixed Frequency of Rebuilding and Releasing

Until now, we assume that the only trading motive faced by the representative investor is the time-varying expected returns. In the real world, investors may choose to rebuild their portfolios at a fixed time frequency. One main thread of liquidity literature (e.g. Amihud and Mendelson 1986 and Acharya and Pedersen 2005) develops based on this assumption and find significant empirical evidence of both liquidity level premium and liquidity risk premium. Following their spirits, we solve the dynamic problem under the assumption that investor releases and rebuilds his/her portfolio at a fixed time frequency in this subsection, to see whether this assumption helps to generate a liquidity risk premium comparable to that found empirically in term of magnitude.

[Insert Figure 5 about here]

We solve the problems for different frequencies of rebuilding the portfolio, per 1 year, 2 years, 5 years and 10 years. And for each frequency, we solve the problem for 3 values of the correlation between return shocks and the shocks on $F_t$, 0, -0.3 and -0.6. As an example, Figure 5 plots the trajectory of the optimal weights invested in risky asset for rebuilding and releasing the portfolio every 10 years. The case with zero correlation is plotted. It shows that it is optimal for investor to trade gradually during the rebuilding and releasing of the portfolio to reduce the price impact of trades, as predicted in Galeanue and Pedersen (2013).

Using the same partial equilibrium approach, we calculate the liquidity level premiums and liquidity risk premium for different frequencies of rebuilding the portfolio. Table 3 shows that liquidity risk premiums are still very small, from 0.847 bps to 2.659 bps out of a total liquidity premium from 94.98 bps to 212.15 bps. The assumption of fixed frequency of releasing and rebuilding the portfolio does not help to generate a larger liquidity risk premium. Therefore, it is no the reason of the large liquidity risk premium found in Acharya and Pedersen (2005).

Besides, similar to the intuition of a higher cost rate, as forced releasing and rebuilding of the portfolio becomes more frequent, it is optimal for investor to invest less into the risky asset and thus trade less and pay less trading costs. As Table 3 shows, liquidity level premium compensating for TC and average trading amount per year both decrease as the rebuilding of portfolio becomes more frequent, and liquidity level premium compensating for the deviation from the optimal weight increases. In addition, the total liquidity premium ranges from 94.98 bps to 212.15 bps, never goes much above 200 bps, since if the trading frequency or cost rate becomes too high, investor can choose to not hold risky asset at all and give up all the excess return provided by the risky asset, 2%.

[Insert Table 3 about here]

### 5.3 Setting with Exogenous Liquidity Shocks

The results in section 5.1 and 5.2 show that the magnitude of liquidity risk premium is negligible under settings of dynamic portfolio choice problem with time-varying expected returns or both time-varying expected returns and rebuilding of the portfolio at a fixed time frequency as trading motives. Does that mean liquidity risk premium is always negligible in the financial market, and all the large liquidity risk premiums documented in the recent liquidity literature are wrong? Not necessary. We all know that during periods of the crisis (e.g., the 1987 market crash, the 1997 Asian crisis, the Russian debt crisis of 1998, the hedge-funds meltdown of 2007, and the 2008 financial crisis), market liquidity goes down, trading-cost rate goes up substantially, at the same time, institutional investors are forced to release large amount of their positions. The large amount of trading costs paid for their forced selling hurts those already wounded investors even harder. Because of this, investors are supposed to worry about the high trading costs during market downturn a lot and require large compensation for that.

### 5.3.1 Assumptions of Exogenous Liquidity Shocks

To investigate how large a liquidity risk premium can be generated by the high trading costs during the market downturn, in this section, we add exogenous liquidity shocks into our benchmark model[9]. To further identify the importance of the correlation between the trading motive and the market conditions, we distinguish two types of liquidity shocks: the liquidity shocks depending on market conditions, and the liquidity shocks independent of the market conditions. Since there is only one risky asset in our model, the changes of the market conditions equivalent to the changes of the stock returns of the risky asset in this paper.

For the cases with an exogenous liquidity shocks *depending* on stock returns, if the risky asset performs badly (with a realized return $r_{t+1}$ more than one standard deviation, $\sigma_r$, lower than the conditional mean $\mu_t$), the investor should release a proportion (or all) of his/her positions in risky asset and repurchase them later. The effect of liquidity shocks on the portfolio weight in risk asset, $\Delta\alpha_{(t+1)-}$, is as below:

*if* $\mu_t - 3\sigma_r \leq r_{t+1} \geq \mu_t - \sigma_r$, *investor releases a proportion of his/her positions in risky asset, the positions released expressed as a weight in risky asset is*

$\Delta\alpha_{(t+1)-} = \alpha_{(t+1)-} * \frac{r_{t+1}-(\mu_t-\sigma_r)}{2\sigma_r}$,

$\Delta\alpha_{(t+1)-} = 0$, *if* $r_{t+1} = \mu_t - \sigma_r$;

$\Delta\alpha_{(t+1)-} = -\alpha_{(t+1)-}$, *if* $r_{t+1} = \mu_t - 3\sigma_r$.

*if* $r_{t+1} < \mu_t - 3\sigma_r$, *investor releases all his positions in risky asset*

$\Delta\alpha_{(t+1)-} = -\alpha_{(t+1)-}$.

For the cases with an exogenous liquidity shocks *independent* of stock returns, we substitute the stock return $r_{t+1}$ by a random variable $\epsilon_{t+1}$ which follows normal distribution with a mean of 0 and a standard deviation of 1:

$\epsilon_{t+1} \sim N(0, 1)$;

[9]In the real world, investors are usually forced to release part of their positions when market goes down. For example, the mutual fund literature has a long history of documenting the flow-performance sensitivity (e.g. Warther 1995, Sirri and Tufano 1998, Froot, O'connell and Seasholes 2001, Huang, Wei and Yan 2007 etc.), they all show there are more fund outflows during market downturn; Brunnermeier and Pedersen (2009) claim that investors' capital and margin requirements are binding when market deteriorates, thus they are forced to reduce their positions and

*if* $-3 \leq \epsilon_{t+1} \geq -1$*, investor releases a proportion of his/her positions in risky asset, the positions released expressed as a weight in risky asset is*

$\Delta\alpha_{(t+1)-} = \alpha_{(t+1)-} * \frac{\epsilon_{t+1}-(-1)}{2}$,

$\Delta\alpha_{(t+1)-} = 0$, if $\epsilon_{t+1} = -1$;

$\Delta\alpha_{(t+1)-} = -\alpha_{(t+1)-}$*, if* $\epsilon_{t+1} = -3$.

*if* $\epsilon_{t+1} < -3$*, investor releases all his positions in risky asset*

$\Delta\alpha_{(t+1)-} = -\alpha_{(t+1)-}$.

### 5.3.2 Calculation of Liquidity Level and Liquidity Risk Premiums

To calculate the liquidity level premium and liquidity risk premium, similar to the benchmark setting, we solve four different cases of the portfolio choice problem:

*Case 1:* with constant expected return, exogenous liquidity shocks *independent* of stock returns, and *no* trading costs[10]

$(a = 0, b = 0, \lambda_t = \lambda_c = 0, Cov_t(V_{t+1}^2, r_{t+1}) = 0)$

*Case 2:* with *time-varying* expected returns, exogenous liquidity shocks *independent* of stock returns, and *no* trading costs

$(a = 0.01, b = 0, \lambda_t = \lambda_c = 0, Cov_t(V_{t+1}^2, r_{t+1}) = 0)$

*Case 3:* with time-varying expected returns, exogenous liquidity shocks *independent* of stock returns, and *constant* trading-cost rate

$(a = 0.01, b = 0, \lambda_t = \lambda_c = 26.88, Cov_t(V_{t+1}^2, r_{t+1}) = 0)$

*Case 4:* with time-varying expected returns, exogenous liquidity shocks *depending* on stock returns, and *time-varying* trading-cost rates

$(a = 0.01, b = 0.315, \lambda_c = 26.88, Cov_t(V_{t+1}^2, r_{t+1}) > 0)$

As before, the initial position in risky asset is assumed as 100%. *Case 1*, the case with constant expected return, exogenous liquidity shocks independent of stock returns, and no trading costs is used as the benchmark case for the calculation of liquidity level and liquidity risk premiums. For each of *Case 2,3,4*, we find the corresponding level of expected return in

[10]Since there is no trading cost, invest will instantly trade back to the optimal weight after liquidity shocks without any loss of utility and additional costs. Therefore, *Case 1,2* with liquidity shocks are exactly the same as the *Case 1,2* in benchmark setting. In our setting, liquidity shocks affect investors only if there are trading costs.

the benchmark case (*Case 1*) which makes the investor indifferent between holding the risky asset in this case and the one in the benchmark case. Then the difference of corresponding expected returns between *Case 2* and *Case 3* is recorded as liquidity level premium, and the difference of the corresponding expected returns between *Case 3* and *Case 4* is recorded as liquidity risk premium.

It is worth noting that under the benchmark setting, the difference between *Case 3* and *Case 4* is only that *constant* trading-cost rate becomes *time-varying*. But under this setting with liquidity shocks, in addition, exogenous liquidity shocks *independent of* stock returns become *depending on* stock returns. The reason is that besides the covariance between trading-cost rates and stock returns $Cov_t(\lambda_{t+1}, r_{t+1})$, the negative covariance between the square of trading amount (forced selling) and stock returns $Cov_t(V_{t+1}^2, r_{t+1})$ also generates liquidity risk premium. To calculate the total liquidity risk premium, we need to include both effects, and the interaction of these two effects.

### 5.3.3 Relation to liquidity-adjusted CAPM in Acharya and Pedersen (2005)

The liquidity-adjusted CAPM proposed by Acharya and Pedersen (2005) assumes that investor has fixed investment horizon with end releasing and no rebalancing in between (the same as the assumption in section 5.2), and they use the ILLIQ, which equivalents to our $\lambda$, as a measure of the effective percentage trading costs. Under their assumptions, the liquidity risk premium can be measured by the covariance between the trading-cost rates and the stock returns, $Cov_t(\lambda_{t+1}, r_{t+1})$, since $\lambda_t$ is proportional to the effective percentage trading costs $c_t$. But it is more realistic to assume that investor has a trading motive which depends on the market condition, like under our setting with liquidity shocks. If we relax the assumptions in Acharya and Pedersen (2005) by allowing the trading amount to be different from the holding amount, (assume the holdings of the investor at time t is $H_t$, and the trading amount is $V_t$ which is smaller than $H_t$ and varying over time), then we have the actual trading costs $\hat{c}_t$ as a percentage of previous holdings $H_{t-1}$ as

$$\hat{c}_t = \frac{c_t V_t}{H_{t-1}} = \frac{1}{2} \frac{\sigma_r^2 \lambda_t V_t^2}{H_{t-1}} \neq \lambda_t, s.t. 0 \leq V_t \leq H_t \tag{18}$$

Then following the logic of the liquidity-adjusted CAPM in Acharya and Pedersen (2005), the liquidity risk should be priced by $Cov_t(\hat{c}_{t+1}, r_{t+1})$, the covariance between the actual trading costs paid as a percentage of holdings and stock returns, instead of $Cov_t(\lambda_{t+1}, r_{t+1})$, the covariance between cost rates and stock returns. Since trading costs $\hat{c}_{t+1}$ depends on both cost rate $\lambda_t$ and trading amount $V_t$, if investor has to trade a lot when the stock return $r_t$ is low, the liquidity risk $Cov_t(\hat{c}_{t+1}, r_{t+1})$ is high even if the cost rate $\lambda_t$ does not change with stock return $r_t$. Therefore, the correlation between trading amount and stock return $Cov_t(V_{t+1}^2, r_{t+1})$ [11] is also an important element of liquidity risk which is not covered by the liquidity-adjusted CAPM in Acharya and Pedersen (2005). In addition, the interaction between $Cov_t(\lambda_{t+1}, r_{t+1})$ and $Cov_t(V_{t+1}^2, r_{t+1})$ could lead to even higher level of liquidity risk than the simple sum of these two.

### 5.3.4 Decomposition of Liquidity Risk Premium

Previous empirical literature shows that when the market goes down, stock liquidity tends to go down ($\lambda_t$ increases) and investors sell more ($V_t^2$ increases). It means stock liquidity $\lambda_t$ and trading amount $V_t^2$ are highly correlated. Since liquidity-adjusted CAPM only has $Cov_t(\lambda_{t+1}, r_{t+1})$ on the right side of the equation, the liquidity risk premium calibrated by Acharya and Pedersen (2005) captures not only the covariance between stock liquidity and returns as they claim, but also a large part of $Cov_t(V_{t+1}^2, r_{t+1})$, and the interaction effect. To separate the liquidity risk premium induced by $Cov_t(\lambda_{t+1}, r_{t+1})$, $Cov_t(V_{t+1}^2, r_{t+1})$ and the interaction effect included in $Cov_t(\lambda_{t+1} V_{t+1}^2, r_{t+1})$, we solve 2 additional cases for the portfolio choice problem with liquidity shocks:

*Case 4-1*: with time-varying expected returns, exogenous liquidity shocks *depending on* stock returns, and *constant* trading-cost rate

$(a = 0.01, b = 0, \lambda_t = \lambda_c = 26.88, Cov_t(V_{t+1}^2, r_{t+1}) > 0)$

*Case 4-2*: with time-varying expected returns, exogenous liquidity shocks *independent of* stock returns, and *time-varying* trading-cost rates

[11] According to Equation 18, we could see trading costs as a percentage of holdings $\hat{c}_t$ actually depends on $Cov_t(V_{t+1}^2, r_{t+1})$, rather than $Cov_t(V_{t+1}, r_{t+1})$ since $\hat{c}_t$ increase with both trading amount $V_t$ and the price impact, which increases with the trading amount $V_t$ as well.

$(a = 0.01, b = 0.315, \lambda_c = 26.88, Cov_t(V_{t+1}^2, r_{t+1}) = 0)$

Again, for *Case 4-1* and *Case 4-2*, we find the corresponding level of expected return in the benchmark case (*Case 1*) which makes the investor indifferent between holding the risky asset in this case and the one in the benchmark case. Since the only difference between *Case 4-1* and *Case 3* is that liquidity shocks depend on stock returns, the difference of corresponding expected returns between these 2 cases is recorded as liquidity risk premium for $Cov_t(V_{t+1}^2, r_{t+1})$. Similarly, since the only difference between *Case 4-2* and *Case 3* is that trading-cost rate becomes time-varying, the difference of the corresponding expected returns between these 2 cases is the total liquidity risk premium induced by the time variation of trading-cost rates. And using the case with zero correlation between return shocks and shocks on state variable $F_t$ ($Corr_t(u_{t+1}, v_{t+1}) = 0$), liquidity risk premium for $Cov_t(\lambda_{t+1}, r_{t+1})$ can be calculated; and the difference of the corresponding expected returns between *Case 4* and *Case 3* is the total liquidity risk premium induced by the liquidity shocks depending on stock returns and the time variation of trading-cost rates together. The liquidity risk premium for $Cov_t(\lambda_{t+1} V_{t+1}^2, r_{t+1})$ is also calculated using the case with zero correlation between return shocks and shocks on state variable $F_t$.

### 5.3.5 Liquidity Risk Premiums for the Setting with Liquidity Shocks

We solve the dynamic portfolio choice problems with exogenous liquidity shocks for 3 different values of the correlation between return shocks and shocks on trading-cost rates, 0, -0.3, -0.6. Table 4 shows liquidity level premiums and liquidity risk premiums calculated using the partial equilibrium approach mentioned before. Here we could see that the total liquidity risk premium under the setting with exogenous liquidity shocks is significantly larger than before, 11.53 bps for the case with a correlation of -0.3 between stock returns and cost-rates, and 20.83 bps for the case with a correlation of -0.6. And it accounts for a substantial fraction of the total liquidity premium, 18% and 28% correspondingly. Although the liquidity level and liquidity risk premiums generated here are still substantially smaller than the 4%-7% documented empirically, in term of relative importance, this result is comparable to that in Acharya and Pedersen (2005), who find that 1.1% out of 4.6% liquidity premiums compensates for liquidity risk. More interestingly, though the total liquidity risk premium can be as large as 20.83 bps, the liquidity risk premiums for $Cov_t(V_{t+1}^2, r_{t+1})$ and $Cov_t(\lambda_{t+1}, r_{t+1})$ individually are very small, only about 6.03 bps for $Cov_t(V_{t+1}^2, r_{t+1})$, and 2.13 bps for $Cov_t(\lambda_{t+1}, r_{t+1})$. The large total liquidity risk premium mainly comes from the interacted covariance, $Cov_t(\lambda_{t+1} V_{t+1}^2, r_{t+1})$, 18.61 bps out of 20.83. Therefore, it is the large trading amount and high trading-cost rate during the market downturn together that hurt the investor and make him/her require a large liquidity risk premium. None of these two effects itself is sufficient to generate a large liquidity risk premium. However, previous researches about liquidity risk premium, such as Amihud (2002), Pastor and Stambaugh (2003), Acharya and Pedersen (2005) etc., attribute the liquidity risk premiums purely to the covariance between the trading-cost rates and stock returns, $Cov_t(\lambda_{t+1}, r_{t+1})$, and neglect the important role of the covariance between the trading amounts and stock returns, $Cov_t(V_{t+1}^2, r_{t+1})$ and their interacting effect, which is the main source of the liquidity risk premium according to our analysis.

[Insert Table 4 about here]

Besides, under the setting with exogenous liquidity shocks, the liquidity level premium mainly compensates for the actual trading costs, about 30 out of 50 bps, rather than the deviation from the optimal weight if there is no trading costs. It is because the forced selling caused by liquidity shocks is inelastic to the level of trading-cost rate, investor can only reduce the amount of the forced sales by investing less in risky assets. But it sacrifices too much of the expected returns when compared with the possible trading costs caused by the potential liquidity shocks.

Now, we have shown that the liquidity risk premium can account for as large as 28% of the total liquidity premium when investor is forced to sell during the market downturn at a cost rate higher than during the normal time. Next, to check the robustness of this finding, we investigate how the relative importance of liquidity risk premium changes with the level of trading-cost rate and the frequency of the liquidity shocks.

First, we solve the same problem with exogenous liquidity shocks for different levels of trading-cost rates $\lambda_c$, from 3.36(1/8*26.88) to 107.52(4*26.88). The correlation between return shocks and shocks on trading-cost rate is set to -0.3 for all cases. We could see from Table 5, both the liquidity level premium and liquidity risk premium increases with the trading-cost rate. As the cost rate becomes 4 times as large as before, the liquidity risk premium increases from 11.53 bps to 21.54 bps, and the relative importance decreases from 18% of total liquidity premium to 12%. Though it decreases slightly, it still accounts for a significant fraction of the total liquidity premium. Moreover, as we predict, the average trading amount decreases as the trading-cost rate becomes higher, and the % of liquidity premium compensating for TC also decreases from 61% to 57%, since investor chooses to invest less into risky asset to reduce the total trading amount and thus bear more losses of utility caused by the under investment.

[Insert Table 5 about here]

Secondly, we solve the same problem for lower frequencies of liquidity shocks, per 2 years and per 5 years. It means instead of facing the probability of forced selling every year in previous setting, investor faces it only every 2 years (or 5 years), We solve it for 3 values of correlation between return shocks and shocks on cost rates each, 0, -0.3, -0.6. It means that every 1, 2 or 5 years, investor releases part or all of his/her positions if the return of the past year is more than one standard deviation below the conditional mean $\mu_t$. We could see from the results reported in Table 6, both liquidity level premium and liquidity risk premium increases as the liquidity shocks become more frequent. The relative importance of liquidity risk premium almost are almost the same for liquidity shocks per year and per 2 years, about 18% of total premium when the correlation is -0.3, and about 28% when the correlation is -0.6. And it decreases slightly to 12% and 22% as the liquidity shocks become indeed infrequent, per 5 years, and it is negligible for the case with no liquidity shocks at all as we have shown in the benchmark setting. Besides, both average trading amount and liquidity level premium compensating for TC increases as liquidity shocks become more frequent, and the relative importance of premium for TC increases.

[Insert Table 6 about here]

To sum up, in this section, using the setting with exogenous liquidity shocks, we find that liquidity risk premium is economically significant if and only if investor is forced to trade during the market downturn, and trading-cost rate goes up at the same time. Liquidity risk premium, instead of originating from the covariance between trading-cost rates and the return shocks, $Cov_t(\lambda_{t+1}, r_{t+1})$, as claimed in most papers of liquidity risk, mainly originates from the covariance between the total trading costs (the product of trading amounts and trading-cost rates) and the return shocks, $Cov_t(\lambda_{t+1}V_{t+1}^2, r_{t+1})$. In addition, we show the importance of liquidity risk premium remains for different levels of trading-cost rates and different frequencies of liquidity shocks.

# 6 The Relation between Market Turnovers and Market Returns

In this section, we document the correlations and the covariances of aggregate market turnovers and market returns using real market data , and we compare them with those of return and turnover trajectories predicted by our model. The correlations and covariances in market data are in general consistent with those in our benchmark setting, and the extremely negative correlation in our setting with exogenous liquidity shocks indicates that our assumption of liquidity risk is very strong. Therefore, based on the small liquidity risk premium in the equilibrium of our setting, our claim that the liquidity risk premium required by long-term investors is very small is conservative.

## 6.1 Market Data

Since both market turnover and market liquidity are highly persistent over time, we firstly do an AR(1) regression for the ln values of both turnover and ILLIQ to capture the innovations in market turnover and liquidity. We use the data from 1966 to 2010.

$$lnTrn_{t+1} = \alpha^{trn} + \rho^{trn} lnTrn_t + \varepsilon_{t+1}^{trn} \tag{19}$$

$$lnILLIQ_{t+1} = \alpha^{ILLIQ} + \rho^{ILLIQ} lnILLIQ_t + \varepsilon_{t+1}^{ILLIQ} \quad (20)$$

We find that both the market turnover and ILLIQ are quite persistent at annually frequency. The time persistency of market turnover, $\rho^{trn}$ in equation (19), is 0.9987, and the R square of equation (19) is 0.9696. The time persistency of ILLIQ, $\rho^{ILLIQ}$ in equation (20), is 0.9735, and the R square of equation (20) is 0.9135.

Table 7 reports the correlations between the annual[12] market excess returns, the innovations in market liquidity and the innovations in market turnover from 1966 to 2010, and the covariance between the annually market excess returns and the innovations in market turnover. In accordance with the forced selling during the market downturn, Panel C of Table 7 reports a negative correlation (-0.170) between market excess returns and market turnovers when market excess returns $R_M - r_f$ are negative, but it is not significant because of the small number of observations. And the correlation between market excess returns and market turnovers is positive for the entire sample (0.203), which is probably because investors on average trade more during the bull market than bear market. In addition, Panel B of Table 7 reports a significant negative correlation (-0.584) between market excess returns and the innovations in lnILLIQ, which is because market is more liquid during the bull market than the bear market, and a negative correlation (-0.281) between the innovations in lnILLIQ and the innovations in market turnover indicating investors on average trade more when the market is relatively more liquid.

## 6.2 Comparison with Simulated Results

For each setting of our model, we simulate 10,000 trajectories of the stock returns and turnovers. Then for each trajectory of turnovers, we do the AR(1) regression, equation 19, to calculate the innovation in the natural logarithm of turnovers. The correlation and covariance between excess returns and innovations in turnover are calculated across all 10,000 simulations with 10 steps each.

Table 8 reports the correlation between the annual returns and turnover for the simula-

[12]The correlations of monthly data please refer to the Appendix 8.2.

tions of our model, and Table 9 reports the covariance. There are mainly 4 effects affecting these values of correlation and covariance:

1. Time-varying expected returns: Investor trades more when conditional expected return is either higher or lower than the average level (high realized return usually comes with low expected return, vice versa). We could see column $Corr(u_t, v_t) = -0.6$ in the Panel A of Table 8 and Table 9 for the benchmark setting where time-varying expected returns play the most crucial role. Both $Corr(R_m - r_f, Trn)$ and $Cov(R_m - r_f, Trn)$ are positive for the positive sample and negative for the negative sample as we expected. In addition, in the Panel B of Table 8 and Table 9 for the setting with exogenous liquidity shocks, both $Corr(R_m - r_f, Trn)$ and $Cov(R_m - r_f, Trn)$ are positive for the positive sample where there is no liquidity shock.

2. Time-varying price impact of trades: Investor trades more when the market is more liquid (high realized return comes with high market liquidity when $Corr(u_t, v_t)$ is negative in our model). We could see row 'Entire sample' in both Panel A and B of Table 8 and Table 9. Both $Corr(R_m - r_f, Trn)$ and $Cov(R_m - r_f, Trn)$ increases as $Corr(u_t, v_t)$ becomes more negative from 0 to -0.6.

3. Forced sales during crisis: Investor is forced to sell when the realized return is too low. We could see Panel B of Table 8 and Table 9 for the setting with exogenous liquidity shocks. Both $Corr(R_m - r_f, Trn)$ and $Cov(R_m - r_f, Trn)$ are negative for the entire sample and the negative sample. It is the most dominant effect in our setting.

4. Wealth effect: A higher wealth level means larger price impact for the same level of turnover (high realized returns lead to higher wealth level). We could see column $Corr(u_t, v_t) = 0$ in the Panel A of Table 8 and Table 9 for the benchmark setting. Both $Corr(R_m - r_f, Trn)$ and $Cov(R_m - r_f, Trn)$ are negative when $Corr(u_t, v_t) = 0$ for the benchmark setting for the entire sample.

Now we compare the simulated results with the market data. For the level of the market turnover, the market data reports an average annual market turnover about 66%, which is substantially larger than the 5% - 10% in the simulations of our model. Therefore our model is not able to capture the high turnover in stock market. Previous literature suggests it could be caused by the noise trades of investors and the large variation of investment sentiment, and those are not included in our setting for the concision of the model.

For the correlation between the excess returns and innovations in turnover, $Corr(R_m - r_f, \Delta lnTrn)$, Panel A in Table 8 reports that for the benchmark setting with the correlation between shocks on returns and trading-cost rates $Corr(u_t, v_t)$ between -0.4 and -0.6, the $Corr(R_m - r_f, \Delta lnTrn)$ for the entire sample ranges from -0.010 to 0.023, which is smaller than the 0.203 in market data. This positive correlation partial caused by the fact that investor trade more when the market is liquid, but the even higher correlation in market data might be caused by higher investment sentiment during the bull market than bear market. The $Corr(R_m - r_f, \Delta lnTrn)$ for the positive sample ranges from -0.014 to 0.049, higher than the -0.056 in market data, and the $Corr(R_m - r_f, \Delta lnTrn)$ for the negative sample ranges from -0.087 to -0.110, smaller than the -0.170 in market data in term of magnitude, which is because the forced selling is not included in the benchmark setting. And the magnitude of the covariance between the excess returns and innovations in turnover, $Cov(R_m - r_f, \Delta lnTrn)$, in our simulated data, Panel A in Table 9, is comparable to that in the market data, ranging from $-51.6$ to 16.2 (*$10^{-4}$).

Panel B in Table 8 reports that for the setting with exogenous liquidity shocks and a correlation between shocks on returns and trading-cost rates $Corr(u_t, v_t)$ as -0.6, the $Corr(R_m - r_f, \Delta lnTrn)$ shoots up to -0.220 for the entire sample, and -0.673 for the negative sample only. It means our assumption of forced selling is extremely strong. The fact that such a strong assumption of liquidity risk can only generate a 20 bps liquidity risk premium strengthens our claim that the actual liquidity risk premium in the market required by long-term investors is very small. Consistently, the magnitude of the covariances reported in Panel B of Table 9 is substantially larger than that in the market data in term of magnitude.

## 7 Conclusions

In this paper, to investigate the sources of liquidity level premium and liquidity risk premium, we solve a dynamic portfolio choice problem with time-varying trading-cost rates, CRRA utility and a time-varying investment opportunity set, under the assumptions of the largest possible trading-cost rate and the time variation in it. We calculate these premiums using a partial equilibrium approach.

We find that the liquidity risk premium generated by the covariance between trading-cost rates and stock returns, $Cov_t(\lambda_{t+1}, r_{t+1})$, which is documented as the main source of liquidity risk (e.g. Amihud 2002, Pastor and Stambaugh 2003 and Acharya and Pedersen 2005 etc.), is negligible, less than 3 bps per year, under our benchmark setting with time varying expected returns. Larger trading amounts and higher trading frequency increase the premium for the level of trading costs (liquidity level premium) only but not the liquidity risk premium.

However, once we add exogenous liquidity shocks into the setting, the liquidity risk premium become economically significant and accounts for a large fraction of the total liquidity premium. Forced selling and high trading-cost rate during the market downturn together hurt the investor hardly and thus generate large liquidity risk premium. But the liquidity risk premium generated by any of these two itself is still negligible. It indicates that large liquidity risk premiums documented in previous empirical papers, such as Pastor and Stambaugh (2003) and Acharya and Pedersen (2005), might largely compensate for the forced selling and high trading-cost rate during market downturn together, rather than simply the high trading-cost rate itself as they claim. Moreover, even with forced selling, the largest liquidity risk premium required by long-term investors in our setting, 20 bps, is still substantially smaller than those documented in empirical literature (7% in Pastor and Stambaugh 2003 and 1.1% in Acharya and Pedersen 2005). Therefore, it is either that long-term investors have not arbitrage away the liquidity risk premium in the market, or those large liquidity risk premiums documented empirically capture liquidity level premiums and other risk premiums as well.

# 8 Appendix

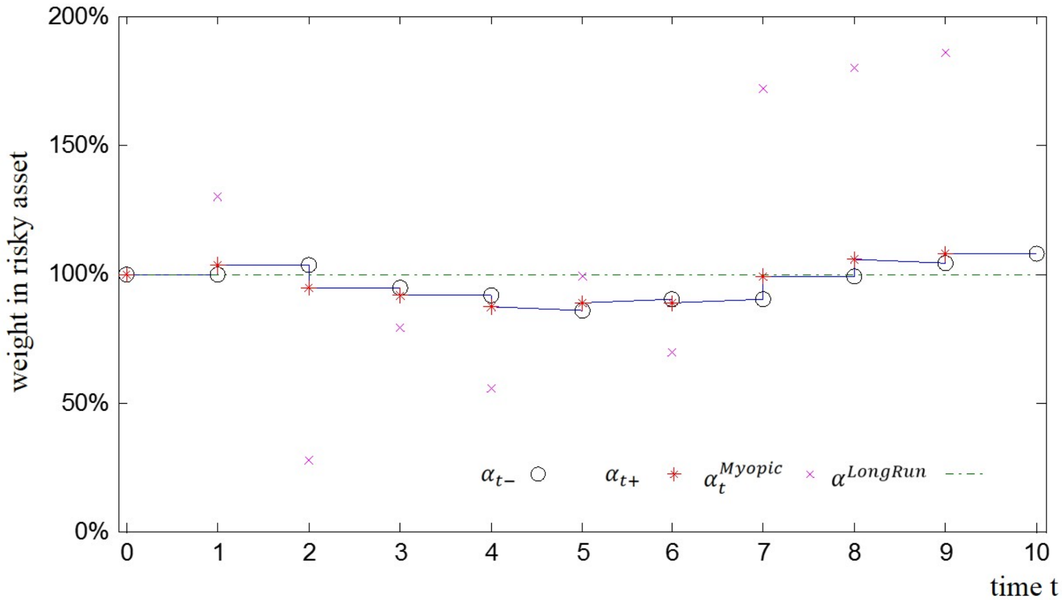


Figure 1: Weights in risky asset under benchmark setting (1 simulation)

This figure plots one simulation of the weights in risky asset from time 0 to 10 (10 years), for the benchmark case with time constant trading-cost rate and 0 correlation between return shocks and state variable $F_t$. It starts from initial weight $\alpha_{0-} = 100\%$. In each time step $t$, we plot both the weight before rebalancing $\alpha_{t-}$, the black circle, and the weight after rebalancing and trading costs $\alpha_{t+}$, the red star. The pink cross denotes the myopic optimal weight in each time step, $\alpha_t^{Myopic}$, and the green dash line is the long-run optimal weight, $\alpha^{LongRun}$ which equals 100%.

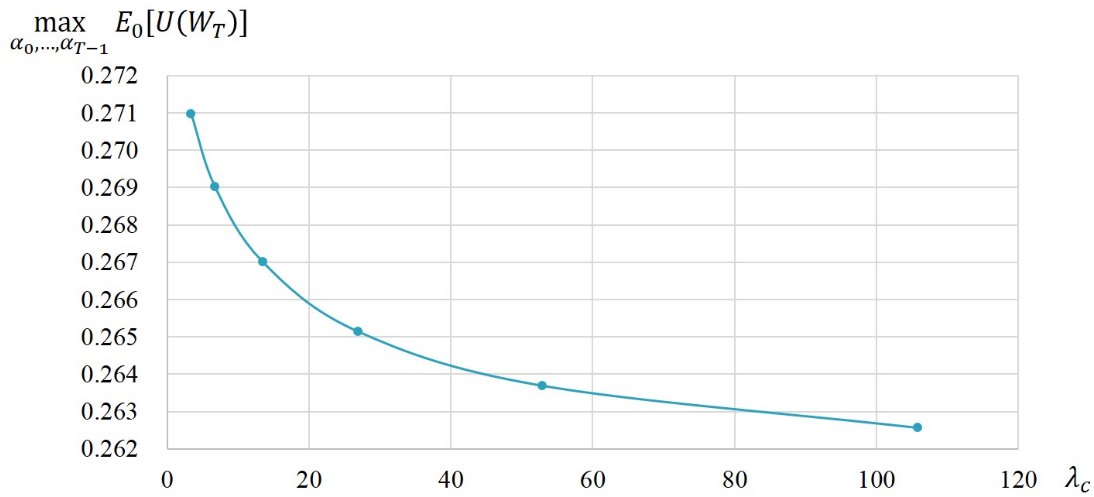


Figure 2: Expected utility for different level of time constant trading-cost rate

This figure plots the expected utility of the final wealth as a function of the trading-cost parameter $\lambda_c$ under the optimal strategy, $\max_{\alpha_0,\ldots,\alpha_{T-1}} E_0 U(W_T)$, for the benchmark case with time constant trading-cost rate. The fact that expected utility is convex in trading-cost rate indicates that the uncertainty in trading-cost parameter $\lambda_c$ always increases the expected utility under the benchmark setting when there is no correlation between shocks on realized returns and trading-cost rates, $Corr(u_t, v_t) = 0$.

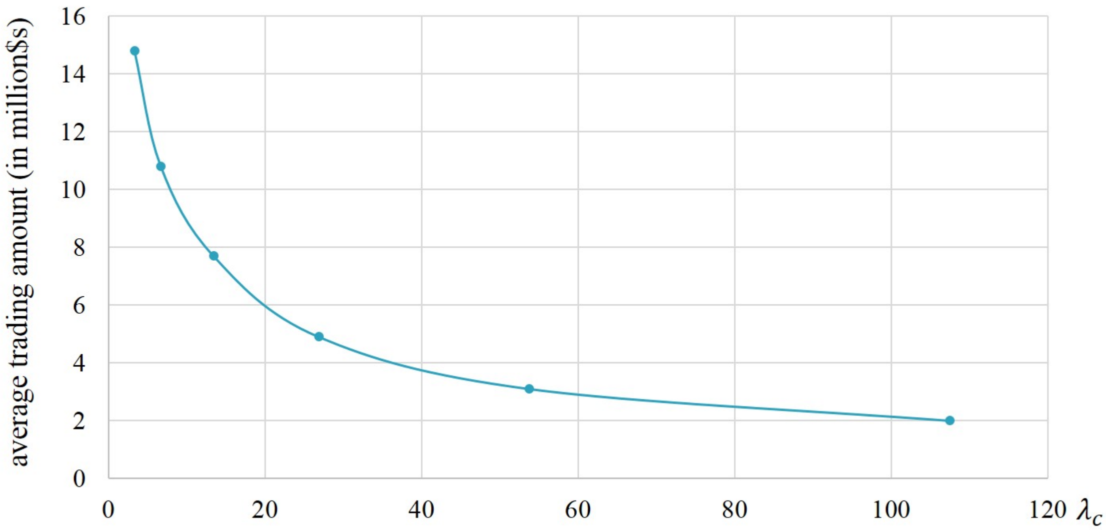


Figure 3: Average trading amount as a function of the trading-cost rate

This figure plots the average trading amount as a function of the trading-costs parameter $\lambda_c$ under the optimal strategy, for the benchmark case with time constant trading-cost rate. The correlation between the return shocks and shocks on $F_t$ $Corr(u_t, v_t) = -0.3$.

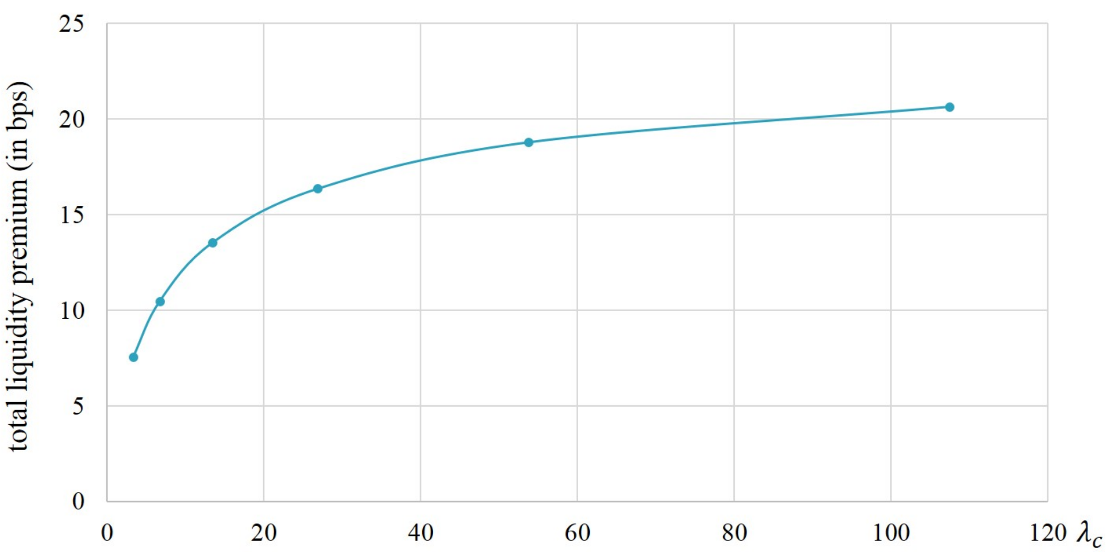


Figure 4: Total liquidity premium as a function of trading-cost rate

This figure plots the total liquidity premium as a function of the trading-costs parameter $\lambda_c$ under the optimal strategy, for the benchmark case with time constant trading-cost rate. The correlation between the return shocks and shocks on $F_t$ $Corr(u_t, v_t) = -0.3$.

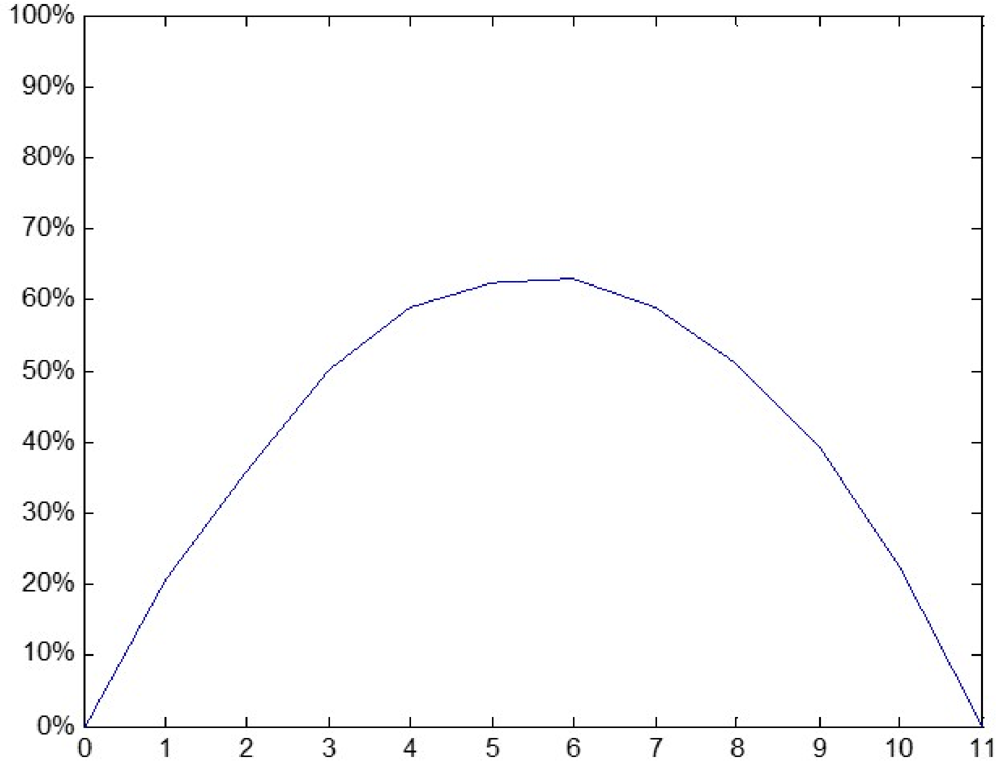


Figure 5: Optimal weights for building and releasing the portfolio every 10 years

This figure plots the average trajectory of the optimal weights for building the portfolio from the beginning and releasing all the positions before the end of time period 10. The correlation between the return shocks and shocks on $F_t$ $Corr(u_t, v_t) = 0$. The weight is averaged across 10,000 simulations.

Table 1: Liquidity risk premium for the benchmark setting

This table reports the liquidity level premium and liquidity risk premium under the benchmark setting, for different values of the correlation between the return shocks and shocks on state variable $F_t$, $Corr(u_t, v_t)$, 0, -0.2, -0.4, -0.6. The liquidity level premium is defined as the decrease in the long-term level of the expected return, $\mu_0$, on the trading-cost-free risky asset that the investor requires to be indifferent between having access to the risky asset without rather than with trading costs. The liquidity risk premium is the decrease in $\mu_0$ on the trading-cost-free risky asset that the investor requires to be indifferent between having access to the risky asset without rather than with *time variation* in trading costs. The premium for $Cov(\lambda_t, r_t)$ is calculated as the liquidity risk premium additional to the liquidity risk premium for the case with zero correlation between return shocks and shocks on trading-cost rate, $Corr(u_t, v_t) = 0$. All premiums are reported in basis points (bps).

| *in bps* | $Corr(u_t, v_t)$ | | | |
|---|---|---|---|---|
| | ***0*** | ***-0.2*** | ***-0.4*** | ***-0.6*** |
| Liquidity Level Premium (Total) | 16.43 | 15.86 | 16.48 | 15.68 |
| Liquidity Risk Premium (Total) | -0.623 | -0.582 | -0.353 | -0.104 |
| **Total Liquidity Premium** | 15.80 | 15.27 | 16.12 | 15.58 |
| *liquidity risk premium for* $Cov(\lambda_t, r_t)$ | *0.000* | *0.042* | *0.270* | *0.519* |
| *premium for* $Cov(\lambda_t, r_t)$ *as % of total premium* | *0.0%* | *0.3%* | *1.7%* | *3.3%* |

Table 2: Liquidity level premium for difference levels of trading-cost rate

This table reports liquidity level premium, for the parts compensating for TC and compensating for the deviation from optimal weight separately, liquidity risk premium and average trading amount across different trading-cost rates $\lambda_c$, from 3.36(1/8*26.88) to 107.52(4*26.88). Liquidity level premium for TC is the premium compensating for actual trading costs expected, and liquidity level premium compensating for the deviation from optimal weight is the premium compensating for the expected loss of utility caused by deviating from the none-trading-cost optimal weight. Both liquidity level premium compensating for TC and average trading amount per year are calculated through 10,000 simulations with 10 steps each. Liquidity level premium for TC as % of total liquidity premium is also reported. All premiums are reported in basis points (bps).

| *in bps,* $Corr(u_t, v_t) = -0.3$ | Average value of trading-cost parameter $\lambda_c$ | | | | | |
|---|---|---|---|---|---|---|
| | ***3.36*** | ***6.72*** | ***13.44*** | ***26.88*** | ***53.76*** | ***107.52*** |
| Liquidity Level Premium (compensating for TC) | 4.83 | 5.30 | 5.35 | 4.18 | 3.45 | 2.88 |
| Liquidity Level Premium (compensating for the deviation from the optimal weight) | 2.88 | 5.45 | 8.56 | 12.65 | 15.69 | 18.05 |
| Liquidity Risk Premium (Total) | -0.167 | -0.316 | -0.393 | -0.494 | -0.370 | -0.297 |
| **Total Liquidity Premium** | 7.54 | 10.44 | 13.51 | 16.34 | 18.77 | 20.63 |
| *liquidity level premium for TC as % of total liquidity premium* | *64%* | *51%* | *40%* | *26%* | *18%* | *14%* |
| *avg. trading amount (million$s)* | *14.8* | *10.8* | *7.7* | *4.9* | *3.1* | *2.0* |

Table 3: Liquidity risk premium with fixed releasing and rebuilding of the portfolio

This table reports the liquidity level premium (compensating for TC and for the deviation from the none-trading-cost optimal weight separately), liquidity risk premium and average trading amount per year for different frequencies of rebuilding the portfolio (per 1, 2, 5 and 10 years), and for 3 values of the correlation between returns shocks and shocks on trading-cost rates (0, -0.3 and -0.6) for each frequency. Both liquidity level premium compensating for TC and average trading amount per year are calculated through 10,000 simulations with 10 steps each. Liquidity level premium for TC as % of total liquidity premium is also reported. All premiums are reported in basis points (bps).

| *in bps* | $Corr(u_t, v_t)$ | Frequency of rebuilding (per $X$ years) | | | |
|---|---|---|---|---|---|
| | | ***1*** | ***2*** | ***5*** | ***10*** |
| Liquidity Level Premium (compensating for TC) | 0 | 8.37 | 11.00 | 24.97 | 33.63 |
| | -0.3 | 7.98 | 11.26 | 25.22 | 36.90 |
| | -0.6 | 7.85 | 11.27 | 25.85 | 38.14 |
| Liquidity Level Premium (compensating for the deviation from the optimal weight) | 0 | 184.35 | 171.56 | 121.10 | 60.49 |
| | -0.3 | 195.38 | 181.77 | 128.46 | 64.38 |
| | -0.6 | 202.44 | 188.73 | 134.99 | 68.28 |
| Liquidity Risk Premium (Total) | 0 | 1.509 | 0.907 | 0.847 | 0.861 |
| | -0.3 | 1.645 | 1.163 | 1.925 | 1.439 |
| | -0.6 | 1.859 | 1.447 | 2.659 | 1.846 |
| **Total Liquidity Premium** | 0 | 194.23 | 183.47 | 146.92 | 94.98 |
| | -0.3 | 205.01 | 194.19 | 155.61 | 102.72 |
| | -0.6 | 212.15 | 201.45 | 163.50 | 108.27 |
| *liquidity level premium for TC as % of total liquidity premium* | *0* | *4%* | *6%* | *17%* | *35%* |
| | *-0.3* | *4%* | *6%* | *16%* | *36%* |
| | *-0.6* | *4%* | *6%* | *16%* | *36%* |
| *avg. trading amount (million $s)* | *0* | *9.49* | *8.75* | *11.39* | *12.43* |
| | *-0.3* | *9.26* | *8.83* | *11.46* | *13.06* |
| | *-0.6* | *9.18* | *8.87* | *11.68* | *13.39* |

Table 4: Liquidity risk premium with exogenous liquidity shocks

This table reports liquidity level premiums and liquidity risk premiums for the setting with exogenous liquidity shocks. We report it for 3 different values of the correlation between return shocks and shocks state variable $F_t$ (0, -0.3, -0.6) separately. Liquidity level premiums and liquidity risk premiums are reported based on their sources. We report the liquidity level premiums compensating for TC and the deviation from the none-trading-cost optimal weight separately, the total liquidity risk premium, and the liquidity risk premiums for the covariance between trading amounts and stock returns, $Cov_t(V_{t+1}^2, r_{t+1})$, the covariance between trading-cost rates and stock returns, $Cov_t(\lambda_{t+1}, r_{t+1})$, and total covariance including the interaction of these two, $Cov_t(\lambda_{t+1} V_{t+1}^2, r_{t+1})$, separately. Liquidity risk premium as % of total liquidity premium is also reported. All premiums are reported in basis points (bps).

| *in bps* | $Corr(u_t, v_t)$ | | |
|---|---|---|---|
| | ***0*** | ***-0.3*** | ***-0.6*** |
| Liquidity Level Premium (compensating for TC) | 29.92 | 31.94 | 33.19 |
| Liquidity Level Premium (compensating for the deviation from the optimal weight) | 19.47 | 20.72 | 21.41 |
| Liquidity Risk Premium (Total) | 2.22 | 11.53 | 20.83 |
| **Total Liquidity Premium** | 51.61 | 64.19 | 75.44 |
| *liquidity risk premium as % of total liquidity premium* | *4%* | *18%* | *28%* |
| *liquidity risk premium for* $Cov_t(V_{t+1}^2, r_{t+1})$ | *1.17* | *3.80* | *6.03* |
| *liquidity risk premium for* $Cov_t(\lambda_{t+1}, r_{t+1})$ | *0.00* | *0.75* | *0.93* |
| *liquidity risk premium for* $Cov_t(\lambda_{t+1} V_{t+1}^2, r_{t+1})$ | *0.00* | *9.31* | *18.61* |

Table 5: Liquidity risk premium with exogenous liquidity shocks for different levels of trading-cost rate

This table reports liquidity level premiums, liquidity risk premiums and average trading amount per year for the setting with exogenous liquidity shocks for different levels of trading-cost rate $\lambda_c$, from 3.36 (1/8*26.88) to 107.52 (4*26.88). We assume the correlation between returns shocks and shocks on state variable $F_t$ is -0.3. Liquidity level premiums and liquidity risk premiums are reported based on their sources. We report the liquidity level premiums (compensating for TC and the deviation from the none-trading-cost optimal weight separately) and the total liquidity risk premiums. Both liquidity level premium compensating for TC and average trading amount per year are calculated through 10,000 simulations with 10 steps each. Liquidity risk premium as % of total liquidity premium, and liquidity level premium for TC as % of total liquidity premium are also reported. All premiums are reported in basis points (bps).

| *in bps*, $Corr(u_t, v_t) = -0.3$ | Average value of trading-cost parameter $\lambda_c$ | | | | | |
|---|---|---|---|---|---|---|
| | ***3.36*** | ***6.72*** | ***13.44*** | ***26.88*** | ***53.76*** | ***107.52*** |
| Liquidity Level Premium (compensating for TC) | 10.93 | 15.48 | 21.79 | 31.94 | 49.51 | 86.53 |
| Liquidity Level Premium (compensating for the deviation from the optimal weight) | 2.17 | 5.91 | 11.96 | 20.72 | 34.58 | 66.56 |
| Liquidity Risk Premium (Total) | 4.88 | 7.52 | 9.92 | 11.53 | 13.71 | 21.54 |
| **Total Liquidity Premium** | 17.98 | 28.91 | 43.67 | 64.19 | 97.81 | 174.62 |
| *liquidity risk premium as % of total premium* | *27%* | *26%* | *23%* | *18%* | *14%* | *12%* |
| *liquidity level premium for TC as % of total liquidity premium* | *83%* | *72%* | *65%* | *61%* | *59%* | *57%* |
| *avg. trading amount (million $s)* | *20.69* | *16.04* | *12.22* | *9.58* | *8.03* | *7.68* |

Table 6: Liquidity risk premium for different frequencies of liquidity shocks

This table reports liquidity level premiums and liquidity risk premiums and average trading amount per year for the setting for different frequencies of liquidity shocks (per 1, 2 and 5 years), and for 3 values of the correlation between returns shocks and shocks on state variable $F_t$, (0, -0.3 and -0.6) for each frequency. Liquidity level premiums and liquidity risk premiums are reported based on their sources. We report the liquidity level premiums compensating for TC and the deviation from the none-trading-cost optimal weight separately, the total liquidity risk premium. Both liquidity level premium compensating for TC and average trading amount per year are calculated through 10,000 simulations with 10 steps each. Liquidity risk premium as % of total liquidity premium, and liquidity level premium for TC as % of total liquidity premium are also reported. All premiums are reported in basis points (bps).

| *in bps* | $Corr(u_t, v_t)$ | Frequency of liquidity shocks (per $X$ years) | | | |
|---|---|---|---|---|---|
| | | ***1*** | ***2*** | ***5*** | ***Never*** |
| Liquidity Level Premium (compensating for TC) | 0 | 29.92 | 20.50 | 12.28 | 3.57 |
| | -0.3 | 31.94 | 22.13 | 13.55 | 4.17 |
| | -0.6 | 33.19 | 22.76 | 13.91 | 3.93 |
| Liquidity Level Premium (compensating for the deviation from the optimal weight) | 0 | 19.47 | 13.84 | 11.47 | 12.85 |
| | -0.3 | 20.72 | 14.14 | 11.41 | 12.67 |
| | -0.6 | 21.41 | 14.17 | 10.71 | 11.75 |
| Liquidity Risk Premium (Total) | 0 | 2.22 | 1.64 | 0.47 | -0.62 |
| | -0.3 | 11.53 | 7.87 | 3.47 | -0.49 |
| | -0.6 | 20.83 | 14.49 | 7.06 | -0.10 |
| **Total Liquidity Premium** | 0 | 51.61 | 35.97 | 24.21 | 15.80 |
| | -0.3 | 64.19 | 44.13 | 28.43 | 16.34 |
| | -0.6 | 75.44 | 51.43 | 31.68 | 15.58 |
| *liquidity risk premium as % of total premium* | *0* | *4%* | *5%* | *2%* | *-4%* |
| | *-0.3* | *18%* | *18%* | *12%* | *-3%* |
| | *-0.6* | *28%* | *28%* | *22%* | *-1%* |
| *liquidity level premium for TC as % of total liquidity premium* | *0* | *58%* | *57%* | *51%* | *23%* |
| | *-0.3* | *50%* | *50%* | *48%* | *26%* |
| | *-0.6* | *44%* | *44%* | *44%* | *25%* |
| *avg. trading amount (million $s)* | *0* | *9.25* | *7.70* | *6.13* | *4.43* |
| | *-0.3* | *9.58* | *7.57* | *6.40* | *4.89* |
| | *-0.6* | *9.65* | *7.60* | *6.53* | *4.81* |

Table 7: Correlation and Covariance between Market Returns, Innovations in ILLIQ and Turnover (annually)

This table reports summary statistics, the correlations between the annually market excess returns, the innovations in market liquidity and the innovations in market turnover from 1966 to 2010, and the covariance between the annually market excess returns and the innovations in market turnover. Panel A for the summary statistics, Panel B for the correlations, and Panel C for the correlations and covariance of the annually market excess returns and the innovations in market turnover for the entire sample, sample with positive returns only, and sample with negative returns only separately.

*Panel A: Summary Statistics*

| | Mean | Std.Dev | # obs | Min | Max |
|---|---|---|---|---|---|
| Rm-rf | 0.056 | 0.185 | 45 | -0.399 | 0.321 |
| $\Delta lnTrn$ | 0.000 | 0.139 | 44 | -0.318 | 0.252 |
| $\Delta lnILLIQ$ | 0.000 | 0.244 | 44 | -0.431 | 0.760 |

*Panel B: Correlations for Entire Sample*

| | Rm-rf | $\Delta lnILLIQ$ | $\Delta lnTrn$ |
|---|---|---|---|
| Rm-rf | 1 | -.584*** | .203 |
| $\Delta lnILLIQ$ | | 1 | -.281* |
| $\Delta lnTrn$ | | | 1 |

*Panel C: Correlation and Covariance of Returns and Turnovers*

| | $Corr(R_m - r_f, \Delta lnTrn)$ | $Cov(R_m - r_f, \Delta lnTrn)$ (*$10^{-4}$) | # obs |
|---|---|---|---|
| Entire sample | .203 | 51.9 | 44 |
| $R_m - r_f > 0$ | -.056 | -6.6 | 30 |
| $R_m - r_f < 0$ | -.170 | -25.9 | 14 |

Table 8: Correlation of the Returns and Innovations in Turnovers (simulation results)

This table reports the correlation between the annual returns and the innovations in turnovers for different settings of our model. Panel A for the benchmark setting with time-varying trading-cost rates, Panel B for the setting with exogenous liquidity shocks and time-varying trading cost rate. For each setting, we report the correlation values for cases with different values of the correlation between the return shocks and shocks on state variable $F_t$, $Corr(u_t, v_t)$ separately. We also report them for the entire sample, sample with positive returns only, and sample with negative returns only separately. We do 10,000 simulations for each case within each setting.

*Panel A: Benchmark setting with time-varying trading-cost rates*

| $Corr(R_m - r_f, \Delta lnTrn)$ | $Corr(u_t, v_t)$ | | | |
|---|---|---|---|---|
| | ***0*** | ***-0.2*** | ***-0.4*** | ***-0.6*** |
| Entire sample | -0.100*** | -0.058*** | -0.010*** | 0.023*** |
| $R_m - r_f > 0$ | -0.074*** | -0.031*** | -0.014*** | 0.049*** |
| $R_m - r_f < 0$ | -0.075*** | -0.039*** | -0.087*** | -0.110*** |

*Panel B: Setting with liquidity shocks and time-varying trading-cost rates*

| $Corr(R_m - r_f, \Delta lnTrn)$ | $Corr(u_t, v_t)$ | | |
|---|---|---|---|
| | ***0*** | ***-0.3*** | ***-0.6*** |
| Entire sample | -0.328*** | -0.303*** | -0.220*** |
| $R_m - r_f > 0$ | -0.087*** | -0.047*** | 0.091*** |
| $R_m - r_f < 0$ | -0.683*** | -0.662*** | -0.673*** |

Table 9: Covariance of the Returns and Innovations in Turnover (simulation results)

This table reports the covariance between the annual returns and the innovations in turnovers for different settings of our model. Panel A for the benchmark setting with time-varying trading-cost rates, Panel B for the setting with exogenous liquidity shocks and time-varying trading cost rate. For each setting, we report the values of covariance for cases with different values of the correlation between the return shocks and shocks on state variable $F_t$, $Corr(u_t, v_t)$ separately. We also report them for the entire sample, sample with positive returns only, and sample with negative returns only separately. We do 10,000 simulations for each case within each setting.

*Panel A: Benchmark setting with time-varying trading-cost rates*

| Cov($R_m - r_f, \Delta lnTrn$) ($*10^{-4}$) | $Corr(u_t, v_t)$ | | | |
|---|---|---|---|---|
| | ***0*** | ***-0.2*** | ***-0.4*** | ***-0.6*** |
| Entire sample | -93.5 | -61.0 | -10.9 | 16.2 |
| $R_m - r_f > 0$ | -33.2 | -15.9 | -7.7 | 25.7 |
| $R_m - r_f < 0$ | -32.1 | -18.5 | -44.1 | -51.6 |

*Panel B: Setting with liquidity shocks and time-varying trading-cost rates*

| Cov($R_m - r_f, \Delta lnTrn$) ($*10^{-4}$) | $Corr(u_t, v_t)$ | | |
|---|---|---|---|
| | ***0*** | ***-0.3*** | ***-0.6*** |
| Entire sample | -351.6 | -327.3 | -235.9 |
| $R_m - r_f > 0$ | -36.4 | -19.9 | 38.4 |
| $R_m - r_f < 0$ | -393.2 | -384.3 | -391.9 |

## 8.1 Robustness Check for the Effect of Rebalancing on Liquidity Risk Premium

Table A1 and A2 show that the liquidity risk premium are still negligible even if the long-run optimal weight is 50% ($\gamma = 5$) and 150% ($\gamma = 5/3$). The need to rebalance does not affect our conclusion.

Table A1: Liquidity risk premium for the benchmark setting with optimal weight as 50%

This table reports the liquidity level premium and liquidity risk premium under the benchmark setting with optimal weight of 50% on risky asset ($\gamma = 5$), for different values of the correlation between the return shocks and shocks on state variable $F_t$, $Corr(u_t, v_t)$, 0, -0.2, -0.4, -0.6. The liquidity level premium is defined as the decrease in the long-term level of the expected return, $\mu_0$, on the trading-cost-free risky asset that the investor requires to be indifferent between having access to the risky asset without rather than with trading costs. The liquidity risk premium is the decrease in $\mu_0$ on the trading-cost-free risky asset that the investor requires to be indifferent between having access to the risky asset without rather than with *time variation* in trading costs. The premium for $Cov(\lambda_t, r_t)$ is calculated as the liquidity risk premium additional to the liquidity risk premium for the case with zero correlation between return shocks and shocks on trading-cost rate, $Corr(u_t, v_t) = 0$. All premiums are reported in basis points (bps).

| *in bps* | $Corr(u_t, v_t)$ | | | |
|---|---|---|---|---|
| | ***0*** | ***-0.2*** | ***-0.4*** | ***-0.6*** |
| Liquidity Level Premium (Total) | 17.14 | 17.76 | 19.26 | 18.75 |
| Liquidity Risk Premium (Total) | -0.682 | -0.635 | -0.398 | 0.069 |
| **Total Liquidity Premium** | 16.46 | 17.12 | 18.86 | 18.82 |
| *liquidity risk premium for* $Cov(\lambda_t, r_t)$ | *0.000* | *0.047* | *0.283* | *0.750* |
| *premium for* $Cov(\lambda_t, r_t)$ *as % of total premium* | *0.0%* | *0.3%* | *1.5%* | *4.0%* |

Table A2: Liquidity risk premium for the benchmark setting with optimal weight as 150%

This table reports the liquidity level premium and liquidity risk premium under the benchmark setting with optimal weight of 150% on risky asset ($\gamma = 5/3$), for different values of the correlation between the return shocks and shocks on state variable $F_t$, $Corr(u_t, v_t)$, 0, -0.2, -0.4, -0.6. The liquidity level premium is defined as the decrease in the long-term level of the expected return, $\mu_0$, on the trading-cost-free risky asset that the investor requires to be indifferent between having access to the risky asset without rather than with trading costs. The liquidity risk premium is the decrease in $\mu_0$ on the trading-cost-free risky asset that the investor requires to be indifferent between having access to the risky asset without rather than with *time variation* in trading costs. The premium for $Cov(\lambda_t, r_t)$ is calculated as the liquidity risk premium additional to the liquidity risk premium for the case with zero correlation between return shocks and shocks on trading-cost rate, $Corr(u_t, v_t) = 0$. All premiums are reported in basis points (bps).

| *in bps* | $Corr(u_t, v_t)$ | | | |
|---|---|---|---|---|
| | ***0*** | ***-0.2*** | ***-0.4*** | ***-0.6*** |
| Liquidity Level Premium (Total) | 15.12 | 13.70 | 12.98 | 11.31 |
| Liquidity Risk Premium (Total) | -0.569 | -0.473 | -0.354 | -0.077 |
| **Total Liquidity Premium** | 14.55 | 13.22 | 12.62 | 11.24 |
| *liquidity risk premium for* $Cov(\lambda_t, r_t)$ | *0.000* | *0.096* | *0.215* | *0.492* |
| *premium for* $Cov(\lambda_t, r_t)$ *as % of total premium* | *0.0%* | *0.7%* | *1.7%* | *4.4%* |

## 8.2 Relation between Monthly Market Turnovers and Monthly Market Returns

Since small and active investors usually react to the price changes within a month, the correlation between market returns and innovations in market turnover can be higher in monthly frequency than in annual frequency. In this section, we also document the correlation between monthly market returns and monthly innovations in market turnover for comparison.

Similarly, we do an AR(1) regression for the ln values of both turnover and ILLIQ to capture the monthly innovations in market turnover and liquidity. Both the market turnover and ILLIQ are quite persistent at monthly frequency. The time persistency of monthly market turnover, $\rho^{trn}$ in equation (19), is 0.980, and the R square of equation (19) is 0.959. The time persistency of monthly ILLIQ, $\rho^{MILLIQ}$ in equation (20), is 0.993, and the R square of equation (20) is 0.980.

Figure A1 plots the innovations in monthly market turnover and their corresponding market excess returns for each month from 1966 January to 2010 December, and Figure A2 plots the innovations in monthly market turnover and their corresponding innovations in market ILLIQ. The red dots are the observations in 2008 financial crisis (2008 September to 2009 June). In Figure A1, we could see that on average innovation in market turnover becomes larger when the market excess return becomes either more positive or more negative, and Figure A2 shows there is a weak negative correlation between the innovations in market turnover and market ILLIQ.

Consistent with the figures, Table A3 reports a correlation of 0.106 between the monthly market returns and the innovations in market turnover, a correlation of -0.255 for negative returns, and a correlation of 0.309 for positive returns. Consistent with our expectation, the magnitudes of correlations are larger for monthly frequency, and the correlation for positive returns is more positive since there are more active trades at the monthly frequency.

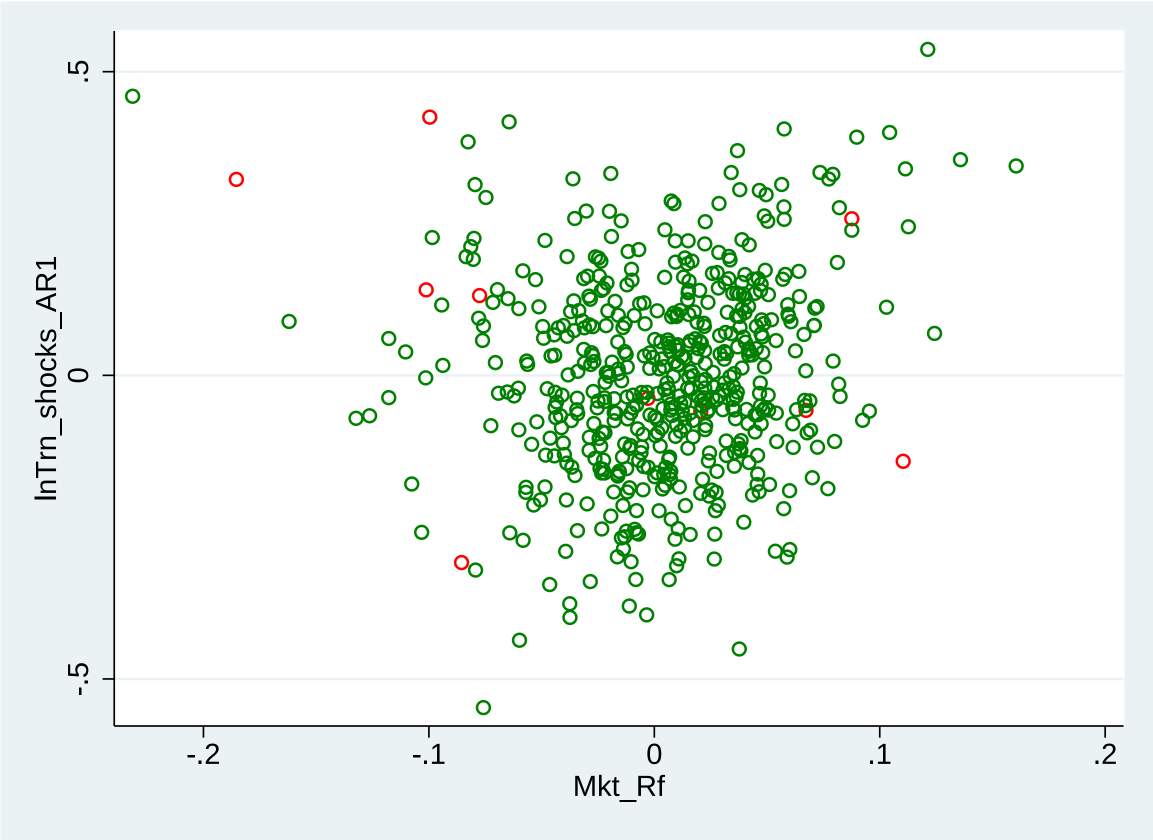


Figure A1: Relation between Innovations of Turnover and Market Returns

This figure plots the innovations in monthly market turnover and their corresponding market excess returns for each month from 1966 January to 2010 December. The red dots are the observations in 2008 financial crisis (2008 September to 2009 June).

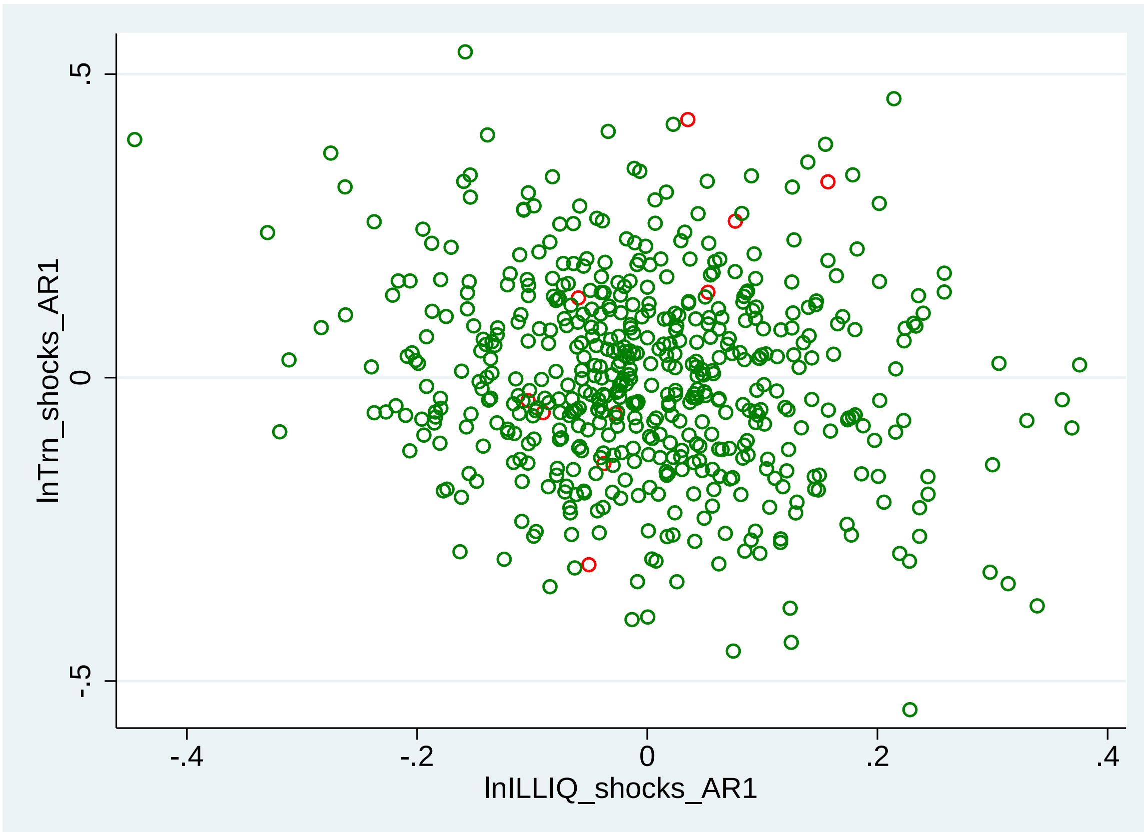


Figure A2: Relation between Innovations of Turnover and Market Returns

This figure plots the innovations in monthly market turnover and their corresponding innovations in monthly ILLIQ for each month from 1966 January to 2010 December. The red dots are the observations in 2008 financial crisis (2008 September to 2009 June).

Table A3: Correlation between Market Return, Innovations in ILLIQ and Turnover (monthly)

This table reports the correlation between the monthly market excess returns, innovations in monthly ILLIQ and innovations in market turnover from 1966 January to 2010 December. Panel A for the entire sample, Panel B for the observations with negative market excess returns, and Panel C for the observations with positive market excess returns.

*Panel A: Entire sample*

| | Rm-rf | $\Delta lnILLIQ$ | $\Delta lnTrn$ |
|---|---|---|---|
| Rm-rf | 1 | $-.411^{***}$ | $.106^{**}$ |
| $\Delta lnILLIQ$ | | 1 | $-.156^{***}$ |
| $\Delta lnTrn$ | | | 1 |

*Panel B:* $Rm - Rf < 0$

| | Rm-rf | $\Delta lnILLIQ$ | $\Delta lnTrn$ |
|---|---|---|---|
| Rm-rf | 1 | $-.278^{***}$ | $-.255^{***}$ |
| $\Delta lnILLIQ$ | | 1 | $-.054$ |
| $\Delta lnTrn$ | | | 1 |

*Panel C:* $Rm - Rf > 0$

| | Rm-rf | $\Delta lnILLIQ$ | $\Delta lnTrn$ |
|---|---|---|---|
| Rm-rf | 1 | $-.139^{**}$ | $.309^{***}$ |
| $\Delta lnILLIQ$ | | 1 | $-.172^{***}$ |
| $\Delta lnTrn$ | | | 1 |

## References


[1] Adrian, T., and Shin, H., 2010, Liquidity and leverage, *Journal of Financial Intermediation* 19, 418437.

[2] Acharya, V., and Pedersen, L., 2005. Asset pricing with liquidity risk, *Journal of Financial Economics*, Elsevier, vol. 77(2), 375-410

[3] Amihud, Y., 2002. Illiquidity and stock returns: cross-section and time-series effects. *Journal of Financial Markets* 5, 31-56.

[4] Amihud, Y., and Mendelson, H. 1986. Asset pricing and the bid-ask spread. *Journal of Financial Economics*, 17(2), 223-249.

[5] Ang, A., S. Gorovyy, and G. B. van Inwegen. 2011. Hedge Fund Leverage. *Journal of Financial Economics.*

[6] Ang, A., Papanikolaou, D., and Westerfield, M. M. 2014. Portfolio choice with illiquid assets. *Management Science*, 60(11), 2737-2761.

[7] Aragon, G. O., and P. Strahan. 2012. Hedge Funds as Liquidity Providers: Evidence from the Lehman Bankruptcy. *Journal of Financial Economics.*

[8] Ben_David, I., Franzoni, F. and Moussawi, R. 2012. Hedge Fund Stock Trading in the Financial Crisis of 20072009. *Review of Financial Studies.*

[9] Brown, D. B., B. I. Carlin, and M. S. Lobo. 2010. Optimal Portfolio Liquidation with Distress Risk. *Management Science* 56:19972014.

[10] Brunnermeier, M. K., and S. Nagel. 2004. Hedge Funds and the Technology Bubble. *Journal of Finance* 59:201340.

[11] Brunnermeier, M. K., and L. H. Pedersen. 2009. Market Liquidity and Funding Liquidity. *Review of Financial Studies* 22:220138.

[12] Cella, C., Ellul, A., and Giannetti, M. 2013. Investors' horizons and the amplification of market shocks. *Review of Financial Studies*, hht023.

[13] Chan, L. K., and Lakonishok, J. 1997. Institutional equity trading costs: NYSE versus Nasdaq. *The Journal of Finance*, 52(2), 713-735.

[14] Chordia, T., A. Sarkar, and A. Subrahmanyam. 2005. An Empirical Analysis of Stock and Bond Market Liquidity. *Review of Financial Studies* 18:85129.

[15] Collin-Dufresne, P., Daniel, K., Moallemi, C. C., and Saglam, M. 2012. Dynamic Asset Allocation with Predictable Returns and Transaction Costs. Working Paper, SFI.

[16] Constantinides, G. M. 1986. Capital market equilibrium with transaction costs. *The Journal of Political Economy*, 842-862.

[17] Diamond, P. A. 1982. Aggregate demand management in search equilibrium. *The Journal of Political Economy*, 881-894.

[18] Fama, E.F., and French, K.R., 1992. The cross-section of expected stock returns. *Journal of Finance* 47, 427-465.

[19] Grleanu, N., and Pedersen, L. H. 2013. Dynamic trading with predictable returns and transaction costs. *The Journal of Finance*, 68(6), 2309-2340.

[20] Gromb, D., and Vayanos, D. 2009. Financially constrained arbitrage and cross-market contagion. Department of Finance, London School of Economics and Political Science.

[21] Gromb, D., and D. Vayanos. 2010. Limits of Arbitrage: The State of the Theory. *Annual Review of Financial Economics* 2:25175.

[22] Grossman, S.J., Laroque, G. 1990 Asset pricing and optimal portfolio choice in the presence of illiquid durable consumption goods. *Econometrica* 58(1):2551.

[23] He, Z., Khang, I. G., and Krishnamurthy, A. 2010. Balance sheet adjustments during the 2008 crisis. *IMF Economic Review*, 58(1), 118-156.

[24] Huang, M. 2003. Liquidity shocks and equilibrium liquidity premia. *Journal of Economic Theory*, 109(1), 104-129.

[25] Jang, B., Koo, H., Liu, H., and Loewenstein, M. 2007. Liquidity premia and transaction costs. *The Journal of Finance*, 62(5), 2329-2366.

[26] Kahl, M., Liu, J., Longstaff, F.A. (2003) Paper millionaires: How valuable is stock to a stockholder who is restricted from selling it? *Journal of Financial Economics.* 67(3):385410.

[27] Keim, D. B., and Madhavan, A. 1997. Transactions costs and investment style: An inter-exchange analysis of institutional equity trades. *Journal of Financial Economics*, 46(3), 265-292.

[28] Koren, M., Szeidl, A. 2003 Portfolio choice with illiquid assets. CEPR Discussion Paper 3795, Centre for Economic Policy Research, London

[29] Liu, H. 2004. Optimal consumption and investment with transaction costs and multiple risky assets. *The Journal of Finance*, 59(1), 289-338.

[30] Lo, A.W., Mamaysky, H., and Wang, J. 2004 Asset prices and trading volume under fixed transactions costs. *Journal of Political Economy.* 112(5): 10541090.

[31] Longstaff, F. A. 2001. Optimal portfolio choice and the valuation of illiquid securities. *Review of Financial Studies*, 14(2), 407-431.

[32] Longstaff, F.A. 2009 Portfolio claustrophobia: Asset pricing in markets with illiquid assets. *American Economic Review.* 99(4):11191144

[33] Lynch, A. W., and Tan, S. 2011. Explaining the Magnitude of Liquidity Premia: The Roles of Return Predictability, Wealth Shocks, and StateDependent Transaction Costs. *The Journal of Finance*, 66(4), 1329-1368.

[34] Manconi, A., Massa, M., and Yasuda, A. 2012. The role of institutional investors in propagating the crisis of 20072008. *Journal of Financial Economics*, 104(3), 491-518.

[35] Mitchell, M., L. Pedersen, and T. Pulvino. 2007. Slow-moving Capital. *American Economic Review PP* 97:21520.

[36] Nagel, S. 2011. Evaporating Liquidity. Working Paper, Stanford University.

[37] Pastor, L., and Stambaugh, R.F., 2003. Liquidity risk and expected stock returns. *Journal of Political Economy*, 111, 642-685.

[38] Scholes, M. 2000. Crisis and Risk Management. *American Economic Review* 90:1721.

[39] Schwartz ES, Tebaldi C 2006 Illiquid assets and optimal portfolio choice. NBER Working Paper 12633, National Bureau of Economic Research, Cambridge, MA.

[40] Vayanos, D. 1998 Transaction costs and asset prices: A dynamic equilibrium model. *Review of Financial Studies*. 11(1):158.

[41] Vayanos, D. 2004. Flight to quality, flight to liquidity, and the pricing of risk. Working paper, London School of Economics.