

# Inequalities in an OLG economy with heterogeneous cohorts and pension systems\*

Marcin Bielecki<sup>†</sup>  
University of Warsaw

Joanna Tyrowicz  
University of Warsaw  
National Bank of Poland

Krzysztof Makarski  
National Bank of Poland  
Warsaw School of Economics

Marcin Waniek  
University of Warsaw

First version: October 2014

This version: April 2015

## Abstract

While the inequalities of endowments are widely recognized as areas of policy intervention, the dispersion in preferences may also imply inequalities of outcomes. In this paper, we analyze the inequalities in an OLG model with obligatory pension systems. We model both policy relevant pension systems (a defined benefit system – DB – and a transition from a DB to a defined contribution system, DC). Our framework features within cohort heterogeneity of endowments (individual productivities) and heterogeneity of preferences (preference for leisure and time preference). We introduce two policy instruments, which are widely used: a contribution cap and a minimum pension. We show four main results. First, longevity increases aggregate consumption inequalities substantially in both pension systems, whereas the effect of a pension system reform works to reinforce the consumption inequalities and reduce the wealth inequalities. Second, the contribution cap has negligible effect on inequalities, but the role for minimum pension benefit guarantee is more pronounced. Third, the reduction in inequalities due to minimum pension benefit guarantee is achieved with virtually no effect on capital accumulation. The fourth result and the main policy implication of our study, is demonstrating that the minimum pension benefit guarantee addresses mostly the inequalities which stem from differentiated endowments and not those that stem from differentiated preferences.

**Key words:** inequality, longevity, defined contribution, defined benefit, Gini

**JEL Codes:** H55, E17, C60 C68, E21, D63

---

\*Earlier version of this paper has received extremely valuable comments from Fabian Kindermann, Patrick Puhani, Dirk Niepelt, Hans Fehr, Martyna Kobus, Jan Hagemeyer, Lukasz Drozd, Borys Grochulski and Jaromir Nosal as well as participants of GRAPE, NBP and SGH seminars for insightful suggestions. The support of National Science Centre (grant UMO-2012/01/D/HS4/04039) is gratefully acknowledged. All opinions expressed are those of the authors and have not been endorsed by NSC nor NBP. The remaining errors are ours.

<sup>†</sup>Corresponding author. E-mail: mbielecki@wne.uw.edu.pl.

# 1 Introduction

It is not controversial to state that pension systems are an important force behind the changes of income distribution. It is an empirical regularity that income and consumption inequalities increase with age, and a big part of this divergence can be explained by redistributive properties of the pension system, see Storesletten et al. (2004). In fact, as demonstrated by Castaneda et al. (2003) an economy populated with identical agents faced with uninsured idiosyncratic shocks to individual productivity cannot reproduce some of the actual inequality distributions unless the quantitative features of the social security system are accounted for. These features comprise contribution rates (taxes) and pension benefits (transfers). In fact, optimal marginal tax rates should vary with age, in response to changing wealth as well as survival probabilities, whereas – as demonstrated by Gervais (2012) – they vary only in response to the earned income age profile, thus making the progressivity observed in many advanced economies a highly imperfect substitute to the optimal taxation scheme.

Clearly, the structure of the pension system affects not only income and consumption inequalities, but – possibly more directly – wealth inequalities. Domeij and Klein (2002) show that the generosity of the Swedish pension system actually *reduces to zero* private savings among a large fraction of the population, making wealth inequalities twice as high as income inequalities in this country. Clearly, the fact that a fraction of population accumulates non-zero wealth also suggests there are important within cohort differences in ability and/or willingness to save. Indeed, Hairault and Langot (2008) show that low productivity individuals have little room to adjust *via* savings to increases in tax rates, thus boosting wealth and welfare inequalities in an economy that undertakes a parametric reform of a pay-as-you-go system. Similar effect is confirmed by Song (2011). Consequently, the effect of pension system on inequalities seems asymmetric – the impulse to change in inequality is lower if benefits are reduced than if contribution rates are increased.

But what about the role of the redistributing features of the pension system itself? Is it effective to introduce redistribution in the retirement period rather than in the working period? Indeed, many pension systems involve caps, minimum pension benefit guarantees or other forms of instruments designed to compress the benefit distribution, and may either encourage or discourage both intensive and extensive labor supply adjustments. Naturally, such instruments affect individuals differently, depending on their time preference, leisure preference and productivity endowment – but are they effective?

While microsimulations are increasingly used as a tool for *ex ante* policy analysis – see Bourguignon and Spadaro (2006) – in the analyses of pension systems and their reforms with the use of the overlapping generations framework, the dominant approach relies on relatively restrictive assumption of within cohort homogeneity, see Fehr (2009). With individuals identical within cohorts, life-time patterns of income are the only sources of potential consumption and wealth inequality, but the profession typically models agents to prefer smooth consumption and wealth to constitute a stock of savings facilitating that smoothing process. While there is a fast growing strand of papers that attempt to introduce within cohort heterogeneity – see for example Fehr et al. (2008), Hairault and Langot (2008), Fehr and Kindermann (2010), Bucciol (2011), Cremer and Pestieau (2011), Kumru and Thanopoulos (2011), Fehr and Uhde (2014), St-Amant and Garon (2014), Kindermann and Krueger (2014) – they rely on idiosyncratic lifespan or income shocks. This way of operationalizing heterogeneity is valuable from many research perspectives, but yields only a limited scope of insights for the analysis of inequality. Most notably, one cannot answer the question on whether or not redistribution instruments within the pension system are effective at achieving their goal for reducing inequalities at older ages. Neither are they able to evaluate the efficiency cost associated with such redistribution.

Our paper contributes to the literature by filling this gap. There are three main contributions. First, we compare explicitly two instruments – contribution cap and minimum pension benefit guarantee<sup>1</sup>. We

---

<sup>1</sup>In the remainder of the paper, for brevity, we often use the term *minimum pension* to refer to this instrument.

do that in two settings: a defined benefit system (DB) and a transition from DB to a defined contribution system (DC). In both settings the economy experiences longevity which gradually changes the population structure. Such choice of settings is motivated by policy relevance: most of the economies have either a DB system or are already in transition to a DC system, see Gruber and Wise (2004), while gradual longevity processes are faced literally around the globe.

Second, we explicitly control for the inequality which stems from differentiated productivity endowments and the inequality which stems from heterogeneous preferences. Hence, we decompose the changes in the inequality resulting from these two instruments into those that help to alleviate dispersion in endowments and those that help to dispersion which stems from differences in preferences. While both of the analyzed instruments are aimed at reducing the inequalities (in terms of income, consumption and wealth) their general equilibrium effects are not likely to be similar. Moreover, they address different percentiles of earned income distribution. Thus, it is particularly relevant to observe, which *types of inequalities* these instruments address best. Assume a world with an entirely private social security with perfect annuities and homogeneous survival probability within a cohort. In such a setting, consumption inequality at older ages can only result from two sources: agents did not want to save the same amount (differentiated preferences) or they could not save the same amount (differentiated endowments). To address this point, we propose a partial equilibrium approach to isolate the effect of each type of inequalities.

Third, we discuss both the efficiency and the inequality consequences of the two instruments, which is of direct policy relevance. Namely, we calibrate our economy to replicate the features of a country which actually underwent a reform from a defined benefit to a defined contribution system, Poland. While the DB system had only the minimum pension benefit guarantee, the DC system imposes also a contribution cap. Thanks to the design of this study, we can also answer the question if the implemented reform has contributed to fostering or limiting the scope of consumption and wealth inequalities in this economy.

We rely on an overlapping generations model, but with an additional feature of a within-cohort heterogeneity. Thanks to this design we can explicitly compare an economy with contribution cap and minimum pension benefit to an economy without such instruments. The standard approach to within cohort heterogeneity in this literature typically relies on random income or productivity shocks.<sup>2</sup> However, the objective in this study is to distinguish specifically inequalities of endowments and differences in preferences. Thus, instead of idiosyncratic productivity shocks, we rely on differentiated productivity endowment – similar to Hénin and Weitzenblum (2005), McGrattan and Prescott (2013). This paper differentiates from the previous ones because it explicitly tackles heterogeneous preferences.<sup>3</sup> In fact, agents share a family of utility functions, but the actual trade-off between consumption and leisure (the intra-temporal choice) as well as preference for the future (the inter-temporal choice) are heterogeneous within cohort. Our agents are fully rational in a sense that the decision heuristic for the agents follows directly from solving the lifetime utility optimization problem and the same holds for the firms. Thus, this approach is similar to multi-agent systems (MAS) with all advantages of the general equilibrium setting. Whereas a standard OLG model aggregates over cohorts to obtain general equilibrium conditions, an OLG model with MAS aggregates over classes of economic agents within a cohort and only then over cohorts, see Ferber (1999), Tesfatsion (2002), Wooldridge (2009). Thus, the uniqueness of the equilibrium relies on the same premises as in standard OLG with a representative

---

<sup>2</sup>As noted by Nishiyama and Smetters (2007), while privatizing social security can improve labor supply incentives, it can also reduce risk sharing. With randomized and non-insurable shocks to individual productivity, the original conclusions of Feldstein’s highly stylized model do not necessarily hold. Similar conclusions originate from models incorporating time inconsistency into the consumer choice, Imrohroglu et al. (2003), Bassi (2008), Fehr et al. (2008), van de Ven and Weale (2010), Fehr and Kindermann (2010), Kumru and Thanopoulos (2011). Alternative approach has been proposed by Gul and Pesendorfer (2004) with recursive and separable dynamic self-control preferences. Specifically, the pension system is viewed as a disciplining device or technology (in terms of savings), whereas PAYG component of the pension systems usually replaces the otherwise absent insurance mechanism (at the expense of inefficiency).

<sup>3</sup>In addition to differentiated productivities, some studies – e.g. Hénin and Weitzenblum (2005) – allow for differentiated mortality risk within cohort. We are unaware of any study that would allow for heterogeneity in preferences.

agent, as long as agents share the family function for preferences.<sup>4</sup>

The theoretical predictions within the scope of this study are relatively straight forward and scarce from a policy perspective. **First**, a DC system should result in higher consumption inequality between cohorts than a DB system, but it is not clear if by definition it will result in higher wealth and consumption inequality within cohort, either pre or post-retirement. We show that the biggest force behind changing the aggregate inequalities is the changing population structure due to longevity – in fact the shift in inequalities due to the pension system reform is minor. **Second**, agents with perfect foresight, if subjected to a contribution cap, will compensate with private voluntary savings, this raising wealth inequalities even in the DB system. We demonstrate that the actual effects of contribution cap are negligible on both wealth and consumption inequality. **Third**, optimal response to a minimum pension benefit guarantee is to lower the lifetime savings and labor supply. Thus, one should expect lower consumption disparities post-retirement but the effect on pre-retirement consumption will depend also on the general equilibrium effects. We find a large positive effect for consumption inequalities post-retirement, large negative effect for wealth inequalities post-retirement and virtually no negative effect on aggregate savings. This instrument has a relatively small effect numerically, but cuts in half the growth in the consumption inequalities due to the pension system reform. Yet, the fiscal costs are considerable. **Finally**, one should expect particularly large strategic response to introducing the minimum pension benefit – e.g. high preference for leisure should translate to much lower individual labor supply and savings if retirement income is guaranteed irrespectively of lifetime income. Thus, it is possible that the minimum pension benefit guarantee will address mainly the inequalities which stem from preferences. We find, however, that the feedback from future taxation dominates the free-riding incentives in the case of the preferences, whereas the minimum pension benefit guarantee addresses mostly the inequalities which stem from differentiated endowments.

This paper is structured as follows. First, we present the model in section 2 and its calibration in section 3. While our setting is fairly standard, there are some features of the model that respond to the characteristic of the actual reform that was undertaken and which we use for calibrating the model – both of which are explained in detail in these sections. Subsequently, in section 4 we analyze the effectiveness of two policy instruments. Step by step we show the results for the DB systems and DC systems, comparing them explicitly. Finally, in section 5 we discuss the results of our mental experiment. As much as reduction in inequalities could be desirable from a policy perspective, one should be inclined more closely towards instruments allowing to alleviate unequal distribution of endowments rather than differentiated preferences. The paper is concluded by a policy recommendation section.

## 2 Model

We develop a general equilibrium overlapping generations model, with exogenous but time varying growth as well as fertility and longevity. Economy is populated by  $k$  classes of agents with differentiated endowments and preferences (within one function family), who live for  $j = 1, 2, \dots, J$  periods facing a time and age specific mortality rate  $\pi_{j,t}$ . Agents have no bequest motive, but since survival rates until the age of  $j$  at time  $t$  – i.e.  $\pi_{j,t}$  – are lower than one, in each period  $t$  certain fraction of subcohort  $(j, k)$  leaves unintentional bequests, which are distributed within their subcohort.

---

<sup>4</sup>The reliance on the MAS has recently become more and more common, see as it could seem, Axtell (2000), Windrum et al. (2007), Wooldridge (2009). An agent is a program/routine capable of optimizing, as suggested by standard representative agent first order conditions. Such program can be run on a system populated by 'agents' with differentiated preferences and endowments, see Russell et al. (1995), Wooldridge and Jennings (1995), Kirman (1997).

## 2.1 Consumers

Consumers are born at the age of 20, which we denote  $j = 1$ , at which time they are randomly assigned with individual productivity multiplier  $\omega_k$  as well as utility function parameters. These values do not change until the agent dies. Unlike in Hénin and Weitzelblum (2005), McGrattan and Prescott (2013) or Kindermann and Krueger (2014), there are no idiosyncratic income or productivity shocks. Thus, a subcohort  $k = 1, 2, 3, \dots, K$  of agents within cohort  $i$  is described uniquely by assigned values of  $\phi$ ,  $\omega$  and  $\delta$ , see section 3.

The year of birth determines fully the survival probabilities at each age  $j$ . At all points in time, consumers who survive until the age of  $J = 80$  die with certitude. The share of population surviving until older age is increasing, to reflect changes in longevity. Decreasing fertility is operationalized by a falling number of births. The data for mortality and births come from a demographic projection until 2060 and is subsequently treated as stationary until the final steady state, see section 3. This modeling choice is conservative in the sense that PAYG DB systems are more fiscally viable if population stabilizes.

At each point in time  $t$  an individual of age  $j$  and subcohort  $k$  born at time  $t - j + 1$  consumes a non-negative quantity of a composite good  $c_{j,k,t}$  and allocates  $l_{j,k,t}$  time to work (total time endowment is normalized to one). In each period  $t$  agents at the age of  $j = \bar{J}_t$  retire exogenously. Consumers can accumulate voluntary savings  $s_{j,k,t}$  that earn the interest rate  $r_t$ . Consequently, consumers lifetime utility is as follows:

$$U_{j,k,t} = u_k(c_{j,k,t}, l_{j,k,t}) + \sum_{s=1}^{J-j} \delta_k^s \frac{\pi_{j+s,t+s}}{\pi_{j,t}} u_k(c_{j+s,k,t+s}, l_{j+s,k,t+s}) \quad (1)$$

where discounting takes into account time preference  $\delta_k$  and probability of survival. The instantaneous utility function is given by:

$$u_k(c_{j,k,t}, l_{j,k,t}) = c_{j,k,t}^{\phi_k} (1 - l_{j,k,t})^{1-\phi_k} \quad (2)$$

and  $l_{j,k,t} = 0$  for  $j \geq \bar{J}_t$ . In this specification  $\phi_k$  determines steady state labor supply. Labor income tax  $\tau^l$  and social security contributions  $\tau$  are deducted from gross earned labor income to yield disposable labor income. Following the Polish legislation, we assume that the labor income tax  $\tau^l$  is deducted from gross pension benefit to yield disposable pension benefit. Interest earned on savings  $r$  are taxed with  $\tau_k$ . In addition, there is a consumption tax  $\tau^c$  as well as a lump sum tax/transfer  $\Upsilon$  equal for all subcohorts, which we use to set the budget deficit in concordance with the data. Received bequests are denoted by  $beq$ . When working, the agents are constrained by earned disposable income, bequests and savings from previous periods with net interest. When retired, the agents are constrained by disposable pension benefit, bequests and savings from previous period with net interest. Thus, agent of age  $j$  in period  $t$  maximizes her lifetime utility function  $U_{j,k,t}$  subject to the following sequence of budget constraints:

$$(1 + \tau_t^c)c_{j,k,t} + s_{j,k,t} = (1 + (1 - \tau_t^k)r_t)s_{j-1,k,t-1} + (1 - \tau_t^l)(1 - \tau)w_t\omega_k l_{j,k,t} + (1 - \tau_t^l)b_{j,k,t} + beq_{j,k,t} - \Upsilon_t \quad (3)$$

## 2.2 Production

Individuals supply labor (time) to the firms. The amount of effective labor of age  $j$  in subcohort  $k$  used at time  $t$  by a production firm is  $L_t = \sum_{j=0}^{\bar{J}-1} \sum_{k=1}^K N_{j,k,t} \omega_k l_{j,k,t}$ , where  $N_{j,k,t}$  is the size of a  $(j, k)$  subcohort at time  $t$ .

Perfectly competitive producers supply a composite final good with the Cobb-Douglas production function  $Y_t = K_t^\alpha (z_t L_t)^{1-\alpha}$  that features labor augmenting exogenous technological progress denoted as  $\gamma_t = z_{t+1}/z_t$ . Standard maximization problem of the firm yields the return on capital  $r_t^k = \alpha K_t^{\alpha-1} (z_t L_t)^{1-\alpha} - d$  and real wage  $w_t = (1 - \alpha) K_t^\alpha z_t^{1-\alpha} L_t^{-\alpha}$ , where  $d$  denotes the depreciation rate on capital.

### 2.3 Pension system

The pre-reform initial steady state pension system is a PAYG DB system, with an exogenous contribution rate  $\tau$  and an exogenous replacement rate  $\rho$  with  $b_{\bar{j},k,t} = \rho \cdot w_{t-1} \cdot \omega_k \cdot l_{\bar{j}-1,k,t-1}$  holding for all  $j, k$ . The benefits are indexed annually.<sup>5</sup> The system collects contributions from the working and pays benefits to the retired:

$$\sum_{j=\bar{j}_t}^J \sum_{k=1}^K N_{j,k,t} b_{j,k,t} = \tau \sum_{j=1}^{\bar{j}_t-1} \sum_{k=1}^K \omega_k w_t N_{j,k,t} l_{j,k,t} + \textit{subsidy}_t \quad (4)$$

where  $\textit{subsidy}_t$  is a subsidy/transfer from the government to balance the pension system.

The post-reform final steady state pension system is PAYG DC. The obligatory contribution rate  $\tau$  is kept the same as in the initial steady state. Thus, the economy we analyze is in transition from a defined benefit to a defined contribution system. The DC pension system collects contributions and uses them to cover for contemporaneous benefits, but pays out pensions computed on the basis of accumulated contributions, as given by equation:

$$b_{\bar{j}_t,k,t} = \frac{\sum_{s=1}^{\bar{j}_t-1} \left[ \prod_{\iota=1}^s (1 + r_{t-j+\iota}^I) \right] \tau_{t-j+s-1} \omega_k w_{t-j+s-1} l_{s,k,t-j+s-1}}{\prod_{s=\bar{j}}^J \pi_{s,t}} \quad (5)$$

where  $r_t^I$  is defined by the rate of the payroll growth. Analogously to the PAYG DB case, the benefits are subsequently indexed annually.

### 2.4 The government

The government collects taxes ( $\tau^k$  on capital,  $\tau^l$  on labor and  $\tau^c$  on consumption, as well as a lump-sum tax/transfer  $\Upsilon$ ) and spends a fixed share of GDP on unproductive yet necessary consumption  $G = g \cdot Y$ . Government balances the pension system. Given that the government is indebted, it naturally also services the debt outstanding.

$$T_t = \sum_{j=\bar{j}_t}^J \sum_{k=1}^K N_{j,k,t} \left[ \tau_t^l ((1 - \tau) w_t \omega_k l_{j,k,t} + b_{j,k,t}) + \tau_t^c c_{j,k,t} + \tau_t^k r_t s_{j-1,k,t-1} + \Upsilon_t \right] \quad (6)$$

$$G_t + \textit{subsidy}_t + r_t D_{t-1} = T_t + (D_t - D_{t-1}) \quad (7)$$

We set the initial steady state debt  $D_t$  at the initial data level, and final steady state at around 45% of GDP, which was the actual value of debt to GDP ratio in 1999. We calibrate  $\Upsilon_t$  in the steady state to match the deficits and debt to maintain the long run debt/GDP ratio fixed and keep it unchanged throughout the whole path. The overall debt to GDP ratio is not allowed to exceed 60% in the model, which is the constitutional limit in Poland. Once this threshold is hit, consumption tax adjusts. On the transition path the values of  $\Upsilon$  and  $G$  are held fixed at the level from the initial steady state (per effective unit of labor).

### 2.5 Market clearing and equilibrium conditions

In the equilibrium the goods market clearing condition is defined as  $\sum_{j=1}^J \sum_{k=1}^K N_{j,k,t} c_{j,k,t} + G_t + K_{t+1} = Y_t + (1 - d)K_t$ . This equation is equivalent to stating that at each point in time the price for capital and labor would be set such that the demand for the goods from the consumers, the government and the producers would be met. This necessitates clearing in the labor and capital markets. Thus labor

<sup>5</sup>The model is calibrated to the case of Poland. Indexation is stipulated by legislation as the rate of the payroll growth.

is supplied according to:  $L_t = \sum_{j=1}^{\bar{J}-1} \sum_{k=1}^K N_{j,k,t} \omega_k l_{j,k,t}$  and capital accumulates according to  $K_{t+1} = (1-d)K_t + \sum_{j=1}^J \sum_{k=1}^K N_{j,k,t} s_{j,k,t}$ , where  $s_{j,k,t}$  denotes private savings.

An equilibrium is an allocation  $\{(c_{1,k,t}, \dots, c_{J,k,t}), (s_{1,k,t}, \dots, s_{J,k,t}), (l_{1,k,t}, \dots, l_{J,k,t}), K_t, Y_t, L_t\}_{t=0}^{\infty}$  and prices  $\{w_t, r_t, r_t^k, r_t^G\}_{t=0}^{\infty}$  such that:

- for all  $t \geq 1$ , for all  $j \in [1, J]$  for all  $k \in [1, K]$   $((c_{j,t}, \dots, c_{J,t+J-j}), (s_{j,t}, \dots, s_{J,t+J-j}), (l_{1,t}, \dots, l_{J,t+J-j}))$  solves the problem of an agent at the age of  $j$  from subcohort  $k$  in period  $t$ , given prices;
- prices are given by:  $r_t = \alpha K_t^{\alpha-1} (z_t L_t)^{1-\alpha} - d$  and  $w_t = (1-\alpha) K_t^\alpha z_t^{1-\alpha} L_t^{-\alpha}$
- government sector is balanced, i.e. (4), (6) and (7) are satisfied;
- markets clear.

## 2.6 Model solving

This section describes construction of the multi-agent model used to simulate the scenarios of pension system reform. We present all classes of agents in the system and the way they interact in the process of simulation. The multi-agent system created to simulate effects of pension system reform consists of three different types of agents, representing various elements of the domain – government agent, private sector agent and multiple subcohorts, i.e. heterogeneity within a single cohort.

**Subcohorts** represent a part of a cohort that shares the same parametrization, that is productivity  $\omega$ , preference for leisure  $\phi$  and time preference  $\delta$ . In each year of the simulation each subcohort either divides her time between labour and leisure given equations (1)-(2), and receives salary from the private sector agent (if she is working) or receives benefits from the government agent (if she is retired), depending on the age of this cohort, following the budget constraint (3). In both cases she pays taxes to the government and spends some of its resources on consumption. Every subcohort decides about intratemporal division of time to work and leisure and intertemporal division of income into consumption and savings in order to maximize lifetime utility.

**Government agent** represents pension system and tax-collecting. Her responsibilities in the system consists of choosing values of different taxes and retirement age, keeping track of these rates, collecting taxes from all the subcohorts and paying benefits to retired subcohorts. The government agent can be PAYG DB or PAYG DC, with or without the two analyzed instruments – in total: six classes of this agent. Only one of them participates in a given instance of the simulation. Government agent is given a certain fiscal closure rule, i. e. a rule that translates government expenditures and tax base to tax rates, see equations (6)-(7).

**Private sector** represents companies from the private sector. She is the recipient of labor supplied by subcohorts, she keeps track of capital value in the system and calculates salaries for subcohorts given the production function.

In the **simulation process** the objective is to calculate the effects of different pension systems on inequality in a society. To achieve this objective, the system calculates two steady states, representing initial and final year of modeled period, and then computes transition path between them. Since general algorithm for computing both steady state and transition path is very similar, we describe it just once, introducing necessary distinctions.

Finding the problem solution is an iterative process using the Gauss-Seidel method. In each iteration, choices of agents are updated. The process stops when the difference between the capital from the new iteration is indiscernible from the previous iteration, i.e. smaller than a given parameter  $\epsilon$ . On a transition path the optimization criterion relies on a sum of  $\epsilon$  from each period. The parameter  $\epsilon$  has been set to  $0.1^{14}$  in the steady states ( $L_\infty < 0.1^{14}$ ) and a sum over all  $T$  set to  $0.1^3$  for a transition path ( $L_1 < 0.1^3$ ).

Every iteration consists of the same major steps. Government, basing on capital calculated in the previous iteration (or initial capital in first iteration) and parametrization of the model, computes tax rates. Given these rates and the structure of the pension system, government also computes pension benefits for the retired cohorts. Given the amount of capital and labor, firms set interest rates and wages. Given the tax rates, interest rates, and wages (as well as received bequests), subcohorts choose labor supplied for each year of his life in the system, as well as consumption and savings. Given these choices, savings are aggregated to capital, to be compared with capital from the previous iteration. If the two values satisfy the norm condition, the process finishes. Otherwise, a new iteration starts.

## 2.7 Simulation scenarios

The baseline scenario of no policy change involves the transition from the initial state of DB PAYG to a final steady state of PAYG DC system. This scenario is subjected to the demographic change and exogenous productivity growth slowdown, see Section 3. This simulation yields income and consumption distributions for each point in time. In the reform scenarios, we introduce either of the two mechanisms designed to curb income inequalities: the minimum pension and the contribution cap.

The **minimum pension** stipulates that an individual receives a minimum pension irrespectively of individual pre-retirement earnings. In the PAYG DB system that would imply that if  $\rho \cdot w_{t-1} \cdot \omega_k \cdot l_{\bar{j}-1,k,t}$  falls short of the threshold  $\underline{b}_t$ , an individual receives  $\underline{b}_t$  as the first pension benefit to be indexed in the next years for the surviving cohorts. In the DC system that would imply that if the combined annuity described in equation (5) falls short of  $\underline{b}_t$ , an individual receives  $\underline{b}_t$  as the first pension to be indexed in the next years for the surviving cohorts. The value of  $\underline{b}_t$  has been computed from actual values, as a share of wage income in  $t = 1$  and kept at the same ratio for the subsequent periods. Note, that this instrument automatically redistributes towards low productivity and low patience subcohorts. Since in principle pure DC systems are balanced, this redistribution necessitates taxation surge.

The **contribution cap** stipulates that annual contributions to the pension system are capped, i.e. if for a given subcohort  $(j, k)$  the contributions defined as  $\tau w_t \cdot \omega_k \cdot l_{j,k,t}$  exceed a certain limit, the contribution rate  $\tau$  is set to 0 for earnings above this threshold. In the legislation the specified value is  $2.5 \cdot \bar{w}_t$ , where  $\bar{w}_t$  is the average gross compensation. However, according to the data, this cap affects approximately 2% of the working-age population. We thus calibrate the percentage rather than the number in the model, thus yielding  $1.7 \cdot \bar{w}_t$ .<sup>6</sup> Note that this instrument automatically reduces the social security contributions among high productivity subcohorts, which allows them to consume/save more (depending on time preference). Naturally, the “missing” contributions negatively impact the balance in the social security system, but also lower the expected received pensions, thus fostering private savings, *ceteris paribus*.

## 3 Calibration

This section describes the calibration of all structural parameters in the model. We discuss below the demographics, technological progress, depreciation, taxation as well as endowments and preferences.

### 3.1 Calibration of the structural parameters

First we describe the structural parameters of the model, i.e. those that do not change between the baseline and the reform scenarios.

---

<sup>6</sup>Kindermann and Krueger (2014) choose the cap at 2.0 for the US economy. As can be seen from our result, this decision is of a minor quantitative importance.



**Demographics.** The Eurostat demographic projection for Poland serves as a source for the size of new cohort arriving each year in the economy, i.e.  $j = 1$  at each point in time  $t$ . The same source provides size of each cohort at consecutive ages, which serves for computing the survival probabilities  $\pi_{j,t}$ . The projection is available until 2060. We assume both birth rate and survival rates flat after that period, so our population stabilizes after 2140.

**Technological progress** The model specifies labor augmenting growth of technological progress. The values for 50 years ahead projection were taken from the forecast by the Aging Work Group of the European Commission, which comprises of such time series for all EU Member States.<sup>7</sup>

**Taxes.** There are no tax redemptions on capital income tax, so *de iure* and *de facto* tax rates were set equal, which implies  $\tau_k = 19\%$ . Labor income tax ( $\tau_l$ ) was set at effective 11%, which matches the rate of labor income tax revenues in the aggregate employment fund. Consumption tax  $\tau_c$  was set at 11%, which matches the rate of revenues from this tax in aggregate consumption in 1999. This tax is allowed to increase in the baseline and reform scenarios if debt hits the threshold of 60% share in GDP.

**Pension system.** The original replacement rate  $\rho$  in the DB PAYG system was set to match the share of the pension benefits in GDP in 1999, i.e. 5%. Subsequently, the social security contributions were set to reflect the size of deficit in the pension system (denoted in our model as *subsidy*), which amounted to 0.8% of GDP in 1999.

For cohorts already working under the defined benefits system, there is no data on accrued savings that could be used to computing pension benefits under defined contribution. We use the records of Social Insurance Fund to recover these implied savings, based on the released sample of 1% of the population. To assure comparability with the model, initial capital is expressed in terms of average wage ( $w$ ). Hence, we mitigate any within cohort heterogeneity there may have been in the data by imposing equal starting point to all individuals within a cohort.

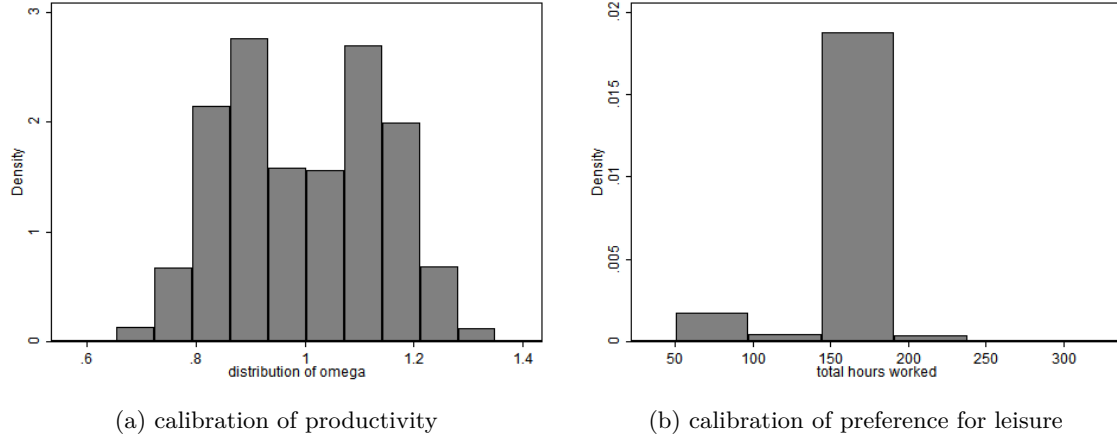
### 3.2 Calibration of the productivity endowment and leisure preference

We have three dimensions of individual within cohort heterogeneity: two for preferences (impatience and preference for leisure) and one for endowments(productivity). Following the previous insights from Hénin and Weitzblum (2005), McGrattan and Prescott (2013) as well as Kindermann and Krueger (2014), we calibrate them using micro datasets. Unlike McGrattan and Prescott (2013), we rely on individual rather than household data for three main reasons. First, we cannot obtain reliable indicators of individual productivity from household budget surveys, whereas family earned income is not recoverable in the labor force survey for the families where at least one family member has other than wage income. Second, both household budget survey data and the labor force survey data are self-reported, thus featuring all the well known problems such as rounding the reported values of earnings and hours. Third, in the real economy wage earners are employed in both the productive sector and the directly unproductive sector, the latter in our model is in fact expressed in terms of government consumption. For these three reasons we rely on Structure of Earnings Survey. It covers the enterprise sector and comprises a sample app. 20 times bigger than LFS or HBS. The values of hours worked as well as earnings are reported in actual terms by the employers, which results in a substantially smoother distribution of the two variables. Finally, this way we also avoid confusion of wage income and capital income, see McGrattan and Prescott (2013).

---

<sup>7</sup>The overall assumption behind these forecasts is that countries with lower *per capita* income will continue to catch up but around 2030 all countries exogenous productivity growth will be converging slowly towards the steady state value of 1.7% *per annum*.

Figure 1: Calibrations based on Structure of Earnings Survey, 1998



**Productivity endowment ( $\omega_k$ ).** Since in our model the productivity endowment is allocated once for the entire lifetime, we use the early years in the career to obtain the distribution of these endowments. We estimate a standard Mincerian wage regression with education, occupation, industry and region controls, as well as the form of contract (fixed term or indefinite duration), form of employment (part-time, full-time, weekends, etc.). We use total hourly wage, including overtime and bonuses. The Mincerian wage regression was estimated for all individuals in the sample, so we had controls for age and experience (both linear and squared). Finally, we use fitted value of log earned hourly wage against the mean of this prediction for the individuals, who were active in the labor market for up to five years after graduation. This final distribution of productivity multipliers of  $\omega$ , i.e.  $\omega_k \in \langle 0.7\omega, 0.8\omega, 0.9\omega, 1.0\omega, 1.05\omega, 1.1\omega, 1.15\omega, 1.2\omega \rangle$ <sup>8</sup> is depicted in Figure 1a and has been used for calibration. These individual productivity endowment multipliers do not change during the lifetime, i.e. age-productivity profile is flat. This choice is irrelevant for the purpose of our study, because it is neutral to the reform scenario, but it is also fairly consistent with the data, see Deaton (1997), Boersch-Supan and Weiss (2011).

**Preference for leisure ( $\phi_k$ ).** Agents' preference for leisure/consumption is directly responsible for the labor supply decisions, so we calibrate it to replicate labor market participation rate of 56.8% in 1999. The final value amounts to 0.5, which seems reasonable: average hours worked in Polish economy amount to app. 2050<sup>9</sup>, i.e. 51.5% of the total workable time.

However, individual preference for leisure is likely to be heterogeneous, with a fraction of population working part-time or not at all. Since preference for leisure is set once for the whole lifetime, we cannot directly replicate the distribution of working/non-working population (i.e. pick from data a share of individuals who do not *currently* participate in the labor market to proxy for a share of individuals who in the model *never* participate in the labor market), because we would automatically translate the initial structure of inequalities to the future via non-participation. Thus, we rely on reported hours actually worked in the Structure of Earnings Survey which ranges from 31% to 206% of the regular working time. We thus obtain the individual multipliers of the preference for leisure  $\phi$ , i.e.  $\phi_k \in \langle 0.5\phi, 1.0\phi, 1.5\phi, 2.0\phi \rangle$ . The distribution scaled by the mean hours worked is depicted in Figure 1b.

<sup>8</sup>We run a similar analysis of median fitted value was to be the metric of endowments, the distribution is similar, the results are available upon request.

<sup>9</sup>Conference Board, averaged for 1999-2012 (*ahwpol* from The Conference Board Total Economy Database).

**The impatience  $\delta_k$ .** The aggregate value  $\delta$  was chosen to 0.975 match the interest rate of 7.7% on the asset portfolio, as observed in the data.<sup>10</sup> Depreciation rate  $d$  is calibrated to match the investment rate in the economy given  $\delta$ , i.e. app. 21%<sup>11</sup>.

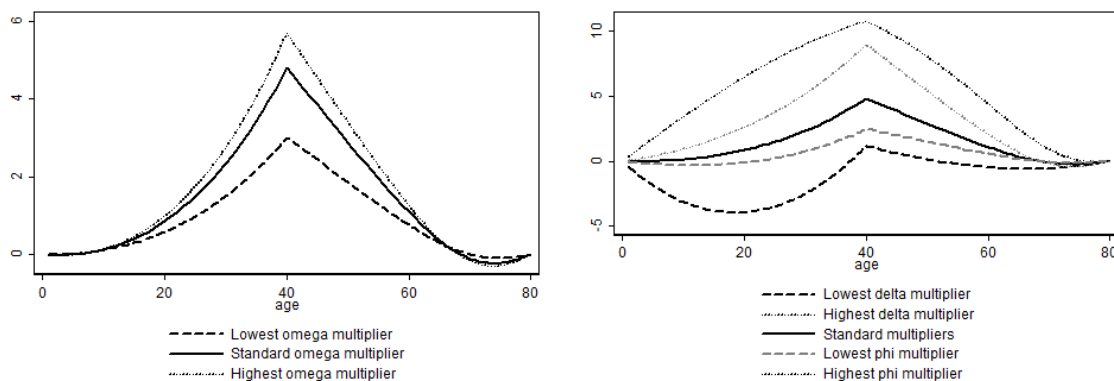
There are no empirical counterparts for the individual  $\delta_k$  for Poland. We thus chose the distribution to be uniform within cohort, i.e. the multipliers for  $\delta$  are given by  $\delta_k \in (0.98\delta, 1.0\delta, 1.02\delta)$ , with 98% of  $\delta$  and 102% of  $\delta$  each pertinent to 0.3 of the population within a cohort, whereas the remaining 0.4 of the population has simply  $\delta$ . While this choice may seem conservative, it already translates to substantial differentiation in the propensity to save. In fact, at the retirement age, patient agents have wealth (i.e. accumulated lifetime savings) 2.3 times larger than average agents, whose wealth is 4 times larger than for the impatient agents (computed for cohorts with multiplier of 1 for  $\omega$  and  $\phi$ ).

### 3.3 The implied within-cohort heterogeneity

The adopted small differences in the parameters of the utility function generate sufficient variation in income. In fact, the Gini coefficient for consumption in the initial steady state reaches approximately 25.5, which is a fair approximation of what is observed in the data, see Brzezinski (2011).<sup>12</sup> In terms of individual level differences, they are best observed in life cycle wealth profiles. More patient subcohorts with higher endowments and lower preference for leisure have substantially higher savings path than less patient subcohorts with lower endowments and higher preference for leisure, see Figure 2.

We depict two extreme scenarios. In addition to a standard subcohort, which has no multipliers on preference for time  $\delta$ , for leisure  $\phi$  and endowments  $\omega$ , we show extreme types of subcohorts. First, in Figure 2a we show life cycle for subcohorts which all multipliers over preferences set to 1.0, but have the highest or the lowest multiplier on endowments. Analogously, in Figure 2b we keep endowments multipliers at 1.0 and display subcohorts with differentiated preferences.

Figure 2: Life cycle path of wealth accumulation – calibration



(a) different endowments, identical preferences

(b) identical endowments, different preferences

<sup>10</sup>While this value may seem high, please note that we are calibrating to the case of a converging economy. For example, Nishiyama and Smetters (2007) calibrate interest rate to 6.25% for the US economy. Also, the average real annual rate of return at the level of 7.5% with a balanced portfolio strategy was achieved on average by the open pension funds in the period 1999-2009. Thus, this value is not excessive, when compared to data or to the literature

<sup>11</sup>Depending on the period over which the average is taken, it ranges from 20.8% for five years ahead and five years post reform, 23.1% for 2 years before-after span and 24.1% for a 1 year before-after span. The average for the period between 1995 (first reliable post-transition data) and 2010 amounts to 20.7%.

<sup>12</sup>For the wealth inequality there is no empirical counterpart for Poland.

## 4 Results

In this section we show the coverage of the analyzed instruments and discuss to what extent they are able to compress consumption inequalities. Clearly, during the retirement period consumption inequalities may depend to a considerable extent on accumulated wealth, thus the wealth inequalities are taken into account.

### 4.1 Coverage

In 1999, according to the data published by Social Insurance Fund, about 4% of the retirees collected a minimum pension benefit. In addition, according to the current data published by the tax authorities, about 2% of the tax payers are subject to the cut off in pension contributions. Our calibration of endowments and preferences replicate these features. However, longevity as well as a change in the pension system (from a defined benefit to a defined contribution) are likely to affect these proportions for two major reasons. First, the incentives coming from the minimum pension may imply that for some individuals lower labor supply becomes optimal.<sup>13</sup> Second, since contribution cap implies a lower effective labor tax rate, there is less distortion to supply labor for individuals with high endowments and/or low disutility of labor.

Figure 3: Coverage of instruments in various pension systems

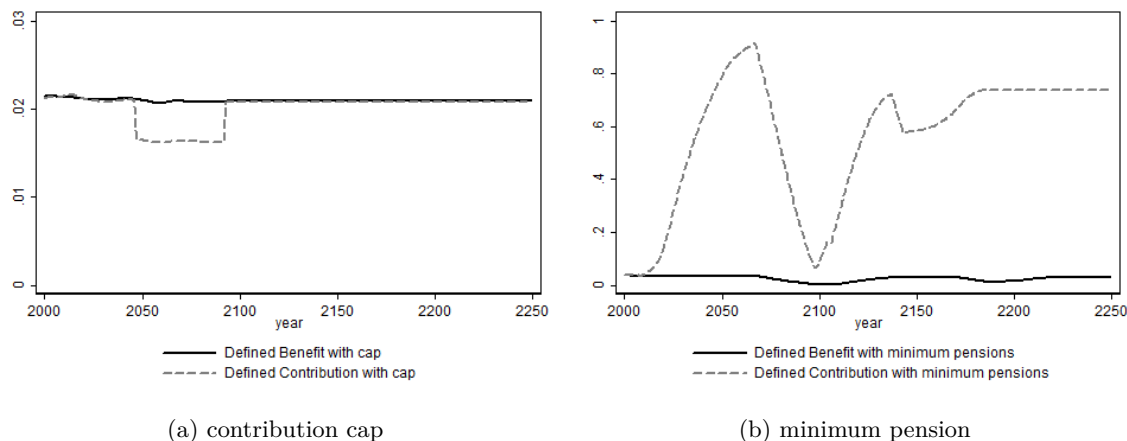


Figure 3 displays the changes in the coverage by the minimum pension for both the defined benefit pension system and a transition from the defined benefit to the defined contribution one. Since the distribution of endowments and preferences towards leisure do not change throughout the simulations, there are no significant changes in the coverage of the contribution cap in the defined benefit system. However, in the DC system we find a small decrease in the coverage rate for about 40 cohorts, see Figure 3a. This adjustments stems from the fact that subsequent to the tax increase – in both systems – few subcohorts with standard endowments but high preference for work temporarily record earnings somewhat below the contribution cap, because they lower labor supply in response to the increased taxes. That effect is transitory.

In the case of minimum pensions, the demographic transition necessitates substantial adjustment in the coverage rate for the DC system. In fact, the coverage rate increases from about 4% to as much as nearly 80% in the final steady state. This implies that the forced savings in the obligatory pension system are not sufficient to provide for the old age security. Please note, that the life expectancy increases in our simulations

<sup>13</sup>Please note, that agents do not see a the link between the labor supply decision and the pension. In fact, in individual optimization the derivative from pension benefit with respect to labor supply is zero. It is an interesting future avenue for research to analyze agents internalizing the pension benefit as a function of labor supply.

by 9 years in total, but we keep the retirement age constant.<sup>14</sup> The variation in the coverage rate reflects the waves of the demographic transition. The scope of the variation stems also from the fact that the minimum pension is computed as a share in average wage on the transition path, whereas the adjustment in capital and debt translate also to wages, which inflates the pure effect of demographics.

## 4.2 Aggregate inequalities

The overall feature of perfect foresight models is that agents adjust savings to the expected path of future incomes. This implies that longevity translates to increased savings regardless of the pension system, although the reasons differ. Under the defined benefit system, agents expect a stark increase in taxation, which lowers their net income. In the defined contribution scheme agents expecting to live longer need higher savings to supplement the relatively low pension benefits. Consequently, there are likely to be substantial wealth effects of the longevity as well as policy instruments analyzed, whereas the consumption will be smoothed in the life-cycle which should lead to lower consumption inequalities. Indeed, this is what we observe. In the DB system, the wealth inequalities are substantially larger during the demographic transition, to level off at higher levels due to the changed population structure, see Figure 4b. Please note that values of Gini coefficient above 1 in the case of wealth inequalities come from a relatively large share of subcohorts with negative savings at some stage of their life.<sup>15</sup>

The transition to a defined contribution system necessitates additional adjustments in individual savings, leading to a starker increase in consumption inequalities and a lower number of subcohorts with negative savings. This is also the first case when either of the instruments displays noticeable effect on inequalities. The minimum pension – with high coverage rates – increases the wealth inequalities and reduces consumption inequalities permanently. This suggest that the incentives from the minimum pension are rather strong, lowering the individual voluntary savings, and thus slowing down the rate of capital accumulation, permanently. On the other hand, wealth inequalities are lower with DC system than in the case of the DB system, which suggests that impatient subcohorts increase savings more in response to demographic changes under a DC system than in the case of a DB system. This effect stems from lower expected pension benefits under the DC system.

The deterrence from private savings is confirmed in Figure 6. Given the OLG setting of our model, we compute inequalities measures at retirement (for subsequent cohorts reaching that age). The majority of the effect on consumption inequalities comes from the demographic transition (changes in life expectancy at retirement). While the introduction of the DC system leaves older cohorts with less room to adjust to the new rules, gradually the difference in the consumption Gini decreases to stabilize with the new demographic structure. Wealth inequalities do not converge between DB and DC systems. The instrument with the strongest power to reduce inequalities in consumption and at the same time foster them in wealth is the minimum pension. In fact, given the wide coverage of the minimum pension, wealth inequalities are substantially higher with the minimum pension benefit than with no instrument.

## 4.3 Macroeconomic effects

Although wealth inequalities grow, the aggregate effect on capital is negligible. The fiscal consequences are considerable, though. Overall, the fiscal cost of demographic transition is about 2.5 pp as a share in GDP with an unchanged pension system (DB). In principle, pure DC system is balanced, so the final steady state has no deficit in the pension system. Also taxes can be somewhat lowered once DC is implemented. Clearly, this holds for a contribution cap, since in DC systems contributions should be equal to benefits. Yet, if DC

---

<sup>14</sup>Agents do not supply labor past that age.

<sup>15</sup>The transversality condition requires no debt at death, but permits negative savings throughout the life-cycle.

Figure 4: Evolution of Gini coefficients, defined benefit system, all subcohorts

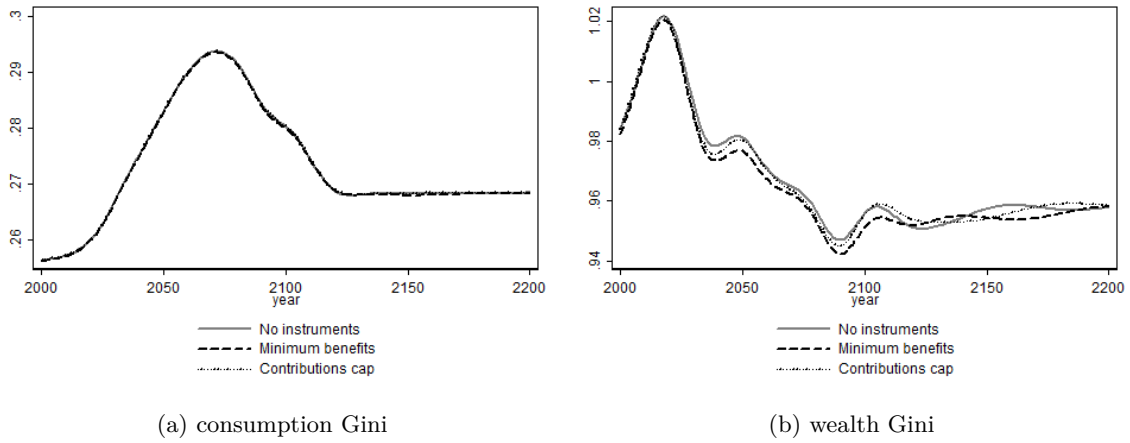


Figure 5: Transition path to a defined contribution system - all subcohorts

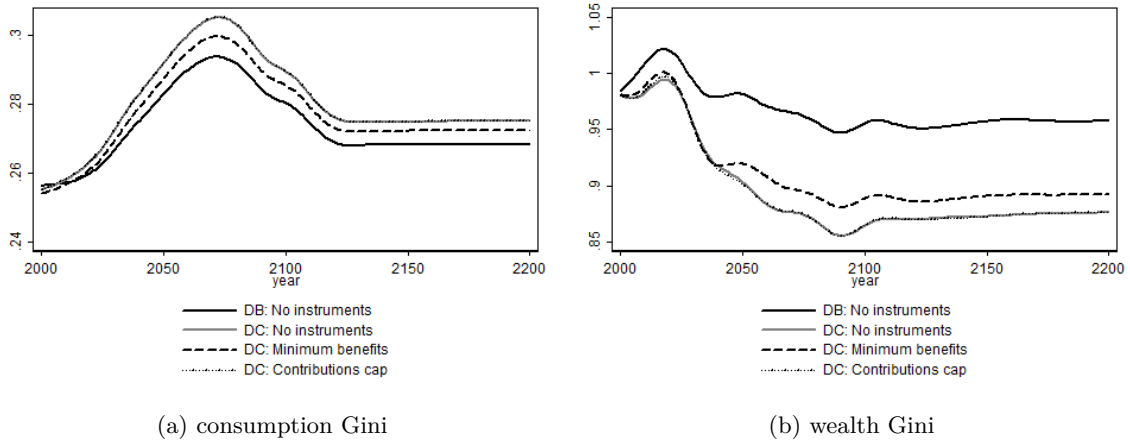


Figure 6: Transition path to a defined contribution system - Gini at retirement

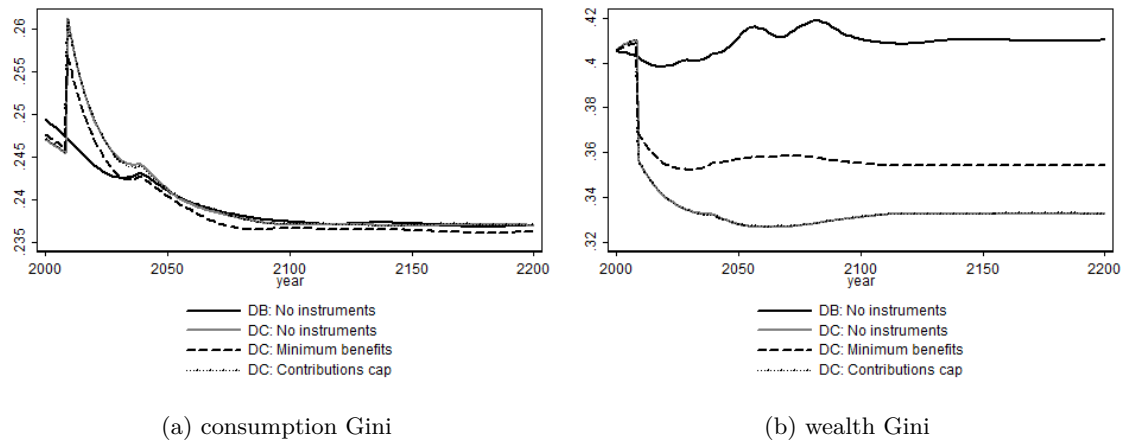
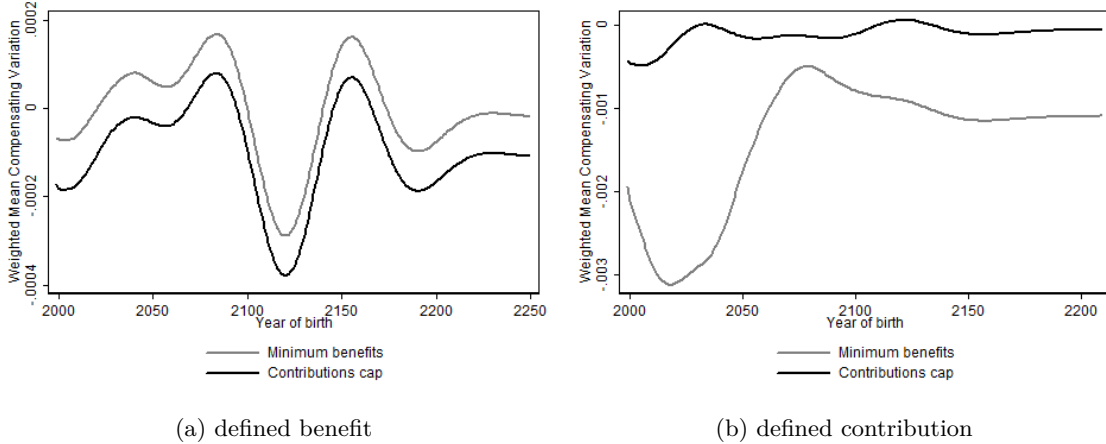


Figure 7: Welfare effects of the two instruments



system is combined with a minimum pension guarantee it generates a fiscal deficit of nearly 1% of GDP. Clearly, minimum pensions are particularly costly from a fiscal perspective in the defined benefit – they reinforce the already weak incentives to work.

Table 1: Macroeconomic effects

	No instrument		Minimum pension		Contribution cap	
	DB	DC (rel to DB)	DB	DC	DB	DC
Capital	52.6%	60.4%	52.7%	60.3%	52.6%	60.5%
	Tax rate					
initial	11.00	11.00	11.00	11.00	11.00	11.00
final	15.44	10.95	15.43	11.99	15.46	10.95
difference (in pp)	4.44	- 0.05	4.43	0.99	4.46	- 0.05
	Pension system deficit					
initial	1.46		1.56		1.46	
final	3.95	0.00	4.02	0.87	3.97	0.00
difference (in pp)	2.49		2.46	-0.69	2.51	

*Notes:* Capital reported in relation to the initial steady state (ratio, in %), expressed per effective unit of labor. Tax rate in pp difference between the initial and final steady state. Pension system deficit as a share of GDP (pp difference between initial and final steady state).

#### 4.4 Aggregate welfare

Overall the welfare effects of these instruments are negative, mostly because they lead to increase in taxation. We display welfare as a comparison of individual utilities in the world with an instrument to those from the world where there are no instruments. We measure change in utility as a compensating variation and express it in terms of lifetime consumption discounted to  $j = 1$ . The recorded compensating variation is subsequently weighted over the subcohorts to obtain the value for the entire cohort in a year when this cohort comes to the model.

As displayed in Figure 7a-7b, since there are virtually no effects of the contribution cap in terms of aggregate change in consumption or wealth, there are also no welfare effects of this instrument. The effects

of minimum pension under the defined benefit system are also minor, because in the DB system there is almost no use of this instrument. The only instrument with a large bearing on the welfare is the minimum pension benefits guarantee in the defined contribution system. Since here, the coverage is large one could expect that this is the source of the welfare gains for some subcohorts. On the other hand, taxes increase substantially, so the overall effect is negative.

Summarizing, there are considerable wealth and thus also consumption consequences of introducing a minimum pension. If consumption equality was the objective, this instrument helps to achieve it in the defined contribution system, but it constitutes a much weaker intervention in the defined benefits system. Also, in the former the wealth inequalities are lowered. Additionally, there are non-negligible fiscal cost in the form of pension system deficit and thus higher taxes to all cohorts. These overall decrease in inequality is not large, though. The noticeable change concerns mostly individuals in their retirement period. Contrary to the minimum pension, the contribution cap has low coverage and thus low overall effects. In the case of a defined contribution system there are no fiscal consequences, but in the case of a defined benefit pension system, lower contributions translate to permanently higher fiscal deficit and thus also higher taxes. However, when compared to the scenario of no policy instruments, these effects are minor.

These overall results help build an intuition on the effects introduced by two instruments: minimum pension benefit and contribution cap. At this point, however, we are unable to judge if these effects – especially in the case of minimum pension – stem from large response to the instrument or rather from the heterogeneity of agents combined with the process of the demographic transition. To put it differently, are minimum pension benefits overcoming the dispersion in endowments? Or maybe the extent of redistribution exceeds these objective sources of heterogeneity and also dispersion in preferences is counteracted? In the next section we separate these two effects.

## 5 Decomposing inequalities to endowments and preferences

In the decomposition experiment, we eliminate one of the channels of heterogeneity and repeat the scenarios of DB and DC systems, with the two analyzed instruments (keeping prices fixed at levels from a respective path with all channels of heterogeneity on). Figures A1 and A2 in the Appendix present the comparisons across these parametrizations without any instruments for consumption and wealth, respectively. When we eliminate any dispersion in endowments, the distributions are fairly similar for both consumption and wealth. However, eliminating the heterogeneity of preferences implies a substantial decrease in inequalities for both aggregates. This general tendency holds for inequalities at retirement, as portrayed by Figures A1c, A1d A2c and A2d. In fact, the role of preference heterogeneity is even more pronounced and the dispersion of endowments matters less for the computed measures of inequality.

In Figures 8 and 9 we demonstrate the effects of the minimum pension benefit for consumption and wealth, respectively. Majority of the change in inequality in the DB system stems from demographic transition and thus there are no visible effects of instruments, whether or not the channels for heterogeneity (individual preferences and endowments) are operational. In the defined contribution system, minimum pension benefits are able to reduce the effects of inequalities due to both preferences and endowments, whereas the contribution cap gives no significant reduction in Gini coefficients.<sup>16</sup> In fact, the reduction in inequalities is larger in the case of a simulation without preference heterogeneity than in the case of a simulation with no endowments heterogeneity. Consequently, while the effects are fairly small, they are bigger for what could be called “equality of opportunities” approach to social policy.

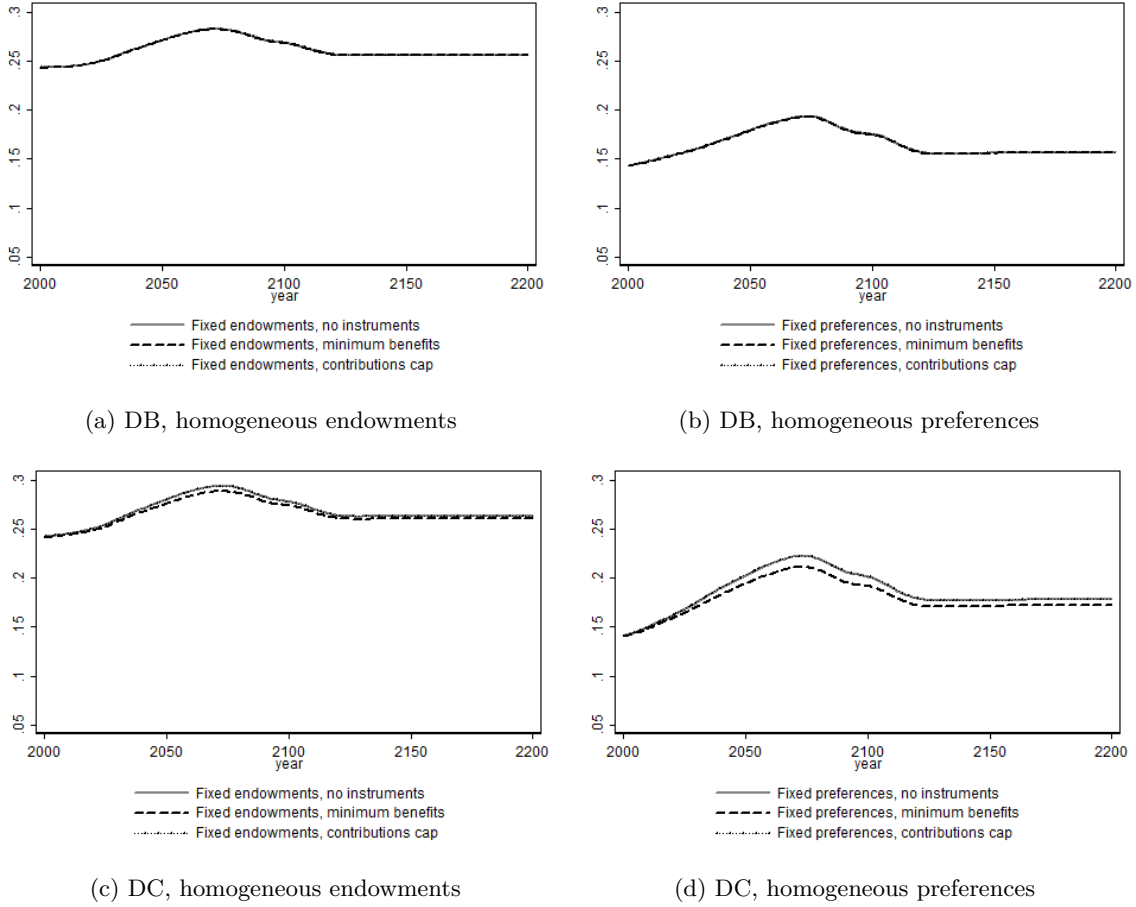
The changes in inequalities follow the same patterns. There are also no visible effects of the contribution cap. The wealth inequality – as expected – is higher when minimum pensions are introduced, but the

---

<sup>16</sup>The same holds for inequalities at retirement, see Figure A4 and A5 in the Appendix.



Figure 8: Decomposition: the effects of instruments - consumption Gini



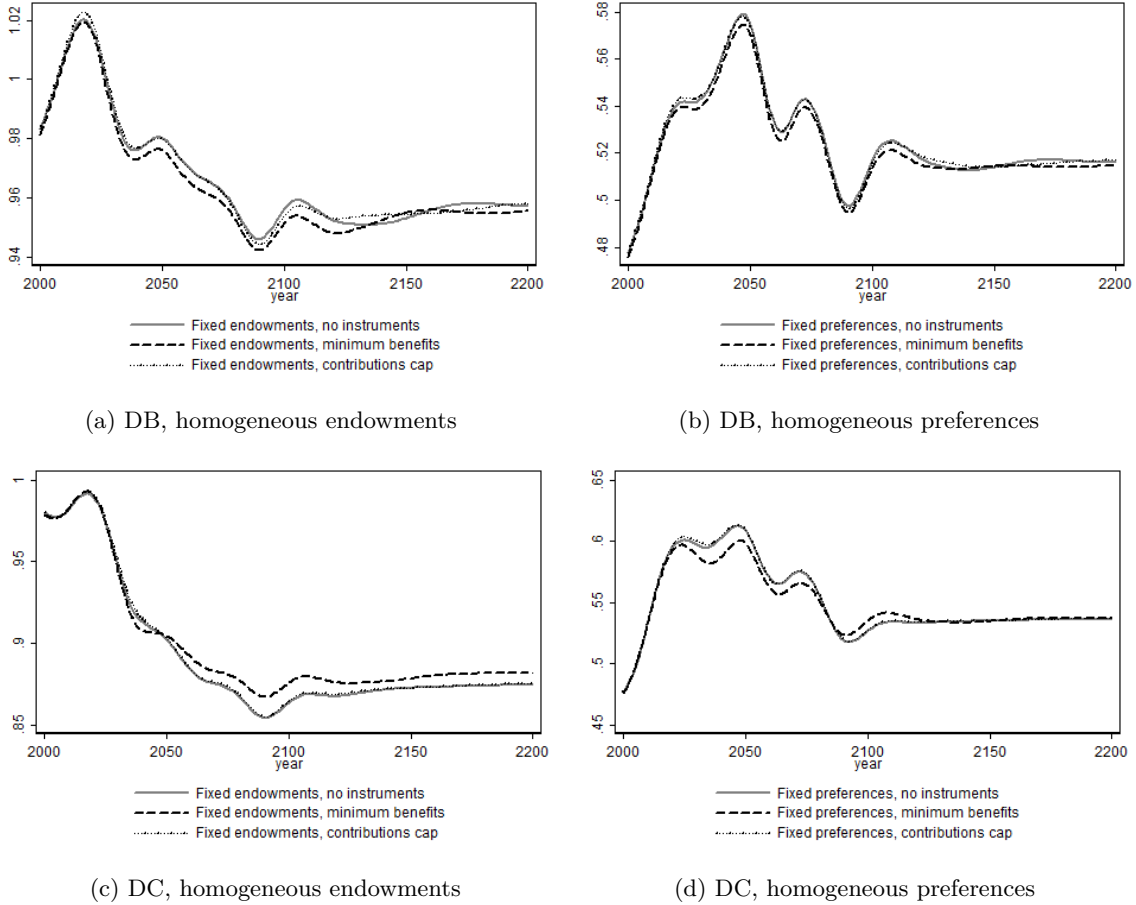
minimum pension benefit guarantee has bigger (though negative) effect on the Gini coefficient in the world with identical endowments than in the world with identical preferences. This confirms that the major source of changes in Gini on consumption stem from changes in individual stock of savings, which are very responsive to the minimum pension benefit guarantee.

The analysis from the previous section as well as the decomposition experiments demonstrate two important results. First, forward looking agents are very responsive to the minimum pension benefit guarantee – even in the expectation of higher taxes, agents save substantially less if the minimum pension benefit is guaranteed. This is more true in the DC system than in the DB system, which implies that the effects are stronger for the intertemporal than for the intratemporal choice. Once we decompose for the heterogeneity due to endowments and heterogeneity due to preferences, we find that this instrument has much bigger effects.

## 6 Conclusions

Inequalities in income and wealth during the working period translate to inequalities in consumption, especially during the retirement. Many countries employ policy instruments with the aim to provide safety nets during the retirement within the pension system. The rationale behind such solutions are both pragmatic

Figure 9: Decomposition: the effects of instruments - wealth Gini



(if redistribution was not rooted in the pension system, it would have to be accounted for in general social benefits) and value based (misfortune during the working period – e.g. prolonged spells of non-employment – *should not* necessitate poverty when one is no longer able to work). In theoretical models and simulations the pragmatic reasoning is equivalent to a minimum pension benefit nested in the system. Indeed, such instruments have been evaluated extensively in previous studies, with an efficiency and welfare analysis. The main contribution of this paper consists of decomposing the inequalities during the retirement period into those that come from misfortune (which we operationalize as endowments) and those that stem from preferences.

Our overlapping generations framework allows to compare two types of pension systems: defined contribution and defined benefit. We account for changing demographics as well as gradually decreasing rate of the technological progress. We nest in the model two instruments which are popular policy solutions: the minimum pension guarantee and the contribution cap. We supplement the standard overlapping generations model with a within-cohort heterogeneity. While some of the previous studies allowed for differences in endowments, our setting allows agents to differ in terms of time preference as well as leisure preference.

In general, redistribution within the pension system is supposed to address not only low earners, but also high earners. For example, the rationale behind the contribution cap is typically that paying out (extremely) high pension benefits to (extremely) highly productive individuals in a general, obligatory pension system is

politically non-viable and economically redundant, because these individuals will be able to smoothen life-time consumption with private savings. This contention depends on a presumption that high productivity goes together with preference for future consumption as well as preference for work. In our framework the distribution of endowments is independent from a distribution of preferences. Likewise, in our setting some of the low productivity individuals have high preference for work and high propensity to save for future consumption.

The consequences of the contribution cap are negligible – any savings prevented in the obligatory public pension system are recorded as private, voluntary savings. Thus, we are unable to find any noticeable benefits or costs of such instrument. This general property holds even for the impatient agents.<sup>17</sup> By contrast, we find that minimum pension guarantee has a large effect on the decision to to save and relatively smaller incentives on labor supply decision. The discouraging effect on savings implies larger wealth inequality with minimum pension but effectively smaller consumption inequality. The reduction in Gini coefficient for consumption is not substantial, however the coverage by this instrument is. The comparison of macroeconomic effects suggest that the discouragement of savings does not translate to lower aggregate savings. Thus, the potential room for inefficiency is small. Clearly, such an instrument can be fiscally costly, up to 1% of GDP.

Decomposing the inequalities to endowments and preferences reveals that the effects of minimum pension benefits are larger in the world with no endowments heterogeneity than in the world with no preferences heterogeneity. This suggest that this instrument could be particularly effective towards overcoming the inequality of opportunities. Furthermore, for a policymaker concerned with consumption equality rather than wealth equality these effects are noticeable.

The literature in the field typically falls into one of the two categories: a theoretical, stylized framework which answers a purely conceptual problem, or a policy evaluation exercise which attempts to provide *ex ante* evaluation of pension or tax policies. Our paper is closer to the latter strand, but it is distinguished by two contributions. First, we attempted to provide an intuition on how big are potential effects of minimum pension benefit and contribution cap. Second, we sought to understand if the potential effects come from overcoming the disparity of opportunities or the heterogeneity in preferences. We find that while differentiated preferences are responsible for majority of the consumption and wealth inequalities *per se*, the minimum pension benefit helps to ameliorate those inequalities which stem from heterogeneous endowments.

---

<sup>17</sup>In the specifications with time inconsistency (quasi-hyperbolic discounting) the conclusions are unaffected.

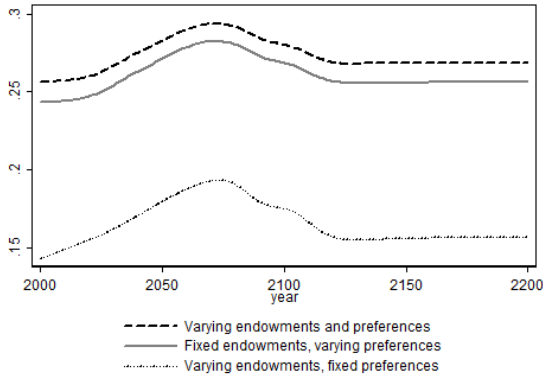
## References

- Axtell, R.: 2000, Why agents? on the varied motivation for agent computing in the social sciences, *Center on Social and Economic Dynamics Report*, Brookings Institution .
- Bassi, M.: 2008, An Egg Today and a Chicken Tomorrow: A Model of Social Security with Quasi-Hyperbolic Discounting, *CSEF Working Paper 205*, Centre for Studies in Economics and Finance (CSEF), University of Naples, Italy.
- Boersch-Supan, A. and Weiss, M.: 2011, Productivity and age: Evidence from work teams at the assembly line, *MEA discussion paper series 148-2007*, Mannheim Research Institute for the Economics of Aging (MEA).
- Bourguignon, F. and Spadaro, A.: 2006, Microsimulation as a tool for evaluating redistribution policies, *The Journal of Economic Inequality* **4**(1), 77–106.
- Brzezinski, M.: 2011, Accounting for recent trends in absolute poverty in Poland: a decomposition analysis, *Post-Communist Economies* **24**(4), 465–475.
- Buccioli, A.: 2011, A note on social security welfare with self-control problems, *Macroeconomic Dynamics* **15**(04), 579–594.
- Castaneda, A., Díaz-Giménez, J. and Ríos-Rull, J.-V.: 2003, Accounting for the US earnings and wealth inequality, *Journal of Political Economy* **111**(4), 818–857.
- Cremer, H. and Pestieau, P.: 2011, Myopia, redistribution and pensions, *European Economic Review* **55**(2), 165–175.
- Deaton, A.: 1997, *The analysis of household surveys: a microeconomic approach to development policy*, Johns Hopkins University Press.
- Domeij, D. and Klein, P.: 2002, Public pensions: To what extent do they account for swedish wealth inequality?, *Review of Economic Dynamics* **5**(3), 503–534.
- Fehr, H.: 2009, Computable stochastic equilibrium models and their use in pension-and ageing research, *The Economist* **157**(4), 359–416.
- Fehr, H., Habermann, C. and Kindermann, F.: 2008, Social security with rational and hyperbolic consumers, *Review of Economic Dynamics* **11**(4), 884–903.
- Fehr, H. and Kindermann, F.: 2010, Pension funding and individual accounts in economies with life-cyclers and myopes, *CESifo Economic Studies* **56**(3), 404–443.
- Fehr, H. and Uhde, J.: 2014, Means-testing and economic efficiency in pension design, *Economic Modelling* **44**, S57–S67.
- Ferber, J.: 1999, *Multi-agent systems: an introduction to distributed artificial intelligence*, Vol. 1, Addison-Wesley Reading.
- Gervais, M.: 2012, On the optimality of age-dependent taxes and the progressive us tax system, *Journal of Economic Dynamics and Control* **36**(4), 682–691.
- Gruber, J. and Wise, D. A.: 2004, Social security programs and retirement around the world: Micro-estimation, *NBER Books* .

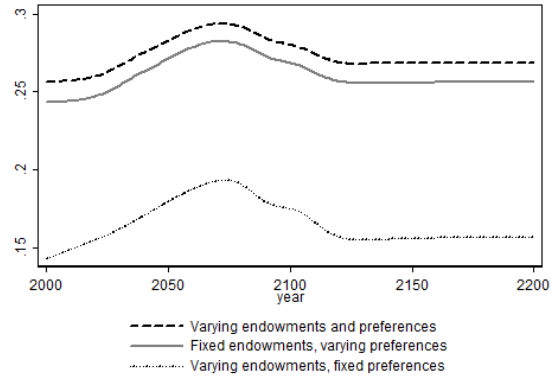
- Gul, F. and Pesendorfer, W.: 2004, Self-Control and the Theory of Consumption, *Econometrica* **72**(1), 119–158.
- Hairault, J.-O. and Langot, F.: 2008, Inequality and social security reforms, *Journal of Economic Dynamics and Control* **32**(2), 386–410.
- Hénin, P.-Y. and Weitzenblum, T.: 2005, Welfare effects of alternative pension reforms: Assessing the transition costs for french socio-occupational groups, *Journal of Pension Economics and Finance* **4**(03), 249–271.
- Imrohoroglu, A., Imrohoroglu, S. and Joines, D. H.: 2003, Time-Inconsistent Preferences and Social Security, *The Quarterly Journal of Economics* **118**(2), 745–784.
- Kindermann, F. and Krueger, D.: 2014, High marginal tax rates on the top 1%? lessons from a life cycle model with idiosyncratic income risk, *NBER Working Paper 20601*, National Bureau of Economic Research, Inc.
- Kirman, A.: 1997, The economy as an evolving network, *Journal of evolutionary economics* **7**(4), 339–353.
- Kumru, C. S. and Thanopoulos, A. C.: 2011, Social security reform with self-control preferences, *Journal of Public Economics* **95**(7-8), 886–899.
- McGrattan, E. R. and Prescott, E. C.: 2013, On financing retirement with an aging population, *NBER Working Paper 18760*, National Bureau of Economic Research.
- Nishiyama, S. and Smetters, K.: 2007, Does Social Security Privatization Produce Efficiency Gains?, *The Quarterly Journal of Economics* **122**, 1677–1719.
- Russell, S., Norvig, P. and Intelligence, A.: 1995, A modern approach, *Artificial Intelligence. Prentice-Hall, Egnlewood Cliffs* **25**.
- Song, Z.: 2011, The dynamics of inequality and social security in general equilibrium, *Review of Economic Dynamics* **14**(4), 613–635.
- St-Amant, P.-A. B. and Garon, J.-D.: 2014, Optimal redistributive pensions and the cost of self-control, *International Tax and Public Finance* pp. 1–18.
- Storesletten, K., Telmer, C. I. and Yaron, A.: 2004, Consumption and risk sharing over the life cycle, *Journal of Monetary Economics* **51**(3), 609–633.
- Tesfatsion, L.: 2002, Agent-based computational economics: Growing economies from the bottom up, *Artificial life* **8**(1), 55–82.
- van de Ven, J. and Weale, M.: 2010, Modelling myopic responses to policy: an enhancement to the NIBAX model, *DWP working papers 88*, Department for Work and Pensions at National Institute of Economic and Social Research.
- Windrum, P., Fagiolo, G. and Moneta, A.: 2007, Empirical validation of agent-based models: Alternatives and prospects, *Journal of Artificial Societies and Social Simulation* **10**(2), 8.
- Wooldridge, M.: 2009, *An introduction to multiagent systems*, John Wiley & Sons.
- Wooldridge, M. and Jennings, N. R.: 1995, Intelligent agents: Theory and practice, *The knowledge engineering review* **10**(02), 115–152.

# A Appendix

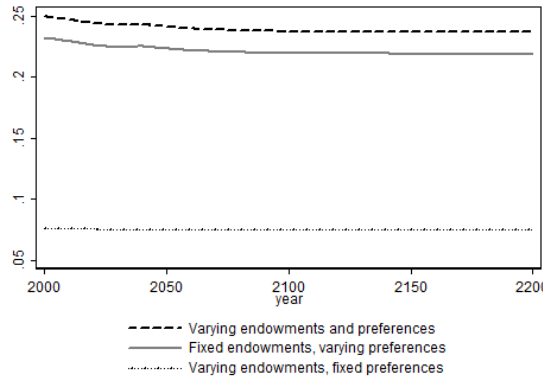
Figure A1: Consumption Gini depending on parametrization



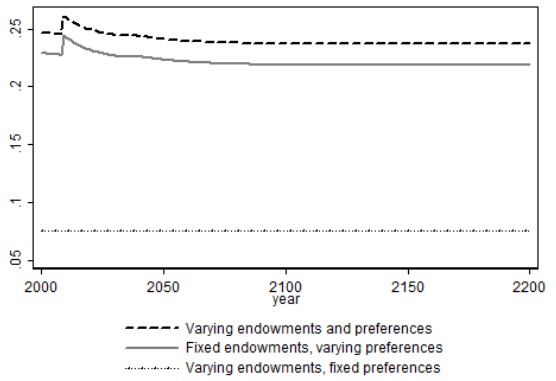
(a) all cohorts, DB



(b) all cohorts, DC

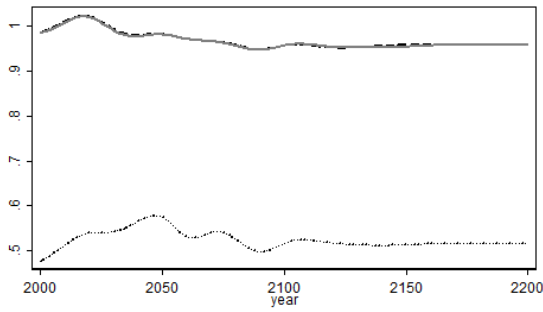


(c) at retirement, DB

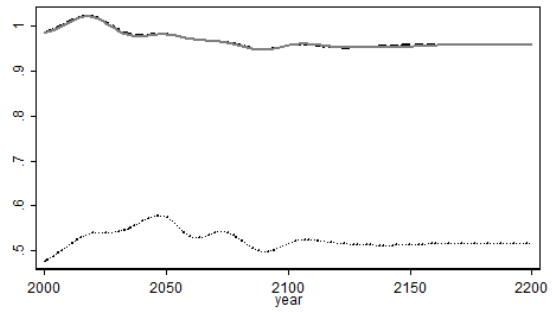


(d) at retirement, DC

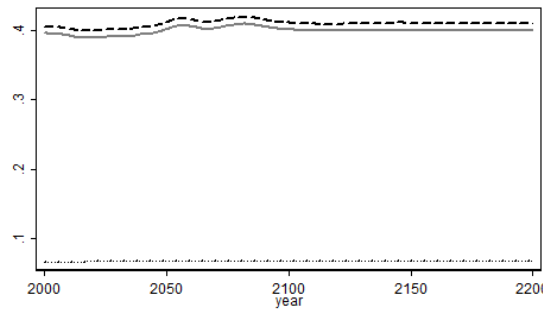
Figure A2: Wealth Gini depending on parametrization



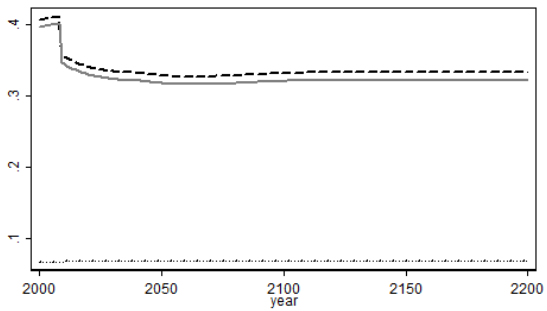
(a) all cohorts, DB



(b) all cohorts, DC

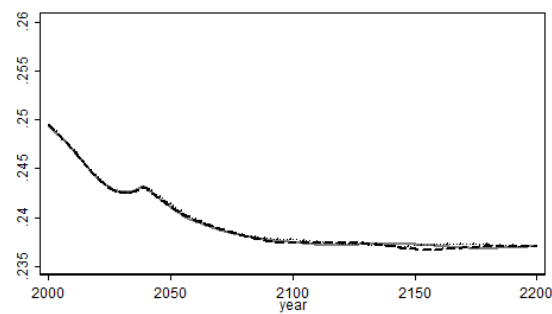


(c) at retirement, DB

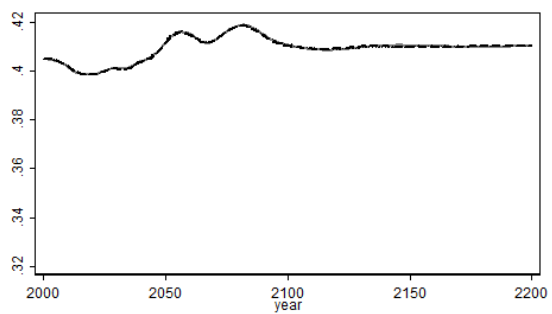


(d) at retirement, DC

Figure A3: Consumption and wealth Gini at retirement, DB system

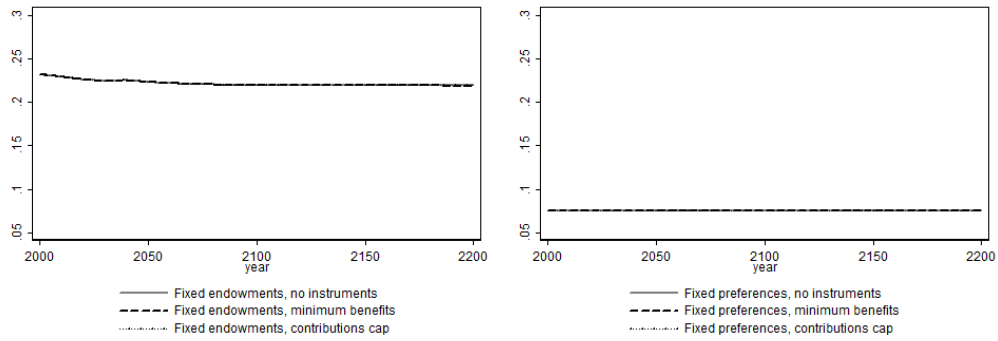


(a) consumption, DB

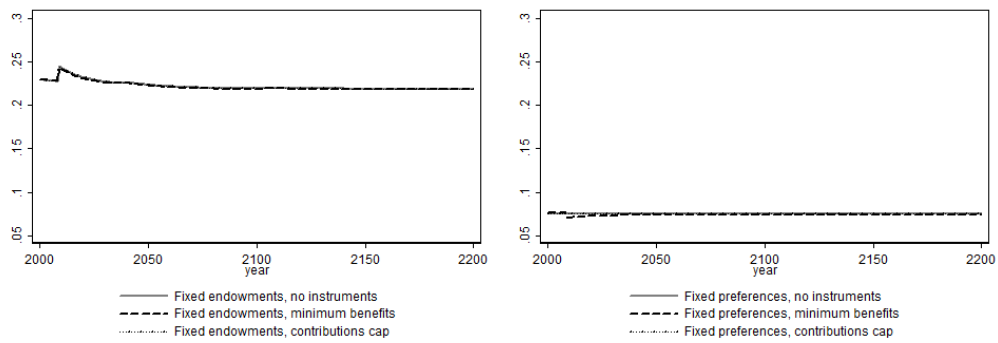


(b) wealth, DB

Figure A4: Decomposition: the effects of policy instruments - at retirement

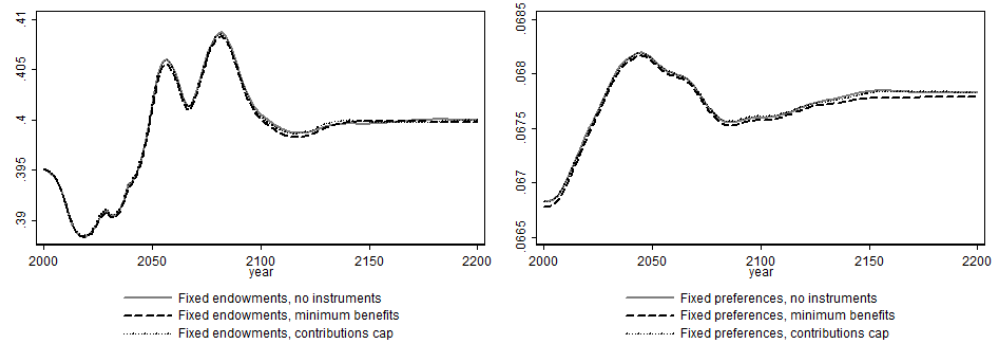


(a) Consumption Gini, defined benefit system

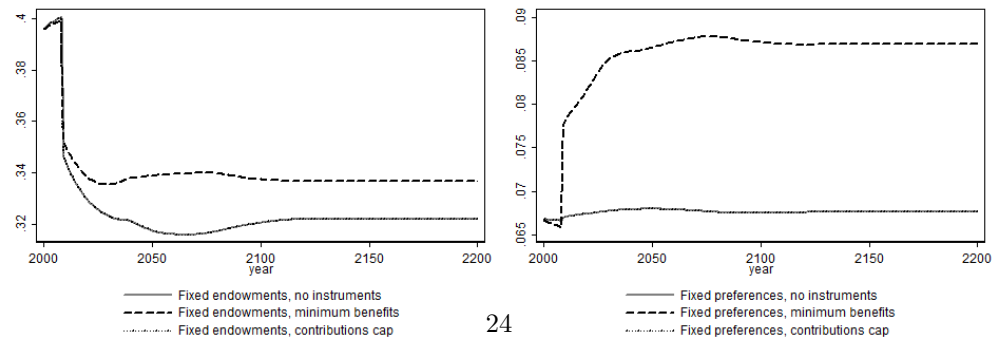


(b) Consumption Gini, defined contribution system

Figure A5: Decomposition: the effects of policy instruments - at retirement



(a) Wealth Gini, defined benefit system



(b) Wealth Gini, defined contribution system