The impact of demographic shocks on the political arrangement of pay-as-you-go pension systems

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Abstract

We analyze the political economy of pay-as-you-go (PAYG) pension systems in the presence of both financial and demographic shocks. A politician responds to shocks in a way that replicates major developments of pension systems around the world. A decrease in the return on capital increases contributions and benefits, while a decrease in the population growth rate increases contributions, but it lowers benefits. A lower mean or a higher variance of population growth causes the pension system to shrink. Contrary to the political arrangement, the Ramsey planner sets a higher average contribution when the demographic shock has a lower mean or a higher variance.

1 Introduction

Developments of pay-as-you-go (PAYG) pension systems are tightly connected with both shocks to the return on capital[1] and demographic shocks. Narrative evidence summarized in Section 2 points to the fact that the introduction of PAYG pension systems in countries such as Germany, Italy, Japan, US or UK could have been triggered by shocks that reduced the savings of old agents or agents close to retirement. In many countries, PAYG pension systems were expanded following other major economic downturns through early retirement schemes and decreases in retirement age.

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[1]We will henceforth refer to shocks to the return on capital by the term "financial shocks", but we bear in mind that these may also represent inflation or capital depreciation shocks.

Beginning with the 1980s, demographic projections worsened the outlook for the financial sustainability of PAYG pension systems, leading many governments to enact substantial reforms. In some countries, these consisted of increases in contributions and cuts in benefits or a tightening of the eligibility conditions for a pension. However, in many Central and Eastern European and Latin American countries, the reforms entailed a partial or complete reduction of the public PAYG pension system in favor of private fully funded pension funds.

The present paper sheds light on what reforms may result from shocks if the consensus between young and old agents is achieved through voting. We focus on the role of PAYG pension systems in sharing financial shocks among different cohorts in an economy with incomplete asset markets. Young agents are prepared to contribute to the pension system because this offers them partial protection against shocks to the real return on capital.

The contributions of this paper are the following. First, we study the interaction between demographic shocks and the role of PAYG pension systems in sharing financial shocks among cohorts in a setting with voting. Second, through the combination of financial and demographic shocks we can account for the most important developments of the PAYG pension systems around the world. The first and second moments of the shocks become explanatory variables for the significant cross-country heterogeneity in the size of the PAYG system and in the types of reforms implemented. While internationally integrated financial markets help to align the moments of the financial shocks, the characteristics of demographic shocks vary considerably. Hence, allowing for demographic shocks is important to account for the aforementioned heterogeneity. Finally, we compare our results to the normative benchmark of the Ramsey planner.

We consider a small open economy in which the return on capital and the population growth rate are stochastic. Young and old agents vote each period over the size of the PAYG contributions. The electoral competition is modeled as a probabilistic voting game as in Persson and Tabellini (2000). We consider differentiable Markov policies, i.e. differentiable policies that depend only on current state variables. Since the contribution to the pension system depends negatively on the amount of savings, young agents incorporate in their voting decision the fact that larger pension contributions imply lower savings, thereby causing a larger PAYG scheme in the next period as well. Hence, this "strategic effect" in the political process weakens the opposition of the young to a social security scheme.

We show that the policy function resulting from the voting process helps overlapping cohorts share both financial and demographic risks by appropriately adjusting contributions each period. More specifically, following a decrease in the return on capital, the politician will increase both contributions and benefits. After a decrease in the population growth rate however, as long as we restrict ourselves to positive contributions, the politician increases contributions but decreases benefits. These results are in line with the major observed developments of the PAYG pension systems.

For the special case when the financial and the demographic shock are independent and the demographic shock has a uniform distribution, we find an analytic solution and we can further analyze the interaction between financial and demographic shocks. We obtain that, in the case of a lower mean or a higher variance of the demographic shock, the politician implements a policy function that is less sensitive to the wealth of the old. Intuitively, when the population growth rate is lower on average or more uncertain, the politician has less room to compensate old agents for their losses from financial shocks.

This result has two opposing implications. On the one hand, the capacity of the PAYG pension system to diversify the financial risks decreases. Consequently, young agents must contribute more to the pension system to maintain the same level of protection against financial shocks. On the other hand, the return of the pension system also decreases due to a weaker strategic effect. This makes young agents less willing to invest in the pension system.

In a calibrated version of the model we show that, for low values of the political weight of the young relative to the old, the impact through the strategic effect dominates. Hence, the average contribution to the PAYG pension system is smaller if the mean population growth rate is lower or its variance is higher.

The results above can explain two stylized facts. First, during the Great Recession, governments did not expand PAYG pension systems to compensate old agents or agents close to retirement for financial losses as they did in previous major recessions[2]. This decrease in the generosity of governments can be explained by the perception of a lower or more uncertain development of demographic indicators. Second, the decision taken by many countries from Latin America and Central and Eastern Europe to (partially) transform the PAYG pension system into a fully funded one can be explained by a lower average or a higher standard deviation of population growth rates in these countries.

Analyzing the Ramsey planner's problem, we find that, following financial and demographic shocks, contributions and benefits change in the same direction as in the politician's case, but the adjustments made are of a smaller magnitude. In the case that the mean of the population growth rate is smaller or its variance is higher, the Ramsey planner implements a larger average contribution to the pension system. This result is contrary to the one

[2] However, many governments took measures to alleviate the conditions of the poorest pensioners, for example, by expanding means tested benefits. See OECD (2012).

obtained in the politician's case. Because in the Ramsey planner's problem the strategic effect plays no role, a lower mean or a higher variance of the demographic shock weakens the pension system's ability of sharing the financial risk without making the investment in capital more attractive. Young agents will agree to expand the size of the pension system in order to better insure against the financial shock.

Previous literature has focused on how either financial or demographic shocks affect PAYG pension systems. In an economy featuring only financial shocks, D'Amato and Galasso (2010) show that both a politician and a Ramsey planner would organize transfers that feature a negative relationship with the wealth of the old. However, the policy rule established by the politician exhibits more persistence and entails higher benefits for the old than the Ramsey planner's. We extend their model by incorporating demographic shocks since these are empirically relevant for the development of PAYG pension systems. In a model featuring only demographic shocks, Gonzalez-Eiras and Niepelt (2008) find that a decrease in the population growth rate leads to an increase in the contributions and a decrease in the pension benefits established by a politician. In their model however, the policy function of the politician does not depend on the moments of the demographic shock. Hence, it cannot explain the considerable cross-country heterogeneity in PAYG pension provision. Moreover, we show that the interaction between financial and demographic shocks plays an important role in determining the equilibrium size of the PAYG pension system. Bohn (2001) considers both financial and fertility shocks but does not analyze the role of the financial shock in determining the contributions to and benefits from the PAYG pension system. Also, he only addresses the policy implemented by a Ramsey planner, while we focus mainly on the politician's policy function.

The remainder of the paper is organized as follows. In Section 2 we present a number of stylized facts motivating the analysis in this paper. Section 3 lays out the model and derives the voting outcome for contributions to and benefits from a PAYG pension system in an economy affected by both financial and demographic shocks. Section 4 presents, for comparison, the policy implemented by a Ramsey planner. Section 5 concludes the main body of this paper. The proofs of the Propositions are included in Appendix 1.

2 Motivating facts

The narrative evidence on the beginnings of PAYG pension systems shows that, in many countries, these were introduced after events that had large negative effects on existing retirement savings (Flora (1987), Gordon and Varian (1988), Perotti and Schwienbacher

(2009)). Table 1 summarizes the key moments in the development of PAYG pension systems in a number of selected countries. The emergence of PAYG pension systems is related to shocks to the real rate of return (UK, US) or the depletion of the capital accumulated in fully funded pension funds during WWII (Japan, Germany, Italy). The reforms summarized in Table 1 also indicate that PAYG pension systems have been expanded following other economic downturns. Coverage has been extended, early retirement schemes were introduced and the retirement age was reduced during recessions in France, Netherlands, Spain, US and Poland.

Beginning with the 1980s, when all the countries considered here started to experience a decrease in their population growth rate (see Figure 1), substantial reforms aimed at restoring the viability of the PAYG pension systems were enacted. These pension reforms involved increases in contributions and cuts in benefits. The reductions in pension benefits were achieved through a variety of means: decreases in accrual rates, indexation or valorization, increases in the number of reference years or the retirement age or strengthening of work incentives, increases in the number of years of contribution. A few countries (Germany, Spain, all countries that shifted to a Notional Defined Contribution system) have established a direct link between benefits paid and demographic indicators.

The reforms aimed at restoring the viability of pension systems have persisted even during the Great Recession. Unlike in previous major recessions, most of the adjustments to PAYG pension systems were made in the view of the ongoing adverse demographic developments and entailed increases in contributions or in the retirement age.

While many countries retained their public PAYG pension systems but tried to restore their financial sustainability, a number of countries decided to partially or completely transform them in private fully funded ones in the face of demographic shocks. Many Central and Eastern European and Latin American countries downsized their PAYG pension systems in this way.

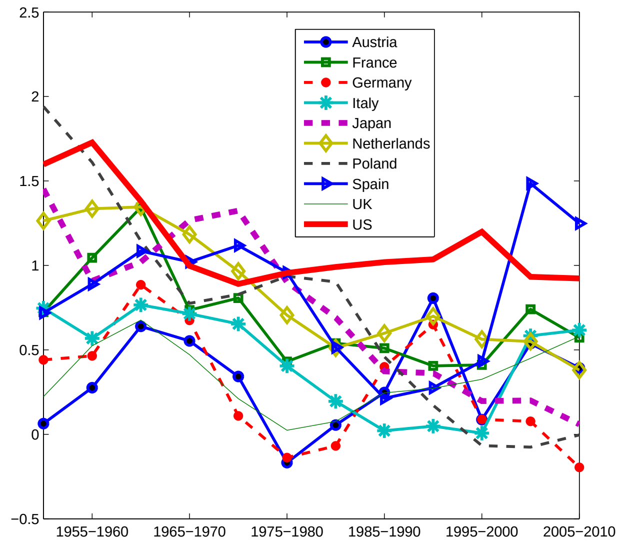
Table 1: PAYG pension system reforms in selected countries

Country	Year of introduction and context	1960-1985	1985-2008	Great Recession
Austria	1906 - Pension system for white collar workers 1926;1935;1939 - Pension system for blue collar workers	1958;1961-1966 - Introduction of early retirement schemes 1984 – Number of reference years extended from 5 to 10	1993 - Increase in contributions of civil servants; net wage instead of gross wage indexation 1997;2000;2003 - Strengthening work incentives; reforms aimed at cutting benefits (including a reduction in accrual rate) 2004 - Switch to individual accounts	
France	1945 - Aftermath of WWII	1982 - Retirement age decreased to 60 years	1993 - Pension indexation with inflation; reference years extended from 10 to 25 years; increase in required contribution period 2003 - Increase in contribution years to 42; incentives to work after retirement age; indexation according to cost of living in the public sector.	Increase in contributions
Germany	1957-1969 - Funded system transformed into PAYG after massive destruction of capital stock in WWII	1972 - Expansion of the statutory pension scheme	1989 - Increase in retirement age; indexation linked to net wage growth (instead of gross) 2004 - Sustainability factor links pension benefits to demographic factors; taxation of pension benefits 2007 - Increase in retirement age	
Italy	1952 - Funded system transformed in PAYG - the previously accumulated capital was used for war spending or eroded by inflation	1956;1965 - Introduction of ”seniority pensions”	1992 - Increase in retirement age and the number of reference years, change of indexation from wages to prices, increase in years of contribution 1995 - Gradual switch to NDC system; sustainability factor that links pension benefits to the age of retirement, economic trends and demographic dynamics; increase in qualifying period for seniority pensions, increase in contribution rates 1997 - Tightening conditions for seniority pensions, partial compatibility between pensions and income from work 2004 - Fixed higher retirement age, bonus for deferred retirement, extra tax on very high pensions	Increase in retirement age
Japan	1954 - Rebuilding of the Kosei Nenkin Hoken (KNH) pension system whose assets were eroded by post war inflation	1961- Kokumin Nenkin (KN) pension system introduced - covers all categories not included in KNH 1965-1973 - Increase in contributions and benefits of KNH 1973 - Indexation of pension entitlements and benefits	1985 - Decrease in the accrual rate of pension benefit (KNH); Increase in the number of contribution years (KN) 1994 - Gradual phasing out of basic pensions; indexation with net wages 1999 - Decrease in accrual rate, shift to CPI indexation, new earnings test introduced, pensionable age increased	Eligibility period for KN shortened Insurance extended to more part time workers

Country	Year of introduction and context	1960-1985	1985-2008	Great Recession
Netherlands	1947-1957 - Gradual introduction of basic pension following WWII	1976-1985 - Introduction of early retirement schemes on a PAYG basis	1997;2006 - Early retirement schemes integrated in the fully funded occupational schemes. Tax advantages for early retirement abolished	Increase in retirement age
Poland	1950 - Funded pension system becomes PAYG during communist regime	1980 - Coverage extended to rural population	1990-1996 - Early retirement facilitated to fight unemployment during the restructuring of the economy 1999 - Three pillar pension system with a mandatory private defined contribution (DC) pillar	2009 - Early retirement eliminated for certain categories of agents 2011 - Reduction in contribution diverted to the DC pillar 2012 - Retirement age raised
Spain	1963;1972 - Introduction of PAYG pension	1978-1982 - Increase in coverage of PAYG system 1985 - Increase in contribution years and reference period, indexation with estimated CPI increase	1990 - Introduction of mean-tested non-contributory pensions 1997 - Switch back to past CPI indexation; reduction in accrual rates, increase in reference period for benefit calculation 2002 - Incentives to work longer	Increase in normal pension age, incentives to work after retirement Sustainability factor linked to life expectancy
UK	1908 - Means tested non-contributory benefits 1948 - Basic State Pension - flat pension paid on a PAYG basis	1959 - Graduated retirement benefit (GRB) pensions that depend on earnings introduced 1975 - State Earnings Related Pension Scheme (SERPS) replaces and expands GRB; Basic pension indexed with inflation or wages whichever is higher	1986 - Earnings-related benefit calculation includes lifelong earnings; Reduction in SERPS replacement rate and survivors' pensions Extension of means tested supplements 1996 - Second State Pension replaces SERPS 2004 - Rewards for late retirement	Increase in retirement age Increase in contributions
US	1935 - During the Great Depression, due to the increase in the poverty of old agents 1939 - Social security extended to the survivors of the contributors	1950 - Benefits increased with cost of living at certain intervals 1961- Eligibility for early retirement at 62 years 1972 - Automatic Cost of Living Adjustment of benefits 1977- Increase in payroll tax, small decrease in benefits 1983 - Taxation of benefits, increase in retirement age	Changes in early and late retirement	

Sources: Immergut et al. (2006), Talos (2006), Rurup (2002), Bonin (2009), Franco (2002), Conrad (2001), Takayama (2003), Euwals et al. (2011), Bozio et al. (2010), the site of the Social Security Administration, OECD (2007), OECD (2009), OECD (2011), OECD (2012), OECD (2013).

Figure 1: Annual average population growth rate in selected countries



In conclusion, narrative evidence supports the view that both financial and demographic shocks were important in the development of PAYG pension systems. The present paper develops a model that can rationalize the reforms enacted following financial and demographic shocks. In the model, the arrangement of the PAYG pension system is established through voting after the shocks have materialized. The politicians that run in the election determine the size of the PAYG pension system by trading off the welfare of young and old agents. Keeping everything else constant, a decrease of the return on capital causes the equilibrium size of the pension system to increase. This is consistent with the stylized facts regarding changes in PAYG pension systems after major economic downturns or large shocks to the gross return on capital. A decrease in the population growth rate on the other hand implies higher contributions but lower benefits. This result is also consistent with the reforms enacted in all countries after the 1980s. The model can also explain the downsizing of PAYG pension systems in Central and Eastern European and Latin American countries through a decrease in the mean or an increase in the variability of the process driving the population growth rate.

Another puzzling fact about pension provision is the considerable cross-country heterogeneity in the size of contributions to and benefits from PAYG pension system (see table 2). Tabellini (2000) explains this by the proportion of the elderly in the economy and the pre-tax income inequality. In his model, higher income inequality of young people leads to larger social security. Chen and Song (2014) also single out income inequality but they argue in favor of a negative relation with the size of the PAYG pension system. Song (2011) explains the development of social security in the US through the wealth inequality among old agents. The narrative evidence presented above shows little support for these determining factors.

Perotti and Schwienbacher (2009) explain this heterogeneity by the fact that different countries experienced inflation shocks of different magnitudes prior to the introduction of the PAYG pension systems. While this may be a good explanation for the introduction of PAYG pension systems, it cannot explain the subsequent expansion of the programs or the reforms enacted after the 1980s. Our model brings an important addition to this strand of literature by exploring other factors that can explain this cross country heterogeneity: the size of the financial and demographic shocks hitting the economy, the mean and variance of these shocks and the political power of the young agents relative to the old.

Table 2: Size of the PAYG pension system in selected countries

Country	Austria	France	Germany	Italy	Japan	Netherlands	Poland	Spain	UK	US
Contribution rate (%)	22.8	16.7	19.6	33	16.8	17.9	19.5	28.3	-	10.4
Gross replacement rate (%)	77	59	42	71	36	30	25	74	33	38

Source: OECD (2013). The contribution rates reported are for the year 2012. For Poland the figure includes the contribution diverted to the DC pillar. The replacement rate corresponds to an agent which earns an average wage.

3 The model

The set up of the model is closely related to D'Amato and Galasso (2010). We consider an infinite horizon economy inhabited by cohorts of overlapping generations, each living for two periods. The size of the cohort born at time t is N_t. Agents work when young, consume when old and derive utility only from the consumption of goods. Hence, when young, they will work the entire time that they are endowed with, which is equal to one unit. On aggregate, labour supply L_t is equal to the size of the young population N_t.

We assume a quadratic utility function:

$$u(c_{t+1}) = -\frac{(c_{t+1} - \gamma)^2}{2}$$

where γ is the "bliss" level of consumption[3] and c_{t+1} is the per capita consumption of agents born at time t. We choose this type of utility function for two reasons: i) it allows us to interpret the results in a mean-variance framework and ii) it allows us to obtain an analytic solution for particular assumptions on the financial and demographic shocks.

The economy has a constant returns to scale technology which gives each period the output $Y_t = wL_t + R_t S_{t-1}$, where S_{t-1} is the stock of capital at time t given by the amount of savings made in the economy at time $t-1$. Capital depreciates fully every period. We consider the case of a small open economy in which the gross return on capital is exogenous but stochastic, independent in time and distributed according to a cumulative distribution function $F(R_t) \sim (\bar{R}, \sigma_R^2)$. The gross population growth rate $n_t = \frac{N_t}{N_{t-1}}$ is also stochastic, independent in time and distributed according to the cumulative distribution function $G(n_t) \sim (\bar{n}, \sigma_n^2)$. We assume that $\bar{R} > \bar{n}$. Wages w are exogenous, deterministic and normalized to 1.

We consider a PAYG pension system that runs each period a balanced budget. Hence, everything that is collected from the members of the young generation is distributed to the members of the old generation. Denoting by τ_t the per capita contribution to the PAYG pension system paid by the young and by b_t the per capita social security benefit distributed at time t, the balanced budget condition becomes:

$$b_t N_{t-1} = \tau_t N_t \Rightarrow b_t = \tau_t n_t$$

In the economy described above, the budget constraints of the agents are:

$$s_t + \tau_t = 1 \tag{1}$$

$$c_{t+1} = R_{t+1} s_t + \tau_{t+1} n_{t+1} \tag{2}$$

where $s_t = \frac{S_t}{N_t}$ is the per capita savings of the young.

3.1 The politician's problem

The contribution to the pension system is determined every period by a voting process that takes place after the shocks R_t and n_t have materialized. Since agents consume only when they are old, when young they will pay the contribution to the pension system and save the rest of the endowment. Because young agents make no economic choices, we can directly define the political equilibrium of the model.

[3]Expected utility can be equivalently written: $E_t c_{t+1} - \frac{1}{\gamma} E_t c_{t+1}^2$. In this form it becomes apparent that the parameter γ is inversely related to the degree of risk aversion.

We consider the setting of a probabilistic voting game[4] in which two candidates compete each period in an election. The candidates are office seeking so they do not care about the welfare of current and future generations but only about maximizing their probability of being elected. They are also non-partisan. Since there is a voting process organized in the beginning of each period, the elected politician cannot commit to implement the policy chosen in his program in the next period as well. We assume however that he can commit to implement the policy outlined in his electoral platform after he is elected. In the voting game, the two candidates take into account the preferences of agents with respect to the policies but also the ideological views of the agents which can make them more inclined to vote for one candidate or another.

We assume that ideology is uniformly distributed in the group of young and old agents with density ϕ_1 and ϕ_0, respectively. In the equilibrium of this probabilistic voting game, both candidates choose the same policy for their electoral program. They do so by maximizing the joint welfare of young and old individuals weighed with the density distribution of their ideologies and their relative size in the population:

$$W^P = \frac{1}{1+n_t}(\phi_0 u(c_t) + \phi_1 n_t E_t u(c_{t+1})) \tag{3}$$

Intuitively, in order to maximize the probability of winning the election, the candidates must design their policies according to the preferences of agents that are more likely to vote according to policies announced than to ideology. Hence, a group which is politically more homogeneous (higher ϕ_i, so lower dispersion of ideology in the group i) will receive a higher weight in the objective function of the candidates and their preferences will matter more for the policy outcome. This is because there is a higher probability that a member of this group is a swing voter. Of course, the outcome will also depend on the relative size of the two groups. The ideology parameters ϕ_i can also be tailored to reflect the historical vote turnout of group i.

Note that, since τ_{t+1} will result from the electoral competition taking place in the beginning of the next period, young agents and, consequently, current politicians must have at time t a perception regarding the policy's law of motion. We will restrict our attention in the following to differentiable Markov policy functions, i.e. to policies that depend on the current state of the economy defined in our model by the savings of the old, the financial shock and the demographic shock. Hence, we assume the following form for the policy

[4] The set up of the probabilistic voting game used in this paper is the one presented in Persson and Tabellini (2000).

function:

$$\tau_t = f(s_{t-1}, R_t, n_t)$$

3.1.1 The intergenerational conflict settled by the voting process

It is worth discussing at this point the intergenerational conflict that may arise in an economy where a policy τ_t must be implemented at time t. The marginal utility with respect to the policy function in the case of an old agent is:

$$\frac{\partial U^o}{\partial \tau_t} = n_t u'(c_t) > 0 \tag{4}$$

where the relation comes from the fact that the utility function is strictly increasing[5]. The old strictly prefer higher contributions irrespective of the state of the economy. Consequently, the higher the weight of the old relative to the young in the political process, the higher will be the contribution implemented.

In the case of a young agent, his preferences over the policy that must be implemented at time t are less clear cut. The change in utility determined by an increase in the contribution at time t is:

$$\frac{\partial U^y}{\partial \tau_t} = E_t\left[u'(c_{t+1})\left(n_{t+1}\frac{\partial \tau_{t+1}}{\partial \tau_t} - R_{t+1}\right)\right] \tag{5}$$

The term $n_{t+1}\frac{\partial \tau_{t+1}}{\partial \tau_t}$ captures the "strategic effect" which works through the savings of the young generation. An increase in τ_t will leave the current young generation with a smaller amount of savings. But the contribution set at period $t+1$ depends on the amount of savings prevailing at the beginning of that period (s_t) and this will lead the next period politician to set a higher contribution τ_{t+1}[6].

The support of the young agents towards a higher contribution to the PAYG pension system depends on the strength of the strategic effect relative to the return on capital. The stronger is the strategic effect, the higher is the return of the PAYG pension system compared to the return on capital. This makes young agents willing to expand the pension system.

[5]We assume that the condition $c_{t+1} < \gamma$ holds in every state. There is no formal restriction that we can impose on the parameters of the model to insure that this condition holds. However, using numerical simulations, we found that for low values of ϕ the condition is violated in less that 1% of cases.

[6]Forni (2005) proves that the policy function is a decreasing function of the savings, meaning that the strategic effect must be positive in the equilibrium of the game between subsequent political administrations. The argument is as follows: if the contribution is an increasing function of savings, then the young generation has an incentive to save more in order to get a higher benefit from the pension system when old. But this cannot be an equilibrium, because a higher benefit of the old reduces the savings of the young.

Using the assumption of quadratic utility, relation (5) becomes:

$$\frac{\partial U^y}{\partial \tau_t} = cov\left[c_{t+1}, R_{t+1}\right] - cov\left(c_{t+1}, n_{t+1}\frac{\partial \tau_{t+1}}{\partial \tau_t}\right) + E_t c_{t+1} E_t\left(R_{t+1} - n_{t+1}\frac{\partial \tau_{t+1}}{\partial \tau_t}\right) -$$
$$- \gamma E_t\left[R_{t+1} - n_{t+1}\frac{\partial \tau_{t+1}}{\partial \tau_t}\right] \tag{6}$$

The first and last term of relation (6) dominate the preferences of young agents towards higher or lower contributions to the pension system.

The first term is the covariance between consumption and the financial shock. We will denote this term by "the risk sharing component" because it captures the property of the PAYG pension system of sharing shocks between young and old agents. To see this, we substitute the budget constraint in the covariance between consumption and return on capital:

$$cov\left[c_{t+1}, R_{t+1}\right] = (\bar{R}^2 + \sigma_R^2)(1 - \tau_t) + cov(b_{t+1}, R_{t+1}) \tag{7}$$

A higher contribution to the pension system lowers the covariance between consumption and return on capital. The pension system has an additional risk sharing property if the covariance between benefits and the financial shock is negative. The more negative this covariance, the less young agents must invest in social security in order to achieve the same level of risk sharing.

The last term of relation (6) shows how the preferences of young agents for the PAYG pension system are determined by the return on capital (R_{t+1}) in excess of the return of the pension system measured by the strategic effect $\left(n_{t+1}\frac{\partial \tau_{t+1}}{\partial \tau_t}\right)$. We will henceforth denote this term "the risk aversion component". A weaker strategic effect or a higher mean return on capital makes the investment in capital more attractive and, hence, weakens the support of young agents for the PAYG pension system. The risk aversion component will dominate the incentives of young agents for high values of γ (low risk aversion).

3.1.2 The political equilibrium

The politicians running in the election maximize the objective function (3) with respect to the contribution to the pension system, subject to the budget constraints of the agents:

$$\max_{\tau_t} \ \phi_0 u(c_t) + \phi_1 n_t E_t u(c_{t+1}) \tag{8}$$

$$c_t = R_t s_{t-1} + \tau_t n_t \tag{9}$$

$$s_t + \tau_t = 1 \tag{10}$$

$$c_{t+1} = R_{t+1} s_t + \tau_{t+1} n_{t+1} \tag{11}$$

At the optimum, the politicians equate the welfare gain of a marginal transfer through the pension system coming from the increase in the utility of the old with the welfare loss coming from the decrease in the utility of the young:

$$u'(c_t) = \phi E_t \left[\left(R_{t+1} - n_{t+1} \frac{\partial \tau_{t+1}}{\partial \tau_t} \right) u'(c_{t+1}) \right] \tag{12}$$

where we denote with $\phi = \frac{\phi_1}{\phi_0}$ the political weight of the young generation relative to the old generation.

The resulting contribution to the pension system depends on the weight that the two groups of agents have in the voting process. A higher dispersion of ideology among the young (relative to the old) gives them a lower political weight. In this case, the politician implements a higher contribution that implies lower consumption for the young agents.

Given a perceived policy function $\tau_{t+1} = f(s_t, R_{t+1}, n_{t+1})$, the current contribution to the pension system will be set by solving the politician's problem formulated in relations (8)-(11). The solution will yield the actual policy function $\tau_t = \bar{f}(s_{t-1}, R_t, n_t)$.

Definition 1. *A Markov perfect equilibrium of the game between subsequent election candidates is obtained if the perceived and the actual policy function are equal, i.e.* $f = \bar{f}$.

In order to have a meaningful model for analysing PAYG pension systems we make two assumptions. The first one rules out cases in which young agents are so risk adverse that they are willing to participate in the PAYG pension system even if they expect no benefits from the pension system next period.

Assumption 1. *Young agents will not sustain positive contributions to the pension system if these do not bring them benefits when old. This is equivalent to*[7]*:* $\gamma > \gamma_{min} = {}^{\Gamma}/_{\bar{R}}$, *where* $\Gamma = \bar{R}^2 + \sigma_R^2$.

The second assumption is related to the fact that our main aim is to study pension systems. While transfers from old to young are also possible in reality - for example through taxes on wealth -, our aim is to analyze transfers from young to old. That is why we impose the restriction that, on average, the contribution to the PAYG pension system is positive. We also rule out the possibility that old agents expropriate the young through the pension system by imposing that the contribution to the PAYG pension system is, on average, less than 1.

Assumption 2. *The average contribution to the pension system is strictly positive and non-expropriating:* $E(\tau_t) \in (0, 1)$.

[7] We obtain this relation by imposing the condition $\frac{\partial U^y}{\partial \tau_t}|_{\tau_t = 0, b_{t+1} = 0} < 0$.

The following proposition lays down a closed form solution to the politician's problem.

Proposition 1. *The politician's optimization problem is solved by the following policy function:*

$$\tau_t^P = \frac{A^P - R_t s_{t-1}}{B^P + n_t} \tag{13}$$

The coefficients A^P and B^P are obtained as a solution to the following system of equations[8]*:*

$$1 = \phi B^P E_t \frac{R_{t+1}^2}{(B^P + n_{t+1})^2} \tag{14}$$

$$A^P = \frac{\gamma - \phi\gamma B^P E_t \frac{R_{t+1}}{B^P + n_{t+1}} + B^P}{1 - \phi B^P E_t \frac{R_{t+1} n_{t+1}}{(B^P + n_{t+1})^2}} \tag{15}$$

Equation (14) can have no solution, one or multiple solutions in the domain of real numbers. However, we can conclude that, if a solution exists, it must be strictly positive. With this result, we can already analyse the way that the policy function can help agents share both financial and demographic risks.

Since $B^P + n_t > 0$, the contribution is inversely related to the wealth of the old ($R_t s_{t-1}$). This means that, at the same level of savings, following a decrease in the return on capital, the politician establishes a higher contribution for the young and hence a higher level of benefits for the old. Consequently, the benefits of the PAYG pension system are negatively correlated with the return on capital. This helps agents to partially protect against financial shocks. Also, a smaller amount of savings of the old leads to a higher contribution required by the politician and, hence, to a higher benefit paid to the old.

The impact of the demographic shock on the contributions to and the benefits from the pension system is established in the following proposition.

Proposition 2. *An increase in the population growth rate lowers the contribution to the pension system and increases the benefits from the pension system if and only if the equilibrium contribution is positive.*

The increase in the population growth rate benefits both young and old agents: contributions are reduced and benefits are increased. This is the way that the PAYG pension system helps young and the old agents share the demographic risk.

[8] A trivial solution to the policy function is given by $A^P = \gamma, B^P = 0$. However, this solution is explosive, so we will not consider it further.

This impact of the demographic shock on the political arrangement of the PAYG pension system is the same as the one obtained by Gonzalez-Eiras and Niepelt (2008). In our paper, this result holds in an economy hit by both financial and demographic shocks. Hence, we establish the result that the PAYG pension system helps in sharing both types of shocks, if the contribution is adjusted correspondingly each period.

3.1.3 A special case

In this subsection we analyse a special case by imposing a number of assumptions that allow us to obtain an analytic solution to the system of equations (14)-(15).

Assumption 3. *The financial and demographic shock are independent and the demographic shock has a uniform distribution on the interval* $[n_{min}; n_{max}]$ *with* $n_{min} > 0$.

Note that we make no assumption on the distribution of the financial shock. In the setup we considered, of a small open economy in which the return on capital is determined at global level, the assumption that the financial and demographic shock are independent is not implausible. With these assumptions, the coefficients of the policy functions A^P and B^P can be easily determined.

As we mentioned above, the political equilibrium can have no solution or multiple solutions depending on the calibration of the parameters of the economy. We impose a restriction on the parameter defining the political weight of the young relative to the old agents in order to insure the existence of a unique equilibrium featuring a finite average contribution to the PAYG pension system.

Proposition 3. *Under Assumption 3, for* $\phi > \phi_{min} = \frac{(\Delta n)^2}{\Gamma\left(1-e^{-\Delta n/\bar{R}}\right)\left(n_{max}-n_{min}e^{\Delta n/\bar{R}}\right)}$, *there exists a unique political equilibrium that has a finite average contribution. The coefficients of the policy function are:*

$$B^P = \frac{-(2\bar{n} - \phi\Gamma) + \sqrt{(2\bar{n} - \phi\Gamma)^2 - 4n_{max}n_{min}}}{2} \tag{16}$$

$$A^P = \frac{\gamma - \phi\gamma\frac{B^P\bar{R}}{\Delta n}\ln\frac{B^P+n_{max}}{B^P+n_{min}} + B^P}{1 - \phi\frac{B^P\bar{R}}{\Delta n}\ln\frac{B^P+n_{max}}{B^P+n_{min}} + \frac{B^P\bar{R}}{\Gamma}} \tag{17}$$

where $\Delta n \equiv n_{max} - n_{min}$.

For this unique politico-economic equilibrium we analyse how the moments of the demographic shock influence the policy function. We first determine the impact of the mean and variance of the demographic shock on the sensitivity of the policy function with respect to the wealth of the old and on the average strategic effect.

Proposition 4. *Under the assumptions of Proposition 3, in an economy with a lower mean or a higher standard deviation of the demographic shock, the politician will implement a policy function that is less sensitive to the wealth of the old and for which the average strategic effect is lower, i.e.* $\frac{\partial B^P}{\partial \bar{n}} < 0, \frac{\partial B^P}{\partial \sigma_n} > 0, \frac{\partial E \frac{n_t}{B^P + n_t}}{\partial \bar{n}} > 0, \frac{\partial E \frac{n_t}{B^P + n_t}}{\partial \sigma_n} < 0$. *The impact on the average contribution to the pension system is ambiguous.*

A straightforward implication of the result stated in Proposition 4 is that in countries with a lower mean of the demographic shock or a higher variance, the PAYG pension system will be designed to be less generous to old agents in the case of wealth losses.

The fact that the policy function is less sensitive with respect to the wealth of the old has two implications. On the one hand, this means that the benefits from the pension system will have a weaker negative correlation with the financial shock. Hence young agents should invest more in the pension system in order to achieve the same level of risk sharing.

On the other hand, this lower sensitivity of the policy function with respect to the wealth of the old also implies a lower strategic effect and, consequently a lower return of the pension system. From this point of view, young agents oppose social security more. As we mentioned in section 3.1.1, whether the risk sharing or the risk aversion component dominates the incentives of the young depends on the relative size of the two components and on how risk averse young agents are.

As a result, the impact of a decrease in the mean or of an increase in the variance of the population growth rate on the average contribution to the PAYG pension system is ambiguous. As we pointed out in section 3.1.1, the politician balances the preferences of the old and young agents when establishing the policy function. Since old agents are always in favor of a higher pension system, the preferences of the young are crucial in determining the equilibrium contribution.

The computations presented in Appendix 2 explain the channels through which the mean and variance of the demographic shock influence the average contribution. We also explain there how the political weight and the risk aversion of the young impact on the relationship between the mean and variance of the demographic shock, on one hand, and the average contribution, on the other hand. In the next subsection we analyse a calibrated version of the model in order to determine how the average contribution to the PAYG pension system depends on the mean and the variance of the demographic shock.

3.1.4 A numerical analysis of the policy function

In this subsection we analyze a calibrated version of the model. We vary the parameters defining the political weight of the young relative to the old and the risk aversion of the young agents $\{\phi, \gamma\}$ starting with the lowest admissible values $\{\phi_{min}, \gamma_{min}\}$.

We choose the calibration for the financial shock based on the estimations presented in OECD (2013) regarding the distribution of outcomes and probabilities for a 40-year investment horizon[9]. We use the values for the 50% and 90% percentile (4.3% p.a., respectively 6% p.a.) to calibrate the mean and maximum value of the financial shock. We eliminate the technological progress (1.76% p.a. following Gonzalez-Eiras and Niepelt (2008)) from these values. We compute the real rates of return on a 40-year horizon. We obtain the standard deviation of the financial shock by assuming a uniform distribution for it. This gives us the calibration of the financial shock: $\bar{R} = 2.6262, \sigma_R = 0.6907$.

For the calibration of the demographic shock, we use the projected demographics from United Nations, Department of Economic and Social Affairs, Population Division (2012). We take as the minimum and the maximum value of the annual population growth rate the average forecast for US for the period 2015-2100 under the low fertility scenario and the high fertility scenario, respectively. We compound the resulting population growth rates on a 40-year horizon. Assuming a uniform distribution, we obtain the mean and the standard deviation of the population growth rate: $\bar{n} = 1.19, \sigma_n = 0.127$.

Figures 2 and 3 show how the mean and the variance of the demographic process influence the average contribution to the pension system for different combinations of the $\{\phi, \gamma\}$ parameters.

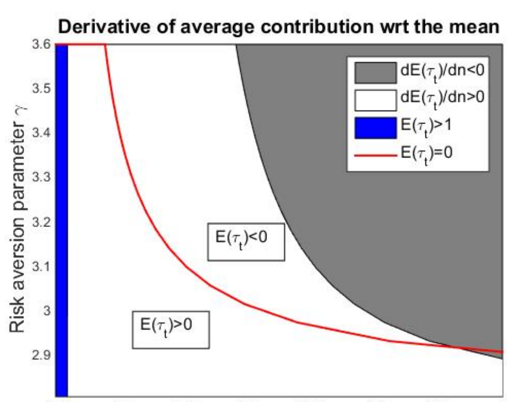


Figure 2

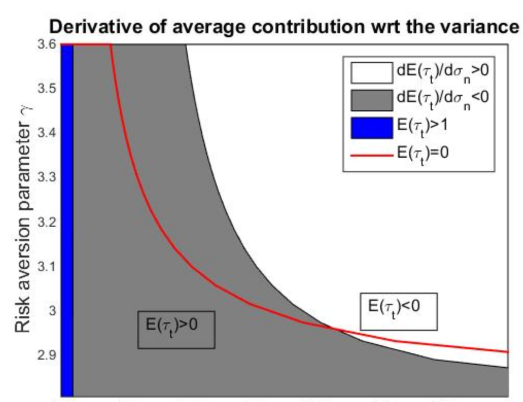


Figure 3

[9] We used the distribution of returns for a portfolio consisting of 50% equity and 50% government bonds.

We analyse the $\{\phi, \gamma\}$ parameter space that is consistent with the restrictions imposed in Assumption 2[10]. For low values of ϕ, in an economy with a lower population growth rate or a higher variance, the politico-economic equilibrium will yield a lower average contribution to the pension system. This means that the risk aversion component dominates the risk sharing component for values close to ϕ_{min}.

As $\{\phi, \gamma\}$ become very high, this result reverses. The situation of high ϕ and high γ coincides with a very low size of the PAYG pension system. In Figure 3, the reversal in the sign of the derivative takes place close to the boundary where the average contribution turns negative. For the calibration we assumed, the sign reversal happens for values of $\{\phi, \gamma\}$ that are consistent with contributions of less than 2.5%.

The sign reversal takes place when the political power of the young becomes very high. In this case the average strategic effect is relatively constant with respect to the mean and variance of the demographic shock. Hence, the return on capital in excess of the pension system varies very little when the characteristics of the demographic shock change. Consequently, the lower negative correlation between benefits and the financial shock dominates and young agents agree to an increase in the size of the pension system.

3.1.5 Robustness check

In this subsection we analyse whether the assumption of a uniform distribution for the demographic shock is important for the results we obtain. We consider two cases: i) a log-normal distribution of the demographic shock $n_t \sim lnN(\mu, \sigma^2)$ and ii) an unspecified distribution of the demographic shock for which only the mean $\bar{n}$ and the variance σ_n^2 are known. For the latter case, we use a second order Taylor expansion of the right hand side of equation (14). For these two specifications of the demographic shock, equation (14) becomes:

i) $1 = \phi \Gamma B^P \frac{((B^P+\bar{n})^2+\sigma_n^2)^3}{(B^P+\bar{n})^8}$, where $\bar{n} = e^{\mu+\frac{\sigma^2}{2}}$ and $\sigma_n^2 = (e^{\sigma^2}-1)e^{2\mu+\sigma^2}$ are the mean and variance of the demographic shock;

ii) $1 = \phi \Gamma B^P \left(\frac{1}{(B^P+\bar{n})^2} + \frac{3\sigma_n^2}{(B^P+\bar{n})^4} \right)$.

We can solve for B^P only numerically. For this purpose, we use the calibration in section 3.1.4. We obtain in each case two solutions for B^P. Consistent with the case of the uniform distribution, we consider only the solution which yields a finite average contribution to the PAYG pension system. Table 3 presents the coefficients of the policy function obtained

[10] This is situated below the $E(\tau_t) = 0$ line until the $E(\tau_t) > 1$ area.

using the uniform distribution, the log-normal distribution and the approximation by second order Taylor expansion. In order to solve for the coefficient A^P, we take $\gamma = 3$.

Table 3: The values of the coefficients A^P and B^P

A^P

ϕ	0.67	0.69	0.71	0.73	0.75	0.77
Uniform	2.7867	2.7577	2.7317	2.7075	2.6844	2.6620
Log-normal	2.7870	2.7581	2.7321	2.7079	2.6848	2.6625
Taylor approx	2.7867	2.7577	2.7317	2.7075	2.6844	2.6620

B^P

ϕ	0.67	0.69	0.71	0.73	0.75	0.77
Uniform	1.8013	2.0364	2.2468	2.4432	2.6306	2.8115
Log-normal	1.8011	2.0363	2.2467	2.4432	2.6305	2.8115
Taylor approx	1.8010	2.0363	2.2467	2.4431	2.6305	2.8114

The values obtained for the coefficients of the policy function are quite similar. Consequently, the assumption regarding the distribution of the demographic shock does not generate substantial differences between the policy functions.

4 The Ramsey planner's problem

In this section we consider the problem of a Ramsey planner who has a commitment technology and we compare the resulting policy function with the one of the politician. In the absence of an endogeneous labour supply decision, the contribution is non-distortionary. Hence, the solution to the Ramsey planner's problem is consistent with the allocations chosen by a social planner.

In order to determine the optimal contributions to the PAYG pension system, the Ramsey planner maximizes the social welfare function, subject to the budget constraints of the agents.

$$\max_{\{\tau_t\}_{t=0}^{\infty}} \sum_{t=-1}^{\infty} \rho^{t+1} E_0[N_t u(c_{t+1})]$$

$$\text{s.t. } s_t + \tau_t = 1 \tag{18}$$

$$c_{t+1} = R_{t+1}s_t + \tau_{t+1}n_{t+1} \tag{19}$$

given N_{-1}, s_{-1}, R_0 and N_0. The welfare weight of the generation born at time t is ρ^{t+1}. We impose $\rho < \frac{1}{\bar{n}}$ in order to ensure that the social welfare function is bounded.

The first order condition of the Ramsey planner's problem is:

$$u'(c_t) = \rho E_t[R_{t+1}u'(c_{t+1})] \tag{20}$$

The form of the policy function implemented by the Ramsey planner is identical to that of the politician, but the coefficients of the policy function will be different.

Proposition 5. *The Ramsey planner's optimization problem is solved by the following policy function:*

$$\tau_t^R = \frac{A^R - R_t s_{t-1}}{B^R + n_t} \tag{21}$$

The coefficients A^R and B^R are obtained as a solution to the following system of equations:

$$1 = \rho E_t \frac{R_{t+1}^2}{B^R + n_{t+1}} \tag{22}$$

$$A^R = \frac{\gamma - \rho\gamma\bar{R} + \rho B^R E_t \frac{R_{t+1}^2}{B^R + n_{t+1}}}{1 - \rho E_t \frac{R_{t+1} n_{t+1}}{B^R + n_{t+1}}} \tag{23}$$

The right hand side of equation (22) is decreasing in B^R while the left hand side is constant [11]. Hence, there always exists a solution for $B^R + n_t > 0$ and this is unique.

Without making any assumptions regarding the demographic and financial shocks, we can already compare the politician and the Ramsey planner's policy functions. The two policy functions are different due to the absence of the strategic effect in the Ramsey planner's problem but also because the weights they use may be distinct. In order to study only the implications of the strategic effect we consider $\rho = \phi$[12].

Proposition 6. *If the Ramsey planner and the politician use the same weights for the young generation ($\rho = \phi$), the following relations between the policy functions of the politician and the Ramsey planner hold:*

1. *The contribution set by the politician is higher than the contribution set by the Ramsey planner, i.e. $\tau^P > \tau^R$;*

2. *Following a financial or a demographic shock, the politician and the Ramsey planner change the contributions and benefits into the same direction;*

[11] As in the case of the political equilibrium, we will not consider further the solution $B^R = 0$ which features an explosive dynamics.

[12] We implicitly assume that $\rho = \phi \geq \underline{\phi}$, where $\underline{\phi}$ is the lowest political weight for which at least one solution for the political equilibrium exists. However, the Ramsey planner's problem admits solutions also for $\rho < \underline{\phi}$.

3. *Following a financial or a demographic shock, the politician adjusts the contributions more than the Ramsey planner, i.e.* $|\frac{\partial \tau_t^P}{\partial R_t s_{t-1}}| > |\frac{\partial \tau_t^R}{\partial R_t s_{t-1}}|$ *and* $|\frac{\partial \tau_t^P}{\partial n_t}| > |\frac{\partial \tau_t^R}{\partial n_t}|$.

The above proposition implies that the response of the politician and the Ramsey planner is qualitatively the same following a financial or a demographic shock. Quantitatively, however, their reaction is different. The politician changes the contributions more than the Ramsey planner because of the existence of the strategic effect.

In order to be able to study further the Ramsey planner's policy function, we impose the assumptions summarized in 3. We also look for a restriction on the parameters of the model that insure that the policy function yields a finite average contribution.

Proposition 7. *Under Assumption 3, the solution to the Ramsey planner's problem has the following coefficients:*

$$B^R = \frac{n_{max} - n_{min} e^{\frac{\Delta n}{\rho \Gamma}}}{e^{\frac{\Delta n}{\rho \Gamma}} - 1} \tag{24}$$

$$A^R = \frac{\gamma - \rho\gamma\bar{R} + B^R}{1 - \rho\bar{R} + \frac{B^R \bar{R}}{\Gamma}} \tag{25}$$

where $\Delta n \equiv n_{max} - n_{min}$.

The average contribution to the pension system is finite for $\rho > \frac{\bar{R}}{\Gamma}$.

The following proposition lays down the result regarding the way in which the mean and variance of the demographic shock affect the policy function of the Ramsey planner.

Proposition 8. *Under Assumptions 1-3, in an economy with a lower mean or a higher variance of the demographic process, the Ramsey planner will impose a higher average contribution, i.e.* $\frac{\partial E(\tau_t^R)}{\partial \bar{n}} < 0, \frac{\partial E(\tau_t^R)}{\partial \sigma_n} > 0$.

This result is exactly the opposite to the one found in the case of the politico-economic equilibrium. Intuitively, in the absence of the strategic effect, when the mean of the population growth rate is lower or the variance is higher the incentives of the old and young agents change in the same direction of sustaining a higher size of the pension system. For the young agents, the return on capital in excess of the pension system does not change when the characteristics of the demographic shock change. But the covariance between the benefits and the financial shock becomes less negative and this makes them sustain a higher size of the pension system.

5 Conclusions

The present paper determines how PAYG pension systems are arranged through a voting process in economies featuring both financial and demographic shocks. We find that, after a decrease in the return on capital, the politician increases contributions and benefits. In response to a fall in the population growth rate, contributions are raised, but benefits are reduced. These responses correspond with major developments of PAYG pension systems observed in reality.

The compensation for financial losses offered to old agents through the pension system is smaller if the mean of the demographic shock is lower or its variance is higher. This result can account for the stylized fact discussed in Section 2 that during the Great Recession, unlike in previous major downturns, governments did not increase the benefits of the pension system. In the political equilibrium, a weaker sensitivity of the policy function with respect to the wealth of the old leads to a lower average contribution to the pension system. This effect can explain reforms that involve the (partial) transition from PAYG pension systems to fully funded ones.

Following financial and demographic shocks, the Ramsey planner changes contributions and benefits in the same direction as the politician. However, the adjustments made are of a smaller magnitude. Changes in the mean or variance of the population growth rate have a qualitatively different impact on the policy function of the politician and the Ramsey planner. Unlike the politician, if the mean of the demographic shock is smaller or the variance is higher, the Ramsey planner increases the average contribution to the pension system.

The analysis in this paper can be extended into several interesting directions. First, it would be important to introduce uninsurable worker specific productivity shocks. In such a framework we can study the provision of means and asset tested old-age benefits alongside earnings related pensions. We can also analyze how the relative size of these two pension pillars is influenced by the business cycle. This is a relevant policy question since we observe that during the Great Recession many governments expanded means and asset tested benefits but kept earnings related pension benefits unchanged or even downsized them[13]. Second, the model can be used to study the political arrangement of pension systems that have a mandatory fully funded component. The downsizing of this component that took place in most of the Central and Eastern European countries during the Great Recession shows that financial shocks may influence its size.

[13] Through increases in the retirement age for example. See OECD (2012).

Appendix 1

Proof of Proposition 1

The strategic effect based on the solution we guessed in (13) is:

$$\frac{\partial \tau^P_{t+1}}{\partial \tau^P_t} = \frac{\partial \tau^P_{t+1}}{\partial s_t}\frac{\partial s_t}{\partial \tau^P_t} = -\frac{R_{t+1}}{B^P + n_{t+1}}\frac{\partial(1-\tau^P_t)}{\partial \tau^P_t} = \frac{R_{t+1}}{B^P + n_{t+1}} \tag{26}$$

We substitute the above expression of the strategic effect, the guess for the policy function (13) and the budget constraints (1) and (2) in relation (12). We obtain the following expression for the contribution:

$$\tau^P_t = \frac{\gamma - \phi\gamma B^P E_t \frac{R_{t+1}}{B^P+n_{t+1}} + \phi B^P A^P E_t \frac{R_{t+1}n_{t+1}}{(B^P+n_{t+1})^2} + \phi (B^P)^2 E_t \frac{R^2_{t+1}}{(B^P+n_{t+1})^2} - R_t s_{t-1}}{\phi (B^P)^2 E_t \frac{R^2_{t+1}}{(B^P+n_{t+1})^2} + n_t}$$

Comparing the above with the solution we proposed in (13), we obtain the system of equations for the coefficients A^P and B^P.

Proof of Proposition 2

$$\frac{\partial \tau^P_t}{\partial n_t} = -\frac{\tau^P_t}{B^P + n_t} < 0 \text{ if } \tau^P_t > 0$$
$$\frac{\partial b^P_t}{\partial n_t} = \frac{B^P \tau^P_t}{B^P + n_t} > 0 \text{ if } \tau^P_t > 0$$

In the last inequality we used the fact that $B^P > 0$.

Proof of Proposition 3

We will provide the proof in steps.

First, under Assumption (3), equation (14) becomes:

$$1 = \phi B^P \Gamma \int_{n_{min}}^{n_{max}} \frac{1}{(B^P + n_{t+1})^2 (n_{max} - n_{min})} dn_{t+1} = \frac{\phi \Gamma B^P}{(B^P + n_{max})(B^P + n_{min})}$$

We obtain the solution for B^P:

$$B^P = \frac{-(2\bar{n} - \phi\Gamma) \pm \sqrt{\delta}}{2}$$

where $\delta = (2\bar{n} - \phi\Gamma)^2 - 4 n_{max} n_{min}$. We used the following property of a uniform distribution: $n_{max} + n_{min} = 2\bar{n}$. From (15) we obtain the solution for A^P.

Second, we notice that the political equilibrium can have no solution, one or two solutions in the domain of real numbers depending on the sign of δ. We impose the condition for having at least one equilibrium: $\delta \geq 0 \Leftrightarrow \phi\Gamma \in [0,(\sqrt{n_{max}}-\sqrt{n_{min}})^2]\cup[(\sqrt{n_{max}}+\sqrt{n_{min}})^2,\infty)$.

Third, we impose the condition that the policy function has a finite mean. We iterate backwards the solution of the politician's problem (13):

$$\tau_t^P = \frac{A^P - R_t}{B^P + n_t} + \frac{R_t(A^P - R_{t-1})}{(B^P + n_t)(B^P + n_{t-1})} + \frac{R_t R_{t-1}(A^P - R_{t-2})}{(B^P + n_t)(B^P + n_{t-1})(B^P + n_{t-2})} + ...$$
$$+ \frac{R_t R_{t-1} R_{t-2} ... R_0}{(B^P + n_t)(B^P + n_{t-1})(B^P + n_{t-2})...(B^P + n_0)} \tau_0^P$$

Using the independence between the financial and demographic shock and the fact that each shock is independent across time, the expected contribution becomes:

$$E(\tau_t^P) = (A^P - \bar{R})E\frac{1}{B^P + n_t}\left[\bar{R}E\frac{1}{B^P + n_t} + \left(\bar{R}E\frac{1}{B^P + n_t}\right)^2 + ...\right] + \left(\bar{R}E\frac{1}{B^P + n_t}\right)^t E(\tau_0^P)$$

It follows that the mean of the process defining the contribution to the pension system is finite if and only if $E\frac{1}{B^P+n_t} \in (-1/\bar{R}; 1/\bar{R})$. Plugging in the expressions of the two values obtained for B^P, we obtain that, for $\phi\Gamma > \frac{(\Delta n)^2}{\left(1-e^{-\Delta n/\bar{R}}\right)\left(n_{max}-n_{min}e^{\Delta n/\bar{R}}\right)}$, only the policy function defined by $B^P = \frac{-(2\bar{n}-\phi\Gamma)+\sqrt{\delta}}{2}$ respects the condition for a finite mean.

Proof of Proposition 4

The sensitivity of the politician's policy function with respect to the wealth of the old is given by $\frac{1}{B^P+n_t}$, while the average strategic effect is equal to $\bar{R}E\frac{n_t}{B^P+n_t}$. We express the bounds of the demographic process in terms of the mean ($\bar{n}$) and standard deviation (σ_n) of the distribution:

$$n_{max} = \bar{n} + \sigma_n\sqrt{3}$$
$$n_{min} = \bar{n} - \sigma_n\sqrt{3}$$

The formulas for B^P and $E\frac{1}{B^P+n_t}$ become:

$$B^P = -\bar{n} + \frac{\phi\Gamma + \sqrt{(\phi\Gamma - 2\bar{n})^2 - 4(\bar{n}^2 - 3\sigma_n^2)}}{2} \quad (27)$$

$$E\frac{1}{B^P + n_t} = \frac{1}{2\sigma_n\sqrt{3}} ln\frac{B^P + \bar{n} + \sigma_n\sqrt{3}}{B^P + \bar{n} - \sigma_n\sqrt{3}} \quad (28)$$

We can immediately obtain the following derivatives:

$$\frac{\partial B^P}{\partial \bar{n}} = -1 - \frac{\phi\Gamma}{\sqrt{\Delta}} < 0$$
$$\frac{\partial E\frac{1}{B^P+n_t}}{\partial \bar{n}} = -\left(\frac{\partial B^P}{\partial \bar{n}} + 1\right)\frac{1}{(B^P+\bar{n}+\sigma_n\sqrt{3})(B^P+\bar{n}-\sigma_n\sqrt{3})} = \frac{\phi\Gamma}{\sqrt{\Delta}(B^P+\bar{n}+\sigma_n\sqrt{3})(B^P+\bar{n}-\sigma_n\sqrt{3})} > 0$$
$$\frac{\partial E\frac{n_t}{B^P+n_t}}{\partial \bar{n}} = -\frac{\partial B^P}{\partial \bar{n}}\left(E\frac{1}{B^P+n_t} - \frac{1}{\phi\Gamma}\right) + \frac{1}{\phi\Gamma} > 0$$
$$\frac{\partial B^P}{\partial \sigma_n} = \frac{6\sigma_n}{\sqrt{\Delta}} > 0$$
$$\frac{\partial E\frac{1}{B^P+n_t}}{\partial \sigma_n} = -\frac{E\frac{1}{B^P+n_t}}{\sigma_n} + \frac{B^P+\bar{n}-\sigma_n\frac{\partial B^P}{\partial \sigma_n}}{\sigma_n(B^P+\bar{n}+\sigma_n\sqrt{3})(B^P+\bar{n}-\sigma_n\sqrt{3})} < 0$$
$$\frac{\partial E\frac{n_t}{B^P+n_t}}{\partial \sigma_n} = \frac{\partial B^P}{\partial \sigma_n}\left(-E\frac{1}{B^P+n_t} + \frac{1}{\phi\Gamma}\right) + \frac{B^P}{\sigma_n}\left(E\frac{1}{B^P+n_t} - \frac{1}{\phi\Gamma} - \frac{\bar{n}}{\phi\Gamma B^P}\right) < 0$$

To sign the derivatives we used the following results:

- $E\frac{1}{B^P+n_t} - \frac{1}{\phi\Gamma} > 0$. We obtain this from an alternative writing of (12):

$$E_t\frac{n_{t+1}}{(B^P+n_{t+1})^2} = E_t\frac{1}{B^P+n_{t+1}} - \frac{1}{\phi\Gamma} > 0$$

- the monotonicity of $\frac{\partial E\frac{1}{B^P+n_t}}{\partial \sigma_n} \equiv \Omega_1$ and $\left(E\frac{1}{B^P+n_t} - \frac{1}{\phi\Gamma} - \frac{\bar{n}}{\phi\Gamma B^P}\right) \equiv \Omega_2$ with respect to ϕ:

$$\frac{\partial \Omega_1}{\partial \phi} = \frac{12\sigma_n}{\phi\Delta\sqrt{\Delta}} > 0$$
$$\frac{\partial \Omega_2}{\partial \phi} = \frac{6\sigma_n^2}{\Gamma\phi^2 B^P\sqrt{\Delta}} > 0$$

  We determine that $\lim_{\phi\to\infty}\Omega_1 = 0$ and $\lim_{\phi\to\infty}\Omega_2 = 0$. In conclusion, Ω_1 and Ω_2 are increasing with ϕ towards 0, hence they are negative everywhere.

The sensitivity of the contribution to the pension system with respect to the wealth of old is inversely related with B^P, hence we get the first result of the proposition.

Proof of Proposition 5

We substitute (18), (19) and (21) in (20). We obtain:

$$\tau_t^R = \frac{\gamma - \rho\gamma\bar{R} + \rho A^R E_t\frac{R_{t+1}n_{t+1}}{B^R+n_{t+1}} + \rho B^R E_t\frac{R_{t+1}^2}{B^R+n_{t+1}}}{B^R\rho E_t\frac{R_{t+1}^2}{B^R+n_{t+1}} + n_t} \tag{29}$$

Comparing this solution with the initial guess in (21), we obtain the system of equations defining the solutions for B^R and A^R.

Proof of Proposition 6

1. We compare the first order conditions of the politician and the Ramsey planner (relations (12) and (20)) for $\phi = \rho$. Since $n_{t+1}\frac{\partial \tau_{t+1}}{\partial \tau_t} > 0$, $u'(c_t^P) < u'(c_t^R) \Rightarrow c_t^P > c_t^R$. The latter relation implies that a higher size of contributions and, hence, benefits prevail in the political equilibrium.

2. Since $B^P + n_t > 0$ and $B^R + n_t > 0$, both the politician and the Ramsey planner decrease the contribution to and the benefits from the pension system when the return on capital increases.

Focusing only on positive equilibrium contributions to the pension system, as shown in the proof of Proposition 2, the fact that $B^P + n_t > 0$ and $B^R + n_t > 0$ also insures that both the politician and the Ramsey planner decrease the contribution to the pension system when the population growth rate increases. In order for both the politician and the Ramsey planner to increase benefits following an increase in the population growth rate, the additional conditions $B^R, B^P > 0$ are necessary. We already know that $B^P > 0$ always. The proof for subpoint 3) of this Proposition establishes that $B^R > B^P$. Hence $B^R > 0$ when $\rho = \phi$. This completes the proof.

3. We compare equations (12) and (20). For $\rho = \phi$, we obtain:

$$E_t \frac{B^P}{(B^P + n_{t+1})^2} = E_t \frac{1}{B^R + n_{t+1}} \Leftrightarrow E_t \frac{1}{B^P + n_{t+1}} - E_t \frac{n_{t+1}}{(B^P + n_{t+1})^2} = E_t \frac{1}{B^R + n_{t+1}}$$

Since $E_t \frac{n_{t+1}}{(B^P + n_{t+1})^2} > 0$, we have:

$$E_t \frac{1}{B^P + n_{t+1}} > E_t \frac{1}{B^R + n_{t+1}} \Leftrightarrow (B^R - B^P) E_t \frac{1}{(B^P + n_{t+1})(B^R + n_{t+1})} > 0 \Leftrightarrow B^R > B^P$$

The last inequality shows that the politician decreases contributions to a higher extent than the Ramsey planner when there is an increase in the return on capital or in the population growth rate.

Proof of Proposition 7

We solve for B^R and A^R from (22) and (23) using the assumption of independence between R_t and n_t and the assumption of a uniform distribution for n_t. The condition for a finite mean is the same as in the politician's case $E\frac{1}{B^R+n_t} \in (-1/\bar{R}; 1/\bar{R})$. But now, $E\frac{1}{B^R+n_t} = \frac{1}{\rho\Gamma}$. Hence we must have $\rho > \frac{\bar{R}}{\Gamma}$.

Proof of Proposition 8

The derivatives of B^R with respect to $\bar{n}$ and σ_n are:

$$\frac{\partial B^R}{\partial \bar{n}} = -1 < 0$$
$$\frac{\partial B^R}{\partial \sigma_n} = \frac{\sqrt{3}e^{\frac{4\sqrt{3}\sigma_n}{\rho\Gamma}} - \frac{12\sigma_n}{\rho\Gamma}e^{\frac{2\sqrt{3}\sigma_n}{\rho\Gamma}} - \sqrt{3}}{(e^{\frac{2\sqrt{3}\sigma_n}{\rho\Gamma}} - 1)^2} > 0$$

To prove the sign of the second derivative, we make the notation $\frac{2\sqrt{3}\sigma_n}{\rho\Gamma} \equiv x$ and rewrite the denominator as:

$$f(x) = \sqrt{3}(e^{2x} - 2xe^x - 1)$$
$$f'(x) = 2\sqrt{3}e^x(e^x - x - 1) > 0$$

Consequently $f(x)$ is an increasing function, so $f(x) > f(0) = 0$.

The average contribution to the pension system has the form:

$$E(\tau_t^R) = \frac{(A^R - \bar{R})E\frac{1}{B^R+n_t}}{1 - \bar{R}E\frac{1}{B^R+n_t}} = \frac{A^R - \bar{R}}{\rho\Gamma - \bar{R}}$$

The following partial derivatives help us determine the change of $E(\tau_t^R)$ with respect to $\bar{n}$ and σ_n:

$$\frac{\partial E(\tau_t^R)}{\partial A^R} = \frac{1}{\rho\Gamma - \bar{R}} > 0$$
$$\frac{\partial A^R}{\partial B^R} = \frac{\left(\bar{R}\rho - 1\right)\left(\frac{\gamma\bar{R}}{\Gamma} - 1\right)}{\left(1 - \rho\bar{R} + \frac{\bar{R}B^R}{\Gamma}\right)^2} > 0$$

To sign the above partial derivatives we use:

- Assumption 1 ($\gamma > \Gamma/\bar{R}$);
- the result of Proposition 7 ($\rho > \frac{\bar{R}}{\Gamma}$);
- an additional restriction on the welfare weight that insures Assumption 2 is satisfied ($\rho > 1/\bar{R}$). The proof for this last restriction entails the following steps.

  First, we first establish that $E(\tau_t^R) < 1 \Leftrightarrow \frac{(\gamma+B^R-\rho\Gamma)(1-\rho\bar{R})}{1-\rho\bar{R}+\frac{\bar{R}B^R}{\Gamma}} < 0$.

Second, using the properties of $\frac{\partial B^R}{\partial \sigma_n}$ derived above we can establish that $B^R > \lim_{\sigma_n \to 0} B^R = \rho\Gamma - \bar{n}$. Based on this inequality we can prove the relations:

$$1 - \rho\bar{R} + \frac{\bar{R}B^R}{\Gamma} > 1 - \rho\bar{R} + \frac{\bar{R}(\rho\Gamma - \bar{n})}{\Gamma} = \frac{\Gamma - \bar{R}\bar{n}}{\Gamma} > 0$$
$$\gamma + B^R - \rho\Gamma > \gamma - \bar{n} > \frac{\Gamma}{\bar{R}} - \bar{n} > 0$$

Consequently, for Assumption 2 to hold it must be the case that $1 - \rho\bar{R} < 0$.

Using the above partial derivatives, we can sign the derivative of $E(\tau_t^R)$ with respect to $\bar{n}$ and σ_n.

$$\frac{\partial E(\tau_t^R)}{\partial \bar{n}} = \underbrace{\frac{\partial E(\tau_t^R)}{\partial A^R}}_{>0} \underbrace{\frac{\partial A^R}{\partial B^R}}_{>0} \underbrace{\frac{\partial B^R}{\partial \bar{n}}}_{<0} < 0$$
$$\frac{\partial E(\tau_t^R)}{\partial \sigma_n} = \underbrace{\frac{\partial E(\tau_t^R)}{\partial A^R}}_{>0} \underbrace{\frac{\partial A^R}{\partial B^R}}_{>0} \underbrace{\frac{\partial B^R}{\partial \sigma_n}}_{>0} > 0$$

Appendix 2

In the case it is finite, the average contribution set by the politician is equal to:

$$E(\tau_t^P) = \frac{(A^P - \bar{R})E\frac{1}{B^P + n_t}}{1 - \bar{R}E\frac{1}{B^P + n_t}} \tag{30}$$

We analyse the change in A^P and $E(\tau_t^P)$ with respect to the mean and variance of the demographic shock.

$$\frac{\partial A^P}{\partial \bar{n}} = \underbrace{\frac{\Gamma(\gamma\bar{R} - \Gamma)}{(\Gamma - \phi\Gamma B^P \bar{R} E\frac{1}{B^P + n_t} + B^P\bar{R})^2}}_{>0} \left(\underbrace{\frac{\partial B^P}{\partial \bar{n}}}_{\equiv\Omega_1<0} \underbrace{\left(-1 + \phi\bar{R}B^P E\frac{1}{B^P + n_t}\right)}_{\equiv\Omega_3>0} + \phi B^P \bar{R} \underbrace{\frac{\partial E\frac{n_t}{B^P + n_t}}{\partial \bar{n}}}_{\equiv\Omega_2>0} \right) \tag{31}$$

$$\frac{\partial E(\tau_t^P)}{\partial \bar{n}} = \frac{1}{(1 - \bar{R}E\frac{1}{B^P + n_t})^2} \left(\frac{\partial A^P}{\partial \bar{n}} \underbrace{E\frac{1}{B^P + n_t}}_{>0} \underbrace{\left(1 - \bar{R}E\frac{1}{B^P + n_t}\right)}_{\equiv\Omega_4>0} + \underbrace{(A^P - \bar{R})}_{\equiv\Omega_5>0} \underbrace{\frac{\partial E\frac{1}{B^P + n_t}}{\partial \bar{n}}}_{>0} \right) \tag{32}$$

$$\frac{\partial A^P}{\partial \sigma_n} = \underbrace{\frac{\Gamma(\gamma\bar{R} - \Gamma)}{(\Gamma - \phi\Gamma B^P \bar{R} E\frac{1}{B^P + n_t} + B^P\bar{R})^2}}_{>0} \left(\underbrace{\frac{\partial B^P}{\partial \sigma_n}}_{\equiv\Omega_1>0} \underbrace{\left(-1 + \phi\bar{R}B^P E\frac{1}{B^P + n_t}\right)}_{>0} + \phi B^P \bar{R} \underbrace{\frac{\partial E\frac{n_t}{B^P + n_t}}{\partial \sigma_n}}_{\equiv\Omega_2<0} \right) \tag{33}$$

$$\frac{\partial E(\tau_t^P)}{\partial \sigma_n} = \frac{1}{(1 - \bar{R}E\frac{1}{B^P + n_t})^2} \left(\frac{\partial A^P}{\partial \sigma_n} \underbrace{E\frac{1}{B^P + n_t}}_{>0} \underbrace{\left(1 - \bar{R}E\frac{1}{B^P + n_t}\right)}_{>0} + \underbrace{(A^P - \bar{R})}_{>0} \underbrace{\frac{\partial E\frac{1}{B^P + n_t}}{\partial \sigma_n}}_{<0} \right) \tag{34}$$

To sign the different components of the above derivatives we used:

– Assumption 1 ($\gamma > \Gamma/\bar{R}$);

– for Ω_1 and Ω_2, the derivatives we already computed in the proof of Proposition 4;

– for Ω_4, the assumption of a finite average contribution to the pension system;

– for Ω_5, the restriction imposed by Assumption 2 ($E(\tau_t) > 0$).

The relation $\Omega_3 > 0$ does not hold for every ϕ. However, we can prove that under Assumption 2 ($E(\tau_t) < 1$) and for $\sigma_n \to 0$ this relation holds. But if $\sigma_n > 0$, the term Ω_3 can be negative even under Assumption 2. Numerical simulations show that in this region the sign of Ω_2 dominates anyway.

A decrease (increase) in $\bar{n}$ (σ_n) impacts on A^P through two different channels:

– through B^P that increases, thus making the policy function less sensitive to the wealth of the old. This leads to an increase also in A^P meaning that young agents desire a higher size of the PAYG pension system.

– through the strategic effect that decreases. This reduces the support of the young generation for the PAYG pension system and leads to a decrease in A^P.

The resulting influence on A^P is not clear cut. However, given the mean and variance of the shocks, the derivative depends only on parameter ϕ. Numerical simulations show that for low values of ϕ, the sign of the derivative of A^P and hence the sign of the derivative of $E(\tau_t^P)$ with respect to $\bar{n}$ (σ_n) is given by Ω_2. As ϕ increases, the strategic effect becomes less sensitive to changes in $\bar{n}$ (σ_n), so the sign of Ω_1 starts to dominate.

Through A^P, the mean and variance of the demographic shock further impact on $E(\tau_t^P)$. Although under Assumption 1 the value of γ does not influence the sign of the derivative of A^P, it does influence its size. That is why for the overall impact on $E(\tau_t^P)$, the size of γ also plays a role. More specifically, a higher γ increases the absolute value of the derivative of A^P with respect to $\bar{n}$ (σ_n) for the same value of ϕ.

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