Optimal Design of Funded Pension Schemes under Financial Fairness, with Applications to the Dutch Pension Reform

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Abstract

We introduce a dynamic risk sharing strategy for a collective DC system that enables collective risk sharing under the principle of Pareto efficiency and financial fairness. The risk-sharing rule is calculated on a rolling basis based on the projected pension system which reflects the current information set and the best predictions of economic environments over the next several years. Utility functions specify inter-temporal risk preferences and a risk-neutral measure helps to make the market values of the pension payments match the desired value profile. The model is flexible and some parameters serve as policy instruments for tuning the model. For the discontinuity problem, we introduce the concept of tolerance band: as long as the buffer size stays within the tolerance band, both the existing and the incoming cohorts shall find it still welfare improving to stay in the collective system.

Keywords: intergenerational risk sharing, funded pension schemes, Pareto efficiency, financial fairness

Preliminary version; do not circulate.

1 Introduction

The financial crisis in the last decade have witnessed sharp and sudden falls in the funding ratio of many pension funds in the Netherlands. As a result, there has been a trend of transforming the traditional defined-benefit schemes towards the defined-contribution patterns in

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the second pillar part - the funded part - of the pension system in the Netherlands, which will mean that the sponsor is retrieving from their role and the pensioners will have to bear more investment risks than ever. The Dutch pension reform and the relevant debates have been going on to settle the innovations and the resulted transitions in the pension system. Recent development involves the approval of the nFTK ("nieuw Financieel Toetsingskader" in Dutch) which provides a regulatory framework for the pension funds.

Collectivity has been among the most predominant features in the Dutch pension system; as a result, there is large space for collective risk sharing, both inter- and intra-generationally. Conventionally, the funding ratio plays a pivotal role in determining how the risks shall be shared. Among the most cutting-edge developments in reality, the nFTK restricts the use of adjusting the contributions as a means of funding status recovery, and the risk sharing rules are mainly in the form of benefit indexation based on an average version of the funding ratio. However, the nFTK does not provide too much reasoning from a technical point of view; as a supplement, it might be useful to explore the optimal risk sharing rules based on some principles in a systematic way. This is exactly what this paper comes for.

We will mainly work under the notion of "PEFF" - Pareto efficiency and financial fairness - when exploring the optimal way of intergenerational risk sharing in the funded pension schemes that allow for it. Utility-wise, the system should be Pareto efficient inter-temporally regarding the pension payments, which means that there should be no possibility of welfare improvement by adjusting the intergenerational transfers. Value-wise, the pension contract should be fairly priced, in the sense that the market value of contributions should equal the market value of the corresponding benefits. The valuation of the benefits should be consistent with the market, in line with the value-based asset-liability management within the pension funds; see Ponds [11].

The notion of PEFF is motivated from the dual properties of the funded pension system. On one hand, it is a multilateral bargaining system, where the Pareto efficiency is a fundamental principle for risk sharing among different participants. On the other hand, it is essentially a financial contract for each pensioner. In a funded pension system, each pensioner pays contributions in his early life in exchange for pension payments when he becomes old. It should be important to make the contract as financially fair as possible in terms of market value. Bao et al. [2] have shown that there always exists a unique risk sharing rule to a multi-period risk sharing system that is PEFF. This provides a solid theoretical support for working under the notion of PEFF.

As for the realistic pension system, we will work with a dynamic version of the PEFF algorithm: the moving-horizon PEFF approach, or the *Mohopeff*. At each time point, a projected pension system is established over a fixed horizon, say 10 years. The optimal risk distribution for the next pension payment is calculated within this projected system based on the PEFF principle. When we go on to the next year, a new projected pension system is set up again and we do the same for the following pension payment. The philosophy behind is that the decision we make today is based on, and shall totally reflect, our current information set and our expectations on both the economic environment and the pension system. A dynamic algorithm allows us to update our estimations, and by using a relatively short horizon we avoid making long-term projections. This is in accordance with the final purpose of the Mohopeff approach, that is, to find a reasonable and sustainable risk sharing rule based on some general principles.

There are some significant differences between the Mohopeff approach and the existing framework like the nFTK. The biggest difference is that in the existing framework the risk sharing rule is pre-specified and the market values of the pension payments come as a result, while in the Mohopeff approach the market values of the pension payments are first specified and the risk sharing rule comes as a result. Second, the exact form of the risk sharing rule is also determined by the specification of risk preferences, e.g. utility functions, in the Mohopeff approach. The Mohopeff approach leads to a time-varying and scenario-dependent risk sharing rules while the nFTK gives time-invariant indexation rules. The Mohopeff provides a different perspective and we hope that these innovations can add some extra fuel to the engine of Dutch pension reform.

The rest of the paper is structured as follows. We will first describe the pension system and the economic environment. Then the PEFF principle will be introduced and we will introduce in detail how the Mohopeff model will work. A CDC scheme will then be investigated; we consider the decumulation phase only, where participants will only make a lump-sum contribution at the time they retire and then get a stream of nominal variable annuity payments. The Mohopeff approach will be tested in a simulated economic environment; we shall focus on the performance of the Mohopeff approach if different policy intruments are implemented. We then move on to the discussion of the discontinuity problem and we introduce the concept of tolerance band for the buffer. Some remarks will conclude this paper in the end.

2 Model Framework

We consider describing the realistic pension system within an OLG model. We will mainly focus on pension systems of collective defined-contribution type. Only the decumulation phase will be considered; this means that each generation makes a lump-sum contribution at the time of entry, and in the next n years they get benefits, which will be nominal variable annuity in this context. This may only be a fraction of the second-pillar pension. The accumulation phase follows an IDC type; this allows large space for, amongst other things, individualization in investment decisions and clear definition of ownership rights. Alongside the generations there is a buffer installed in the system which enables inter-temporal transfer.

Different from the existing framework, no risk-sharing rules (e.g. the indexation rules) will be presumed. Rather, the risk sharing rules will be calculated as the output according to the PEFF principle which will be introduced later. The PEFF technique will function on a moving-horizon basis. We will always denote N as the length of the moving horizon.

ALM techniques will be used to simulate the economic environment. We will thus have to distinguish two types of models: the *underlying system (U-system)* which is the simulated economic environment and acts as the "pseudo-real-world", and the *projected system (P-system)* which is the projection of the system for calculating the optimal risk-sharing rules based on the PEFF principle. The U-system will generate a number of economic scenarios over N_S years which we will call a *path* or *trajectory*. At each time point of each path, a P-system will be established in order to calculate the optimal asset allocation between the current pension payments and the buffer. The target type of risk in the P-system is mainly the investment risk, i.e. the return from investing in the capital market, while in the U-system more sources of randomness can be included. In the P-system, only investment returns are allowed to be stochastic; other variables will be set as deterministic, based on the best predictions. The PEFF principle plays the key role in the P-system. In this section we will mainly introduce the general setting of this paper. The P-system will be introduced in next section.

We consider a discrete-time environment with $\tau = 0, 1, \cdots, N_S$ standing for the time points. We always use a tilde ($\tilde{\cdot}$) as an indication if some variable is a prediction.

The economy. We assume an economy where all the prices are given exogenously. Assume that there exist some observable rates that can be regarded as the reference nominal and real risk-free rates. $R_{\tau;s_1,s_2}$ denotes the gross nominal forward rate from time s_1 to $s_2 > s_1$ along the term structure observed at time τ. $\tilde{\eta}_{\tau;s_1,s_2}$ denotes as the predicted gross inflation from time s_1 to s_2 observed at time τ. It is the difference between the nominal and real term structure. η_τ is the realized gross inflation from time $\tau - 1$ to τ. We further assume that the wage growth rate is the same as the price inflation, so we can use the term "inflation" unambiguously.

The financial market. There is a simple financial market over an underlying finite probability space $(\Omega, \mathcal{F}, \mathbb{P}, \mathbb{Q})$, where $\mathbb{P}$ is the given objective probability measure while $\mathbb{Q}$ is a given risk-neutral measure for pricing. It is not required that the market is arbitrage-free and complete; we only need that there is some $\mathbb{P}$ and $\mathbb{Q}$ installed in the system.

It is assumed that the pension fund will always invest all its asset in a fixed asset mix which provides a stream of stochastic returns. The fund has access to short-term and long-term bonds whose prices correspond to the observed nominal term structure, and a stock index. No investment decisions are involved.

The population. The generations will be numbered by the time they enter the system. Each generation comes into the system only when they have retired. n denotes the maximal years they can exist in the system. Thus at time τ there are n generations simultaneously as pensioners: $G_{\tau-(n-1)}, \cdots, G_{\tau-1}, G_\tau$. Let $P_{s,t}$ be the actual population size of generation s at time $t \geq s$.

The pension arrangement. Each generation pays a lump-sum contribution at entry in exchange for a nominal variable annuity. Contributions are paid in at the beginning of each period while benefits are paid out by the end of each period. Generation G_τ pays their contribution at time τ and they expect their variable annuity payments at time points $\tau+1, \cdots, \tau+n$. C_τ denotes the aggregate contribution generation G_τ makes at time τ. This variable annuity pension takes the actuarially fair fixed nominal annuity as a benchmark reference. For the actuarially fair annuity, the conversion is made at the time of entry:

$$C_\tau = b_\tau \sum_{s=1}^{n} \left(\frac{1}{R_{\tau;\tau,\tau+s}} \frac{\tilde{P}_{\tau;\tau,\tau+s}}{P_{\tau,\tau}} \right) \tag{2.1}$$

where b_τ is called the *benchmark annuity ambition* (BAA) for generation G_τ in the context. It is not the annuity that will be actually paid; the actual pension payments will fluctuate around this level according to the funding status and the current return from the asset mix. Ideally, in each year the generations shall get the stochastic gross return from the asset mix whose market value equals the BAA level. It is just a "benchmark" as later certain adjustments will be made to determine the actual market values of the benefits. Note that in equation 2.1 predictions on future demography is used: $\tilde{P}_{\tau;\tau,\tau+s}$ stands for the prediction, made at time τ, of the population of generation G_τ at time $\tau+s$. The part $\frac{\tilde{P}_{\tau;\tau,\tau+s}}{P_{\tau,\tau}}$ is actually the unconditional survival probability for the whole generation.

$B_{s,\tau}$ denotes the total benefit for generation G_s at time τ. Define

$$B_\tau := \sum_{s=\tau-n}^{\tau-1} B_{s,\tau}$$

as the aggregate benefit paid out at time τ. Correspondingly, let

$$v_b(B_\tau) = \sum_{s=\tau-n}^{\tau-1} b_s$$

be the *benchmark ambition* of the aggregate benefit B_τ, which serves as the benchmark market value.

F_τ denotes the buffer size at time τ after pension payment but before new contribution comes in, and A_τ denotes the end-of-period total asset to be divided between the current pension payments and the buffer for future use:

$$A_\tau = B_\tau + F_\tau. \tag{2.2}$$

Funding ratio is among the most important indicators within pension funds. In this context, as the benefits are essentially contingent claims, the actual market value of the liability can be difficult to compute if it is dependent on the current funding status. It may be convenient to use the *benchmark liability* as an alternative. The benchmark liability is defined as the present value of all the unpaid BAAs discounted according to the current term structure. L_τ denotes the benchmark liability at time τ after the aggregate benefit B_τ is paid but before new contribution C_τ comes in. Specifically,

$$L_\tau = \sum_{t=1}^{n-1} \frac{\left(\sum_{j=t}^{n-1} b_{\tau-n+j}\right)}{R_{\tau;\tau,\tau+t}}.$$

The funding ratio is a continuous process in reality; in our discrete environment, we are interested in two kinds of funding ratios in particular: the *pre-payment benchmark funding ratio* (pre-BFR), denoted as π_τ, and the *post-payment benchmark funding ratio* (post-BFR), denoted as κ_τ. The pre-payment BFR is defined as the benchmark funding ratio at time τ calculated just before the benefit B_τ is paid:

$$\pi_\tau = \frac{A_\tau}{v_b(B_\tau) + L_\tau} = \frac{B_\tau + F_\tau}{v_b(B_\tau) + L_\tau}.$$

The post-payment BFR is the benchmark funding ratio at time τ calculated just after the benefit B_τ is paid, but before new contribution comes in:

$$\kappa_\tau = \frac{F_\tau}{L_\tau}.$$

The U-system. The U-system aims to simulate the economic environment, maximally mimicking the real world. Certain econometric model will be used to generate economic scenarios according to presumed dynamics which are delicately calibrated to real data. Usually a VAR model will be used to produce discrete-time results. At each time point along every scenarios, the output of the U-system include the nominal and real term structures of some benchmark risk-free rate, the realized inflation where we always assume that the price and wage inflation are the same, the demography information and the return from the stock index.

The key problem. According to Equation 2.2, at each time point, it has to be determined how much money shall be paid out as the current benefits and how much shall be left in the buffer for future liabilities. The distribution rules should be reasonable, flexible and sustainable. In the next section, we will introduce extensively how the Mohopeff approach works and what it indicates for the key problem.

3 The Notion of Pareto Efficiency and Financial Fairness and the Mohopeff Approach

This section deals with the principle of Pareto efficiency and financial fairness and the Mohopeff approach. We first introduce the notion of PEFF and the algorithm in a multi-period setting, then we discuss in detail how the Mohopeff – the moving-horizon version of the PEFF approach – will work.

The term of risk sharing in this context can be decomposed into two levels: inter-temporally and intra-temporally (intra-group). The first level, inter-temporal risk sharing, is essentially inter-temporal risk smoothing, which will mean that there shouldn't be any space for Pareto improvements by adjusting the inter-temporal transfer. Aggregate benefits will thus be considered; it is defined as the sum of all the current pension payments at each time point. Once those aggregate benefits are determined, then comes the second level: how to distribute the aggregate benefits among current pensioners. The risk sharing in this level is due to the intra-group heterogeneity among the current pensioners regarding the risk preferences. In this paper we will mainly talk about the inter-temporal risk sharing; we simply assume that the pensioners are equally risk-averse and the aggregate benefit shall be distributed proportionally among the pensioners according to their contributions.

3.1 The Setting of a P-System

As we have mentioned, while the U-system simulates the global underlying economic environment, the P-system will be the local environment for calculating the risk sharing rules. The P-system is the projected pension system reflecting our best predictions on the economic environment based on which we calculate the best risk sharing solutions for the current pension payments. In other words, we try to find the optimal balance between the present and the future in the P-system. We first describe the assumptions of the P-system, then we show how the concept of PEFF can work within the P-system.

Compared with the U-system, we will make simpler assumptions in the P-system that are different from the U-system, even though we now have full knowledge about the underlying U-system. This is to avoid the *God mode* when you can make perfect decisions based on all the information from a clairvoyant God. Such a discrepancy also leads to sub-optimal solutions; however, this reflects exactly the philosophy behind the Mohopeff model, that we should always make the optimal decision based on our current incomplete information set. We shall see later how well the Mohopeff can perform under such a situation.

Consider the P-system at time τ with horizon N. This means that we consider the projected pension system from year τ to $\tau + N$. The current buffer size is F_τ. It ends at time $\tau + N$ with the undistributed capital in the buffer, $F_{\tau;\tau+N}$, for further liabilities. Denote $B_{\tau;s,t}$ as the total benefit generation G_s will get at time t. Here s is the time when the generation entered the system and t is the time when the generation gets the benefit, $t = \tau + 1, \cdots, \tau + N$. s will range from $\tau + (t - n)$ to $\tau + (t - 1)$. The τ and the semi coma indicates that they are local variables within the P-system at time τ. Define

$$B_{\tau;t} := \sum_{s=\tau+(t-n)}^{\tau+(t-1)} B_{\tau;s,t}$$

as the aggregate benefit paid out from the system at time point t. The τ in the subscript suggests that it is calculated at time τ.

	t=0 to 1	1 to 2	...	N-2 to N-1	N-1 to N
G(-(n-1))	B				
G(-(n-2))	B	B			
...	...	...	...		
G(-N)	B	B	...	B	
...	...	...	...	...	...
G(-1)	B	B	...	B	B
G(0)	C B	B	...	B	B
G(1)		C B	...	B	B
...			...	...	...
G(N-2)				C B	B
G(N-1)					C B

Figure 1: Projected Pension Arrangement at Time 0

We also need the predictions on cash inflows over the horizon, i.e. the aggregate contributions into the system by future generations. Under the assumption of fixed contribution rate, the stream of contributions $\tilde{C}_{\tau;\tau+1}, \cdots, \tilde{C}_{\tau;\tau+(N-1)}$ will be determined jointly by the projected demography, predicted inflation and rate of return on the investments of the corresponding generations. Note that again the tilde indicates the nature of prediction. Please note that $\tilde{C}_{\tau;\tau}$ is C_τ which has already realized at time τ.

$X_{\tau;t}$ denotes the stochastic annual gross per-unit-of-money return from the fixed asset mix investment from year $t-1$ to t. They are seen as random variables. It is assumed that the risk stream $\{X_{\tau;t} | t = \tau+1, \cdots, \tau+N\}$ is sequentially independent, i.e. $X_{\tau;t}$ and $X_{\tau;s}$ are independent for $t \neq s$. We will have to give an estimated probability distribution to each $X_{\tau;t}$, both under the given objective probability measure $\mathbb{P}$ and risk-neutral measure $\mathbb{Q}$. It is also possible to introduce in the P-system dynamics for stochastic interest rates. However, for the moment we will assume the absence of stochastic interest rates in order to keep the P-system simple.

The budget constraints within the horizon is rather straightforward. At the end of each period, it has to be determined how to distribute the investment proceeds between the current aggregate benefit and the buffer, i.e.

$$B_{\tau;t} + F_{\tau;t} = (\tilde{C}_{\tau;t-1} + F_{\tau;t-1})X_{\tau;t} \quad \forall\ t = \tau+1, \cdots, \tau+N.$$

In this setting, the investment proceed $(\tilde{C}_{\tau;t-1} + F_{\tau;t-1})X_{\tau;t}$ is the total asset available to all the beneficiaries at time point t before $B_{\tau;t}$ is paid out. This will be denoted as $A_{\tau;\tau+t}$.

There can be many ways of splitting $A_{\tau;\tau+t}$ into $B_{\tau;\tau+t}$ and $F_{\tau;\tau+t}$. The following part of this paper is dedicated to explore the risk sharing rules under the criteria of PEFF.

To talk about risk sharing we employ utility functions to represent the risk preferences. Each aggregate benefit shall be regarded as a separate agent in this setting. It is assumed that at time point t, the current pensioners as a whole will use a utility function $u_{\tau;t}$ to evaluate the aggregate benefit $B_{\tau;t}$. Such an assumption is specifically tailored for defining Pareto efficiency in this multi-period setting where inter-temporal risk sharing is involved. We will also have to give a special utility function $u_{\tau;p}$ to the final buffer $F_{\tau;\tau+N}$ so that any Ponzi-scheme solution will be prevented: the volatility cannot be shifted indefinitely into the future.

We will work with the so-called *stereotype utility function* $u(\cdot)$ which is defined as follows:

1. it is continuous and differentiable;
2. it is strictly concave;
3. the marginal utility satisfies the *Inada conditions*

$$\lim_{x \downarrow lb} u'(x) = +\infty, \qquad \lim_{x \to \infty} u'(x) = 0$$

   where lb is the lower bound of the domain of u which is not attainable, $lb \in [-\infty, +\infty)$.

The utility functions here characterize the risk preferences of all the pensioners as a whole with regard to the aggregate benefits.

3.2 Pareto Efficiency and Financial Fairness in a Multi-Period Setting

Consider the P-system with horizon length N. A risk-sharing rule $(B_{\tau;\tau+1}, B_{\tau;\tau+2}, \cdots, B_{\tau;\tau+N}, F_{\tau;\tau+N})$ is called *Pareto efficient*, or *Pareto optimal*, if there does not exist another risk-sharing rule $(B'_{\tau;\tau+1}, B'_{\tau;\tau+2}, \cdots, B'_{\tau;\tau+N}, F'_{\tau;\tau+N})$ such that

$$\Big(\mathbb{E}^{\mathbb{P}}_{\tau} u_{\tau;\tau+1}(B'_{\tau;\tau+1}), \cdots, \mathbb{E}^{\mathbb{P}}_{\tau} u_{\tau;\tau+N}(B'_{\tau;\tau+N}), \mathbb{E}^{\mathbb{P}}_{\tau} u_{\tau;p}(F'_{\tau;\tau+N})\Big) \gneqq \Big(\mathbb{E}^{\mathbb{P}}_{\tau} u_{\tau;\tau+1}(B_{\tau;\tau+1}), \cdots, \mathbb{E}^{\mathbb{P}}_{\tau} u_{\tau;\tau+N}(B_{\tau;\tau+N}), \mathbb{E}^{\mathbb{P}}_{\tau} u_{\tau;p}(F_{\tau;\tau+N})\Big)$$

for the given utility functions $u_{\tau;\tau+1}, \cdots, u_{\tau;p}$, where

$$\mathbb{E}^{\mathbb{P}}_{\tau} = \mathbb{E}^{\mathbb{P}}[\,\cdot\,|\mathcal{F}_{\tau}].$$

Note that all the expectations here are taken at time τ, i.e. the beginning of the P-system. Thus the PE defined here is in an *ex ante* sense w.r.t. the projected cash outflows (including the end buffer). The utility functions will specify the risk preferences among the cash outflows, thus have an impact on the risk redistribution. There are many ways to specify the risk aversion levels. We will give 3 ways as examples which we will call the *utility knob* implying that it is a kind of policy instrument and the system designer can tune the model by making use of it.

1. front-protection: the aggregate benefits are assumed to be more risk averse than the end buffer, thus the end buffer will absorb more volatility.

2. back-protection: the aggregate benefits are assumed to be less risk averse than the end buffer, thus the end buffer will absorb less volatility.
3. balanced-protection: assume equal risk aversion levels to both the aggregate benefits and the end buffer.

In each P-system, the notion of financial fairness is linked to the concept of *value profile* which is the vector of ex-ante undiscounted market values of the aggregate benefits

$$V_\tau = \left(\mathbb{E}_\tau^\mathbb{Q} B_{\tau;\tau+1}, \mathbb{E}_\tau^\mathbb{Q} B_{\tau;\tau+2}, \cdots, \mathbb{E}_\tau^\mathbb{Q} B_{\tau;\tau+N}, \mathbb{E}_\tau^\mathbb{Q} F_{\tau;\tau+N}\right).$$

Financial fairness requires that the market values of the cash outflows shall match some given value profile vector. In other words, the risk redistribution shall not change the market values of the cash outflows.

The main result in [2] tells that in a P-system, the PEFF risk sharing rule will always exist and is unique. The PEFF risk sharing rules are increasing functions $\{f_{\tau;t}\}$, which tells how to distribute the total assets between the current benefit and the buffer at the end of each period. The benefits are calculated as, for $t = \tau+1, \cdots, \tau+N$

$$\begin{aligned} B_{\tau;t} &= f_{\tau;t}(A_{\tau;t}), \\ F_{\tau;t} &= A_{\tau;t} - B_{\tau;t}. \end{aligned}$$

In most cases, the PEFF will not lead to analytical solutions. An iterative numerical algorithm is proposed in [2] to compute the risk sharing rule numerically.

To apply the PEFF algorithm the needed direct inputs are

- The deterministic stream of projected lump-sum contributions;
- The utility functions for evaluating the (aggregate) benefits and the final buffer;
- The distributions of all the investment risks which are to be shared, both under $\mathbb{P}$ and under $\mathbb{Q}$;
- The value profile.

3.3 Value Adjustment

To apply the PEFF algorithm in the P-system one needs to reasonably determine the value profile. This section introduces a way to determine the value profile. The main idea is that the (undiscounted) market value of each cash outflow shall be anchored to their benchmark ambition level, and is adjusted as if the funding status can only recover to some certain level within the horizon of the P-system.

First note that we have a global budget constraint for the value profile, that is, the present market values of cash inflows and outflows should be equal:

$$\sum_{s=1}^{N} \frac{\mathbb{E}_\tau^\mathbb{Q} B_{\tau;\tau+s}}{R_{\tau;\tau,\tau+s}} + \frac{\mathbb{E}_\tau^\mathbb{Q} F_{\tau;\tau+N}}{R_{\tau;\tau,\tau+N}} = F_\tau + C_\tau + \sum_{s=1}^{N-1} \frac{\tilde{C}_{\tau;\tau+s}}{R_{\tau;\tau,\tau+s}}. \tag{3.1}$$

To determine the value profile we need to define first the *benchmark ambition profile* of the aggregate benefits. For a P-system at time τ, any generation G_t with $t < \tau$ has already been assigned its BAA b_t, which is the actuarially fair nominal annuity level. The conversion is done based on the interest rates at the time when G_t entered the system. The conversion 2.1 can also be generalized for all generations G_t's with $t > \tau$ within the same P-system. However, as those generations have not yet stepped into the system, predictions on contributions and demography information will be used. The current term structure will be used for discounting. That is, define the corresponding BAA $\tilde{b}_{\tau;t}$ by

$$\tilde{C}_{\tau;t} = \tilde{b}_{\tau;t} \sum_{s=1}^{n} \left(\frac{1}{R_{\tau;t,t+s}} \frac{\tilde{P}_{\tau;t,t+s}}{\tilde{P}_{\tau;t,t}} \right) \tag{3.2}$$

The $\tilde{b}_{\tau;t}$'s calculated in such a way are the best prediction on the future annuity ambitions based on the current information set.

Recall that for $t = \tau + 1, \cdots, \tau + N$

$$B_{\tau;t} := \sum_{s=t-n}^{t-1} B_{\tau;s,t} = \sum_{s=t-n}^{\tau} B_{\tau;s,t} + \sum_{s=\tau+1}^{t-1} B_{\tau;s,t}.$$

The benchmark ambition for $B_{\tau;t}$ is defined by summing up the corresponding BAAs:

$$v_b(B_{\tau;t}) := \sum_{s=t-n}^{\tau} b_s + \sum_{s=\tau+1}^{t-1} \tilde{b}_{\tau;s}.$$

The benchmark liability $L_{\tau;t}$ can also be calculated as the present value of all the unpaid actual and predicted BAAs, discounted at the risk-free rates suggested by the term structure at time τ. It indicates the projected total liability right after the benefit $B_{\tau;t}$ is paid, but before new contributions $\tilde{C}_{\tau;t}$ comes into the system.

There are several ways to link the benchmark ambition to the actual value profile. The naive way is to let

$$\mathbb{E}_{\tau}^{\mathbb{Q}} B_{\tau;t} = v_b(B_{\tau;t}) \quad \forall t = \tau + 1, \cdots, \tau + N.$$

This means that seen at the beginning of the P-system, the ex-ante market values of the aggregate benefits equal the actuarially fair level. However, this will also mean that the buffer is left unprotected. The pensioners will not help resolving the problem if the funding status is too high or too low, as the market values of the pension payments will not be adjusted.

Another way is to give full protection to the funding ratio. First let

$$\mathbb{E}_{\tau}^{\mathbb{Q}} F_{\tau;\tau+N} = \kappa L_{\tau;\tau+N}, \tag{3.3}$$

that is, seen from now, the market value of the buffer $F_{\tau;\tau+N}$ will meet the solvency requirement for the future projected liabilities. The κ is some target funding ratio level, e.g. 100%

or 110%. If we also make $\mathbb{E}_\tau^{\mathbb{Q}} B_{\tau;t} = v_b(B_{\tau;t})$ then the global budget constraint 3.1 may not hold, as it will imply a value F'_τ that may be different from the starting buffer F_τ. To resolve this problem we will spread any difference between F'_τ and F_τ proportionally among all the $\mathbb{E}_\tau^{\mathbb{Q}} B_{\tau;t}$'s, i.e. we will look for an adjustment ratio δ_τ s.t.

$$\mathbb{E}_\tau^{\mathbb{Q}} B_{\tau;t} = \delta_\tau v_b(B_{\tau;t}) \quad \forall t = \tau+1, \cdots, \tau+N.$$

This δ will be uniquely determined by

$$\delta_\tau \left(\sum_{s=1}^{N} \frac{v_b(B_{\tau;\tau+s})}{R_{\tau;\tau,\tau+s}} \right) + \frac{\mathbb{E}_\tau^{\mathbb{Q}} F_{\tau;\tau+N}}{R_{\tau;\tau,\tau+N}} = F_\tau + \sum_{s=0}^{N-1} \frac{\tilde{C}_{\tau;\tau+s}}{R_{\tau;\tau,\tau+s}}. \tag{3.4}$$

where, as we have specified

$$\mathbb{E}_\tau^{\mathbb{Q}} F_{\tau;\tau+N} = \kappa L_{\tau;\tau+N}.$$

The philosophy behind this is that if the starting buffer size is too high or too low, the surplus or deficit will be smoothed only within the N-year horizon in the form of market value. The N-year horizon is thus seen as a recovery period and we call this adjustment method *full recovery.*

The words “financial fairness” in this context then have several meanings. For each generation, the financial fairness is achieved by that the variable annuity is benchmarked to the actuarially fair level. For each annuity payment, financial fairness means that the risk redistribution will not change the market values of the annuity payments which equal the value profile. The adjustment ratio δ_τ is jointly determined by the current funding status and the projected cash flows, and it measures the deviation of the actual market value of the pension payment from its BAA level.

To conclude, we have proposed 2 ways of adjusting the benchmark ambition in order to get the value profile, which we call the *value knob* in accordance with the utility knob introduced above.

1. no recovery: set $\delta_\tau = 1$ regardless of the current funding status.
2. full recovery: fully aim at the target post-payment BFR κ in the projected pension system, at the cost of adjusting the market values of the current benefits. That is, determine δ_τ by equations 3.4 and 3.3.

3.4 Mohopeff: a Dynamic Strategy

If we are in a stylized theoretical economic environment where the interest rates, inflation and demography all remain constant over time and we have perfect information over the distributions of the investment risks, then the multi-period PEFF algorithm allows us to calculate the risk sharing rule at time 0 and the rule will still remain optimal from an ex-ante point of view at any future time points. However, if one considers reality where the economic

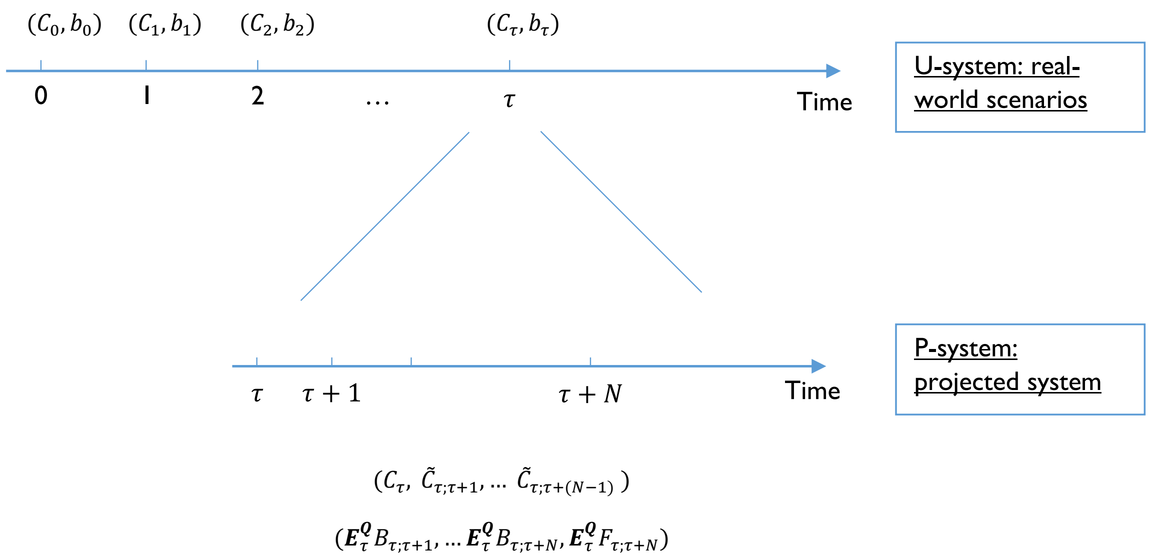

Figure 2: Mohopeff: U-system and P-system

environment will always change and our predictions cannot be fully accurate, then any risk sharing rules calculated at time 0 will not be optimal any more as time goes on, since the rules are calculated based on past information and expectations.

To allow updating the PEFF approach is developed to be on a moving horizon basis, thus the name *Mohopeff*. In the Mohopeff approach, the P-system will be established every year to utilize the most updated information. Regarding the P-system at time τ, only $f_{\tau;\tau+1}$ will be actually implemented; for $t > \tau+1$, the $f_{\tau,t}$'s are only auxiliary and will not be executed. When we move on to the next year $\tau+1$, another P-system will be established. Thus we can say that Mohopeff gives a situation-dependent risk sharing strategy.

The idea of funding status recovery over a rolling recovery period is among the most interesting topics under the Dutch pension reform. The Mohopeff approach also adopts similar approach in terms of market value. In the Mohopeff approach, the adjustment ratio δ_τ is interpreted as the implied proper level of adjustment if the funding status can only be recovered to the desired level within the next N years.

3.5 Illustrations from Approximated Explicit Solution

In most situations we have to rely on a numerical algorithm to get the risk sharing rules. However, an approximated explicit solution exists when we assume exponential utility functions to all the aggregate benefits

$$u_{\tau;t}(x) = 1 - e^{-\alpha_{\tau;t}x}$$

as well as the final buffer

$$u_{\tau;p}(x) = 1 - e^{-\alpha_{\tau;p}x}.$$

In the P-system at time τ, the explicit solution gives, in line with Theorem 7.1 in [2]

$$B_{\tau;t} = \mathbb{E}^{\mathbb{Q}}_{\tau} B_{\tau;t} + \beta_{\tau;t}(\tilde{C}_{\tau;t-1} + F_{\tau;t-1})(X_{\tau;t} - R_{\tau;t}) \quad \forall t = \tau+1, \cdots, \tau+N \tag{3.5}$$

where the coefficients $\beta_{\tau;t}$'s are determined recursively by

$$\beta_{\tau;\tau+N} = \frac{\alpha_{\tau;p}}{\alpha_{\tau;\tau+N} + \alpha_{\tau;p}},$$
$$\beta_{\tau;t} = \frac{\beta_{\tau;t+1}\mu_{\tau;t+1}\alpha_{\tau;t+1}}{\alpha_{\tau;t} + \beta_{\tau;t+1}\mu_{\tau;t+1}\alpha_{\tau;t+1}} \quad \forall t = \tau+1, \cdots, \tau+(N-1).$$

Here $R_{\tau;t}$ stands for the risk-free 1-year gross forward rate over time $t-1$ to t, and $\mu_{\tau;t}$ is the expected gross return from the fixed asset over time $t-1$ to t.

Note that under the Mohopeff approach, only the formula for $B_{\tau;\tau+1}$ will be executed. This explicit solution tells that under exponential utility assumption, the aggregate benefit payment consists of 2 parts:

1. the first part $\mathbb{E}^{\mathbb{Q}}_{\tau} B_{\tau;\tau+1}$, which is the benchmark ambition adjusted to the initial funding status and the projected liabilities. If the current benchmark funding ratio is high, then the benchmark ambition will be adjusted upwards and pensioners will on average get better paid than when the funding status is worse, and vice versa.
2. the second part $\beta_{\tau;\tau+1}(C_\tau + F_\tau)(X_{\tau;\tau+1} - R_{\tau;\tau+1})$, which is a proportion of the excess return of the total investment over the benchmark risk-free rate. The slope is determined by the expected return in the coming years: a more aggressive estimation will lead to a larger proportion and vice versa. Furthermore, if the risk aversion assigned to the buffer increases, the proportion also becomes larger.

We may compare this linear solution with one which says that any surplus or deficit in the funding ratio compared to the target ratio should be smoothed within a N-year horizon by adjusting the indexation. That is, the indexation is a linear function of the pre-payment BFR in this context[1]:

$$B_\tau = v_b(B_\tau)\left(1 + \frac{\pi_\tau - \pi^*}{N}\right).$$

We shall call this method the *smoothing via indexation* (SvI).

The solution in this section suggests

$$B_\tau = v_b(B_\tau)\left[\delta_{\tau-1} + \beta_{\tau-1;\tau}\frac{L_\tau + v_b(B_\tau)}{v_b(B_\tau)}\left(1 - \frac{R_{\tau-1;\tau-1,\tau}}{X_\tau}\right)\pi_\tau\right].$$

The differences are that the Mohopeff risk sharing rule involves more adjustments and are situation-dependent. The idea of N-year recovery period is reflected by the adjustment ratio $\delta_{\tau-1}$.

[1] Ideally the actual funding ratio shall be used instead of the benchmark funding ratio. However, we still choose to use the BFR in order to keep things comparable.

4 The Mohopeff Approach in Collective DC: an ALM Study

This section tests the Mohopeff model by means of an ALM study. Under the U-system, different economic scenarios can be simulated and the corresponding pension arrangements will be calculated by the Mohopeff approach. To be exact, along each simulated economic scenario, P-systems can be established and will work as processors. The output will be the Mohopeff risk sharing rules and thus the pension arrangement will be known.

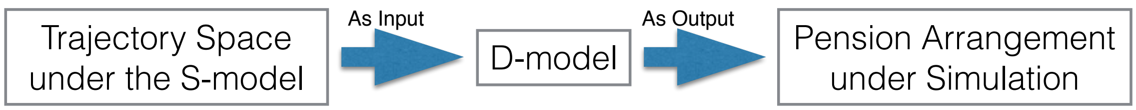

Figure 3: Framework of the ALM Study

4.1 Calibration

Regarding the U-system, we first assume that the population size of each generation at all time will always be the same and is normalized to 1, thus there is no need to simulate any demography scenarios. We can then totally focus on the financial risks that will be shared and the intergenerational risk sharing will not be further perplexed by the demography risks. Each generation will stay in the system for 20 years during their retirement. Also, for simplicity, it is assumed that the incoming aggregate contribution is deterministic and grows at the rate of realized inflation, thus the contribution flow can be totally normalized and represented by the price index.

For the scenario data we employ the financial market model proposed by Draper [6] from CPB the Netherlands bureau for economic policy analysis. We use the calibration that is consistent with the Dutch Committee Parameters 2014 (see e.g. Figure 4 in [6]). A number of scenarios will be generated, each consisting of 51 years. The Mohopeff model starts from the beginning of the 20th year; the first 19 years are used to generate the BAAs for the older generations as well as the benchmark liability L_0 when Mohopeff starts. The starting benchmark funding ratio F_0/L_0 is set to some κ_0 at the beginning.

At time point τ along each scenario, the model generates a nominal term structure in gross form $\{R_{\tau;\tau,\tau+s}|s = 0, 1, 5, 10, 30, 100\}$ where $R_{\tau;\tau,\tau+100}$ is the ultimate forward rate, term structure of future gross inflation $\tilde{\eta}_{\tau;\tau,\tau+s}$ implied by the difference between the nominal and real term structure, the actual gross inflation from last year η_τ and the realized return from the fixed asset S_τ during the last year. The fund is assumed to invest in a fixed asset mix consisting of positions in bonds and the stock index. Given the constant population size

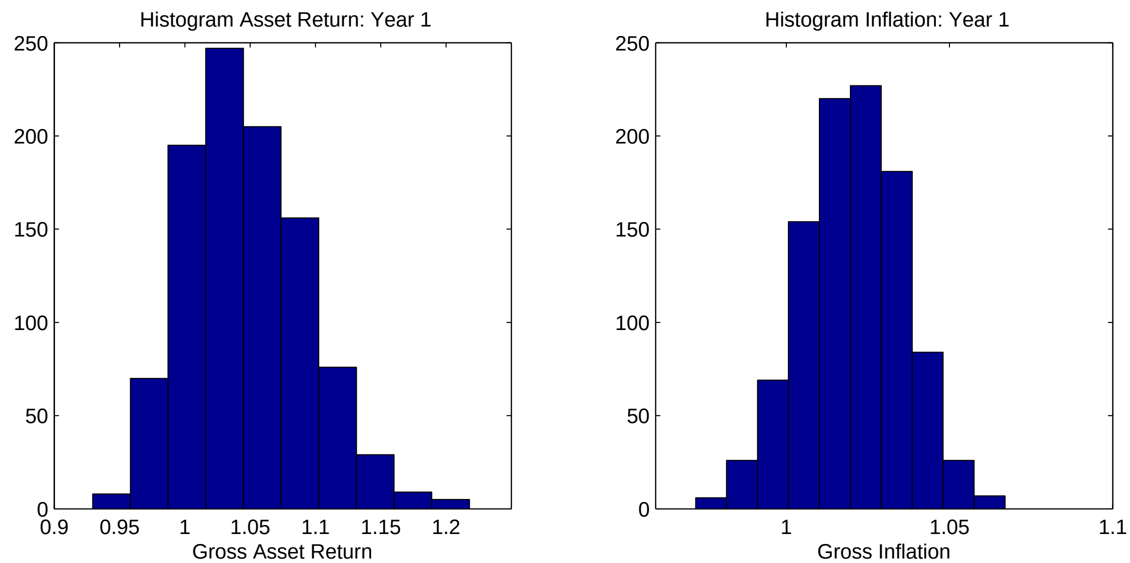

Figure 4: Gross Asset Return and Inflation

assumption, the annuity conversion factor becomes

$$\sum_{t=\tau+1}^{\tau+20} \frac{1}{R_{\tau;\tau,t}}.$$

The pension fund is assumed to have a fixed asset mix. The fund invests 80% of its capital in bonds and 20% in a stock index. The 80% investment in fixed income instruments is distributed equally in bonds with maturities 1, 5, 10 and 30 years. The positions in bonds will be rolled-over each year and there are no held-to-maturity positions.

Each P-system will be calibrated as follows. We consider a moving horizon of 10 years. The contributions grow at the rate of expected inflation, and the asset returns are assumed to be independent from year to year. Returns from stock market follow finitely discretized log-normal distributions with the same expected excess return 3% over the risk-free rates along the observed term structure and the same standard deviation 20%.

It is assumed that the pensioners will evaluate the aggregate benefit normalized by the value profile under a time-consistent utility function u. That is, using the notations as before,

$$u_{\tau;t}(x) = u\left(\frac{x}{\mathbb{E}_\tau^{\mathbb{Q}} B_{\tau;t}}\right).$$

Analogy holds for the final buffer size, i.e.

$$u_{\tau;p}(x) = u_p\left(\frac{x}{\mathbb{E}_\tau^{\mathbb{Q}} F_{\tau;\tau+N}}\right).$$

In such a way the utility function actually evaluates the percentage of change instead of the aggregate benefit itself.

4.2 Impact of Value Knob under CARA Utility

In this section we implement the Mohopeff approach using 1000 economic scenarios by the U-system. The starting post-BFR is set to 110%. We proceed first with the CARA utility, which means we implement the linear risk sharing rules given by equation 3.5. The P-systems always aim at

$$\mathbb{E}_{\tau}^{\mathbb{Q}} F_{\tau;\tau+N} = L_{\tau;\tau+N},$$

i.e. the market value of the projected end buffer equals the projected benchmark liability and a 10-year recovery period is adopted to smooth the risks.

4 ratios will be reported as result:

- post-payment benchmark funding ratio ("funding ratio" in short)
$$\kappa_{\tau} = \frac{F_{\tau}}{L_{\tau}};$$
- benefit ratio
$$\rho_{\tau} = \frac{B_{\tau}}{v_b(B_{\tau})};$$
- adjustment ratio δ_{τ};
- risk-sharing ratio, which is the slope of the risk sharing rule. Under the assumption of CARA utility, it is the $\beta_{\tau-1;\tau}$ at time τ.

Figure 5 shows the result when no recovery is implemented. One can see that the buffer becomes more divergent as time goes by. If the funding ratio gets too high or too low, the pensioners will not help pulling it back to the desired level. The risk-sharing ratio suggests that the current pensioners will generally take 4% to 7% of the excess return of the total investment. For the pensioners, the aggregate benefits can be as several times higher than the corresponding benchmark ambitions. This is due to two reasons. First, in the U-system, the financial asset generally grows much faster than the wage growth; see Figure 4. Second, the difference between the distributions of the asset returns under $\mathbb{P}$ and $\mathbb{Q}$ is large, and in the U-system the equity market usually outperforms the risk-free bonds to a large extent.

Figure 6 shows the result if the pension fund implements the full-recovery strategy. A striking fact is that the funding ratio stabilizes within some interval after several years. The pensioners now have to sacrifice if the funding status needs recovery. This can be seen from the adjustment ratio: the participants may need to accept up to 20% cut in market value of their variable annuity payments in bad situations. They may even face further cut in actual payments if the current financial market performs badly.

4.3 Comparison to Smoothing via Indexation: Recovery from Low Funding Status

This section discusses the situation if we start with a low funding status. Suppose now the starting post-BFR is 90%. Figure 7 shows that under the Mohopeff full-recovery strategy,

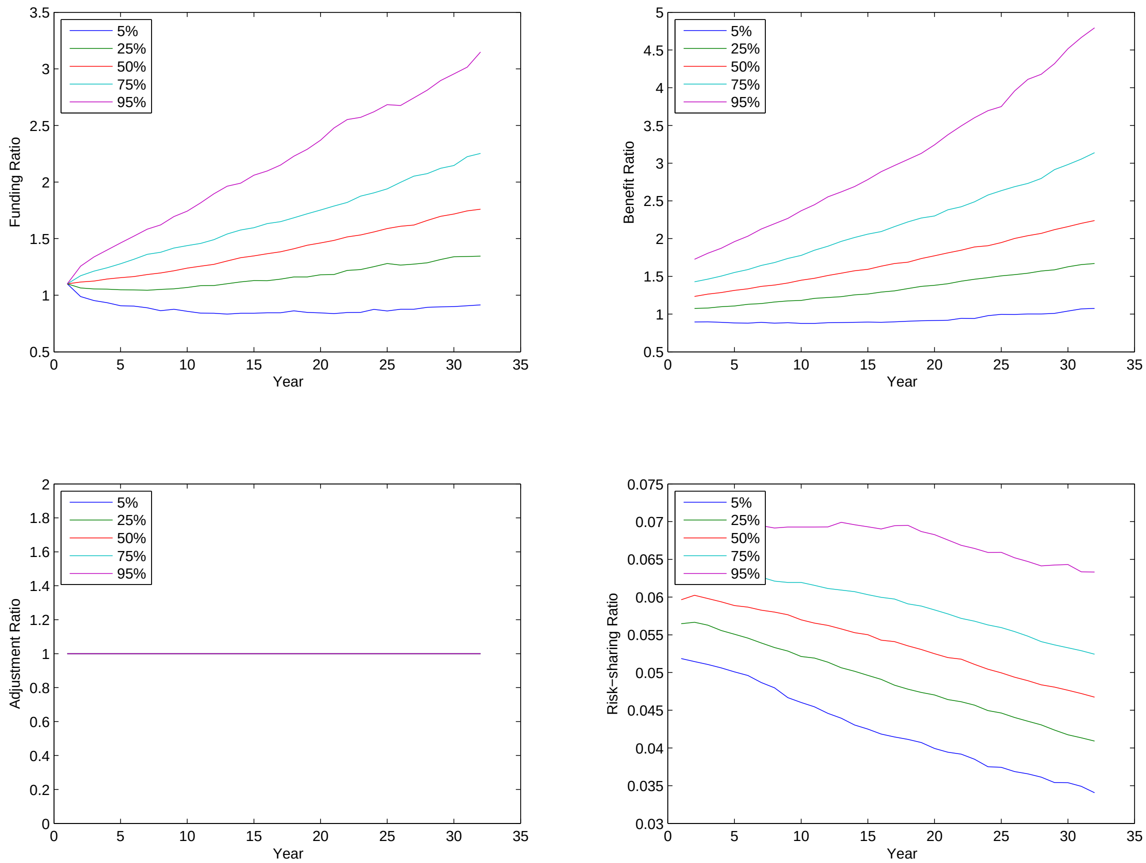

Figure 5: CARA Utility, No Recovery, 110% Starting Post-BFR

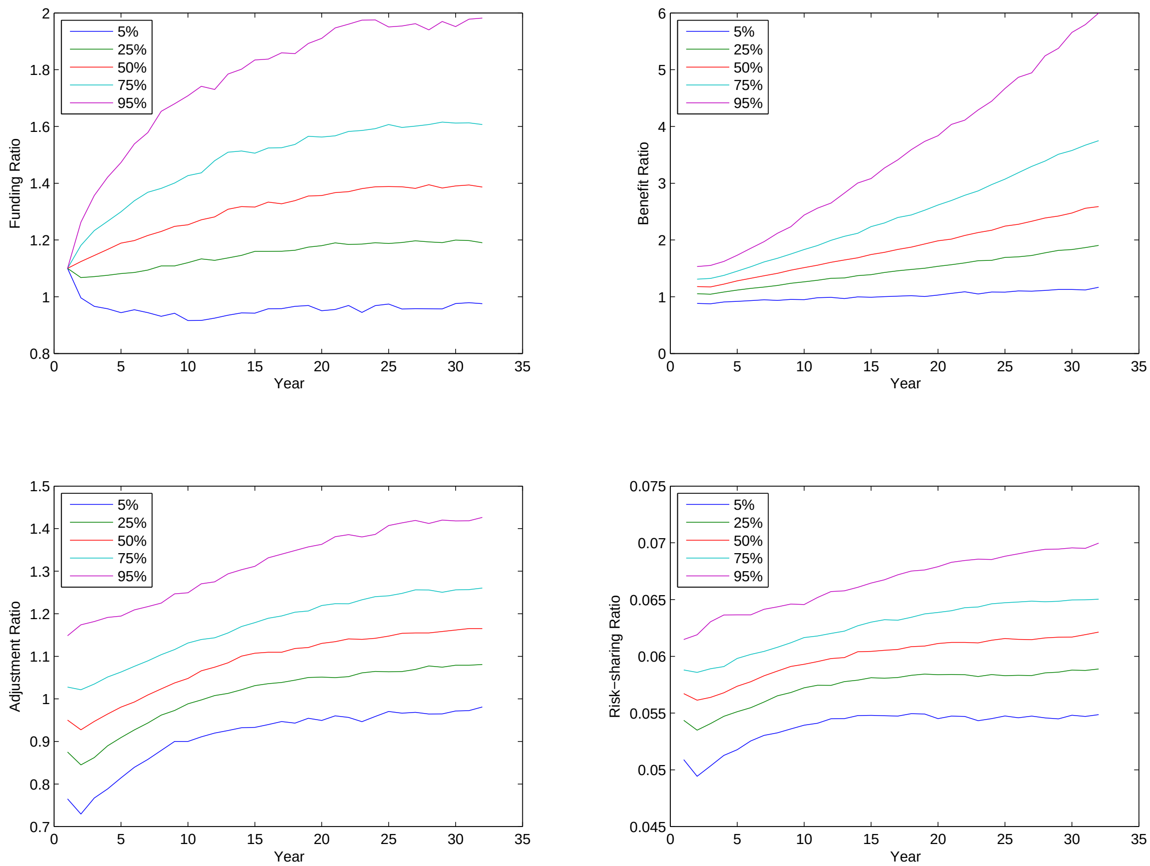

Figure 6: CARA Utility, Full Recovery, 110% Starting Post-BFR

the pension fund revives quickly to a stable distribution; however, the cost is then by the current pensioners. They may face up to 40% cut in the market value of their current pension payments, which may render the CDC system far less attractive to them.

One may be interested how the SvI risk sharing strategy will perform in the same situation. Figure 8 and 9 show the funding ratio and the benefit ratio under the SvI strategy: the strategy gives highly similar results even though the starting funding status is quite different; it doesn't take drastic measures to help recovering from low funding ratio situation. When the system is underfunded, the recovery process is less effective than under the Mohopeff full-recovery strategy.

4.4 Impact of Utility Knob under Full Recovery

The results above are presented under the CARA utility, which describes the risk preference as "constant absolute risk aversion". Although it offers an explicit solution which also has a clear interpretation how the risk sharing takes place, people may argue that the CARA utility doesn't give a good picture of the risk preference of the pensioners. In this section we

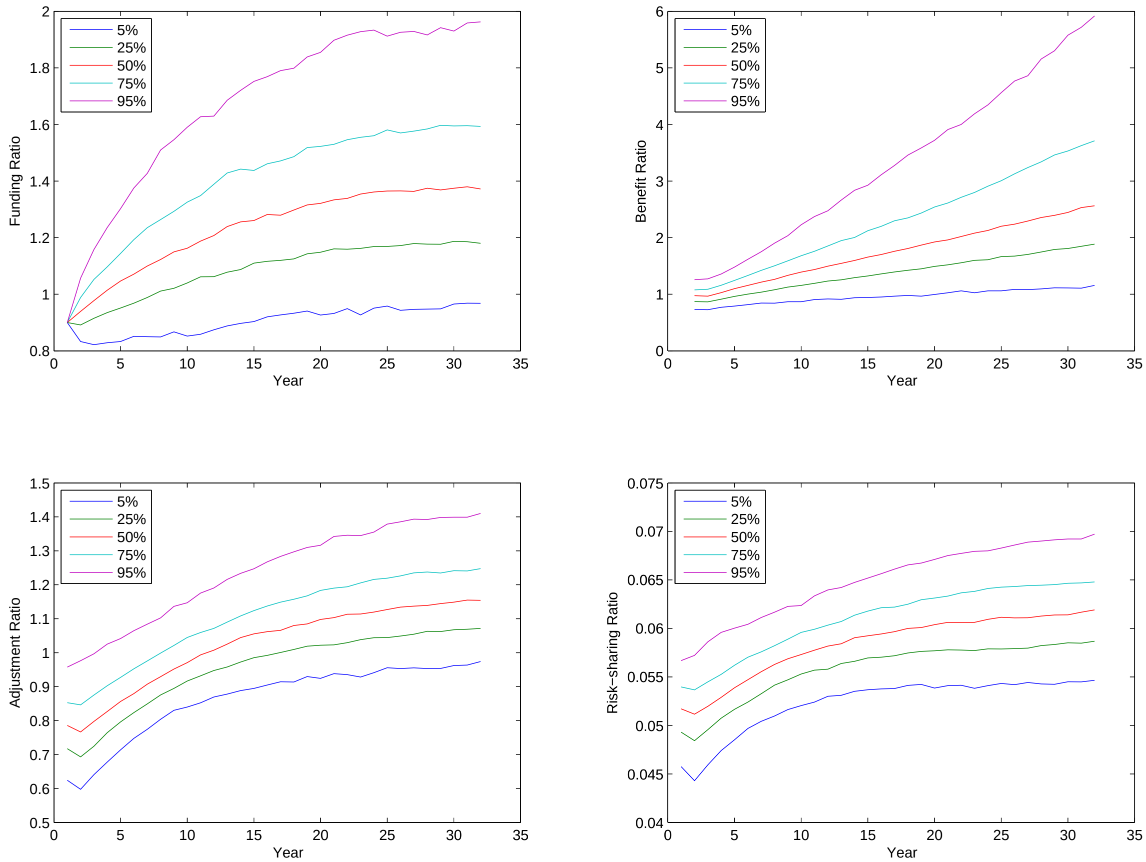

Figure 7: CARA Utility, Full Recovery, 90% Starting Post-BFR

will see the impact of utility knob on the pension arrangements using other kinds of utility functions.

We first work with balanced protection, i.e. we set the same utility function for both the aggregate benefits and the end buffer in each P-system. The starting post-BFR is set to be 110%. It is interesting whether different utility functions in this situation will produce different risk sharing rules.

As an example, Figure 10 shows the risk sharing rule of 3 kinds of utility functions at the first time point along the first scenario: power utility with $\gamma = 3$, power utility with $\gamma = 5$, and CARA utility. The risk sharing rules show how much we should pay out as the next year's aggregate benefit as a function of the pre-payment BFR at that time. Two important things can be read from the figure. First, all sorts of utilities produce risk sharing rules that look like linear; second, the three lines almost coincide with each other and the figure on the right tells that the difference is almost negligible.

Figure 11 gives the quantiles of the four ratios of our interest when one uses power utility with $\gamma = 3$, under the situation when the starting post-BFR is 110% and full-recovery is adopted. Compared to Figure 6 where CARA utility is used, they both produce highly

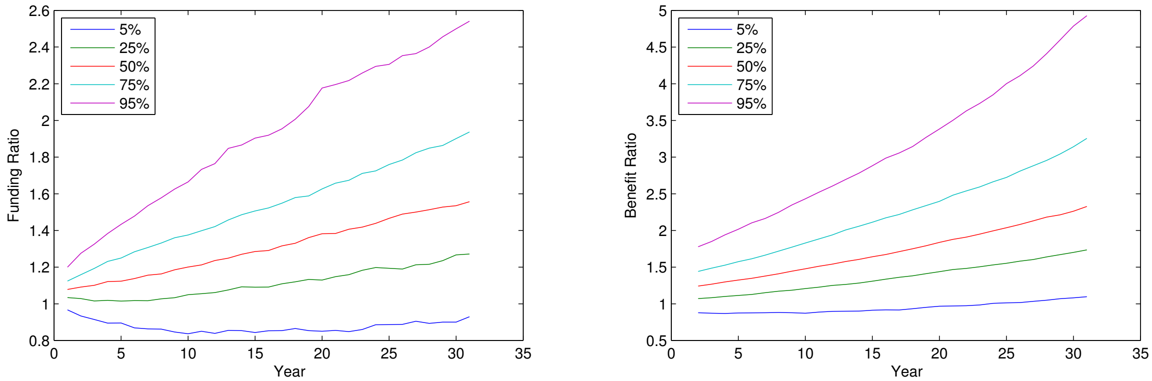

Figure 8: SvI Indexation, 110% Starting Post-BFR

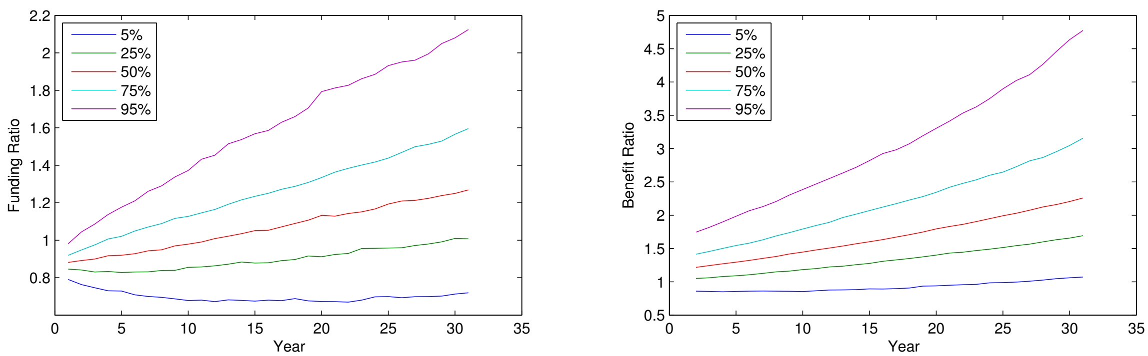

Figure 9: SvI Indexation, 90% Starting Post-BFR

similar FR and BR distributions. The only remarkable difference is that power utility seems to encourage a higher risk sharing ratio, which means that the current pensioners will generally take 1% more out of the excess return from the total investment of the fund.

Besides the common CARA and power utility, other kinds of utility functions can also be experimented. We propose to use *kinked utility function* which assumes that risk aversion rises to another level when the benefit is low. Figure 12 shows an example in the form of marginal utility. The x-axis stands for the aggregate benefits normalized to its market value. At the level of 0.9 there is a kink; below and above this level we assume power utilities with $\gamma = 5$ and 3 respectively. The motivation behind is that people become more risk averse when they face probable benefit cuts. Figure 13 shows the risk sharing rule of 3 kinds of utility functions at the first time point along the first scenario: power utility with $\gamma = 3$, the kinked utility as is mentioned above, and the CARA utility. Interestingly, the kinked utility produces a "valley" when the pre-BFR approaches the critical point. The slope is flatter below and steeper above compared to the CARA benchmark.

We then proceed to the cases of front- or back-protection. This entails specifying one kind of utility for the aggregate benefits and another kind for the end buffer. Figure 14 shows the

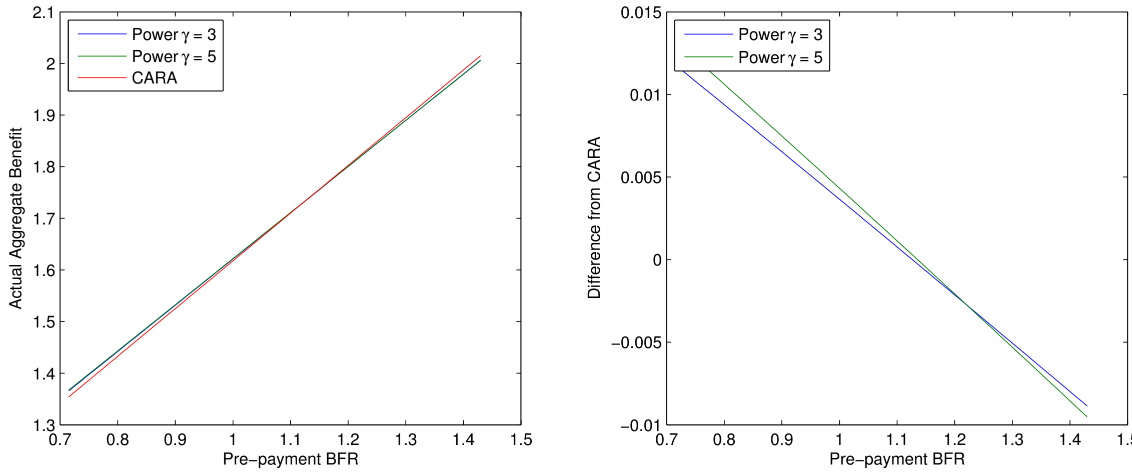

Figure 10: Comparison of Risk-Sharing Rules under Power Utility with Different Risk Aversion

resulted risk sharing rules under

1. Front-protection: power utility with $\gamma = 5$ for the aggregate benefits and power utility with $\gamma = 3$ for the end buffer.
2. Front-protection: power utility with $\gamma = 3$ for the aggregate benefits and power utility with $\gamma = 5$ for the end buffer.
3. Balanced-protection: power utility with $\gamma = 3$ for both the aggregate benefits and the end buffer.

The results fall within our expectation. Front protection leads a flatter line while back protection leads a steeper one, for the aggregate benefits. Compared to the balanced protection, the current pensioners get protected when the BFR is low in the case of front protection, and in return they get less when the BFR is high.

It makes some difference when kinked utility is involved. We consider using both the power utility and the kinked utility in different utility policies:

1. Front-protection: kinked utility with $\gamma = 5, 3$ for the aggregate benefits, and power utility with $\gamma = 3$ for the end buffer.
2. Back-protection: power utility with $\gamma = 3$ for the aggregate benefits and kinked utility with $\gamma = 5, 3$ for the end buffer.
3. Balanced-protection: power utility with $\gamma = 3$ for both the aggregate benefits and the end buffer.

The lines in Figure 15 vary from Figure 14 where only power utilities are employed. In the case when the current aggregate benefits are assigned the kinked utility, the current

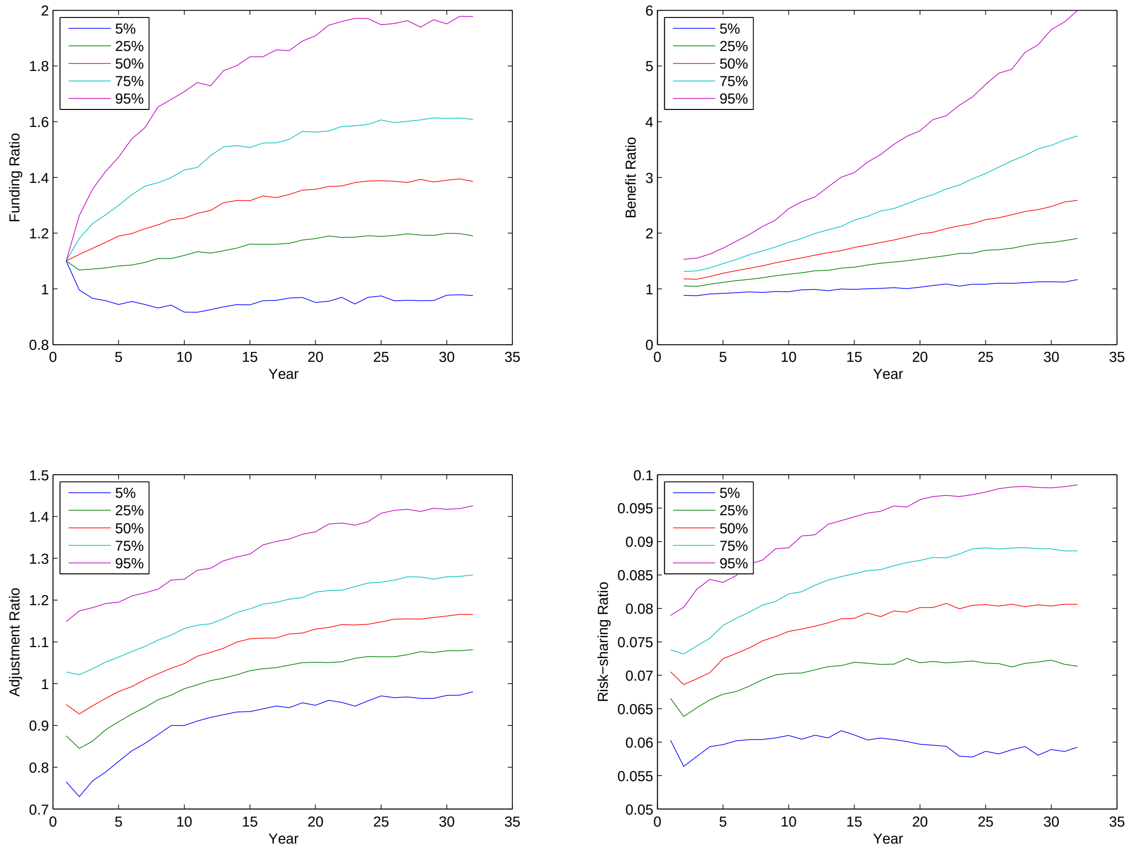

Figure 11: Power Utility, Full Recovery, 110% Starting Post-BFR

pensioners are protected when the funding ratio decreases to a dangerous level. Conversely, in the back-protection case, the current pensioners need to sacrefice in order to help the low funding ratio recover.

4.5 Impact of the Horizon Length

The length of the horizon is also a parameter which can be tuned, though in the results above we have made it as default to set the horizon to be 10 years. Figure 16 shows the quantiles of the benchmark funding ratio under a 5-year, 10-year and 15-year horizon. As can be expected, using a shorter horizon for the P-system, the BFR will be pulled back faster if it gets too high or too low, and as a result the BFR stabilizes faster than when the horizon length is longer. If we start with a lower BFR i.e. 0.9, the recovery comes faster when a shorter horizon is adopted.

However, as the other side of the same coin, a shorter horizon also means less space for inter-temporal risk sharing, because the current pension payment has to share a larger proportion of any surplus or deficit in the funding status.

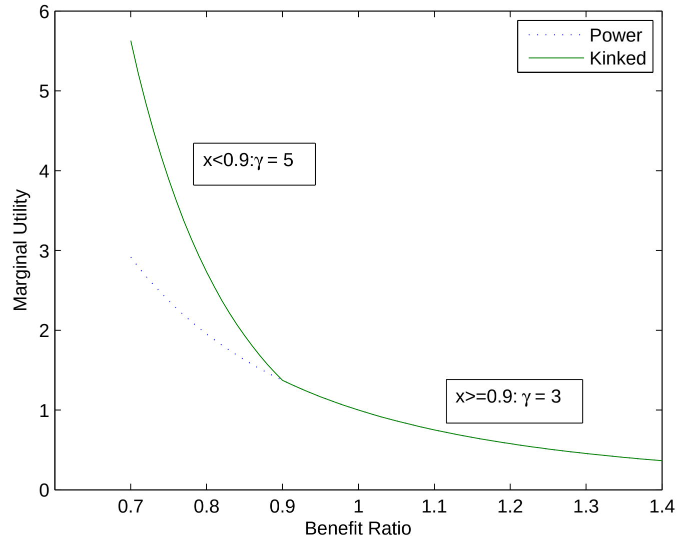

Figure 12: Construction of Kinked Utility Function

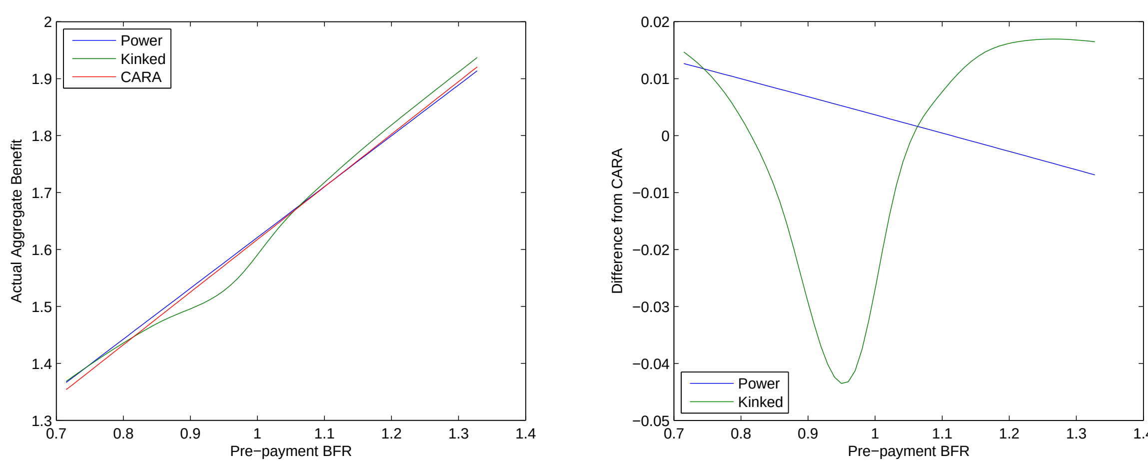

Figure 13: Comparison of Risk-Sharing Rules under Different Utility Functions

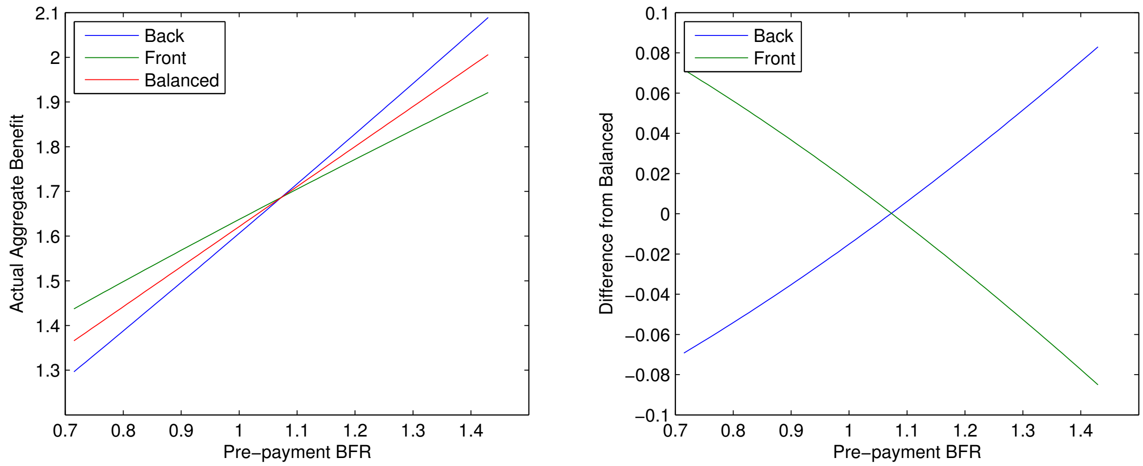

Figure 14: Comparison of Risk-Sharing Rules under Different Utility Policies

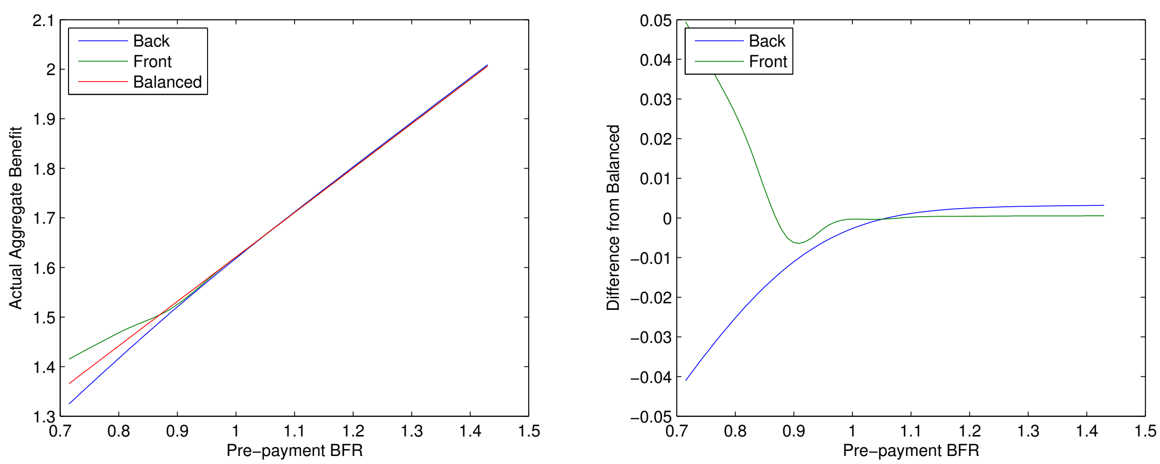

Figure 15: Comparison of Risk-Sharing Rules under Different Utility Policies, with Kinked Utility.

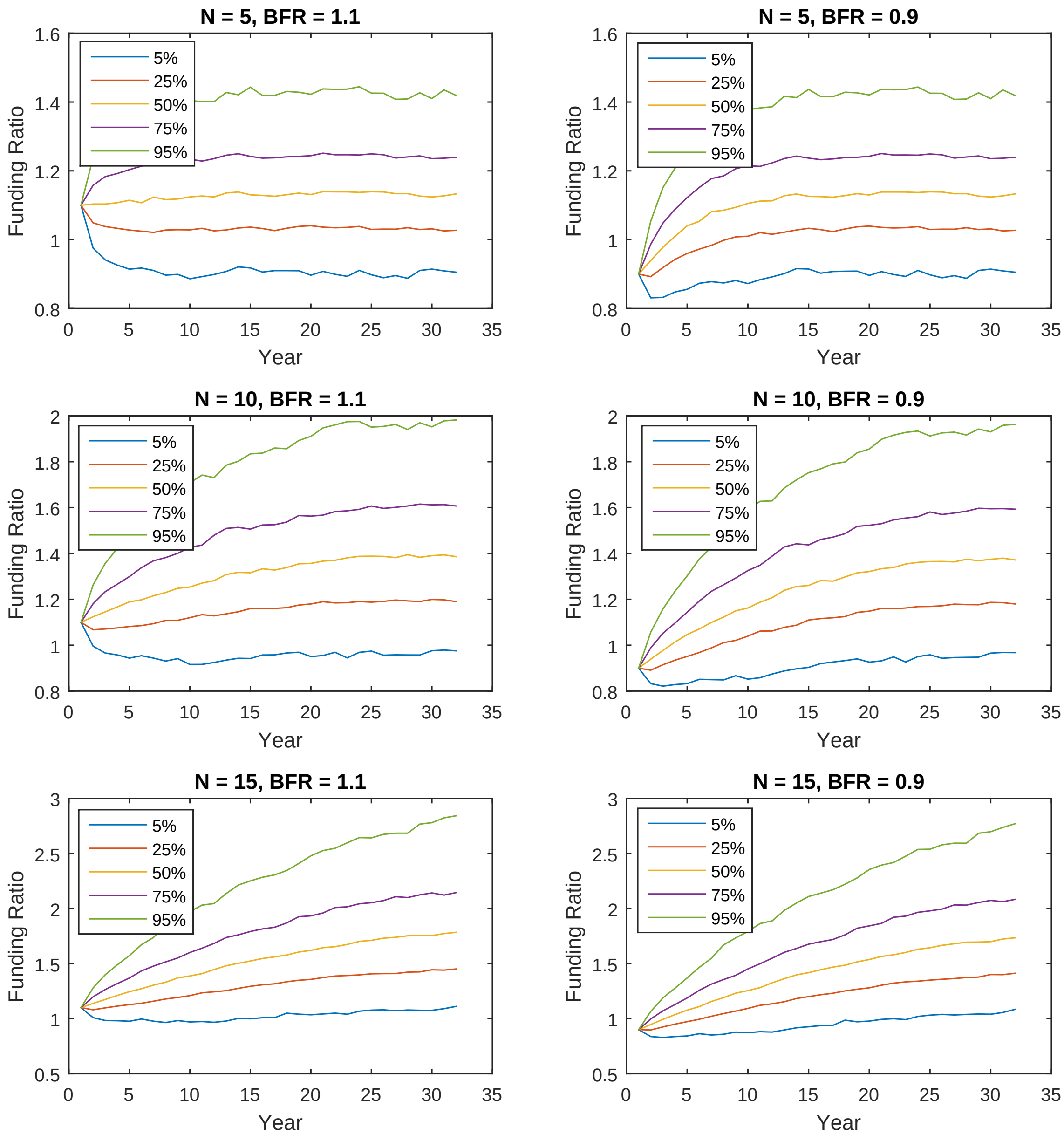

Figure 16: Distribution of Benchmark Funding Ratio with Different Choices of Horizon Length

5 Tolerance Band and Sustainability

The discussion above has implicitly assumed compulsory participation and has taken into no consideration the *discontinuity problem*, that is, if the funding ratio of the fund is too low which leads to serious benefit cut, the prospective cohorts may find the scheme unattractive and they may stay out of the system by looking for alternatives like individual annuity product from insurance companies. It is then natural to ask which level of the buffer shall be called "too low". Symmetrically, when the funding ratio is too high, the existing cohorts may want to terminate the system immediately and distribute all the wealth accumulated in the buffer. In this section we propose a way to find this break-up point based on a utility analysis.

Individuals participate in the system to share risks within the collective and this results in welfare improvement for them compared to the stand-alone situation. Thus the individual is still willing to accept benefit cuts to some certain extent as long as he still foresees welfare improvement in the coming years. Conversely, he is also willing to stay in the system instead of terminating the fund when the funding status is high, as long as he still expects to be better off in the system. This idea motivates the concept of the *tolerance band* of the buffer: as long as the buffer size stays within a certain range, both the existing and the prospective cohorts will still like to participate and the pension system will not break up.

The tolerance band is calculated as follows. It is assumed that we work under a *stationarity condition*, that is, given the same ex-interim BFR level, the tolerance band computed at the current time point should also hold for later time points, thus we only have to consider the first time point. The downside of the tolerance band entails considering the cohort who needs to decide whether to step into this Mohopeff pension system at time 0 by comparing the total utility between taking part in the system and purchasing an alternative annuity product from the open market. One possibility to describe the total utility of this cohort is to assume that the cohort uses aggregate expected utility of the form

$$EU = \mathbb{E}^{\mathbb{P}} \sum_{t=1}^{20} d^{t-1} u_I(B_t^I),$$

where d is a subjective discount factor, u_I a utility function and B_t^I the pension payment that the cohort gets at time t.

For the purpose of comparison we need an alternative annuity product from the open market. We choose the simplest fixed nominal annuity from some insurance company; the annuity payment is calculated in the actuarially fair way thus it equals the BAA level of the corresponding pension payment from the Mohopeff system. Following a conventional way we let $d = 0.95$ and $u_I(x) = \frac{x^{1-\gamma}}{1-\gamma}$ with $\gamma = 3$.

Figure 17 shows the difference in aggregate expected utility, as a function of starting benchmark funding ratio, between the Mohopeff system and the individual nominal annuity arrangement. It suggests that the critical breaking-up point of the BFR is roughly 0.814 – that is, if the BFR goes below 0.814, the about-to-enter cohort may choose not to step in

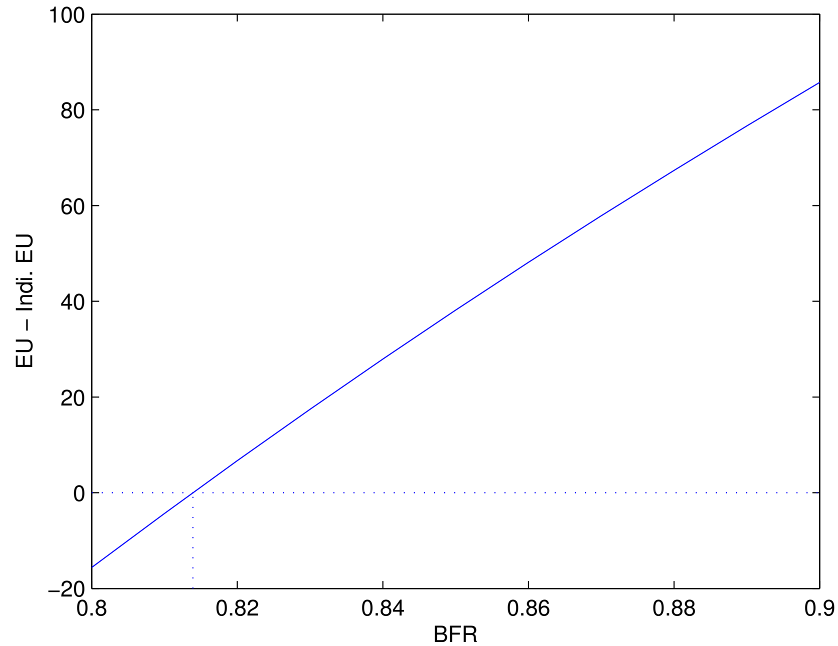

Figure 17: Mohopeff Aggregate Expected Utility, Normalized by Individual Aggregate Expected Utility

and they may go for the individual annuity products. The collective system then faces the danger of collapse, or "ruin".

Figure 18 shows the probability of downside ruin and the expected time to ruin given collapse. As can be expected, the probability of ruin stays around 5% when the BFR is above 1.1, and can rise to 17% if the BFR approaches 0.9 downwards. Given collapse, the expected time to ruin ranges from 8 years to 22 years depending on the funding status.

Regarding the upside of the tolerance band, we employ the same approach as for the floor of the tolerance band, i.e. we assume that the existing generations will compare the aggregate utility between staying in the system or terminating the system and immediately converting what they get into a fixed annuity for the rest of their lives. We further assume that upon termination, what each generation shall get equals the current BFR times the discounted sum of their corresponding unpaid BAAs, that is, the generations dismiss the fund by taking the capital in the buffer proportionally to the present value of the unpaid BAAs, discounted at the current term structure. This is equivalent to say that after termination, each generation shall get a fixed nominal annuity which is equal to the BFR times the BAA level.

Figure 19 shows the number of generations in favor of termination as a function of the starting BFR. If we assume that the fund can be terminated only when over 2/3 of the existing generations are in favor of termination, then the cap of the tolerance band is roughly 1.35 when 13 generations out of 19 find it beneficial to dismiss the fund.

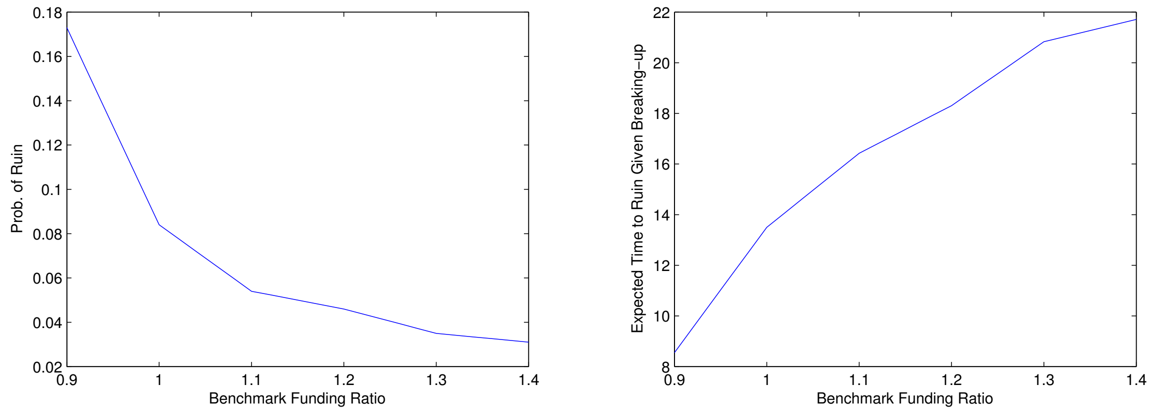

Figure 18: Statistics of Ruin: Floor

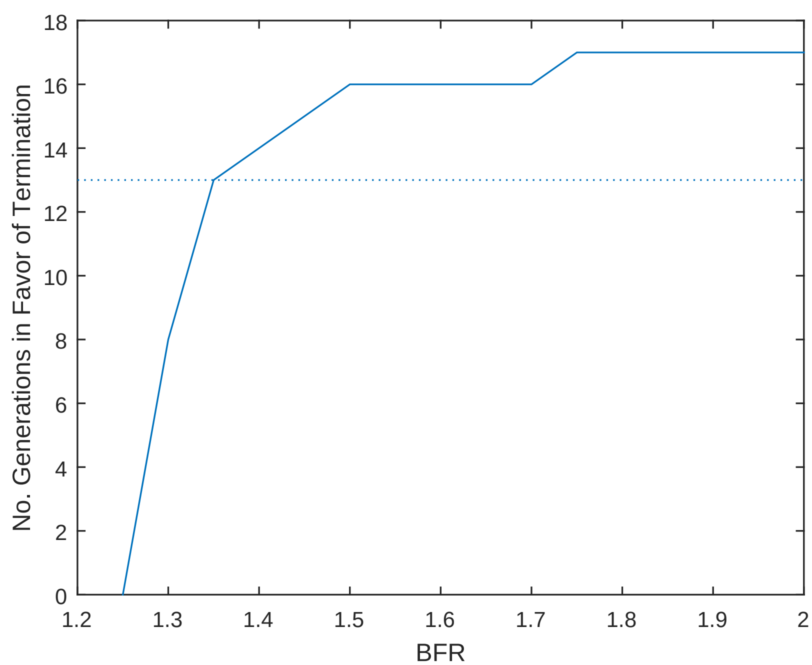

Figure 19: Number of Generations in Favor of Termination

Several comments follow.

- Seen from the figures in Section 4, the Mohopeff approach tends to be stable w.r.t. the downside of the tolerance band, but not the upside. The probability that the BFR goes above 1.35 is significant. This suggests that one may choose the back-protection utility knob or a shorter horizon for the P-system to restrain the use of the buffer.
- To determine the cap of the tolerance band we applied a 2/3 majority rule. However, in the context it is assumed that there is no change in demography profile: each generation keeps the same size and exists in the system for exactly 20 years. If the mortality rates are considered the results shall be different. We find, as expected, that the older generations are much easier to support termination than younger generations who will receive more annuity payments. Thus, if the 2/3 majority rule still apply while the mortality rates are present, the tolerance cap shall be higher as the young generations have a larger population size.
- We didn't take into consideration any transaction costs of getting a fixed nominal annuity from the open market. Usually insurance companies charge a higher premium for individual pension products. The result is that the tolerance band shall be wider when transaction costs are considered.

6 Concluding Remarks

In this paper we have investigated the design of a collective pension system with collective risk sharing under the notion of Pareto efficiency and financial fairness. The Mohopeff approach is introduced and is tested on the economic scenarios generated from the underlying VAR model. To apply the Mohopeff approach one works with projected pension systems on a rolling basis. On one hand, utility functions describe the inter-temporal risk preferences, hence one can talk about Pareto efficienct risk sharing; on the other hand, the market values of the current aggregate benefits are adjusted from its benchmark in such a way as if any surplus or deficit in the funding status can only be smoothed within an N-year recovery period, and financial fairness requires that the value profile should be matched. Numerical results in Section 4 have shown that the Mohopeff performs well under a realistic setting. Furthermore, the Mohopeff approach can easily be tuned via utility knobs and value knobs, thus it offers large space for policy flexibility. We also explored the concept of the tolerance band; as long as the buffer size stays within the tolerance band, both the new and the old generations have no incentives of leaving the system.

There are many directions that can be further explored. First, some work still needs to be done before the Mohopeff approach can be applied in reality. In this paper we calibrate the P-system in a rather simple way. The stock index are assumed to be log-normal distributed and no interest rate risk is present. We also totally ignore the changes in demography and assumes a simple dynamics of the contribution growth. The purpose of doing so is manifold: first, simpler assumptions facilitate the computational procedure; secondly, the absence of demography changes will not complicate how risk sharing can be organized; and lastly, the discrepancy in the quantitative model features between the P-system and the U-system helps to mimic the fact that we cannot totally model the reality and thus the approach should be tested as stable under misspecifications. These stylized assumptions need to be relaxed or removed. The Mohopeff approach has the potential to handle more complicated model settings.

Secondly, the Mohopeff approach has a close relationship with the Dutch pension reform, and there can be further applications. For example, a pension system where personal pension accounts can be established while collective risk sharing can still be present is proposed by Bovenberg and Nijman [4]. The Mohopeff approach can make a contribution, as it enables to define clearly the ownership rights in terms of market value and there can be large space for collective risk sharing. The idea of a rolling recovery period from the Mohopeff suits perfectly the need of smoothing financial shocks among generations. Furthermore, Pazdera et al. [10] shed some light on how intra-group risk sharing can be arranged among current pensioners, and this will help completing the whole picture of the decumulation phase. Hence we see this application of Mohopeff as a very promising future topic.

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