The Correlation Risk Premium: Term Structure and Hedging

Gonçalo Faria[(1),*] and **Robert Kosowski**[(2),*]

[(1)]CEF.UP, University of Porto;
[(2)] Imperial College Business School, CEPR, Oxford-Man Institute of Quantitative Finance.

Netspar International Pension Workshop
Amsterdam

January 30th, 2015

* We gratefully acknowledge financial support from Netspar, INQUIRE Europe and the BNP Paribas Hedge Fund Centre.

Presentation Outline

I. Motivation

*"**Traders are speculating on correlation** among equities, already the highest since the crash of 1987, **will increase** as the threat of a **banking crisis in Europe drowns out news about individual companies**. Equity prices moving in unison have **hurt returns for money managers who seek relative value among stocks and industries**, leaving hedge fund managers with fewer ways to beat their benchmark measures…."*

*"**Fear drives correlations**".*

Bloomberg News, 16th Sep 2011.

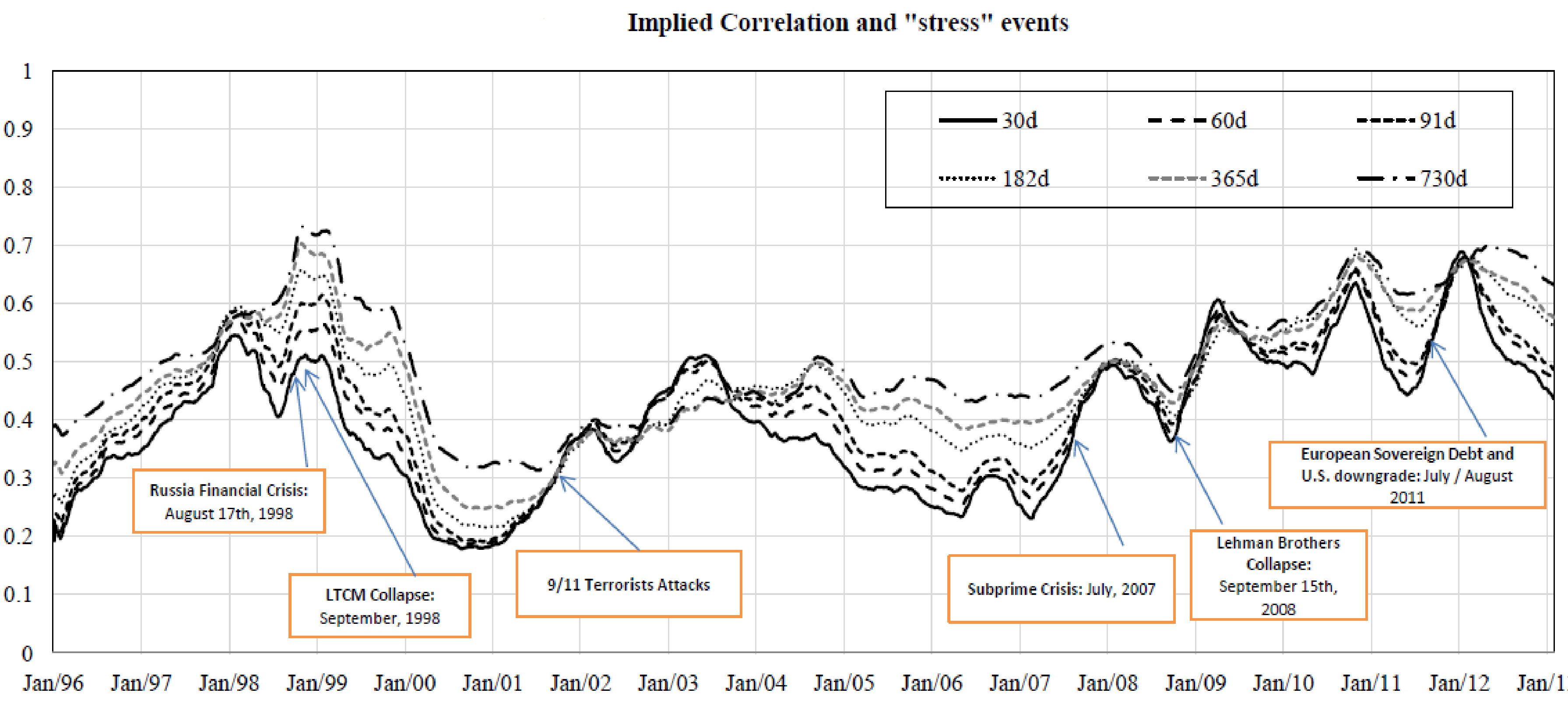
I. Motivation

1. **Diversification benefits can suddenly evaporate** when **correlations unexpectedly increase:**

2. **Need** for early warning “stress” signals. **Promising area**: **derivatives reflecting market expectations of volatility and correlation;**

3. Volatility indices (e.g. VIX) have been extensively used. But the **area of correlation swaps is relatively unexplored**. This is **surprising!**

I. Motivation



II. Main findings

1. **Document and compare different correlation risk measures** and their dynamics, for the S&P500 Index;

2. **Analysis of the term structure of implied correlation and correlation risk factor (**30d, 60d, 91d, 182d, 365d and 730d) reveals shifts in the slope of correlation term structure;

3. **CBOE Implied Correlation** Indices can be accurately replicated by means of synthetic correlation swap rates, **despite differences regarding maturities and constituents**;

4. Analysis of unconditional and conditional correlation hedging strategies shows **only some conditional correlation hedging strategies add value.**

III. Related literature

- **Structural models:**
  - endogenous correlation risk carrying a risk premium (**Buraschi, Trojani and Vedolin (2014), Piatti (2014)** and **Martin (2013)**);

- **Empirical evidence**:
  - significant risk premium, systemic nature, diversification constrains (**Driessen, Maenhout and Vilkov (2012)** and **Buraschi, Kosowski and Trojani (2014));**

- **Optimal portfolio decisions**:
  - Relevance of the absolute **correlation hedging demand** (**Buraschi, Porchia and Trojani (2010)**).

IV. Calculation of the correlation risk factor

- For an equity index, the **correlation risk factor** CR for the time period (t,T) :

$$CR = RC_{t,T} - E_t^Q\left(RC_{t,T}\right) \quad (1)$$

  where: $RC_{t,T}$ is the average "pair-wise" realized correlation of equity index constituents and $E_t^Q\left(RC_{t,T}\right)$ its risk-neutral expectation.

- $E_t^Q\left(RC_{t,T}\right)$ computation:
  - equal to the correlation swap rate quote $SC_{t,T}$, if available;
  - If $SC_{t,T}$ is not available: "synthesized" from the cross-section of index and individual stock options.

    => Methodology we use to obtain synthetic correlation swap rates.

(more in Appendix I)

V. Data

- **Time window**: January 1996 until January 2013 (daily frequency).
- **Focus**: S&P500 Index and its constituents. Robustness test: 100 largest constituents (by market capitalization) of the S&P500 Index.
- **Data resources include**:
  - **(i)** a comprehensive data set of options on the S&P500 Index and its constituents;
  - **(ii)** Daily quotation of the S&P500 Index and its constituents;
  - **(iii)** set of OTC correlation swap quotes with different maturities;
  - **(iv)** CBOE Implied Correlation Indices quotations;
  - **(v)** T-Bill rates time series.
- **Databases**:
  - For **(i)** and **(ii)**: Compustat, CRSP and Optionmetrics;
  - For **(iii)**: unique data set of correlation swap quotes for various maturities (since March 2000 until July 2012) from a major bank in London;
  - For **(iv)** and **(v)**: publicly available information from CBOE and Federal Reserve.

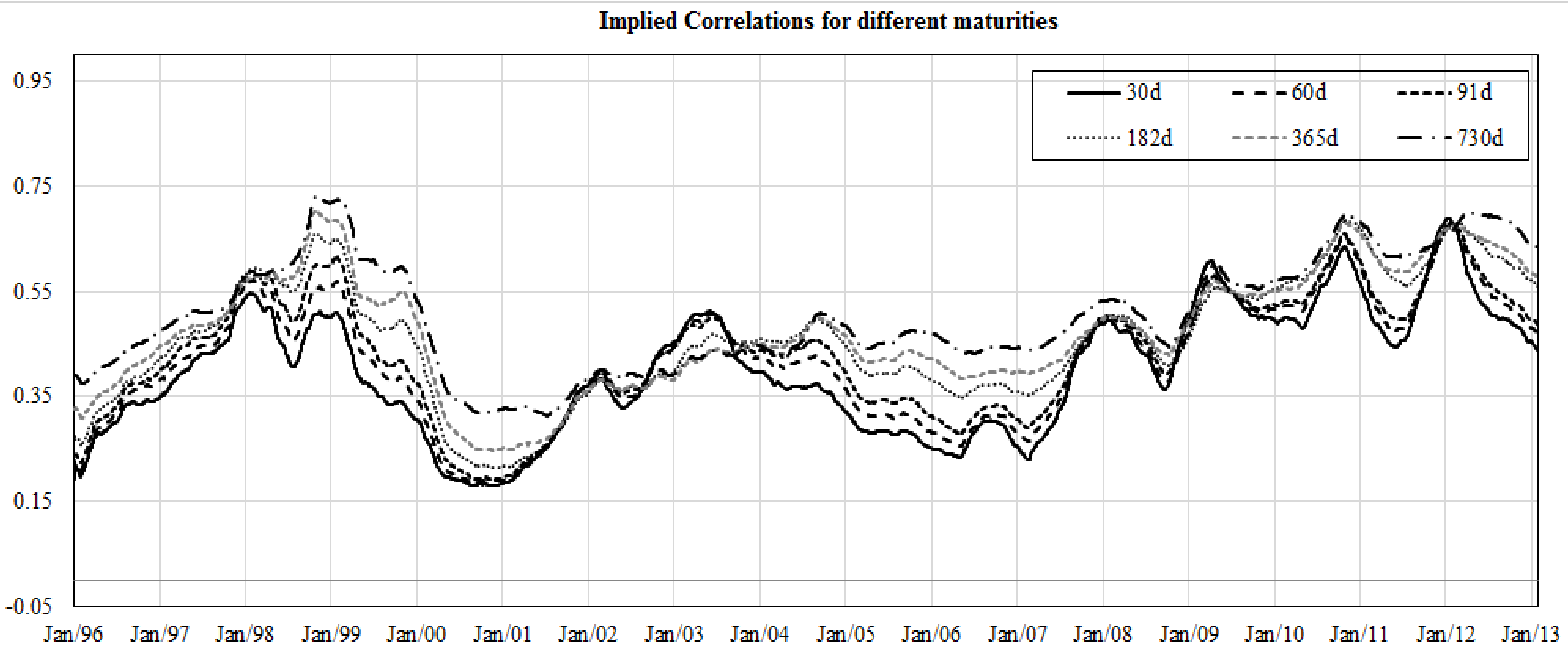
VI. Main results: Real *vs.* Synthetic Correlation Swap Rates

1. Level and dynamics of **correlation swap quotes** (**Q.IC**) for the S&P500 Index for different maturities are accurately replicated by the **synthetic correlation swap rates** (**S.IC**) estimated from option prices. **For 03.2000 – 07.2012**:

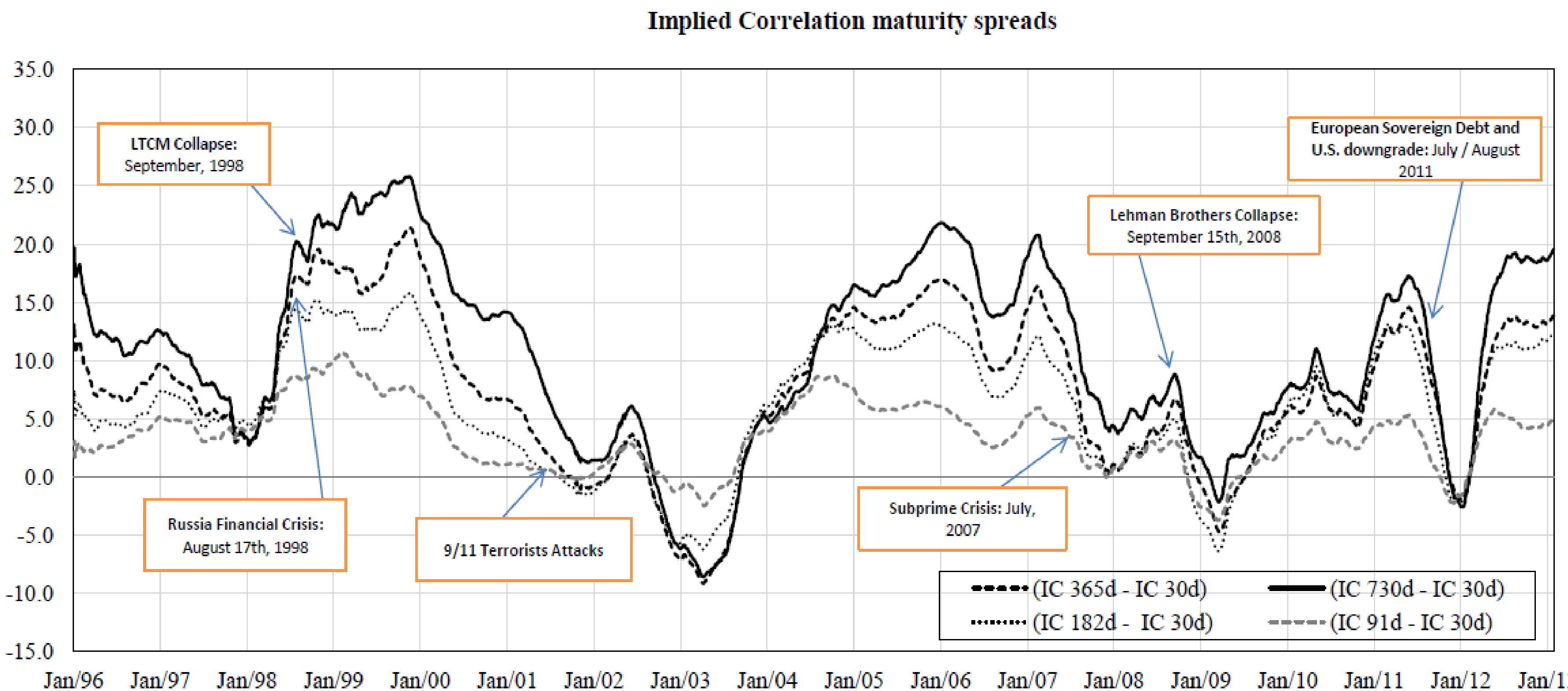
	30d			182d			365d		
	S.IC	Q.IC	S.IC - Q.IC	S.IC	Q.IC	S.IC - Q.IC	S.IC	Q.IC	S.IC - Q.IC
Mean	0.4013	0.4004	0.0009	0.4438	0.4467	-0.0029	0.4447	0.4434	0.0013
(t-stat)			0.00			0.00			0.00
Median	0.3844	0.3822		0.4327	0.4350		0.4331	0.4327	
10th Percentile	0.2202	0.2267		0.2695	0.3234		0.2841	0.3359	
90th Percentile	0.6059	0.5989		0.5997	0.5839		0.5891	0.5624	
Std. Dev.	0.1470	0.1445		0.1237	0.1082		0.1085	0.0958	
Correlation	**0.9535**			**0.9803**			**0.9712**		

VI. Main results: Implied Correlation (IC) term structure

2. Upward sloping during "normal" market regimes; IC term structure flattens or even with negative slope during "turbulent" periods. The IC and its term structure change significantly over time.

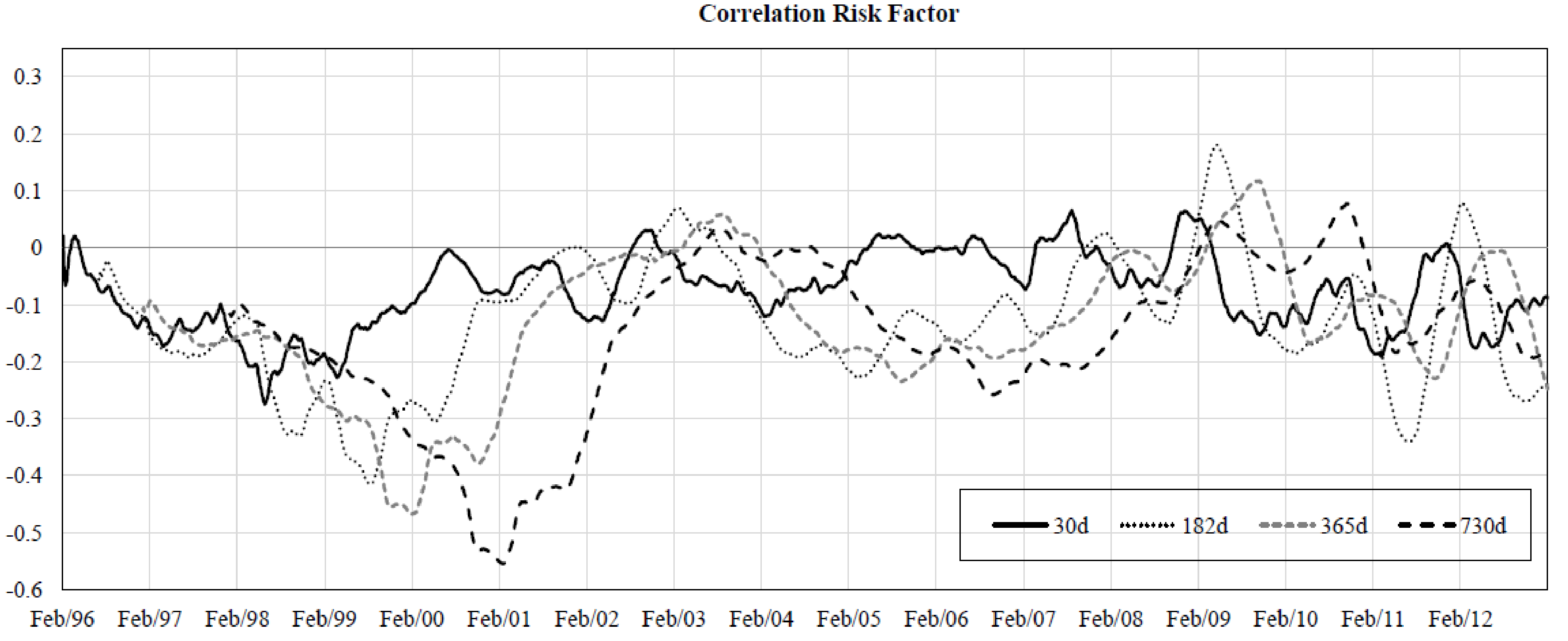


VI. Main results: Implied Correlation (IC) term structure



VI. Main results: Correlation Risk factor (CR) term structure

3. Downward sloping during "normal" market regimes; during "turbulent" periods, **CR** term structure flattens or even with positive slope. **CR** factor and its term structure change significantly over time. [**CR** given by equation (1)]



VI. Main results: Replication of CBOE IC Indices

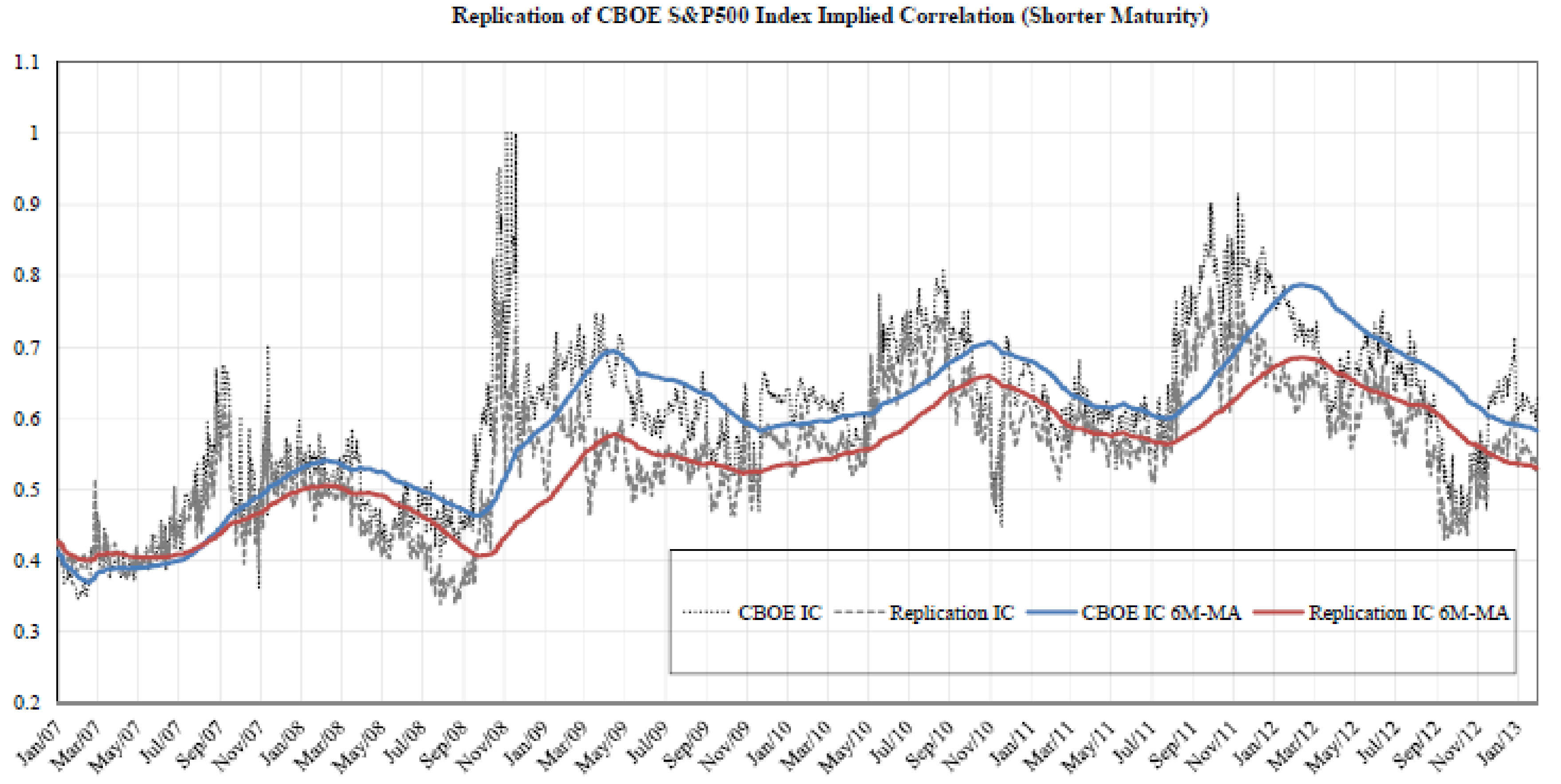
4. CBOE Implied Correlation Indices can be accurately replicated by means of synthetic correlation swap rates, **despite differences regarding maturities and constituents**.

CBOE Implied Correlation Index with Shorter Maturity versus Replication

01/2007 : 01/2013	CBOE IC	Replication IC	Replication Error
Mean	0.6024	0.5464	-0.0559
			0.00
Median	0.6137	0.5541	-0.0549
10th Percentile	0.4442	0.4141	-0.1108
90th Percentile	0.7450	0.6650	-0.0061
Std. Dev.	0.1132	0.0905	0.0434
Correlation of time series	**0.93**		

Note: CBOE Implied Correlation Indices includes only options on 50 largest capitalization stocks of the S&P500 Index.

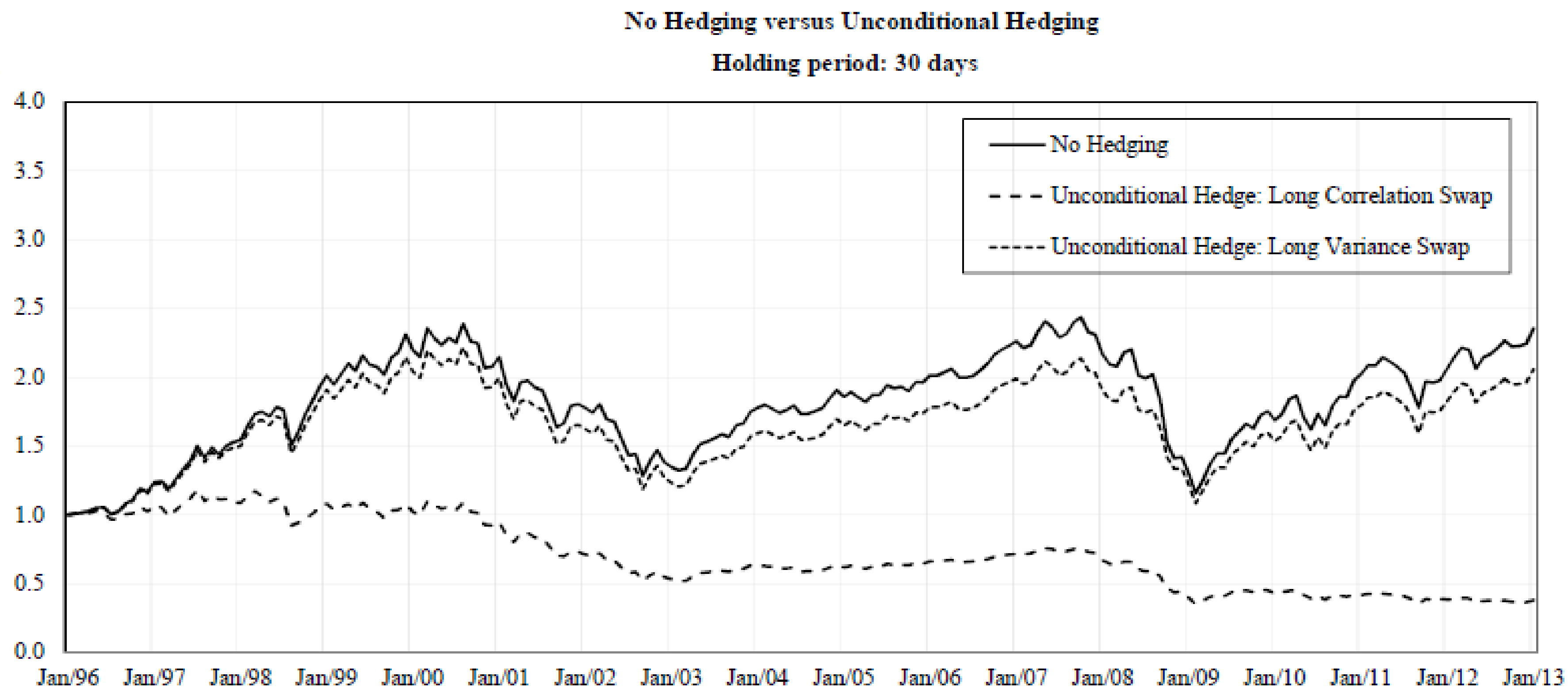
VI. Main results: Replication of CBOE IC Indices



VI. Main results: Correlation Hedging Strategies

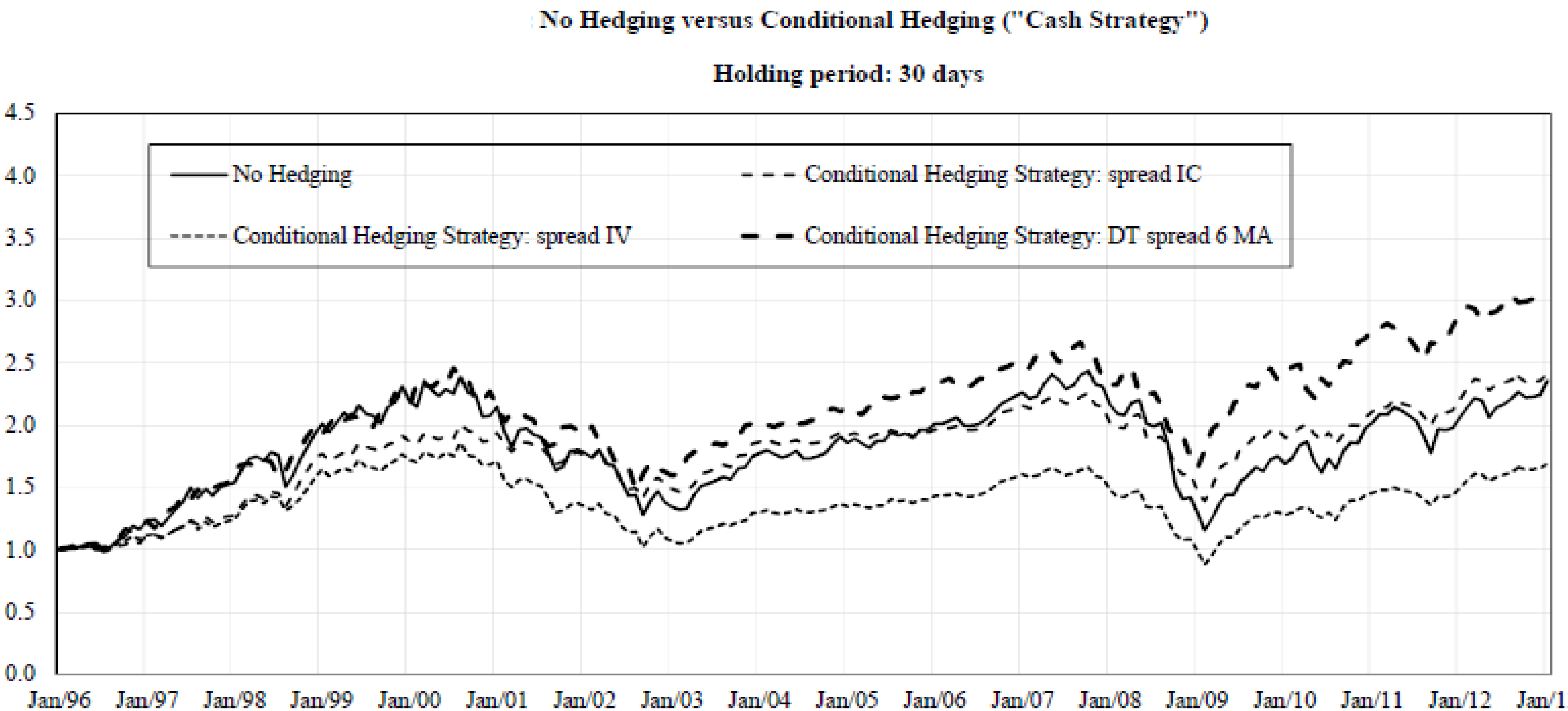
Compared with a **strategy long in the S&P500 with no hedging (full sample).**

5. <u>Unconditional</u> **correlation hedging** strategies using correlation swaps strongly underperform. Cumulative growth of a $1 investment starting in January 1996:



VI. Main results: Correlation Hedging Strategies

6. The **conditional** ***"Cash"*** **strategy delivers good results**, specially with trading signals related with the level of the correlation risk factor and with the dispersion trade returns. Cumulative growth of a $1 investment starting in January 1996:



VII. Relevance of this research project

1. **Informational content in IC and CR term structure in equity markets**: useful for the design of early warning signals of financial stress;

2. **For the design of risk management strategies** by asset managers, particularly:
   - Long term oriented, such as pension funds,
   - Those more exposed to the correlation risk, such as hedge funds.

3. **Important for** regulators and supervisors when assessing:
   - **the systemic risk** at macro level;
   - **risk management policies** at micro level.

4. Correlation risk carries a significant premium: some institutional investors may be interested in **supplying correlation protection to the market place**.

VIII. Concluding Remarks

1. This paper is a **contribution** to the recent literature **on early warning indicators of financial stress** and equity market correlation risk premium;

2. **Analysis of alternative measures of correlation risk and their term structure**;

3. Analysis of unconditional and conditional hedging strategies: **only some conditional correlation hedging strategies add value**;

4. **Extensions:**
   - Develop a structural model (general equilibrium model) that derives correlation risk endogenously for different maturities;
   - Additional performance measures to evaluate different correlation hedging strategies.

References

- **Bakshi, G. S., Kapadia, N. and D.B. Madan, 2003**, Stock Returns Characteristics, Skew Laws, and the Differential Pricing of Individual Equity Options, The Review of Financial Studies, 16, 101-143.
- **Britten-Jones, M. and A. Neuberger, 2000**, Option Prices, Implied Price Processes, and Stochastic Volatility, Journal of Finance 55, 839-866.
- **Buraschi, A., Kosowski,R. and F. Trojani, 2014,** 'When there is no place to hide': Correlation risk and the Cross section of Hedge Fund Returns, Review of Financial Studies 27, 581-616.
- **Buraschi, A., Porchia, P. and F. Trojani, 2010**, Correlation Risk and Optimal Portfolio Choice, Journal of Finance 65, 393-420.
- **Buraschi, A., Trojani, F. and A. Vedolin, 2014**, When uncertainty blows in the orchard: comovement and equilibrium variance risk premia, Journal of Finance, 69 (1) ,101-137.
- **Carr, P. and L. Wu, 2009**, Variance Risk Premiums. The Review of Financial Studies 22(3), 1311-1341
- **Driessen, J., Maenhout, P. and G. Vilkov, 2009**, The price of correlation risk: evidence from equity options, Journal of Finance 64 (3), 1377-1406.
- **Driessen, J., Maenhout, P. and G. Vilkov, 2012**, Option-Implied Correlations and the Price of Correlation Risk, Working paper.
- **Martin, I., 2013,** The Lucas Orchard, Econometrica 81(1), 55-111.
- **Piatti, I., 2014,** Heterogeneous Beliefs about Rare Event Risk in the Lucas Orchard, Working paper.

Appendix I - Synthetic correlation swap quote

When correlation swap quotes are not available, it can be approximated using the concept of implied correlation IC_t (e.g. **Buraschi, Kosowski and Trojani (2014)** and **Driessen, Maenhout and Vilkov (2009)**):

$$IC_t = \frac{E_t^Q\left[RV_{t,T}^I\right] - \sum_{i=1}^n w_i^2 E_t^Q\left[RV_{t,T}^i\right]}{\sum_{i \neq j} w_i w_j \sqrt{E_t^Q\left[RV_{t,T}^i\right] E_t^Q\left[RV_{t,T}^j\right]}} = \frac{SV_{t,T}^I - \sum_{i=1}^n w_i^2 SV_{t,T}^i}{\sum_{i \neq j} w_i w_j \sqrt{SV_{t,T}^i SV_{t,T}^j}} \quad (2)$$

where:

$$RV_{t,T}^I = \text{Realized variance of Index over time span } [t,T];$$

$$RV_{t,T}^i = \text{Realized variance of stock } i \text{ over time span } [t,T];$$

$$SV_{t,T}^I = \text{Variance swap rate for Index over time span } [t,T];$$

$$SV_{t,T}^i = \text{Variance swap rate for stock } i \text{ over time span } [t,T];$$

$$w_i = \text{market capitalization weight of stock } i;$$

Appendix I - Synthetic correlation swap quote

and the variance swap rates are computed using the methodology of **Bakshi, Kapadia and Madan (2003)**:

$$SV_{t,T} = \int_{S_t}^{\infty} \left[\frac{2\left(1 - \ln\left(\frac{K}{S_t}\right)\right)}{K^2} \right] C(t, T-t; K)\, dK + \int_0^{S_t} \left[\frac{2\left(1 + \ln\left(\frac{S_t}{K}\right)\right)}{K^2} \right] P(t, T-t; K)\, dK, \quad (3)$$

where:

- $C(t,T-t;K)$ is the market price of OTM European Call at time *t,* with time to maturity of *(T - t)*, and with strike price *K*.
- $P(t,T-t;K)$ is the market price of OTM European Put at time *t,* with time to maturity of *(T - t)*, and with strike price *K*.

Appendix I - Synthetic correlation swap quote

If variance swap quotes are not available, the variance swap rate $SV_{t,T}$ for the index (or individual stocks) can be synthesized from listed vanilla options prices (see, for e.g., **Britten-Jones and Neuberger (2000)**, **Bakshia, Kapadia and Madan (2003)** and **Carr and Wu (2009)**) as well as using interpolated implied volatility surfaces for a range of standard maturities and set of option delta points (for, e.g, as computed by Optionmetrics).

What we do:

- We use Optionmetrics volatility surface to obtain a smoothed implied volatility surface for a range of maturities and option delta points;
- We only use OTM calls (delta <= 0.5) and OTM puts (delta >= - 0.5);
- After applying those filters we use 13 OTM call and 13 OTM put implied volatility from the surface data for each maturity and each day;
- Then a total of 1001 grid points in the moneyness range from 1/3 to 3 is filled in;
- Then option prices are calculated from interpolated and extrapolated volatilities, using the know interest rate for a given maturity;
- Those are the option prices used to compute the synthetic variance swap rate using **Bakshia, Kapadia and Madan (2003)** formula (3).

Appendix II - IC and CR: Summary Statistics

01/1996 : 01/2013	IC						RC - IC					
	30	60	91	182	365	730	30	60	91	182	365	730
Synthetic quotes												
Mean	0.4018	0.4210	0.4358	0.4606	0.4696	0.4889	-0.0768	-0.0971	-0.1113	-0.1319	-0.1345	-0.1552
Median	0.3889	0.4187	0.4382	0.4592	0.4576	0.4701	-0.0748	-0.0967	-0.1130	-0.1359	-0.1367	-0.1468
10th Percentile	0.2283	0.2502	0.2659	0.2955	0.3148	0.3547	-0.2209	-0.2417	-0.2583	-0.2989	-0.3078	-0.3861
90th Percentile	0.5935	0.5964	0.6010	0.6209	0.6207	0.6334	0.0595	0.0487	0.0484	0.0424	0.0289	0.0220
Std. Dev.	0.1408	0.1329	0.1301	0.1265	0.1191	0.1087	0.1219	0.1250	0.1282	0.1360	0.1347	0.1500