The Correlation Risk Premium: Term Structure and Hedging

GONÇALO FARIA and ROBERT KOSOWSKI *

This Version: 18th August 2014

ABSTRACT

As the recent financial crisis has shown, diversification benefits can suddenly evaporate when correlations unexpectedly increase. We analyse alternative measures of correlation risk and their term structure, based on S&P500 correlation swap quotes, synthetic correlation swap rates estimated from option prices and the CBOE Implied Correlation Indices. An analysis of unconditional and conditional correlation hedging strategies shows that only some conditional correlation hedging strategies add value. Among the conditional hedge strategy’s conditioning variables we find that the level of the correlation risk factor and dispersion trade returns deliver the best results, while the CBOE Implied Correlation Indices perform poorly.

* We would like to thank Netspar for helpful comments. We gratefully acknowledge financial support from Netspar, Tilburg University, INQUIRE Europe and the BNP Paribas Hedge Fund Centre. The usual disclaimer applies. Contact addresses: Gonçalo Faria, CEF.UP, University of Porto, g.faria@imperial.ac.uk and Robert Kosowski, Imperial College Business School and Oxford-Man Institute of Quantitative Finance, r.kosowski@imperial.ac.uk.
I. Introduction

This paper contributes to the recent literature on early warning indicators of financial stress and equity market correlation risk by exploiting a unique dataset of correlation swap quotes and studying the term structure of correlation risk and its relationship to the recently introduced CBOE S&P500 Implied Correlation Indices. As the recent financial crisis has shown, diversification benefits can suddenly evaporate when correlations unexpectedly increase. Such correlation spikes are associated with significant welfare losses. The ability to understand and forecast correlations is therefore of great importance to institutional investment managers, such as hedge funds, pension funds and insurance companies, and to central bankers and economic policy makers in general. Following the recent financial crisis, there have been calls for early warning systems that could help investors and regulators anticipate impending financial crises. One promising area for such signals is in the form of derivatives whose prices reflect market expectations of volatility and correlation. Volatility indices in the form of the VIX, for example, have been extensively studied and used by various market participants and regulators, but the area of correlation swaps is relatively unexplored. This is surprising since the variance risk premium is largely due to a correlation risk premium (Driessen, Maenhout and Vilkov (2009), Buraschi, Trojani and Vedolin (2014)). Moreover, as we will document in this paper, dynamics of correlation and of the correlation risk factor have more inertia than that of variance and variance risk factor: this translates into a much more persistent effect from correlation shocks than those from variance shocks.

There are several possible ways of measuring and trading exposure to correlation risk. The first contribution of this paper is, therefore, to rigorously examine different measures of correlation risk and compare their dynamics over time as well as their information content with a view to forecasting correlation. In a correlation swap the counterparties exchange realized versus implied correlation. Such a contract is therefore the ‘purest’ way of generating correlation risk exposure (compared to alternatives such as dispersion trades using options). When such swap contracts are not available or not sufficiently liquid a natural alternative approach is to construct
a synthetic correlation contract using listed options on an index and its constituents. We obtain a unique dataset of correlation swap quotes for various maturities from March 2000 until 2012 from a major international bank. This allows us to study the relationship between market values (quotes) and synthetic correlation swaps prices. We document and compare the summary statistics and dynamics of different correlation risk measures based on correlation swap rates, synthetic correlation swap rates and the publicly available CBOE Implied Correlation Indices. This exercise is carried out for different sub-samples during the time period from January 1996 until January 2013 for the S&P500 Index as well as the 100 largest constituents of the S&P500 Index.

The second contribution of this paper is the analysis of the term structure of correlation risk. Implied correlation measures may contain valuable information about future correlation. The extant literature has not analyzed the term structure of correlation risk in detail and we fill this research gap. Implied correlation at different horizons reveals important information about correlation expectations. Egloff, Leippold and Wu (2010) and Ait-Sahalia, Karaman and Mancini (2013), analyze the variance term structure for the S&P 500 Index. We contribute to this literature by studying the correlation risk term structure.

We compare the correlation risk term structure estimated from correlation swap quotes, a portfolio of options and the CBOE Implied Correlation Indices, which consist of two fixed maturities. We also study how different the hedging properties of the CBOE Implied Correlation Indices are from those obtained from other implied correlation measures. This set of questions has not been addressed in the literature before but is relevant for practitioners since the CBOE Correlation Indices are publicly available which, in the future, may facilitate the design of hedging instruments.

Our empirical analysis leads to four main findings. First, both the level and the dynamics of the correlation swap quotes for S&P500 index with maturities of 30 days, 91 days, 182 days and 365 days are accurately replicated by the synthetic correlation swap rates estimated from option prices. Second, during “normal” market regimes, the implied correlation term structure is upward sloping since implied correlations for higher maturities are higher than for shorter
maturities. When financial markets go through turbulent periods a common pattern emerges: implied correlations increase for all maturities, and this increase is much more pronounced for shorter maturities. As a result the slope of implied correlation term structure becomes flat or even negative. Third, on average, the correlation risk factor term structure has a negative slope, that is the average ex-post correlation risk premium, measured by the difference between realized correlation and implied correlation, is more negative for higher maturities. The same downward sloping shape has been found by Ait-Sahalia, Karaman and Mancini (2013) for the variance risk factor term structure. Our evidence shows that the correlation risk factor term structure changes significantly over time: during periods when the implied correlation term structure is negative the correlations risk factor term structure tends to be positive. The corollary is that the correlation risk factor term structure dynamics seems to be driven fundamentally by the implied correlation term structure dynamics. Fourth, the CBOE Implied Correlation Indices and their dynamics can be accurately replicated by means of synthetic correlation swap rates.

Moreover, we test and compare three different ways of implementing a correlation hedging strategy. The first is unconditional and consists of a long position in a correlation swap contract or a long position in a variance swap contract. We consider the correlation swap contract as it is a pure correlation derivative. The variance swap contract is a pure variance derivative that partially reflects the correlation dynamics. It is tested as a second best. The second is conditional and is implemented by means of a long position in a correlation swap contract or a variance swap contract which depends on specific trading signals related to (2a) the implied correlation term structure, (2b) the correlation risk factor term structure, (2c) the level of the correlation risk factor and (2d) the implied correlation term structure embedded in the CBOE Implied Correlation Indices. The third hedging strategy, which we name “Cash” Hedging strategy, is also conditional but it involves reducing the exposure to the S&P500 Index and increasing the allocation to cash, depending on the conditioning variables (2a-2d) as well as other predictors such as the level of dispersion trade returns, the implied variance term structure, the variance risk factor term structure and the level of the variance risk factor. We find that unconditional correlation hedging strategies strongly underperform a strategy with no hedging. Among the
conditional correlation hedging strategies those that involve a long position in swaps, particularly in correlation swap contracts, underperform, in most cases, relative to a strategy with no hedging. The conditional “Cash Hedging” strategy delivers good results, i.e. higher average returns coupled with lower standard deviations than a strategy with no hedging. Among the different predictors, the level of the correlation risk factor and of the dispersion trade returns deliver the best results, while CBOE Implied Correlation Indices perform poorly.

Related literature

Our research is related to three streams of the literature. First, recent theoretical economic models explain how correlation risk can arise endogenously and why it should carry a risk premium. Martin (2013) investigates a model that consists of a collection of Lucas trees that generates return correlations that vary endogenously, spiking in times of crisis. Since disasters spread across assets, the model generates large risk premia even for assets with stable cash flows. Buraschi, Trojani and Vedolin (2014) consider a multi-asset Lucas economy but with agents that have heterogeneous beliefs about a firm’s fundamentals and the business cycle. They show how correlation risk can arise endogenously due to agents’ optimal risk sharing. Moreover, Buraschi, Porchia and Trojani (2010) find evidence that the absolute correlation hedging demand increases with the investment horizon, which implies that correlation risk becomes particularly relevant for optimal life-cycle decisions as well as pension fund managers.

Second, empirical work by Driessen, Maenhout and Vilkov (2012) shows that correlation risk commands a significant risk premium. Third, Buraschi, Kosowski and Trojani (2014) study the relation between correlation risk, hedge fund characteristics and their risk-return profile. The authors provide evidence supporting the fact that correlation risk is a systematic risk (“there is no place to hide”). The authors show that correlation risk affects hedge fund portfolios consisting of different asset classes including equities, futures and options which feature prominently in many institutional investor portfolios. Evidence of the systematic nature of correlation risk highlights its importance for institutional investors and policy makers. Buraschi, Kosowski and Trojani
find a pronounced nonlinear relation between correlation risk exposure and the tail risk of hedge fund returns, with correlation risk being the most significant risk factor in explaining the cross-section of hedge funds performance.

Understanding different measures of the correlation risk factor and their properties (key statistics, dynamics, and term structure) is highly relevant in two areas in practice. First, it is important for the design of risk management strategies by asset managers and particularly for those that are long term oriented, such as pension funds, and those that are particularly highly exposed to correlation risk, such as hedge funds. Second, for macro and micro-prudential regulation and supervision activities by regulators and supervisors, namely for assessing systemic risk at the macro level and risk management policies at the micro level. The fact that we focus on implied correlation risk which reflects market expectations of future correlations makes this work particularly useful for the design of early warning signals of financial stress and therefore a relevant input for financial stability policies.

The results presented in Buraschi, Kosowski and Trojani (2014) may lead to investors and supervisors’ reassessing the risk implicit in long-short strategies. The authors find that the hedge funds’ ability to create market neutral returns is often associated with a significant exposure to correlation risk. Thus, investment strategies traditionally considered “conservative” such as Relative Value and Long/Short Equity strategies, are in fact substantially more risky than it seems because they are strongly exposed to correlation risk.

The rest of the paper is structured as follows. Section II discusses alternative measures of implied correlation and correlation risk. Section III describes the data and how the synthetic correlation swap rate is estimated. Section IV summarizes the main empirical findings about the relation between correlation swap rates and synthetic correlation swap rates estimated from option prices, about the implied correlation term structure, about the correlation risk factor term structure and about the CBOE Implied Correlation Indexes replication. Section V describes different correlation hedging trading strategies. Section VI reports the result of robustness exercise by making the analysis for the 100 largest constituents of the S&P500 Index. Section VII concludes.
II. Methodology: alternatives measures of correlation risk

One of the three main contributions of our paper is to examine different measures of correlation risk. Therefore, we first focus on “pure” correlation risk proxies such as correlation swap rates, since alternative ways of trading correlation such as dispersion trades are subject to potential hedging errors resulting from model misspecifications and may generate undesired exposure to other risk factors. The term “pure” is used to highlight that correlation swaps payoffs are exclusively driven by correlation dynamics as apposed to movements in underlying equity market due to delta exposures, for example.

In the case of an equity index, such as the S&P500, for example, the correlation risk factor, with respect to the time period \((t, T)\), is typically represented by the difference between the average “pair-wise” realized correlation of equity index constituents during that time period, \(RC_{t,T}\), and its risk-neutral expected value at time \(t\), \(E^Q(RC_{t,T})\). This difference is also commonly referred as the (ex-post) correlation risk premium, and effectively corresponds to the realization of the correlation risk factor, representing the cost (excluding transaction costs) incurred by an investor aiming to hedge the exposure to eventual correlation spikes.

\[
CR_t \equiv RC_{t,T} - E^Q(RC_{t,T}). \tag{1}
\]

Different measures of the correlation risk factor essentially correspond to different approaches to the calculation of implied correlation, that is the risk-neutral expected value \(E^Q(RC_{t,T})\) in Equation (1). We study three alternative approaches.

The first and most direct way of computing \(E^Q(RC_{t,T})\) is to use the correlation swap rate \(SC_{t,T}\), if available in the form of correlation swap quotes. In a correlation swap contract, the buyer pays implied correlation until the maturity \(T\) of the contract, that is the correlation swap rate \(SC_{t,T}\), and receives the correlation \(RC_{t,T}\), realized from the beginning to the maturity of the
contract. Since the starting price of a correlation swap is zero, the correlation swap rate equals
the risk neutral expected value, \( \text{SC}_{t,T} = E^Q(\text{RC}_{t,T}) \), and consequently the correlation risk factor
(Approach 1) in Equation (1) is given by:

\[
CR_t = RC_{t,T} - \text{SC}_{t,T}.
\]  

(2)

We have access to equity market correlation swap quotes for different maturities which
were made available to us by a major investment bank in London.

The second method of computing \( E^Q(\text{RC}_{t,T}) \) in Equation (1), is to approximate the
correlation swap rate \( \text{SC}_{t,T} \) by using a cross-section of index and individual stock variance swaps,
which in turn can be “synthesized” from the cross-section of index and individual stock options.
As in Buraschi, Kosowski and Trojani (2014), an approximation to the correlation swap rate
\( \text{SC}_{t,T} \) is given by an implied correlation rate \( \text{IC}_{t,T} \) given by:

\[
\text{IC}_{t,T} = \frac{E^Q[RV_{t,T}^I] - \sum_{i=1}^{n} w_i^2 E^Q[RV_{t,T}^I]}{\sum_{i,j} w_i w_j \sqrt{E^Q[RV_{t,T}^I] E^Q[RV_{t,T}^I]}} = \frac{\text{SV}_{t,T}^i - \sum_{i=1}^{n} w_i^2 \text{SV}_{t,T}^i}{\sum_{i,j} w_i w_j \sqrt{\text{SV}_{t,T}^i \text{SV}_{t,T}^j}},
\]

(3)

where \( \text{SV}_{t,T}^I \) and \( \text{SV}_{t,T}^i \) are the index and single stock variance swap rates over the period
\((t,T)\), which correspond to the risk-neutral expected value for variance of index and each single
stock respectively, and \( w_i \) is the market capitalization of stock \( i \). In order to compute \( \text{SV}_{t,T}^I \) and
\( \text{SV}_{t,T}^i \), one can synthesize them from listed vanilla options prices (see, for e.g., Britten-Jones and
Neuberger (2000), Bakshi Kapadia and Madan (2003) and Carr and Wu (2009)) as well as using
interpolated implied volatility surfaces for a range of standard maturities and a range of option
deltas (for example, as computed by Optionmetrics).

In this paper, for the purpose of estimating the index and single stock variance swap
rates, \( \text{SV}_{t,T}^I \) and \( \text{SV}_{t,T}^i \) in (3), we follow the methodology of Bakshi, Kapadia, and Madan (2003)
(BKM for the remainder of the paper). As long as prices are continuous and volatility is stochastic, this method delivers the estimate of the risk-neutral, or option-implied, integrated variance up until the option’s maturity. It is often called as a model-free implied variance approach. It is extracted from markets prices of out-of-the-money (OTM) European calls and puts:

\[
SV_{T,t} = \int_{S_t}^{\infty} \left[ \frac{2 \left( 1 - \ln \left( \frac{K}{S_t} \right) \right)}{K^2} \right] C(t,T-t;K)dK + \int_{0}^{S_t} \left[ \frac{2 \left( 1 + \ln \left( \frac{S_t}{K} \right) \right)}{K^2} \right] P(t,T-t;K)dK, \quad (4)
\]

where \( C(t,T-t;K) \) and \( P(t,T-t;K) \) are the market prices of European calls and European puts at time \( t \), with time to maturity of \((T-t)\), and with strike price \( K \). In order to obtain these option prices we use volatility surfaces data from IvyBD (Optionmetrics), which we describe in detail in Section III.

Using the synthetic correlation swap rate \( IC_{t,T} \) from Equation (3) the correlation risk factor (Approach 2) in Equation (1) is given by:

\[
CR_t = RC_{t,T} - IC_{t,T}. \quad (5)
\]

The third potential source of an implied correlation risk proxy in Equation (1) is to use the CBOE S&P500 Index Implied Correlation Indices launched by the Chicago Board of Options Exchange (CBOE) in July 2009, with historical values back to 2007.\(^1\) On a daily basis the CBOE disseminates two indexes tied to two different fixed maturities. For example, beginning in July 2009, the CBOE calculated two implied correlation indexes: one with maturity of January 2010 (“ICJ”) and another with maturity of January 2011 (“JCJ”). Regarding the options that are selected to compute implied correlation a CBOE document (CBOE, (2009)) explains:

“The options used to calculate ICJ (January 2010) are SPX options [S&P500 options] expiring in December 2009 and individual stock LEAPS expiring in January 2010. Likewise, JCJ (January 2011) uses SPX options expiring in December 2010 and LEAPS expiring in January 2011. ICJ will be calculated through November 2009 option expiration; JCJ will be calculated through November 2010 option expiration. On the business day immediately following November expiration, CBOE will introduce a new maturity of the CBOE S&P 500 Correlation Index. For example, on Monday, November 23, 2009, CBOE will begin calculating KCJ (January 2012), using SPX options expiring in December 2011 and stock LEAPS expiring in January 2012.”

Note that both implied correlation indices computed by the CBOE are measures of the risk-neutral expected average “pair-wise” realized correlation of returns of S&P500 index constituents, implied through S&P500 option prices and prices of individual options on the 50 largest constituents of the S&P500 Index.

In this case (Approach 3) the correlation risk factor in Equation (1) is given by:

$$CR_t = R_C^t - CBOE_{IC_t}.$$  \hspace{1cm} (6)

For the remainder of the paper we will refer to a “shorter maturity” $CBOE_{IC_t}$ and a “longer maturity” $CBOE_{IC_{T,T}}$ implied correlation, corresponding to the respective maturities of the two implied correlation figures disclosed by the CBOE on a daily basis.

To compute the implied correlation term structure, we use the spreads between the implied correlation from options with higher maturities and the implied correlation from options with the shorter maturity (30 days). The same procedure is used to compute the correlation risk premium term structure.

III. Data

Our sample period is from January 1996 until January 2013 (daily frequency). This
period includes many important macroeconomic events including the Asian crisis (1997), the Russian crisis (1998), the burst of the Tech Bubble (2000), the subprime and Lehman Brothers crisis (2007) and the Sovereign debt crisis (since 2011). Our sample includes turbulent episodes during which correlation risk increased significantly.

Our analysis is focused on the S&P500 Index and its constituents. However, in the robustness test described in section VI, we also carry out an analysis based only on the largest 100 constituents of the S&P500 Index. The daily index composition is obtained from Compustat Index Constituents file. Using each day market capitalization of the stocks from Center for Research in Security Prices (CRSP) we obtain value weights on each day for each index constituent for the entire sample. We use daily data on stock and index returns from the CRSP. For stock and index options we use IvyBD (Optionmetrics). Additionally, for the S&P500 Index, we also make use of a unique data set of correlation swap quotes and realized correlations for different maturities, provided by an Investment Bank, for the sample period from March 2000 to July 2012. At last, we download time series of the CBOE S&P500 Index Implied Correlation Indices from January 2007 (date after which data is available) to January 2013 from the CBOE webpage.

In order to estimate the synthetic correlation swap rates, we make use of the OptionMetrics Volatility surface file to obtain standardized volatilities for maturities of 30, 60, 91, 182 and 365 days. The volatility surface file contains a smoothed implied-volatility surface for a range of maturities and option delta points. We only use out-of-the-money (OTM) calls (delta smaller or equal to 0.5) and puts (deltas larger than -0.5). As it results from integrals in formula (4), it is necessary to have a continuum of option prices to obtain synthetic variance swap rates that will be used to compute the implied correlation in (3). We adopt the following approach: integrals in (4) are discretized and approximated using the options available through the Volatility Surface. After applying the above mentioned delta filters, we use 13 OTM call and 13 OTM put implied volatilities from the surface data for each maturity and each day. Then a total of 1001 grid points in the moneyness range from 1/3 to 3 is filled in, by interpolating the available implied volatilities inside the available moneyness range and extrapolating using the
last known (boundary for each side) value. Interpolation is done using cubic spline data interpolation.\(^2\) Then option prices are calculated from interpolated and extrapolated volatilities using the known interest rate for a given maturity. Those are the option prices used to compute the variance swap rate using BKM’s method.

After computing the daily series of model-free implied variances for the index and individual options and the index weights \((w_t)\), the model-free implied correlation \(IC_{t,T}\) for day \(t\) with maturity \(T\) is given by Equation (3) in section II. On days when there are missing implied variances, particularly for the index constituents, weights of the available stocks are normalized so that they sum up to one.

In order to obtain the realized variance and realized correlations time series the procedure is as follows. For day \(t\), daily returns for the index and the stocks from day \(t+1\) until the end of the maturity window are considered. The realized variance is compute for the respective period and then annualized through linear approach.

Realized correlations \(RC\) between the stocks in the index are calculated using CRSP stock returns. For a given estimation window, from \(t\) to \(T\), we compute the correlations pairwise, that is, for each pair of stocks, for each day \(t\). Then these pairwise correlations are aggregated into a cross-sectional weighted average across all pairs of stocks, using their index weights, to obtain a realized correlation from day \(t\) to \(T\), \(RC_{t,T}\). The procedure is repeated for all days in the estimation period.

Following Buraschi, Kosowski and Trojani (2014), we compute the correlation risk factor \((CR_{t,T})\) for each day and each maturity by computing the difference between the realized correlation \((RC_{t,T})\) and implied correlation \((IC_{t,T})\).

IV. Empirical Results

We begin our empirical analysis by comparing the summary statistics and dynamics of correlation swap quotes obtained from an investment bank to the synthetic correlation swap rates

---

\(^2\) An alternative interpolation method, pchip (piecewise cubic hermite interpolating polynomial), was also tested with no material differences.
that we estimate from option prices. We then use the estimated implied correlation from option prices for different maturities to document the implied correlation term structure and analyse its dynamics. We estimate the realized correlation risk factor for different maturities and document its term structure and its dynamics.

Finally, building on our implied correlation term structure analysis we examine the CBOE Implied Correlation Indices and whether they can be replicated using synthetic correlation swap rates obtained from option prices.

a. Synthetic Correlation Swap rate versus Correlation Swap Quotes

The starting point of our empirical analysis is the comparison between two measures of the risk-neutral expectation of realized correlation: the “real world” correlation swap quote, obtained from an investment bank, and the synthetic correlation swap rate constructed using option prices, as in Britten-Jones and Neuberger (2000), Bakshi, Kapadia and Madan (2003) and Driessen, Maenhout and Vilkov (2009, 2012). Our objective is to study how well we can replicate the correlation swap quotes using synthetic swap rates for the maturities for which we have correlation swap quotes: 30 days, 91 days, 182 days and 365 days. This analysis is of interest for a couple reasons. First, in order to study the term structure dynamics of implied correlation ($IC_{t,T}$) and of the correlation risk factor ($CR_{t,T}$), it is important to have as long and complete a time series as possible. Unfortunately, correlation swap data is not publicly available and is difficult to obtain. Moreover, conversations with derivatives traders in London and New York reveal that the correlation swap market is relatively illiquid and its liquidity is cyclical as large premia are required to entice traders to sell correlation. If synthetic correlation swap rates are a good approximation of correlation swap quotes that we obtained privately, then further research can be carried out to study the information obtained in correlation swap quotes.

Second, the illiquidity and high unconditional costs of correlation swaps imply that market participants that are interested in trading correlations, either for hedging reasons or to take a directional position, may be interested in alternatives and option based approximations to correlation swaps. If the correlation swap rate can be synthetically approximated using publicly
available option prices with a low tracking error, then such proxies could be used to trade correlation.

Table 1 reports the summary statistics for (i) the synthetic correlation swap rate ($S_{IC_{t,T}}$), calculated from daily observations of model-free implied variances for the index and index components, using Equation (3), for four different maturities (30, 91, 182 and 365 calendar days) and (ii) for the correlation swap rates ($Q_{IC_{t,T}}$) directly obtained from correlation swaps with four different maturities (30, 91, 182 and 365 days). The correlation between the $S_{IC_{t,T}}$ and $Q_{IC_{t,T}}$ time series is also reported, for the different maturity periods considered. The sample period is from March 2000 to July 2012, since this is the period of time for which we have correlation swap rates.

[ INSERT TABLE 1 ABOUT HERE ]

For the full sample period from March 2000 to July 2012, we find that the difference between the average level of $S_{IC_{t,T}}$ and $Q_{IC_{t,T}}$ ranges between minus 15 basis points (for 182 days) and 31 basis points (for 365 days). Importantly, the correlation between the time series for the synthetic correlation proxy and the correlation quotes is very high: 95 percent for 30 days, 98 percent for 91 days, 98 percent for 182 days and 97 percent for 365 days. The conclusions remain qualitatively the same when we examine sub-samples, from March 2000 until December 2005 and from January 2006 until July 2012.

The above results show that it is possible to closely replicate the level and the dynamics of the correlation swap quotes for the S&P500 index (for maturities of 30 days, 91 days, 182 days and 365 days) by means of synthetic correlation swap rates computed from option prices and the Optionmetrics volatility surface. Since the Optionmetrics data is available from 1996 to 2013, which is 5 years longer than the data for which we have correlation swap quotes, we use synthetic correlation swap rates in the subsequent empirical analysis.
b. Implied Correlation Term structure

Figure 1A plots the 6-month moving average of the estimated implied correlations ($IC_{t,T}$) for the S&P500 Index for different maturities: 30 days, 60 days, 91 days, 182 days and 365 days. It therefore represents a daily series of the IC term structure. We observe that (i) the implied correlation at different maturities changes significantly during the sample period, and (ii) the implied correlations strongly comove which suggests an occasional parallel shifts of the entire term structure during the sample period.

Panel A of Figure 1 shows that the slope of IC term structure changes through time: during most of the sample period IC increases with maturity, that is, the IC term structure has a positive slope, but during some subperiods the slope of the IC term structure becomes flat or even negative. Panel B of Figure 1 further illustrates this by plotting the spread of IC with higher maturities (365d, 182d and 91d) relative to the IC for 30 days.$^3$ During most of the sample period these IC maturity spreads are positive, with the exception of four periods of time: (1) after the 9/11 terrorist attacks, (2) during the U.S. economic slowdown between 2002 and 2003, (2) the beginning of the subprime crisis in the summer of 2007, (3) after the Lehman Brothers collapse in September 2008, and (4) more recently during the peak of the sovereign debt crisis in Europe and the U.S. sovereign debt downgrade during the second half of 2011.

Panels A and B of Figure 2 suggest a common pattern for implied correlation dynamics around periods of turbulence in financial markets. First, implied correlations increase at all maturities. Second, for shorter maturities implied correlations increase more than for longer maturities, that is, spreads (long term – short term) decrease and eventually become negative.

$^3$ The 95% confidence intervals for the spreads IC 365d – 30d, IC 182d – 30d and IC 91d – 30d, for the full sample are [7.37;7.90], [5.99;6.44] and [3.38;3.66], respectively. This suggests that for the three cases, considering the full sample, the IC spreads are all statistically different from zero. Moreover, the boundaries of the three intervals do not overlap which means that, for the level of confidence considered the three spreads are statistically different between them.
The 30 days implied correlation line overlaps or even “crosses” higher maturities’ implied correlation lines during periods of increased stress in markets. When the lines overlap, this means that the implied correlation term structure is flattening, and when they cross, this results in an inverted implied correlation term structure.

[[ INSERT FIGURE 2 ABOUT HERE ]]

In Panels A and B of Figure 3 we document the implied correlation dynamics for different maturities around key U.S. monetary policy events. We consider a subsample period from January 2007 to January 2013 as this was a particularly busy period of monetary policy action, following the subprime crisis and Lehman Brothers collapse: Quantitative Easing 1 (QE1, 2008 and 2009), Quantitative Easing 2 (QE2, 2010), Operation Twist (OT1, 2011) and its extension (OT2, 2012). Interestingly, Figure 3A shows that the Implied Correlation term structure reacts differently to different monetary policy events.

[[ INSERT FIGURE 3 ABOUT HERE ]]

At the announcement of QE1, in November 2008, the 30 days implied correlation “crossed” all other implied correlation curves, resulting in an inverted implied correlation term structure. This is consistent with an interpretation that the QE1 announcement did not straightaway reduce risk aversion and uncertainty in financial markets. Implied correlations continued to increase across all maturities with the spread widening between short term maturities and long term maturities, as shown in Panel B of Figure 3. On the contrary, the reinforcement of QE1 at March 2009 was followed by decreasing implied correlations across all maturities. Importantly, this decrease was relatively more pronounced for shorter maturities than longer maturities which lead to a positively sloped IC term structure. The figure suggests that only after this second intervention in the form of QE2 calm returned to equity markets.

Following a period of increasing implied correlation for all maturities, the announcement
of QE2, in November 2010, coincides with an inflection point for that trend. However, there was no change in the IC term structure slope, which continued to be positive and even increased, as documented in Panel B of Figure 3. Implied Correlations decreased for all maturities until June 2011. By the middle of July 2011 concerns around the Euro zone as well as about the U.S. debt ceiling started to dominate equity markets. Implied correlations started to increase at all maturities with that increase being relatively higher for shorter maturities. The Fed announced Operation Twist (OT1) on September 21st but implied correlations continued to increase and, by the beginning of October the term structure of IC became downward sloping, as can also be seen in Panel B of Figure 3. At the beginning of January 2012 implied correlations started to decrease at all maturities and the term structure recovered its normal positive slope. When the extension of operation twist (OT2) was announced no relevant change occurred both with respect to the trend of IC at all maturities and the term structure.

In summary, our main empirical findings regarding the dynamics of implied correlation are as follows:

- Implied correlations from equity options (S&P500 index) are always positive and they change significantly. This applies to all maturities.
- In “normal” market circumstances, implied correlations for higher maturities are higher than for shorter maturities, implying a positively sloped implied correlation term structure. This is the “normal” IC regime.
- When financial markets enter into turbulent periods a common pattern emerges: (i) implied correlations increase for all maturities, (ii) that increase is significantly more aggressive for shorter maturities, implying that the implied correlation term structure flattens or even becomes negatively sloped. This could be labelled the “stressed” IC regime.
- Evidence points to fact that, in the U.S., implied correlations from option prices react

---

4 See for example, “Euro zone crisis hits euro, stocks for third day” by Caroline Valetkevitch, Reuters, July 12, 2011. Excerpt from this news: “The euro and stocks fell for a third day on Tuesday as moves by officials to stem the European debt crisis failed to allay concerns that the risk was spreading to Italy and Spain. Gold rallied to near its all-time high and U.S. bond prices rose as worries about contagion pushed investors into safe-haven assets.”
to relevant announcements regarding monetary policy. However that reaction does not follow a common pattern: in some cases the monetary policy stimulus (QE1, OT1) is followed by the “stressed” IC regime, in other cases (QE1 reinforcement, QE2) it is followed by the “normal” IC regime with decreasing implied correlations for all maturities, and in other cases (OT2) there seems to be no change at all in terms of IC dynamics and their term structure.

c. Correlation Risk Term structure

Figure 4 plots implied correlations and realized correlations for the full sample period of analysis considering three constant maturities: 30 days, 91 days and 365 days. Independent of the maturity, for almost the entire sample period it is apparent that Implied correlations exceed Realized Correlations. This suggests that for almost the entire sample period the realized correlation risk factor, measured as the difference between the realized and implied correlation, is negative. This is confirmed by the results in Tables 2 and 3.

Panel A of Table 2 reports implied correlations and realized correlations for the full sample period of analysis considering five constant maturities: 30 days, 60 days, 91 days, 182 days and 365 days. Panel A of Table 2 reports the 95% confidence intervals for the average implied correlation (IC) for all maturities considered. It suggests that the IC estimates for all maturities are relatively precisely estimated. Interestingly, there is no overlapping of the bounds of these confidence intervals, which further supports the maturity dependence of IC estimates and therefore the relevance of studying its term structure. Table 2 also shows the average correlation risk factor (CR) for each maturity, calculated as the difference between the average realized correlation (RC) and the average implied correlation (IC). The table shows that (i) there exists a significant average correlation risk at all maturities, between minus 7.68 percent for 30 days maturity and minus 14.22 percent for 365 days and (ii) the average correlation risk term
structure for the full sample period has a negative slope, i.e. the average correlation risk is more negative the higher is the maturity. Table 3 reports the implied correlations and realized correlations for different sub-sample periods considering the same five constant maturities: 30 days, 60 days, 91 days, 182 days and 365 days. The qualitative conclusions from Table 2 and 3 are the same.

[INSERT TABLE 2 ABOUT HERE]
[INSERT TABLE 3 ABOUT HERE]

The average level of the correlation risk factor and its term structure is not the only variable of interest to market participants. The correlation risk factor for any maturity can be expected to change throughout time, as Figure 4 shows, and the realized premia for different maturities do not necessarily comove. In other words, there is no guarantee that the correlation risk term structure does not flatten or even becomes positively sloped.

Panel A of Figure 5 plots the 6-month moving average correlation risk factor (CR) for the S&P500 Index, for the full sample period from 01/1996 to 01/2013 and for 30, 60, 91, 182 and 365 days. Panel B of Figure 5 plots exactly the same information but only for 30, 182 and 365 days.

[INSERT FIGURE 5 ABOUT HERE]

Panels A and B of Figure 5 illustrate the fact that the correlation risk factor for all maturities is negative during almost the entire sample period and that it changes significantly: for example, considering the 30 days maturity the correlation risk premium ranges between minus 27 percent and plus 6 percent. Additionally, Panels A and B of Figure 5 also illustrate the fact that the higher the maturity the larger is the range of values the correlation risk factor assumes.

The results in Panels A and B of Figure 5 suggest that the correlation risk term structure changes significantly. Considering the spreads between higher maturities’ correlation risk factor
and the 30 days correlation risk factor, as plotted in Figure 5C, three facts emerge. First, spreads between the correlation risk for higher maturities and the correlation risk for 30 days are negative during almost the entire sample. Second, for almost the entire sample period, that spread is more negative the higher is the longer maturity “leg” of the spread computation. Third, those spreads change significantly during the time period of the sample: in some moments they become positive and their relative position changes.

The first two facts imply that the higher the maturity the more negative is the correlation risk factor, i.e, the ex-post correlation risk premium term structure is negatively sloped for almost the entire subsample. This is consistent with the information reported in Panel A of Table 1 with respect to the average correlation risk factor for different maturities.

The third fact implies that there are some periods of time during which the 30 days correlation risk factor is more negative than the correlation risk factor for higher maturities. Panel C of Figure 5 shows that this happens essentially in periods of high turbulence in financial markets: around 9/11 terrorist attacks and the posterior economic depressing period in U.S., beginning of subprime crisis in the summer of 2007, around Lehman Brothers collapse in September 2008 and the peak of sovereign debt in Europe and also with the downgrade of U.S. sovereign debt. Recalling reported evidence in section IV.b about the dynamics of Implied Correlation, it is immediate to conclude that periods when the slope of IC term structure becomes negative tend to be periods where the slope of correlation risk factor term structure becomes positive. The corollary is that correlation risk factor term structure dynamics seems to be driven fundamentally by implied correlation term structure dynamics.

d. CBOE Implied Correlation Indices and their replication

We analyse the relationship between the estimated implied correlation described in section IV.a and IV.b with the CBOE Implied Correlation Indices described in section II. The driving force of our analysis is to understand if and how the CBOE Implied Correlation Indices can be replicated by synthetic correlation swap rates which, from evidence disclosed in section IV.a, we know replicate well the correlation swap rates from correlation swaps.
There are two major differences between the estimated synthetic correlation swap rates using option prices and the CBOE implied correlation rates. The first one is with respect to maturities. When estimating synthetic correlation swap rates there are constant maturities (30 days, 60 days, 91 days, 181 days, 365 days) while the CBOE Implied Correlation rates are for fixed maturities. The second one is with respect to the fact that CBOE Implied Correlation Indices only consider the sub-sample of 50 highest constituents, while in our estimate of synthetic correlation swaps we consider all constituents of the S&P 500 Index.

In order to replicate the CBOE Implied Correlation Indices, we first address the maturity issue. The purpose is to replicate the time series of implied correlation from “shorter maturity” CBOE Implied Correlation index and of implied correlation from “longer maturity” CBOE Implied Correlation Index. The replication strategy consists of choosing from the available synthetic swaps (30 days, 60 days, 91 days, 182 days and 365 days) the one whose maturity is closest to that day’s time to maturity of the “shorter maturity” CBOE Implied Correlation index and of the “longer maturity” CBOE Implied Correlation Index. Making this exercise for all trading days during the sub-sample period between January 2007, the starting date for CBOE Implied Correlation Index series, and January 2013 we obtain a time-series for the implied correlation corresponding to synthetic correlation swaps of different maturities. We compared this implied correlation time series of this replication strategy with that of CBOE Implied Correlation Indexes.

[ INSERT TABLE 4 ABOUT HERE ]

Panel A of Table 4 reports summary statistics for the time series of “shorter maturity” CBOE implied correlation and the time series of implied correlation associated with the replication strategy. The difference between implied correlation associated with the replication strategy and the CBOE implied correlation is named “replication error” and is also reported in the table. On average, the replication error with respect to the “shorter maturity” CBOE Implied
correlation index is minus 5.6%, i.e., the replication strategy implies on average a lower implied correlation than the CBOE Implied Correlation Index with shorter maturity. Importantly, the correlation between the two implied correlation time series is high at 93%, which suggests the replication strategy accurately reproduces the dynamics of “shorter maturity” CBOE Implied Correlation Index. This is illustrated in Panel A of Figure 6 where it is clear that (i) the two series co-move and (ii) that during most part of the sample the CBOE implied correlation is higher than that from the replication strategy.

[ INSERT FIGURE 6 ABOUT HERE ]

Panel B of Figure 6 compares the effective time to maturity of the “shorter maturity” CBOE correlation index and the maturity of the estimated synthetic correlation swap used in the replication strategy. It is apparent from the figure that during each year there are periods of time where the maturity of CBOE Correlation Index is higher than of the replication strategy and other periods were the opposite occurs. Panel C of Figure 6 plots the time series of the replication error and two facts emerge. First, for almost the entire sample the replication error is negative. This implies that replication strategy consistently underestimates the CBOE implied correlation. Second, during periods of higher turbulence, as in the summer 2007 when the subprime crisis began and especially the period following Lehman Brothers collapse in September 2008, the error from the replication strategy rises significantly.

Panel B of Table 4 reports summary statistics for the time series of “longer maturity” CBOE implied correlation and the time series of implied correlation associated with the replication strategy. The “replication error” is also reported in the Table. On average, the replication error with respect to the “longer maturity” CBOE Implied correlation index is minus 5.5%, which is very close to the one found for the replication of the “shorter maturity” CBOE implied correlation. The correlation between the time series for the two implied correlation measures is high at 89%, even if slightly below the one found for the shorter maturity CBOE implied correlation. Panel A of Figure 7 plots the corresponding time series and what is apparent
is the time series’ strong co-movement and the fact that in our sample the CBOE implied correlation is higher than that from the replication strategy. Panel B of Figure 7 shows the “composition” of the replication strategy (including which synthetic swap to use), and the differences over time between the CBOE Correlation Index maturities and those of the replication strategy. Panel C of Figure 7 plots the replication error: compared with the replication error for the “shorter maturity” there is a relatively higher period of time where the replication error is positive but during almost the entire sample the replication error is negative.

Our empirical findings regarding the replication of the CBOE Implied Correlation Indices can be summarized as follows.

- Both for the “shorter maturity” and “longer maturity” CBOE Implied Correlation Index, the replication strategy accurately reproduces the dynamics of the indexes, especially for the “shorter maturity”;
- Regarding levels, both for the “shorter maturity” and “longer maturity” CBOE Implied Correlation Index, the replication strategy underestimates CBOE indexes. Our prior to explain this difference is related to differences at the idiosyncratic volatility level: synthetic correlation swap rates are computed using options of the S&P500 Index and of all its constituents while CBOE Implied Correlation Indices only consider a sub-sample of the fifty largest constituents. This suggests that returns among larger capitalizations have, on average, higher correlation than returns of the market as a whole. This is confirmed in the robustness exercise presented in Section VI;
- Since the replication error increases in periods of enhanced uncertainty, this suggests that the difference between average correlations among large capitalizations stocks relative to the overall market increases in periods of equity market turbulence.
V. Correlation Hedging strategies

The empirical findings presented in the previous sections have implications for hedging strategies and motivate studying different trading strategies involving swap or option implied correlation. The importance of this is corroborated by the high persistence of correlation. We compare the autocorrelation properties of the implied correlation (IC), of the realized correlation (RC) and of the correlation risk factor (CR) for different maturities (30d, 91d, 182d and 365d) with those of the implied variance (IV), of the realized variance (RV) and of the variance risk factor (VR) for the same maturities. As illustrated in Figure 8, correlation measures are more persistent than the corresponding variances, for all maturities analyzed. For asset managers with sufficiently long holding periods and for policy makers in general, the relevant insight from this fact is that adverse correlation shocks in financial markets can be expected to have much more persistent effects than adverse variance shocks.

[[ INSERT FIGURE 8 ABOUT HERE ]]  

“The significance of implied correlation is that it reflects changes in the relative premium between index options and single-stock options, providing trading signals for a strategy known as volatility dispersion (correlation) trading. Commonly, a long volatility dispersion trade is characterized by selling at-the-money index option straddles and purchasing at-the-money straddles in options on index components. One interpretation of this strategy is that when implied correlation is high, index option premiums are rich relative to single-stock options. Therefore, it may be profitable to sell the rich index options and buy the relatively inexpensive equity options.” (CBOE (2009))

Does the data support the above statements? This statement highlights two common views of correlation trading:

1. Due to the negative values of the correlation risk factor, a profitable strategy is to short implied correlation in index options while being long in the realized correlation
considering the pairwise correlation among the components of the index;

2. A standard way of doing this is through a dispersion trade strategy, i.e., selling at-the-money index option straddles and purchasing at-the-money straddles in options on index components.

In this section we present results for three correlation hedging strategies. All are tested when applied conditionally to some conditioning variables and, for two of them, also when applied unconditionally. Two holding periods are considered: one month (30 days) and six months (180 days). All strategies are buy-and-hold during the holding period and hedging policies have the same maturity of the holding period. When applied conditionally, at the beginning of each (holding) period, time $t$, the investor decides to implement the hedging policy using the information available at time $t$ regarding a particular predictive variable. When applied unconditionally, there is a continuous rolling over of the hedge. The benchmark portfolio is that of an unhedged investor who invests only in the S&P500 Index.

The first correlation hedging strategy studied corresponds to a long position in a correlation swap contract (long the implied correlation (IC) in index options and short the realized correlation (RC)). The investor trading correlation with a hedging motive is looking for protection against correlation spikes, which occur in periods of turbulence, as the realized correlation dynamics plotted in Figure 4 show. Correlation spikes imply decreased diversification benefits. Based on the evidence in Figure 5 the correlation risk factor (RC – IC) is negative during almost the entire sample period. This implies that an investor that wishes to hedge correlation risk will effectively pay a premium, given by the correlation risk factor. The expectation is that in periods of increased realized correlation she is compensated in the form of higher payoffs.

The second hedging strategy tested consists of being long the implied variance (IV) of index options and short the realized variance (RV) of the index. This is equivalent to a long position in a variance swap contract. Why do we use index variance swap contracts if our focus is on hedging correlation risk? First, the ex-post variance risk premium is largely due to a ex-post correlation risk premium (Driessen, Maenhout and Vilkov (2009), Buraschi, Trojani and Vedolin
(2014)). For the full sample period of our analysis (from 01/1996 to 01/2013) the regression of the index variance risk factor (VR) for 30 days (182 days) against the correlation risk factor (CR) for 30 days (182 days) has a R-square of 30% (44%). Moreover, the correlation between the index VR and the CR series two series for 30 days (182 days) is 55% (66%). These statistics suggest that, although far from perfect, the index VR dynamics reflects partially the CR dynamics. Second, for the full sample period, the average level of CR for 30 days (182 days) is minus 7.68% (minus 13.46%), respectively (Panel A of Table 2), which compares with the average level of index VR for 30 days (182 days) of minus 0.9% (minus 1%).

This suggests that, in practice, the premium that the investor has to pay is much higher when using correlation swaps than when using variance swaps. Consequently, although variance swaps imperfectly reflect correlation dynamics, it may be the case that from a cost-benefit perspective it is a second best to use variance swaps to hedge correlation risk due to the significantly lower ex-post variance risk premium.

However, the cost of implementing such unconditional strategies may be prohibitive if the ex-post correlation risk premium or the ex-post variance risk premium is too high on average. This motivates the comparison of unconditional hedging strategies with conditional strategies that are conditional on some conditioning variable or signal. We study four signals (or criteria) to conditionally enter into a long position in a correlation or variance swap contract:

- **Criterion 1**, related to the IC term structure: activate the hedging strategy when the spread between the IC for 90 days and the IC for 30 days is lower than the spread between the six month moving average (6-MA) of the IC for 90 days and the IC for 30 days. The rational for this criterion is the evidence described in section IV.b;

- **Criterion 2**, related to the CR term structure: activate the hedging strategy when the CR for 30 days is higher than the CR for 180 days. The rational for this criterion is

---

5 We are not disclosing summary statistics for the index and individual implied variance, realized variance and ex-post variance risk premium (variance risk factor) due to space constrains. The same applies to the simple regression analysis between the variance risk factor and the correlation risk factor. However, all results are available upon request.

6 We tested the strategy payoffs for several alternative signals. Results are available upon request.
the evidence disclosed in section IV.c;

- **Criterion 3**, related to the level of CR for 30 days: activate the hedging strategy when CR for 30 days is higher than a threshold level defined as the 90th percentile level of CR for 30 days for the full sample period. The rational for this criterion is the evidence disclosed in section IV.c;

- **Criterion 4**, related to the information provided by the CBOE Implied Correlation Indices: activate the hedging strategy when the level of the CBOE Implied Correlation Index for the S&P500 Index with longer maturity is lower than that for shorter maturity. The rational supporting this criterion is the evidence disclosed in sections IV.b and IV.d;

However, even for the conditional implementation of those two hedging strategies there are caveats associated with the availability of suitably liquid correlation and variance swaps or options. If swaps were unavailable the question arises of whether options are available, especially for Index constituents, in order to synthetically replicate the correlation and variance swap rate.

This leads us to the third hedging strategy to be tested: can the investor use information about market correlation and market variance as a signal for asset allocation? Concretely, we test the return from a strategy, that we call the “Cash” Hedging strategy, that is 100% long in the S&P500 Index (risky asset) reducing this exposure by 50% and keeping the proceeds in cash conditional to some signals. The first group of signals make use of correlation related information. It includes the four criteria disclosed above and an additional one:

- **Criterion 5**, related with returns from the dispersion trade strategy: activate the hedging strategy when the dispersion trade strategy payoff (DT) is lower than the six month moving average (6-MA) of DT.

Dispersion trading strategies are popular among practitioners, because their implementation only requires traded options. However, one caveat is that in contrast to correlation swaps, a dispersion trade strategy is not a pure correlation strategy. The reason is that as the strategy is implemented through options in order to produce a payoff exclusively driven by
correlation it would be necessary to construct a continuous perfect hedge based on the options’ “greeks”. A continuous rebalancing of the options portfolio would be necessary but this is obviously unfeasible.

The second group of criteria to implement the “Cash” Hedging strategy, which uses variance related information, includes:

- **Criterion 6**, related with the IV term structure: activate the hedging strategy when the spread between the IV for 90 days and the IV for 30 days is lower than the spread between the six month moving average (6-MA) of the IV for 90 days and the IV for 30 days;
- **Criterion 7**, related with the VR term structure: activate the hedging strategy when the VRP for 30 days is higher than the VR for 180 days;
- **Criterion 8**, related with the level of VR for 30 days: activate the hedging strategy when VR for 30 days is higher than a threshold level defined as the 90th percentile level of VR 30 days for the full sample period;\(^7\)

### a. No Hedging versus Unconditional Hedging with Correlation and Variance Swaps

As reported in Table 5, the average annualized monthly return of an investor with a 30-day (182-day) holding period invested in the S&P500 Index between January 1996 and January 2013, without any hedging policy, would have been 6.54% (6.64%) with an annualized Sharpe ratio of 0.40 (0.39). How does this performance compare to that of an investor that is long the S&P500 Index and that employs an unconditional correlation hedging policy?

\( [[ \text{INSERT TABLE 5 ABOUT HERE} ]] \)

\(^7\) This criterion and criterion 3 makes use of information not available at date \( t \), as the 90th percentile can only be computed at the end of the time period to which the sample respects. However, the rational supporting the criterion is to have a sufficiently high threshold which can eventually be defined arbitrarily. An alternative, to guarantee that all criteria are based on information available at time \( t \), is to have a dynamic threshold level: for example, the 90th percentile of the year before \( t \). We will test this alternative in a next stage of this project.
Starting with the correlation swap contract based strategy, results reported in Panel A of Table 5 for the full sample period (01/1996 to 01/2013) show that if the investor with a 30-day (182-day) holding period was long in the S&P500 Index and unconditionally long in a 30-day (182-day) correlation swap contract, the average annualized monthly return would have been minus 4.52% (minus 7.66%) with an annualized Sharpe ratio of minus 0.32 (minus 0.63). Panel B of Table 5 reports results for an investor with a 30-day (182-day) holding period that is long in the S&P500 Index and unconditionally long in a 30-day (182-day) variance swap contract: the average annualized monthly return, considering the full sample period, would have been 5.59% (5.57%) with an annualized Sharpe ratio of 0.35 (0.37). For the sub-sample period from 01/2007 to 01/2012 results do not change significantly.

This set of results is complemented by evidence in Figure 9, which plots the cumulative growth of a $1 investment in the S&P 500 Index starting in January 1996 with (i) no hedging, (ii) unconditionally long a correlation swap contracts (30-days and 182-days in Panel A and B, respectively) and (iii) unconditionally long a variance swap contracts (30-days and 182-days in Panel A and B, respectively).

[[ INSERT FIGURE 9 ABOUT HERE ]] 

Considering the summary statistics reported in Table 5 and the dollar example in Figure 9, two conclusions emerge. First, an investor that is long in the S&P500 Index is better off, from a risk-adjusted return perspective, by not applying any hedging policy versus the alternative of being unconditionally hedged using either correlation or variance swaps. This means the premia (that the investor who is long swaps has to pay) are, on average, excessive relative to the payoffs that the hedge generates. This implies that only a conditional use of these correlation and variance swaps may be useful. Second, the underperformance generated by unconditional hedging strategies is much more expressive when making use of correlation swaps than when using variance swaps. We propose two explanations for this. First, the ex-post correlation risk premium is on average much higher than the ex-post variance risk premium, as previously
pointed out. An interesting question is why this is the case, in particular if this is essentially due to demand and supply forces. The answer to this question is beyond the scope of this paper and is left for future research. Second, as shown in Figure 8, correlation is a much more persistent variable than variance. This means that negative effects from an undesirable hedging policy, that is unconditionally active, persist much more when using correlation swaps than when using variance swaps.

If unconditional hedging policies underperform a strategy with no hedging, the next step is to evaluate the performance of those hedging policies when applied conditionally. In the next sub-section we report the results for this.

b. No Hedging versus Conditional Hedging with Correlation and Variance Swaps

Starting with the correlation swap based hedging strategy, the results reported in Panel A of Table 5 show that there is a clear improvement in the Sharpe ratio of the hedging strategies when applied conditionally versus the scenario when they are applied unconditionally. However, only when using the CRP 90th percentile signal ( Criterion 3) the conditional correlation hedging strategy generates a (slightly) better performance for the investor with no hedging, especially for the 30-day holding period. For all other signals, the conditional hedging strategies that employ correlation swaps continue to add no value to the investor that is long in the S&P500 Index. Broadly speaking, exactly the same applies for conditional strategies that make use of variance swaps (Panel B of Table 5).

These results about conditional hedging strategies based on correlation or variance swaps provide a note of caution. A further caveat is that transaction costs associated with these strategies, in the form of bid-ask spreads are not being taken into account, for simplicity. Transaction costs would reduce the performance of hedging strategies even further. In fact, the effective correlation swap rate (IC) or variance swap rate (IV) to be paid by the investor can increase significantly in periods of enhanced uncertainty when bid-ask spreads are expected to rise. These periods are likely to be those that are selected by different conditioning variables as
times to activate the hedging strategies. Related to this, as previously referred to, there are caveats associated with the availability of suitably liquid correlation and variance swaps or options. If swaps were unavailable the question arises of whether options are available, especially for Index constituents, in order to synthetically replicate the correlation and variance swap rate.

c. No Hedging versus “Cash” Hedging strategy

Results from the previous two sub sections V.a and V.b suggest that if the objective is to maximize risk-adjusted returns or the Sharpe ratio of the resulting portfolio, then directly trading in correlation and variance swaps may not be the most efficient correlation hedging policy. However, an investor seeking protection against adverse shocks during turbulent market periods may follow a different strategy. She could use information about market correlation and market variance as signals to trigger a reduction of the investment in risky assets and increase the cash position. This motivates the study of the “Cash” Hedging strategy that, as explained before, is 100% long in the S&P500 Index (risky asset) reducing this exposure by 50% and keeping the proceeds in cash conditional to some signals. Summary statistics of the “Cash” Hedging strategy using signals based on correlation related information (Criteria 1, 2, 3, 4 and 5) are disclosed in Panel A of Table 6 and based on variance related information (Criteria 6, 7 and 8) are disclosed in Panel B of Table 6.

[[ INSERT TABLE 6 ABOUT HERE ]]

Results reported in Panel A of Table 6 show that the “Cash” Hedging strategy using the correlation related information can, using the appropriate signals, clearly outperform a strategy with no hedging. For both holding periods and for both sample periods. Using the Sharpe ratio as an evaluation criterion, the “Cash” Hedging strategy that employs the signal based on the IC term structure (Criterion 1) outperforms the strategy with no hedging by reducing the standard deviation of returns. However, the “Cash” Hedging strategy making use of the signals based on
the CRP threshold level (Criterion 3) and signals based on the Dispersion trade strategies returns (Criterion 5), in most of the cases, reach higher average returns with lower standard deviations than the strategy with no hedging. For example, for the 30-day holding period investor, using the Dispersion Trade strategies returns as a signal, the average monthly return annualized of the "Cash" Hedging strategy in the full sample period would have been 7.62% (6.54% with no hedging), with annualized standard deviation of 12.88% (16.03% with no hedging) and corresponding annualized Sharpe ratio of 0.57 (0.40 with no hedging).

The results in Panel B of Table 6 show that the "Cash" Hedging strategy implemented using variance related information also outperforms, in most of the cases, the strategy with no hedging. Interestingly, when using the "Cash" Hedging strategy based on signals from the IV term structure (Criterion 6) and the VRP term structure (Criterion 7) the reason for the outperformance relative to the strategy with no hedging stems from the lower standard deviations of returns. When using a signal based on a threshold value for the VRP (Criterion 8) the same happens for a 182-day holding period investor but for a 30-day holding period investor higher returns are obtained with lower standard deviations.

These results suggest that, generally speaking, the performance of the "Cash" Hedging strategy when using correlation related information is equal or even better than when using variance related information. This is interesting given the underperformance reported in Table 5 for unconditional and conditional hedging strategies that make use of a "pure" correlation instrument as the correlation swap when compared when using a variance swap. This leads to the conclusion that for hedging purposes, although trading correlation directly through swaps seems to be unattractive, the dynamics of correlation related variables is very informative and useful for the design of hedging strategies.

Panel A of Figure 10 plots the cumulative growth of a $1 investment starting in January 1996 resulting from four strategies for the 30-day holding period investor: no hedging, "Cash" Hedging strategy using a signal based in the IC term structure (Criterion 1), "Cash" Hedging strategy using a signal based in the Dispersion Trade strategy returns (Criterion 5) and "Cash" Hedging strategy using a signal based in the IV term structure (Criterion 6). This evidence
complements and reinforces the conclusions obtained from Table 6. The strategy based on signals from Dispersion Trade strategy returns clearly outperforms the other strategies in terms of cumulative gains and, importantly, versus a “non-hedging” strategy with a less volatile path. The strategy based on the IC term structure, versus the “no hedging” strategy, posts a similar cumulative return dollar but with a smoother path during the sample period. And the strategy based on the IV term structure clearly underperforms in terms of cumulative returns versus all the remaining strategies. For a 182-day holding period investor, results are plotted in Panel B of Figure 10 for the same strategies with exception of the one based on dispersion trade results.\(^8\) The main conclusion is that the “Cash” Strategy based on the IC term structure generates the highest cumulative return. Compared with the “no hedging” strategy this is achieved keeping a smoother path.

The main insight from this section V.c is that by using the information content in both correlation and variance related variables it is possible to design simple and implementable strategies, based in risky asset/cash allocation, with higher returns and smoother paths than a strategy with no hedging. Moreover, the transaction costs associated with those strategies are expected to be much lower than those associated with conditional hedging strategies based on correlation and variance swaps explored in section V.b: the signals used to activate the hedging strategy make use of publicly available information and the investment decisions, which consists of selling or buying the risky asset (S&P500 Index), should imply lower transaction costs than trading on swaps, as the former should be significantly more liquid than the later.

d. CBOE Implied Correlation Indices and Hedging Strategies

As documented in section IV.d, the CBOE Implied Correlation Indices can be accurately

\(^8\) For our analysis we consider only options with maturities of 30 days to construct straddles. The reason is that for 182 days relevant data is missing.
replicated through synthetic correlation swap rates. Since both the correlation swaps and the options required to replicate the synthetic correlation swap rates may be illiquid, this is relevant for practitioners since the CBOE Implied Correlation Indices are publicly available which may facilitate the design of hedging instruments based on them. The CBOE already offers futures on the VIX and one could envisage futures and options on the CBOE Implied Correlation Indices being launched by exchanges in the future. Such products could significantly reduce the transaction costs associated with implementing correlation hedging strategies.

However, summary statistics reported in Tables 5 and 6 suggest that conditional hedging strategies based on information from the CBOE Implied Correlation Indices underperform the strategy without hedging and alternative conditional hedging strategies. Figure 11 plots the cumulative growth of a $1 investment starting in January 2007, date after which the CBOE Implied Correlation Indices are available, using different conditional hedging strategies based on signals extracted from the term structure embedded in the CBOE Implied Correlation Indices.

Panel A and Panel B of Figure 11 illustrate that the dynamics of returns from conditional hedging strategies. The CBOE Implied Correlation Indices signal, the one based on variance swaps and the “Cash” strategy are an almost perfect replication of the dynamics of returns from a strategy with no hedging. In the case of the conditional hedging strategy that invests in correlation swaps using the CBOE Implied Correlation Indices signal the underperformance is clear. One potential reason for this poor performance of CBOE Implied Correlation Indices as instruments to design hedging strategies may be related with the way maturities in these Indices is defined. In fact, having fixed maturity dates and not fixed time-windows (as for example the VIX index has – 30 days), certainly induces differences in the information content of the index. Therefore, it would be interesting to if the CBOE published additional Implied Correlation Indices for fixed time-windows, for example, 30 days, 91 days, 182 days and 730 days which would represent a continuously updated and publicly available IC term structure with fixed times
VI. Robustness check

In this section we present the main results of a robustness test which repeats the analysis for the 100 largest constituents of the S&P500 Index. The main purpose of this robustness test is to understand if the pattern of the correlation risk factor (that is its levels, dynamics and term structure) changes when the subsample of 100 largest constituents of the S&P500 Index is considered. Additionally, for the purposes of replication of the CBOE Implied Correlation Indices, which consider for the individual “leg” the largest 50 constituents of the S&P500 Index, our prior is that by using synthetic correlation swaps rates when making this filter (largest 100 constituents) versus when the full S&P 500 Index is considered (base case scenario presented in section IV.d) the replication of the CBOE Implied Correlation Indices should be even more accurate than the one described in section IV.d.

The pattern of the correlation risk factor with respect to dynamics and term structure does not change significantly versus when the full S&P500 Index is used. Figure 12 plots the six month moving average CRP using the largest 100 constituents of the S&P500 Index (for the individual leg) and, as it becomes clear when comparing with Figure 5, there are no material differences.

Table 12 reports summary statistics of the implied correlation (IC), the realized correlation (RC) and the correlation risk factor (CR) for all maturities (30d, 60d, 91d, 182d and 365d), for the full sample period (from 01/1996 to 01/2013) and for different sub-sample periods. Comparing with results in Table 2 and Table 3, for the full S&P500 Index, it is clear that the major difference when considering the subset of 100 largest constituents of the S&P500 Index respects to the levels of CR: although both the IC and RC increase for all maturities, the IC increases relatively more than RC implying that for all maturities the average CR becomes more
At last, regarding the accuracy of the replication exercise of the CBOE Implied Correlation Indices there is a slight improvement, as expected: for the shorter maturity the average replication IC is 0.589 (which compares with 0.5464 when using the full S&P500 Index – Panel A of Table 4) implying that the average replication error decreases from minus 0.0559 (Panel A of Table 4) to minus 0.0134; for the longer maturity the average replication IC is 0.5758 (which compares with 0.5613 when using the full S&P500 Index – Panel B of Table 4) implying that the average replication error decreases from minus 0.055 (Panel B of Table 4) to minus 0.0405.

VII. Concluding Remarks

This paper contributes to the recent literature on early warning indicators of financial stress and the equity market correlation risk premium. First, we document and compare the dynamics and key statistics of correlation risk measures using correlation swap rates, synthetic correlation swap rates and the publicly available CBOE Implied Correlation Indices. Second, we analyze and compare the term structure estimated using these different correlation risk measures. We also study how different the hedging properties of the CBOE Implied Correlation Indices are from those obtained from other implied correlation measures.

Our main empirical findings are as follows. First, both the level and the dynamics of the correlation swap quotes for S&P500 index for different maturities are accurately replicated by the synthetic correlation swap rates estimated from option prices. Second, during normal market regimes, the implied correlation term structure has a positive slope. When financial markets go through turbulent periods the implied correlation term structure becomes flat or even negatively sloped. Third, on average, the correlation risk factor term structure has a negative slope but changes significantly throughout time. Fourth, the CBOE Implied Correlation Indices and their
dynamics can be accurately replicated by means of synthetic correlation swap rates.

Moreover, we compare different correlation hedging strategies. We start to test strategies that are unconditionally long in a correlation swap contract or in a variance swap contract. Second, we test strategies that are long in a correlation or variance swap contract depending on specific trading signals. Third, we study a “Cash” Hedging strategy, which is also conditional and involves reducing the exposure to the S&P500 Index and increasing the allocation to cash, depending on specific trading signals. We conclude that only some conditional correlation hedging strategies add value. Among the conditional hedge strategy’s conditioning variables, the trading signals, we find that the level of the correlation risk premium and dispersion trade returns deliver the best results, while the CBOE Implied Correlation Indices perform poorly.

The theoretical literature about correlation risk is still in its infancy. To the best of our knowledge there is still no model that derives correlation risk endogenously for different maturities in a general equilibrium model. Moreover, the structural dynamic relationship between the correlation risk factor and its term structure, macroeconomic uncertainty and monetary policy hasn’t also been addressed in the literature. We plan to extend our analysis in these directions in a follow up paper.
References


Table 1: Synthetic Correlation Quotes vs Correlation Swap Quotes

The table reports in Panel A the summary statistics (time-series mean, median, 10th and 90th percentiles, and standard deviation) for (i) the synthetic correlation swap rate (S.IC), i.e., the implied correlation calculated from daily observations of model-free implied variances for the index and index components, using Equation (3) in the text, for four different maturities (30, 91, 182 and 365 (calendar) days) and for (ii) the correlation swap rates (Q.IC) directly obtained from correlation swaps with four different maturities (30, 91, 182 and 365 days). The sample period is from 03/2000 to 07/2012. We also report the correlation between S.IC and Q.IC time series. Panels B and C report the mean of S.IC and of Q.IC, as well as the correlation between them, for two sub-sample periods from 03/2000 to 12/2005 and from 01/2006 until 07/2012.

<table>
<thead>
<tr>
<th></th>
<th>30d</th>
<th>91d</th>
<th>182d</th>
<th>365d</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel A: 03/2000 - 07/2012</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Mean</strong></td>
<td>S.IC</td>
<td>Q.IC</td>
<td>S.IC - Q.IC</td>
<td>S.IC</td>
</tr>
<tr>
<td></td>
<td>Mean</td>
<td>0.4025</td>
<td>0.4014</td>
<td>0.0010</td>
</tr>
<tr>
<td></td>
<td>(t-stat)</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td><strong>Median</strong></td>
<td>0.3862</td>
<td>0.3832</td>
<td>0.4259</td>
<td>0.4225</td>
</tr>
<tr>
<td>10th Percentile</td>
<td>0.2207</td>
<td>0.2269</td>
<td>0.2615</td>
<td>0.2799</td>
</tr>
<tr>
<td>90th Percentile</td>
<td>0.6078</td>
<td>0.5990</td>
<td>0.6011</td>
<td>0.5861</td>
</tr>
<tr>
<td><strong>Std. Dev.</strong></td>
<td>0.1474</td>
<td>0.1446</td>
<td>0.1316</td>
<td>0.1162</td>
</tr>
<tr>
<td><strong>Correlation</strong></td>
<td><strong>0.9535</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>30d</th>
<th>91d</th>
<th>182d</th>
<th>365d</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel B: 03/2000 - 12/2005</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Mean</strong></td>
<td>S.IC</td>
<td>Q.IC</td>
<td>S.IC - Q.IC</td>
<td>S.IC</td>
</tr>
<tr>
<td></td>
<td>Mean</td>
<td>0.3297</td>
<td>0.3358</td>
<td>-0.0062</td>
</tr>
<tr>
<td></td>
<td>(t-stat)</td>
<td>0.02</td>
<td>0.02</td>
<td>0.02</td>
</tr>
<tr>
<td><strong>Correlation</strong></td>
<td><strong>0.9618</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>30d</th>
<th>91d</th>
<th>182d</th>
<th>365d</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel C: 01/2006 - 07/2012</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Mean</strong></td>
<td>S.IC</td>
<td>Q.IC</td>
<td>S.IC - Q.IC</td>
<td>S.IC</td>
</tr>
<tr>
<td></td>
<td>Mean</td>
<td>0.4675</td>
<td>0.4506</td>
<td>0.0169</td>
</tr>
<tr>
<td></td>
<td>(t-stat)</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td><strong>Correlation</strong></td>
<td><strong>0.9388</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
The table reports summary statistics (time-series mean, median, 10th and 90th percentiles, and standard deviation) for the implied correlation (IC), realized correlation (RC), and for the difference between them (RC - IC), for the S&P500 Index constituents. In Panel A, the sample period is from 01/1996 to 01/2013, and IC is calculated from daily observations of model-free implied variances for the index and index components, using Equation (3) in the text, for five different maturities (30, 60, 91, 182 and 365 (calendar) days). For each maturity, it is reported the 95% confidence interval for the IC estimate. In Panel B, the sample period is from 03/2000 to 07/2012, and IC is the correlation swap rate directly obtained from correlation swaps with five different maturities (30, 91, 182 and 365 days). In both Panels, RC is a cross-sectional weighted average (using the weights from the index) of all historical pairwise correlations at time t, each calculated over a 30, 60, 91, 182 and 365-day (calendar) window of daily stock returns.

### Panel A

<table>
<thead>
<tr>
<th></th>
<th>IC</th>
<th>RC</th>
<th>RC - IC</th>
</tr>
</thead>
<tbody>
<tr>
<td>01/1996 : 01/2013</td>
<td>30 60 91 182 365</td>
<td>30 60 91 182 365</td>
<td>30 60 91 182 365</td>
</tr>
<tr>
<td>Synthetic quotes</td>
<td>Mean</td>
<td>0.4018</td>
<td>0.3249</td>
</tr>
<tr>
<td></td>
<td>Median</td>
<td>0.3889</td>
<td>0.2961</td>
</tr>
<tr>
<td></td>
<td>10th Percentile</td>
<td>0.5935</td>
<td>0.5237</td>
</tr>
<tr>
<td></td>
<td>Std. Dev.</td>
<td>0.1408</td>
<td>0.1426</td>
</tr>
<tr>
<td>95% confidence interval</td>
<td>[0.3975;0.4060]</td>
<td>[0.4173;0.4253]</td>
<td>[0.4594;0.4670]</td>
</tr>
</tbody>
</table>

### Panel B

<table>
<thead>
<tr>
<th></th>
<th>IC</th>
<th>RC</th>
<th>RC - IC</th>
</tr>
</thead>
<tbody>
<tr>
<td>03/2000 : 07/2012</td>
<td>30 60 91 182 365</td>
<td>30 60 91 182 365</td>
<td>30 60 91 182 365</td>
</tr>
<tr>
<td>Swap quotes</td>
<td>Mean</td>
<td>0.4014</td>
<td>0.3473</td>
</tr>
<tr>
<td></td>
<td>Median</td>
<td>0.3832</td>
<td>0.3230</td>
</tr>
<tr>
<td></td>
<td>10th Percentile</td>
<td>0.2269</td>
<td>0.1718</td>
</tr>
<tr>
<td></td>
<td>Std. Dev.</td>
<td>0.1446</td>
<td>0.1495</td>
</tr>
<tr>
<td>Synthetic quotes</td>
<td>Mean</td>
<td>0.4025</td>
<td>0.3473</td>
</tr>
<tr>
<td></td>
<td>Median</td>
<td>0.3862</td>
<td>0.3230</td>
</tr>
<tr>
<td></td>
<td>10th Percentile</td>
<td>0.2207</td>
<td>0.1718</td>
</tr>
<tr>
<td></td>
<td>Std. Dev.</td>
<td>0.1474</td>
<td>0.1495</td>
</tr>
</tbody>
</table>
Table 3: Implied and Realized Correlations: Sub sample periods

The table reports the mean values for the implied correlation (IC), realized correlation (RC), and for the difference between them (RC - IC), for the S&P500 Index constituents and considering five sample periods (subsamples of the full sample period of 01/1996 and 01/2013). In Panel A, IC is calculated from daily observations of model-free implied variances for the index components, using Equation (3) in the text, for five different maturities (30, 60, 91, 182 and 365 (calendar) days). In Panel B, IC is the correlation swap rate directly obtained from correlation swaps with five different maturities (30, 91, 182 and 365 days). In both panels, RC is a cross-sectional weighted average (using the weights from the index) of all historical pairwise correlations at time t, each calculated over a 30, 60, 91, 182 and 365-day (calendar) window of daily stock returns.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>IC</td>
<td>RC</td>
<td>RC - IC</td>
<td></td>
<td>IC</td>
<td>RC</td>
<td>RC - IC</td>
<td></td>
<td>IC</td>
<td>RC</td>
</tr>
<tr>
<td>30</td>
<td>0.359</td>
<td>0.234</td>
<td>-0.125</td>
<td></td>
<td>0.354</td>
<td>0.310</td>
<td>-0.045</td>
<td></td>
<td>0.443</td>
<td>0.387</td>
</tr>
<tr>
<td>60</td>
<td>0.387</td>
<td>0.235</td>
<td>-0.152</td>
<td></td>
<td>0.372</td>
<td>0.307</td>
<td>-0.065</td>
<td></td>
<td>0.455</td>
<td>0.386</td>
</tr>
<tr>
<td>91</td>
<td>0.410</td>
<td>0.235</td>
<td>-0.175</td>
<td></td>
<td>0.386</td>
<td>0.305</td>
<td>-0.081</td>
<td></td>
<td>0.465</td>
<td>0.389</td>
</tr>
<tr>
<td>182</td>
<td>0.441</td>
<td>0.236</td>
<td>-0.205</td>
<td></td>
<td>0.400</td>
<td>0.303</td>
<td>-0.097</td>
<td></td>
<td>0.491</td>
<td>0.398</td>
</tr>
<tr>
<td>365</td>
<td>0.468</td>
<td>0.236</td>
<td>-0.232</td>
<td></td>
<td>0.405</td>
<td>0.303</td>
<td>-0.102</td>
<td></td>
<td>0.502</td>
<td>0.429</td>
</tr>
</tbody>
</table>

Syntethic Quotes

Swap Quotes

30   -  -  -  0.333  0.310  -0.024  0.434  0.387  -0.047  0.502  0.460  -0.043  -  -  -
60   -  -  -  -  -  -  -  -  -  -  -  -  -  -  -  -  -  -
91   -  -  -  0.371  0.305  -0.065  0.466  0.389  -0.077  0.532  0.469  -0.063  -  -  -
182  -  -  -  0.396  0.303  -0.093  0.483  0.398  -0.086  0.573  0.491  -0.082  -  -  -
365  -  -  -  0.402  0.303  -0.099  0.490  0.429  -0.061  0.597  0.510  -0.087  -  -  -
The table reports summary statistics (time-series mean, median, 10th and 90th percentiles, and standard deviation) for the implied correlation level given by each of the two CBOE Correlation Index (CBOE IC) available each day (shorter and longer maturity), for a replicating strategy of that CBOE Implied Correlation Index (Replication IC) and for the replication error (Replication IC - CBOE IC). This replication strategy consists in choosing, each day, a correlation swap with one of six maturities: the maturity chosen is the closest to the time to maturity period implied in the CBOE Implied Correlation Index. In Panel A, we considered the CBOE Implied Correlation Index with shorter maturity and the correlation swap is for one of five maturities: 30, 60, 91, 182 and 365 (calendar) days. In Panel B, we considered the CBOE Implied Correlation Index with longer maturity and the correlation swap is for one of six maturities: 30, 60, 91, 182, 365 and 730 (calendar) days. For the 30, 60, 91, 182 and 365 fixed maturities we used the synthetic correlation swap rate computed using Equation (3) in the text, and for the 730 fixed maturity we used the correlation swap quote for that maturity.

### Table 4: CBOE S&P500 Implied Correlation Index and Replication

#### Panel A : CBOE Implied Correlation Index with Shorter Maturity versus Replication

<table>
<thead>
<tr>
<th></th>
<th>CBOE IC Mean</th>
<th>Replication IC</th>
<th>Replication Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.6024</td>
<td>0.5464</td>
<td>-0.0559</td>
</tr>
<tr>
<td>Median</td>
<td>0.6137</td>
<td>0.5541</td>
<td>-0.0549</td>
</tr>
<tr>
<td>10th Percentile</td>
<td>0.4442</td>
<td>0.4141</td>
<td>-0.1108</td>
</tr>
<tr>
<td>90th Percentile</td>
<td>0.7450</td>
<td>0.6650</td>
<td>-0.0061</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>0.1132</td>
<td>0.0905</td>
<td>0.0434</td>
</tr>
</tbody>
</table>

**Correlation of time series** 0.93

#### Panel B : CBOE Implied Correlation Index with Longer Maturity versus Replication

<table>
<thead>
<tr>
<th></th>
<th>CBOE IC Mean</th>
<th>Replication IC</th>
<th>Replication Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.6163</td>
<td>0.5613</td>
<td>-0.0550</td>
</tr>
<tr>
<td>Median</td>
<td>0.6329</td>
<td>0.5598</td>
<td>-0.0630</td>
</tr>
<tr>
<td>10th Percentile</td>
<td>0.4491</td>
<td>0.4526</td>
<td>-0.1269</td>
</tr>
<tr>
<td>90th Percentile</td>
<td>0.7548</td>
<td>0.6706</td>
<td>0.0263</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>0.1142</td>
<td>0.0802</td>
<td>0.0561</td>
</tr>
</tbody>
</table>

**Correlation of time series** 0.89
Table 5: Hedging policies using correlation and variance swap contracts

The table reports annualized returns, standard deviations and Sharpe ratios for different trading strategies. Two holding periods are considered for all strategies: 30 days and 182 days. Results are based on non-overlapping monthly and semester returns, corresponding to the two holding periods. The maturity of hedging strategies equals the investor holding period. Two sample periods are considered: from 01/1996 to 01/2013 and from 01/2007 to 01/2013. "No Hedging" represents the exposure to the S&P500 Index without any hedging. In Panel A, Unconditional Hedging is a strategy long in the S&P500 Index and in a correlation swap contract with maturity equal to the holding period. Conditional Hedging strategies are long in the S&P500 Index and in a correlation swap contract with maturity equal to the holding period but the investor only activates a long position in the correlation swap contract conditional to the following criteria: 

Criterion 1: Spread between implied correlation (IC) 90 days and IC 30 days is lower than the spread between six month moving average (6-MA) of IC 90 days and 6-MA IC 30 days; 
Criterion 2: Correlation Risk factor (CR), measured as the difference between realized correlation (RC) and IC, for 30 days is higher than CR for 180 days; 
Criterion 3: CR 30 days is higher than the 90th percentile level (01/1996 - 01/2013 and 01/2007 - 01/2013 sample periods); 
Criterion 4: CBOE S&P500 Implied Correlation Index for longer maturity is lower than for shorter maturity.

Panel A: Correlation Swap Contracts

<table>
<thead>
<tr>
<th>Holding Period</th>
<th>No Hedging</th>
<th>Unconditional</th>
<th>Conditional</th>
<th>Conditional</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>01/1996 - 01/2013</td>
<td>01/2007 - 01/2013</td>
<td></td>
</tr>
<tr>
<td>Return</td>
<td></td>
<td>30d 182d</td>
<td>30d 182d</td>
<td>30d 182d</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td></td>
<td>16.03% 16.59%</td>
<td>14.54% 12.68%</td>
<td>15.33% 14.69%</td>
</tr>
<tr>
<td>Sharpe ratio</td>
<td></td>
<td>0.40 0.39</td>
<td>-0.32 -0.63</td>
<td>0.01 0.13</td>
</tr>
</tbody>
</table>

Panel B: Variance Swap Contracts

<table>
<thead>
<tr>
<th>Holding Period</th>
<th>No Hedging</th>
<th>Unconditional</th>
<th>Conditional</th>
<th>Conditional</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>01/1996 - 01/2013</td>
<td>01/2007 - 01/2013</td>
<td></td>
</tr>
<tr>
<td>Return</td>
<td></td>
<td>30d 182d</td>
<td>30d 182d</td>
<td>30d 182d</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td></td>
<td>16.03% 16.59%</td>
<td>15.39% 14.55%</td>
<td>15.52% 14.82%</td>
</tr>
<tr>
<td>Sharpe ratio</td>
<td></td>
<td>0.40 0.39</td>
<td>0.35 0.37</td>
<td>0.39 0.46</td>
</tr>
</tbody>
</table>
Table 6: "Cash" Hedging strategy

The table reports annualized returns, standard deviations and Sharpe ratios for different trading strategies. Two holding periods are considered: 30 days and 182 days. Results are based on non-overlapping monthly and semester returns, corresponding to the two holding periods. The maturity of hedging strategies equals the investor holding period. Two sample periods are considered: from 01/1996 to 01/2013 and from 01/2007 to 01/2013. "No Hedging" represents the exposure to the S&P500 Index without any hedging. In Panel A, Conditional Hedging strategies are long in the S&P500 Index with this exposure being reduced to 50%, with the remaining 50% allocated to cash, conditional on the following criteria related with correlation information:

**Criterion 1**: Spread between implied correlation (IC) 90 days and IC 30 days is lower than the spread between six month moving average (6-MA) of IC 90 days and 6-MA IC 30 days;

**Criterion 2**: Correlation Risk factor (CR), measured as the difference between realized correlation (RC) and IC, for 30 days is higher than CR for 180 days;

**Criterion 3**: CR 30 days is higher than the 90th percentile level (01/1996 - 01/2013 and 01/2007 - 01/2013 sample periods);

**Criterion 4**: CBOE S&P500 Implied Correlation Index for longer maturity is lower than for shorter maturity;

The dispersion trade strategy return (DT) is lower than the 6-MA DT. The implemented dispersion trade strategy consists in selling at-the-money index option straddles and purchasing at-the-money straddles in options on index components, with all options having maturity of 30-days. In Panel B, Conditional Hedging strategies are long in the S&P500 Index with this exposure being reduced to 50%, with the remaining 50% allocated to cash, conditional on the following criteria related with variance information:

**Criterion 6**: Spread between implied variance (IV) 90 days and IV 30 days is lower than the spread between 6-MA of IV 90 days and 6-MA IV 30 days;

**Criterion 7**: Variance Risk factor (VR), measured as the difference between realized variance (RV) and IV, for 30 days is higher than VR for 180 days;

**Criterion 8**: VR 30 days is higher than the 90th percentile level (01/1996 - 01/2013 and 01/2007 - 01/2013 sample periods).

<table>
<thead>
<tr>
<th>No Hedging</th>
<th>Panel A: Using market correlation information</th>
</tr>
</thead>
<tbody>
<tr>
<td>Criteria 1</td>
<td>01/1996 - 01/2013</td>
</tr>
<tr>
<td>Holding Period</td>
<td>30d</td>
</tr>
<tr>
<td>Return</td>
<td>6.54%</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>16.03%</td>
</tr>
<tr>
<td>Sharpe ratio</td>
<td>0.40</td>
</tr>
<tr>
<td>Panel B: Using market variance information</td>
<td></td>
</tr>
<tr>
<td>Criteria 6</td>
<td>01/1996 - 01/2013</td>
</tr>
<tr>
<td>Holding Period</td>
<td>30d</td>
</tr>
<tr>
<td>Return</td>
<td>6.54%</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>16.03%</td>
</tr>
<tr>
<td>Sharpe ratio</td>
<td>0.40</td>
</tr>
</tbody>
</table>

| Criteria 7 | 01/1996 - 01/2013 | 01/2007 - 01/2013 |
| Holding Period | 30d | 182d | 30d | 182d | 30d | 182d | 30d | 182d |
| Return | 2.52% | 4.52% | 2.85% | 3.95% | -0.95% | 1.29% | 8.24% | 4.31% |
| Std. Dev. | 17.78% | 19.38% | 12.27% | 11.25% | 15.07% | 17.30% | 15.25% | 19.39% |
| Sharpe ratio | 0.14 | 0.23 | 0.23 | 0.35 | -0.06 | 0.07 | 0.52 | 0.22 |

| Criteria 8 | 01/1996 - 01/2013 | 01/2007 - 01/2013 |
| Holding Period | 30d | 182d | 30d | 182d | 30d | 182d | 30d | 182d |
| Return | 17.78% | 19.38% | 13.67% | 14.07% | 13.12% | 12.51% | 15.04% | 19.38% |
| Std. Dev. | 17.78% | 19.38% | 13.67% | 14.07% | 13.12% | 12.51% | 15.04% | 19.38% |
| Sharpe ratio | 0.14 | 0.23 | 0.15 | 0.45 | 0.20 | 0.46 | 0.45 | 0.31 |
Table 7: Implied and Realized Correlations: Summary Statistics (100 largest constituents of S&P500 Index)

The table reports summary statistics (time-series mean, median, 10th and 90th percentiles, and standard deviation) for the implied correlation (IC), realized correlation (RC), and for the difference between them (RC - IC), for the S&P500 Index constituents (100 largest constituents). In Panel A, the sample period is from 01/1996 to 01/2013, and IC is calculated from daily observations of model-free implied variances for the index and index components (100 largest constituents), using Equation (3) in the text, for five different maturities (30, 60, 91, 182 and 365 (calendar) days). For each maturity, it is reported the 95% confidence interval for the IC estimate. In Panel B, the sample period is from 03/2000 to 07/2012, which is the period for which there are correlation swaps quotes available. In Panel C, five subsample periods, within the full sample period from 01/1996 to 01/2013, are considered. In all Panels, RC is a cross-sectional weighted average (reweighting the weights of the 100 largest constituents from the index) of all historical pairwise correlations at time t, each calculated over a 30, 60, 91, 182 and 365-day (calendar) window of daily stock returns.

### Panel A

<table>
<thead>
<tr>
<th></th>
<th>IC</th>
<th>RC</th>
<th>RC - IC</th>
<th></th>
<th>IC</th>
<th>RC</th>
<th>RC - IC</th>
<th></th>
<th>IC</th>
<th>RC</th>
<th>RC - IC</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>0.4327</td>
<td>0.4499</td>
<td>0.4634</td>
<td>0.4891</td>
<td>0.5000</td>
<td>0.3378</td>
<td>0.3373</td>
<td>0.3382</td>
<td>0.3425</td>
<td>0.3490</td>
<td>-0.0949</td>
</tr>
<tr>
<td>95% confidence interval</td>
<td>[0.4280,0.4373]</td>
<td>[0.4456,0.4542]</td>
<td>[0.4592,0.4676]</td>
<td>[0.4830,0.4932]</td>
<td>[0.4962,0.5039]</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Median</td>
<td>0.4160</td>
<td>0.4440</td>
<td>0.4632</td>
<td>0.4809</td>
<td>0.4873</td>
<td>0.3107</td>
<td>0.3124</td>
<td>0.3183</td>
<td>0.3155</td>
<td>0.3334</td>
<td>-0.0869</td>
</tr>
<tr>
<td>10th Percentile</td>
<td>0.2424</td>
<td>0.2673</td>
<td>0.2844</td>
<td>0.3085</td>
<td>0.3334</td>
<td>0.1719</td>
<td>0.1896</td>
<td>0.2007</td>
<td>0.2101</td>
<td>0.2145</td>
<td>-0.2461</td>
</tr>
<tr>
<td>90th Percentile</td>
<td>0.6487</td>
<td>0.6465</td>
<td>0.6408</td>
<td>0.6691</td>
<td>0.6681</td>
<td>0.5383</td>
<td>0.5195</td>
<td>0.5037</td>
<td>0.5166</td>
<td>0.5482</td>
<td>0.0473</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>0.1548</td>
<td>0.1448</td>
<td>0.1400</td>
<td>0.1359</td>
<td>0.1274</td>
<td>0.1430</td>
<td>0.1314</td>
<td>0.1257</td>
<td>0.1204</td>
<td>0.1159</td>
<td>0.1247</td>
</tr>
</tbody>
</table>

### Panel B

<table>
<thead>
<tr>
<th></th>
<th>IC</th>
<th>RC</th>
<th>RC - IC</th>
<th></th>
<th>IC</th>
<th>RC</th>
<th>RC - IC</th>
<th></th>
<th>IC</th>
<th>RC</th>
<th>RC - IC</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>0.4386</td>
<td>0.4514</td>
<td>0.4615</td>
<td>0.4830</td>
<td>0.4891</td>
<td>0.3563</td>
<td>0.3556</td>
<td>0.3573</td>
<td>0.3642</td>
<td>0.3642</td>
<td>0.1773</td>
</tr>
<tr>
<td>Median</td>
<td>0.4196</td>
<td>0.4441</td>
<td>0.4599</td>
<td>0.4696</td>
<td>0.4707</td>
<td>0.3324</td>
<td>0.3378</td>
<td>0.3414</td>
<td>0.3448</td>
<td>0.3488</td>
<td>0.3858</td>
</tr>
<tr>
<td>10th Percentile</td>
<td>0.2362</td>
<td>0.2598</td>
<td>0.2729</td>
<td>0.2842</td>
<td>0.2955</td>
<td>0.1737</td>
<td>0.1919</td>
<td>0.2119</td>
<td>0.2349</td>
<td>0.2424</td>
<td>-0.2413</td>
</tr>
<tr>
<td>90th Percentile</td>
<td>0.6689</td>
<td>0.6580</td>
<td>0.6517</td>
<td>0.6736</td>
<td>0.6737</td>
<td>0.5573</td>
<td>0.5569</td>
<td>0.5218</td>
<td>0.5527</td>
<td>0.5547</td>
<td>0.0600</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>0.1643</td>
<td>0.1531</td>
<td>0.1460</td>
<td>0.1402</td>
<td>0.1295</td>
<td>0.1505</td>
<td>0.1389</td>
<td>0.1325</td>
<td>0.1257</td>
<td>0.1162</td>
<td>0.1236</td>
</tr>
</tbody>
</table>

### Panel C

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Days</td>
<td>IC</td>
<td>RC</td>
<td>RC - IC</td>
<td>IC</td>
<td>RC</td>
<td>RC - IC</td>
<td>IC</td>
<td>RC</td>
<td>RC - IC</td>
<td>IC</td>
</tr>
<tr>
<td>30</td>
<td>0.3681</td>
<td>0.2548</td>
<td>-0.1133</td>
<td>0.3803</td>
<td>0.3217</td>
<td>-0.0587</td>
<td>0.4877</td>
<td>0.3941</td>
<td>-0.0936</td>
<td>0.6116</td>
</tr>
<tr>
<td>60</td>
<td>0.3938</td>
<td>0.2561</td>
<td>-0.1376</td>
<td>0.3972</td>
<td>0.3191</td>
<td>-0.0781</td>
<td>0.4989</td>
<td>0.3936</td>
<td>-0.1053</td>
<td>0.6211</td>
</tr>
<tr>
<td>91</td>
<td>0.4144</td>
<td>0.2560</td>
<td>-0.1584</td>
<td>0.4107</td>
<td>0.3176</td>
<td>-0.0993</td>
<td>0.5074</td>
<td>0.3977</td>
<td>-0.1097</td>
<td>0.6275</td>
</tr>
<tr>
<td>182</td>
<td>0.4452</td>
<td>0.2579</td>
<td>-0.1873</td>
<td>0.4245</td>
<td>0.3152</td>
<td>-0.1093</td>
<td>0.5313</td>
<td>0.4068</td>
<td>-0.1245</td>
<td>0.6647</td>
</tr>
<tr>
<td>365</td>
<td>0.4750</td>
<td>0.2566</td>
<td>-0.2163</td>
<td>0.4272</td>
<td>0.3142</td>
<td>-0.1130</td>
<td>0.5558</td>
<td>0.4392</td>
<td>-0.0966</td>
<td>0.6720</td>
</tr>
</tbody>
</table>
Figure 1: Implied Correlation Term Structure

The figure plots the 6-month moving averages of the implied correlation (IC) for the S&P500 Index for the period from 01/1996 to 01/2013. The implied correlation is calculated from daily observations of model-free implied variances for the index and index components, using Equation (3) in the text. Each model-free implied variance is calculated from 30, 60, 91, 182 and 365 days options. Panel A shows a plot of IC for all maturities. Panel B plots the daily series of the spread between IC for 365 days and IC for 30 days, IC for 182 days and IC for 30 days and IC for 91 days and IC for 30 days.

Panel A: Implied Correlations for different maturities

Panel B: Spreads between Implied correlations for different maturities
Panel A of this figure plots the 6-month moving averages of the implied correlation (IC) for the S&P500 Index from 01/1996 to 01/2013. The implied correlation is calculated from daily observations of model-free implied variances for the index and index components, using Equation (3) in the text. Each model-free implied variance is calculated from 30, 60, 91, 182 and 365 days options. In Panel B, for the same sample period, the figure plots the daily series of the spread between the 6-month moving average Implied Correlation for the S&P500 Index (IC) for 365 days and IC for 30 days, IC for 182 days and IC for 92 days and IC for 30 days.
Panel A of this figure plots the 6-month moving averages of the implied correlation (IC) for the S&P500 Index from 01/2007 to 01/2013. The implied correlation is calculated from daily observations of model-free implied variances for the index and index components, using Equation (3) in the text. Each model-free implied variance is calculated from 30, 60, 91, 182 and 365 days options. In Panel B, for the same sample period, the figure plots the daily series of the spread between the 6-month moving average Implied Correlation for the S&P500 Index (IC) for 365 days and IC for 30 days, IC for 182 days and IC for 30 days and IC for 92 days and IC for 30 days.

Panel A: Implied Correlation and US Monetary Policy events

Panel B: Implied Correlation maturity spreads
The figure plots the 6-month moving averages of the implied correlation (IC) and realized correlation (RC) for the S&P500 Index for the period from 01/1996 to 01/2013. The implied correlation is calculated from daily observations of model-free implied variances for the index and index components, using Equation (3) in the text. Each model-free implied variance is calculated from 30 days (Panel A), 91 days (Panel B) and 365 days (Panel C) options. The realized correlation (RC) is a cross-sectional weighted average (using the weights from the index) of all historical pairwise correlations at time t, each calculated over a 30, 91, and 365-day (calendar) window of daily stock returns.
This figure plots the 6-month moving average Correlation Risk Factor (CR) for the S&P500 Index, for the sample period from 01/1996 to 01/2013, using index and index constituents options with fixed maturity of 30, 60, 91, 182 and 365 days. At date t, the correlation risk factor is the difference between the implied correlation (IC) at date t for each maturity and realized correlation (RC) at date t for the time window with the same maturity. The implied correlation (IC) is calculated from daily observations of model-free implied variances for the index and index components, using Equation (3) in the text. The realized correlation (RC) is a cross-sectional weighted average (using the weights from the index) of all historical pairwise correlations at time t, each calculated over a 30, 60, 91, 182 and 365-day (calendar) window of daily stock returns. Panel A of this figure plots the CR for 30, 60, 91, 182 and 365 days; Panel B plots the CR for 30, 182 and 365 days and Panel C plots the spread between CR for 365 days and CR for 30 days, between CR for 182 days and CR for 30 days and between CR for 91 days and 30 days.
Panel A of this figure plots the daily series of implied correlation level given by the CBOE Implied Correlation Index (CBOE IC) available each day with shorter maturity, and the implied correlation for a replicating strategy of that CBOE Implied correlation index (Replication IC). This replication strategy consists in choosing, each day, a correlation swap with one of five maturities: 30, 60, 91, 182 and 365 (calendar) days. The maturity chosen is the closest to the time to maturity period implied in the CBOE Implied Correlation Index. Replication IC is obtained using the model-free implied variances for the index components, using Equation (3) in the text. It is also plotted the 6 month moving average of CBOE IC and Replication IC: CBOE IC 6M-MA and Replication IC 6M-AM. Panel B plots the daily series of rolling time to maturity of CBOE Implied Correlation Index and the constant maturity of the chosen correlation swap for the replication strategy. Panel C plots the daily series of the replication error, measured as the daily difference between the Replication IC and the CBOE IC in percentage. The sample period is from 01/2007 until 01/2013.
Figure 7: Replication of CBOE S&P500 Index Implied Correlation (Longer Maturity)

Panel A of this figure plots the daily series of implied correlation level given by the CBOE Implied Correlation Index (CBOE IC) available each day with shorter maturity, and the implied correlation for a replicating strategy of that CBOE Implied correlation index (Replication IC). This replication strategy consists in choosing, each day, a correlation swap with one of six maturities: 30, 60, 91, 182, 365 and 730 (calendar) days. The maturity chosen is the closest to the time to maturity period implied in the CBOE Implied Correlation Index. Replication IC is obtained using the model-free implied variances for the index components, using Equation (3) in the text. For 730 days, IC is the correlation swap rate of 2Y Correlation quotes. Panel A also plots the 6 month moving average of CBOE IC and Replication IC: CBOE IC 6M-MA and Replication IC 6M-AM. Panel B plots the daily series of rolling time to maturity of CBOE Implied Correlation Index and the constant maturity of the chosen correlation swap for the replication strategy. Panel C plots the daily series of the replication error, measured as the daily difference between the Replication IC and the CBOE IC in percentage. The sample period is from 01/2007 until 01/2013.
This figure plots the autocorrelation properties of the correlation risk factor (CR), variance risk factor (VR), implied correlation (IC), implied variance (IV), realized correlation (RC) and realized variance (RV) for different maturities: 30d, 91d, 182d and 365d. At date t, the correlation risk factor (CR) is the difference between the realized correlation (RC) at date t for each maturity and the implied correlation (IC) at date t for the time window with the same maturity. The implied correlation is calculated from daily observations of model-free implied variances for the index and index components, using Equation (3) in the text. The realized correlation (RC) is a cross-sectional weighted average (using the weights from the index) of all historical pairwise correlations at time t, each calculated over a 30, 91, 182 and 365-day (calendar) window of daily stock returns. At date t, the variance risk factor (VR) is the difference between the realized variance (RV) at date t for each maturity and implied variance (IV) at date t for the time window with the same maturity. The implied variance (IV) is computed using Equation (4) in the text, and realized variance (RV) at time t is calculated over a 30, 91, 182 and 365-day (calendar) window of daily index returns.
This figure plots the cumulative growth of a $1 investment starting in January 1996. "No Hedging" strategy is invested in the S&P500 Index without any hedging. "Unconditional Hedge" with correlation swap is a strategy long in the S&P500 Index and long in a correlation swap contract with maturity equal to the holding period (receives the realized correlation and pays the correlation swap rate). "Unconditional Hedge" with variance swap is a strategy long in the S&P500 Index and long in a variance swap contract with maturity equal to the holding period (receives the realized variance and pays the variance swap rate). Two holding periods are considered for all strategies: 30 days (Panel A) and 182 days (Panel B).
This figure plots the cumulative growth of a $1 investment starting in January 1996. "No Hedging" strategy is invested in the S&P500 Index without any hedging. Conditional Hedging strategies are long in the S&P500 Index with this exposure being reduced to 50%, with the remaining 50% allocated to cash, conditional on the following three criteria: 1) spread between implied correlation (IC) 90 days and IC 30 days is lower than the spread between six month moving average (6-MA) of IC 90 days and 6-MA IC 30 days (Criterion 1 in the text); 2) spread between implied variance (IV) 90 days and IV 30 days is lower than the spread between 6-MA of IV 90 days and 6-MA IV 30 days (Criterion 6 in the text); 3) the dispersion trade strategy return (DT) is lower than the 6-MA DT (Criterion 5 in the text). The implemented dispersion trade strategy consists in selling at-the-money index option straddles and purchasing at-the-money straddles in options on index components, with all options having maturity of 30-days. Two holding periods are considered for all strategies: 30 days (Panel A) and 182 days (Panel B).
This figure plots the cumulative growth of a $1 investment starting in January 2007. "No Hedging" strategy is invested in the S&P500 Index without any hedging. **Three different conditional hedging strategies** are presented. All are long in the S&P500 Index and, conditional on the CBOE S&P500 Implied Correlation Index for longer maturity being lower than for shorter maturity, the strategy becomes (1) long in a correlation swap contract with maturity equal to the holding period (receives the realized correlation and pays the correlation swap rate), (2) long in a variance swap contract with maturity equal to the holding period (receives the realized variance and pays the variance swap rate) and (3) reduce the investment in the S&P500 Index by 50%, with the remaining 50% allocated to cash. Two holding periods are considered for all strategies: 30 days (Panel A) and 182 days (Panel B).

**Panel A: Holding period: 30 days**

**Panel B: Holding Period: 182 days**
This figure plots the 6-month moving average Correlation risk factor (CR) for the S&P500 Index, for the sample period from 01/1996 to 01/2013, using index and index constituents options (100 highest weights) with fixed maturity of 30, 60, 91, 182 and 365 days. At date \( t \), the correlation risk factor is the difference between the implied correlation (IC) at date \( t \) for each maturity and realized correlation (RC) at date \( t \) for the time window with the same maturity. The implied correlation (IC) is calculated from daily observations of model-free implied variances for the index and index components (100 highest weights), using Equation (3) in the text. The realized correlation (RC) is a cross-sectional weighted average (rebasing the 100 highest weights from the index) of all historical pairwise correlations at time \( t \), each calculated over a 30, 60, 91, 182 and 365-day (calendar) window of daily stock returns. Panel A of this figure plots the CR for 30, 60, 91, 182 and 365 days; Panel B plots the CR for 30, 182 and 365 days and Panel C plots the spread between CR for 365 days and CR for 30 days, between CR for 182 days and CR for 30 days and between CR for 91 days and 30 days.