Stock Market Mean Reversion and Portfolio Choice over the Life Cycle

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- How does stock market predictability affect life cycle asset allocation in the presence of undiversifiable labor income risk and potentially binding liquidity constraints?
- Does "standard" portfolio advice (proliferation of lifestyle funds) continue to hold in the presence of stock market predictability?
- If not, how should advice be modified?

Literature Review

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- Our paper: stock market predictability with labor income over the life cycle with Epstein-Zin-Weil preferences with some evidence that preference specification captures well stockholder preferences
- I.i.d. stock returns model a special case

Model

- Epstein-Zin-Weil preference parameters

$$V_t = \left\{ (1-\beta) C_t^{1-1/\psi} + \beta \left(E_t \left(p_{t+1} V_{t+1}^{1-\gamma} + b(1-p_{t+1}) X_{t+1}^{1-\gamma} \right) \right)^{\frac{1-1/\psi}{1-\gamma}} \right\}$$

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Model: Labor Income

$$Y_{it} = Y_{it}^{p} U_{it}$$

$$Y_{it}^{p} = \exp(g(t, Z_{it}))\, Y_{it-1}^{p} N_{it}$$

Constant replacement rate, no pensions uncertainty (see Bagliano, Fugazza and Nicodano, 2014)

- Hump shape over the life cycle captured by $g(t, Z_{it})$
- Permanent labor income shocks N_{it}
- Tranistory labor income shocks U_{it}

Model: Mean Reversion

$$r_{t+1} - r_f = f_t + z_{t+1}$$

$$f_{t+1} = \mu + \phi(f_t - \mu) + \varepsilon_{t+1}$$

- Negative correlation between (z_{t+1}) and (ε_{t+1})
- IID model: $r_{t+1} - r_f = \mu + z_{t+1}$.
- Important to undertand how correlation structure (especially with permanent labor income shocks, $\ln N_{it}$) affects optimal behavior.

Solution Method: Normalized Value Function

$$x_{it+1} = \frac{Y^p_{it}}{Y^p_{it+1}}(r_{t+1}s_{it} + r_f b_{it}) + U_{it+1}$$

and

$$v_{it}(x_{it}, f_t) = \Bigg[(1-\beta)c_{it}^{1-1/\psi} + \beta\left\{E_t\left[\begin{array}{c} p_{t+1}(v_{it+1}(x_{it+1}, f_{t+1}))^{1-\gamma}(Y^p_{it+1}/Y^p_{it})^{1-\gamma} + \\ b(1-p_{t+1})(x_{it+1})^{1-\gamma}(Y^p_{it+1}/Y^p_{it})^{1-\gamma} \end{array}\right]\right\}^{\frac{1-1/\psi}{1-\gamma}}\Bigg]^{\frac{1}{1-1/\psi}}$$

Approximation of a VAR

- Stack four variables: factor (f_t), stock returns (r_t), permanent component of labor income $\ln N_{it}$, transitory component of labor income $\ln U_{it}$
- Tauchen and Hussey (1991) approximation with sufficient fineness of the grids to accurately capture dynamics of the VAR without making the problem computationally intractable
- Parallelization
- Experiments in the Appendix of paper to capture persistence of factor
- 15 gridpoints for factor, 20 gridpoints for r_t, 5 gridpoints for lnN_t, 3 gridpoints for lnU_t
- 251 gridpoints for continuous state variable (x_{it})

Baseline Calibration

Annual, discrete time, partial equilibrium model

r_f	0.02
μ	0.04
σ_u	0.1
σ_n	0.1
β	0.95
ψ	0.5
γ	5

Baseline Calibration

Factor and correlations

ϕ	0.91
$\rho_{z,\varepsilon}$	-0.8
$\rho_{n,z}$	0.15
$\rho_{n,\varepsilon}$	0.0
σ_{ε}	.000034
σ_{z}	0.18

- Factor uncertainty
- Correlation between stock return and factor innovations
- Correlation between permanent labor income and stocks return shocks
- Correlation between permanent labor income and factor innovations: any empirical evidence?

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Hedging demands

Mean reversion relative to i.i.d. model:

$$hedg(\rho_{z,\varepsilon} = -0.8) = 100 * \{\frac{\alpha(\rho_{z,\varepsilon} = -0.8) - \alpha(I.I.D.)}{\alpha(I.I.D.)}\}$$

Hedging demands from changing correlations within the mean reversion model

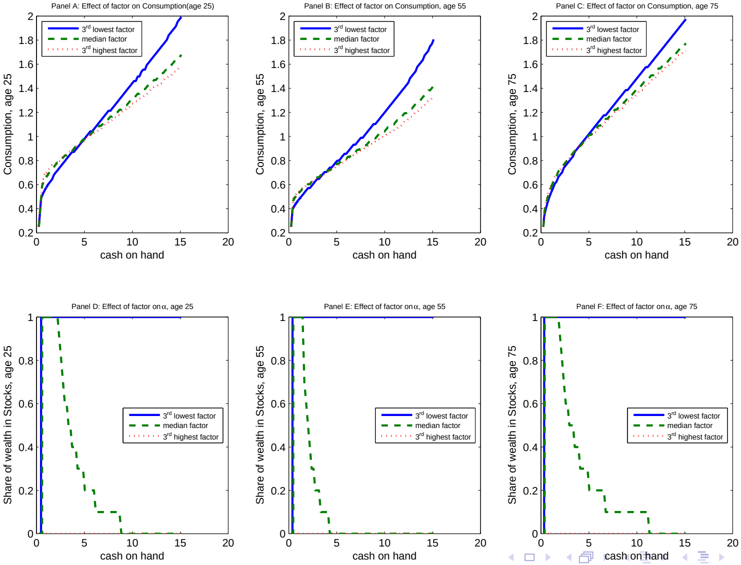
$$hedg(\rho_{z,\varepsilon} = -0.8)_{\text{factor}} = 100 * \{\frac{\alpha(\rho_{z,\varepsilon} = 0.0) - \alpha(\rho_{z,\varepsilon} = -0.8)}{\alpha(\rho_{z,\varepsilon} = -0.8)}\}$$

Hedging demands conditional on initial realization of factor

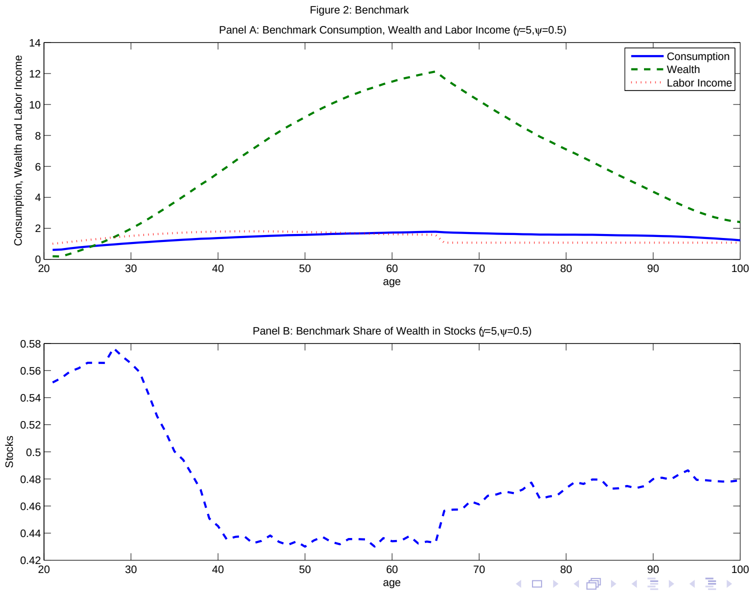
$$hedg(f(t) = i) = 100 * \{\frac{\alpha(f(t) = i) - \alpha(I.I.D.)}{\alpha(I.I.D.)}\}$$

Baseline Results: Policy Functions

Figure 1: Policy Functions: Benchmark case (γ=5,ψ=0.5)

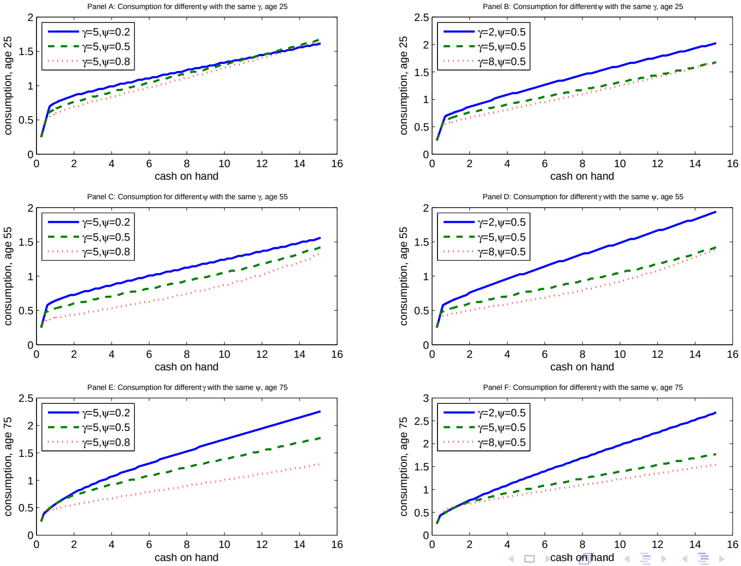


Baseline Results: Average Simulations



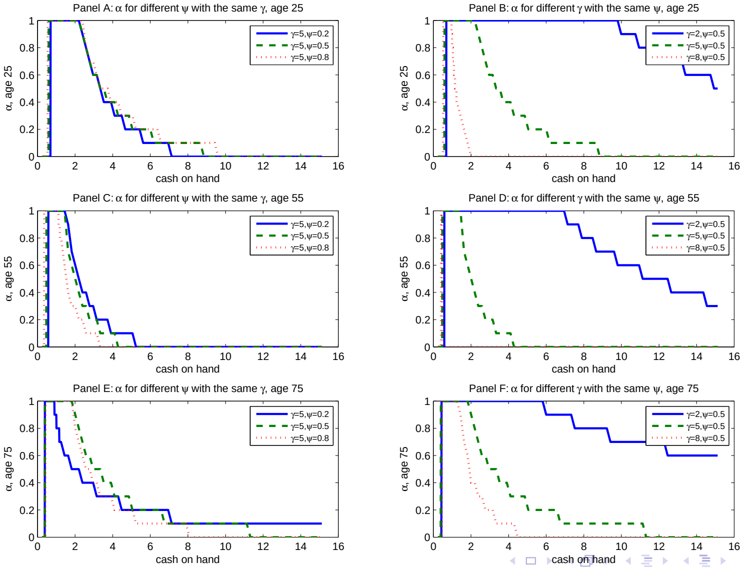
Consumption for different preference parameters

Figure 3: Consumption policy functions for different preference parameters



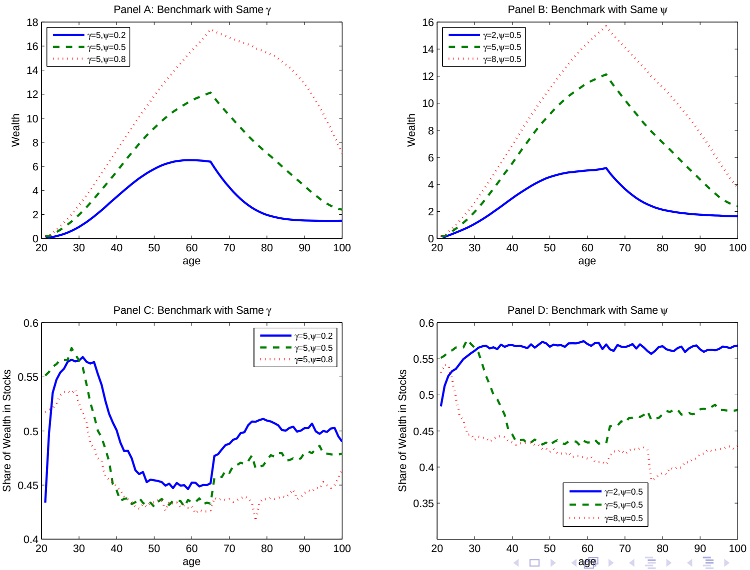
Alpha for different preference parameters

Figure 4: Share of wealth in stocks for different preference parameters



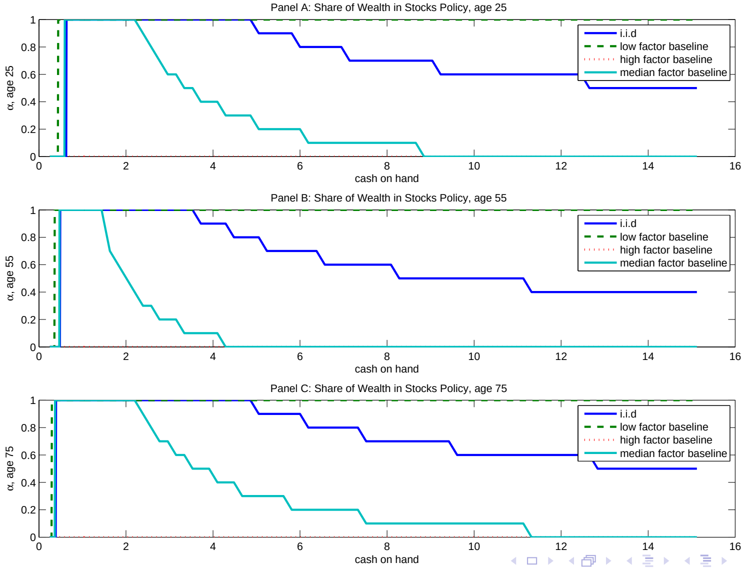
Simulations for different preference parameters

Figure 5: Benchmark with different parameters

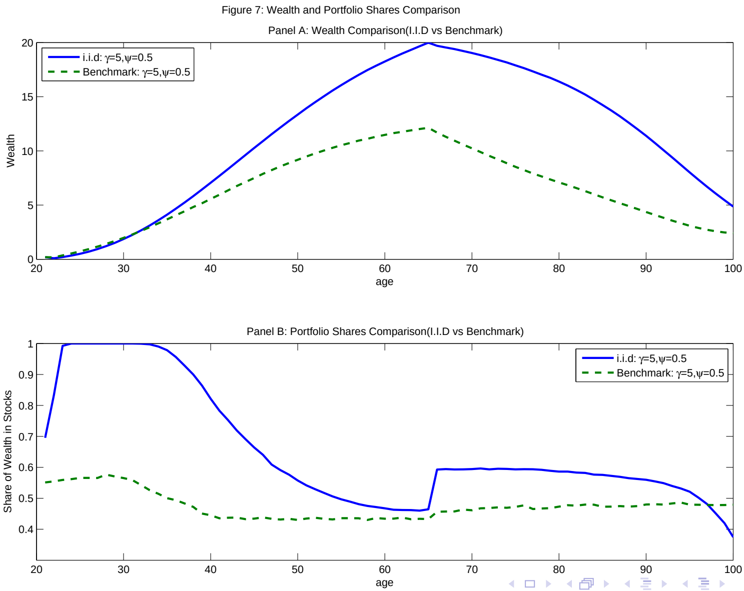


Comparison with i.i.d. model: share of wealth in stocks

Figure 6: Share of Wealth in Stocks Policy Functions: Mean Reversion vs I.I.D. case

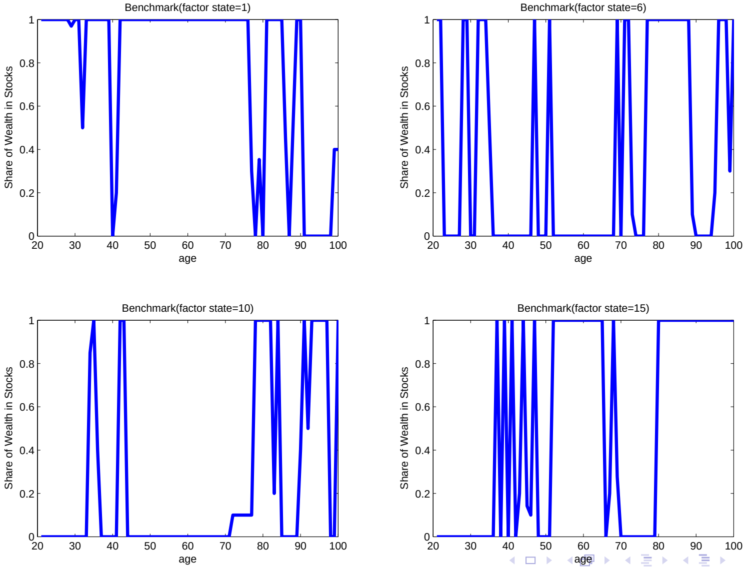


Comparison with i.i.d. model: average life cycle profiles

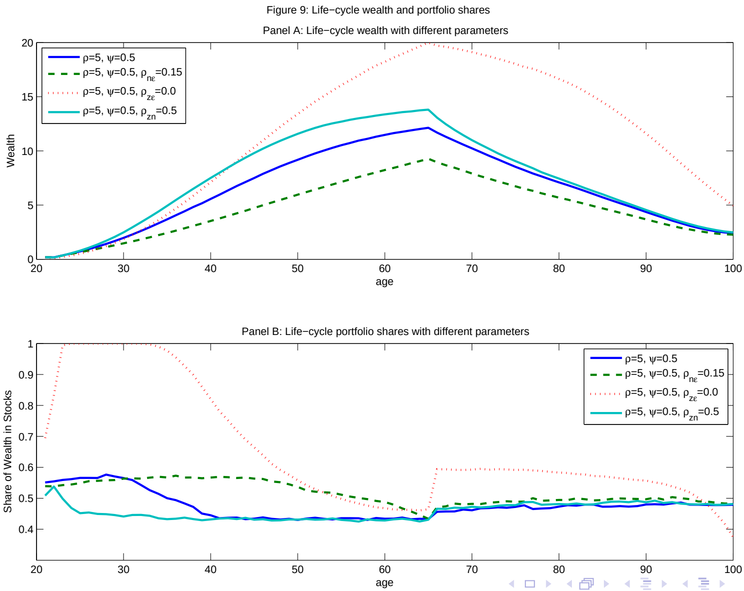


Baseline Results: Typical Simulations

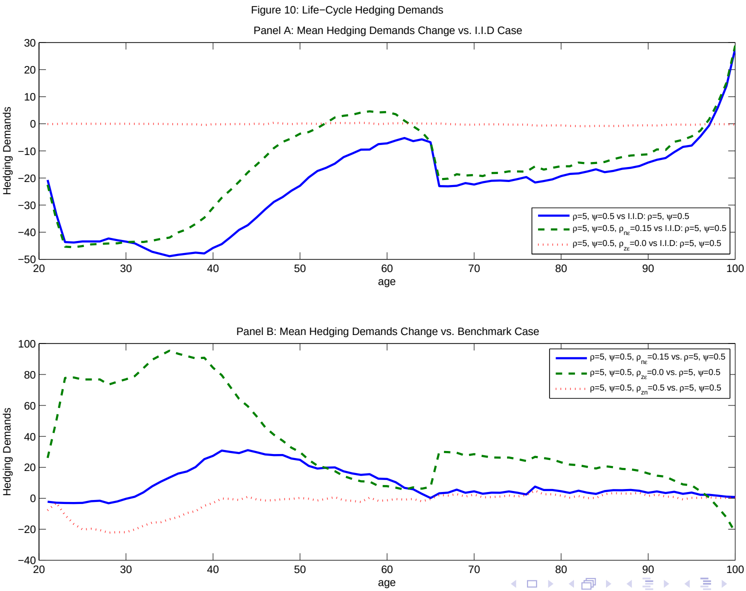
Figure 8: Life−cycle simulated share of wealth in stocks for different initial factor



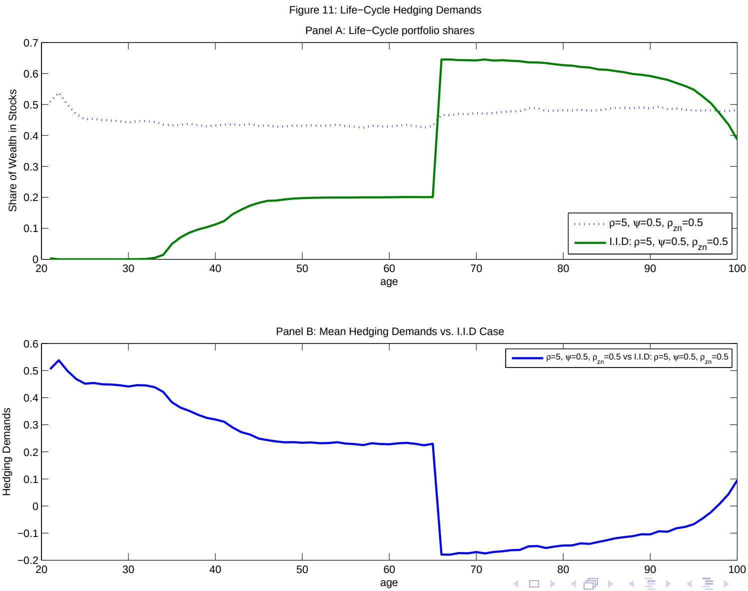
Life-cycle profiles for different correlations



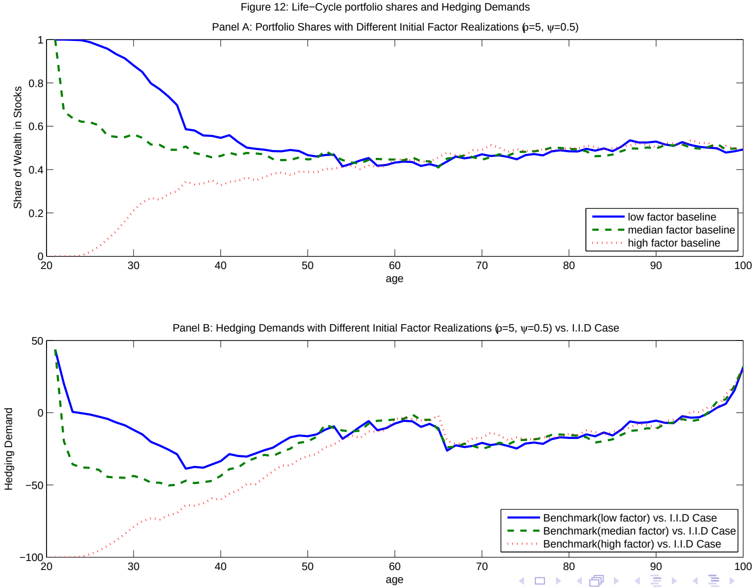
Hedging demands: relative to i.i.d. model



Hedging demands for correlation(N,Z)=0.5



Baseline Results: Conditional Simulations



How does factor affect results?

- What are implications for life style funds?
- Last graph casts doubt on conventional wisdom that households should decrease share of wealth in stocks as households approach retirement
- Application: should retirees have followed the lifestyle funds advice in 2008?

Conclusion

- Strategic asset allocation in presence of labor income risk
- Hedging demands can be substantial
- Important to understand factor dynamics
- Some doubt on optimality of lifestyle funds without regards to market conditions or expectations