Stock Market Mean Reversion and Portfolio Choice over the Life Cycle

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Goal

- Normative paper about life cycle asset allocation
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- How does stock market predictability affect life cycle asset allocation in the presence of undiversifiable labor income risk and potentially binding liquidity constraints?
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Does “standard” portfolio advice (proliferation of lifestyle funds) continue to hold in the presence of stock market predictability?
Goal

- Normative paper about life cycle asset allocation
- How does stock market predictability affect life cycle asset allocation in the presence of undiversifiable labor income risk and potentially binding liquidity constraints?
- Does “standard” portfolio advice (proliferation of lifestyle funds) continue to hold in the presence of stock market predictability?
- If not, how should advice be modified?
Literature Review

- Infinite horizon with stock market predictability models but without labor income risk (Brennan, Schwartz and Lagnado (1997), Campbell and Viceira (1999))

- Life cycle models without stock market predictability replicate financial analyst advice (Cocco, Gomes and Maenhout (2005), Gomes and Michaelides (2005))

- Dynamic models featuring cointegration (Benzoni, Collin-Dufresne and Goldstein (2007)), or time-variation in labor income risk (Lynch and Tan, 2011), or time variation in interest rates (Munk and Sorensen (2010) and Koijen, Nijman and Werker (2009)), or inflation (Brennan and Xia, 2002)

- Our paper: stock market predictability with labor income over the life cycle with Epstein-Zin-Weil preferences with some evidence that preference specification captures well stockholder preferences

I.i.d. stock returns model a special case
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- I.i.d. stock returns model a special case
Epstein-Zin-Weil preference parameters

\[ V_t = \left\{ (1 - \beta) C_t^{1-1/\psi} + \beta \left( E_t(p_{t+1} V_{t+1}^{1-\gamma} + b(1 - p_{t+1}) X_{t+1}^{1-\gamma}) \right)^{\frac{1-1/\psi}{1-\gamma}} \right\} \]
Epstein-Zin-Weil preference parameters

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- \( \beta \) is discount factor

\( \psi \) is elasticity of intertemporal substitution

\( \gamma \) is relative risk aversion coefficient

\( p_t + 1 \) is conditional next period survival probability

\( b \) captures bequest motive
Model

- Epstein-Zin-Weil preference parameters

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- \( X_{t+1} \) is bequest motive captured by
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- \(\beta\) is discount factor
- \(\psi\) is elasticity of intertemporal substitution
- \(\gamma\) is relative risk aversion coefficient
- \(p_{t+1}\) conditional next period survival probability
- bequest motive captured by \(b\)
Model: Labor Income

\[ Y_{it} = Y_{it}^P U_{it} \]

\[ Y_{it}^P = \exp(g(t, Z_{it})) Y_{it-1}^P N_{it} \]

Constant replacement rate, no pensions uncertainty (see Bagliano, Fugazza and Nicodano, 2014)

- Hump shape over the life cycle captured by \( g(t, Z_{it}) \)
- Permanent labor income shocks \( N_{it} \)
- Tranistory labor income shocks \( U_{it} \)
Model: Mean Reversion

\[ r_{t+1} - r_f = f_t + z_{t+1} \]

\[ f_{t+1} = \mu + \phi(f_t - \mu) + \varepsilon_{t+1} \]

- Negative correlation between \((z_{t+1})\) and \((\varepsilon_{t+1})\)
- IID model: \(r_{t+1} - r_f = \mu + z_{t+1}\).
- Important to understand how correlation structure (especially with permanent labor income shocks, \(\ln N_{it}\)) affects optimal behavior.
Solution Method: Normalized Value Function

\[ x_{it+1} = \frac{Y_{it}^p}{Y_{it+1}^p} (r_{t+1}s_{it} + r_f b_{it}) + U_{it+1} \]

and

\[ v_{it}(x_{it}, f_t) = \left[ (1 - \beta)c_{it}^{1-1/\psi} + \right. \]

\[ \beta \left\{ E_t \left[ p_{t+1}(v_{it+1}(x_{it+1}, f_{t+1}))^{1-\gamma}(Y_{it+1}^p / Y_{it}^p)^{1-\gamma} + \right. \right. \]

\[ b(1 - p_{t+1})(x_{it+1})^{1-\gamma}(Y_{it+1}^p / Y_{it}^p)^{1-\gamma} \right\}^{\frac{1-1/\psi}{1-\gamma}} \]
Approximation of a VAR

- Stack four variables: factor ($f_t$), stock returns ($r_t$), permanent component of labor income $\ln N_{it}$, transitory component of labor income $\ln U_{it}$
- Tauchen and Hussey (1991) approximation with sufficient fineness of the grids to accurately capture dynamics of the VAR without making the problem computationally intractable
- Parallelization
- Experiments in the Appendix of paper to capture persistence of factor
- 15 gridpoints for factor, 20 gridpoints for $r_t$, 5 gridpoints for $\ln N_t$, 3 gridpoints for $\ln U_t$
- 251 gridpoints for continuous state variable ($x_{it}$)
Annual, discrete time, partial equilibrium model

\[
\begin{align*}
  r_f & = 0.02 \\
  \mu & = 0.04 \\
  \sigma_u & = 0.1 \\
  \sigma_n & = 0.1 \\
  \beta & = 0.95 \\
  \psi & = 0.5 \\
  \gamma & = 5
\end{align*}
\]
Baseline Calibration

Factor and correlations

\[ \begin{align*}
\phi & = 0.91 \\
\rho_{z,\varepsilon} & = -0.8 \\
\rho_{n,z} & = 0.15 \\
\rho_{n,\varepsilon} & = 0.0 \\
\sigma_{\varepsilon} & = 0.000034 \\
\sigma_{z} & = 0.18
\end{align*} \]

- Factor uncertainty
- Correlation between stock return and factor innovations
- Correlation between permanent labor income and stocks return shocks
- Correlation between permanent labor income and factor innovations: any empirical evidence?
Factor and correlations

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi$</td>
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</tr>
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- **Factor uncertainty**
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Hedging demands

Mean reversion relative to i.i.d. model:

\[ hedg(\rho_{z,\varepsilon} = -0.8) = 100 \times \left\{ \frac{\alpha(\rho_{z,\varepsilon} = -0.8) - \alpha(I.I.D.)}{\alpha(I.I.D.)} \right\} \]

Hedging demands from changing correlations within the mean reversion model

\[ hedg(\rho_{z,\varepsilon} = -0.8)_{\text{factor}} = 100 \times \left\{ \frac{\alpha(\rho_{z,\varepsilon} = 0.0) - \alpha(\rho_{z,\varepsilon} = -0.8)}{\alpha(\rho_{z,\varepsilon} = -0.8)} \right\} \]

Hedging demands conditional on initial realization of factor

\[ hedg(f(t) = i) = 100 \times \left\{ \frac{\alpha(f(t) = i) - \alpha(I.I.D.)}{\alpha(I.I.D.)} \right\} \]
Baseline Results: Policy Functions

Figure 1: Policy Functions: Benchmark case ($\gamma=5, \psi=0.5$)

- Panel A: Effect of factor on Consumption, age 25
- Panel B: Effect of factor on Consumption, age 55
- Panel C: Effect of factor on Consumption, age 75
- Panel D: Effect of factor on $\alpha$, age 25
- Panel E: Effect of factor on $\alpha$, age 55
- Panel F: Effect of factor on $\alpha$, age 75

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Baseline Results: Average Simulations

Figure 2: Benchmark

Panel A: Benchmark Consumption, Wealth and Labor Income ($\gamma=5, \psi=0.5$)

Panel B: Benchmark Share of Wealth in Stocks ($\gamma=5, \psi=0.5$)

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Figure 3: Consumption policy functions for different preference parameters

Panel A: Consumption for different $\psi$ with the same $\gamma$, age 25
- $\gamma = 5$, $\psi = 0.2$
- $\gamma = 5$, $\psi = 0.5$
- $\gamma = 5$, $\psi = 0.8$

Panel B: Consumption for different $\psi$ with the same $\gamma$, age 25
- $\gamma = 2$, $\psi = 0.5$
- $\gamma = 5$, $\psi = 0.5$
- $\gamma = 8$, $\psi = 0.5$

Panel C: Consumption for different $\psi$ with the same $\gamma$, age 55
- $\gamma = 5$, $\psi = 0.2$
- $\gamma = 5$, $\psi = 0.5$
- $\gamma = 5$, $\psi = 0.8$

Panel D: Consumption for different $\gamma$ with the same $\psi$, age 55
- $\gamma = 2$, $\psi = 0.5$
- $\gamma = 5$, $\psi = 0.5$
- $\gamma = 8$, $\psi = 0.5$

Panel E: Consumption for different $\gamma$ with the same $\psi$, age 75
- $\gamma = 5$, $\psi = 0.2$
- $\gamma = 5$, $\psi = 0.5$
- $\gamma = 5$, $\psi = 0.8$

Panel F: Consumption for different $\gamma$ with the same $\psi$, age 75
- $\gamma = 2$, $\psi = 0.5$
- $\gamma = 5$, $\psi = 0.5$
- $\gamma = 8$, $\psi = 0.5$
Figure 4: Share of wealth in stocks for different preference parameters

Panel A: \( \alpha \) for different \( \psi \) with the same \( \gamma \), age 25

Panel B: \( \alpha \) for different \( \gamma \) with the same \( \psi \), age 25

Panel C: \( \alpha \) for different \( \psi \) with the same \( \gamma \), age 55

Panel D: \( \alpha \) for different \( \gamma \) with the same \( \psi \), age 55

Panel E: \( \alpha \) for different \( \psi \) with the same \( \gamma \), age 75

Panel F: \( \alpha \) for different \( \gamma \) with the same \( \psi \), age 75
Simulations for different preference parameters

Figure 5: Benchmark with different parameters

Panel A: Benchmark with Same $\gamma$

Panel B: Benchmark with Same $\psi$

Panel C: Benchmark with Same $\gamma$

Panel D: Benchmark with Same $\psi$

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Comparison with i.i.d. model: share of wealth in stocks

Figure 6: Share of Wealth in Stocks Policy Functions: Mean Reversion vs I.I.D. case

Panel A: Share of Wealth in Stocks Policy, age 25

Panel B: Share of Wealth in Stocks Policy, age 55

Panel C: Share of Wealth in Stocks Policy, age 75
Comparison with i.i.d. model: average life cycle profiles

Figure 7: Wealth and Portfolio Shares Comparison

Panel A: Wealth Comparison (I.I.D vs Benchmark)

- i.i.d.: $\gamma = 5, \psi = 0.5$
- Benchmark: $\gamma = 5, \psi = 0.5$

Panel B: Portfolio Shares Comparison (I.I.D vs Benchmark)

- i.i.d.: $\gamma = 5, \psi = 0.5$
- Benchmark: $\gamma = 5, \psi = 0.5$

Wealth and Portfolio Shares Comparison

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Baseline Results: Typical Simulations

Figure 8: Life-cycle simulated share of wealth in stocks for different initial factor

Benchmark(factor state=1)

Benchmark(factor state=6)

Benchmark(factor state=10)

Benchmark(factor state=15)
Life-cycle profiles for different correlations

Figure 9: Life-cycle wealth and portfolio shares

Panel A: Life-cycle wealth with different parameters

Panel B: Life-cycle portfolio shares with different parameters

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Hedging demands: relative to i.i.d. model

Figure 10: Life-Cycle Hedging Demands

Panel A: Mean Hedging Demands Change vs. I.I.D Case

Panel B: Mean Hedging Demands Change vs. Benchmark Case
Hedging demands for correlation \((N,Z)=0.5\)

Figure 11: Life−Cycle Hedging Demands

Panel A: Life−Cycle portfolio shares

- \(\rho = 5, \psi = 0.5, \rho_{zn} = 0.5\)
- I.I.D: \(\rho = 5, \psi = 0.5, \rho_{zn} = 0.5\)

Panel B: Mean Hedging Demands vs. I.I.D Case

- \(\rho = 5, \psi = 0.5, \rho_{zn} = 0.5\) vs I.I.D: \(\rho = 5, \psi = 0.5, \rho_{zn} = 0.5\)
Baseline Results: Conditional Simulations

Figure 12: Life-Cycle portfolio shares and Hedging Demands

Panel A: Portfolio Shares with Different Initial Factor Realizations ($\rho=5$, $\psi=0.5$)
- low factor baseline
- median factor baseline
- high factor baseline

Panel B: Hedging Demands with Different Initial Factor Realizations ($\rho=5$, $\psi=0.5$) vs. I.I.D Case
- Benchmark(low factor) vs. I.I.D Case
- Benchmark(median factor) vs. I.I.D Case
- Benchmark(high factor) vs. I.I.D Case
How does factor affect results?

- What are implications for life style funds?
- Last graph casts doubt on conventional wisdom that households should decrease share of wealth in stocks as households approach retirement
- Application: should retirees have followed the lifestyle funds advice in 2008?
Conclusion

- Strategic asset allocation in presence of labor income risk
- Hedging demands can be substantial
- Important to understand factor dynamics
- Some doubt on optimality of lifestyle funds without regards to market conditions or expectations