Time-Inconsistent Preferences, Borrowing Costs, and Social Security

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Summary

- Individuals have time-inconsistent preferences
  - Hyperbolic discounting
- Social security can act as a commitment device
- Assumption about credit markets
  - Complete ✓
  - Totally missing ✓
  - Borrowing is costly (credit spreads) ?
Summary

- Individuals have time-inconsistent preferences
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  - Borrowing is costly (credit spreads) ✓
Credit spreads

\[ r(k(t)) = \begin{cases} 
  r_B, & \text{if } k(t) < 0, \\
  r_S, & \text{if } k(t) > 0,
\end{cases} \]

where \( r_B \geq r_S \).

Approximation

\[ r(k(t)) \approx r_S - \frac{r_S - r_B}{1 + \exp[\psi k(t)]}, \]

where \( \psi \) is a large, positive scaler.
Model

\[ r(k) \]

\[ r_B \]

\[ r_S \]

\[ (0, (r_B + r_S)/2) \]
Model

- Consumer Optimization Problem

\[
\max_{c(\nu)} \int_t^{\bar{T}} F(\nu - t) u[c(\nu)] d\nu,
\]

subject to

\[
\frac{d k(\nu)}{d \nu} = r(k(\nu)) k(\nu) + y(\nu) - c(\nu),
\]

\[
y(\nu) = \begin{cases} 
(1 - \tau) w, & \text{for } \nu \in [t, T], \\
 b, & \text{for } \nu \in [T, \bar{T}],
\end{cases}
\]

\[
r(k(t)) = r_S - \frac{r_S - r_B}{1 + \exp[\psi k(t)]},
\]

\[k(t) \text{ given, } k(\bar{T}) = 0.\]
Welfare

Theorem

*Of the full spectrum of credit spreads ranging from zero (perfect credit markets) to infinity (missing credit markets), a fully-funded social security arrangement is irrelevant *only* at the knife edge of perfect credit markets.*

Welfare Metric

\[
g(\tau) \equiv \sqrt{\int_0^{\bar{T}} [c^*(t|\tau) - c_0(t)]^2 dt},
\]

\[
\Delta g \equiv \frac{g(0) - g(\tau)}{g(0)}.
\]
From the Theorem to Welfare:

- Theorem

\[ c(t) \]

\[ \tau > 0 \]

\[ \tau = 0 \]

A credit spread \( r_B > r_S \) is sufficient to alter the distribution of consumption over the life cycle, but it doesn’t guarantee a welfare improvement.
From the Theorem to Welfare (ctd):

- Welfare Metric
  
  **Small** discount rate which values the consumption in old age.

- All that we need is $\rho_w < \rho_c$. In the paper we have:

  $$\rho_w < \rho_c \iff \begin{cases} 
  \rho_c = \beta u - t > 0, \ u \in (t, \bar{T}] \\
  \rho_w = 0,
\end{cases}$$

  since

  $$\begin{align*}
  F_c &= \frac{1}{1 + \rho_c} = \frac{1}{1 + \beta(u - t)}, \ u \in [t, \bar{T}] \\
  F_w &= \frac{1}{1 + \rho_w} = 1.
  \end{align*}$$
Comment

![Graph showing consumption over age with three lines: actual (τ = b = 0), actual (τ = 0.106, b = 0.3743), and first plan (τ = b = 0). The graph plots consumption, c(t), against age, t.]
Comment

- The result is driven by two key assumptions:
  - Social security acts as a commitment device, which requires that the credit market is not complete so that individuals cannot perfectly undo the effects of social security by transacting in their private accounts.
  - The discount rate used in measuring the welfare must be smaller than the discount rate used in making consumption choices, so that old age consumption are highly valued *ex post*.

- Is the assumption of Time-inconsistent Preference relevant?
- Is there a fundamental difference between a positive credit spread and missing credit market?
In the robustness check, $\Delta g$ is negative for some parameter values. Can you explain the intuition behind it?

How do you explain the kinks in the consumption path? Is it sensitive to the initial wealth (e.g. $k(0) > 0$)?

As the tax rate $\tau$ increases, the consumption path moves farther away from the first plan, which is welfare reducing by measuring with Euclidean distance. Is it a upper bound for $\tau$ such that social security is welfare improving?

Please explain the discount function $F(\nu - t)$ in the model description rather than in the numerical examples.