Consumption Behavior, Annuity Income and Mortality Risk of the Elderly*

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Abstract

Previous studies find that individuals do not draw down their assets after retirement which is at odds with the predictions of a simple life cycle model without uncertainty. Hurd (1989, 1999) explains saving behavior of elderly singles and couples by adding lifetime uncertainty and bequest motives to the simple life cycle model. In this paper we aim to test whether predictions of the models proposed by Hurd (1989, 1999) hold for a sample of elderly Americans. We also extend the theoretical model of Hurd (1999) for couples. We use data taken from the Health and Retirement Study (HRS) supplemented with the Consumption and Activities Mail Survey (CAMS). In line with theory we find that, on average, individuals' total consumption is greater than their annuity income after retirement and the difference between total consumption and annuity income increases with the wealth level. Our results also suggest that consumption growth decreases with higher mortality rates for elderly singles. On the other hand, for elderly couples, consumption growth does not respond to changes in the mortality risk of the couple.

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1 Introduction

A simple life cycle model without uncertainty predicts that rational agents' level of consumption is determined by their lifetime income. Under the assumption that individuals' annual earned income is greater than their annual retirement income, this model implies that individuals save when they are young and draw down their assets after retirement. The existing literature shows that the prediction that wealth declines with age is not supported by the empirical evidence (See, e.g., Van Ooijen *et al*. 2014 and Poterba *et al*. 2011). These papers, among the others, find no evidence in favor of wealth decumulation by the elderly even at advanced ages. Extended versions of the life cycle model make attempts to explain this inconsistency. Hurd (1989, 1999) explains saving behavior of elderly singles and couples by adding uncertainty about the date of death and bequest motives. Hubbard *et al*. (1994) find that when uncertainty about lifetime, earnings and out-of-pocket medical expenditures are taken into account, the predictions of the model matches the observed trajectories of wealth and consumption more closely. Boersch-Supan and Stahl (1991) assume that the individuals' marginal utility of consumption is affected by their health status and, therefore, in their model individuals become consumption constrained due to deteriorating health in old age.

In this paper we aim to test the predictions of the life cycle models proposed by Hurd (1989, 1999). Hurd (1989) derives and estimates a model of consumption with mortality risk and bequest motives for elderly singles. This model predicts that the growth rate of consumption decreases as the mortality rate increases since individuals with higher mortality rates increase current consumption at the expense of future consumption. Hurd (1999) proposes a theoretical model to explain the consumption behavior of elderly couples. His model takes into account the mortality risk of both spouses and also allows for bequest motives and predicts that the growth rate of consumption declines as the mortality risk of the couple increases at advanced ages. For the US, and in line with this prediction, Salm (2010) finds that the consumption growth decreases with higher mortality rates for elderly singles. Using the two waves of the CAMS survey for the period 2001-2003, he shows that an increase in subjective mortality by 1 percentage point is associated with a decrease in consumption of nondurables by 1.8 percentage points.

Hurd (1989, 1999) focuses on retired individuals so that retirement decisions do not need to be explained. Models for couples and singles both predict that wealth profiles decline with age after retirement; therefore, we expect to see that total consumption is greater than annuity income unless individuals have a (strong) bequest motive.

Additionally, the difference between total consumption and annuity income depends on the level of initial wealth, i.e. the higher the initial wealth, the higher the difference is.

Our contribution to the empirical literature is twofold. First, we test the models' predictions regarding the relation between wealth and the difference between total consumption and annuity income, a step not taken in previous studies. Second, we test the prediction regarding the consumption change and mortality risk for singles as well as couples. For this purpose we first extend the theoretical model of Hurd (1999) for couples and then estimate the model. For our analysis we use data taken from the Health and Retirement Study (HRS) supplemented with the Consumption and Activities Mail Survey (CAMS). In contrast with Salm (2010), we use five waves of the CAMS survey covering the years from 2001 to 2009. More importantly, we investigate as well whether the model's prediction holds for elderly couples.

The main findings of this paper are as follows: In line with theory we find that, on average, individuals' total consumption is greater than their annuity income after retirement and the difference between total consumption and annuity income increases with the wealth level. Our results also suggest that consumption growth decreases with higher mortality rates for elderly singles. On the other hand, for elderly couples, consumption growth does not respond to changes in the mortality risk of the couple.

The paper is structured as follows: Section 2 outlines the theoretical models. Section 3 describes the data and the descriptive statistics. Section 4 presents the estimation results, and Section 5 offers some concluding remarks.

2 Theoretical models

2.1 The Singles Model

Hurd (1989) analyzes a consumption model with mortality risk and bequest motives for elderly singles. The model includes only retired individuals so that retirement decisions do not have to be explained. Mortality risk is the only source of uncertainty. Households can hold private wealth and annuities to finance consumption during the period of retirement. Annuities are exogenously given and real annuity income is assumed to be constant over time.[1]

[1] In the United States, Social Security benefits are indexed for inflation whereas occupational pensions including defined benefit (DB) and defined contribution (DC) plans are not indexed for inflation.

In addition, individuals face liquidity constraints. That is, the level of net worth is non-negative in each period and individuals enter retirement with a positive amount of assets. This assumption implies that individuals cannot borrow against future Social Security or pension income; therefore, private wealth and annuity wealth are not perfect substitutes. Hurd (1989) assumes that the consumers maximize the following expected utility function starting from the beginning of the retirement phase, $t = 1$, until the certain time of death, $t = L$:

$$\sum_{\tau=t}^{L} (1+\rho)^{1-\tau} a_{\tau}^{t} u(c_{\tau}) + \sum_{\tau=t}^{L} (1+\rho)^{-\tau} m_{\tau+1}^{t} V((1+r)A_{\tau}), \quad t = 1,...,L \tag{1}$$

where $u(.)$ is an increasing, concave utility function, ρ is the rate of time preference, A_{τ} is the net worth at the end of period τ, a_{τ}^{t} is the probability that a person lives in period τ given that he/she survives the period t, $m_{\tau+1}^{t}$ is the probability that a person dies at the beginning of $\tau+1$ given that he/she survives the period t, $V(.)$ denotes the utility from bequests and it is increasing in A_{τ}, r is the real interest rate. In this model, the period at which the person dies is a random variable. L is the maximum age after which the person dies with certainty, i.e. $m_{L+1}^{L} = 1$.

The first term in the objective function gives the expected discounted utility from consumption, c_{τ}, if the person is alive in period τ. The second term shows the expected discounted utility from leaving a bequest if the person dies at the beginning of period $\tau+1$. The utility of leaving a bequest is a function of the net worth at the beginning of period $\tau+1$, i.e. $(1+r)A_{\tau}$. The utility function in equation (1) is maximized subject to the following asset accumulation constraints and the liquidity constraints:

$$A_t = (1+r)A_{t-1} + y - c_t, \quad t = 1,...,L \tag{2a}$$

$$A_t \geq 0, \qquad t = 1,...,L \tag{2b}$$

where y is the non-capital income (in real terms), consisting only of annuities which are assumed to be constant over time. If we ignore the liquidity constraints (2b), the first order condition of this maximization can be written as (see section A.1 for the details):

$$u'(c_t) = \frac{1+r}{1+\rho}\left((1-m_{t+1}^{t})u'(c_{t+1}) + m_{t+1}^{t} V'((1+r)A_t)\right) \tag{3}$$

where $u'(.)$ is the marginal utility of consumption in period t, $V'(.)$ is the marginal utility of leaving a bequest in period t, m_{t+1}^{t} is the instantaneous mortality rate which is the probability that a person dies at the beginning of period $t+1$ given that he/she survives the previous period. Equation (3) is an Euler equation that describes the rule for the allocation of resources over time under lifetime uncertainty. According to this rule, the individual's marginal utility in period t, which depends on both consumption and leaving a bequest, is equal to the discounted expected marginal utility in period $t+1$. In case of a constant relative risk aversion (CRRA) utility function $u(c_t) = c_t^{1-\gamma}/(1-\gamma)$, and no bequest motive ($V'(.) = 0$) equation (3) becomes:

$$\left(\frac{c_{t+1}}{c_t}\right)^{\gamma} = \frac{1+r}{1+\rho}\left(1-m_{t+1}^{t}\right) \tag{4}$$

where γ is the coefficient of risk aversion. After taking the natural logarithm of both sides, equation (4) can be re-written as:

$$\Delta \ln c_{t+1} = \frac{1}{\gamma}\ln\left(\frac{1+r}{1+\rho}\right) + \frac{1}{\gamma}\ln\left(1-m_{t+1}^{t}\right) \tag{5}$$

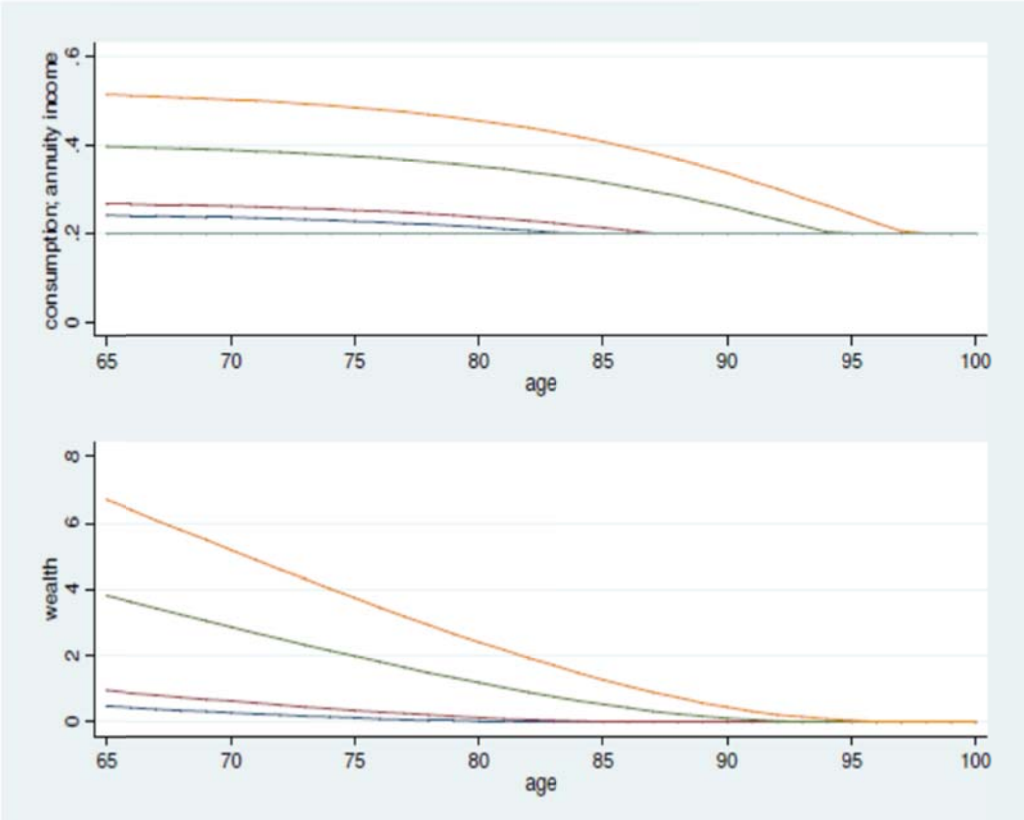
where $\Delta \ln c_{t+1} = \ln c_{t+1} - \ln c_t$. Equation (5) suggests that consumption growth increases with higher real interest rates while it decreases with higher rates of time preference and higher subjective mortality rates. If individuals are impatient ($\rho > r$), the consumption growth is negative since they prefer consuming today rather than consuming tomorrow. Since the survival probability declines exponentially with age, individuals behave even more impatient as they become older, and consumption declines at a faster rate. If individuals are more risk averse, the consumption path becomes flatter and consumption will decline slower with age.

This model assumes that individuals enter the retirement period with positive amount of assets, so that they are not liquidity constrained at the beginning of the retirement. However, they may become constrained at a certain period t ($A_t = 0, t = t_0,..,L$). In that case, consumption will be equal to annuity income from t_0 onwards. This is illustrated in Figure 1.

Figure 1

Consumption and wealth in a model without bequests: variation in A_0

(y=0.2; r=0.001; ρ =0.001; γ =4)



The flat line stands for the constant annuity stream after retirement. Individuals enter the retirement period at age 65 with positive amount of assets (level of initial wealth is positive). As they draw down their assets, they consume more than their annuity income. However, individuals may become constrained at a certain period and from this period onwards their consumption is equal to their annuity income. As a result, this model predicts that consumption is never smaller than annuity income, ($c_t \geq y$). This prediction still holds if we add additional uncertainty to the model such as uncertainty about out-of-pocket medical expenses (de Nardi *et al.*, 2010). On the other hand, if people have a (strong) bequest motive, consumption may not exceed annuity income. This prediction also may not hold if the marginal utility of consumption depends on health status (Boersch-Supan and Stahl, 1991). Figure 1 also shows that different consumption profiles can be obtained for different levels of initial wealth. The level of consumption is an increasing function of initial wealth and the difference between total consumption and annuity income depends on the level of initial wealth, i.e. the higher the initial wealth, the higher the difference is. For high level of initial wealth the wealth-age profile is much steeper, suggesting that at the early stage of the retirement phase consumption is considerably larger than the annuity income. For low level of initial wealth the wealth-age profile is rather flat and individuals draw down their assets at a slow rate.

2.2 The Couples Model

Hurd (1999) proposes a consumption model with mortality risk and bequest motives for elderly couples. In this model the couple receives utility from consumption when both spouses are alive and from leaving bequests. There are two types of bequests. The wealth is first transferred to the surviving spouse and after the death of the surviving spouse; the wealth is inherited by children or others. This model is an extension of the singles model and the assumptions of the singles model are maintained. Both spouses are retired and the couple maximizes the following expected utility function starting from the beginning of the retirement phase, $t = 1$, until the certain time of death of the surviving spouse, $t = L_{\max}$:

$$\sum_{\tau=t}^{L_{\min}} (1+\rho)^{1-\tau} a_\tau^t u(C_\tau) + \sum_{\tau=t}^{L_m} (1+\rho)^{-\tau} pm_{\tau+1}^t M((1+r)A_\tau) + \sum_{\tau=t}^{L_f} (1+\rho)^{-\tau} pf_{\tau+1}^t F((1+r)A_\tau) + \sum_{\tau=t}^{L_{\max}} (1+\rho)^{-\tau} h_{\tau+1}^t V((1+r)A_\tau) \tag{6}$$

where $u(C_\tau)$ shows the couple's utility from consumption, ρ is the rate of time preference of the couple, a_τ^t is the probability that both spouses will be alive in period τ given that they survive the period t, $L_{\min}$ denotes the lifespan of the couple after which one of the spouses dies with certainty, A_τ is the net worth at the end of period τ, r is the real interest rate, $pm_{\tau+1}^t$ is the probability that the husband becomes a widower at the beginning of period $\tau+1$, L_m is the maximum lifespan of the husband, $pf_{\tau+1}^t$ is the probability that the wife becomes a widow at the beginning of period $\tau+1$, L_f is the maximum life span of the wife, $M((1+r)A_\tau)$ is the widower's utility of wealth, $F((1+r)A_\tau)$ is the widow's utility of wealth, $h_{\tau+1}^t$ is the probability that both spouses die at the beginning of period $\tau+1$, $V(.)$ is the utility from leaving bequests to the children or others, and $L_{\max}$ is the maximum lifespan of the surviving spouse.

The utility function in equation (6) is maximized subject to the asset accumulation constraints and the liquidity constraints as introduced in the singles problem:

$$A_t = (1+r)A_{t-1} + y - c_t\,,\ t = 1,...,L_{\max} \tag{7a}$$

$$A_t \geq 0\,, \qquad t = 1,...,L_{\max} \tag{7b}$$

The solution of this model depends on the widower's marginal utility of wealth, $M'((1+r)A_\tau)$, and the widow's marginal utility of wealth, $F'((1+r)A_\tau)$ (Hurd 1999, equation 5 on page 16). In order to estimate this model, one should explicitly define these marginal utilities. In this paper we aim to extend the model proposed by Hurd (1999) by deriving a unitary model for the couple. This model allows us to find a solution which is independent of the widower's and the widow's marginal utilities of wealth so that it can be estimated directly using the subjective survival probabilities of the husband and the wife. In our model the couple maximizes the following expected utility function starting from the beginning of the retirement phase, $t = 1$, until the certain time of death of the surviving spouse, $t = L_{\max}$:

$$\sum_{\tau=t}^{L_{\min}} (1+\rho)^{1-\tau} a_\tau^t u\left(\frac{c_\tau}{\sqrt{2}}\right) + \sum_{\tau=t}^{L_m} (1+\rho)^{-\tau} pm_{\tau+1}^t M\left((1+r)A_\tau\right) + \sum_{\tau=t}^{L_f} (1+\rho)^{-\tau} pf_{\tau+1}^t F\left((1+r)A_\tau\right) + \sum_{\tau=t}^{L_{\max}} (1+\rho)^{-\tau} h_{\tau+1}^t V\left((1+r)A_\tau\right) \tag{8}$$

where $u\left(\frac{c_\tau}{\sqrt{2}}\right)$ shows the couple's utility from consumption divided by an equivalent scale (the often used OECD square root scale)[2], $M\left((1+r)A_\tau\right)$ is the widower's utility of wealth, i.e. $\sum_{\tau=t+1}^{L_m} (1+\rho)^{t+1-\tau} a_\tau^{m,t+1} u(c_\tau)$, $F\left((1+r)A_\tau\right)$ is the widow's utility of wealth, i.e. $\sum_{\tau=t+1}^{L_f} (1+\rho)^{t+1-\tau} a_\tau^{f,t+1} u(c_\tau)$. The utility function in equation (8) is maximized subject to the asset accumulation constraints and the liquidity constraints in (7a) and (7b).

In case of no bequest motive to the children or others and no liquidity constraints, we solve the couple's maximization problem and find the following Euler equation (see section A.2 for the details):

$$u'\left(\frac{c_t}{\sqrt{2}}\right) = \left(\frac{1+r}{1+\rho}\right)\left[\left(1 - cm_{t+1}^t\right)u'\left(\frac{c_{t+1}}{\sqrt{2}}\right) + \sqrt{2}\left(pm_{t+1}^t + pf_{t+1}^t\right)u'(c_{t+1})\right] \tag{9}$$

[2] The OECD uses a scale which divides household income by the square root of household size when comparing income inequality and poverty across countries (e.g.OECD 2008).

where cm_{t+1}^{t} is the instantaneous mortality rate of the couple which is the probability that one of the spouses dies at the beginning of period $t+1$ given that both spouses were alive in the previous period. The mortality risk of the couple, cm_{t+1}^{t}, is the sum of mortality risk of the wife and mortality risk of the husband minus the probability that both spouses die in period $t+1$ given that both spouses were alive in the previous period.[3] pm_{t+1}^{t} is the probability that the husband becomes a widower at the beginning of period $t+1$ and it is equal to the probability that the wife dies at the beginning of $t+1$ times the probability that the husband survives the period t+1. Similarly, pf_{t+1}^{t} is the probability that the wife becomes a widow at the beginning of period $t+1$. Hurd (1999, p. 16) points out that the Euler equation does not include the marginal utility of leaving bequests to the children or others ,$V'(.)$, since the probability that both spouses dies in the near future is equal to zero. In case of a constant relative risk aversion (CRRA) utility function $u(c_t)=c_t^{1-\gamma}/(1-\gamma)$ for couples, equation (9) becomes:

$$\left(\frac{c_{t+1}}{c_t}\right)^{\gamma}=\frac{1+r}{1+\rho}\left[\left(1-cm_{t+1}^{t}\right)+\sqrt{2}^{1-\gamma}\left(pm_{t+1}^{t}+pf_{t+1}^{t}\right)\right] \tag{10}$$

By using this type of utility function, we implicitly assume that the husband and the wife have the same coefficient of risk aversion, γ. After taking the natural logarithm of both sides, equation (10) can be written as:

$$\Delta\ln c_{t+1}=\frac{1}{\gamma}\ln\left(\frac{1+r}{1+\rho}\right)+\frac{1}{\gamma}\ln\left[1-\left(cm_{t+1}^{t}-\sqrt{2}^{1-\gamma}\left(pm_{t+1}^{t}+pf_{t+1}^{t}\right)\right)\right] \tag{11}$$

$\Delta\ln c_{t+1}=\ln c_{t+1}-\ln c_t$. We call the term inside the logarithm, $cm_{t+1}^{t}-\sqrt{2}^{1-\gamma}\left(pm_{t+1}^{t}+pf_{t+1}^{t}\right)$, the couple's adjusted mortality rate and one can show that it is positive as long as the coefficient of risk aversion is larger than one and it increases with an increase in the mortality rates of the wife and the husband. Equation (11) shows, therefore, that the consumption growth of a couple is negatively related to the couple's adjusted mortality rate. As the mortality risk of the couple increases at advanced ages, we expect the consumption to decline with age. In the empirical part of this paper we assume that the coefficient of risk aversion is equal to 3, which is a reasonable value obtained by the

[3] We assume that the death of the husband is independent of the death of the wife.

previous studies (See, e.g., Palumbo 1999) and calculate the couple's adjusted mortality rate accordingly.[4]

3 Data

The HRS is a biennial panel survey of Americans and its respondents were first interviewed in 1992 (Juster and Suzman, 1995). The HRS is well-suited for the purpose of this study since it is a large sample of elderly population and it includes detailed information on employment status, annuity income, household wealth, marital status, subjective survival probabilities, and health status of the respondents and their spouses. The data on consumption come from the Consumption and Activities Mail Survey (CAMS) which is a supplemental survey to the HRS. In 2001 the CAMS survey sent questionnaires to a subsample of the households who were interviewed in the HRS 2000 core survey. If household members are married or have a partnership, the questionnaire was sent to one of the spouses, selected randomly. In the initial wave of the CAMS survey 3,866 households answered questions about household spending in 26 categories of nondurables and 6 categories of durables (see Hurd and Rohwedder, 2008 for details). This survey has smaller number of households than the HRS and covers the period from 2001 to 2011.

In this study we use five waves of the CAMS survey for the period from 2001 to 2009. The CAMS survey is matched to most of the information in the previous HRS wave, i.e. CAMS 2001 is matched to the HRS 2000. However, information on financial variables such as wealth and income can be obtained from the next HRS wave. For example, HRS 2002 collects information on total income for the year 2001 which coincides with the information on consumption in CAMS 2001.[5]

Individuals' annuity income (before-tax) is defined as the sum of income from employer pension and/or annuity, social security disability, supplemental security income, social security retirement, spouse or widow benefits, unemployment and worker's compensation, and other income including veteran's benefits, welfare and food stamps. After-tax annuity income is obtained by deducting total taxes paid (federal taxes, state taxes, and the Federal Insurance Contributions Act (FICA) tax which includes social security tax and Medicare tax) from before-tax annuity income. Federal taxes, state income taxes, and the FICA tax for each household in each year are calculated based on the NBER

[4] One can show that in this case the couple's adjusted mortality rate is equal to the average of wife's and husband's mortality rate.

[5] Since HRS 2012 is not available yet, we exclude CAMS 2011 from our analysis.

tax calculator, TAXSIM (see Feenberg and Coutts, 1993). This program calculates tax liabilities for these three categories for each household in our sample once we provide required information about the respondents such as their marital status, income from all sources, deductions etc.[6]

To measure individuals' mortality risk we use subjective mortality rates instead of life table mortality rates because of two reasons. Firstly, life table mortality rates do not show much individual variation since they are aggregated to allow only for differences by age, gender and race. Subjective mortality rates are correlated with individual characteristics such as level of education, wealth, and income, as well as behavioral factors such as smoking, alcohol consumption and obesity. Consequently, subjective mortality rates contain information which is not contained in life table mortality rates. Secondly, individuals make decisions based on their own chances of survival. For example, smokers may think that they would die earlier than an average person in the population and behave accordingly. In this case, life table mortality rates may not explain smokers' economic decisions.

Subjective mortality rates for each respondent and his/her spouse (if present) are calculated in each wave from 2000 to 2008 based on a question about individuals' probability of survival to a certain age. More explicitly, the HRS asks about respondents' self-reported probability of living to age T where T depends on the current age of the respondents and it is more than 10 years above the respondents' current age. The question is:

"[Using any] number from 0 to 100 where "0" means that you think there is absolutely no chance and "100" means that you think the event is absolutely sure to happen…What is the percent chance that you will live to be 80/85/90/95/100) or more?"

Hurd and McGarry (2002) show that these probabilities are good predictors of individuals' actual mortality within the sample and individuals who expect to live longer are less likely to die. Following Gan et al. (2003) and Salm (2010) we derive annual subjective mortality rates by assuming that individuals' subjective mortality rate ($m_{i,\tau}^{\tau-1}$) is proportionate to the life table mortality rate ($m_{0,\tau}^{\tau-1}$) as follows:

$$m_{i,\tau}^{\tau-1} = \xi_i m_{0,\tau}^{\tau-1} \qquad , \ \tau = t+1, t+2, ..., L+1 \tag{12}$$

[6]For more information see http://hrsonline.isr.umich.edu/index.php?p=shownews3x1&hfyle=news198.

Under this assumption the subjective probability $s_{i,t,T}$ of individual i to survive from age t to age T can be shown as:

$$s_{i,t,T} = \prod_{\tau=t-1}^{T-1}(1-m_{i,\tau+1}^{\tau}) = \prod_{\tau=t-1}^{T-1}(1-\xi_i m_{0,\tau+1}^{\tau}) \qquad (13)$$

Then individual mortality factor ξ_i in equation (12) is obtained by minimizing the following expression for each individual:

$$\min_{\xi_i}\left(s_{i,t,T} - \prod_{\tau=t-1}^{T-1}(1-\xi_i m_{0,\tau+1}^{\tau}) \right)^2 \qquad (14)$$

This method does not produce meaningful mortality factors if individuals reported zero or one answers to probability of survival to age T. To include these individuals in our analysis, we change the answers from 0 to 0.01 and 1 to 0.99, respectively. Life table mortality rates are taken from race, gender, and age specific life-tables of National Vital Statistics Reports which are available for the years from 2000 to 2008.

3.1 Sample selection

The CAMS survey has questions about household spending which are answered by the respondents who also participated in the HRS core survey in 2000. If two respondents are married or have a partnership, one of the spouses is selected randomly to answer the questions about household spending. We start with an unbalanced panel sample which includes 5,402 households who answered questions in the CAMS survey at least one year in the period from 2001-2009. We restrict the sample to 3,615 households in which the respondent and, if present, his/her spouse are aged 65 and over. Age 65 is the legal retirement age in the United States and at age 65 individuals become eligible for Medicare. Although most individuals are retired by the age of 65, we observe in the data that some individuals have income from earnings after age 65. Since we focus on retired individuals, we further restrict the sample to 3,264 households in which the respondent and, if present, his/her spouse do not earn any wage income. Next, we focus on individuals who are not liquidity constrained in the sense that they hold positive amount of household wealth in the first year they entered the survey. This restriction is necessary because we derive the Euler equations for singles and couples by assuming that there are no liquidity constraints. As a result, we estimate the Euler equations on a sample of individuals who are not liquidity constrained. After excluding households with zero or negative wealth holdings, we are left with 3,033 households. We further focus on 2,428 households which are single-person or two-person households.[7] In our sample a two-person household may become a one-person household during the observation period as a result of divorce, separation or the death of the spouse. However, we do not allow a one-person household to become a two-person household during the observation period. After excluding households who married or remarried during the observation period, we are left with 2,378 households. Next, we restrict the sample to 2,095 households in which the respondent and, if present, his/her spouse provided non-missing information on subjective survival probabilities. 1,183 households also have non-missing information on the change in nondurable consumption. Our final sample includes 1,154 households and 3,692 household-year observations with non-missing information on other covariates. Although we select households with positive level of wealth in the first year they entered the survey, for some households wealth is non-positive in the subsequent years. Among the 1,154 households with positive wealth in the first year of the survey, 54 of them have non-positive wealth in the subsequent years (4.68 percent). The models described above allow that individuals may become constrained at a certain period and from this period onwards consumption is equal to annuity income.

3.2 Descriptive Statistics

Table 1 displays the mean, median and standard deviation of some variables for the sample which includes both single and couple households.

Total consumption is the sum of durables and nondurables excluding spending on cars and mortgages.[8] Categories of durable consumption include refrigerator, washer/dryer, dishwasher, television, and computer and categories of nondurables are home insurance, property tax, rent, electricity, water, heat, home repair services, phone/cable/internet, auto insurance, health insurance, house/yard supplies, home repair supplies and services, food, dining out, clothing, gasoline, vehicle services, drugs, health services, medical supplies, vacations, tickets, hobbies, contributions, and gifts. Total consumption, after-tax annuity

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[7] For the one-person household, the household size is equal to one and the household member is separated/divorced/widowed or never married. For the two-person household, the household size is equal to two and the household members are married or living with a partner.

[8] These two spending categories contain components of savings. Individuals were asked to report total mortgage payments and total car payments which include both interest and principal. To find a pure spending measure for these two components, one needs to remove the saving component from the payments by subtracting the principle. Since the CAMS survey does not include information on principal and interest separately, we cannot calculate the pure spending measures for these components.

income, total net household wealth, total net financial wealth, and total health expenditures are measured at the household level. Total net household wealth excludes Individual Retirement Accounts (IRAs) and the value of 401k/Keogh plans and it is defined as the sum of all wealth components less all debt. The wealth components are the value of the primary residence, real estate, vehicles, stocks, checking accounts, government bonds, bonds, other wealth and the debt components are mortgages, home loans and other debt. Total net financial wealth does not include the value of the primary residence, real estate, mortgages, and home loans. Total health expenditures consist of expenditures on drugs, health services, medical supplies, and health insurance.

Table 1: Summary statistics (one-person or two-person households)

	mean	median	std. deviation
Total consumption (in 2003 dollars)	23380	18950	17240
Annuity income (in 2003 dollars)	18330	16090	13120
Wealth (in 2003 dollars)	289240	156730	590680
Financial wealth (in 2003 dollars)	151010	47510	356180
Total health expenditures (in 2003 dollars)	3510	2610	4100
Wealth/Annuity income	17.243	8.370	41.474
Financial wealth/Annuity income	8.974	2.418	27.982
Total consumption/Annuity income	1.632	1.133	2.119
Total consumption minus annuity income	0.498	0.206	2.017
Dummy (total consumption>=annuity income)	0.587	1	0.492
Age [a]	74.827	75	6.426
Male	0.324	0	0.468
Number of observations (households)	3,692 (1,154)		

[a] Age stands for the age of the respondent who answered the questions about the household consumption. For two-person households one of the spouses is chosen randomly to answer these questions. The variables measured at the household level are divided by the OECD-modified equivalence scale.[9]

According to Table 1, there is a difference between mean and median values for some variables such as total consumption, wealth, and financial wealth which may suggest that some households have very high levels of wealth and/or consumption. The mean ratio of wealth to annuity income is 17, which suggest that the sample has very wealthy households. On the other hand, the main component of wealth for the individuals aged 65 and older is housing wealth which is more difficult to liquidate. The mean ratio of financial wealth, which excludes housing wealth, to annuity income is 9 percent whereas the median

[9] This scale was proposed by Hagenaars *et al.*(1994) and it assigns a value of 1.5 to the households with two adults.

is around 2, showing that the majority of individuals have accumulated financial wealth when they were young. The ratio of total consumption to annuity income is larger than one which suggests that most individuals draw down their assets after retirement. This finding is reinforced by the mean value of the dummy variable which takes one if total consumption is greater than or equal to after-tax annuity income and zero otherwise. The mean value is 0.59 and median value is one for this variable. Table 1 also reveals that the proportion of men in the sample is 32 percent which suggests that women are over-represented in the sample. This finding is not surprising since the sample consists of individuals who are aged 65 and over and women live longer than men.

Table 2 shows the mean, median, and standard deviation of the variables used in the estimation of the Euler equation (equation (5)) for single households.

Table 2: Summary statistics (one-person households)

	mean	median	std. deviation
Nondurable consumption (in 2003 dollars) (all categories)	21992	17968	15381
Nondurable consumption (in 2003 dollars) (Salm (2010), sub-categories)	5911	4666	5820
Nondurable consumption growth ($\Delta \ln c_{t+1}$) (all categories)	-0.022	-0.022	0.266
Nondurable consumption growth (Salm (2010), sub-categories)	-0.042	-0.039	0.368
Subjective mortality rate ($m_{i,t}^{t-1}$)	0.058	0.034	0.055
Life table mortality rate ($m_{0,t}^{t-1}$)	0.047	0.039	0.030
Age	76.809	77	6.582
Male	0.212	0	0.408
Poor health	0.266	0	0.442
Good health	0.400	0	0.490
Years of education	12.517	12	2.475
Any ADL limitations	0.177	0	0.382
Any IADL limitations	0.369	0	0.482
CES-D score	1.566	1	1.877
Number of observations (households)	1,323(646)		

Notes: The number of observations is based on the estimation of the Euler equation. Since consumption expenditures are reported in every two years, the annual change in consumption growth is obtained by dividing biennial consumption growth rate into two.

We focus on nondurable consumption categories because these are easier to adjust for consumers compared to durable consumption categories. All categories of nondurable consumption are home insurance, property tax, rent, electricity, water, heat, home repair services, phone/cable/internet, auto insurance, health insurance, house/yard supplies, home repair supplies and services, food, dining out, clothing, gasoline, vehicle services, drugs, health services, medical supplies, vacations, tickets, hobbies, contributions, and gifts. The expenditures on some of these categories such as home insurance, property tax, rent, electricity, water, heat, and auto insurance are essential for individuals and we may not expect that they change in response to changes in the subjective mortality risk. Medical expenditures can be seen as an investment in health and may not provide direct utility to individuals. Salm (2010) uses seven categories of nondurables which are food, dining out, clothing, gasoline, vacations, tickets, hobbies. The latter categories are more easily adjusted in the response to changes in the subjective mortality risk. In the empirical part of the study we consider both all categories and the sub-categories of nondurables used by Salm (2010).

According to Table 2, the mean annual consumption growth in nondurable spending ($\Delta \ln c_{t+1}$) is negative which indicates that households' current consumption is higher than their consumption in the next year, on average. The mean of the subjective mortality risk ($m_{i,t}^{t-1}$) is 0.058 and the mean of the life table mortality risk ($m_{0,t}^{t-1}$) is 0.047, suggesting that individuals underestimate their survival probabilities compared to life table probabilities, on average. Good health is a binary indicator which takes one if individuals' self-rated health status is excellent and very good. Similarly, poor health is a dummy variable which is equal to one if individual's self-reported health is fair or poor. According to Table 2 the majority of the singles households are in medium or good health. "Any ADL limitations" is a binary indicator which shows whether individuals have any difficulty in activities of daily living such as eating, bathing, walking across a room, dressing, getting in and out of bed, and using the toilet. The variable "Any IADL limitations'' indicates whether individuals have any difficulty in Instrumental Activities of Daily Living task such as using a telephone, taking medication, handling money, shopping for groceries, preparing meals, and using a map. The sample statistics in Table 2 show that single individuals aged 65 and over are more likely to have IADL limitations than ADL limitations. The score on the Center for Epidemiologic Studies Depression (CES-D) is a mental health index which is commonly used by psychologists as well as economists (Hao, 2008; Finkelstein et al, 2008). The CES-D ranges from 0 to 8 and it is the sum of six negative indicators (depression, everything is an effort, sleep is restless, felt alone, felt sad, could not get going) minus two positive indicators (felt happy and enjoyed life); the higher the score, the more negative the respondent's feelings in the past week. According to Table 2, most individuals in the sample seem to be mentally healthy.

Table 3 gives descriptive statistics of the variables used in the estimation of the Euler equation (equation (11)) for couple households. Variables are defined as in Table 2.

Table 3: Summary statistics (two-person households)

	mean	median	std. deviation
Nondurable consumption (in 2003 dollars) (all categories)	34156	27996	23509
Nondurable consumption (in 2003 dollars) (Salm (2010), sub-categories)	10764	8870	8349
Nondurable consumption growth ($\Delta \ln c_{t+1}$) (all categories)	-0.038	-0.023	0.254
Nondurable consumption growth (Salm (2010), sub-categories)	-0.051	-0.052	0.318
Husband's subjective mortality rate	0.053	0.034	0.049
Wife's subjective mortality rate	0.041	0.025	0.045
The couple's adjusted mortality rate, subjective[a]	0.048	0.033	0.037
The couple's adjusted mortality rate from the life table[a]	0.040	0.034	0.020
Age_husband	75.784	75	5.216
Age_wife	73.287	73	5.034
Poor health_husband	0.224	0	0.417
Poor health_wife	0.199	0	0.400
Good health_husband	0.434	0	0.495
Good health_wife	0.441	0	0.496
Years of education_husband	12.991	12	3.116
Years of education_wife	12.792	12	2.311
Any ADL limitations_husband	0.111	0	0.314
Any ADL limitations_wife	0.119	0	0.324
Any IADL limitations_husband	0.344	0	0.475
Any IADL limitations_wife	0.263	0	0.440
CES-D score_husband	0.766	0	1.249
CES-D score_wife	1.048	0	1.521
Number of observations (households)	1,061 (524)		

Notes: The number of observations is based on the estimation of the Euler equation. Since consumption expenditures are reported in every two years, the annual change in consumption growth is obtained by dividing biennial consumption growth rate into two. [a] The coefficient of risk aversion is set equal to 3 to calculate the couple's adjusted mortality rate based on subjective probabilities and life table probabilities.

According to these statistics, subjective mortality rate of the husband, which is calculated using his self-reported probability of survival, is greater than that of the wife, suggesting that individuals are aware of the gender gap in life expectancy. The couple's adjusted mortality rate which is calculated based on subjective mortality risk of the husband and the wife is positive and it is very close to the couple's adjusted mortality rate obtained from the life table. Table 3 also reveals that husbands in the sample are two years older than their wives, on average. Another difference between husbands and wives comes from the health status in the sense that wives seem to be healthier than their husbands.

Table 4 reports the levels of total consumption and nondurable spending across years. According to Table 4, on average, both total and nondurable consumption have decreased over the years 2001-2009.

Table 4: Total consumption per household across years (in 2003 dollars) (equivalised)

		Total consumption		Nondurables		Nondurables (Salm (2010), sub-categories)	
year	No. Obs.	mean	median	mean	median	mean	Median
2001	583	25669	20064	25359	19605	7302	5339
2003	747	24955	20686	24619	20298	7134	5640
2005	826	22723	18155	22440	17968	6997	5229
2007	872	22789	18738	22400	18403	6758	5393
2009	664	20684	17500	20125	17312	5719	4821

Number of observations (households): 3,692 (1,154)

This pattern can be partly explained by the age effect. Individuals in the sample are becoming older over time and, therefore, they increase current consumption at the expense of future consumption. The decline in consumption from 2007 to 2009 could also be driven by the financial crisis which affected both the financial and housing wealth of the elderly.

Now, we check whether total consumption (or expenditures) exceeds after-tax annuity income for elderly singles and couples. Table 5 reports the mean of the dummy variable indicating that total consumption is greater than and equal to after-tax annuity income by different age groups in the sample.

Table 5: The mean of the dummy (consumption>=annuity income) by age groups

years from 2001 to 2009		
age class	mean	No. obs.
65-69	0.562	759
70-74	0.556	901
75-79	0.588	913
80-83	0.623	720
85+	0.641	399
Total	0.587	3,692

Income and expenditures are in 2003 dollars and divided by the OECD-modified equivalence scale.

According to Table 5 for the majority of the respondents total consumption is greater than their annuity income. Table 6 shows the proportions by age groups and years.

Table 6: The mean of the dummy (consumption>=annuity income) by age groups and years

	2001		2003	
age class	mean	No. obs.	mean	No. obs.
65-69	0.649	154	0.606	158
70-74	0.673	141	0.639	177
75-79	0.630	156	0.582	175
80-83	0.649	102	0.686	162
85+	0.657	30	0.723	75
	2005		2007	
age class	mean	No. obs.	mean	No. obs.
65-69	0.528	178	0.543	180
70-74	0.453	203	0.548	215
75-79	0.610	187	0.600	215
80-83	0.603	164	0.596	157
85+	0.638	94	0.622	105
	2009			
age class	mean	No. obs.		
65-69	0.426	89		
70-74	0.503	165		
75-79	0.516	180		
80-83	0.592	135		
85+	0.600	95		

Income and expenditures are is in 2003 dollars and divided by the OECD-modified equivalence scale. Age stands for the age of the respondent who answered the questions about the household consumption. For two-person households one of the spouses is chosen randomly to answer these questions.

These findings also suggest that the majority of the respondents spend more than their annuity income after retirement, although there are some differences across years. For example, in 2009 we find that most respondents in the age groups 65-69 spent less than their annuity income. The drop in consumption in this particular year could reflect the effect of the global financial crisis. Due to a strong stock market decline in 2009 individuals who just entered retirement might experience a loss in their financial assets, which might cause them to consume less than their annuity income. Table 6 also shows that the proportions in 2009 are smaller than those in 2001, which may suggest that younger cohorts have fewer assets than older cohorts.

4 Estimation Results

In the previous section we find that more than half of the individuals in our sample spend more than their annuity income after retirement which indicates most individuals have decreasing wealth profiles in old age. The models that we outlined in Section 2 also predict that the difference between total consumption and annuity income is larger if the level of initial wealth is higher (see Figure 1 in Section 2). In this section we first test whether this prediction holds.

Table 7 gives the estimation results by the Ordinary Least Squares (OLS) for single households. The dependent variable is the logarithm of total consumption minus logarithm of annuity income in models reported under the first two columns. According to the results under the first column, the coefficient of wealth is positive and statistically significant, suggesting that the higher the level of wealth, the larger the difference between total consumption and annuity income is. In other words, individuals (or single households) are more likely to spend more than their annuity income as the level of their assets increases. The results also indicate a significant age effect. As individuals draw down their assets after retirement, their total consumption converges to annuity income at advanced ages. Another interesting finding is that level of education is positively associated with the difference between total consumption and annuity income, suggesting that high educated are able to spend more than their annuity income after retirement.

Table 7: Estimation results based on the OLS (one-person households)

	(1) Log total consumption minus log annuity income	(2) Log total consumption minus log annuity income	(3) Log total consumption (excluding health exp.) minus log annuity income	(4) Log total health expenditures
Year of birth	-0.023***	-0.022***	-0.016***	-0.051***
	(0.005)	(0.005)	(0.005)	(0.009)
Age	-0.019***	-0.018***	-0.015**	-0.032***
	(0.005)	(0.006)	(0.006)	(0.009)
Wealth (in $10,000)	0.003***		0.004***	0.003***
	(0.001)		(0.001)	(0.001)
CES-D score	0.014	0.016	0.015	0.012
	(0.011)	(0.011)	(0.011)	(0.016)
Poor health	0.110**	0.079*	0.069	0.167**
	(0.045)	(0.047)	(0.050)	(0.079)
Good health	0.016	0.044	0.070	-0.088
	(0.041)	(0.043)	(0.043)	(0.061)
Any ADL limitations	0.008	0.019	0.002	0.046
	(0.049)	(0.050)	(0.051)	(0.085)
Any IADL limitations	0.040	-0.012	-0.020	-0.011
	(0.041)	(0.042)	(0.042)	(0.066)
Years of education	0.023***			0.066***
	(0.008)			(0.015)
Constant	47.29***	44.63***	41.70***	108.3***
	(11.58)	(11.72)	(11.88)	(18.12)
Number of observations (households)	1995(646)	1995(646)	1995(646)	1941(645)
p-value Wald test: all health variables	0.015	0.320	0.214	0.017

Robust standard errors in parentheses * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

The p-value of the Wald test for health indicators indicates that the coefficients of health variables are jointly significant at a 5 percent level of significance. Specifically, being in poor health significantly increases the total consumption relative to annuity income, which is probably because total consumption includes health expenditures. The results under the fourth column in Table 7 show that the logarithm of total health expenditures is significantly higher for those who are in poor health. Once we exclude health expenditures from total expenditures (under column 3), the difference between total consumption and annuity income becomes independent of individuals' health status.[10] The results under the fourth column also suggest the logarithm of total health expenditures becomes higher as the level of education increases. At first glance, this finding may seem unexpected because high educated are usually healthier than low educated and, therefore, we expect that they have less spending on health. However, in the United States low-income individuals (who are also low educated) are covered by a social health care program called Medicaid whereas high-income individuals have out-of-pocket expenditures.

Table 8 shows the estimation results by the Ordinary Least Squares (OLS) for couple households. Similar to findings for single households, the level of wealth is positively associated with the difference between total consumption and annuity income, suggesting that couple households with more assets tend to spend more than their household annuity income. The age effect in this model is not as strong as in the singles model.

Neither the age of husband nor the age of wife has a significant effect on the difference between total consumption and annuity income.[11] One explanation can be that couple households are usually younger than single households, and therefore, they may have a flatter consumption profile. The *p*-value of the Wald test for health indicators shows that the coefficients of health variables are not jointly significant. On the other hand, the difference between total consumption and annuity income decreases for households in which the wife has any difficulty in instrumental activities of daily living such as handling money, shopping for groceries preparing meals etc. This finding is in line with the predictions of a model proposed by Boersch-Supan and Stahl (1991). In their model individuals' marginal utility of consumption is affected by their health status and, therefore, individuals become consumption constrained due to deteriorating health in old age.

[10] The Wald test (not reported in Table 7) for the equality of coefficient estimates on poor health in models (1) and (3) suggests that the estimated coefficient on poor health in model (1) is statistically different than that in model (3).

[11] We also tried to estimate the models in Table 8 by controlling for only the age of husband or only the age of wife. In both cases, we did not find a significant age effect.

Table 8: Estimation results based on the OLS (two-person households)

	(1) Log total consumption minus log annuity income	(2) Log total consumption minus log annuity income	(3) Log total consumption (excluding health exp.) minus log annuity income	(4) Log total health expenditures
Year of birth_husband	0.001	0.014	0.030	-0.057
	(0.030)	(0.036)	(0.036)	(0.050)
Year of birth_wife	-0.016	-0.028	-0.055^{*}	0.038
	(0.030)	(0.036)	(0.028)	(0.051)
Age_husband	-0.001	0.013	0.028	-0.056
	(0.030)	(0.037)	(0.036)	(0.050)
Age_wife	-0.010	-0.024	-0.053^{*}	0.046
	(0.030)	(0.037)	(0.029)	(0.051)
Wealth (in $10,000)	0.004^{***}		0.006^{***}	0.002^{***}
	(0.001)		(0.001)	(0.001)
Cesd score_husband	0.003	0.003	0.006	-0.022
	(0.014)	(0.015)	(0.015)	(0.027)
Cesd score_wife	-0.008	-0.014	-0.015	-0.029
	(0.012)	(0.012)	(0.013)	(0.021)
Any IADL limitations_husband	0.021	-0.013	-0.013	-0.031
	(0.038)	(0.044)	(0.044)	(0.060)
Any IADL limitations_wife	-0.051	-0.116^{**}	-0.106^{**}	-0.049
	(0.043)	(0.046)	(0.047)	(0.072)
Poor health_husband	0.053	0.023	0.0011	0.138^{**}
	(0.047)	(0.051)	(0.052)	(0.067)
Poor health_wife	0.010	-0.006	-0.012	-0.042
	(0.051)	(0.054)	(0.055)	(0.079)
Good health_husband	-0.024	-0.001	0.007	-0.030
	(0.039)	(0.047)	(0.047)	(0.059)
Good health_wife	-0.002	0.051	0.076^{*}	-0.066
	(0.041)	(0.047)	(0.043)	(0.064)
Any ADL limitations_husband	0.115^{*}	0.096	0.089	0.010
	(0.063)	(0.067)	(0.069)	(0.104)
Any ADL limitations_wife	0.072	0.098	0.106	0.027
	(0.064)	(0.068)	(0.069)	(0.085)
Years of education_husband	0.010			0.027^{**}
	(0.008)			(0.013)
Years of education_wife	0.011			0.018
	(0.011)			(0.020)
Constant	29.90^{**}	28.89^{**}	38.45^{***}	43.70^{**}
	(12.73)	(13.57)	(13.77)	(18.45)
Number of observations (households)	1617(524)	1617(524)	1617(524)	1606(524)
p-value Wald test: all health variables	0.342	0.427	0.612	0.351
p-value Wald test: age	0.579	0.528	0.300	0.257

Robust standard errors in parentheses * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Next, we investigate the extent to which individuals' subjective mortality risk is associated with the changes in their consumption at older ages. We first estimate the Euler equation as shown in equation (5) where the change in consumption is explained by subjective mortality risk of elderly singles. The estimation results are given in Table 9.

Table 9: Estimation of the Euler Equation by OLS (one-person households)

	(1) Consumption growth (All categories) $\Delta \ln c_{t+1}$	(2) Consumption growth (All categories) $\Delta \ln c_{t+1}$	(3) Consumption growth (Salm (2010, sub-categories) $\Delta \ln c_{t+1}$	(4) Consumption growth (Salm (2010, sub-categories) $\Delta \ln c_{t+1}$
$\ln\left(1-m_{i,t}^{t-1}\right)$	0.119	0.170	0.294	0.343*
	(0.119)	(0.123)	(0.190)	(0.200)
Change in self-rated health	-0.005		-0.005	
	(0.009)		(0.013)	
Change in any ADL	0.027		0.019	
	(0.020)		(0.028)	
Change in any IADL	0.019		-0.022	
	(0.016)		(0.025)	
Change in CES-D	0.003		0.001	
	(0.004)		(0.006)	
Years of education	0.0003	0.001	-0.002	-0.001
	(0.003)	(0.003)	(0.004)	(0.004)
Poor health		0.023		-0.001
		(0.021)		(0.030)
Good health		0.001		-0.001
		(0.017)		(0.023)
Any IADL limitations		0.003		-0.033
		(0.016)		(0.024)
Any ADL limitations		0.015		0.078**
		(0.023)		(0.034)
CES-D score		0.003		0.009
		(0.004)		(0.006)
Constant	-0.020	-0.046	0.006	-0.014
	(0.045)	(0.047)	(0.064)	(0.064)
Number of observations (households)	1306(641)	1323(646)	1306(641)	1323(646)
p-value Wald test: all health variables	0.327	0.501	0.847	0.084

Robust standard errors are in parentheses * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

In all regressions, the dependent variable is the growth rate of real annual consumption expenditures on nondurables. In the first two models we use all categories of nondurables whereas in the last two models we only include seven categories of nondurables as explained in section 3.2. We also control for individuals' health status and education level when we estimate the relationship between the consumption growth and subjective mortality risk. Following Salm (2010) we include health indicators in levels (model 2 and model 4). We also run regressions in which we control for the change in individuals' health status. The latter model is theoretically possible once we assume that individuals' marginal utility of consumption is affected by their health status (See, e.g., Boersch-Supan and Stahl 1991).

Note that all variables on the right-hand side in Table 9 are measured in year between t and t+1, therefore, they are exogenous in the sense that they are not correlated with the error term in year t+1.[12] According to Table 9 in all specifications the coefficient of subjective mortality risk has an expected sign but it is statistically significant at a 10 percent level of significance only in the fourth model. . The size of the estimate shows that an increase in subjective mortality by 1 %-point is associated with a decrease in consumption of nondurables by approximately 0.34 %-points. This finding suggests that the growth rate of consumption expenditures on sub-categories of nondurables is lower for individuals with higher subjective mortality rates in line with the prediction of the life cycle model outlined in Section 2.1. On the other hand, the subjective mortality risk does not explain the growth rate of consumption expenditures on all categories of nondurables probably because some of these categories such as home insurance, property tax, rent, electricity, water, heat etc. are not adjusted in response to changes in the mortality risk. The estimated coefficient on the variable $\ln\left(1-m_{i,t}^{t-1}\right)$ in the fourth model corresponds to a parameter of relative risk aversion which is equal to 2.9. This estimate is consistent with the previous estimates in the literature (See, e.g., Skinner 1985 and Palumbo 1999).

The results under the fourth column also suggest that the growth rate of nondurable consumption is higher for the individuals who have any difficulty in activities of daily living such as eating, bathing, walking etc. In other words, these individuals' current expenditures on sub-categories of nondurables are smaller than their future expenditures on the same categories. The reason behind this finding can be that individuals with difficulty

[12] The CAMS survey is matched to most of the information in the previous HRS wave, i.e. CAMS 2001 is matched to the HRS 2000. The consumption growth between 2003 and 2001 is matched to variables from the HRS dataset in 2002.

in daily activities would spend more money on health services and medical supplies and less money on other categories such as food, dining out, vacations and hobbies. The sub-categories of nondurables in model (4) exclude expenditures on health; therefore, we find that the current expenditures on non-health related categories are lower for unhealthy individuals.

Table 10 report the results from the estimation of the Euler equation as shown in equation (11) where the change in consumption is explained by the couple's adjusted mortality rate, $cm_{t+1}^{t} - \sqrt{2}^{1-\gamma}\left(pm_{t+1}^{t} + pf_{t+1}^{t}\right)$, and a set of control variables including health indicators (in levels or in first differences) and the level of education for the husband and the wife. In the first two models in Table 10, in which we use all categories of nondurable spending, the coefficient on the couple's adjusted mortality rate has an expected sign, although it is not statistically significant. In the last two models, we find an unexpected sign for the coefficient on the couple's adjusted mortality rate, yet the estimate is not statically different from zero. The control variables also do not seem to play a role in determining the growth rate of consumption of elderly couples. On the other hand, the possible correlation between the variables of the husband and the variables of the wife might explain the insignificant coefficient estimates.

One of the reasons why the prediction of the life-cycle model outlined in section 2.2 does not hold can be that the assumption of the model regarding the same parameter of relative risk aversion for both spouses may not be supported by the empirical evidence.[13] The existing literature shows that women are more risk averse in financial decision-making than are men.[14] The model can be extended by assuming a different coefficient of relative risk aversion for the husband and the wife. Moreover, instead of estimating the risk aversion parameter directly we replace it with a specific value. Although we run some robustness checks by assigning different values (see footnote 13), the complete analysis requires estimating this parameter directly.

[13] The coefficient of risk aversion is set equal to 3 to calculate the couple's adjusted mortality rate. We also tried the values between 1 and 3 but the estimation results did not change substantially.

[14] See Croson and Gneezy (2009) for a review of the literature on this topic.

Table 10: Estimation of the Euler Equation by OLS (two-person households)

	(1) Consumption growth (All categories) $\Delta \ln c_{t+1}$	(2) Consumption growth (All categories) $\Delta \ln c_{t+1}$	(3) Consumption growth (Salm (2010, sub-categories) $\Delta \ln c_{t+1}$	(4) Consumption growth (Salm (2010, sub-categories) $\Delta \ln c_{t+1}$
$\ln\left[1-\left(cm_t^{t-1}-\sqrt{2}^{1-\gamma}\left(pm_t^{t-1}+pf_t^{t-1}\right)\right)\right]$ [a]	0.263	0.267	-0.064	-0.122
	(0.188)	(0.187)	(0.261)	(0.266)
Change in self health_husband	0.011		0.018	
	(0.009)		(0.012)	
Change in self health_wife	0.011		0.009	
	(0.010)		(0.013)	
Change in any ADL_husband	0.023		-0.014	
	(0.031)		(0.036)	
Change in any ADL_wife	0.013		-0.033	
	(0.025)		(0.030)	
Change in any IADL_husband	0.0001		0.034*	
	(0.017)		(0.020)	
Change in any IADL_wife	-0.009		-0.010	
	(0.018)		(0.023)	
Change in CES-D_husband	-0.0004		0.003	
	(0.006)		(0.009)	
Change in CES-D_wife	-0.001		0.002	
	(0.005)		(0.007)	
Years of education_husband	-0.0004	0.001	-0.002	-0.002
	(0.003)	(0.003)	(0.004)	(0.003)
Years of education_wife	-0.001	-0.002	-0.005	-0.006
	(0.004)	(0.004)	(0.006)	(0.006)
Poor health_husband		0.006		0.015
		(0.023)		(0.030)
Poor health_wife		0.002		-0.022
		(0.024)		(0.031)
Good health_husband		-0.026		0.004
		(0.017)		(0.021)
Good health_wife		-0.009		-0.017
		(0.017)		(0.021)
Any IADL limitations _husband		-0.003		0.030
		(0.016)		(0.021)
Any IADL limitations _wife		-0.011		-0.012
		(0.019)		(0.025)
Any ADL limitations _husband		-0.018		-0.042
		(0.028)		(0.036)
Any ADL limitations _wife		0.011		-0.024
		(0.027)		(0.036)
CES-D score_husband		-0.003		-0.005
		(0.007)		(0.009)
CES-D score_wife		-0.001		0.002
		(0.006)		(0.008)
Constant	-0.011	0.017	0.049	0.060
	(0.048)	(0.055)	(0.073)	(0.078)
Number of observations (households)	1004(506)	1061(524)	1004(506)	1061(524)
p-value Wald test: all health variables	0.845	0.921	0.460	0.802

Robust standard errors are in parentheses * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$. [a] The coefficient of risk aversion is set equal to 3 to calculate this variable.

4.1 Sensitivity checks

Following Salm (2010), we instrument subjective mortality rate to account for measurement error and focal point responses. Salm (2010) uses the instrumental variables which are indicators for mother's age at death 76 or younger, mother's age at death 77 to 84, mother's age at death 85 or older, father's age at death 72 or younger, father's age at death 73 to 80, and father's age at death 81 or older and the reference categories are mother is still alive and father is still alive, respectively. For elderly singles, we find that father variables do not significantly predict individuals' subjective mortality risk; therefore, we did not use these variables in our estimation. Instead, we instrument subjective mortality rate with life table mortality rate ($m_{0,t}^{t-1}$) and the mother variables. Results are given in Table 11.

According to the first stage results under the columns (2) and (4), life table mortality rate is strongly correlated with subjective mortality rate. The null hypothesis of the Hansen's J statistic in Table 11 states that overidentifying restrictions are valid. Failing to reject the null hypothesis indicates that the instruments are exogenous in the sense that they are not correlated with the error term of the consumption growth equation for elderly singles. The F statistics in models (2) and (4) are quite high, showing the strength of the relationship between subjective mortality rate and excluded instruments in the first stage regressions. The last row in Table 11 reports Wooldridge's (1995) robust score test statistic to determine whether the endogenous right-hand-side variable in the model, which is subjective mortality rate in our case, is in fact exogenous. The p-value of the exogeneity test statistic in Table 11 indicates that the exogeneity of the right-hand-side variable cannot be rejected; suggesting that the IV estimation is not needed and we can rely on the OLS results in Table 9.

Table 12 shows the IV estimation results of the Euler equation for couple households. We instrument the couple's adjusted mortality rate based on subjective mortality risk of the husband and the wife with the couple's adjusted mortality rate obtained from the life table. We do not use parental longevity variables (the mother's and the father's ages at death) as additional instruments because we find that these variables do not significantly predict the couple's adjusted mortality risk. According to the first stage results in Table 12, the couple's adjusted mortality risk obtained from the life table is strongly correlated with the same variable calculated using subjective mortality risk of the husband and the wife.

Table 11: Instrumental Variable (IV) Estimation of the Euler Equation (one-person households)

	(1) Consumption growth (Salm (2010, sub-categories) IV $\Delta \ln c_{t+1}$	(2) $\ln\left(1-m_{i,t}^{t-1}\right)$ First Stage	(3) Consumption growth (Salm (2010, sub-categories) IV $\Delta \ln c_{t+1}$	(4) $\ln\left(1-m_{i,t}^{t-1}\right)$ First Stage
$\ln\left(1-m_{i,t}^{t-1}\right)$	0.760*		0.873*	
	(0.434)		(0.470)	
Change in self-rated health	-0.005	-0.0003		
	(0.013)	(0.001)		
Change in any ADL	0.030	-0.013***		
	(0.028)	(0.004)		
Change in any IADL	-0.015	0.001		
	(0.023)	(0.003)		
Change in CES-D	0.001	0.0002		
	(0.005)	(0.001)		
Years of education	-0.003	0.001**	-0.002	0.001
	(0.004)	(0.000)	(0.004)	(0.001)
$\ln\left(1-m_{0,t}^{t-1}\right)$		0.705***		0.659***
		(0.060)		(0.060)
Mother is still alive (ref.)		-		-
Mother's age at death<= 76		-0.020**		-0.017*
		(0.009)		(0.009)
77<=Mother's age at death<=84		-0.009		-0.007
		(0.009)		(0.009)
Mother's age at death>= 85		-0.0005		0.001
		(0.009)		(0.009)
Poor health			0.019	-0.023***
			(0.031)	(0.004)
Good health			-0.007	0.010***
			(0.023)	(0.003)
Any IADL limitations			-0.030	0.002
			(0.023)	(0.003)
Any ADL limitations			0.085**	-0.005
			(0.034)	(0.004)
CES-D score			0.008	0.0003
			(0.007)	(0.001)
Constant	0.050	-0.039***	0.022	-0.031***
	(0.073)	(0.012)	(0.072)	(0.012)
Number of obs.(households)	1290(630)	1290(630)	1307(635)	1307(635)
F test (p-value)		47.296 (0.00)		41.77 (0.00)
Hansen's J test(p-value)		0.795 (0.85)		1.012(0.798)
Test of exogeneity- χ_1^2 *(p-value)*		1.242 (0.265)		1.279(0.257)

Robust standard errors in parentheses * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 12: Instrumental Variable (IV) Estimation of the Euler Equation (two-person households)

	(1) Consumption growth (All categories) IV $\Delta \ln c_{t+1}$	(2) First Stage	(3) Consumption growth (All categories) IV $\Delta \ln c_{t+1}$	(4) First Stage
$\ln\left[1-\left(cm_t^{t-1}-\sqrt{2}^{1-\gamma}\left(pm_t^{t-1}+pf_t^{t-1}\right)\right)\right]$ [a]	0.427		0.241	
	(0.433)		(0.472)	
Change in self health_husband	0.011	-0.001		
	(0.009)	(0.001)		
Change in self health_wife	0.011	-0.002		
	(0.010)	(0.002)		
Change in any ADL_husband	0.023	0.001		
	(0.030)	(0.004)		
Change in any ADL_wife	0.013	-0.002		
	(0.025)	(0.003)		
Change in any IADL_husband	0.0003	-0.0003		
	(0.017)	(0.002)		
Change in any IADL_wife	-0.009	0.001		
	(0.018)	(0.002)		
Change in CES-D_husband	0.0000	-0.002**		
	(0.006)	(0.001)		
Change in CES-D_wife	-0.001	0.0003		
	(0.005)	(0.001)		
Years of education_husband	-0.001	0.001**	0.001	0.0001
	(0.003)	(0.0004)	(0.003)	(0.0004)
Years of education_wife	-0.001	0.0006	-0.002	0.00001
	(0.004)	(0.0006)	(0.004)	(0.0005)
$\ln\left[1-\left(cm_{0,t}^{t-1}-\sqrt{2}^{1-\gamma}\left(pm_{0,t}^{t-1}+pf_{0,t}^{t-1}\right)\right)\right]$ [a]		0.833***		0.765***
		(0.0739)		(0.068)
Poor health_husband			0.006	-0.010***
			(0.023)	(0.003)
Poor health_wife			0.002	-0.006*
			(0.025)	(0.003)
Good health_husband			-0.026	0.004*
			(0.017)	(0.002)
Good health_wife			-0.009	0.008***
			(0.018)	(0.002)
Any IADL limitations _husband			-0.003	0.001
			(0.016)	(0.002)
Any IADL limitations _wife			-0.011	-0.001
			(0.019)	(0.002)
Any ADL limitations _husband			-0.018	-0.006*
			(0.028)	(0.003)
Any ADL limitations _wife			0.011	-0.002
			(0.027)	(0.004)
CES-D score_husband			-0.003	-0.003***
			(0.007)	(0.001)
CES-D score_wife			-0.001	-0.0004
			(0.005)	(0.0009)
Constant	0.0002	-0.036***	0.016	-0.017**
	(0.059)	(0.008)	(0.059)	(0.008)
Number of observations (households)	1004(506)	1004(506)	1061(524)	1061(524)
Test of exogeneity- χ_1^2 (p-value)		0.159		0.003
		(0.689)		(0.954)

Robust standard errors are in parentheses * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$. [a] The coefficient of risk aversion is set equal to 3 to calculate this variable.

The second stage results in Table 12 suggest that the coefficient on the couple's adjusted mortality rate has an expected sign, yet it is not statistically significant. In line with the estimation results in Table 10 we find that the prediction of the life-cycle model for elderly couples does not hold even after accounting for measurement error and focal point responses in subjective survival probabilities of the husband and the wife.

5 Conclusions

Previous studies find that individuals do not draw down their assets after retirement which is at odds with the predictions of a simple life cycle model without uncertainty. Hurd (1989, 1999) explain saving behavior of elderly singles and couples by adding lifetime uncertainty and bequest motives to the simple life cycle model. In this paper we test the predictions of the models proposed by Hurd (1989, 1999) for elderly Americans. We also extend the theoretical model of Hurd (1999) for couples. For this purpose we use data taken from Health and Retirement Study (HRS) supplemented with the Consumption and Activities Mail Survey (CAMS). More than half of the individuals in our sample spend more than their annuity income after retirement which indicates most individuals have decreasing wealth profiles in old age as predicted by the theory. Moreover, we find that the difference between total consumption and annuity income increases with the level of wealth for both elderly singles and couples. The findings of the singles model also suggest that the growth rate of consumption expenditures on sub-categories of nondurables is lower for individuals with higher subjective mortality rates. On the other hand, the subjective mortality risk does not explain the growth rate of consumption expenditures on all categories of nondurables probably because some of these categories such as home insurance, property tax, rent, electricity, water, heat etc. are not adjusted in response to changes in the mortality risk.

We also find that the growth rate of consumption of elderly couples does not depend on their adjusted mortality risk, contrary to the prediction of the theory. One of the reasons behind this finding can be that the assumption of the model regarding the same coefficient of relative risk aversion for both spouses may not be supported by the empirical evidence since women tend to be more risk averse than men in financial decision-making.

Overall this paper finds some evidence in favor of wealth decumulation by the elderly after retirement. As a future research we can extend the couples model by assuming a different coefficient of relative risk aversion for the husband and the wife. Another extension would be adding additional uncertainty to the model such as uncertainty about

out-of-pocket medical expenses which could be an important factor to explain savings of the elderly.

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A Derivation of the Euler Equation

A.1 Singles model

Derivation of equation (3):

$$\max_{c_t, A_t} \sum_{\tau=t}^{L} (1+\rho)^{1-\tau} a_\tau^t u(c_\tau) + \sum_{\tau=t}^{L} (1+\rho)^{-\tau} m_{\tau+1}^t V((1+r)A_\tau), \quad t = 1,\dots,L$$

$$\text{s.t.} \quad A_t = (1+r)A_{t-1} + y - c_t, \quad t = 1,\dots,L$$

$$A_t \geq 0, \qquad t = 1,\dots,L$$

where

A_t = the net worth at the end of period t

y = the non-capital income (in real terms) in period t, consisting only of annuities which are assumed to be constant over time.

ρ = the rate of time preference

r = the real interest rate

c_t = consumption in period t

a_τ^t = the probability that a person lives in period τ given that he/she survives the period t ,and $a_t^t = 1$.

$m_{\tau+1}^t$ = the probability that a person dies at the beginning of $\tau+1$ given that he/she survives the period t.

L = the maximum age after which the person dies with certainty, i.e. $m_{L+1}^L = 1$.

If we ignore the liquidity constraints, the Lagrangean equation of this problem can be written as:

$$\mathrm{L} = \sum_{\tau=t}^{L}(1+\rho)^{1-\tau} a_\tau^t u(c_\tau) + \sum_{\tau=t}^{L}(1+\rho)^{-\tau} m_{\tau+1}^t V((1+r)A_\tau) - \sum_{\tau=t}^{L}\lambda_\tau [A_\tau - (1+r)A_{\tau-1} - y + c_\tau]$$

where λ_τ is the Lagrange multiplier in period τ. By differentiating the Lagrangean equation with respect to c_t, c_{t+1}, and A_t and setting the derivatives equal to zero, we obtain the following first order conditions:

$$\frac{\partial \mathcal{L}}{\partial c_t} = 0, \quad (1+\rho)^{1-t} a_t^t u'(c_t) - \lambda_t = 0 \qquad (A.1)$$

$$\frac{\partial \mathcal{L}}{\partial c_{t+1}} = 0, \quad (1+\rho)^{-t} a_{t+1}^t u'(c_{t+1}) - \lambda_{t+1} = 0 \qquad (A.2)$$

$$\frac{\partial \mathcal{L}}{\partial A_t} = 0, \quad (1+\rho)^{-t} m_{t+1}^t V'((1+r)A_t)(1+r) - \lambda_t + \lambda_{t+1}(1+r) = 0 \qquad (A.3)$$

By combining equations (A1)-(A3), we find the Euler equation as follows:

$$u'(c_t) = \frac{1+r}{1+\rho}\left[\frac{a_{t+1}^t}{a_t^t}u'(c_{t+1}) + \frac{m_{t+1}^t}{a_t^t}V'((1+r)A_t)\right] \qquad (A.4)$$

Let the random variable T denote the period at which the person dies. Then $m_{t+1}^t = P(T = t+1 | T > t)$ is called the "instantaneous mortality rate'' and it gives the probability that person dies at the beginning of period $t+1$ given that he/she survives period t, $t = 1,\dots,L$. The probability that the person lives in period $t+1$ given that he/she survives the period t is equal to

$a_{t+1}^{t} = P(T > t+1 | T > t) = 1 - m_{t+1}^{t}$ and $a_t^t = 1$, $t = 1,\ldots,L$

Then the equation (A.4) can be re-written as:

$$u'(c_t) = \frac{1+r}{1+\rho}\left[(1-m_{t+1}^{t})u'(c_{t+1}) + m_{t+1}^{t}V'((1+r)A_t)\right] \tag{A.5}$$

A.2 Couples model

Derivation of equation (9)

$$\max_{c_t, A_t} \sum_{\tau=t}^{L_{\min}} (1+\rho)^{1-\tau} a_\tau^t u\left(\frac{c_\tau}{\sqrt{2}}\right) + \sum_{\tau=t}^{L_m} (1+\rho)^{-\tau} pm_{\tau+1}^{t} M((1+r)A_\tau) +$$
$$\sum_{\tau=t}^{L_f} (1+\rho)^{-\tau} pf_{\tau+1}^{t} F((1+r)A_\tau) + \sum_{\tau=t}^{L_{\max}} (1+\rho)^{-\tau} h_{\tau+1}^{t} V((1+r)A_\tau)$$

s.t. $A_t = (1+r)A_{t-1} + y - c_t$, $t = 1,\ldots,L_{\max}$

$A_t \geq 0$, $t = 1,\ldots,L_{\max}$

where

$u\left(\frac{c_\tau}{\sqrt{2}}\right)$ = the couple's utility from consumption divided by an equivalent scale

ρ = the rate of time preference of the couple

A_t = the net worth at the end of period t

r = the real interest rate

a_τ^t = the probability that both spouses will be alive in period τ given that they survive the period t ,and a_t^t =1.

$L_{\min}$ = the lifespan of the couple after which one of the spouses dies with certainty

$pm_{\tau+1}^{t}$ = the probability that the husband becomes a widower at the beginning of $\tau+1$

L_m = the maximum lifespan of the husband

$pf_{\tau+1}^{t}$ = the probability that the wife becomes a widow at the beginning of $\tau+1$

L_f = the maximum life span of the wife

$M((1+r)A_\tau)$ = widower's utility of wealth= $\sum_{\tau=t+1}^{L_m} (1+\rho)^{t+1-\tau} a_\tau^{m,t+1} u(c_\tau)$

$F((1+r)A_\tau)$ = widow's utility of wealth= $\sum_{\tau=t+1}^{L_f} (1+\rho)^{t+1-\tau} a_\tau^{f,t+1} u(c_\tau)$

$h_{\tau+1}^{t}$ = the probability that both spouses die at the beginning of $\tau+1$, $V(.)$ is the utility from leaving bequests to the children or others

$L_{\max}$ = the maximum lifespan of the surviving spouse

In case of no bequest motive to the children or others and no liquidity constraints, we can write the Lagrangean equation of this problem as follows:

$$\mathcal{L} = \sum_{\tau=t}^{L_{\min}} (1+\rho)^{1-\tau} a_{\tau}^{t} u\left(\frac{c_{\tau}}{\sqrt{2}}\right) + \sum_{\tau=t}^{L_m} (1+\rho)^{-\tau} pm_{\tau+1}^{t} \left[\sum_{\tau=t+1}^{L_m} (1+\rho)^{t+1-\tau} a_{\tau}^{m,t+1} u(c_{\tau})\right]$$

$$+\sum_{\tau=t}^{L_f} (1+\rho)^{-\tau} pf_{\tau+1}^{t} \left[\sum_{\tau=t+1}^{L_f} (1+\rho)^{t+1-\tau} a_{\tau}^{f,t+1} u(c_{\tau})\right] - \sum_{\tau=t}^{L_{\max}} \lambda_{\tau} \left[A_{\tau} - (1+r)A_{\tau-1} - y + c_{\tau}\right]$$

where λ_{τ} is the Lagrange multiplier in period τ. The first order conditions of this problem are:

$$\frac{\partial \mathcal{L}}{\partial c_t} = 0, \quad (1+\rho)^{1-t} a_t^t \frac{1}{\sqrt{2}} u'\left(\frac{c_t}{\sqrt{2}}\right) - \lambda_t = 0 \qquad \text{(A. 6)}$$

$$\frac{\partial \mathcal{L}}{\partial c_{t+1}} = 0, \quad (1+\rho)^{-t} a_{t+1}^{t} \frac{1}{\sqrt{2}} u'\left(\frac{c_{t+1}}{\sqrt{2}}\right) + (1+\rho)^{-t} pm_{t+1}^{t} u'(c_{t+1})$$

$$+ (1+\rho)^{-t} pf_{t+1}^{t} u'(c_{t+1}) - \lambda_{t+1} = 0 \qquad \text{(A. 7)}$$

$$\frac{\partial \mathcal{L}}{\partial A_t} = 0, \quad -\lambda_t + \lambda_{t+1}(1+r) = 0 \qquad \text{(A. 8)}$$

By combining equations (A.6)-(A.8), we find the following Euler equation:

$$u'\left(\frac{c_t}{\sqrt{2}}\right) = \left(\frac{1+r}{1+\rho}\right)\left[\frac{a_{t+1}^{t}}{a_t^t} u'\left(\frac{c_{t+1}}{\sqrt{2}}\right) + \sqrt{2}\,\frac{pm_{t+1}^{t} + pf_{t+1}^{t}}{a_t^t} u'(c_{t+1})\right] \qquad \text{(A.9)}$$

where a_t^t =1, a_{t+1}^{t} is the probability that both spouses will be alive in period $t+1$ given that they survive the period t and it is equal to one minus the probability that one of the spouses dies at the beginning of period $t+1$ given that both spouses were alive in period t ($a_{t+1}^{t} = 1 - m_{t+1}^{t}$). pm_{t+1}^{t} is the probability that the husband becomes a widower at the beginning of $t+1$ and pf_{t+1}^{t} is the probability that the wife becomes a widow at the beginning of $t+1$. Then the equation (A.9) can be re-written as:

$$u'\left(\frac{c_t}{\sqrt{2}}\right) = \left(\frac{1+r}{1+\rho}\right)\left[\left(1 - cm_{t+1}^{t}\right)u'\left(\frac{c_{t+1}}{\sqrt{2}}\right) + \sqrt{2}\left(pm_{t+1}^{t} + pf_{t+1}^{t}\right)u'\left(c_{t+1}\right)\right] \qquad \text{(A.10)}$$