Strategic Asset Allocation and Longevity Risk

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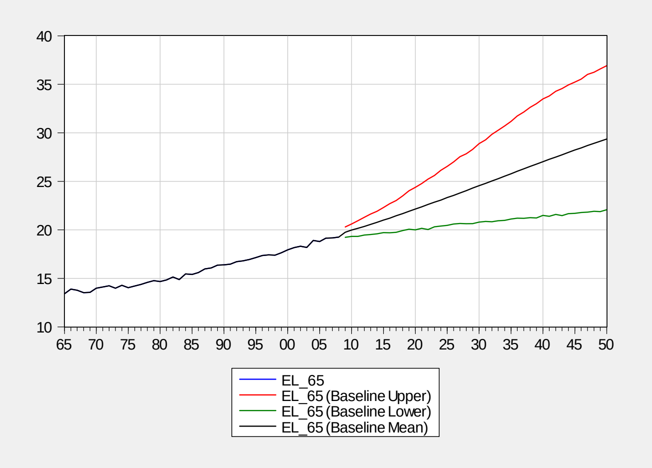
Realized and Projected(Lee-Carter) Expected life at 65



Life expectancy at 65

Four Hours a Day

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- **Economic Theory**: **issuing and trading longevity linked securities in financial market can improve aggregate longevity risk-sharing and offers a viable solution to mitigate the negative welfare effects of risk concentration.**

Market practice: a Sharpe Ratio of 0.76

- Looking at all U.S. stocks from 1926 to 2011 that have been traded for more than 30 years Berkshire Hathaway has a higher Sharpe ratio than all U.S. mutual funds.
- Frazzini et al.(2014) report that 36% of Berkshire's liabilities consist of insurance float, with an estimated cost which is only 2.2%, more than 3 percentage points below the average T-bill rate.
- This reduction in the cost of leverage comes at a risk:
  - In December 2010 Berkshire Hathaway opened a dispute with the reinsurer Swiss Re. Financial press reported that management felt that the losses it had suffered on the book of life reinsurance business were greater than had been expected. It recorded a pre-tax loss of $642m in 2011 due to mortality rates exceeding expectations under the quota share type deal[1].

[1] As on march 28 2013, the reinsurer Swiss Re has announced that its dispute with Warren Buffett's Berkshire Hathaway regarding a 2010 life retrocession deal has been settled.

The Question

- Conventional asset allocation models do not include actuarial risks among the stochastic factors that determine the investor's opportunity set, therefore they cannot be informative for optimal asset - insurance liabilities allocation.
- It is important to propose a framework to quantify the impact of longevity risk reinsurance business on the risk-return trade-off for traditional portfolios.
- A fortiori this exercise involves the description of the risk return tradeoff over a range of time horizons ranging from short term (e.g. monthly) financial market price fluctuations up to demographic (below business cycle) secular trends.

The State of the Art: an incomplete list of contribution areas (Sorry!)

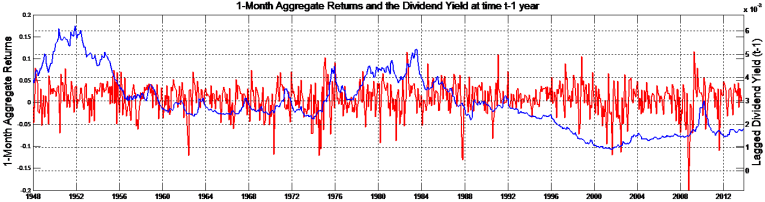
The approach lies at the interesection between different research areas

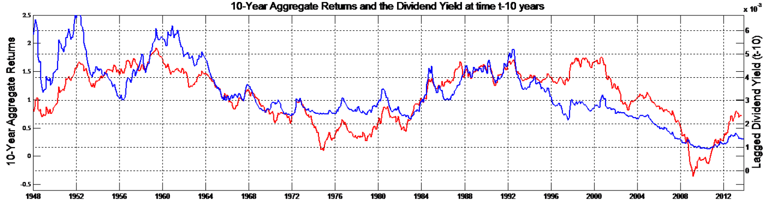
- Actuarial Science:
  - design and valuation of longevity linked securities
  - annuity fair pricing
  - longevity risk premium estimation.
- Financial Management for Insurance Business.
- Life-Cycle Portfolio Allocation.
- Asset Allocation in the presence of risks for the Long Run.

The Framework

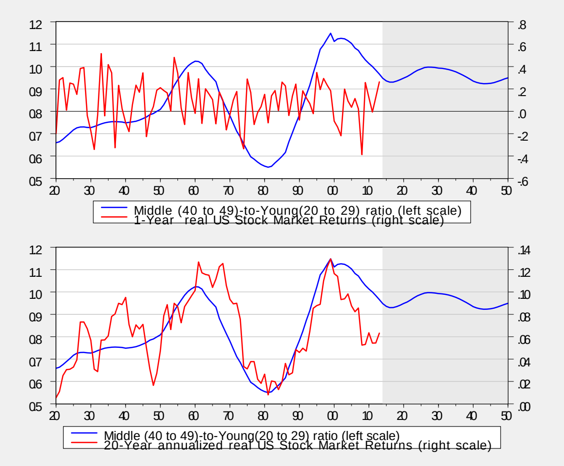
- Include longevity linked securities (instruments designed to reduce the impact of systematic longevity risk on public and private balance sheets) in traditional portfolios
- Consider explicitly the risk term structure.
    - The risk term structure describes the dependence of the risk-return tradeoff on the investor's holding period.
    - The optimal blend of financial securities vary with the investor's holding period due to changes in opportunity set driven by predictability effects

Predictability and the Investment Horizon





Age Structure of US Population and Stock Market Returns (Favero et al. 2011, Ortu Tamoni T. 2013)



The Risk Term Structure

- Uncertainty cannot be properly measured by period volatility (e.g. yearly).
- the return on traditional assets (stocks) are very little predictable at short-horizons, but their predictability increases with the horizon.
- Longevity shocks are known to be driven by low frequency trends on time scales comparable to business cycles. Their impact is negligible for short-term investors but can be substantial for a long term investor like a pension fund
- Short-run returns on traditional assets are orthogonal to longevity trends while long-run returns are not.

The Model

- To investigate the impact of longevity risks on the term structure of risk return trade-offs of optimal investment allocations we extend the Vector Autoregression (VAR) framework originally proposed by Campbell-Viceira(2005)
- We estimate a VAR model including
  - Traditional assets: US stocks, Treasury bonds, T-bills.
  - Annuity Linked Security, a synthetic security exposed to longevity risk whose price is indexed to Annuity Prices which are publicly reported by North American insurance companies.
  - CPredictors for traditional assets: the dividend-price ratio, the spread between long-term and short-term bonds and the nominal T-bill yield.
  - Mortality predictor: innovations in the common mortality factor in the Lee-Carter (1992) stochastic mortality model.

Annuity Accounting

$$R^A_{t+1} = \frac{(P_{x+1,t+1} + C)s_{x+1,t+1}}{P_{x,t}s_{x,t}} - 1$$

after log-linearization and differencing:

$$\Delta p_{x,t} = \rho\left(\Delta p_{x+1,t+1}\right) - \Delta r^A_{t+1} + b_x\left(-e_t\right)$$

- $\Delta p_{x,t}$ Annuity Linked Security period return on a (nontraded) tontine insurance in which contracts are terminated and then possibly renegotiated
- R^A_{t+1} Yearly return to annuitant from an annuity contract individual-specific whose purchase is irreversible)
- C coupon, $P_{x,t}$ price at time t for a standardized annuity sold to an individual with age x
- $s_{\overline{x}+i,t+i}$ probability to be alive in year $t+i$, of age $\overline{x}+i$, conditional on being alive at age $\overline{x}$,
- $s_{x+1,t+1}/s_{x,t}$ mortality credit component

Lee Carter stochastic mortality modeling

Lee Carter (1992) assume that

$$\begin{aligned} q_{x,t} &= \exp\left(a_x + b_x k_t + \varepsilon_{x,t}\right), \\ k_t &= c_0 + c_1 k_{t-1} + e_t, \end{aligned}$$

and for small variations in mortality rates

$$\Delta \ln\left(1 - q_{x,t}\right) \simeq -\left[b_x e_t + \varepsilon_{x,t+1} - \varepsilon_{x,t}\right].$$

and assuming $\sum_{x=65}^{110} b_x = 1$ and $\sum_{x=65}^{110}\left(\varepsilon_{x,t+1} - \varepsilon_{x,t}\right) \approx 0$ then it is possible to define an aggregate longevity (mortality) shock qk_t (e_t):

$$qk_t \simeq -\sum_{x=65}^{110}\left(b_x e_t + \varepsilon_{x,t}\right) \simeq -e_t.$$

VAR equation for the Annuity Linked Security return

Iteration of the

$$\Delta p_{x,t} \simeq -\sum_{j=0}^{m} \rho^j E_t \Delta r^A_{t+1+j} + \sum_{j=0}^{m} \rho^j b_x E_t \left(-e_{t+j+1}\right)$$

Rationality of the annuitant forces the (log) holding period return r_t^A to be competitive (but for the mortality credit) to the compensation one would get by investing in a portfolio of traded financial securities with similar risk and return characteristics. Assuming a stationary evolution for r_t^A and integrated evolution for k_t:

$$\begin{aligned} \Delta p_{x,t} &\simeq \phi_0^A + \phi_1^{A,Mkt} \mathbf{z}_t^{Mkt} + \phi_2 q k_t, \\ x\Delta p_{t+1} &= \phi_0^A + \phi_1^{A,Mkt} \mathbf{z}_t^{Mkt} + \phi_2 q k_t + \nu^A_{t+1}, \end{aligned}$$

The full VAR(1) Specification

We now analyze the effect of extending the traditional portfolio to include excess annuity prices by considering the following augmented VAR specification:

$$\begin{aligned} \mathbf{z}_t &= \Phi_0 + \Phi_1 \mathbf{z}_{t-1} + \nu_t \\ \nu_t &\sim \mathcal{N}(0, \Sigma_\nu), \end{aligned} \tag{1}$$

where

$$\mathbf{z}_t = \begin{bmatrix} r_{0t} \\ \mathbf{x}_t^{Mkt} \\ x\Delta p_t \\ \mathbf{s}_t^{Mkt} \\ qk_t \end{bmatrix}$$

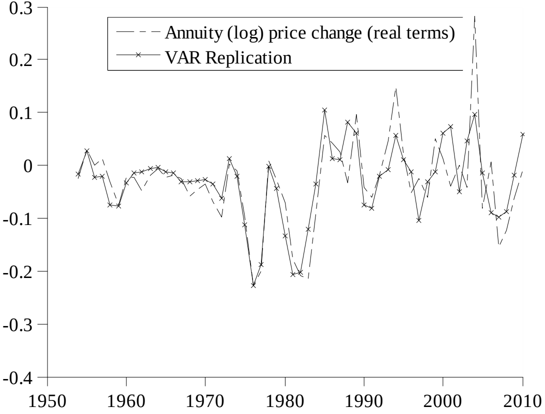
VAR(1) estimation results

	rtb_t	xr_t	xb_t	$x\Delta pr_t$	y_t	$(d-p)_t$	spr_t	$(qk)_t$	R^2
	(t)	(t)	(t)	(t)	(t)	(t)	(t)	(t)	
rtb_{t+1}	0.477	0.007	-0.106	0.074	0.590	-0.010	-0.680	-0.016	0.475
	(3.265)	(0.203)	(-1.753)	(0.640)	(2.534)	(-0.568)	(-1.452)	(-0.116)	
xr_{t+1}	0.190	0.048	0.131	0.929	-0.215	0.154	-0.892	-0.236	0.167
	(0.388)	(0.363)	(0.679)	(1.660)	(-0.225)	(2.423)	(-0.462)	(-0.372)	
xb_{t+1}	0.917	-0.116	-0.502	0.116	0.267	-0.054	4.362	0.588	0.562
	(4.121)	(-1.972)	(-4.311)	(0.370)	(0.550)	(-1.921)	(4.203)	(2.721)	
$x\Delta pr_{t+1}$	0.531	-0.099	0.057	-0.188	-1.146	-0.034	-0.446	0.271	0.516
	(4.294)	(-2.228)	(0.970)	(-0.968)	(-3.950)	(-1.871)	(-0.675)	(2.199)	
y_{t+1}	-0.204	0.025	0.038	-0.046	0.940	0.007	0.286	-0.064	0.794
	(-3.702)	(2.477)	(1.702)	(-0.841)	(9.253)	(1.896)	(1.315)	(-1.728)	
$(d-p)_{t+1}$	-0.812	0.149	-0.081	-0.898	-0.038	0.885	2.475	0.148	0.845
	(-1.392)	(0.808)	(-0.363)	(-1.504)	(-0.035)	(13.163)	(1.141)	(0.221)	
spr_{t+1}	0.104	-0.017	0.014	0.048	0.036	-0.002	0.339	0.011	0.456
	(2.452)	(-1.775)	(1.040)	(1.069)	(0.466)	(-0.585)	(2.076)	(0.404)	
$(qk)_{t+1}$	-0.113	-0.051	0.000	-0.059	-0.008	-0.009	0.373	0.747	0.575
	(-1.341)	(-1.671)	(0.002)	(-0.617)	(-0.034)	(-1.172)	(1.292)	(6.625)	

Cross-Correlations of Residuals

	rtb	xr	xb	Δpr	y	$(d-p)$	spr	$(-qk)$
rtb	3.763	0.206	0.117	0.192	-0.298	-0.234	0.318	0.104
xr	-	14.584	0.040	-0.063	-0.248	-0.970	0.295	0.118
xb	-	-	7.184	0.176	-0.620	-0.089	0.132	-0.014
Δpr	-	-	-	3.927	-0.547	0.022	0.583	-0.044
y	-	-	-	-	1.368	0.262	-0.843	-0.041
$(d-p)$	-	-	-	-	-	15.768	-0.277	-0.098
spr	-	-	-	-	-	-	1.047	0.047
$(-qk)$	-	-	-	-	-	-	-	2.617

A reduced model of log annuity price changes



Results

- The Term structure of risk
- The composition of the GMV Portfolio at different horizons
- Optimal Portfolio weights with a target of 10 % returns
- Term Structure of Sharpe Ratios

The term structure of risk

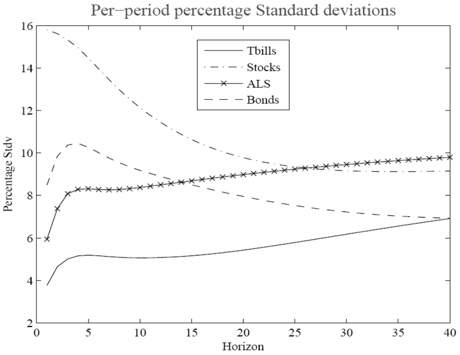


Figure 3: Term structure of risks for the securities included in the Extended VAR model.

The composition of the Global Minimum Variance Portfolio at different horizons

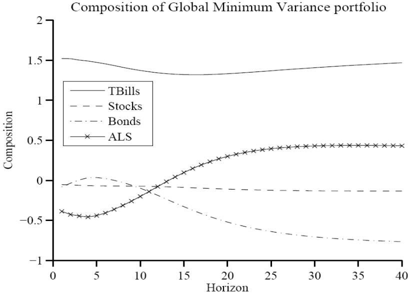


Figure 5: Term structure of allocations forming the GMV portfolio at different horizons.

Risk of TBills and GMV

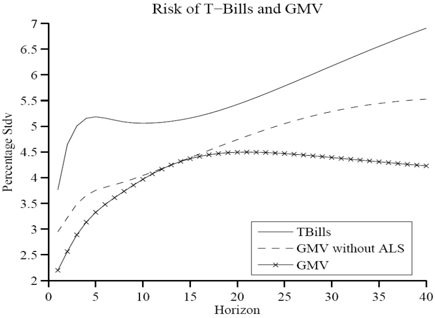


Figure 6: Term structure of risks for an allocation in T-Bill (continuous line), in the GMV portfolio restricted to financial securities (dashed line), in the GMV portfolio including also the Annuity-Linked Security.

Optimal Portfolio weights with a target of 10 % returns

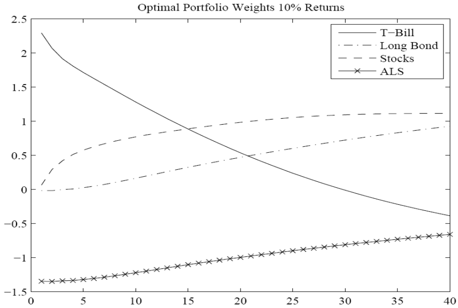
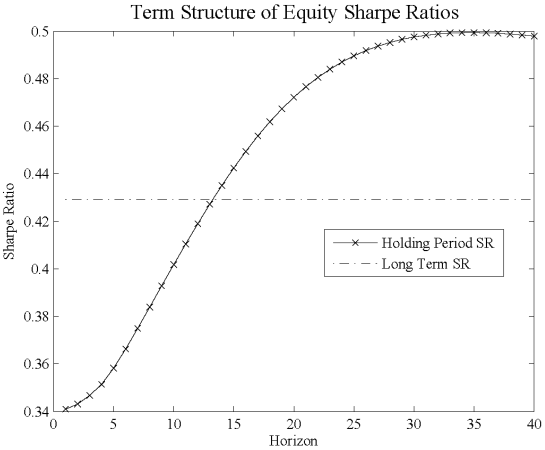


Figure 8: Term structure of the efficient allocation with target expected return of 10%.

The term structure of Sharpe Ratios



Longevity securitization and inter-temporal hedging of the aggregate longevity risk

$$\min_{\mathbf{w}} Var_{t-1}\left[R_t^{qk}\left(\mathbf{w}_{t-1}\right) - qk_t\right]$$

$$\begin{aligned} s.t. \quad &: \quad R_t^{qk}\left(\mathbf{w}_{t-1}\right) = \mathbf{w}_{t-1} \cdot \mathbf{x}_t + W_{0,t-1} rtb_t \\ \mathbf{w}_{t-1} &= \left[w_{xr,t-1}, w_{xb,t-1}, w_{x\Delta p,t-1}\right], \\ \mathbf{x}_t &= \left[xr_t, xb_t, x\Delta p_t\right], \end{aligned}$$

with solution

$$\begin{aligned} \mathbf{w}^* &= Var\left[\mathbf{x}_t\right]^{-1} Cov\left[\mathbf{x}_t, qk_t\right]^T, \\ w_{rtb}^* &= W_0 - \mathbf{w}^* \cdot \mathbf{1} \end{aligned}$$

Numerical Results

$HedgingPortfolio$	w_{xr}	w_{rtb}	w_{xb}	$w_{x\Delta pr}$
$Unconstrained$	-0.042	-0.014	0.009	0.047
$Constrained\_1\_w_{xr} = 0$	0	-0.062	0.004	0.058
$Constrained\_2\_w_{rtb} = w_{xr} = 0$	0	0	-0.059	0.059

SR_∞^{Equity}	$SR_\infty^{LongBond}$	SR_∞^{ALS}
0.429	0.0323	(-) 0.437

SR_∞^{Unc}	$SR_\infty^{Cons\_1}$	$SR_\infty^{Cons\_2}$
(-) 0.504	(-) 0.421	(-) 0.333

Conclusions

- Our analysis shows that efficiency of longevity risk sharing can be achieved via integration between insurance and financial markets.
  - The long term nature of the longevity risk requires an accurate analysis of the term structure of the risk return trade-offs generated by including a longevity-linked security in the set of investments.
  - A potential large number of short term investors would be willing to increase their exposure to longevity risk without increasing their investment horizon. This requires the organization of a maturity transformation activity by financial intermediaries that seems to be a crucial step to increase the interest of the market for longevity-linked securities, as well as their liquidity.
  - An integrated market for insurance and financial contracts with a publicly traded longevity index would also imply a more transparent and efficient pricing of life annuities with a direct benefit for annuity subscribers.