A Multivariate Model of Strategic Asset Allocation with Longevity Risk

by

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Discussion by

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Overview

- Extension “Term Structure Risk-Return Tradeoff” (Campbell & Viceira, 2005) by including longevity-risk sensitive securities
  - VAR-model extended by including
    - A mortality predictor
    - A synthetic financial security exposed to longevity risk

- Quantitative assessment longevity risk compensation

- Interesting & Relevant Study

- But: Also somewhat confusing at several places…
Term Structure of Risks

Figure D.3: Term structure of risks for the securities included in the Extended VAR model.

ALS = Annuity Linked Security.
Risks of T-Bills and of Global Minimum Variance Portfolios

Figure D.7: Term structure of risks for an allocation in T-Bill (continuous line), in the GMV portfolio restricted to financial securities (dashed line), in the GMV portfolio including also the Annuity-Linked Security.
Lee and Carter (1992)

Model paper

\[ \log(q_{xt}) = a_x + b_x k_t + \varepsilon_{xt} \]

Lee-Carter:

\[ \log(m_{xt}) = a_x + b_x k_t + \varepsilon_{xt}, \quad m_{xt} = \frac{D_{xt}}{E_{xt}} \]

Link:

\[ q_{xt} \approx 1 - \exp(-m_{xt}) \text{ or } m_{xt} \approx -\log(1 - q_{xt}) \]

(for \( q_{xt} \approx 0 \): \( q_{xt} \approx m_{xt} \); but: \( q_{xt} \in [0, 1], m_{xt} \in [0, \infty) \))
Mortality Shock

Mortality shock based on Lee-Carter Model

\[
\log(q_{xt}) = a_x + b_x k_t + \varepsilon_{xt}
\]

\[
k_t = c_0 + c_1 k_{t-1} + e_t
\]

Different mortality models with all kinds of sensitivities
(to age groups included, to sample period used, to …)

Lee-Carter model based shocks represent market perspective?
Some regressions

Paper derives as approximate relationship:

$$\Delta p_{x,t} = \rho \Delta p_{x+1,t+1} - \Delta r^A_{t+1} + b_x (-e_t)$$

Iterate forward:

$$\Delta p_{x,t} = -\sum_{j=0}^{m} \rho^j \Delta r^A_{t+1+j} + b_x \sum_{j=0}^{m} \rho^j (-e_{t+j}) + \rho^{m+1} \Delta p_{x+m+1,t+m+1} \approx 0$$

Regressions:

$$-\sum_{j=0}^{m} \rho^j \Delta r^A_{t+1+j} = \alpha_{0r} + \alpha_{1r} \Delta p_{x,t} + \eta_{rt}$$

$$b_x \sum_{j=0}^{m} \rho^j (-e_{t+j}) = \alpha_{0e} + \alpha_{1e} \Delta p_{x,t} + \eta_{et}$$

What to expect: $\alpha_{1r} = 0, \alpha_{1e} = 1$?

with $\alpha_{1r} + \alpha_{1e} \approx 1$
Timing & Direction

Paper uses approximate relationship

\[ \Delta p_{x,t} = \rho \Delta p_{x+1,t+1} - \Delta r^A_{t+1} + b_x(-e_t) \]

to motivate inclusion of \((- e_t)\) in VAR-model as mortality predictor.

However, VAR-model does not include \((- e_t)\) but \((- e_{t-1})\).

Moreover, VAR-estimates include variable \(- qk_{t-1} = +e_{t-1}\), with

\[ x\Delta p_{x,t+1} = \cdots + 0.271(-qk_t) + \cdots = \cdots + 0.271(+e_t) + \cdots : \]

“a positive shock to longevity increases the price of annuities.”
Persistent Mortality Shock

From VAR-model:

\[
(-qk)_{t+1} = \begin{pmatrix}
-0.113 \\
(-1.341)
\end{pmatrix}
\begin{pmatrix}
-0.051 \\
(-1.671)
\end{pmatrix}
\begin{pmatrix}
0.000 \\
(0.002)
\end{pmatrix}
\begin{pmatrix}
-0.059 \\
(-0.617)
\end{pmatrix}
\begin{pmatrix}
-0.008 \\
(-0.034)
\end{pmatrix}
\begin{pmatrix}
-0.009 \\
(-1.172)
\end{pmatrix}
\begin{pmatrix}
0.373 \\
(1.292)
\end{pmatrix}
\begin{pmatrix}
0.747 \\
(6.625)
\end{pmatrix}
\]

We have \(- qk_t = +e_t\).

Lee-Carter Model Assumptions:

\[
\ln (q_{x,t}) = a_x + b_x k_t + \epsilon_{x,t}
\]

\[
k_t = c_0 + c_1 k_{t-1} + e_t
\]

\[
\epsilon_{x,t} \sim NID \left(0, \sigma_{\epsilon}^2 \right)
\]

\[
e_t \sim NID \left(0, \sigma_e^2 \right)
\]