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A Multivariate Model of Strategic Asset Allocation with Longevity Risk

by

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Overview

- Extension “Term Structure Risk-Return Tradeoff” (Campbell & Viceira, 2005) by including longevity-risk sensitive securities
  - VAR-model extended by including
    - A mortality predictor
    - A synthetic financial security exposed to longevity risk
- Quantitative assessment longevity risk compensation
- Interesting & Relevant Study
- But: Also somewhat confusing at several places…

Term Structure of Risks

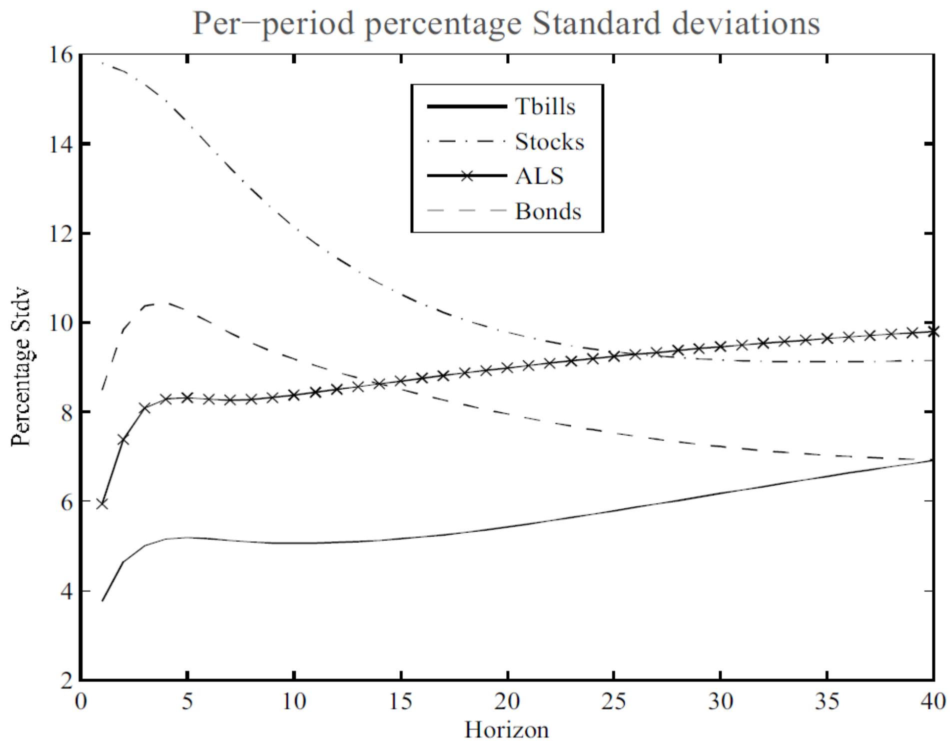

Figure D.3: Term structure of risks for the securities included in the Extended VAR model.

ALS = Annuity Linked Security.

Risks of T-Bills and of Global Minimum Variance Portfolios

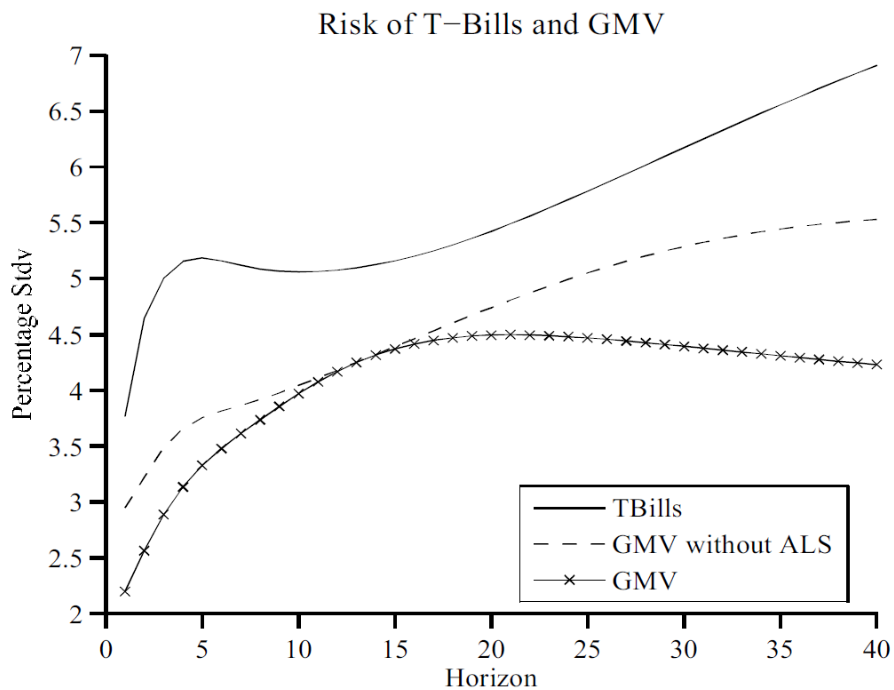

Figure D.7: Term structure of risks for an allocation in T-Bill (continuous line), in the GMV portfolio restricted to financial securities (dashed line), in the GMV portfolio including also the Annuity-Linked Security.

Lee and Carter (1992)

Model paper

$$\log(q_{xt}) = a_x + b_x k_t + \varepsilon_{xt}$$

Lee-Carter:

$$\log(m_{xt}) = a_x + b_x k_t + \varepsilon_{xt}, \;\; m_{xt} = \frac{D_{xt}}{E_{xt}}$$

Link:

$$q_{xt} \approx 1 - \exp(-m_{xt}) \text{ or } m_{xt} \approx -\log(1 - q_{xt})$$
$$(\text{for } \; q_{xt} \approx 0: \; q_{xt} \approx m_{xt}; \text{ but}: \; q_{xt} \in [0,1], \; m_{xt} \in [0,\infty))$$

Mortality Shock

Mortality shock based on Lee-Carter Model

$$\log(q_{xt}) = a_x + b_x k_t + \varepsilon_{xt}$$
$$k_t = c_0 + c_1 k_{t-1} + e_t$$
$$\uparrow$$

Different mortality models with all kinds of sensitivities (to age groups included, to sample period used, to …)

Lee-Carter model based shocks represent market perspective?

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Some regressions

Paper derives as approximate relationship:

$$\Delta p_{x,t} = \rho \Delta p_{x+1,t+1} - \Delta r_{t+1}^{A} + b_x \left(-e_t\right)$$

Iterate forward:

$$\Delta p_{x,t} = -\sum_{j=0}^{m} \rho^j \Delta r_{t+1+j}^{A} + b_x \sum_{j=0}^{m} \rho^j \left(-e_{t+j}\right) + \underbrace{\rho^{m+1} \Delta p_{x+m+1,t+m+1}}_{\approx 0}$$

$$\approx -\sum_{j=0}^{m} \rho^j \Delta r_{t+1+j}^{A} + b_x \sum_{j=0}^{m} \rho^j \left(-e_{t+j}\right)$$

Regressions:

$$\left.\begin{aligned} -\sum_{j=0}^{m} \rho^j \Delta r_{t+1+j}^{A} &= \alpha_{0r} + \alpha_{1r} \Delta p_{x,t} + \eta_{rt} \\ b_x \sum_{j=0}^{m} \rho^j \left(-e_{t+j}\right) &= \alpha_{0e} + \alpha_{1e} \Delta p_{x,t} + \eta_{et} \\ \textit{with} \;\; \alpha_{1r} + \alpha_{1e} &\approx 1 \end{aligned}\right\} \text{What to expect: } \alpha_{1r} = 0,\ \alpha_{1e} = 1?$$

Timing & Direction

Paper uses approximate relationship

$$\Delta p_{x,t} = \rho \Delta p_{x+1,t+1} - \Delta r_{t+1}^{A} + b_x(-e_t)$$

to motivate inclusion of ($-\, e_t$) in VAR-model as mortality predictor.

However, VAR-model does not include ($-\, e_t$) but ($-\, e_{t-1}$).

Moreover, VAR-estimates include variable $-\, qk_{t-1} = +e_{t-1}$, with

$$x\Delta p_{x,t+1} = \cdots + 0.271(-qk_t) + \cdots = \cdots + 0.271(+e_t) + \cdots \;\; :$$

“a positive shock to longevity increases the price of annuities.”

Persistent Mortality Shock

From VAR-model:

								$(-qk)_t$ (t)
$(-qk)_{t+1}$	-0.113	-0.051	0.000	-0.059	-0.008	-0.009	0.373	0.747
	(-1.341)	(-1.671)	(0.002)	(-0.617)	(-0.034)	(-1.172)	(1.292)	(6.625)

We have $-qk_t = +e_t$.

Lee-Carter Model Assumptions:

$$\begin{aligned} \ln(q_{x,t}) &= a_x + b_x k_t + \epsilon_{x,t} \\ k_t &= c_0 + c_1 k_{t-1} + e_t \\ \epsilon_{x,t} &\sim NID\left(0, \sigma_\epsilon^2\right) \\ e_t &\sim NID\left(0, \sigma_e^2\right) \end{aligned}$$