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Two Essays on Adverse Selection in Annuity Markets

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Two essays on adverse selection in annuity markets

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Preface

In the summer of 2009 I started to work on my Research Project for the Honours Bachelor Economics and the Honours Bachelor Econometrics & Operations Research under the supervision of Ben J. Heijdra. At the University of Groningen, this Research Project is only available for Honours students and is meant to be a substitute for the Bachelor Thesis. It offers students much more freedom in terms of subject choice and presentation of results.

Heijdra and I decided to delve into the subject of adverse selection in annuity markets. We wanted to know what the effects are on the macroeconomic equilibrium of an imperfect longevity insurance market. Along the way, we hoped to shed some light on the so-called 'annuity puzzle': the observation that annuities are rarely used in practice even though economists claim their superiority over alternative investment products. I have reported the results of our first joint project in the essay entitled 'Adverse selection and risk pooling in annuity markets'. A journal version has already been published as a working paper (Heijdra and Reijnders, 2009). We found that adverse selection may cause some groups in the population to rationally decide to drop out of the annuity market, thereby providing a possible (albeit not complete) explanation for the annuity puzzle. Moreover, our simulations showed that the welfare effects of adverse selection in the annuity market compared to perfect insurance were not very large, and that an imperfect insurance market was still vastly better than the complete absence of one.

Our work on this first paper conjured up more questions that demanded our attention. Hence, instead of finishing the Research Project after this one essay, we decided to move on. A robustness check revealed that the welfare ranking of different equilibria might depend on the type of redistribution scheme that is used for accidental bequests when annuity markets are absent. In order to fully grasp the underlying mechanisms we decided to simplify the model developed in the first essay substantially by casting it in discrete time. This allowed us to obtain more analytical results and to study the transition between steady states. Moreover, we added a social security system to the model, following the suggestion in the literature that crowding out of private by social annuities might provide a rationale for the annuity puzzle. My second essay, entitled 'The welfare effects of adverse selection in annuity markets and the role of mandatory social annuitization', describes our findings. We show that social annuities are immune to informational asymmetry and may therefore appear to be an attractive policy option for the government. However, their introduction leads to a higher degree of adverse selection in private annuity markets, a decrease in the savings rate, and a lower level of long-run welfare. Currently we are working on a (more elaborate) journal version of this paper.

Finally, the insights gained during this Research Project have also led to another working paper, written together with Jochen O. Mierau (see Heijdra, Mierau and Reijnders, 2010). It deals with a phenomenon which we call 'the tragedy of annuitization'. We show that even though full annuitization of wealth is optimal and fully rational for individuals it may lead to less welfare for future generations.

The Research Project is finished, but only for now. In the future I hope to be able to continue doing research on the macroeconomics of health and annuitization in a setting with informational asymmetries and incomplete markets, as I believe this to be a intruiging and policy relevant topic.

Laurie Reijnders

Groningen, August 16 2010

Heijdra, B.J., Mierau, J.O., and Reijnders, L.S.M. (2010). *The tragedy of annuitization* (Working Paper No. 3141). CESifo, München.

Heijdra, B.J. and Reijnders, L.S.M. (2009). *Economic growth and longevity risk with adverse selection* (Working Paper No. 2898), CESifo, München.

Adverse selection and risk pooling in annuity markets

Abstract

We study a closed economy featuring endogenous growth and overlapping generations of finitely-lived agents. Individuals differ in their health type and thereby their mortality profile. We show that if health status is private information then a pooling equilibrium emerges in the annuity market. Healthy agents receive a better than actuarially fair return while the unhealthy individuals get less than they are entitled to. This gives an incentive to unhealthy agents to borrow money in the last stages of their life. However, they will instead impose a voluntary borrowing constraint on themselves in order not to reveal their health status. For a plausibly parameterized version of the model the welfare loss of having a pooling equilibrium instead of a fair separating equilibrium is small, while the welfare gain relative to the total absence of annuities is much larger.

1 Introduction

In a seminal paper, Yaari (1965) argues that in the face of life span uncertainty individuals will fully annuitize their financial wealth. That is, they will invest all their savings in the annuity market, thereby insuring themselves against the risk of outliving their assets. However, empirical evidence has revealed a so-called annuity puzzle: despite the theoretical attractiveness of annuities, in practice people tend not to invest much of their financial wealth in annuity markets.

Several explanations for this puzzle have been given in the literature. First of all, individuals may have a bequest motive, in that they wish to leave an inheritance to those they leave behind. If so, they will want to keep part of their financial assets outside the annuity market. Secondly, psychological factors may play a role. According to Cannon and Tonks (2008), people might feel uncomfortable to 'bet on a long life'. Investing in annuities only seems attractive if you expect to live long enough, as most of us would hate to die before having received at least our initial outlay back in periodic payments. Third, private annuity demand may be crowded out by a system of social security benefits.

A fourth explanation is that in reality annuities may not be actuarially fair, in the sense that individuals are insufficiently compensated for their risk of dying. This may be due to administrative costs and taxes, or monopoly profits as a result of imperfect competition among annuity firms. The implications for macroeconomic growth and welfare of a loading factor on annuities proportional to the mortality rate is investigated in Heijdra and Mierau (2009a). Another reason for annuity market imperfection is adverse selection. The healthier someone believes herself to be, the more likely she is to buy an annuity. As a consequence, low-mortality (and thus high-risk) individuals are overrepresented in the annuity market. Annuity firms will have to take this selection effect into account when pricing their products, as they will incur a loss if they offer a rate based on average survival probabilities in the population. The resulting higher prices (or lower return) will induce high-mortality (low-risk) individuals to invest less in the annuity market.

In this paper we abstract from bequest motives, administrative costs and imperfect competition and focus solely on the adverse selection channel. In contrast to the other explanations for annuity market imperfection, the concept of adverse selection can help us understand differences in annuity purchases between groups in the population. We show that under asymmetric information concerning health status there will be a pooling equilibrium in the annuity market in which individuals of all health types receive the same rate of return. Despite the fact that this equilibrium is worse for everyone in terms of expected lifetime utility than a separating equilibrium, it will nevertheless be the inevitable market outcome. This follows from the fact that health status cannot be credibly signalled, as a healthy agent has an incentive to misrepresent herself in order to receive a higher return. Another important implication of our model is that unhealthy agents will want to borrow in the last years of their lives. However, as by doing so they would reveal their health status and be forced to pay a much higher rate, they will instead impose a voluntary credit constraint on themselves.

Our conclusion that a pooling equilibrium will prevail in the annuity market appears opposite to that of Rothschild and Stiglitz (1976), who show that a pooling equilibrium does not exist in an insurance market. However, their result relies heavily on the assumption that customers can buy only one insurance contract such that the insurer sets both price and quantity. They admit themselves that this may be an objectionable assumption in some cases, and Walliser (2000) argues that it indeed does not apply to the annuity market. We therefore assume instead that individuals can buy multiple contracts at different annuity firms without this information becoming public.

Papers closely related to ours are Abel (1986) and Walliser (2000). Both specify their model in discrete time, while ours is set in continuous time. In Abel (1986) a two-period exogenous growth model is developed in which agents have privately known heterogenous mortality profiles and a bequest motive. Due to adverse selection, the rate of return on private annuities is less than actuarially fair. In this context, the introduction of an actuarially fair social security system further decreases the return on annuities in the steady state. Walliser (2000) builds on the work of Abel (1986), but calibrates his model with 75 instead of only 2 periods. The paper investigates the effects of social security benefits on private annuity demand and shows that privatization of social security lowers the loading factor on annuities resulting from adverse selection. In his model, he exogenously imposes a nonnegativity constraint on asset holdings. In contrast, this result arises endogenously in the model developed here.

The remainder of this paper is structured as follows. Section 2 describes the model. In Section 3 the steady state of the model will be analyzed using a suitable parameterization. Section 4 concludes. In addition there are two appendices, containing the mathematical proofs of the propositions stated in the text and a description of the estimation procedure used for the mortality process.

2 The model

2.1 Production

We consider a closed economy in which the production side is characterized by a large number of perfectly competitive firms, say N. The technology of firm $i \in \{1, \ldots, N\}$ can be described by the following Cobb-Douglas production function:

$$Y_i(t) = \Omega(t) K_i(t)^{\varepsilon} L_i(t)^{1-\varepsilon}, \quad 0 < \varepsilon < 1, \tag{1}$$

where $Y_i(t)$ is the ouput of firm i at time t, $K_i(t)$ denotes its capital stock and $L_i(t)$ is labour input. Furthermore, $\Omega(t)$ is an index of the general level of factor productivity in the economy, which the firm takes as given. Note that the production technology features diminishing returns to scale for each production factor separately, but constant returns to scale at the firm level.

The cash-flow of the firm at time $t \in [0, \infty)$ is the difference between the revenues it receives and the amount it spends on capital investment and wages:

$$\Pi_i(t) \equiv Y_i(t) - w(t) L_i(t) - I_i(t), \tag{2}$$

where $\Pi_i(t)$ denotes the cash-flow, $I_i(t)$ is investment, and $w(t)$ is the return to labour at time t (to be determined below). Note that we have normalized the output price to unity, so that all variables can be considered real. At each point in time, the firm strives to maximize its fundamental stock market value, which is defined as:

$$V_i(t) \equiv \int_t^{\infty} \Pi_i(\tau) e^{-\int_t^{\tau} r(s) ds} \, d\tau, \tag{3}$$

where $r(s)$ is the rate of return on capital (to be determined below). That is, the firm's stock market value equals the present discounted value of future cash flows over its entire (infinite) time horizon.

We assume that the inter-firm externality $\Omega(t)$ takes the following form:

$$\Omega(t) = \Omega_0 k(t)^{1-\varepsilon}, \tag{4}$$

where Ω_0 is a positive constant, $k(t) \equiv K(t)/L(t)$ is the economy-wide capital intensity, $L(t) = \sum_{i=1}^{N} L_i(t)$ is aggregate employment, and $K(t) = \sum_{i=1}^{N} K_i(t)$ is the aggregate capital stock. That is, investment in capital of a given firm benefits all firms in the economy by raising the general productivity indicator. This specification is a generalization of the one suggested by Romer (1989), who assumed a constant labour force and made $\Omega(t)$ dependent on the total capital stock $K(t)$ instead of the per-capita capital stock $k(t)$. Intuitively, he argued that private investment has a positive external effect in that it leads to transfers of knowledge to other firms.

Solving the firm's optimization problem (see Box 1) and aggregating over firms, we find the economy-wide factor demand equations:

$$w(t)L(t) = (1-\varepsilon)Y(t), \tag{5}$$

$$r(t) + \delta = \varepsilon\Omega, \tag{6}$$

where $Y(t) = \sum_{i=1}^{N} Y_i(t)$ is aggregate output and we assume that the capital stock is sufficiently productive such that $\varepsilon\Omega_0 > \delta$. Since the right-hand side of equation (6) is constant it follows that $r(t) = r \equiv \varepsilon\Omega_0 - \delta$ for all $t \in [0, \infty)$, i.e. the rental rate on capital is constant over time.

Box 1

The factor demand equations. At each point in time, the representative firm wishes to maximize its stock market value (3) subject to the capital accumulation identity $\dot{K}_i(t) = I_i(t) - \delta K_i(t)$. The corresponding current value Hamiltonian is defined as:

$$\mathcal{H}(K_i(t), L_i(t), I_i(t), q(t)) = \Omega(t)K_i(t)^{\varepsilon}L_i(t)^{1-\varepsilon} - I_i(t) - w(t)L_i(t) + q(t)[I_i(t) - \delta K_i(t)],$$

where $I_i(t)$ and $L_i(t)$ are the control variables, $K_i(t)$ is the state variable and $q(t)$ is the co-state variable. Assuming a strictly positive solution for investment and labour use, the first-order conditions of the Maximum Principle of the theory of optimal control are given by:

$$\begin{aligned}
\frac{\partial \mathcal{H}(\cdot)}{\partial I_i(t)} &= -1 + q(t) = 0, \\
\frac{\partial \mathcal{H}(\cdot)}{\partial L_i(t)} &= (1-\varepsilon)\Omega(t) K_i(t)^{\varepsilon} L_i(t)^{-\varepsilon} - w(t) = 0, \\
\dot{K}(t) = \frac{\partial \mathcal{H}(\cdot)}{\partial q(t)} &= I_i(t) - \delta K_i(t), \\
\dot{q}(t) - r(t)q(t) = -\frac{\partial \mathcal{H}(\cdot)}{\partial K_i(t)} &= -\varepsilon \Omega(t) K_i(t)^{\varepsilon-1} L_i(t)^{1-\varepsilon} + \delta q(t),
\end{aligned}$$

and the transversality condition $\lim_{\tau\to\infty} K_i(\tau)q(\tau)e^{-\int_t^\tau r(s)ds} = 0$. Rewriting yields the firm's marginal productivity conditions:

$$\begin{aligned}
w(t) &= (1-\varepsilon)\Omega(t) k_i(t)^{\varepsilon}, \\
r(t) + \delta &= \varepsilon \Omega(t) k_i(t)^{\varepsilon-1},
\end{aligned}$$

where $k_i(t) \equiv K_i(t)/L_i(t)$ is the capital-labour ratio of firm i. By symmetry, $k_i(t) = k(t)$ for all i, i.e. all firms choose the same capital intensity. Using (4), we can now aggregate over all firms to find total output:

$$\begin{aligned}
Y(t) &\equiv \sum_{i=1}^{N} Y_i(t), \\
&= \Omega(t) \sum_{i=1}^{N} \left(\frac{K_i(t)}{L_i(t)}\right)^{\varepsilon} L_i(t), \\
&= \Omega_0 k(t)^{1-\varepsilon} k(t)^{\varepsilon} L(t), \\
&= \Omega_0 K(t).
\end{aligned}$$

It follows that, due to spillovers between firms as a result of private capital investments, the diminishing returns to capital at the firm level are negated at the aggregate level. Total output in the economy features constant returns to capital. This is a standard result in the so-called 'AK' model (even though 'ZK' would have been more appropriate here).

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2.2 Households

The population consists of overlapping generations of finitely-lived agents who are identical in every respect except for their health type. Each agent learns at birth whether she is 'healthy' (indexed by the subscript H) or 'unhealthy' (U). Health status is assumed to be private information. That is, agents know their own health type, but it is not observable for others such as the government and annuity firms.

Let D denote the time of death of a given consumer. We assume that $D \in [0, \bar{D}_j]$ where $\bar{D}_j$ is the (finite) maximum age for persons of health type j. Here '0' does not refer to the age at biological birth, but rather to the age of an economic newborn. Effectively this implies that we do not consider a consumer's childhood (in which her economic contribution is negligible).

We adopt the mortality process introduced in Boucekkine et al. (2002). This process is characterized by the following survival function $1 - \Phi_j(u)$ and instantaneous mortality rate $\mu_j(u)$ (for other the properties of this mortality process, see Box 2 below):

$$1 - \Phi_j(u) = \frac{\eta_{0j} - e^{\eta_{1j} u}}{\eta_{0j} - 1}, \tag{7}$$

$$\mu_j(u) = \frac{\eta_{1j} e^{\eta_{1j} u}}{\eta_{0j} - e^{\eta_{1j} u}}, \tag{8}$$

for $0 \leq u \leq \bar{D}_j \equiv (1/\eta_{1j}) \ln \eta_{0j}$.

If we postulate that $\mu_U(u) > \mu_H(u)$ for all u, i.e. unhealthy agents have higher risk of dying at each age than do agents in good health, then $\bar{D}_U < \bar{D}_H$. It is possible to choose parameters η_{0U}, η_{0H}, η_{1U}, and η_{1H} such that $\bar{D}_U = \bar{D}_H$, but then the mortality profiles will necessarily have to cross. This would give the undesirable and counterintuitive result that for high ages $\mu_U(u) < \mu_H(u)$.

We have chosen to set $\eta_{0U} = \eta_{0H} = \eta_0$, and $\eta_{1U} = \theta\eta_{1H}$ for $\theta > 1$. In order to find empirically relevant values for the parameters we use the life table for the cohort born in 1960 in the Netherlands from age 18 onward.[1] We estimate the parameters in such a way that the average over the health groups corresponds to the data. For more information about the estimation procedure, see Appendix B.

We find $\hat{\eta}_0 = 187.8646$, $\hat{\eta}_{1H} = 0.0696$, $\hat{\eta}_{1U} = 0.0799$, $\hat{\bar{D}}_H = 75.2148$, and $\hat{\bar{D}}_U = 65.5224$.[2] Figure 1 gives the corresponding surving fractions and the instantaneous mortality rate profiles.

[1] The data can be found in the online Human Mortality Database (www.mortality.org). We make no distinction between males and females, even though it is an empirical fact that females tend to live longer in most developed countries.

[2] Recall that these are economic ages. Add 18 years in order to determine the implied biological age. For example, the maximum biological age for unhealthy agents is $65.5224 + 18 = 73.5224$.

(a) Surviving fraction

$e^{-M_j(u)}$

(b) Instantaneous mortality rate

$\mu_j(u)$

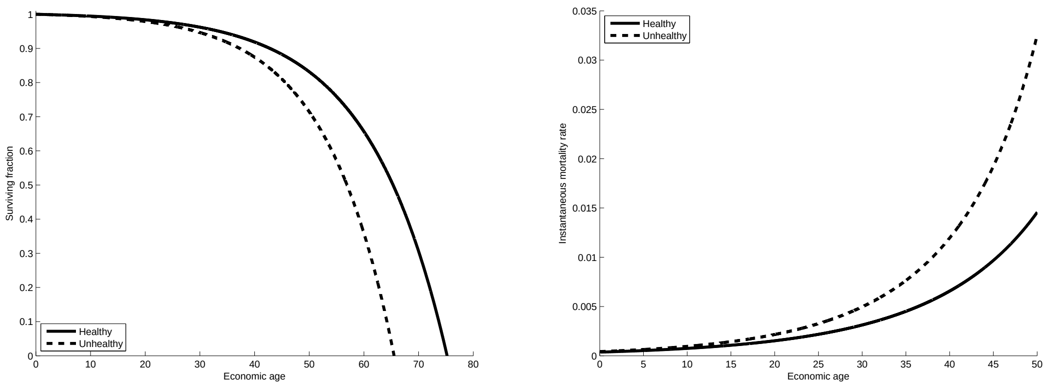

Figure 1: Demographics

Box 2

Mortality process. The mortality process suggested by Boucekkine et al. (2002) has the following properties.

- The unconditional cumulative distribution function:

$$\Phi_j(u) \equiv P\{D \leq u\} = \frac{e^{\eta_{1j}u} - 1}{\eta_0 - 1}, \quad 0 \leq u \leq \bar{D}_j,$$

  such that $\Phi_j(0) = 0$ and $\Phi_j(\bar{D}_j) = 1$ (see below).

- The unconditional probability density function:

$$\phi_j(u) \equiv \Phi_j'(u) = \frac{\eta_{1j}e^{\eta_{1j}u}}{\eta_0 - 1}, \quad 0 \leq u \leq \bar{D}_j.$$

- The unconditional survival function:

$$1 - \Phi_j(u) = \frac{\eta_0 - e^{\eta_{1j}u}}{\eta_0 - 1}, \quad 0 \leq u \leq \bar{D}_j.$$

- The maximum age:

$$\Phi_j(\bar{D}_j) = 1 \quad \Leftrightarrow \quad \bar{D}_j = \frac{\ln \eta_0}{\eta_{1j}}.$$

- The instantaneous mortality rate, force of mortality or hazard rate:

$$\mu_j(u) \equiv \frac{\phi_j(u)}{1 - \Phi_j(u)} = \frac{\eta_{1j}e^{\eta_{1j}u}}{\eta_0 - e^{\eta_{1j}u}}, \quad 0 \leq u \leq \bar{D}_j.$$

That is, $\mu_j(u)$ can be interpreted as the 'probability' that a person of age u in health group j dies instantaneously. Note however that this is not truly a probability, as its value can exceed 1. For example, as $u \to \bar{D}_j$, $\mu_j(u) \to \infty$.

The instantaneous mortality rate function $\mu_j(t)$ uniquely determines the distribution of D. Define the cumulative mortality rate $M_j(u)$ by:

$$M_j(u) \equiv \int_0^u \mu_j(s)\, ds,$$

such that $dM_j(u)/du = \mu_j(u)$. We can then write the unconditional distribution function in terms of $M_j(u)$:

$$\Phi_j(u) = 1 - e^{-M_j(u)}.$$

- The life expectancy at birth:

$$E_j(D) = \int_0^{\bar{D}_j} e^{-M_j(u)}\, du = \frac{1}{\eta_0 - 1}\left[\eta_0 \bar{D}_j + \frac{1 - e^{\eta_{1j}\bar{D}_j}}{\eta_{1j}}\right].$$

- The conditional distribution:

$$\Phi_j(u|D \geq D_0) = P\{D \leq u | D \geq D_0\} = \frac{\Phi_j(u) - \Phi_j(D_0)}{1 - \Phi_j(D_0)},$$
$$\phi_j(u|D \geq D_0) = \frac{d\Phi_j(u|D \geq D_0)}{du} = \frac{\phi_j(u)}{1 - \Phi_j(D_0)}, \quad D_0 \leq u \leq \bar{D}_j.$$

These are the cumulative distribution function and probability density function conditional upon surivival up to age D_0.

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2.2.1 Demography

Let $L(v,t)$ be the size at time $t \in [0,\infty)$ of the cohort born at time $v \leq t$. This cohort consist of both healthy and unhealthy individuals. We write:

$$L(v,t) = L_H(v,t) + L_U(v,t). \tag{9}$$

The total population alive at a given time t can be found by aggregating over all cohorts:

$$L_j(t) = \int_{t-\bar{D}_j}^{t} L_j(v,t)\, dv, \tag{10}$$
$$L(t) = L_H(t) + L_U(t). \tag{11}$$

The proportion of healthy and unhealthy agents in the population is assumed to stay constant over time and equal to π_H and π_U, respectively, with $\pi_H + \pi_U = 1$. If we would not impose this then the unhealthy class might 'die out', in which case we get the theoretically trivial (but perhaps practically appealing) result that the future population consists of healthy people only. For the estimated mortality profile above we find $\hat{\pi}_H = 1 - \hat{\pi}_U = 0.5077$, i.e. the population shares of healthy and unhealthy agents are approximately equal.

We postulate that the number of agents that are born into a given health group at each point in time is proportional to the size of that health group at birth:

$$L_j(v,v) = \beta_j L_j(v) = \beta_j \pi_j L(v), \tag{12}$$

where β_j is the crude birth rate of group j.[3]. The overall birth rate is $\bar{\beta} \equiv L(v,v)/L(v) = \beta_H \pi_H + \beta_U \pi_U$.

Finally, we assume the population is large such that there is no aggregate uncertainty and probabilities and frequencies coincide. For example, the probability of survival up to a given age u times the initial number of people in the cohort gives the size of the cohort at age u.[4] The cohort evolution can thus be written as:

$$L_j(v,t) = L_j(v,v) e^{-M_j(t-v)}, \tag{13}$$

where the cumulative mortality rate equals $M_j(t-v) \equiv \int_0^{t-v} \mu_j(u)\, du$ (see Box 2).

In the demographic steady state the population grows at a constant rate $n \equiv \dot{L}(t)/L(t)$. We can determine the population at a given time, say t, if we know the number of people alive at an earlier point in time, say $v \leq t$:

$$L(t) = L(v) e^{n(t-v)}. \tag{14}$$

For future reference, and in order to gain more insight in the demography just described, we now define the following population fractions:

- The population fraction of healthy people by cohort:

$$\frac{L_H(v,t)}{L(v,t)} = \frac{\beta_H \pi_H e^{-M_H(t-v)}}{\beta_U \pi_U e^{-M_U(t-v)} + \beta_H \pi_H e^{-M_H(t-v)}}. \tag{15}$$

  As $\mu_U(u) > \mu_H(u)$ for all u, we find that $L_H(v,t)/L(v,t) \to 1$ as $u \to \bar{D}_U$. That is, as unhealthy people have a higher probability of dying at each point in time, in the long run a given cohort will tend to consist of healthy agents only.

[3]The advantage of a constant birth rate is that it simplifies computations, yet we are aware that it is not entirely realistic to assume that eighty-year-old women produce offspring at the same rate as their eighteen-year-old granddaughters.

[4]Note that the large population assumption may be unjustified for the last surviving members of a cohort. However, this will not substantially affect the results, as this group is very small relative to the total population.

- The steady state population fraction of each health group:

$$\frac{L_j(t)}{L(t)} = \frac{\beta_j \pi_j \int_{t-\bar{D}_j}^{t} L(t) e^{-n(t-v)-M_j(t-v)}\, dv}{L(t)},$$
$$= \beta_j \pi_j \int_0^{\bar{D}_j} e^{-nu-M_j(u)}\, du. \tag{16}$$

  Since by definition $L_j(t)/L(t) = \pi_j$, we find that for $j \in \{H, U\}$:

$$\beta_j \int_0^{\bar{D}_j} e^{-nu-M_j(u)}\, du = 1. \tag{17}$$

  This implicitly defines the birth rate β_j which, for a given mortality profile and population growth rate, keeps the fraction of unhealthy and healthy people in the population constant in the demographic steady state. As $\mu_U(u) > \mu_H(u)$ for all $u \in [0, \bar{D}_U]$, we have $\beta_U > \beta_H$. That is, the birth rate must be higher among unhealthy individuals in order to keep the population as a whole balanced, since they tend to die younger. For the estimated mortality profile above we find $\hat{\beta}_H = 0.0221$ and $\hat{\beta}_U = 0.0244$.

  If we define $\bar{\mu}_j$ to be the average mortality rate of health group j, then it can be shown that $n = \beta_j - \bar{\mu}_j$ for each j (see Box 3).

- The relative cohort sizes:

$$l_j(v,t) \equiv \frac{L_j(v,t)}{L(t)} = \beta_j \pi_j e^{-n(t-v)-M_j(t-v)}. \tag{18}$$

Box 3

Population growth rate. Let $\bar{\mu}_j$ be the average mortality rate of health group $j \in \{H, U\}$. That is:

$$\bar{\mu}_j \equiv \frac{1}{L_j(t)} \int_{t-\bar{D}_j}^{t} \mu_j(t-v) L_j(v,t)\, dv,$$
$$= \frac{L(t)}{L_j(t)} \int_{t-\bar{D}_j}^{t} \mu_j(t-v) l_j(v,t)\, dv,$$
$$= \frac{\beta_j \pi_j}{\pi_j} \int_{t-\bar{D}_j}^{t} \mu_j(t-v) e^{-n(t-v)-M_j(t-v)}\, dv,$$
$$= \beta_j \int_0^{\bar{D}_j} \mu_j(u) e^{-nu-M_j(u)}\, du,$$

where $u \equiv t - v$ is the agent's age. Using this expression and the implicit definition of the birth rate given in (17) we derive:

$$\begin{aligned}
-\frac{d}{du}\left[e^{-nu-M_j(u)}\right] &= ne^{-nu-M_j(u)} + \mu_j(u)e^{-nu-M_j(u)},\\
-\left. e^{-nu-M_j(u)}\right|_0^{\bar{D}_j} &= n\int_0^{\bar{D}_j} e^{-nu-M_j(u)}\,du + \int_0^{\bar{D}_j} \mu_j(u)e^{-nu-M_j(u)}\,du,\\
1 &= \frac{n}{\beta_j} + \frac{\bar{\mu}_j}{\beta_j}.
\end{aligned}$$

As a result, $n = \beta_j - \bar{\mu}_j$, as stated in the text.

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2.2.2 Individual behaviour

We now turn to the utility maximization problem of individuals, who are assumed to be identical in every respect *except for age and health status*. In our discussion we refrain from schooling, such that agents do not differ in their human capital endowment and the wage rate is the same across ages and health groups.[5] Moreover, we do not take the labour supply decision nor retirement into account but instead assume that each individual inelastically supplies one unit of labour during her (economic) life.

At time $t \in [0, \infty)$, the remaining lifetime utility of an individual of health type j and vintage $v \leq t$ is given by:

$$\Lambda_j(v,t) \equiv \int_t^{v+D} u(\bar{c}_j(v,\tau))e^{-\rho(\tau-t)}\,d\tau, \tag{19}$$

where $\bar{c}_j(v,\tau)$ is individual consumption and $\rho > 0$ is the pure rate of time preference. The felicity function $u : \mathbb{R}_{++} \to \mathbb{R}$ is chosen to be logarithmic for analytical convenience, such that $u(\bar{c}_j(v,\tau)) = \ln(\bar{c}_j(v,\tau))$. As such, it is twice continuously differentiable and features positive but diminishing marginal utility of consumption, i.e. $u'(\bar{c}_j(v,\tau)) > 0$ and $u''(\bar{c}_j(v,\tau)) < 0$ for all $\bar{c}_j(v,\tau) > 0$. Note that in this specification the intertemporal substitution elasticity and the rate of relative risk aversion are both constant at unity.

Even though eventual death is a fact of life, the exact time at which it takes places remains unpredictable. In terms of our model, the age at death D is not known with certainty. However, each individual knows her own health status $j \in \{H, U\}$ and the associated probability distribution

[5] Note that the same result would be obtained if we assume that differences in health type among agents do not become prevalent before age 0. That is, we set $M = 0$ in Sheshinski (2008). If we in addition postulate that agents complete their schooling period before economic birth, then the schooling decision is not affected by health and human capital is equal across the entire population.

of D on the interval $[0, \bar{D}_j]$. By the expected utility hypothesis, the best she can do is to maximize her *expected* remaining lifetime utility, which is given by:

$$
\begin{aligned}
E(\Lambda_j(v,t)) &= \int_{t-v}^{\bar{D}_j} \phi_j(u|D \geq t-v) \left[\int_t^{v+D} \ln \bar{c}_j(v,\tau) e^{-\rho(\tau-t)} \, d\tau \right] du, \\
&= \int_{t-v}^{\bar{D}_j} \left[\int_t^{v+D} \frac{\phi_j(u)}{1-\Phi_j(t-v)} \ln \bar{c}_j(v,\tau) e^{-\rho(\tau-t)} \, d\tau \right] du, \\
&= \frac{1}{1-\Phi_j(t-v)} \int_t^{v+\bar{D}_j} \left[\int_{\tau-v}^{\bar{D}_j} \phi_j(u) \, du \right] \ln \bar{c}_j(v,\tau) e^{-\rho(\tau-t)} \, d\tau, \\
&= \frac{1}{1-\Phi_j(t-v)} \int_t^{v+\bar{D}_j} [1-\Phi_j(\tau-v)] \ln \bar{c}_j(v,\tau) e^{-\rho(\tau-t)} \, d\tau, \\
&= e^{M_j(t-v)} \int_t^{v+\bar{D}_j} \ln \bar{c}_j(v,\tau) e^{-\rho(\tau-t)-M_j(\tau-v)} \, d\tau.
\end{aligned} \tag{20}
$$

Note that we use the conditional distribution of D, given the fact that the consumer has already survived up to age $t-v$. An interesting fact of life is that the expected total lifetime increases with every year lived. After living for only one year, the total expected life time already exceeds the life expectancy at birth.

At every moment t in time, the agent earns wage $w(t)$ which can be spent on consumption $\bar{c}_j(v,t)$ or saved. If more is consumed than is earned in a given period, the agent will have to decrease her stock of savings or borrow money. There are two possible markets for these monetary transactions. We assume that all outstanding loans and savings accounts are recontracted at every moment in time such that the rate of return paid or earned is fully flexible.

Capital market. In the capital market the rate of return equals r. If the agent is a net saver then she might die before having the opportunity to deplete her accumulated stock of savings. In that case there is an (unintended) bequest. With finite lives it is not possible to borrow money in the capital market, as the agent is not allowed to die indebted.

Annuity market. In its original definition an annuity is an asset which pays an annual income. The current use of the term in economics is much broader and captures a wide range of investment products. When we refer to an annuity here, we in fact mean a life annuity: an asset which pays a stipulated return contingent upon survival of the annuitant. Moreover, the focus is on private annuities (which people buy for themselves) and not on social annuities (such as social security benefits).

In order to compete with other investment products, annuities have to provide a rate of return $r + p_j(u)$ which exceeds the market rate of interest in order to compensate for the risk of death (upon which the payments end). The annuity premium $p_j(u)$ might depend on both health status and age. The annuity firm is willing to pay this additional return on savings under the condition that in case of death of the annuitant it is held free of any obligation. Conversely, an agent who sells an actuarial note gets a life-insured loan[6] and will have her debts acquitted when she dies prematurely. Hence, an agent who exclusively uses the annuity market for financial transactions will never die indebted or leave an (unintended) bequest.

We assume that the agent does not wish to leave any bequest. This can happen for several reasons. For example, parents might derive insufficient utility from the welfare of their children (which might be unrealistic), or believe that their children will be born into a richer world and will thus be able to make do for themselves (which might sound more plausible). Combined with the fact that the return on annuities exceeds the rental rate of capital, it follows that the agent will completely annuitize. That is, she will invest all her wealth in annuities. This is the famous result found by Yaari (1965).

We can now derive the budget identity of a type j individual:

$$\dot{\bar{a}}_j(v,t) = [r + p_j(t-v)]\bar{a}_j(v,t) + w(t) - \bar{c}_j(v,t), \tag{21}$$

where $\bar{a}_j(v,t)$ is the agent's stock of financial assets (or demand for annuities), and $\dot{\bar{a}}(v,t) \equiv \partial\bar{a}(v,t)/\partial t$.

Solving the budget identity gives the consolidated lifetime budget constraint at birth:

$$\int_v^{v+\bar{D}_j} \bar{c}_j(v,\tau)e^{-r(\tau-v)-P_j(\tau-v)}\, d\tau = \bar{h}_j(v,v), \tag{22}$$

where $P_j(\tau-v) = \int_0^{\tau-v} p_j(u)\, du$ is the cumulative annuity premium and $h_j(v,v)$ is human wealth at birth:

$$\bar{h}_j(v,v) \equiv \int_v^{v+\bar{D}_j} w(\tau)e^{-r(\tau-v)-P_j(\tau-v)}\, d\tau. \tag{23}$$

For the derivations, see Box 4.

Box 4

The budget identity and the budget constraint. The budget identity, as it name suggests, holds by definition. It is given in (21) but restated here for convenience:

$$\dot{\bar{a}}(v,t) = [r + p_j(t-v)]\bar{a}_j(v,t) + w(t) - \bar{c}_j(v,t).$$

[6]Taking out a life-insured loan implies borrowing money and simultaneously buying a life insurance policy to cover the loan in case of death.

The consolidated lifetime budget constraint at birth can be found by solving the budget identity. In order to economize on notation, we define the cumulative annuity premium as:

$$P_j(t-v) = \int_0^{t-v} p_j(u)du,$$

and note that $dP_j(t-v)/dt = p_j(t-v)$. Since agents are assumed to be born without assets (as their parents do not leave them any bequests), we have the initial condition $\bar{a}_j(v,v) = 0$. We also need a terminal condition on assets, in order to avoid so-called 'pyramid schemes' or 'Ponzi games' in which a consumer can beat the system by borrowing unrestrictedly while paying interest obligations by taking out even more life-insured loans. Hence, we impose the terminal condition $\bar{a}_j(v, v+\bar{D}_j)e^{-r(\tau-v)-P_j(\tau-v)} = 0$, which is equivalent to $\bar{a}_j(v, v+\bar{D}_j) \geq 0$ in case of nonsaturation. We find:

$$[\dot{\bar{a}}_j(v,\tau) - [r + p_j(\tau-v)]\bar{a}_j(v,\tau)]e^{-r(\tau-v)-P_j(\tau-v)} = [w(\tau) - \bar{c}_j(v,\tau)]e^{-r(\tau-v)-P_j(\tau-v)},$$

$$\int_v^{v+\bar{D}_j} \frac{d}{d\tau}\left[\bar{a}_j(v,\tau)e^{-r(\tau-v)-P_j(\tau-v)}\right] d\tau = \int_v^{v+\bar{D}_j} [w(\tau) - \bar{c}_j(v,\tau)]e^{-r(\tau-v)-P_j(v,\tau)} d\tau,$$

$$\lim_{(\tau-v)\to\bar{D}_j} \bar{a}_j(v,\tau)e^{-r(\tau-v)-P_j(\tau-v)} - \bar{a}_j(v,v) = \int_v^{v+\bar{D}_j} [w(\tau) - \bar{c}_j(v,\tau)]e^{-r(\tau-v)-P_j(v,\tau)} d\tau,$$

$$\int_v^{v+\bar{D}_j} \bar{c}_j(v,\tau)e^{-r(\tau-v)-P_j(\tau-v)} d\tau = \bar{h}_j(v,v),$$

where $\bar{h}_j(v,v)$ is a measure of human wealth at birth:

$$\bar{h}_j(v,v) \equiv \int_v^{v+\bar{D}_j} w(\tau)e^{-r(\tau-v)-P_j(\tau-v)} d\tau.$$

It can be interpreted as the discounted value of all future earnings. Note that discounting occurs for two reasons: both the time value of money and lifetime uncertainty are taken into account.

****

We can now consider the individual's utility maximization problem. Since preferences are of the time-consistent form, it is sufficient to determine the optimality conditions from the perspective of birth (i.e. at time v). This simplifies the computations involved. The goal is to maximize expected lifetime utility (20) subject to the budget constraint (22). The Lagrangian is defined by:[7]

$$\mathcal{L}(\bar{c}_j(v,\tau),\lambda) = e^{M_j(t-v)} \int_v^{v+\bar{D}_j} \ln \bar{c}_j(v,\tau) e^{-\rho(\tau-v)-M_j(\tau-v)} \\ - \lambda \left[\int_v^{v+\bar{D}_j} \bar{c}_j(v,\tau) e^{-r(\tau-v)-P_j(\tau-v)}\, d\tau - \bar{h}_j(v,v) \right], \tag{24}$$

where λ is the marginal utility of wealth. Assuming an interior solution, the first-order necessary conditions are the budget constraint and:

$$\frac{\partial \mathcal{L}(\cdot)}{\partial \bar{c}_j(v,\tau)} = \frac{1}{\bar{c}_j(v,\tau)} e^{-\rho(\tau-v)-M_j(\tau-v)} - \lambda e^{-r(\tau-v)-P_j(\tau-v)} = 0, \quad \forall\, \tau \in [v, \bar{D}_j]. \tag{25}$$

In order to eliminate λ we can take the time derivative of (25) and divide the result by the original equation. This yields the consumption Euler equation:

$$\frac{\dot{\bar{c}}_j(v,\tau)}{\bar{c}_j(v,\tau)} = r + p_j(\tau - v) - \rho - \mu_j(\tau - v). \tag{26}$$

Box 5

Consumption. Using the Euler equation (26) and the budget constraint (22), we can derive the following relations between consumption at birth, consumption at a given moment during life, and human wealth.

- Consumption at time $t \geq v$ in terms of consumption at birth:

$$\bar{c}_j(v,t) = \bar{c}_j(v,v) e^{(r-\rho)(t-v)-M_j(t-v)+P_j(t-v)}.$$

- The level of consumption at birth (rather than the relative change) in terms of human wealth:

$$\begin{aligned} h_j(v,v) &= \int_v^{v+\bar{D}_j} \bar{c}_j(v,\tau) e^{-r(\tau-v)-P_j(\tau-v)}\, d\tau, \\ &= \bar{c}_j(v,v) \int_v^{v+\bar{D}_j} e^{-\rho(\tau-v)-M_j(\tau-v)}\, d\tau, \\ \bar{c}_j(v,v) &= \left[\int_v^{v+\bar{D}_j} e^{-\rho(\tau-v)-M_j(\tau-v)}\, d\tau \right]^{-1} h_j(v,v). \end{aligned}$$

That is, the fraction of financial and human wealth consumed at birth depends on the degree of felicity discounting of the consumer. Felicity discounting increases with the rate of time preference or impatience (ρ) and the instantaneous probability of dying ($\mu_j(u)$). As $\mu_U(u) > \mu_H(u)$ and $\bar{D}_U < \bar{D}_H$, it follows that unhealthy

[7] The problem can also be solved by using the Hamiltonian and the budget identity (21). This yields the same result.

individuals consume more of their human wealth at birth than do healthy agents, and as a consequence they save less in the early years of economic life.

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2.2.3 Per capita behaviour

We can aggregate the individual behaviour to per capita behaviour. In general, we define the per capita value at time $t \in [0, \infty)$ of a variable $\bar{x}_j(v,t)$ as:

$$x_j(t) \equiv \int_{t-\bar{D}_j}^{t} l_j(v,t)\bar{x}_j(v,t)\ dv, \tag{27}$$

$$x(t) \equiv \sum_{j\in\{H,U\}} x_j(t). \tag{28}$$

In addition, we define cohort aggregate per capita assets as:

$$a_j(v,t) \equiv l_j(v,t)\bar{a}_j(v,t), \tag{29}$$

$$a(v,t) \equiv \sum_{j\in\{H,U\}} a_j(v,t). \tag{30}$$

2.3 Annuity market

We make the following assumptions regarding the market for annuities:

(A1) The annuity market is perfectly competitive. There is a large number of risk neutral firms offering annuities to individuals, and firms can freely enter or exit the market.

(A2) Annuity firms do not use up any real resources.

(A3) Health status is private information of the annuitant. The distribution of health types in the population and the corresponding mortality rates are common knowledge.

(A4) Age, or equivalently date of birth, is public information.

(A5) Annuitants can buy multiple annuities for different amounts and from different annuity firms. Individual annuity firms cannot monitor an annuitant's holdings with other firms.

Under these assumptions, the equilibrium in the annuity market is a pooling equilibrium. The existence of this pooling equilibrium depends critically on (A3): as annuity firms cannot distinguish between healthy and unhealthy agents they will offer a single rate of return which applies to both groups. They can exploit their knowledge about the mortality distribution in the population in determining the annuity rate. As a consequence of (A4), there is market segmentation. The annuity market consists of separate submarkets for each age group or cohort. By (A1), in each submarket the expected profit equals zero. Note that if (A5) would not hold then annuity firms could indirectly deduce an agent's health group by the amount of wealth she has invested. Healthy individuals tend to be wealthier, which exacerbates the degree of adverse selection in the annuity market (see below).

An implicit assumption in the model is that annuitants cannot credibly signal their health status to the market. In the absence of cheap and credible medical tests, this is not a strong assumption at all. When asked about her health type, each individual has a clear incentive to claim to be unhealthy in order to get the highest possible return in a separated market. However, firms know this and will therefore not believe the annuitant's claim. Hence, even though part of their clients tell the truth, the fact that some have an incentive to lie is enough for the annuity firm to assume that everyone will be a fraud.

We now turn to the determination of the competitive annuity rate in the market. An annuity firm sells annuities to agents of age u which offer a rate of return $r + p_j(u)$ that exceeds the rental rate on capital. The return the firm makes on this investment depends on the health group j to which the client belongs, and amounts to $r + \mu_j(u)$. The excess return above the market rate is the 'mortality bonus': some clients will die young and will subsequently lose their claim at an early stage, and their assets can be redistributed among the surviving clients.

If an individual's health type could be observed by annuity firms then $p_j(u) = \mu_j(u)$ would be the actuarially fair annuity premium in a separating equilibrium. In a pooling equilibrium, however, both health types receive the same premium which we denote by $\bar{p}(u)$. Under the assumption that both health types are net savers the zero profit condition in the annuity market for age $u \in [0, \bar{D}_U]$ is given by:[8]

$$L_H(v, v+u)[\bar{p}(u) - \mu_H(u)]\bar{a}_H(v, v+u) + L_U(v, v+u)[\bar{p}(u) - \mu_U(u)]\bar{a}_U(v, v+u) = 0, \quad (31)$$

or equivalently:

$$[\bar{p}(u) - \mu_H(u)]a_H(v, v+u) + [\bar{p}(u) - \mu_U(u)]a_U(v, v+u) = 0. \quad (32)$$

Solving for the pooling premium on annuities yields:

$$\bar{p}(u) = \mu_H(u)\frac{a_H(v, v+u)}{a(v, v+u)} + \mu_U(u)\frac{a_U(v, v+u)}{a(v, v+u)}. \quad (33)$$

Hence, the excess return on annuities can be interpreted as an annuity-demand-weighted average of the mortality rates of the two health groups. As such, the age-dependency of the annuity rate

[8] Note that the pooling equilibrium does not exist for $u \in (\bar{D}_U, \bar{D}_H]$. Only healthy agents are able to survive past age $\bar{D}_U$ such that there is no longer scope for risk pooling.

is a consequence both the age-dependency of mortality and the annuity demand composition of the cohort of vintage v at age u. This result has also been found in a partial equilibrium context by Sheshinski (2008) and relates to the linear equilibrium concept of Pauly (1974). As noted by Walliser (2000), it can alternatively be interpreted as a Nash equilibrium among annuity firms in which each firm that deviates from the zero profit price incurs a loss.

From this result we can immediately see the effects of the adverse selection mechanism. If there would be no adverse selection then the demand for annuities at a given age would be independent of health status. In that case, the actuarially fair pooling premium is a weighted average of the relative health group sizes in a given cohort. That is, (33) changes to:

$$\bar{p}^{AF}(u) = \mu_H(u)\frac{L_H(v,v+u)}{L(v,v+u)} + \mu_U(u)\frac{L_U(v,v+u)}{L(v,v+u)}, \tag{34}$$

where 'AF' stands for 'actuarially fair'. However, this rate is not sustainable. As $\mu_H(u) \leq \bar{p}^{AF}(u) \leq \mu_U(u)$, healthy individuals will tend to overinvest in annuities while unhealthy individuals invest less than in a seperated market. This cannot be an equilibrium as annuity firms will make a loss. They will adjust their expectations about the demand for annuities by members of both health groups. As a consequence of this information feedback, the annuity rate in the competitive equilibrium will reflect the adverse selection effect such that $\bar{p}(u) \leq \bar{p}^{AF}(u)$ for all ages $u \in [0, \bar{D}_U]$.

3 Steady state analysis

We now bring all elements of the model together and consider the equilibrium that results when the production sector, households, and annuity firms interact. We focus on the steady state as the transitional dynamics of the model are too complicated to handle. In the steady state the economy proceeds along a balanced growth path. Such a path is characterized by a constant (exponential) growth rate of the per capita capital stock. We denote this constant rate by γ. Thus:

$$\gamma \equiv \frac{\dot{k}(t)}{k(t)}. \tag{35}$$

As we have a closed economy and no government debt, equilibrium in the financial markets implies that $a(t) = k(t)$. That is, per capita asset holdings by households are equal to the per capita capital stock used in the production process. We now find:

$$\gamma = r - n + \frac{w(t)}{k(t)} - \frac{c(t)}{w(t)}\frac{w(t)}{k(t)}. \tag{36}$$

For the derivation, see Box 6.

Box 6

Derivation of the balanced growth rate. Recall that by definition of per capita assets, $a(t) = a_H(t) + a_U(t)$. Taking the time derivative of this equation yields $\dot{a}(t) = \dot{a}_H(t) + \dot{a}_U(t)$. We find:

$$\begin{aligned}
\dot{a}_j(t) &= \int_{t-\bar{D}_j}^{t} l_j(v,t)\dot{\bar{a}}_j(v,t)\ dv + \int_{t-\bar{D}_j}^{t} \dot{l}_j(v,t)\bar{a}_j(v,t)\ dv, \\
&= \int_{t-\bar{D}_j}^{t} l_j(v,t)\left[[r + p_j(t-v)]\bar{a}(v,t) + w(t) - \bar{c}_j(v,t)]\right]\ dv, \\
&\quad - \int_{t-\bar{D}_j}^{t} l_j(v,t)[n + \mu_j(t-v)]\ dv, \\
&= \int_{t-\bar{D}_j}^{t} l_j(v,t)[p_j(t-v) - \mu_j(t-v)]\bar{a}_j(v,t)\ dv, \\
&\quad + \int_{t-\bar{D}_j}^{t} l_j(v,t)\left[(r-n)\bar{a}_j(v,t) + w(t) - \bar{c}_j(v,t)\right]\ dv, \\
&= \int_{t-\bar{D}_j}^{t} l_j(v,t)[p_j(t-v) - \mu_j(t-v)]\bar{a}_j(v,t)\ dv + (r-n)a_j(t) + \pi_j w(t) - c_j(t),
\end{aligned}$$

since:

$$\dot{l}_j(v,t) = \frac{\dot{L}_j(v,t)}{L_j(v,t)} l_j(v,t) - \frac{\dot{L}(t)}{L(t)} l_j(v,t) = -[\mu_j(t-v) + n] l_j(v,t).$$

Combining yields:

$$\dot{a}(t) = (r-n)a(t) + w(t) - c(t) + F(t),$$

where $F : \mathbb{R} \to \mathbb{R}$ is defined by:

$$F(t) = \sum_{j\in\{H,U\}} \int_{t-\bar{D}_j}^{t} l_j(v,t)[p_j(t-v) - \mu_j(t-v)]\bar{a}_j(v,t)\ dv.$$

This function represents the redistribution between health groups. It immediately follows that $F(t) = 0$ for all t in case of a separating equilibrium (when $p_j(t-v) = \mu_j(t-v)$), and in a pooling equilibrium (when $p_j(t-v) = \bar{p}(t-v)$ as given in (33)). In the pooling equilibrium there is redistribution on a microeconomic level from unhealthy to healthy and from the dead to those who are still alive. However, it cancels out in the aggregate as annuity firms break even.
As $a(t) = k(t)$ we find:

$$\gamma = r - n + \frac{w(t)}{k(t)} - \frac{c(t)}{k(t)}.$$

Multiplying the last fraction on the right-hand side by $w(t)/w(t)$ gives (36).

****

As $w(t) = (1-\varepsilon)\Omega_0 k(t)$ (see Box 1 and equation (5)), we have:

$$\frac{w(t)}{k(t)} = (1-\varepsilon)\Omega. \tag{37}$$

It follows that the steady state growth in the wage rate equals γ. Thus, for a given $t \geq v$ we can write:

$$w(t) = w(v)e^{\gamma(t-v)}. \tag{38}$$

Using this relation, we can show that the following steady state expressions do not depend on vintage or moment in time, but only on age. That is, they are stationary over time.

- Steady state value of scaled individual consumption at birth:

$$\frac{\bar{c}_j(v,v)}{w(v)} = \left[\int_0^{\bar{D}_j} e^{-\rho u - M_j(u)}\, du\right]^{-1} \left[\int_0^{\bar{D}_j} e^{-(r-\gamma)u - P_j(u)}\, du\right]. \tag{39}$$

  Observe that, due to growth in the economy, agents who are born later are born richer. Scaled consumption at birth remains constant over time, but its unscaled value grows with exponential rate γ.

- Steady state path for scaled aggregate consumption:

$$\frac{c(t)}{w(t)} = \sum_{j\in\{H,U\}} \beta_j \pi_j \frac{\bar{c}_j(v,v)}{w(v)} \int_0^{\bar{D}_j} e^{(r-n-\gamma-\rho)u - 2M_j(u) + P_j(u)}\, du. \tag{40}$$

- Steady state path for scaled individual assets:

$$\frac{\bar{a}_j(v,v+u)}{w(v)} e^{-ru-P_j(u)} = \int_0^u e^{-(r-\gamma)s - P_j(s)}\, ds - \frac{\bar{c}_j(v,v)}{w(v)} \int_0^u e^{-\rho s - M_j(s)}\, ds. \tag{41}$$

- Steady state path for scaled cohort per capita assets:

$$\begin{aligned}\frac{a_j(v,v+u)}{w(v)} e^{-(r-n)u + M_j(u) - P_j(u)} = \beta_j \pi_j \Bigg[&\int_0^u e^{-(r-\gamma)s - P_j(s)}\, ds \\ &- \frac{\bar{c}_j(v,v)}{w(v)} \int_0^u e^{-\rho s - M_j(s)}\, ds\Bigg].\end{aligned} \tag{42}$$

The growth model now consists of equations (36), (37), (39) (for $j = H$ and $j = U$), (40), and (42). Furthermore, it requires an expression for the annuity rate of return. The general model has been summarized in Table 1.

3.1 Simulation results

In case of a pooling equilibrium the rate of return on annuities is defined in terms of cohort assets, while the level of assets in turn depends on this rate. Hence, the pooling equilibrium can only

Table 1: General model

(a) Microeconomic relationships:

$$\frac{\bar{c}_j(v,v)}{w(v)} = \frac{\int_0^{\bar{D}_j} e^{-(r-\gamma)u-P_j(u)}\,du}{\int_0^{\bar{D}_j} e^{-\rho u-M_j(u)}\,du}, \quad j \in \{H,U\} \tag{T1.1}$$

$$\begin{aligned}\frac{a_j(v,v+u)}{w(v)} &= \beta_j \pi_j e^{(r-n)u-M_j(u)+P_j(u)} \left[\int_0^u e^{-(r-\gamma)s-P_j(s)}\,ds \right. \\ &\quad \left. - \frac{\bar{c}_j(v,v)}{w(v)} \int_0^u e^{-\rho s-M_j(s)}\,ds \right], \quad j \in \{H,U\}\end{aligned} \tag{T1.2}$$

(b) Macroeconomic relationships:

$$\frac{c(t)}{w(t)} = \sum_{j\in\{H,U\}} \beta_j \pi_j \frac{\bar{c}_j(v,v)}{w(v)} \int_0^{\bar{D}_j} e^{(r-n-\gamma-\rho)u-2M_j(u)+P_j(u)}\,du \tag{T1.3}$$

$$\gamma = r - n + \left[1 - \frac{c(t)}{w(t)}\right] \frac{w(t)}{k(t)} \tag{T1.4}$$

$$\frac{w(t)}{k(t)} = (1-\varepsilon)\Omega \tag{T1.5}$$

(c) Annuity market:

$$p_j(u) = \begin{cases} \mu_j(u) & \text{Separating equilibrium} \\ \bar{p}_j(u) \equiv \mu_H(u)\dfrac{a_H(v,v+u)}{a(v,v+u)} + \mu_U(u)\dfrac{a_U(v,v+u)}{a(v,v+u)} & \text{Pooling equilibrium} \end{cases} \tag{T1.6}$$

Notes. The endogenous variables are $\bar{c}_j(v,v)/w(v)$, $a_j(v,v+u)/w(v)$, $p_j(u)$, γ, $w(t)/k(t)$, and $c(t)/w(t)$.

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be found by applying an iterative procedure. Such a procedure requires initial values close to the final solution in order to converge. We therefore first calibrate the separating equilibrium, under the assumption that health status can be observed by annuity firms. It then follows that each health group receives its own actuarially fair annuity premium, equal to the instantaneous mortality rate. We calculate the corresponding asset paths and the implied pooling premium. (Note that this pooling premium does not constitute an equilibrium in the pooled annuity market as it is based on asset holdings which are derived under conflicting assumptions.)

For the numerical simulation we use the mortality distribution estimated in Section 2.2 and set $r = 0.06$, $\rho = 0.035$ and $n = 0.01$. The resulting asset paths, consumption profiles, and the 'implied' relative pooling rate are given in Figure 2(a)-(d). We find that scaled individual consumption grows over time, and is nearly the same for healthy and unhealthy agents. Empirically, the consumption profile tends to be hump-shaped such that consumption decreases again in the last years of life. Hence, in this aspect the model is not consistent with real-life data. Cohort assets path are nonnegative everywhere, but the unhealthy save less than the healthy. Before continuing, we provide a proposition which states that under very general conditions this result always holds true.

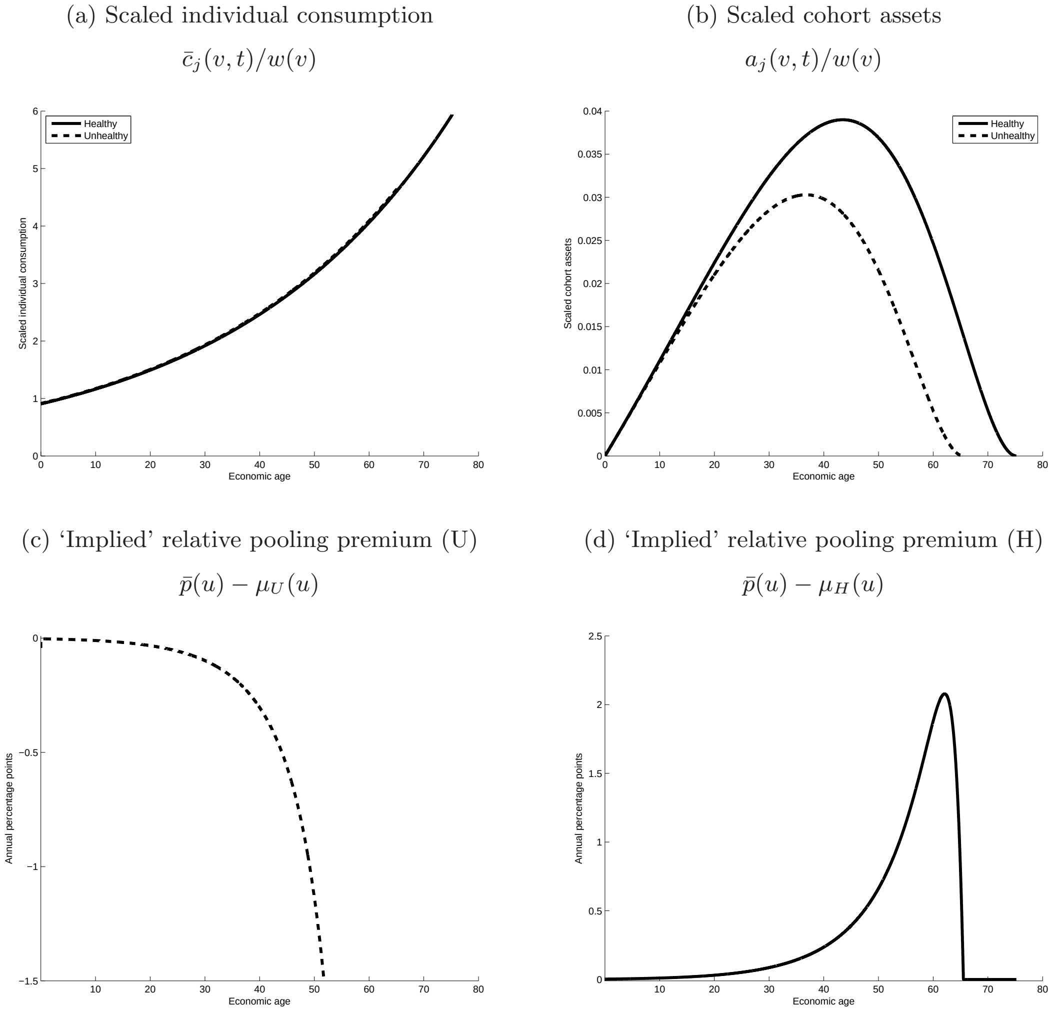
Proposition 1. *Consider the case in which annuity firms can observe the health type of annuitants, such that the annuity market is characterized by a separating equilibrium. Provided the growth-corrected interest rate exceeds the pure rate of time preference, $r - \gamma > \rho$, agents of both health types are net savers throughout life, i.e. $\bar{a}_j(v, v) = \bar{a}_j(v, v + \bar{D}_j) = 0$ and $\bar{a}_j(v, v + u) > 0$ for all $u \in (0, \bar{D}_j)$.*

Proof. See Appendix A. □

Panels (c) and (d) show the difference between the implied relative pooling premium and the mortality rate for each health group. As was to be expected, we find that the pooling premium on annuities is smaller than $\mu_U(u)$ and larger than $\mu_H(u)$ for all $u \in [0, \bar{D}_U]$. As $u \to \bar{D}_U$, $\bar{p}_U(u) - \mu_U(u)$ decreases at an accelerating rate.

Using the implied pooling premium of the separating equilibrium we can calculate the next iteration. It turns out that, given this rate, the unhealthy agents wish to have negative annuity holdings (life-insured loans) in the last part of their lives. Since the pooling premium is (much) lower than their mortality rate, borrowing in the annuity market has become an attractive option. Note however, that the calculation of the annuity premium in (33) requires nonnegative assets, and therefore continuing the calculations hereafter would give insensible results.

Yet, as the healthy continue to save throughout life, an unhealthy agent who wishes to borrow immediately reveals her health status. Consequently, the annuity market will separate from that



(a) Scaled individual consumption

$\bar{c}_j(v,t)/w(v)$

(b) Scaled cohort assets

$a_j(v,t)/w(v)$

(c) 'Implied' relative pooling premium (U)

$\bar{p}(u) - \mu_U(u)$

(d) 'Implied' relative pooling premium (H)

$\bar{p}(u) - \mu_H(u)$

Figure 2: Separating equilibrium

moment on (as assumption (A2) is violated), and the agent will have to pay her own actuarially fair rate. But this borrowing rate is much less attractive than the pooling rate, and by applying Proposition 1 we find that the agent does not wish to borrow at this rate. Rather, she will impose a voluntary borrowing constraint on herself.

We denote the age at which the self-imposed borrowing constraint becomes binding by $\bar{u}$, with $\bar{u} \in [0, \bar{D}_U]$. Note that $\bar{u}$ is an endogenous variable, as the age at which the borrowing constraint becomes effective can be optimally chosen by the unhealthy agent. The following proposition shows that in fact $\bar{u} \in (0, \bar{D}_U)$, such that the agent faces a binding constraint for all $u \in [\bar{u}, \bar{D}_U]$.

Proposition 2. *Consider the case in which annuity firms cannot observe the health type of annuitants. Assume that the growth-corrected interest rate exceeds the pure rate of time preference, $r - \gamma > \rho$. Suppose there exists a pooling equilibrium in the annuity market. Then:*

- *(a) Healthy agents are net savers throughout life, i.e. $\bar{a}_H(v, v) = \bar{a}_H(v, v + \bar{D}_H) = 0$ and $\bar{a}_H(v, v + u) > 0$ for all $u \in (0, \bar{D}_H)$.*
- *(b) Unhealthy agents are net savers until age $\bar{u} \in (0, \bar{D}_U)$ and face a binding self-imposed borrowing constraint after age $\bar{u}$, i.e. $\bar{a}_U(v, v) = 0$, $\bar{a}_U(v, v + u) > 0$ for $u \in (0, \bar{u})$, and $\bar{a}_U(v, v + u) = 0$ for $u \in [\bar{u}, \bar{D}_U]$.*

Proof. See Appendix A. □

The model presented in Table 1 requires two modifications. First of all, for $u \in [\bar{u}, \bar{D}_U]$ we now have $a_U(v, v + u) = 0$. Secondly, as the consumption profile cannot be discontinuous (i.e it cannot feature jumps)[9] we need the following 'smooth connection' condition:

$$e^{\gamma \bar{u}} = \frac{\bar{c}_U(v, v)}{w(v)} e^{(r-\rho)\bar{u} - M_U(\bar{u}) + P_U(\bar{u})}, \tag{43}$$

where the left-hand side gives the value of the new consumption profile at age $\bar{u}$ and the right-hand side states the corresponding value for the old profile. For a 'smooth connection' these values should be equal. Note that from age $\bar{u}$ onwards, the unhealthy agent consumes exactly her wage income in each period:

$$\bar{c}_U(v, v + u) = w(v)e^{\gamma u}, \quad \bar{u} \leq u \leq \bar{D}_U. \tag{44}$$

In the numerical simulation we find $\bar{u} = 54.89$. This implies that an unhealthy agent lives, at most, the last 10.63 years of her life without any financial assets. However, with a life expectancy

[9]This is a consequence of the fact that we have specified a concave utility function. In that case, agents will always prefer to 'smooth' their consumption over time.

at birth of 53.36 years the probability of encountering the borrowing constraint is only 0.5755. The simulation results are given in Figure 3(a)-(d). We see that scaled individual consumption is still upward sloping everywhere for healthy agents. Unhealthy agents, however, have a temporary decrease in consumption just before $\bar{u}$. From age $\bar{u}$ onward they exactly consume their wage income (given by the dotted line) at each moment in time.[10] The scaled cohort asset profiles show that, compared to the separating equilibrium, the healthy have more assets at all ages and the unhealthy less. This has consequences for the equilibrium pooling premium, which is lower than the implied rate from the separating equilibrium. The difference between the pooling premium and the instantaneous mortality rate of a healthy individual peaks at around 0.17 percentage points, while the absolute difference with the actuarially fair rate might be more than 1 percentage point for an unhealthy agent.

It follows that the healthy individuals benefit for part of their lives from the presence of the unhealthy in the annuity market. Due to asymmetric information they receive a better than actuarially fair return on their investments. This shows that imperfections in the annuity market do not exclusively lead to less than actuarially fair rates, as is often assumed in the literature.

3.2 Welfare analysis

We would like to determine the welfare effects of asymmetric information in the annuity market. How well off are agents in a pooling equilibrium compared to a (hypothetical) separating equilibrium? And what is the difference with the case that there are no annuities at all? (For a derivation of the key equations of the no annuities equilibrium, see Box 7 below.)

Arguably the best measure of welfare we have in our model is lifetime utility. We can write the expected lifetime utility at birth of an agent in health group $j \in \{H, U\}$ and given equilibrium type $i \in \{SE, PE, NAE\}$ as:

$$\Lambda_j^i(v, v) = \int_v^{v+\bar{D}_j} \ln \bar{c}_j(v, \tau) e^{-\rho(\tau-v)-M_j(\tau-v)} \, d\tau, \tag{45}$$

$$= \int_0^{\bar{D}_j} \ln\left(\frac{\bar{c}_j(v, v+u)}{w(v)}\right) e^{-\rho u - M_j(u)} \, du + \ln w(v) \int_0^{\bar{D}_j} e^{-\rho u - M_j(u)} \, du. \tag{46}$$

As we have shown above that scaled consumption is stationary over time, the first term on the right-hand side does not depend on the year of birth. The second term, however, does depend on vintage v. The implication of growth in the economy is that the later in time an agent is born, the higher is the wage she receives at the start of her economic life. Therefore we need to choose

[10]Note that from this moment on, the Euler equation given in (26) is no longer relevant for the unhealthy agents. Since their assets are identically zero, they are 'forced' to consume exactly their wage income each period. There is no longer scope for intertemporal substitution of consumption.

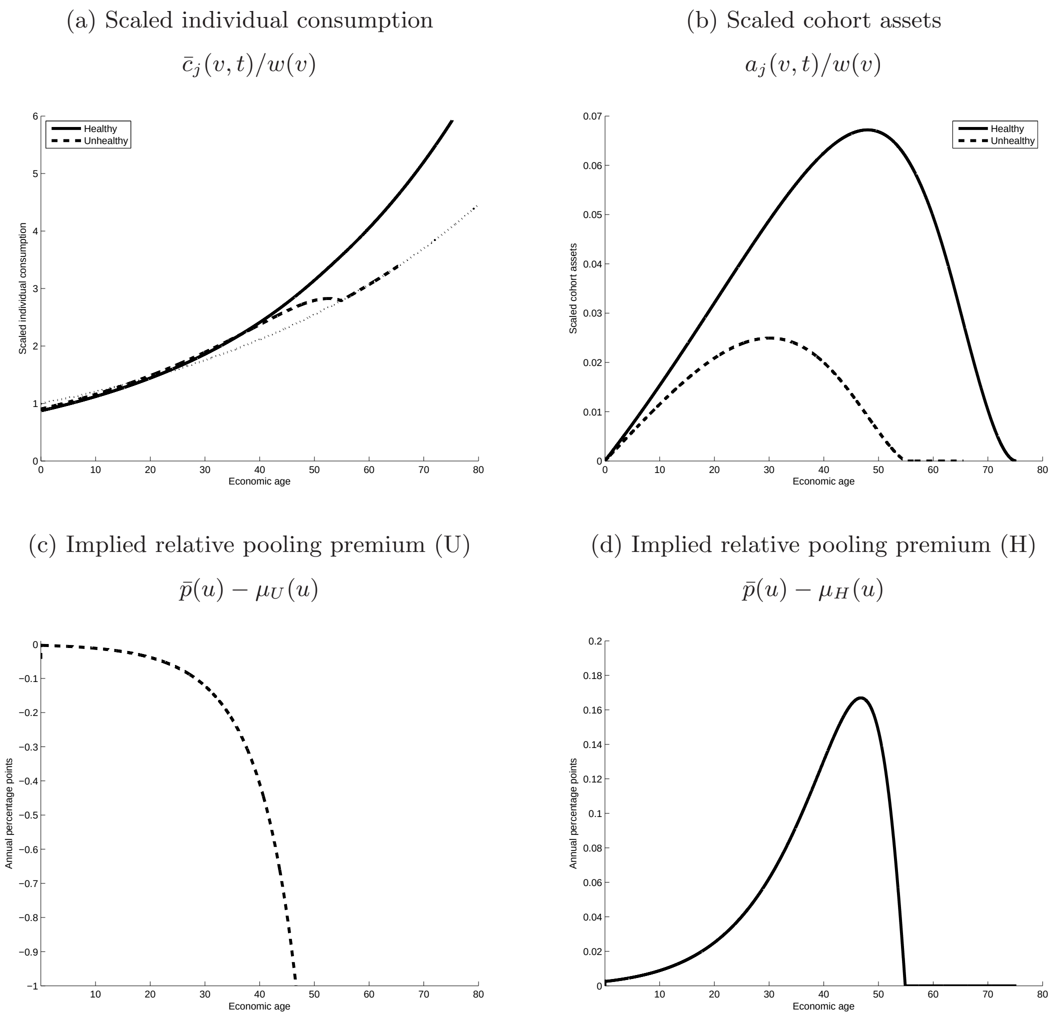


Figure 3: Pooling equilibrium

a benchmark case, and index all other values to it. We take some year v_0 as the benchmark year and set $w(v_0) = 1$ such that $\ln w(v_0) = 0$.

The resulting welfare indicators for the pooling equilibrium, the separating equilibrium, and the case with no annuities are given in Table 2. The indicators are measured in 'utils', units of utility. We cannot attach a direct interpretation to the absolute differences in welfare, but the results do provide us with an ordinal ranking of the equilibria. It follows that a pooling equilibrium is slightly worse than a separating equilibrium, yet any annuity market is significantly better than the complete absence of annuities.

Table 2: Welfare analysis

	Pooling	Seperating	No annuities
Healthy	9.4113	10.1844	7.6191
Unhealthy	8.3599	9.0134	6.7343

In order to give a more intuitive interpretation of these values we develop the following metric. Suppose you are an individual in health class j and we have a pooling equilibrium or no annuities at all. How many years in the future would you have need to have been born in order to achieve the same level of lifetime utility at birth as a base-year v_0 newborn under a separating equilibrium? The required wage level at the year-of-birth v_1 can be calculated from:

$$\ln w(v_1) \int_0^{\bar{D}_j} e^{-\rho u - M_j(u)}\, du = \Lambda_j^{SE}(v_0, v_0) - \Lambda_j^i(v_0, v_0). \tag{47}$$

The corresponding "lost growth years" can then be computed as:

$$LGY_j^i = \frac{\ln w(v_1)}{\gamma^i},$$

where γ^i is the growth rate in equilibrium i. For example, under a pooling equilibrium we find that the unhealthy lose 1.4763 years and the healthy 1.6729 years. Given respective life expectancies of 53.36 and 61.25 years this does not seem too large a loss. For the unhealthy agents the difference between the consumption profile in the pooling and the separating equilibrium is largest in the last stages of life, but these consumption levels are discounted heavily in the utility function. In contrast, the welfare loss is more pronounced for the healthy agents as the better than actuarially fair return they receive on annuities prompts them to save considerably more (and thus consume less) right from the start. In the no annuities scenario the distortion is much worse and amounts to 6.7790 years of lost growth for unhealthy and 7.3093 years for healthy agents.

At first sight it might appear as though the above results imply that the pooling equilibrium does not exist. Both the unhealthy agents and the healthy agents *as a group* are better of by truthfully signaling their health status to the annuity firms. As a separating equilibrium gives them higher utility, this announcement would be credible. However, each healthy agent *as an individual* has an incentive to deviate from the optimal group strategy. Once the separating equilibrium is realized, posing as an unhealthy agent and receiving the higher annuity rate is optimal given that the other agents are honest in their health claim. Simulations show that 'cheating' gives the healthy agent a lifetime utility of 12.1032,[11] which clearly exceeds the 10.1844 payoff of being honest. This leads to a free-rider problem: as each healthy agent has this incentive and they cannot coordinate their actions, the pooling equilibrium will be the inevitable, yet suboptimal, outcome. Hence, when health is not observable the separating equilibrium can never be attained.

Box 7

No annuities. When the annuity market does not exist, some people will die leaving unintended bequests. In order to avoid draining capital from the economy, we have to make an assumption about how these funds are redistributed among the agents who are still alive. Following Heijdra and Mierau (2009a), we assume the presence of a government sector which pays out a lump-sum transfer indexed to the wage rate. That is, for an agent of vintage v, the transfer is given by:

$$z(v,t) = z \cdot w(t),$$

where z is a positive indexing variable. This index is endogenously determined by the redistribution scheme (see below), but taken as given by the households. Note that $z(v,t)$ only depends on the wage rate and not on health. This is inevitable as the government cannot observe health status, and thus cannot discriminate between healthy and unhealthy individuals. The budget identity for a type j individual changes to:

$$\dot{\bar{a}}_j(v,t) = r\bar{a}_j(v,t) + (1+z)w(t) - \bar{c}_j(v,t).$$

The budget constraint at a given time $t \geq v$ is then given by:

$$\bar{a}_j(v,t) = e^{r(t-v)} \int_0^t [(1+z)w(v+u) - \bar{c}_j(v,v+u)]e^{-ru}\, du.$$

The risk of untimely death implies that the probability constraint on wealth $P\{\bar{a}_j(v,v+D) \geq 0\} = 1$ must hold. However, as D is unknown, Yaari (1965) shows that equiva-

[11] This result is obtained by calculating the consumption path for an healthy individual who receives net annuity rate $\mu_U(u)$ for $u \in [0, \bar{D}_U)$ and $\mu_H(u)$ for $u \in [\bar{D}_U, \bar{D}_H]$, keeping everything else constant (including the growth rate and the behaviour of all other agents).

lently we can impose:

$$\bar{a}_j(v, v + \bar{D}_j) = 0, \quad \dot{\bar{a}}_j(v, t) \geq 0 \text{ whenever } \bar{a}_j(v, t) = 0.$$

In our model we can simplify the constraint even further and write that $\bar{a}_j(v, t) \geq 0$ must hold for all $t \geq v$. Agents become increasingly impatient with age due to their upward sloping mortality profile. Hence, once the nonnegativity constraint on assets becomes binding at a certain age $\bar{u}_j$ it will remain so until certain death. Analogously to the case of a self-imposed credit constraint discussed above, the agent now has to optimally determine $\bar{u}_j \in [0, \bar{D}_j]$. We derive two conditions that $\bar{u}_j$ has to satisfy. First of all, since $\bar{a}_j(v, v + \bar{u}_j) = 0$ we find:

$$\bar{c}_j(v, v) \int_0^{\bar{u}_j} e^{-\rho u - M_j(u)} \, du = (1 + z) w(v) \int_0^{\bar{u}_j} e^{-(r-\gamma)u} \, du,$$
$$\frac{\bar{c}_j(v, v)}{w(v)} \int_0^{\bar{u}_j} e^{-\rho u - M_j(u)} \, du = \frac{1 + z}{r - \gamma} \left[1 - e^{-(r-\gamma)\bar{u}_j}\right].$$

Secondly, we know that after $\bar{u}_j$ the agent consumes exactly her wage plus income transfer, i.e. $\bar{c}_j(v, t) = (1 + z)w(t)$ for $t \geq v + \bar{u}_j$. Hence, the following 'smooth connection' condition has to be satisfied:

$$(1 + z)e^{\gamma \bar{u}_j} = \bar{c}_j(v, v)e^{(1-\rho)\bar{u}_j - M_j(\bar{u}_j)}.$$

Combining the two expressions gives the following implicit solution for $\bar{u}_j$:

$$e^{\rho \bar{u}_j + M_j(\bar{u}_j)} \int_0^{\bar{u}_j} e^{-\rho u - M_j(u)} \, du = \frac{e^{(r-\gamma)\bar{u}_j} - 1}{r - \gamma}.$$

The value of z is endogenously determined by the balanced-budget requirement for the redistribution scheme:

$$\sum_{j \in \{H,U\}} \int_{t-\bar{u}_j}^{t} l_j(v, t)\mu_j(t - v)\bar{a}_j(v, t) \, dv = z \cdot w(t).$$

Using the same parameter values as for the numerical simulations above, we find $\bar{u}_H = 60.6345$, $\bar{u}_U = 53.7550$, and a growth rate $\gamma = 0.0139$ which is substantially lower than in both the pooling and the separating equilibrium. For the resulting consumption and asset paths, see Figure I(a)-(b).

(a) Scaled individual consumption $\bar{c}_j(v,t)/w(v)$

(b) Scaled cohort assets $a_j(v,t)/w(v)$

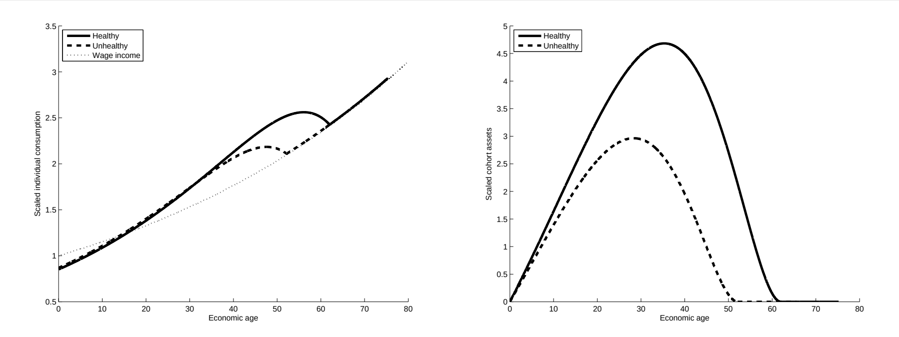


Figure I: No annuities equilibrium

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4 Conclusion

In this paper we have focused on the effects of adverse selection in annuity markets on economic growth and welfare. To that end, we have constructed a general equilibrium model featuring endogenous growth and overlapping generations. The agents who inhabit this closed economy are assumed to have finite lives and can be distinguished based on their health type, either healthy or unhealthy. We have shown that if health is private information, a pooling equilibrium emerges in the annuity market in which the healthy agents receive a better than actuarially fair return while the unhealthy individuals get less than they are entitled to. Even though this gives an incentive to unhealthy agents to borrow money in the last stages of their life, they will instead impose a voluntary borrowing constraint on themselves in order not to reveal their health status. Hence, our model can explain why some individuals rationally choose to stay out of the annuity market.

For a plausibly parameterized version of the model the welfare loss of having a pooling equilibrium instead of a fair separating equilibrium is small, while the welfare gain relative to the total absence of annuities is much larger.

A Proofs

Proposition 1. *Consider the case in which annuity firms can observe the health type of annuitants, such that the annuity market is characterized by a separating equilibrium. Provided the growth-corrected interest rate exceeds the pure rate of time preference, $r-\gamma>\rho$, agents of both health types are net savers throughout life, i.e. $\bar{a}_j(v,v)=\bar{a}_j(v,v+\bar{D}_j)=0$ and $\bar{a}_j(v,v+u)>0$ for all $u\in(0,\bar{D}_j)$.*

Proof. In a separating equilibrium we have $M_j(u)=P_j(u)$ for $0\le u\le\bar{D}_j$ as $p_j(u)=\mu_j(u)$. Since $\rho<r-\gamma$ we find:

$$\frac{\bar{c}_j(v,v)}{w(v)}=\frac{\int_0^{\bar{D}_j}e^{-(r-\gamma)u-M_j(u)}\,du}{\int_0^{\bar{D}_j}e^{-\rho u-M_j(u)}\,du}<1. \tag{A.1}$$

Let $u\in[0,\bar{D}_j]$ be the age of the consumer. Then we can write:

$$\frac{\bar{a}_j(v,v+u)}{w(v)}e^{-ru-M_j(u)}=\Gamma_j(u), \tag{A.2}$$

where $\Gamma_j:[0,\bar{D}_j]\to\mathbb{R}$ is defined by:

$$\Gamma_j(u)=\int_0^u e^{-(r-\gamma)s-M_j(s)}ds-\frac{\bar{c}_j(v,v)}{w(v)}\int_0^u e^{-\rho s-M_j(s)}ds. \tag{A.3}$$

As Γ_j is a continuous function defined on a closed and bounded interval we know that Γ_j has a global maximum and a global minimum on its domain. Candidates for these extreme points are the boundaries of the domain and the interior critical points. For the boundary points we find $\Gamma_j(0)=\Gamma_j(\bar{D}_j)=0$ as $\bar{a}_j(v,v)=\bar{a}_j(v,v+\bar{D}_j)=0$ by the initial condition and the property of nonsaturation.

Using Leibnitz' rule, we find that the first order derivative of Γ is given by:

$$\begin{aligned}\Gamma_j'(u)&=e^{-(r-\gamma)u-M_j(u)}-\frac{\bar{c}_j(v,v)}{w(v)}e^{-\rho u-M_j(u)}\\&=e^{-M_j(u)}\left[e^{-(r-\gamma)u}-\frac{\bar{c}_j(v,v)}{w(v)}e^{-\rho u}\right].\end{aligned} \tag{A.4}$$

The unique interior root of this equation is:

$$u^*\equiv-\frac{1}{r-\gamma-\rho}\ln\left(\frac{\bar{c}_j(v,v)}{w(v)}\right), \tag{A.5}$$

where $u^*>0$ as $\bar{c}_j(v,v)/w(v)<1$ and $r-\gamma>\rho$ by assumption. We find that $\Gamma_j'(u)>0$ for $0\le u<u^*$ and $\Gamma_j'(u)<0$ for $u^*<u<\bar{D}_j$. We conclude that Γ_j has a global maximum at u^* and a global minimum at 0 and $\bar{D}_j$. As this global minimum is strict and equals zero, we find $\bar{a}_j(v,v+u)>0$ for all $u\in(0,\bar{D}_j)$. □

Proposition 2. *Consider the case in which annuity firms cannot observe the health type of annuitants. Assume that the growth-corrected interest rate exceeds the pure rate of time preference, $r - \gamma > \rho$. Suppose there exists a pooling equilibrium in the annuity market. Then:*

(a) Healthy agents are net savers throughout life, i.e. $\bar{a}_H(v,v) = \bar{a}_H(v, v + \bar{D}_H) = 0$ and $\bar{a}_H(v, v+u) > 0$ for all $u \in (0, \bar{D}_H)$.

(b) Unhealthy agents are net savers until age $\bar{u} \in (0, \bar{D}_U)$ and face a binding self-imposed borrowing constraint after age $\bar{u}$, i.e. $\bar{a}_U(v,v) = 0$, $\bar{a}_U(v, v+u) > 0$ for $u \in (0, \bar{u})$, and $\bar{a}_U(v, v+u) = 0$ for $u \in [\bar{u}, \bar{D}_U]$.

Proof. We assume that there exists a pooling equilibrium in the annuity market. This is only possible if the asset holdings of both health groups have the same sign everywhere. However, they cannot both be negative as that would imply that nobody saves in this closed economy. In that case there is no capital stock and wages are zero, which contradicts the existence of an equilibrium. Hence the asset holdings of the healthy and unhealthy agents will both have to be nonnegative: $\bar{a}_H(v, v+u) \geq 0$ and $\bar{a}_U(v, v+u) \geq 0$ for $0 \leq u \leq \bar{D}_U$. The corresponding pooling premium is given by:

$$\bar{p}(u) = \begin{cases} \dfrac{\mu_H(u)a_H(v,v+u) + \mu_U(u)a_U(v,v+u)}{a_H(v,v+u) + a_U(v,v+u)} & \text{for } 0 \leq u \leq \bar{D}_U \\ \mu_H(u) & \text{for } \bar{D}_U < u \leq \bar{D}_H \end{cases}.$$

Write $\bar{P}(u) \equiv \int_0^u \bar{p}(s)\, ds$. It follows that:

$$\begin{aligned} &\mu_H(u) \leq \bar{p}(u) \leq \mu_U(u), && \text{for } 0 \leq u \leq \bar{D}_U, \\ &M_U(u) \geq \bar{P}(u), && \text{for } 0 \leq u \leq \bar{D}_U, \\ &M_H(u) \leq \bar{P}(u), && \text{for } 0 \leq u \leq \bar{D}_H. \end{aligned}$$

Now consider the two statements made in the proposition.

(a) Take a healthy agent. Define $f : [0, \bar{D}_H] \to \mathbb{R}$ by:

$$f(u) = e^{M_H(u) - \bar{P}(u)}. \tag{A.6}$$

It follows that f is a differentiable function, that $f(0) = 1$ and that $f(u) \leq 1$ for all $u \in (0, \bar{D}_H]$. The first-order derivative of f is given by:

$$f'(u) = [\mu_H(u) - \bar{p}(u)]f(u) = \begin{cases} [\mu_H(u) - \bar{p}(u)]f(u) & \text{for } 0 \leq u \leq \bar{D}_U \\ 0 & \text{for } \bar{D}_U < u \leq \bar{D}_H \end{cases}, \tag{A.7}$$

such that $f'(u) \leq 0$ for all $u \in [0, \bar{D}_H]$. Using the function f, we can write consumption at birth as:

$$\frac{\bar{c}_H(v,v)}{w(v)} = \frac{\int_0^{\bar{D}_H} f(u) e^{-(r-\gamma)u - M_H(u)}\, du}{\int_0^{\bar{D}_H} e^{-\rho u - M_H(u)}\, du}. \tag{A.8}$$

By the properties of f and the assumption that $r - \gamma > \rho$ it immediately follows that:

$$\frac{\bar{c}_H(v,v)}{w(v)} < 1. \tag{A.9}$$

We now write:

$$\frac{\bar{a}_H(v, v+u)}{w(v)} e^{-ru - \bar{P}(u)} = \Gamma_H(u), \tag{A.10}$$

where $\Gamma_H : [0, \bar{D}_H] \to \mathbb{R}$ is defined by:

$$\Gamma_H(u) = \int_0^u f(s) e^{-(r-\gamma)s - M_H(s)}\, ds - \frac{\bar{c}_H(v,v)}{w(v)} \int_0^u e^{-\rho s - M_H(s)}\, ds. \tag{A.11}$$

As Γ_H is a continuous function defined on a closed and bounded interval we know that Γ_H has a global maximum and a global minimum on its domain. Candidates for these extreme points are the boundaries of the domain and the interior critical points. For the boundary points we find $\Gamma_H(0) = \Gamma_H(\bar{D}_H) = 0$ as $\bar{a}_H(v,v) = \bar{a}_H(v, v+\bar{D}_H) = 0$.

The first-order derivative of Γ_H is given by:

$$\Gamma_H'(u) = e^{-M_H(u)} \left[f(u) e^{-(r-\gamma)u} - \frac{\bar{c}_H(v,v)}{w(v)} e^{-\rho u} \right]. \tag{A.12}$$

It follows that a stationary point u_H^* of Γ_H satisfies:

$$\frac{\bar{c}_H(v,v)}{w(v)} e^{(r-\gamma-\rho)u_H^*} = f(u_H^*). \tag{A.13}$$

Since $r - \gamma > \rho$, the left-hand side of this equation is increasing in u. Combined with the fact that $\bar{c}_H(v,v)/w(v) < 1$, $f(0) = 1$ and $f'(u) \leq 0$ for all $u \in [0, \bar{D}_H]$ we find that the stationary point is unique. The second-order derivative of Γ_H evaluated in u_H^* is:

$$\begin{aligned}
\Gamma_H''(u_H^*) = e^{-M_H(u_H^*)} &\left[f'(u_H^*) e^{-(r-\gamma)u_H^*} - (r-\gamma) f(u_H^*) e^{-(r-\gamma)u_H^*} + \rho \frac{\bar{c}_H(v,v)}{w(v)} e^{-\rho u_H^*} \right] \\
&- \mu_H(u_H^*) e^{-M_H(u_H^*)} \left[f(u_H^*) e^{-(r-\gamma)u_H^*} - \frac{\bar{c}_H(v, v+u_H^*)}{w(v)} e^{-\rho u_H^*} \right], \\
&= -e^{-(r-\gamma)u_H^* - M_H(u_H^*)} \left[(r-\gamma-\rho) f(u_H^*) - f'(u_H^*) \right].
\end{aligned} \tag{A.14}$$

As $f'(u_H^*) \leq 0$, it follows that $\Gamma_H''(u_H^*) < 0$. We conclude that Γ_H has a global maximum at u_H^* and a global minimum at 0 and $\bar{D}_H$. As this global maximum is strict and equals zero, we find $\bar{a}_H(v, v+u) > 0$ for all $u \in (0, \bar{D}_H)$.

(b) Take an unhealthy agent. Define $h : [0, \bar{D}_U] \to \mathbb{R}$ by:

$$h(u) = e^{M_U(u) - \bar{P}(u)}. \tag{A.15}$$

It follows that h is a differentiable function, that $h(0) = 1$ and that $h(u) \geq 1$ for all $u \in (0, \bar{D}_U]$. The first-order derivative of h is given by:

$$h'(u) = [\mu_U(u) - \bar{p}(u)]h(u). \tag{A.16}$$

Since we have shown above that $\bar{a}_H(v, v+u) > 0$ for all $u \in (0, \bar{D}_H)$ and $\mu_U(u) > \mu_H(u)$ for all $u \in [0, \bar{D}_U]$ by assumption, it follows that $\bar{p}(u) < \mu_U(u)$ for all $u \in (0, \bar{D}_U]$. As a consequence, we find that $h'(u) > 0$ on its domain. Using the function h, we can write consumption at birth as:

$$\frac{\bar{c}_U(v,v)}{w(v)} = \frac{\int_0^{\bar{D}_U} h(u) e^{-(r-\gamma)u - M_U(u)}\, du}{\int_0^{\bar{D}_U} e^{-\rho u - M_U(u)}\, du}. \tag{A.17}$$

If $\bar{c}_U(v,v)/w(v) > 1$ then there exists $\varepsilon > 0$ such that $\bar{a}_U(v, v+\varepsilon) < 0$, which contradicts the assumption that a pooling equilibrium exists. Hence:

$$\frac{\bar{c}_U(v,v)}{w(v)} < 1. \tag{A.18}$$

It follows that $\bar{a}_U(v, v+u)$ is positive for small values of u. Now suppose to the contrary that there does not exist an age $\bar{u} \in (0, \bar{D}_U)$ such that $\bar{a}_U(v, v+u) = 0$ for $u \in [\bar{u}, \bar{D}_U]$. Then we would have $\bar{a}_U(v, v+u) > 0$ for $u \in (0, \bar{D}_U)$. In that case we can write:

$$\frac{\bar{a}_U(v,v)}{w(v)} e^{-ru - \bar{P}(u)} = \Gamma_U(u), \tag{A.19}$$

where $\Gamma_U : [0, \bar{D}_U] \to \mathbb{R}$ is defined by:

$$\Gamma_U(u) = \int_0^u h(s) e^{-(r-\gamma)s - M_U(s)}\, ds - \frac{\bar{c}_U(v,v)}{w(v)} \int_0^u e^{-\rho s - M_U(s)}\, ds. \tag{A.20}$$

As Γ_U is a continuous function defined on a closed and bounded interval we know that Γ_U has a global maximum and a global minimum on its domain. Candidates for these extreme points are the boundaries of the domain and the interior critical points. For the boundary points we find $\Gamma_U(0) = \Gamma_U(\bar{D}_U) = 0$ as $\bar{a}_U(v,v) = \bar{a}_U(v, v+\bar{D}_U) = 0$.

The first-order derivative of Γ_U is given by:

$$\Gamma_U'(u) = e^{-M_U(u)} \left[h(u) e^{-(r-\gamma)u} - \frac{\bar{c}_U(v,v)}{w(v)} e^{-\rho u} \right]. \tag{A.21}$$

It follows that a stationary point u_U^* of Γ_U satisfies:

$$\frac{\bar{c}_U(v,v)}{w(v)} e^{(r-\gamma-\rho)u_U^*} = h(u_U^*). \tag{A.22}$$

Both the left-hand side and the right-hand side of this equation are increasing and convex in u, opening the possibility of multiple stationary points. Define $g : [0, \bar{D}_U] \to \mathbb{R}$ by:

$$g(u) = \frac{\bar{c}_U(v,v)}{w(v)} e^{(r-\gamma-\rho)u}. \tag{A.23}$$

As $\bar{c}_U(v,v)/w(v) < 1$ it follows that $g(0) < h(0)$. Since both functions are strictly increasing and $\lim_{u \to \bar{D}_U} h(u) > g(\bar{D}_U)$, it follows that if g and h cross on $[0, \bar{D}_U]$ then they cross exactly twice. Hence, we conclude Γ_U has two critical points on its domain.

The second-order derivative of Γ_U evaluated in u_U^* is:

$$\begin{aligned}\Gamma_U''(u_U^*) &= -e^{-(r-\gamma)u_U^* - M_U(u_U^*)} \left[(r-\gamma-\rho)h(u_U^*) - h'(u_U^*)\right] \\ &= -h(u_U^*)e^{-(r-\gamma)u_U^* - M_U(u_U^*)} \left[(r-\gamma-\rho) - [\mu_U(u_U^*) - \bar{p}(u_U^*)]\right] \gtreqless 0.\end{aligned} \tag{A.24}$$

where we have used the fact that $h'(u_U^*) = [\mu(u_U^*) - \bar{p}(u_U^*)]h(u_U^*)$. Since $r - \gamma > \rho$ and $\mu_U(u) - \bar{p}(u) \approx 0$ for low values u, we find that the first stationary point is a maximum. As $[\mu_U(u) - \bar{p}(u)] \to \infty$ for $u \to \bar{D}_U$, we find that the second stationary point is a minimum. As $\Gamma_U(0) = \Gamma_U(\bar{D}_U) = 0$ and there are exactly two interior stationary points it follows that the minimum is associated with negative asset holdings. This is a contradiction to the assumption that $\bar{a}_U(v, v+u) > 0$ for all $u \in (0, \bar{D}_U)$. Hence we conclude that there does exist an age $\bar{u} \in (0, \bar{D}_U)$ such that $\bar{a}_U(v, v+u) > 0$ for $u \in (0, \bar{u})$ and $\bar{a}_U(v, v+u) = 0$ for $u \in [\bar{u}, \bar{D}_U]$.

□

B Estimating the mortality profiles

Our goal is to estimate the mortality profiles for the healthy and the unhealthy in such a way that the population average corresponds to the data. We take n as given, and estimate the parameters η_0, η_{1H}, π_H and θ using nonlinear least squares interpolation.

Let S_i be the surviving fraction at age u_i in the life table. Then for $0 \le u \le \ln \eta_0/\eta_{1H}$ we seek to find the best fit of the following equation:

$$S_i \left(\equiv \frac{L(v, v+u_i)}{L(v,v)} \right) = \chi_H(u_i) \frac{\beta_H \pi_H}{\beta_H \pi_H + \beta_U \pi_U} e^{-M_H(u_i)} + \chi_U(u_i) \frac{\beta_U \pi_U}{\beta_H \pi_H + \beta_U \pi_U} e^{-M_U(u_i)} + \varepsilon_i, \quad \text{(C.1)}$$

where:

$$\beta_j = \left[\int_0^{\bar{D}_j} e^{-ns-M_j(s)} \right]^{-1}, \quad \text{(C.2)}$$

$$e^{-M_j(u_i)} = \frac{\eta_0 - e^{\eta_{1j} u_i}}{\eta_0 - 1}, \quad \text{(C.3)}$$

and $\eta_{1U} = \theta \eta_{1H}$. The variables χ_H and χ_U are dummy variables:

$$\chi_H(u_i) = \begin{cases} 1 & 0 \le u_i < \bar{D}_H \\ 0 & \text{otherwise} \end{cases}, \quad \text{(C.4)}$$

$$\chi_U(u_i) = \begin{cases} 1 & 0 \le u_i < \bar{D}_U \\ 0 & \text{otherwise} \end{cases}, \quad \text{(C.5)}$$

where $\bar{D}_H = \ln \eta_0/\eta_{1H}$ and $\bar{D}_U = (1/\theta)\bar{D}_H$.

We use the life table for the cohort born in 1960 in the Netherlands as our data. The data can be found in the online Human Mortality Database (www.mortality.org). Under the assumption that $n = 0.01$ we estimate the following parameter values (t-statistics in brackets):

$$\hat{\eta}_0 = \underset{(16.18)}{187.8646}, \quad \hat{\eta}_{1H} = \underset{(79.63)}{0.0696}, \quad \hat{\pi}_H = \underset{(15.54)}{0.5077}, \quad \hat{\theta} = \underset{(117.63)}{1.1479}$$

From these results we derive estimates for the remaining demographic parameters:

$$\hat{\eta}_{1U} = \underset{(66.14)}{0.0799}, \quad \hat{\beta}_H = \underset{(244.30)}{0.0221}, \quad \hat{\beta}_U = \underset{(274.55)}{0.0244}, \quad \hat{\bar{D}}_H = \underset{(194.68)}{75.2148}, \quad \hat{\bar{D}}_U = \underset{(176.04)}{65.5224}$$

Figure 4 shows the implied surviving fraction and instantaneous mortality rate for both health groups, and compares it with the data.

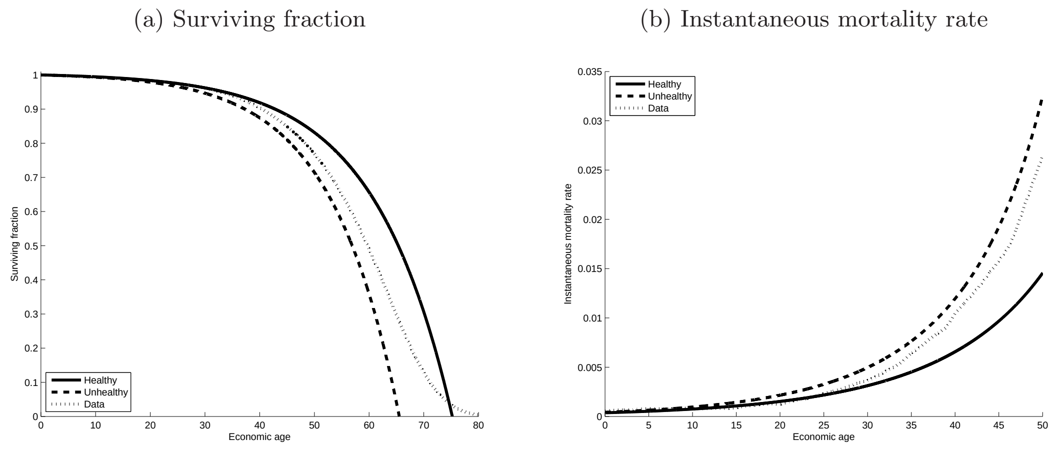

Figure 4: Demographics

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The welfare effects of adverse selection in annuity markets and the role of mandatory social annuitization

Abstract

We study the welfare effects of adverse selection in private annuity markets in an economy inhabited by overlapping generations of heterogeneous agents who are distinguished by their health status. An agent's health type is assumed to be private information. We show that if a private annuity market is introduced then it will be characterized by a pooling equilibrium. Even though it is individually optimal to invest in annuities, agents of both health types are in the long run worse off in welfare terms compared to the case where annuities are absent and accidental bequests are redistributed to the young. We also study the welfare implications of a social security system with fixed mandatory contributions for both health types. These social annuities are immune to adverse selection and therefore offer a higher rate of return than private annuities do. However, they have a negative effect on steady-state welfare. The positive effect of a fair pooled rate of return on a fixed part of savings and a higher return on capital in equilibrium is outweighed by the negative consequences of increased adverse selection in the private annuity market and a lower wage rate.

1 Introduction

In a seminal paper, Yaari (1965) argues that in the face of life span uncertainty individuals will fully annuitize their financial wealth. That is, they will invest all their savings in the annuity market, thereby insuring themselves against the risk of outliving their assets. However, empirical evidence has revealed a so-called annuity puzzle: despite the theoretical attractiveness of annuities, in practice people tend not to invest much of their financial wealth in private annuity markets.

Several explanations for this puzzle have been given in the literature. First of all, individuals may have a bequest motive, in that they wish to leave an inheritance to those they leave behind. If so, they will want to keep part of their financial assets outside the annuity market. Secondly, psychological factors may play a role. According to Cannon and Tonks (2008), people might feel uncomfortable to bet on a long life. Investing in annuities only seems attractive if you expect to live long enough, as most of us would hate to die before having received at least our initial outlay back in periodic payments. Third, private annuity demand may be crowded out by a system of social security benefits.

A fourth explanation is that in reality annuities may not be actuarially fair, in the sense that individuals are insufficiently compensated for their risk of dying. This may be due to administrative costs and taxes or monopoly profits as a result of imperfect competition among annuity firms. The implications for macroeconomic growth and welfare of a loading factor on annuities proportional to the mortality rate are investigated in Heijdra and Mierau (2009a). Another reason for annuity market imperfection is adverse selection. The healthier someone believes herself to be, the more likely she is to buy an annuity. As a consequence, low-mortality (and thus high-risk) individuals are overrepresented in the annuity market. Annuity firms will have to take this selection effect into account when pricing their products, as they will incur a loss if they offer a rate based on average survival probabilities in the population. The resulting higher prices (or lower return) will induce high-mortality (low-risk) individuals to invest less in the annuity market.

In this paper we abstract from bequest motives, administrative costs, and imperfect competition and focus on the adverse selection channel and the role of social annuities. Our work mainly builds on the foundations laid out by Heijdra and Reijnders (2009). They consider a continuous-time endogenous growth model in which agents differ in their health status acquired at birth, which is assumed to be private information. The equilibrium in the annuity market is then characterized by risk pooling among health types which induces the unhealthy agents to drop out of the market in the last stages of their lives. This pooling equilibrium is slightly dominated in welfare terms by a hypothetical separating equilibrium (in which each health type receives its actuarially fair rate of return), while the complete absence of annuities is significantly worse.

We extend the work by Heijdra and Reijnders (2009) in several directions. Instead of a continuous-time model with endogenous growth we work with a discrete-time framework in which long-run growth is exogenously determined. This allows us to study what happens during the transition from one steady state to another. We find that when a private annuity market is opened up, it is individually optimal for agents to invest in it, albeit not socially optimal. Agents of both types are in the long run worse off compared to the benchmark case in which annuities are absent and accidental bequests are redistributed to the newly arrived young. This can be seen as a 'tragedy of annuitization', as described in Heijdra et al. (2010). A mandatory social annuity system, while providing a 'fairer' rate of return than the private annuity market, reduces steady-state welfare even more. It aggravates the degree of adverse selection in the private annuity market and reduces the overall level of savings in the economy.

Other papers closely related to ours are Abel (1986), Walliser (2000), and Palmon & Spivak (2007). All three find that the introduction of social annuities accentuates the problem of adverse selection in the private annuity market. In Abel (1986) a two-period exogenous growth model is developed in which agents have privately known heterogenous mortality profiles and a bequest motive. Due to adverse selection, the rate of return on private annuities is less than actuarially fair. In this context, the introduction of an actuarially fair social security system further decreases the return on annuities in the steady state. Walliser (2000) builds on the work of Abel (1986), but calibrates his model with 75 instead of only 2 periods. The paper investigates the effects of pay-as-you-go social security benefits on private annuity demand and shows that privatization (i.e. elimination) of social security lowers the loading factor on annuities resulting from adverse selection. Finally, Palmon & Spivak (2007) argue that a modest social security system may reduce welfare in an adverse selection economy. The positive effect of providing agents with social annuities at an actuarially fair pooling rate is outweighed by the negative impact of increased adverse selection in the private annuity market.

In contrast to our work, however, both Walliser (2000) and Palmon & Spivak (2007) focus on the features of private annuity markets in isolation and do not take general equilibrium effects into account. Moreover, the latter fail to specify what happens to accidental bequests in the absence of annuities and therefore incorrectly conclude that annuities are always welfare improving. Abel (1986) on the other hand does model a production sector with potentially endogenous factor prices and provides signs for the responses of key economic variables following a change in the rate of contribution to the social security system. Yet he does not give any insight in the magnitude of the effect on consumer welfare, nor how it may differ among risk types. Our contribution lies in providing a consistent general equilibrium framework for studying all relevant aspects related to life annuitization and social security. We are able to both analytically characterize the underlying mechanisms and to quantify their relative importance through a simulation with realistic parameter values.

The remainder of this paper is structured as follows. Section 2 describes the key features of the model in terms of the decisions made by households, firms, and the government sector. In Section 3 we introduce private annuity markets, while Section 4 shows the effects on general equilibrium and welfare when mandatory social annuities are added to the model. Section 5 concludes.

2 Model

2.1 Consumers

The population consists of overlapping generations of finitely-lived agents who are identical in every respect except for their health type. Each agent learns at birth whether she is 'healthy' (indexed by the subscript H) or 'unhealthy' (U). Health status is assumed to be private information.

Agents live for a maximum of two periods, termed 'youth' (superscript y) and 'old age' (o). For an agent of health type $j \in \{H, U\}$ the probability of surviving into the second phase of life is $1-\mu_j$. This is where the difference between health types comes in: unhealthy agents have a higher risk of dying and therefore a shorter expected life span, i.e. $\mu_U > \mu_H$. We assume that cohorts are sufficiently large such that there is no aggregate uncertainty and probabilities and frequencies coincide. For example, the fraction of healthy young agents who die after the first period equals exactly μ_H. Note that from the perspective of an individual agent, lifetime uncertainty is resolved at the start of the second period. When still alive, the agent will live for exactly one additional period.

In both periods the agent inelastically supplies a unit amount of labour. The expected lifetime utility of a representative agent of health type $j \in \{H, U\}$ who is born in period t is given by:

$$E(\Lambda_{j,t}) \equiv \ln C^y_{j,t} + \frac{1-\mu_j}{1+\rho} \ln C^o_{j,t+1}, \tag{1}$$

where $C^y_{j,t}$ is consumption during youth, $C^o_{j,t+1}$ is consumption during old age, and $\rho > 0$ is the pure rate of time preference. The felicity function is chosen to be logarithmic for analytical convenience, implying a unitary intertemporal substitution elasticity and a constant rate of relative risk aversion. We assume that the agent does not have a bequest motive such that she does not derive any utility from wealth that remains after her death.

The agent's periodic budget identities are given by:

$$C^y_{j,t} + S_{j,t} = w_t + Z_t, \tag{2}$$

$$C^o_{j,t+1} = w_{t+1} + (1 + r_{t+1})S_{j,t}, \tag{3}$$

where w_t is the wage rate, r_{t+1} is the rental rate of capital, $S_{j,t}$ is the amount saved, and Z_t denotes a lump-sum income transfer received from the government during youth. Combining the two budget identities yields the consolidated budget constraint:

$$C^y_t + \frac{C^o_{t+1}}{1 + r_{t+1}} = w_t + Z_t + \frac{w_{t+1}}{1 + r_{t+1}}. \tag{4}$$

That is, the present value of total consumption (left-hand side) should equal lifetime income or human wealth at birth (right-hand side). The representative agent maximizes life-time utility (1) subject to the budget constraint (4).

We start by considering the situation in which annuity markets do not exist. In that case, agents cannot insure themselves against life span uncertainty. Their only option is to invest their savings in the capital market at a net rate of return r_{t+1}. Since there is a risk of dying after youth the agent may pass away before being able to consume her savings, thereby leaving an unintended bequest. It is not possible to borrow money in the capital market, as the agent is not allowed to die indebted. Hence we impose the borrowing constraint $S_{j,t} \geq 0$.

Assuming an interior solution (i.e. the borrowing constraint is not binding), the agent's optimal plans are fully characterized by:

$$C^y_{j,t} = \frac{1+\rho}{2+\rho-\mu_j}\left[w_t + Z_t + \frac{w_{t+1}}{1+r_{t+1}}\right], \tag{5}$$

$$S_{j,t} = \frac{1-\mu_j}{2+\rho-\mu_j}[w_t + Z_t] - \frac{1+\rho}{2+\rho-\mu_j}\frac{w_{t+1}}{1+r_{t+1}}, \tag{6}$$

$$C^o_{j,t+1} = \frac{(1-\mu_j)(1+r_{t+1})}{2+\rho-\mu_j}\left[w_t + Z_t + \frac{w_{t+1}}{1+r_{t+1}}\right]. \tag{7}$$

We find that consumption during youth and the present value of consumption during old age are proportional to human wealth at birth. The level of savings depends positively on the total income received during youth and negatively on the discounted value of future wage income.

2.2 Demography

Let L_t denote the size of the cohort born in period t. This cohort consists of both healthy and unhealthy agents:

$$L_t \equiv L_{H,t} + L_{U,t}. \tag{8}$$

The total population alive in period t can be found by aggregating over cohorts and health types:

$$P_t \equiv \sum_{j\in\{H,U\}} \left[(1-\mu_j)L_{j,t-1} + L_{j,t}\right]. \tag{9}$$

The proportion of healthy and unhealthy agents in the population is assumed to stay constant over time and equal to π_H and π_U, respectively, with $\pi_H + \pi_U = 1$. In addition, we postulate that the number of agents that are born into a given health group at each point in time is proportional to the size of that health group at birth:[1]

$$L_{j,t} = (1+\beta_j)\pi_j P_{t-1}, \quad j \in \{H,U\}, \tag{10}$$

[1]This is not meant to imply that only healthy people get healthy children.

where β_j denotes the crude birth rate. Combined with a positive rate of population growth n this furnishes the following consistency condition:

$$1 + \beta_j = \frac{(1+n)^2}{2+n-\mu_j}. \tag{11}$$

2.3 Government

We have to make an assumption about how the accidental bequests left by the dead are redistributed among the agents who are still alive. We therefore introduce a government sector which collects the bequests and uses them to finance lump-sum income transfers Z_t to the young.[2] The government budget constraint is given by:

$$(1+r_t) \sum_{j \in \{H,U\}} \mu_j L_{j,t-1} S_{j,t-1} = L_t Z_t. \tag{12}$$

That is, the total amount of accidental bequests (left-hand side) should equal the sum of income transfers (right-hand side).

2.4 Production

The production side of this closed economy is characterized by a large number of perfectly competitive firms who produce a homogeneous commodity. The technology is represented by the following Cobb-Douglas production function:

$$Y_t = \Omega_0 K_t^{\varepsilon} P_t^{1-\varepsilon}, \quad 0 < \varepsilon < 1, \tag{13}$$

where Y_t is total output, K_t is the aggregate capital stock, P_t is the size of the population or labour force, and $\Omega_0 > 0$ is an exogenously given index of the general level of factor productivity in the economy.[3] By defining $y_t \equiv Y_t/P_t$ and $k_t \equiv K_t/P_t$ we can write the production function in per-capita terms:

$$y_t = \Omega_0 k_t^{\varepsilon}. \tag{14}$$

Profit-maximizing behaviour of firms yields the following factor demand equations:

$$r_t + \delta = \varepsilon \Omega_0 k_t^{\varepsilon-1}, \tag{15}$$

$$w_t = (1-\varepsilon)\Omega_0 k_t^{\varepsilon}, \tag{16}$$

[2] This is a standard assumption in the economic literature when agents do not have a bequest motive. See for example Pecchenino and Pollard (1997).

[3] The endogenous growth scenario, i.e. the case for which Ω_0 is replaced by $\Omega_t = \Omega_0 k_t^{1-\varepsilon}$, is studied in detail in a continuous-time framework by Heijdra and Reijnders (2009).

where $\delta > 0$ is the constant rate of depreciation of the capital stock. In the steady state we have $k_{t+1} = k_t = \hat{k}$ such that the level of income per capita $\hat{y}$ is constant. As a consequence, the long-run rate of economic growth over and above population growth equals zero, which is a standard result in an exogenous growth model without technological process.

The general model without annuities is given in Table 1. The fundamental difference equation (T1.7) is obtained by imposing equilibrium in the savings market and using the cohort size evolutions described in Section 2.2.

Table 1: General model without annuities

(a) Microeconomic relationships:

$$C^y_{j,t} = \frac{1+\rho}{2+\rho-\mu_j}\left[w_t + Z_t + \frac{w_{t+1}}{1+r_{t+1}}\right] \tag{T1.1}$$

$$S_{j,t} = \frac{1-\mu_j}{2+\rho-\mu_j}[w_t + Z_t] - \frac{1+\rho}{2+\rho-\mu_j}\frac{w_{t+1}}{1+r_{t+1}} \tag{T1.2}$$

$$C^o_{j,t+1} = \frac{(1-\mu_j)(1+r_{t+1})}{2+\rho-\mu_j}\left[w_t + Z_t + \frac{w_{t+1}}{1+r_{t+1}}\right] \tag{T1.3}$$

(b) Macroeconomic relationships:

$$r_t + \delta = \varepsilon \Omega_0 k_t^{\varepsilon-1} \tag{T1.4}$$

$$w_t = (1-\varepsilon)\Omega_0 k_t^{\varepsilon} \tag{T1.5}$$

$$L_t Z_t = (1+r_t) \sum_{j\in\{H,U\}} \mu_j L_{j,t-1} S_{j,t-1} \tag{T1.6}$$

(c) Fundamental difference equation:

$$k_{t+1} = \phi_1[w_t + Z_t] - \phi_2 \frac{w_{t+1}}{1+r_{t+1}} \tag{T1.7}$$

$$\phi_1 \equiv \sum_{j\in\{H,U\}} \phi_{1j}, \quad \phi_{1j} \equiv \frac{\pi_j(1-\mu_j)}{(2+n-\mu_j)(2+\rho-\mu_j)}$$

$$\phi_2 \equiv \sum_{j\in\{H,U\}} \phi_{2j}, \quad \phi_{2j} \equiv \frac{\pi_j(1+\rho)}{(2+n-\mu_j)(2+\rho-\mu_j)}$$

Notes. Endogenous variables are $C^y_{j,t}$, $S_{j,t}$, $C^o_{j,t+1}$, r_t, w_t, k_{t+1}, and Z_t.

3 Private annuity markets

Now consider the introduction of a private annuity market in this economy. A (life) annuity is an asset which pays a stipulated return contingent upon survival of the annuitant. In order to compete with other investment products, annuities have to provide a rate of return which exceeds the market rate of interest to compensate for the risk of death. The annuity firm is willing to pay this additional return on savings as it is held free of any obligation after the annuitant has died. Conversely, an agent who sells an actuarial note gets a life-insured loan and will have her debts acquitted when she dies prematurely. Hence, an agent who exclusively uses the annuity market for financial transactions will never die indebted or leave an (unintended) bequest.

It is important to note that an annuity market constitutes just another type of redistribution scheme. Instead of channeling funds from the dead to the newly arrived young by means of lump sum transfers as above, an annuity market redistributes money from the dead to those that survive by offering a premium over and above the rental rate of capital. Therefore, the welfare effects of introducing a private annuity market are not a priori clear.

3.1 Separating equilibrium

We make the following assumptions about the private market for annuities:

(A1) The annuity market is perfectly competitive. There is a large number of risk neutral firms offering annuities to individuals, and firms can freely enter or exit the market.

(A2) Annuity firms do not use up any real resources.

As a consequence of these assumptions the expected profit of each annuity firm is zero. We start by considering the benchmark case in which health status is public information. The resulting equilibrium in the annuity market will be a separating equilibrium in which each health type receives its actuarially fair rate of return and achieves perfect insurance against life span uncertainty. There is market segmentation in the sense that there is a separate market for unhealthy and for healthy agents.

Let $A^p_{j,t}$ denote the private annuity holdings of an agent of health type $j \in \{H, U\}$. As the net rate of return on annuities exceeds the rental rate of capital all agents will completely annuitize their wealth. This is the famous result found by Yaari (1965).

An annuity firm sells annuities to agents of health type j that pay a net rate of return $r^p_{j,t+1}$ contingent upon survival of the annuitant. The firm invests the proceeds in the capital market, thus earning a rate of return r_{t+1}. Since some of its clients will die young and will subsequently lose their claim at an early stage, the annuity firm can redistribute their assets among the surviving clientele. The zero profit conditions for the annuity market are therefore given by:

$$(1 + r_{t+1})L_{j,t}A^p_{j,t} = (1 + r^p_{j,t+1})(1 - \mu_j)L_{j,t}A^p_{j,t}, \quad j \in \{H, U\}. \tag{17}$$

That is, the gross return earned on the amount of savings collected from clients of health type j in period t (left-hand side) should equal the (expected) claim originating from surviving clients of health type j in period $t+1$. It follows that:

$$1 + r^p_{j,t+1} = \frac{1 + r_{t+1}}{1 - \mu_j}, \quad j \in \{H, U\}. \tag{18}$$

Note that $1 + r^p_{U,t+1} > 1 + r^p_{H,t+1}$ since $\mu_U > \mu_H$, i.e. unhealthy agents earn a higher gross rate of return on their investments in the annuity market than do healthy agents.

3.2 Pooling equilibrium

The following assumptions are added to the ones above:

(A3) Health status is private information of the annuitant. The distribution of health types in the population and the corresponding mortality rates are common knowledge.

(A4) Annuitants can buy multiple annuities for different amounts and from different annuity firms. Individual annuity firms cannot monitor an annuitant's holdings with other firms.

Under these assumptions, the equilibrium in the annuity market is a pooling equilibrium. The existence of this pooling equilibrium depends critically on (A3): as annuity firms cannot distinguish between healthy and unhealthy agents they will offer a single rate of return which applies to both groups. They can exploit their knowledge about the distribution of health types in the population in setting the annuity rate. Note that if (A4) would not hold then annuity firms could indirectly deduce an agent's health type by the amount of wealth she has invested. Healthy individuals tend to save more, which exacerbates the degree of adverse selection in the annuity market.

Our conclusion that a pooling equilibrium will prevail in the annuity market appears opposite to that of Rothschild and Stiglitz (1976), who show that a pooling equilibrium does not exist in an insurance market. However, their result relies heavily on the assumption that customers can buy only one insurance contract such that the insurer sets both price and quantity. They admit themselves that this may be an objectionable assumption in some cases, and Walliser (2000) argues that it indeed does not apply to the annuity market. First of all, monitoring the receipts of annuity payments from other annuity firms would be very difficult. Secondly, in contrast to other types of insurance, the death of the annuitant ends the liability of the annuity firm instead of creating it. Hence, withholding payments and investigating compliance is not feasible. We therefore believe it to be justified to assume that the annuity firm can only control the rate of return on annuities and cannot set prices and quantities simultaneously in order to obtain full information revelation.

An implicit assumption in the model is that annuitants cannot credibly signal their health status to the market. In the absence of cheap and credible medical tests, this is not a strong assumption at all. When asked about her health type, each individual has a clear incentive to claim to be unhealthy in order to get the highest possible return in a separated market. However, firms know this and will therefore not believe the annuitant's claim. Hence, even though part of their clientele tells the truth, the fact that some have an incentive to lie is enough for the annuity firm to assume that everyone will be a fraud.

We now turn to the determination of the pooled annuity rate, denoted by r^p_{t+1}. This rate will apply to agents of both health types and does therefore not feature a subscript j. Under the assumption that both health types are net savers,[4] the zero profit condition for the annuity market is given by:

$$(1+r_{t+1})\left[L_{H,t}A^p_{H,t}+L_{U,t}A^p_{U,t}\right]=(1+r^p_{t+1})\left[(1-\mu_H)L_{H,t}A^p_{H,t}+(1-\mu_U)L_{U,t}A^p_{U,t}\right]. \quad (19)$$

As in the case of a separating equilibrium the gross return earned on the amount of savings collected from clients in period t (left-hand side) should equal the (expected) claim originating from surviving clients in period $t+1$. It follows that:

$$1+r^p_{t+1}=\frac{1+r_{t+1}}{1-\bar{\mu}^p_t}, \quad (20)$$

where $1-\bar{\mu}^p_t$ denotes the asset-weighted average survival rate of the healthy and unhealthy agents:

$$1-\bar{\mu}^p_t\equiv(1-\mu_H)\frac{L_{H,t}A^p_{H,t}}{L_{H,t}A^p_{H,t}+L_{U,t}A^p_{U,t}}+(1-\mu_U)\frac{L_{U,t}A^p_{U,t}}{L_{H,t}A^p_{H,t}+L_{U,t}A^p_{U,t}}. \quad (21)$$

This result has also been found in a partial equilibrium context by Sheshinski (2008) and relates to the linear equilibrium concept of Pauly (1974). As noted by Walliser (2000), it can alternatively be interpreted as a Nash equilibrium among annuity firms in which each firm that deviates from the zero profit price incurs a loss.

The pooling rate derived above provides a clear insight in the consequences of information asymmetry in the annuity market. In the absence of adverse selection the demand for annuities by any agent would be independent of her health status. In that case (20) changes to:

$$1+r^f_{t+1}=\frac{1+r_{t+1}}{1-\bar{\mu}^f}, \quad (22)$$

where the superscript f stands for 'actuarially fair' and $1-\bar{\mu}^f$ denotes the average survival rate among the cohort born in period t:

$$1-\bar{\mu}^f\equiv(1-\mu_H)\frac{L_{H,t}}{L_{H,t}+L_{U,t}}+(1-\mu_U)\frac{L_{U,t}}{L_{H,t}+L_{U,t}}. \quad (23)$$

However, this rate is not sustainable. As $1-\mu_U\leq 1-\bar{\mu}^f\leq 1-\mu_H$, healthy individuals will tend to overinvest in annuities while unhealthy individuals invest less than in a seperated market. This

[4] The pooling equilibrium is only valid when either both health types are net savers or both are net borrowers. However, as we model a closed economy the second option is not feasible.

cannot be an equilibrium as annuity firms will make a loss. They will adjust their expectations about the demand for annuities by members of both health groups. As a consequence of this information feedback, the annuity rate in the competitive equilibrium will reflect the adverse selection effect such that $1 + r^p_{t+1} \leq 1 + r^f_{t+1}$ for all t. Still, the healthy agents obtain a better than actuarially fair rate of return on their private annuity holdings. The presence of unhealthy agents in the market allows them to earn an informational rent.

The complete model with private annuities is given in Table 2.

3.3 Welfare analysis

We visualize the properties of the model by means of a simulation with realistic parameter values. The time dimension is important: we assume that one period lasts for 40 years and set the parameters accordingly. For example, in order to have an annual rate of depreciation of 6 percent we need to define $\delta = 1 - (1 - 0.06)^{40} = 0.9158$. The population grows at 1 percent per annum ($n = 0.49$) and consists of equal shares of healthy and unhealthy individuals ($\pi_H = \pi_U = 0.5$). Their respective morality rates are $\mu_H = 0.3$ and $\mu_U = 0.52$. The relative life expectancy $(2 - \mu_H)/(2 - \mu_U)$ then corresponds to the empirical estimate of Heijdra and Reijnders (2009).

The case without annuities and with transfers to the young (abbreviated as TY) is taken to be the benchmark. We parameterize the model in such a way as to control the steady-state values in this scenario. A steady-state annual return on capital of 6 percent ($\hat{r} = 9.2857$) is achieved by setting the annual rate of time preference at 3.25 percent ($\rho = 2.5995$). We normalize the long-run level of income per capita to unity such that the steady-state capital stock is $\hat{k} = 0.0294$ and the technology index equals $\Omega_0 = 2.8805$.

We assume that the economy is initially in the steady state of the model without annuities. At a given time t a private annuity market is introduced. The transition paths of the capital intensity, the wage rate, the gross return on investment, consumption during youth and old age, and the expected lifetime utility of both health types are derived by means of an iterative procedure.

The results are given in Figures 1 to 4 for both a separating equilibrium (SE) and a pooling equilibrium (PE) in the private annuity market. All quantities are scaled by their counterparts in the no annuities case. As such, a value in excess of unity indicates that the variable under consideration increases relative to the benchmark scenario, when it falls short of unity there is a decrease. The only exception is lifetime utility. As the logarithmic felicity function returns negative values for small consumption levels, the ordinal ranking of equilibria would be distorted when we divide by the benchmark utility level. Instead the difference is shown. Table 3 presents a numerical comparison between (unscaled) steady-state values.

Table 2: General model with private annuities

(a) Microeconomic relationships:

$$C^y_{j,t} = \frac{1+\rho}{2+\rho-\mu_j}\left[w_t + \frac{w_{t+1}}{1+r^p_{j,t+1}}\right] \tag{T2.1}$$

$$A^p_{j,t} = \frac{1-\mu_j}{2+\rho-\mu_j}w_t - \frac{1+\rho}{2+\rho-\mu_j}\frac{w_{t+1}}{1+r^p_{j,t+1}} \tag{T2.2}$$

$$C^o_{j,t+1} = \frac{(1-\mu_j)(1+r^p_{j,t+1})}{2+\rho-\mu_j}\left[w_t + \frac{w_{t+1}}{1+r^p_{j,t+1}}\right] \tag{T2.3}$$

(b) Macroeconomic relationships:

$$r_t + \delta = \varepsilon\Omega_0 k_t^{\varepsilon-1} \tag{T2.4}$$

$$w_t = (1-\varepsilon)\Omega_0 k_t^{\varepsilon} \tag{T2.5}$$

(c) Fundamental difference equation:

$$k_{t+1} = \phi_1 w_t - \phi_2 \frac{w_{t+1}}{1+r^p_{j,t+1}} \tag{T2.6}$$

$$\phi_1 \equiv \sum_{j\in\{H,U\}} \phi_{1j}, \quad \phi_{1j} \equiv \frac{\pi_j(1-\mu_j)}{(2+n-\mu_j)(2+\rho-\mu_j)}$$

$$\phi_2 \equiv \sum_{j\in\{H,U\}} \phi_{2j}, \quad \phi_{2j} \equiv \frac{\pi_j(1+\rho)}{(2+n-\mu_j)(2+\rho-\mu_j)}$$

(d) Private annuity market:

$$1+r^p_{j,t+1} = \begin{cases} \dfrac{1+r_{t+1}}{1-\mu_j} & \text{Separating equilibrium} \\ \dfrac{1+r_{t+1}}{1-\bar{\mu}^p_t} & \text{Pooling equilibrium} \end{cases} \tag{T2.7}$$

$$1-\bar{\mu}^p_t \equiv (1-\mu_H)\frac{L_{H,t}A^p_{H,t}}{L_{H,t}A^p_{H,t}+L_{U,t}A^p_{U,t}} + (1-\mu_U)\frac{L_{U,t}A^p_{U,t}}{L_{H,t}A^p_{H,t}+L_{U,t}A^p_{U,t}}$$

Notes. Endogenous variables are $C^y_{j,t}$, $A^p_{j,t}$, $C^o_{j,t+1}$, r_t, w_t, k_t, and $r^p_{j,t+1}$.

	TY	SE	PE	PE + SA*
$\hat{k}$	0.0294	0.0305	0.0296	0.0264
$\hat{r}$	9.2857	9.0398	9.2403	10.0759
$\hat{w}$	0.7000	0.7074	0.7013	0.6780
$\hat{C}_H^y$	0.7763	0.6335	0.6224	0.6060
$\hat{C}_U^y$	0.8180	0.6539	0.6558	0.6386
$\hat{C}_H^o$	0.5528	1.7669	2.0172	1.9493
$\hat{C}_U^o$	1.1241	1.8238	1.4603	1.4111
$E(\hat{\lambda}_H)$	-0.1676	-0.3458	-0.3377	-0.3710
$E(\hat{\lambda}_U)$	-0.1853	-0.3445	-0.3713	-0.4025

$^*\theta = 0.05$

Table 3: Welfare analysis

$k_t/k_t(\mathrm{TY})$

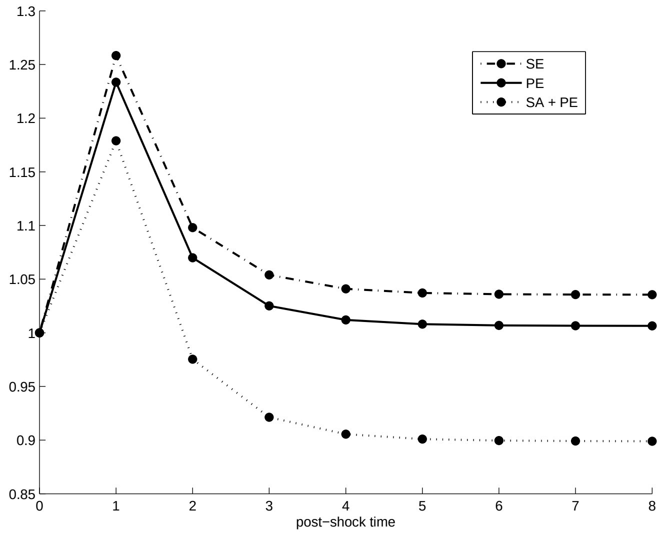


Figure 1: Capital intensity

(a) Wage rate $w_t/w_t(\mathrm{TY})$

(b) Gross return on investment $(1+r_t)/(1+r_t(\mathrm{TY}))$

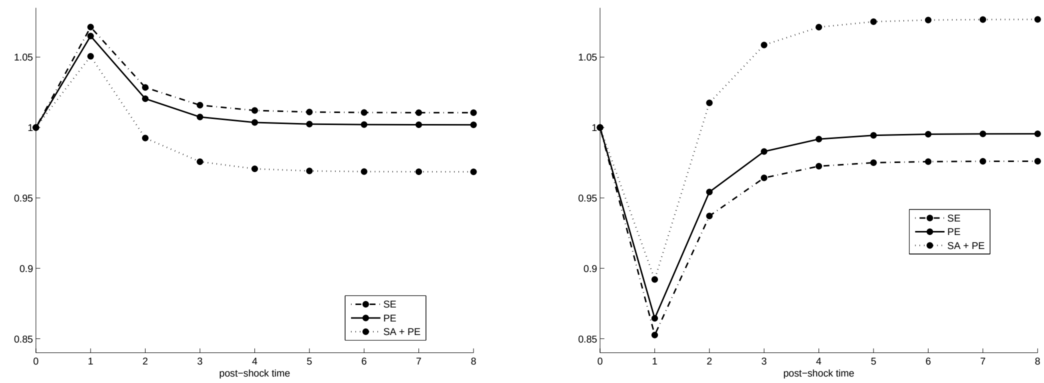
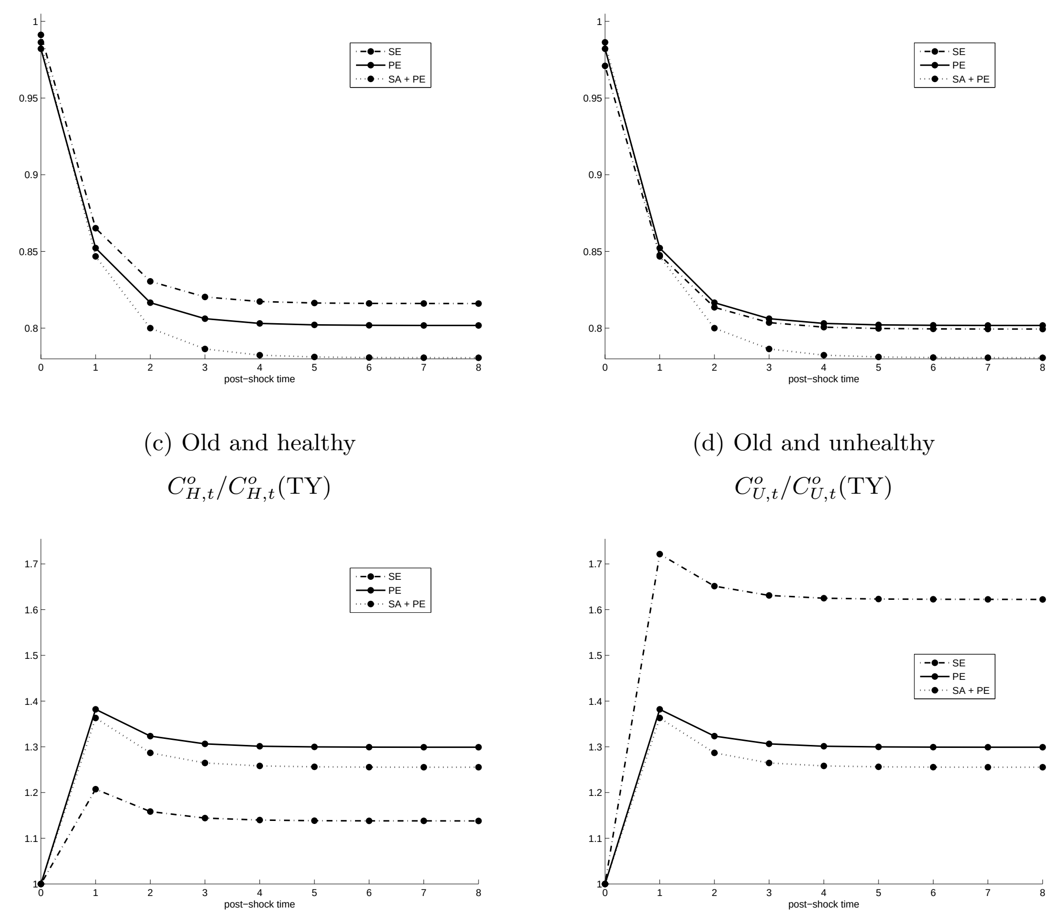


Figure 2: Wage rate and gross return

We find that the introduction of a private annuity market increases the capital intensity both during transition and in the long run. Consequently the wage rate increases and the interest rate drops relative to the benchkmark. The effects are more pronounced for the SE than for the PE. Agents save a larger part of their income in the first period and therefore consume less during youth and more during old age. Comparing the lifetime utility profiles we find that healthy agents prefer the PE to the SE, and vice versa for the unhealthy.

At the time the annuity market is opened up there are still accidental bequests in the economy which have been left by the previous generation. Hence, in the first period of the new regime young agents benefit from the higher return on their savings through full annuitization while also receiving the intergenerational transfer. Their expected utility level increases relative to the benchmark. However, from period $t + 1$ (i.e. post-shock time 1) onwards the transfers are abolished and both health types have a lower utility level under either equilibrium in the private annuity market than in the absence of annuities. This is an example of the so-called 'tragedy of annuitization', as described in Heijdra et al. (2010). Even though it is individually rational to fully annuitize (as the annuity rate of return exceeds the return on capital), this is not optimal from a social point of view. If all agents invests their financial wealth in the annuity market then the resulting long-run equilibrium leaves everyone worse off compared to the case where annuities are absent and accidental bequests are redistributed to the young. In other words, the introduction of an annuity market is welfare improving when considered in isolation, but not when the general equilibrium repercussions as a result of endogenous factor prices are taken into account.

As measures of utility are purely ordinal, the nominal difference in steady-state utility levels as



(a) Young and healthy $C^y_{H,t}/C^y_{H,t}(\mathrm{TY})$

(b) Young and unhealthy $C^y_{U,t}/C^y_{U,t}(\mathrm{TY})$

(c) Old and healthy $C^o_{H,t}/C^o_{H,t}(\mathrm{TY})$

(d) Old and unhealthy $C^o_{U,t}/C^o_{U,t}(\mathrm{TY})$

Figure 3: Consumption

(a) Healthy $E\left(\Lambda_{H,t}\right)-E\left(\Lambda_{H,t}(\mathrm{TY})\right)$ (b) Unhealthy $E\left(\Lambda_{U,t}\right)-E\left(\Lambda_{U,t}(\mathrm{TY})\right)$

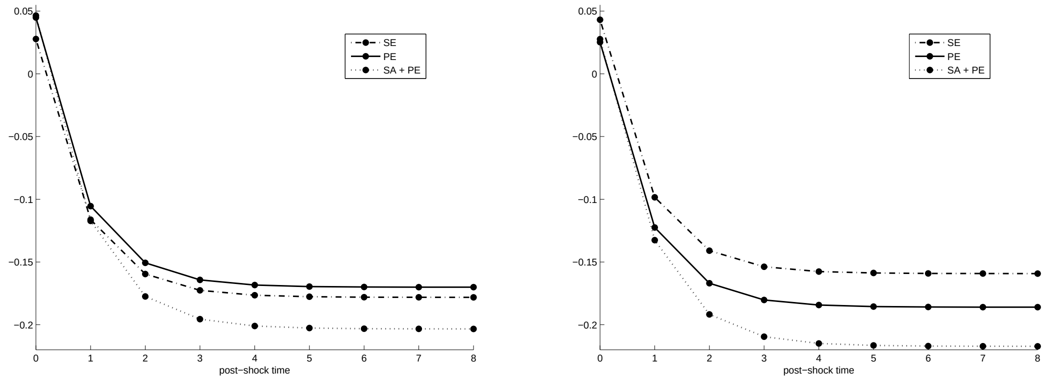


Figure 4: Expected lifetime utility

presented in Table 3 cannot be given a quantitative interpretation. In order to evaluate the magnitude of utility differences in a meaningful way we use the metric of equivalent variations. That is, we determine how much additional consumption during youth we would have to give to an agent of a given health type in one of the private annuity market scenarios in order to make her as well off as in the no annuities case, keeping all prices (i.e. the wage rate and the interest rate) and old-age consumption fixed at their initial level. That is, we find that value of Δ_j which makes the following equation true:

$$\ln(\hat{C}_j^y+\Delta_j)+\frac{1-\mu_j}{1+\rho}\ln\hat{C}_j^o=E(\hat{\Lambda}_j(TY)). \tag{24}$$

where $E(\hat{\Lambda}_j(TY))$ is the steady-state benchmark utility level.

The results are given in Table 4. The first two rows give the nominal value of Δ_H and Δ_U, the last two rows scale the result by steady-state consumption during youth for the benchmark. For example, to ensure that a healthy agent has the same level of utility when there is a pooled private annuity market as when there are no annuities at all we have to give her 0.1154 in terms of youth consumption. This corresponds to 14.87% of the consumption level she would have had in the absence of annuities. These numbers are quite significant, showing that the (negative) welfare effects of introducing a private annuity market in this economy are not inconsequential.

	SE	PE	PE + SA*
Δ_H	0.1236	0.1154	0.1367
Δ_U	0.1129	0.1341	0.1549
$\Delta_H/\hat{C}_H^y(TY)$	0.1592	0.1487	0.1761
$\Delta_U/\hat{C}_U^y(TY)$	0.1380	0.1639	0.1894

* $\theta = 0.05$

Table 4: Equivalent variations

4 Social annuities

Following Abel (1986) we now study the consequences of introducing mandatory social annuities. These can be interpreted, for example, as a fully funded pension system. Agents contribute a fixed amount A_t^s during their youth and receive a benefit $(1 + r_{t+1}^s)A_t^s$ when they survive into the second phase of life, where r_{t+1}^s is the (implicit) net return on social annuities. All contributions are pooled and invested in the capital market. The gross return is then divided equally among the survivors. This implies that the following resource constraint should be satisfied:

$$(1 + r_{t+1})A_t^s[L_{H,t} + L_{U,t}] = (1 + r_{t+1}^s)A_t^s[(1 - \mu_H)L_{H,t} + (1 - \mu_U)L_{U,t}]. \tag{25}$$

Solving for the implied rate of return yields $r_{t+1}^s = r_{t+1}^f$, the actuarially fair pooling rate as defined in (23):

$$1 + r_{t+1}^s = \frac{1 + r_{t+1}}{1 - \bar{\mu}^s}, \quad 1 - \bar{\mu}^s \equiv (1 - \mu_H)\frac{L_{H,t}}{L_{H,t} + L_{U,t}} + (1 - \mu_U)\frac{L_{U,t}}{L_{H,t} + L_{U,t}}. \tag{26}$$

Intuitively, as contributions are mandatory and independent of health type, the social annuity system is immune to adverse selection. This makes the introduction of social annuities appear like an attractive option for the government: it can offer agents a higher rate of return on part of their savings than the private market can.

We assume that the agent can invest her remaining financial wealth in a pooled private annuity market. The lifetime budget constraint can be written as:

$$C_{j,t}^y + \frac{C_{j,t+1}^o}{1 + r_{t+1}^p} = w_t + \frac{w_{t+1}}{1 + r_{t+1}^p} + \frac{\bar{\mu}^s - \bar{\mu}_t^p}{1 - \bar{\mu}^s}A_t^s. \tag{27}$$

Note that, ceteris paribus, the introduction of social annuities has a positive effect on the level of total human wealth as $\bar{\mu}^s \geq \bar{\mu}_t^p$ for all t due to adverse selection in the private annuity market.

The agent's optimal plans are fully characterized by:

$$C^y_{j,t} = \frac{1+\rho}{2+\rho-\mu_j}\left[w_t + \frac{w_{t+1}}{1+r^p_{t+1}} + \frac{\bar{\mu}^s - \bar{\mu}^p_t}{1-\bar{\mu}^s}A^s_t\right], \tag{28}$$

$$A^p_{j,t} + A^s_t = \frac{1-\mu_j}{2+\rho-\mu_j}w_t - \frac{1+\rho}{2+\rho-\mu_j}\left[\frac{w_{t+1}}{1+r^p_{t+1}} + \frac{\bar{\mu}^s - \bar{\mu}^p_t}{1-\bar{\mu}^s}A^s_t\right], \tag{29}$$

$$C^o_{j,t+1} = \frac{(1-\mu_j)(1+r^p_{t+1})}{2+\rho-\mu_j}\left[w_t + \frac{w_{t+1}}{1+r^p_{t+1}} + \frac{\bar{\mu}^s - \bar{\mu}^p_t}{1-\bar{\mu}^s}A^s_t\right]. \tag{30}$$

Compared to the model without social annuities given in Table 2 there are a few noteworthy differences. Consumption during youth and old age are still proportional to human wealth, but the definition of human wealth has been augmented by a term reflecting the relative return on social annuities. From (29) we see that private annuity demand has been replaced by total annuity demand, the sum of private and social annuity holdings. As such, mandatory social annuities do indeed crowd out their private counterparts. Keeping everything else constant, this crowding out is more than one-for-one as the higher level of total income in the second period reduces the incentive for private saving.

We assume that the compulsory contribution to the social annuity system is proportional to wage income during youth, i.e. $A^s_t = \theta w_t$ for $\theta \in [0, 1]$. Note that for high values of θ agents might be overannuitized. That is, they could be forced by the social security system to annuitize a larger portion of their financial wealth than they would voluntarily do. However, we abstain from this scenario in this paper since in real life we observe that most people are still net savers despite the social security premiums they pay each month.

The model with social annuities is summarized in Table 5.

4.1 Welfare analysis

The transition paths of the per capita capital stock, wages, the return on capital, consumption, and lifetime utility after introducing both private and social annuities in the economy (denoted by SA + PE) are given in Figures 1-4 for the case that social security contributions amount to 5% of wage income. We find that, compared to the benchmark of no annuities and transfers to the young, the level of capital intensity increases for one period but afterwards decreases sharply. For period $t + 2$ onwards this results in a decrease in the wage rate and a rise in the interest rate, as shown in Figure 2(a)-(b), respectively. Note that these effects are exactly opposite to those following the introduction of a private annuity market only. Youth consumption drops while consumption during old age goes up. The overall effect on lifetime utility is negative for both health types, except for the impact period in which the transfers have not yet been abolished.

In order to evaluate the magnitude of the relative welfare decrease in the steady state we calculate

Table 5: General model with private and social annuities

(a) Microeconomic relationships:

$$C_{j,t}^{y} = \frac{1+\rho}{2+\rho-\mu_j}\left[w_t + \frac{w_{t+1}}{1+r_{t+1}^{p}} + \frac{\bar{\mu}^{s}-\bar{\mu}_t^{p}}{1-\bar{\mu}^{s}}A_t^{s}\right] \tag{T5.1}$$

$$A_{j,t}^{p} + A_t^{s} = \frac{1-\mu_j}{2+\rho-\mu_j}w_t - \frac{1+\rho}{2+\rho-\mu_j}\left[\frac{w_{t+1}}{1+r_{t+1}^{p}} + \frac{\bar{\mu}^{s}-\bar{\mu}_t^{p}}{1-\bar{\mu}^{s}}A_t^{s}\right] \tag{T5.2}$$

$$C_{j,t+1}^{o} = \frac{(1-\mu_j)(1+r_{t+1}^{p})}{2+\rho-\mu_j}\left[w_t + \frac{w_{t+1}}{1+r_{t+1}^{p}} + \frac{\bar{\mu}^{s}-\bar{\mu}_t^{p}}{1-\bar{\mu}^{s}}A_t^{s}\right] \tag{T5.3}$$

(b) Macroeconomic relationships:

$$r_t + \delta = \varepsilon\Omega_0 k_t^{\varepsilon-1} \tag{T5.4}$$

$$w_t = (1-\varepsilon)\Omega_0 k_t^{\varepsilon} \tag{T5.5}$$

(c) Fundamental difference equation:

$$k_{t+1} = \phi_1 w_t - \phi_2\left[\frac{w_{t+1}}{1+r_{t+1}^{p}} + \frac{\bar{\mu}^{s}-\bar{\mu}_t^{p}}{1-\bar{\mu}^{s}}A_t^{s}\right] \tag{T5.6}$$

$$\phi_1 \equiv \sum_{j\in\{H,U\}}\phi_{1j}, \quad \phi_{1j} \equiv \frac{\pi_j(1-\mu_j)}{(2+n-\mu_j)(2+\rho-\mu_j)}$$

$$\phi_2 \equiv \sum_{j\in\{H,U\}}\phi_{2j}, \quad \phi_{2j} \equiv \frac{\pi_j(1+\rho)}{(2+n-\mu_j)(2+\rho-\mu_j)}$$

(d) Private annuity market:

$$1 + r_{t+1}^{p} = \frac{1+r_{t+1}}{1-\bar{\mu}_t^{p}} \tag{T5.7}$$

$$1-\bar{\mu}_t^{p} \equiv (1-\mu_H)\frac{L_{H,t}A_{H,t}^{p}}{L_{H,t}A_{H,t}^{p}+L_{U,t}A_{U,t}^{p}} + (1-\mu_U)\frac{L_{U,t}A_{U,t}^{p}}{L_{H,t}A_{H,t}^{p}+L_{U,t}A_{U,t}^{p}}$$

(e) Mandatory social annuities:

$$A_t^{s} = \theta w_t \tag{T5.8}$$

$$1 + r_{t+1}^{s} = \frac{1+r_{t+1}}{1-\bar{\mu}^{s}} \tag{T5.9}$$

$$1-\bar{\mu}^{s} \equiv (1-\mu_H)\frac{L_{H,t}}{L_{H,t}+L_{U,t}} + (1-\mu_U)\frac{L_{U,t}}{L_{H,t}+L_{U,t}}$$

Notes. Endogenous variables are $C_{j,t}^{y}$, $A_{j,t}^{p}$, $C_{j,t+1}^{o}$, r_t, w_t, k_t, r_{t+1}^{p}, A_t^{s} and r_{t+1}^{s}.

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the equivalent variations for three different values of θ, see Table 6 and Section 3.3 above. Note that $\theta = 0$ is not given but corresponds to the pooling equilibrium as shown in Table 4. We find that the equivalent variations increase monotonically in θ for both health types, implying that the equilibrium becomes progressively worse in welfare terms when the contribution to the social annuity system increases. This is somewhat counterintuitive, as social annuities offer agents a higher rate of return on a fixed part of their savings than private annuities do, keeping everything else constant. However, it is exactly this ceteris paribus condition which is misleading.

	$\theta = 0.01$	$\theta = 0.03$	$\theta = 0.05$
Δ_H	0.1170	0.1223	0.1367
Δ_U	0.1357	0.1409	0.1549
$\Delta_H/\hat{C}^y_H(TY)$	0.1508	0.1576	0.1761
$\Delta_U/\hat{C}^y_U(TY)$	0.1659	0.1722	0.1894

Table 6: Equivalent variations with social annuities

Importantly, the introduction of social annuities has two opposing effects on the rate of return on private annuities. First of all, there is a partial equilibrium effect. The mandatory investment in social annuities results in a decrease in the demand for private annuities. Unhealthy agents reduce their private annuity demand disproportionally more than healthy agents do. As a consequence, the asset share of healthy agents increases and thereby the degree of adverse selection in the private annuity market. This leads to a decrease in the rate of return on private annuities, in line with the findings of by Abel (1986), Walliser (2000), and Palmon & Spivak (2007).

However, there is also a general equilibrium effect which partly offsets the decrease. The total level of savings decreases in the steady state, leading to a rise in the rental rate of capital relative to the scenario without social annuities, as is apparent from Figure 2(b). This response is not taken into consideration when the focus is restricted to partial equilibrium analysis or when the interest rate is assumed to be exogenously fixed.

The net effect on the private annuity rate is negative in all our simulations. In Figure 5 we show the actuarially fair gross annuity rate $1+r^s_t$ and the pooled private annuity rate $1+r^p_t$ when social annuities are either completely absent or the contribution rate equals 0.05. The private annuity rate is clearly lower in the latter case, both during transition and in the steady state. Overall, we find that in the long run the negative effects of introducing social annuities (a lower wage rate and a lower return on private annuities) outweigh the positive effect (an actuarially fair rate of return on a part of financial wealth) as is evident from the drop in welfare of future generations.

$r_t^p,\ r_t^s$

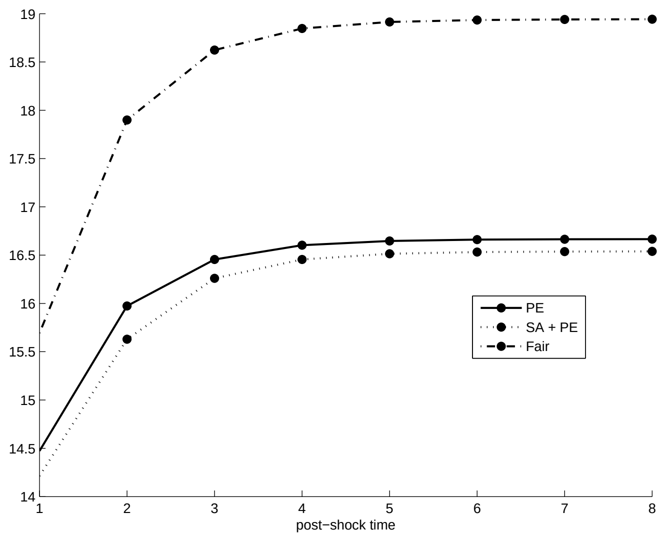

Figure 5: Annuity rate of return

5 Conclusion

In this paper we have constructed a discrete-time general equilibrium model featuring exogenous growth and overlapping generations of heterogeneous agents who are distinguished by their health status. An agent's health type is assumed to be private information. We show that if a private annuity market is introduced in this economy then it will be characterized by a pooling equilibrium. Due to adverse selection, the resulting annuity rate of return is less than the actuarially fair pooling rate. Even though it is individually optimal to invest in annuities, agents of both health types are in the long run worse off in welfare terms compared to the case where annuities are absent and accidental bequests are redistributed to the young.

We have also studied the welfare implications of a social security system with fixed mandatory contributions for both health types. These social annuities are immune to adverse selection and therefore offer a higher rate of return than private annuities do. However, they have a negative effect on steady-state welfare. The positive effect of a fair rate of return on a fixed part of savings and a higher return on capital in equilibrium is outweighed by the negative consequences of increased adverse selection in the private annuity market and a lower wage rate.

Our results suggest that privatization of social security may be welfare improving when annuity markets are characterized by asymmetric information. Elimination of social annuities reduces the degree of adverse selection in the private annuity market, increases the level of savings, and has a positive effect on lifetime utility for both health groups in the population. An even larger welfare gain could materialize when the government would decide to implement a redistribution scheme which transfers accidental bequests to the newly arrived young agents.

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