Optimal Portfolio Choice with Longevity and Health Insurance: A Developing Country Context*

Draft - Please do not distribute without authors' consent

Cheng Wan[†], Hazel Bateman[‡] and Katja Hanewald[§]

January 6, 2021

Abstract

We derive the optimal portfolio for retirees in a developing country context facing uncertain lifespan, catastrophic medical expenditures, and long-term care costs. In the model, retirees can choose from a portfolio consisting of life annuities, critical illness insurance, long-term care insurance, and bonds. The model is calibrated with Chinese data. We find that in emerging economies like China, an optimal portfolio highly depends on a retiree's economic background. For retirees with an average pension, we find a substantial demand for critical illness insurance. An annuity is most important for those with a low pension. The demand for long-term care insurance is small. Choice of a lower health investment provides the larger welfare compared with an adequate health investment, and it reduces the demand for critical illness insurance. These optimal insurance amounts for retirees with different economic backgrounds can inform governments and insurers to design insurance system and to advise retirement planning. Our research also provides insights for governments and individuals to decide which insurance to expand or to purchase under budget constraints. Finally, our study suggests that bundling longevity and health-related insurance products is one way to achieve a lower price for insurers facing tight regulations and it can increase annuity demand for more affluent retirees.

Keywords: Annuities, long-term care insurance, critical illness insurance, life-cycle modelling, China, developing countries

JEL Codes: D14, G11, J14, J32

---

*The authors acknowledge the financial support from the ARC Centre of Excellence in Population Ageing Research (CEPAR) - CE17010005. This research includes computations using the computational cluster Katana supported by Research Technology Services at UNSW Sydney.

[†]Corresponding author. ARC Centre of Excellence in Population Ageing Research (CEPAR), UNSW Sydney, Sydney NSW 2052, Australia; E-mail: cheng.wan@student.unsw.edu.au.

[‡]School of Risk & Actuarial Studies, UNSW Sydney, and CEPAR.

[§]School of Risk & Actuarial Studies, UNSW Sydney, and CEPAR.

1 Introduction

Ageing populations in both developed and developing countries put pressure on the sustainability of public pension and healthcare systems. Uncertainties due to the COVID-19 pandemic bring about additional challenges to retirement planning and increase the awareness of general health risks. Individuals are required to take more responsibilities to manage retirement risks, and can allocate their retirement savings to a range of financial products. Life annuities have been identified as the appropriate product to address longevity risk (Yaari, 1965), yet voluntary annuity markets are thin worldwide. A growing literature investigates behavioural and rational explanations for the lack of interest in life annuities.[1] However, Davidoff et al. (2005) prove that Yaari's result is still robust, considering many of these explanations. One key component missing in Yaari's model is that the impact of health shocks is ignored. The mortality development is assumed to be deterministic given an initial health state, and the probability of death acts only as a weight parameter to discount utility at each age. Sudden reductions in expected life expectancy are not allowed nor large out-of-pocket (OOP) expenditures related to health risks. These health risks are typically covered by national health insurance programs, although coverage differs between the developed and developing world. Long-term care financing remains a gap worldwide. In developing countries, the mortality and monetary shocks due to dreadful diseases are substantial. Critical illness insurance that provides a lump sum benefit is potentially an important component in retirement planning.

In this study, we develop a life-cycle model where a retiree faces the risk of critical illness, long-term care term dependence, and uncertain longevity, as well as stochastic medical and care expenditures. We also consider a health-state dependent utility function under which the utility of non-medical consumption depends on the retiree's current health state. The retiree chooses consumption and allocates his retirement wealth among a portfolio of life annuities, critical illness insurance, long-term care insurance, and a savings account to maximise his expected lifetime utility. We show the optimal portfolios for retirees in developing countries with different economic profiles and health investment strategies.

We set the individual in China and model China's social insurance in retirement.[2] We select China as an example of a rapidly ageing country with a less well-developed social insurance system.[3] Individuals in China face the risk of catastrophic medical expenditures due to a limited public healthcare system (Meng et al., 2012; Hou et al., 2014; Zhang et al., 2017). There is also a large economic inequality among older Chinese (e.g., Hanewald et al., 2019). We use China's official mortality curves and disease incidence curves,

[1] Benartzi et al. (2011) and Brown (2009) provide excellent summaries explaining the role and problems of life annuities in retirement planning.

[2] Section 2 provides a short description of China's retirement system.

[3] Several studies make suggestions for reforms of the social security system in China: the pension system (Barr and Diamond, 2010; Lu et al., 2014), the healthcare system (Sun et al., 2017), and the long-term care system (Lu et al., 2015; Yang et al., 2016).

as well as estimated long-term care transitions from the China Health and Retirement Longitudinal Study (CHARLS) to calibrate the health transition processes.

We are the first to consider life annuities, critical illness insurance, and long-term care insurance together in a life-cycle framework for retirement planning. Our study is closely related to research on the role of uncertain health cost risks on consumption behaviour and portfolio selection in retirement (e.g., Pang and Warshawsky, 2010; Yogo, 2016; Koijen et al., 2016; Peijnenburg et al., 2017; and Ameriks et al., 2020), and our results provide a reference for studies that elicit stated preferences for retirement portfolio allocation (e.g., Wu et al., 2017; Wan et al., 2020). We also build on the literature focusing on stochastic mortality developments and the correlated health costs in explaining the annuity puzzle (e.g., Ai et al., 2017; Reichling and Smetters, 2015).

We find that in emerging economies like China, the optimal portfolio highly depends on a retiree's economic background. Assuming retirees seek adequate medical services to treat critical illnesses, for those with an average pension of ¥3,000 (USD 450) per month, the optimal portfolio includes a substantial amount of critical illness insurance. Retirees with a low pension of ¥1,000 (USD 150) per month should annuitise almost half of their wealth if they have high retirement savings of ¥1 million (USD 150,000), and annuitise fully if they have low retirement savings of ¥150,000 (USD 22,500). There is a small demand for long-term care insurance for retirees with the four pension and savings amounts mentioned above. Allowing for choices of lower health investment reduces the demand for critical illness insurance, and a lower than adequate health investment strategy is optimal across all economic backgrounds considered in this analysis. These optimal portfolios predicted for retirees with different economic backgrounds provide substantial welfare gains compared with no private insurance (the status quo situation for retirees in China). The welfare gains with the optimal portfolios range from approximately 10% to more than 100% of the initial retirement savings for portfolio allocation, and the gain is higher for retirees with less wealth or pension income.

An important contribution is that we model the impact of health investments on optimal portfolio choice. This relates our study to the literature on health-state dependent utility of consumption, i.e., whether the utility of consumption is higher or lower in a poorer health state (e.g., Viscusi and Evans, 1990; Edwards, 2008; Finkelstein et al., 2013; Tengstam, 2014; Gyrd-Hansen, 2017; Gerking et al., 2017). We allow the consumption of medical services, i.e., health investment, to impact post-illness mortality developments, which indirectly affects the lifetime utility, and we model health-state dependent utilities of non-medical consumption. Specifically, we are the first to consider both a lower and a higher weight on the utility of consumption simultaneously when health deteriorates in a life-cycle model. We find that annuity demand increases from zero to at least 10% of the initial retirement savings for the wealthiest retirees when health-state dependent utility of consumption is considered, regardless of whether a higher or lower weight is assumed in poorer health states. For retirees with more pension or savings, higher weights on the utility of consumption in the poorer health states often result in more demand for health-related insurance, while lower weights often decrease it. For retirees with less pension or savings, the optimal portfolios are mainly driven by their financial constraints and considering the impact of health state on utility of consumption slightly modifies the results.

In developing countries like China, insurance industry can face tight regulations of pricing to avoid solvency risks, and there is a large economic variation among individual retirees as well as local governments. Our research finds that an optimal portfolio highly depends on a retiree's economic background. The optimal insurance amounts for retirees with different economic backgrounds provide a reference for governments to develop insurance system and for insurers to advise retirement planning. Our research also provides insights for governments and individuals to decide which insurance to expand or to purchase under budget constraints. Finally, our study suggests that bundling longevity and health-related insurance products is one way to achieve a lower price for insurers facing tight regulations, and it can increase annuity demand for more affluent retirees. These findings provide insights for policymakers in developing countries to consider how to expand their social insurance system for insurance companies to develop long-term insurance products.

Our paper is organised as follows. In Section 2, we provide a background of China's retirement insurance system. In Section 3, we introduce the life-cycle model of consumption and portfolio allocation in retirement and explain how we calibrate the parameters. In Section 4, we present the benchmark results for retirees with different economic backgrounds and examine the impact of health investment strategies. In Section 5, we illustrate the sensitivity of the results to changes in key model parameters. Section 6 discusses the practical implications for the policymakers and insurance industry to design retirement insurance products in a developing country context. Section 7 concludes.

2 Background of China's retirement insurance system

This section provides a brief background of China's public and private insurance for retirement. For detailed information about the history and reform of China's social insurance, please refer to Fang and Feng (2018) for the pension system and Sun et al. (2017) for the healthcare system.

2.1 Pension

2.1.1 Public pension system

In 2020, China's social pension mainly had two systems, the Basic Old Age Insurance that unifies the Government Pension (covering employees in China's government and related institutes) and the Employee

Pension (covering employees in enterprises), and the Resident Pension that unifies the Urban Resident Pension (covering residents in urban areas) and the New Rural Resident Pension (covering residents in rural areas).[4] The Government Pension provides the most generous income while the Resident Pension provides the least support. Zhu and Walker (2018) estimated that the average monthly income from the Government Pension was ¥2,894 while that from the Urban Resident Pension was only ¥267.

The Employee Pension covers most of the working population in China. The average pension payment of the Employee Pension was ¥2,496 per month (Zhu and Walker, 2018). The pension amount is adjusted annually according to a growth rate published by the Ministry of Human Resources and Social Security of China (MHRSS). The annual growth rate was about 10% in 2005 and gradually slowed down to 5% in 2020. However, there is no mechanism to guarantee that China's social pension growth can cover inflation in the future. In more developed cities like Beijing or Shanghai, the average Employee Pension per month is around ¥4,000 (USD 600), while in less developed provinces, the average pension amount is half of that, although the cost of living is also lower. The replacement rate of the Employee Pension has fallen since 1995 from over 80% to around 40% (Zhu and Walker, 2018).

2.1.2 Private annuity

A large number of private annuity products are offered in China, but the annuity ownership is less than 5%, according to the CHARLS survey conducted in 2015. This is partly caused by the regulation that requires using a low discount rate for pricing long-term insurance products and a conservative official mortality table. Among the private annuity holders, 76% were farmers and the self-employed, and 17% were enterprise employees (Zhu and Walker, 2018).

2.2 Health insurance

2.2.1 Public health insurance

After a series of reforms, China had achieved nearly universal health insurance by 2011 (Yip et al., 2012). Currently, the main components of China's public health insurance are the Employee Medical Insurance and the Resident Medical Insurance.[5] Both programs provide basic coverage for major diseases and the Resident Medical Insurance has a lower reimbursement rate compared to the Employee Medical Insurance due to a lower contribution by its participants. Their capacities to reduce financial strains on families caused by critical illnesses are limited: Public health insurance often exclude innovative ex- pensive imported drugs

[4] Since 2014, the New Rural Resident Pension was merged into the Urban Resident Pension, and since 2015 the Government Pension was merged into the Employee Pension (Fang and Feng, 2018). However, at least eight types of social pension exist in China due to a series of reforms (Zhu and Walker (2018)).

[5] Since 2016, China started to integrate the Urban Resident Basic Medical Insurance and the New Rural Cooperative Medical Scheme into the Resident Medical Insurance (Pan et al., 2016).

and medical treatments;[6] Retirees must pay by themselves if they prefer to access advanced medical services in cities where they are not participants of the local insurance programs.[7] Past studies have suggested using private health insurance to finance the medical services that are not covered by the public health insurance (e.g., Li et al., 2013a). For example, China's government recently started to develop supplemental insurance for large medical expenditures. Its management has been outsourced to insurance companies (Liu et al., 2017). Overall, the reimbursement rate for large critical illness medical expenditure is approximately 50% (Zhu et al., 2016). A vast regional difference of economic and healthcare developments has caused patients with more serious illnesses tend to spend more to receive medical treatments in cities like Beijing for their high-quality resources while other patients may prefer to receive average medical treatments from nearby hospitals. [8]

In China, long-term care services are mainly provided by family members (Zhen et al., 2015). China started the first social long-term care insurance pilot program in 2012 (Lu et al., 2015) and there were 29 pilot programs by September 2020. The requirements to receive long-term care services vary and the schemes are heavily funded by local governments. The government guidance emphasises the use of the employee insurance funds; however, it is unlikely to be sustainable (Yin et al., 2019).

2.2.2 Private health insurance

China's private health insurance is growing fast, and critical illness insurance is the most popular health insurance product.[9] However, products specifically designed for the old are still rare. Critical illness insurance products available for the old often have an age limit. Long-term products for medical insurance or care insurance almost do not exist.[10] The overall ownership of private health insurance is less than 5%, according to the CHARLS survey conducted in 2015.[11]

[6] The more expensive drugs or treatments are often not included in the National Essential Medicine Scheme or the local Reimbursement Drug list, which is a catalogue of drugs and treatments that are priced at the manufacturer's cost and has higher reimbursement rates available (Liu et al., 2017; Li et al., 2013c).

[7] A policy in 2015 aimed to reduce such friction to allow public health insurance usable in other cities, but the actual implementation state varies.

[8] For instance, the average inpatient care cost of cancer was tripled in Beijing than that in inland provinces like Anhui, Chongqing or Gansu (Yin et al., 2019; Sun et al., 2018).

[9] According to Ernst & Young, in 2017, the premium of critical illness insurance accounted for approximately 57% of the total premium of the health insurance market in China. Medical insurance accounted for 32%, and care insurance accounted for 111%. See at https://www.ey.com/Publication/vwLUAssets/ey-white-paper-on-china-commercial-health-insurance/$File/ey-white-paper-on-china-commercial-health-insurance.pdf.

[10] A new type of "Million RMB Coverage" medical insurance has become popular since 2016. However, it is mostly a one-year product and often sets an age limit. Since October 2020, the government of Beijing has worked with an insurance company to provide a long-term supplementary medical insurance to reduce catastrophic medical expenditures and the elderly is covered. However, such kind of insurance is still rare in China.

[11] Aside from the traditional insurance channels, a popular "insurance" that emerged recently is a crowd-funded mutual aid program named "Xiang Hu Bao", which aims to provide cover for critical illness. However, this program also has an age limit of 60.

2.3 Summary

China's public insurance for retirement provides universal but basic coverage, and the public long-term care insurance is in a pilot phase and is not widely available in most cities. China's private insurance market offers annuities, medical insurance, critical illness insurance, and long-term care insurance. However, many products have an age limit to purchase or lack of a long-term contract. Overall, the ownership of private insurance is low. Recent insurance market data has shown a growing demand for health insurance, especially for disease-related products. The COVID-19 pandemic has further increased the general awareness of health risks, especially for the elderly (Wan et al., 2020)). There is support from a policy level to advocate innovations to create retirement insurance products.

3 Model

This section describes the individual decision framework in the post-retirement phase. An individual is assumed to have public insurance and retirement savings. The individual faces the risk of critical illness, long-term care term dependence, and uncertain longevity and uncertain health-related cost, and he can choose from a life annuity, critical illness insurance, and long-term care insurance to maximise his expected lifetime utility of retirement consumption. In this section, we first describe the insurance products and the financial market. Next, we describe the health transition models and the health-related cost functions. Finally, we describe the life-cycle model and an individual's optimisation problem.

3.1 Insurance products and financial market

At retirement, an individual can purchase a life annuity, critical illness insurance, and long-term care insurance. The individual is not allowed to sell the insurance or to purchase more in the future.[12] The life annuity pays a fixed amount of real income Annuity_t at each period t until death. The critical illness insurance provides a real lump sum payment CII_t when the insured becomes critically ill for the first time at period t. The long-term care insurance provides a fixed amount of real income LTCI_t each period in which the insured requires long-term care, i.e., the insured cannot independently perform three or more of the following six activities of daily living (ADLs): bathing, dressing, eating, toileting, continence, transferring in and out of bed.[13] Both the annuity and the critical illness insurance are priced in an actuarially fair way according to age and gender using official tables in China.[14] However, there are no official tables for long-term care

[12] This is because: a) the health-related long-term insurance such as critical illness insurance and long-term care insurance usually are not available for purchase after age 60 in China, and b) delayed annuitisation is rare in developed countries.

[13] In China, failing to perform at least three ADLs is usually required to qualify for private long-term care insurance payments.

[14] We use the mortality table for pension business provided by the China Bank and Insurance and Regulatory Commission (CBIRC) to price annuity, and we use the CBIRC table for critical illness incidence rate and the CBIRC mortality table for

insurance, and we have based our estimates on the CHARLS data.[15] The pricing details for long-term care insurance are described in Appendix A. In addition, we assume a 15% insurance loading for all insurance products, which is consistent with Mitchell et al. (1999) for longevity insurance and Karaca-Mandic et al. (2011) for health insurance. We investigate the impact of different pricing assumptions in Section 5.

The remaining retirement savings after insurance purchase is held in a savings account that earns a risk-free rate. We do not consider investment in stocks because stock market participation is very low in China (Chen and Ji, 2017), particularly among the elderly (Zheng et al., 2019).

Throughout the paper, we assume a real interest rate of 2% per year, and set the real discount rate for insurance pricing as 1.5%, in line with insurance regulation in China.[16]

3.2 Health transitions models

In our model, the health status of the individual is assumed to be objective and verifiable by medical underwriting, and we do not assume any residual subjective health information that can influence the demand for insurance. In each period t, an individual's health state is one of the four states $H_t \in \{1, 2, 3, 4\}$, where $H_t = 1$ corresponds to healthy, $H_t = 2$ corresponds to critically ill, $H_t = 3$ corresponds to long-term care dependent, and $H_t = 4$ corresponds to death. Following Yogo (2016) and Koijen et al. (2016), we also use a Markov process to model the health evolution over time. We denote the 4-by-4 transition probability matrix at period t by $\boldsymbol{P}_t$, where its element $\pi_t(i, j)$ denotes the transition probability from the health state $H_t = i$ in period t to the health state $H_{t+1} = j$ in period $t + 1$:

$$\pi_t(i, j) = \text{Prob}(H_{t+1} = j | H_t = i). \tag{1}$$

This stylised health model allows to study the impact of three key risks faced by retirees: longevity, critical illness, and long-term care risk. In our benchmark health transition model, we assume a) there is no recovery to the healthy state from the critically ill or the long-term care state; b) there are transitions between the critically ill and the long-term care state, and we assume these transition probabilities are the same as those from the healthy state to the critically ill or the long-term care state.

The transition probabilities are calibrated using official tables and estimates based on the CHARLS data.[17] Table 1 summarises the calibration method for each health transition. For example, the transition

health insurance business to price critical illness insurance.

[15] The insurance industry in China prices annuity and critical illness insurance separately based on regulated official tables. For long-term care insurance, both the census data and the longitudinal surveys like the CHARLS have been used to estimate the health transitions relevant for pricing. However, the definitions of long-term care state are not consistent.

[16] The CBIRC requires that the nominal pricing interest rate cannot be set above 3.5% for long-term products and we assume an inflation rate of 2% based on a 10-year average of China's national CPI.

[17] A better approach is to estimate the full transition model based on the same data. However, longitudinal surveys like the CHARLS do not provide sufficient information to accurately identify the state of being critically ill.

probability from healthy to critically ill at each age is calibrated by the critical illness incidence rates provided by CBIRC, and the probability of death from the healthy state is calibrated by the CBRIC mortality curve for pension business adjusted to exclude the deaths from being critically ill or in the long-term care state. Long-term care transitions are calibrated with the CHARLS data. A multinomial logit model has been selected based on the Akaike Information Criterion to estimate the transition rates for the male data.[18] The technical details are provided in Appendix A.

Table 1: Calibration methods for health transition probability matrix

	Healthy	Critically ill	Long-term care	Death
Healthy	1-rest	CBIRC incidence rates	CHARLS estimates	Adjusted CBIRC mortality rates for pension business
Critically ill	0	1-rest	CHARLS estimates	Adjusted from CBIRC incidence rates and kx CBIRC mortality rates
Long-term care	0	CBIRC incidence rates	1-rest	CHARLS estimates
Death	0	0	0	1

To investigate the impact of health investment on optimal portfolio, we further extend the health transition probability matrix to allow the amount of medical expenditures spent on treating illnesses to affect the mortality rate in the state of being critically ill. In Section 4.2, we examine the impact of health investment by considering three health investment strategies. In Section 5.1, we relax the restrictions on the health transition matrix to allow for a higher incidence of diagnosing with a critical illness or needing long-term care in poorer health states.

3.3 Health cost models

China's public insurance partly covers the costs of major diseases but the coverage for long-term care expenditures is not widely available (29 cities have pilot long-term care insurance).[19] The health-related costs in our model, that is, the costs due to critical illness at period t, CostCI_t, and the costs due to long-term care at period t, CostLTC_t, are both OOP, and their calibrations are based on empirical studies and industry reports. We assume that the health-related costs are not consumption hence will not directly generate utility. In our benchmark analysis, we assume an individual will always choose to spend enough to receive adequate health-related services and has the lowest mortality in the critically ill state, while in Section 3 we allow for choices of insufficient health investments such that lower medical expenditures will result in a higher mortality when critically ill. Another way to model the impact of health costs is to assume that health costs

[18]We tested an ordered probit model which was used in Koijen et al. (2016) and Yogo (2016), and we tested ordered logit, multinomial logit, and the formulation with the cloglog link function.

[19]Section 2 provides a brief description of the insurance for critical illness and long-term care.

generate utility but do not affect the underlying health transitions (e.g., Pang and Warshawsky, 2010; Yogo, 2016). Our approach differs from this approach in that we can capture the trade-offs among different types of insurance products in a more explicit way by a clearer separation of the utility generating mechanism: directly generated by non-medical consumption, or indirectly generated by medical expenditures via their impact on lifespan such that more consumption is enabled by a longer pension period. This formulation permits an investigation on the health-state dependent utility with less ambiguity for interpretation.

3.3.1 Costs of critical illness

Unlike the costs of long-term care, the costs of critical illness are often catastrophic to a family. A huge amount of costs happens in the first year because of urgent life-extending treatments and hospitalisation, and even the richest 20% in China can experience financial hardship (Li and Feng, 2017). Such a financial shock is likely to affect the retirement planning for an individual. In China, the coverage for critical illness is basic. Patients need to pay by themselves to access more advanced and imported treatments or medicines that are not covered by public insurance (Li et al., 2013a). In practice, the deductibles, cap amounts, reimbursement rates, and the drug lists of advanced medicines and treatments of the local public insurance can be quite different, depending on each city's financial capability. Overall, half of the critical illness costs is OOP (Zhu et al., 2016).

The incidence of significant expenditure caused by critical illness is considerable and the cost distribution is typically right-skewed with a large variation.[20] Therefore, a lognormal distribution is often considered proper to model the costs of critical illness (e.g., Wu et al., 2018; Zhang et al., 2019). To calibrate the lognormal distribution for the OOP costs of critical illness, we choose to base on individual medical expenditure data recorded directly by the healthcare system like hospitals rather than household surveys. This is because a) household survey data is likely to underestimate the costs in that those with critical illnesses are less likely be survey participants, and b) surveys like the CHARLS do not provide enough details about the severity of the diseases, and part of the expense only measured the costs happened in the last month. We calibrate the expectation of the lognormal distribution according to the reported average median of the OOP critical illness costs in Beijing (capital of China) and Zhaoqing (an inland city with a larger reimbursement rate and a larger cap amount than that an average city in China can provide) in 2014 (Fang et al., 2018). We further adjust the cost to 2020 according to medical cost inflation net of general CPI from 2015 to 2020.[21] The adjusted cost is assumed to represent sufficient medical expenditures to access adequate medical services.[22]

[20]The costs exactly for critical illnesses are often difficult to access. The average individual inpatient cost is much larger than the median (Zhang et al., 2019), and the variation of coefficient for personal health expenditure is about 7 (Zhao, 2018).

[21]Willis Towers Watson Global Medical Trends Survey Report 2016, 2017, 2020. The value in 2020 is a forecast. See at https://www.willistowerswatson.com/en-US/Insights/2019/11/2020-global-medical-trends-survey-report.

[22]For example, the total medical expenditure for the COVID-19 infected with severe symptoms on average is approximately

To calibrate the standard deviation, we rely on individual-level inpatient billings recorded by the healthcare system and more than 20,000 hospitalisation cases are provided by Wu et al. (2018). However, the data does not include an individual's age nor the disease severity, but the total time for inpatient is recorded. We include in the estimation sample only those with an inpatient time more than 25 days, which is about the average length of inpatient time for those age beyond 60 (Yin et al., 2019). The calibrated lognormal distribution for the OOP expenditure of critical illness is:

$$\text{CostCI} \sim \text{Lognormal}\Big(11.86,\, 0.92^2\Big), \tag{2}$$

and we assume a maximum OOP cost of ¥800,000 (USD 120,000).[23]

3.3.2 Costs of long-term care

In China, the public long-term care insurance is not widely available, and the Chinese government is conducting 29 pilot long-term care insurance programs with different benefit packages and funding modes (Wang et al., 2017). Regarding to the cost of long-term care, as emphasised by Zhen et al. (2015), the healthcare system in China has rooted in its traditional old-age support and more than 85% of the Chinese old adults receive primary care from spouses or children when sick. Lu et al. (2015) showed that the average time for informal care was approximately 300 hours in rural areas and 426 hours in urban areas for individuals with three or more ADLs. Li et al. (2013b) estimated that the average monthly cost of informal care was ¥2,991 in rural areas compared with ¥2,320 in urban areas. The typical cost of a nursing home charged between ¥1,000 and ¥4,000, while in the Qingdao pilot long-term care insurance program, the daily price was ¥170 for institution care, ¥65 for nursing home service and ¥50 for home care in 2015 (Lu et al., 2017).

In this study, we calibrate the costs of long-term care based on the CHARLS wave 4 data, which was collected in 2015. The data contains the hours of informal care received as well as the monthly wage for nurse. We assume an hourly wage of ¥20 to estimate the monetary cost of informal care and add that to the wage for nurse to calculate the total monthly expenditure due to needing long-term care. We find that a linear model captures the correlation between the logarithm of the total long-term care expenditure and age of the survey participants reasonably well, and the residual plot of the log-linear model shows there is no evidence of heteroscedasticity. The estimated monthly cost of long-term care has the following lognormal

¥400,000, and this amount is considered as adequate. Assuming a 50% reimbursement rate (if the government did not choose to fully cover it), the OOP cost is around ¥200,000, which is comparable to the median expenditure of around ¥150,000 in Beijing in 2014 (Fang et al., 2018).

[23] For example, the largest approved claim for private medical insurance reimbursement in China's history was about ¥1,500,000 in 2019.

distribution:

$$\text{CostLTC(Age)} \sim \text{Lognormal}\Big(6.1313 + 0.0189 \times \text{Age},\, 1.459^2\Big). \tag{3}$$

We multiply this cost by twelve to derive the annual cost and adjust it to 2020 according to China's general CPI. After that, we have applied a maximum monthly long-term care cost of ¥8,000 to filter out outliers.

3.4 Individual profiles

We assume that the individual is an urban retiree and is covered by public insurance in China. Further, we assume that the individual has worked in a private firm, i.e., that he does not have access to the pension for government employees, or to enterprise annuities, which are still rare and typically exist only in large state-owned enterprises. The individual retires with a retirement savings of M, and receives an annual income P_t at period t from the public pension. We study the optimal retirement portfolio problem for four wealth profiles: individuals with i) a low level of retirement savings and a low level of pension income, and ii) a low retirement savings and an average pension income, iii) a high retirement savings and a low pension income, and iv) a high retirement savings and an average pension income. The wealth values are calibrated to about the 30th percentile, and 80th percentile of the wealth distributions based on the CHARLS data, and we set the pension amounts according to Zhu and Walker (2018) to represent a low and an average pension for retirees with Employee Pension. Our target population excludes retirees with more savings or pension as they are less financially vulnerable and their financial portfolios are likely to include more types of assets not covered by our model. The public health insurance is assumed to be the same across the wealth profiles as the coverage of the Urban Employee Basic Medical Insurance (UEBMI) is almost identical.[24] Therefore, the health-related cost in our model is driven by the OOP part described in the previous section.

3.5 Individual decision framework

In our benchmark model, individual preferences are represented by a time-separable utility function with constant relative risk aversion (CRRA):

$$u(c_t) = c_t^{1-\gamma}/(1-\gamma), \tag{4}$$

where c_t is the consumption in period t, β is the subjective discount factor, and γ is the risk aversion coefficient (e.g., Yaari, 1965; Davidoff et al., 2005). Empirical studies found that the utility of consumption depends on the health state (e.g., Finkelstein et al., 2013; Gyrd-Hansen, 2017; Gerking et al., 2017). Therefore, similar

[24] The accumulated amounts in the Individual Medical Account are depending on the salary path, however the contribution period is limited for the current generation close to retirement, therefore we assume its impact is minimal.

to Koijen et al. (2016) and Peijnenburg et al. (2017), we extend our model framework to adopt the following health-state dependent utility function:

$$u(c_t|H_t) = \eta_{H_t} c_t^{1-\gamma}/(1-\gamma), \tag{5}$$

where η_{H_t} is a constant parameter weighting the impact of the individual health state on the utility of consumption c_t. This function distinguishes the utilities of consumption when the individual is in different health states as suggested by Finkelstein et al. (2013), and is a natural extension of Peijnenburg et al. (2017) to allow for different health-state dependent utilities with more than one unhealthy states. Another way to specify a health-state dependent utility is to assume that consumption, rather than utility, is discounted in different health states (Laitner et al., 2018). In our paper we follow the former approach to incorporate health-state dependent utility because i) in our model, the individual chooses consumption after paying health-related costs, therefore consumption does not need to be discounted to reflect non-medical consumption, and ii) we are interested in the impact of health state on consumption and how this affects portfolio choice. Note that the health-state dependent utility given in Equation (5) reduces to the standard case without health-state dependent utility once $\eta_{H_t} = 1$. Without loss of generality, in the following we will use the health-state dependent utility formulation in the optimisation problem.[25] We use $v()$ to value bequest motives:

$$v(M) = b\frac{M^{1-\gamma}}{1-\gamma}, \tag{6}$$

where M is the bequest wealth and b is the strength of the bequest motive. This bequest function has been used in similar simulation studies (see, e.g., Friedman and Warshawsky, 1990; Shen and Sherris, 2018), and its form can be generalised into our framework of health-state dependent utility by considering death is the terminal poor health state.[26]

In our benchmark case, we calibrate the time preference $\beta = 0.999$ and the risk aversion $\rho = 3$ based on İmrohoroğlu and Zhao (2018) who study the long-term care risks and family insurance in China. These values are comparable to those used in the literature focusing on the U.S. For example, in Peijnenburg et al. (2017) these values have been set as 0.96 and 5 with the exact same utility function, respectively. Feldstein and Ranguelova (2001) suggest that a coefficient of relative risk aversion between 2 and 3 is reasonable. We set the strength of the bequest motive $b = 50$ to reflect a relatively strong bequest motive according to Friedman and Warshawsky (1990). Regarding the parameter for health-state dependent utility, the empirical studies

[25] Other studies have questioned the validity of time-separable utility and use recursive preferences (Epstein and Zin, 1989), which separate the relative risk aversion from the elasticity of inter-temporal substitution, see, e.g., Cocco et al. (2005), Pang and Warshawsky (2010), and Blake et al. (2014).

[26] Other studies have used luxury bequest motives, see, e.g., Lockwood (2012) and Ameriks et al. (2020).

are mixed: for example, studies in developed countries find both a higher and a lower value of consumption when health deteriorates (Viscusi and Evans, 1990; Edwards, 2008; Finkelstein et al., 2013; Tengstam, 2014; Gyrd-Hansen, 2017; Gerking et al., 2017). There are limited empirical results for China. Wang and Wang (2020) estimate that, for the elderly Chinese, the value of non-medical consumption is about 20% higher when an individual has several chronic diseases while it is about 30% lower if he is limited by three ADLs. Therefore, we use 1.2 and 0.7 for η_{H_t} if an individual is critically ill or needs long-term care, respectively. We test the sensitivity of the results to these values in Section 5.

3.6 The optimisation problem

In our case, a healthy individual chooses a proportion of his wealth to buy a life annuity, critical illness insurance, and long-term care insurance at retirement and chooses his consumption at each future period until death. His objective is to maximise the expected lifetime utility of consumption in retirement given by the following value function:

$$V = E_0 \left\{ \sum_{t=0}^{T-1} \sum_{j=1}^{3} \beta^t p_t(H_t = j)[u(c_t|H_t = j) + \beta \pi_t(H_t = j, 4) v(M_{t+1})] \right\}, \tag{7}$$

where E_0 is the expectation operator at the initial period $t = 0$. $u(c_t|H_t)$ is the health state-dependent utility of consumption defined in Equation (5); $p_s(H_t)$ is the probability that the individual's health status is H_t (excluding the dead state) at period t; $\pi_t(H_t, 4)$ is the transition probability from H_t to death at period t;

Consumption c_t at each period depends on the proportions $\omega_{\mathrm{a}}, \omega_{\mathrm{c}}, \omega_{\mathrm{l}}$ of the initial retirement wealth M that has been allocated to purchase a regular annuity income $\mathrm{Annuity}_t(\omega_{\mathrm{a}})$, a lump sum payment payment for critical illness $\mathrm{CII}_t(\omega_{\mathrm{c}})$, and a regular income for long-term care $\mathrm{LTCI}_t(\omega_{\mathrm{l}})$, respectively. For a simpler notation, we will omit the notations for the allocated proportions for the corresponding insurance payouts. For example, we will use $\mathrm{Annuity}_t$ for $\mathrm{Annuity}_t(\omega_{\mathrm{a}})$.

In each period, we assume the following constraints for retirement planning, that is, the individual cannot sell or cancel the insurance contracts, borrowing is not allowed, and the consumption floor is the government subsidy. Given the insurance allocation made at the initial period, from period t to $t+1$, an individual starts with available retirement wealth M_t, receives pension P_t and annuity income $\mathrm{Annuity}_t$; Depending on the realised health status H_t, the individual incurs costs of critical illness CostCI_t if he is critically ill and receives a lump sum payment CII_t if the period t is also the first time he is diagnosed with a critical illness; or he incurs long-term care costs $\mathrm{CostLTC}_t$ and receives a long-term care income LTCI_t if he cannot independently perform three or more of the six ADLs. Next, the individual chooses the consumption c_t based on the current cash on hand A_t. His remaining wealth goes to a savings account growing at a real risk-free rate R and becomes the available wealth M_{t+1} in the next period. Additionally, we assume a consumption floor such that once the cash on hand before consumption A_t is less than the government subsidy S, he consumes the subsidy and the remaining wealth is zero. In the last period, after the insured determines the consumption, any remaining wealth in the next period becomes his bequest.

The optimal retirement portfolio problem can be redefined with the following Bellman equation, subject to the constraints introduced earlier and the conditions for insurance payments and health costs described in Section 3.1 and Section 3.3. We solve the optimisation problem numerically by backward induction with the endogenous gridpoint method (EGM) of Carroll (2006). Solution methods are provided at Appendix B.

$$V_t(M_t, H_t) = \max_{c_t,\omega_{\rm a},\omega_{\rm c},\omega_{\rm l}} E_t\left\{u(c_t|H_t) + \beta\Big[\sum_{j=1}^{3}\pi_t(H_t,j)V_{t+1}(M_{t+1},H_{t+1}=j) + \pi_t(H_t,4)v(M_{t+1})\Big]\right\},$$

$$\begin{aligned}
&\text{s.t.}\\
A_t &= M_t + P_t + \text{Annuity}_t + \text{CII}_t + \text{LTCI}_t - \text{CostCI}_t - \text{CostLTC}_t - c_t,\\
M_{t+1} &= RA_t,\\
A_t &\geqslant 0,\\
c_t &\geqslant S,\\
\omega_{\rm a},\omega_{\rm a},\omega_{\rm l} &\geq 0,\\
\omega_{\rm a}+\omega_{\rm c}+\omega_{\rm l} &\leqslant 1.
\end{aligned}$$

4 Results

In this section we provide the optimal portfolio choices predicted by the calibrated life-cycle model. After solving for the optimal consumption function with backward induction, for each scenario, the average lifetime utility obtained with 10,000 Monte-Carlo simulated life paths is used to determine the optimal portfolio allocation.[27] In Section 4.1, we provide the optimal allocation of retirement savings to the life annuity, the critical illness insurance, and the long-term care insurance for retirees with different retirement savings and monthly pension amounts. In Section 4.2, we show the optimal health investment strategy and the optimal portfolio for retirees with different health investment preferences. In Section 4.3 we explore the impact of health-state dependent utility on optimal portfolio selection.

[27]We have tested the optimal portfolio obtained under 100,000 simulations and the results for optimal portfolio are identical and the average total utilities are of minor difference.

4.1 Optimal portfolio

4.1.1 Optimal insurance allocation

We show the optimal amounts of retirement savings allocated to the life annuity, the critical illness insurance, and the long-term care insurance when all three products are available for purchase at retirement. We examine the results for four combinations of retirement savings (high: ¥1 million, low: ¥150,000) and monthly pension amounts (average: ¥3,000, low: ¥1,000). These values cover the range of our target population. We do not consider retirees with higher or lower retirement savings or pensions because they are either likely to have other investment channels not modelled here or are likely to be severely impacted by budget constraints. In our benchmark scenarios, we assume that retirees choose to receive adequate medical services as described in Section 3.3. Table 2 show that, for a male retired at age 60 with ¥1 million retirement savings and a monthly pension of ¥3,000, the optimal amounts allocated to the life annuity, the critical illness insurance, and the long-term care insurance are ¥100,000, ¥300,000, and ¥50,000, respectively. The remaining retirement savings are ¥550,000. The expected lifetime welfare gain with the optimal insurance, compared with no insurance, is 12% of his initial retirement savings. This means that, for a male just retired with ¥1 million retirement savings and ¥3,000 pension, the expected lifetime utility from the optimal portfolio with three insurance, is equivalent to that from a portfolio with ¥1,120,000 savings without any private insurance.[28] If a retiree with high retirement savings only has a monthly pension of ¥1,000, his optimal portfolio allocation consists of ¥400,000 of the annuity, ¥250,000 of the critical illness insurance, ¥50,000 of the long-term care insurance, and ¥300,000 of retirement savings. Comparing with the high-pension scenario, the annuity demand increases significantly from ¥100,000 to ¥400,000. Both profiles have more than 30% of wealth held in a savings account.

In the case that a retiree is not wealthy, for example, he only has a limited savings of ¥150,000 upon retirement, our results suggest that he should allocate most of his retirement savings to the health-related insurance products if he has a monthly pension of ¥3,000. Specifically, he should spend ¥120,000 on the critical illness insurance, ¥20,000 on the long-term care insurance, and zero on the annuity. This suggests that with an average pension of ¥3,000 and limited wealth, the retiree should focus on insuring the OOP health-related costs rather than purchasing more regular income. However, if the retiree has a pension of only ¥1,000, similar to the high-wealth scenario, he should also spend a large portion of the retirement savings, i.e., ¥140,000 for the annuity, with the remaining ¥10,000 for the long-term care insurance and zero for the

[28] We simulate the average lifetime utilities without any insurance for a sequence of wealth values and use splines to obtain a utility curve with respect to wealth. The wealth value corresponding to the utility with optimal insurance is then derived and is used to compare with the initial retirement savings to generate the expected lifetime welfare gain. >100% means the equivalent wealth necessary to generate the same utility with optimal insurance is more than twice of the initial retirement savings, and the exact values are beyond this range (the extrapolation of the splines outside the fitting range is not used for its bias.

Table 2: Optimal portfolio when three insurance products are available to purchase

	Wealth = 1 million Pension = 3,000	Wealth = 1 million Pension = 1,000	Wealth = 150,000 Pension = 3,000	Wealth = 150,000 Pension = 1,000
Annuity	10	40	0	14
CII	30	25	12	0
LTCI	5	5	2	1
Savings account	55	30	1	0
Wealth gain	12%	32%	39%	65%

Note: Results are for males at age 60 at specified wealth and pension profiles. The optimisation is derived with benchmark parameters. Allocation values are in 10,000 CNY. The wealth gain is the extra amount of the initial retirement wealth required for those without insurance to achieve the same utility with optimal insurance.

critical illness insurance. This suggests that a retiree facing limited regular income will have a larger welfare gain by focusing on consumption instead of insuring catastrophic medical costs. The equivalent wealth gain for the retiree with a low pension and low wealth is 65% and it is the largest among the four economic profiles with their corresponding optimal portfolios.

Overall, our results show that a substantial part of retirement savings should be allocated to the annuity and the critical illness insurance, depending on a retiree's economic situation. This demonstrates the importance of regular income stream for those with a low pension, and the impact of uncertain medical expenditure with more pension or wealth in retirement planning. We find that in all four scenarios there is a positive allocation of retirement wealth to long-term care insurance, contrary to the annuity and the critical illness insurance. This signals a unique role of long-term care insurance in retirement planning.

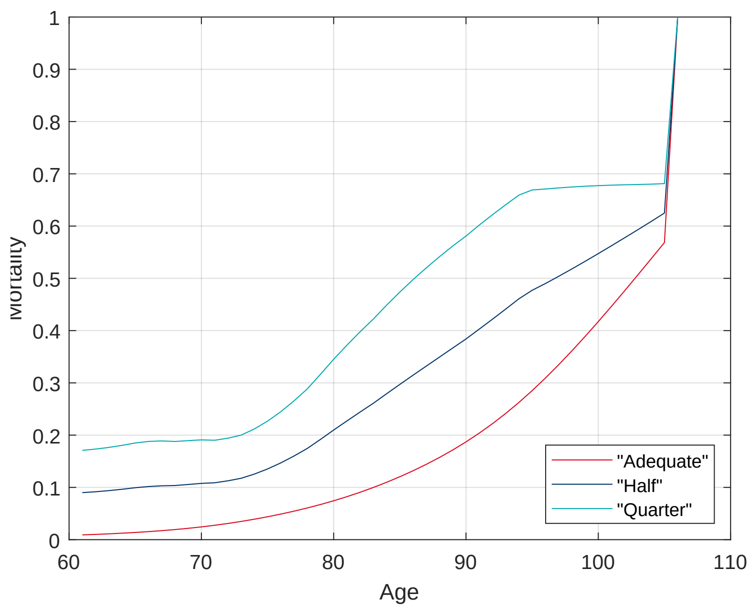
In addition, we find that the relative lifetime welfare gain with optimal insurance is much larger for retirees with lower wealth than those with higher wealth, and for retirees with a lower pension compared those with a higher pension. This means that the optimal retirement planning is more important for those with lower economic profiles, and it can help bridge the gap of economic status built up during work, thereby reducing the impact of wealth inequality in retirement.

4.2 Optimal health investment

The previous results assume that a retiree will always choose to spend sufficiently to receive adequate medical services in case of being critically ill. However, whether such a health investment can provide a reasonable return, for example, whether the expenditure for recovery is expensive, and what if a retiree is not willing to purchase advanced treatments or drugs, are both practical and important considerations that can alter the portfolio choice in retirement. A core issue underlying this problem is how to take into account the relation between the medical expenditures and the health improvements after receiving the purchased medical services.

To simplify the problem, we consider three choices of health investment: the benchmark case "Adequate" where retirees always choose to invest in health sufficiently and incur adequate medical expenditures such that the mortality in the critically ill state follows the official mortality curve for health business; the "Quarter" case where retirees always choose one-fourth[29] of the benchmark adequate medical expenditures such that the mortality in the critically ill state follows a higher mortality with industry mortality curves adjusted by the proportion of deaths due to critical illness; and the "Half" case where retirees always choose to invest half of the benchmark adequate medical expenditures such that the mortality is the average of the previous two. Figure 1 presents the three mortality curves for a male retiree after age 60 with different choices of health investment. The life expectancies at age 60 with the "Adequate", the "Half", and the "Quarter" health investments are 21.2 years, 18.7 years, and 18.3 years, respectively.

Figure 1: Post-illness mortality with different health investments



Interestingly, our results (Table 3) suggest that allowing for the correlation between the health investment and mortality reduces the demand for critical illness insurance substantially. For example, for retirees an average pension and high wealth, the demand for the critical illness insurance decreases from ¥300,000 in the "Adequate" case, to ¥50,000 in the "Half" case and to zero in the "Quarter" case. The impact on annuity demand is mixed: for those with high savings but a low pension, demand increases from ¥400,000 with "Adequate" health investment to ¥650,000 with "Half" health investment, while for those with an average

[29]For example, in 2014, the OOP cost of critical illness was on average about 35,000 in inland cities Yichang and Siping where the annual disposable income is about half of that in Beijing (Fang et al., 2018), and the OOP costs of cancer was on average USD 4947 in 13 provinces across China (Huang et al., 2016). Both values reflect a lower choice of health investment that is about a quarter of the adequate expenditure described in Section 3.3.

Table 3: Optimal portfolio with different choices of health investment

		Choice of health investment Adequate	Half	Quarter
Wealth = 1 million, Pension = 3,000	Annuity	10	0	0
	CII	30	5	0
	LTCI	5	5	5
	Utility	-0.36	-0.23	-0.21
	Wealth gain	12%	26%	54%
Wealth = 1 million, Pension = 1,000	Annuity	40	65	40
	CII	25	5	0
	LTCI	5	5	10
	Utility	-0.83	-0.64	-0.68
	Wealth gain	32%	>100%	>100%
Wealth = 150,000, Pension = 3,000	Annuity	0	0	0
	CII	12	11	4
	LTCI	2	4	5
	Utility	-2.68	-1.39	-1.03
	Wealth gain	39%	78%	>100%
Wealth = 150,000, Pension = 1,000	Annuity	14	15	7
	CII	0	0	5
	LTCI	1	0	3
	Utility	-11.16	-9.89	-9.97
	Wealth gain	65%	97%	>100%

Note: Results are for males at age 60 at specified wealth and pension profiles. The optimisation is derived with the choice of health investment specified in each scenario and other benchmark parameters. Allocation values are in ¥10,000. The wealth gain is the extra amount of the initial retirement wealth required for those without insurance to achieve the same utility with optimal insurance.

pension and high wealth, annuity demand decreases to zero with a lower health investment. Demand for long-term care insurance is generally more stable or increases with a lower health investment in treating critical illness.

Regarding the optimal choice of health investment, we compare the utility achieved with optimal portfolio for each health investment choice. For instance, for the average-pension and high-wealth scenario, the largest utility is achieved at -0.21 in the "Quarter" case, compared with -0.23 in the "Half" case and -0.36 in the "Adequate" case. This demonstrates that the minimal health investment choice "Quarter" is optimal for this wealthy group of retirees. Overall, the best choice of health investment is "Quarter" for retirees with ¥3,000 pension per month and "Half" for those with ¥1,000 pension. This also suggests that with a lower health investment, the individual can use the annuity payments to build up a buffer for the potential cost due to critical illness. The impact on the welfare gain is material, for example, for the retirees with an average pension and high savings, both with optimal insurance, choosing to receive adequate medical services will generate 42% less of the initial ¥1 million wealth compared with choosing to spend only a quarter of the adequate medical expenditures.

4.3 Health-state dependent utility

In our benchmark scenarios we set the weights for the utility of consumption to $\eta_{H_t=2} = 1.2$ if the retiree is critically ill and to $\eta_{H_t=3} = 0.7$ if he needs long-term care. However, empirical studies find mixed results

Table 4: Optimal portfolio with health-state dependent utility

		Weights for utility of consumption in poorer health states							
	Weight at CI	1.2	1	0.8	0.6	0.8	1.2	1.4	1.2
	Weight at LTC	0.7	1	0.8	0.8	0.6	1.2	1.2	1.4
Wealth = 1 million, Pension = 3,000	Annuity	10	0	20	35	25	10	15	15
	CII	30	25	25	15	25	35	70	35
	LTCI	5	5	5	0	5	10	10	15
	Wealth gain	12%	14%	8%	11%	13%	32%	42%	74%
Wealth = 1 million, Pension = 1,000	Annuity	40	45	55	65	50	25	50	20
	CII	25	25	20	20	25	25	35	25
	LTCI	5	5	5	5	10	5	5	10
	Wealth gain	32%	37%	>100%	>100%	>100%	19%	29%	46%
Wealth = 150,000, Pension = 3,000	Annuity	0	0	0	0	0	0	0	0
	CII	12	11	11	10	11	10	12	6
	LTCI	2	3	2	3	2	5	3	9
	Wealth gain	39%	61%	32%	26%	31%	>100%	>100%	>100%
Wealth = 150,000, Pension = 1,000	Annuity	14	13	13	12	13	11	9	10
	CII	0	0	0	0	0	0	0	0
	LTCI	1	2	2	3	2	4	6	5
	Wealth gain	65%	66%	94%	>100%	>100%	71%	>100%	>100%

Note: Results are for males at age 60 at specified wealth and pension profiles. The optimisation is derived with the health-state dependent utility weights η_{H_t} specified in each scenario and other benchmark parameters. Allocation values are in ¥10,000. The wealth gain is the extra amount of the initial retirement wealth required for those without insurance to achieve the same utility with optimal insurance.

regarding whether the utility of consumption will be lower or higher if health deteriorates (Viscusi and Evans, 1990; Edwards, 2008; Finkelstein et al., 2013; Tengstam, 2014; Gyrd-Hansen, 2017; Gerking et al., 2017). To further understand the impact of health-state dependent utility on optimal portfolio in retirement, we compare the benchmark scenario to seven assumptions about the weight parameter η_{H_t} in poorer health states. The first assumption considers the situation where the utility of consumption is the same in the healthy, the critically ill, and the long-term care state, that is, utility is not health-state dependent. The next three assumptions consider lower weights in poorer health states while the last three assumptions consider higher weights. For the scenarios with lower weights, we choose η_{H_t} equals 0.8 and 0.6 according to Koijen et al. (2016) where the utility weight for consumption in poor health is 0.74. For the scenarios with higher weights, symmetrically, we set η_{H_t} to 1.2 and 1.4, respectively.

Comparing with the benchmark results, Table 4 shows that our previous results regarding the optimal portfolio are generally robust under alternative assumptions for health-state dependent utility. For example, we find that there is a substantial demand for the critical illness insurance for retirees with an average pension or high savings, a high annuity demand for retirees with a low pension, and a small but persistent demand for the long-term care insurance. However, one clear difference is that annuity demand increases from zero to at least 10% of the initial retirement wealth when considering health-state dependent utility for those with a higher pension and wealth. In addition, different weights for the utility of consumption in poorer health states can trigger different trade-off mechanisms between the longevity and the health-related insurance products. For retirees with less pension or wealth, the demand for the health-related insurance increases when the weights of utility of consumption are higher in the poorer health states. For example, for wealthier retirees with 1 million retirement savings and ¥3,000 pension, the allocation to the critical illness insurance (long-term care insurance) increases from ¥250,000 (¥50,000) when both weights equal 1 to ¥350,000 (¥100,000) when both weights equal 1.2. If the weights are lower in the poorer health states, we find an increase of the annuity demand and it is occasionally accompanied with a decrease in the demand for health-related insurance. For example, the allocation to the critical illness insurance (long-term care insurance) decreases from ¥250,000 (¥50,000) when both weights equal 1 to ¥150,000 (¥0) when the weight in the critically ill state is 0.6 and 0.8 in the long-term care state, while the annuity demand increases significantly from zero to ¥350,000. For retirees with less pension or wealth, the optimal portfolio is mainly driven by the constraints and considering the health-state dependent utility slightly modify the results. For example, for those with a low pension and low wealth, the main allocation of retirement savings remains to be the annuity, the demand for the critical illness insurance remains low, but the demand for long-term care insurance increases at the expense of the annuity if the weight of utility of consumption in the long-term care state is higher. In addition, the wealth gain with optimal insurance has mixed results with health-state dependent utility, for example, it decreases (increases) with lower (higher) utility weighting parameters for the wealthiest group, while an opposite direction is observed for retirees with a lower pension but higher wealth.

Retirees with different preferences for consumption in different health states are likely to choose different health investment strategies. Therefore, we study how the optimal portfolio results change when we simultaneously consider the choice of health investment and the health-state dependent utility. For instance, for the wealthiest group in our analysis, on the one hand, previously we find that allowing for different weights on the utility of consumption in poorer health states can increase annuity demand (Table 4, top panel), while considering choices of health investment can lower annuity demand (Table 3, top panel). Taking into account the choice of health investment and the health-state dependent utility simultaneously, we find that the annuity demand remains low but the demand for both the critical illness insurance and the long-term care insurance increases mildly. On the other hand, the optimal health investment remains to be "Quarter" under each of the seven weighting mechanisms. Both suggest that the impact of health investment choices is likely to be larger than that from considering health-state dependent utility with plausible weighting parameters, regardless whether the weights are lower or higher than that in the healthy state.

5 Sensitivity analysis

In this section we conduct additional analysis to investigate how different assumptions of health transition matrix, product pricing, risk aversion, time preference, bequest motives, and government subsidy can affect

Table 5: Six additional assumptions of health transition processes

	Assumptions of transitions between critically ill (CI) and needing long-term care (LTC)
Scenario 1 (Benchmark)	Transition from CI state to LTC state = Transition from Healthy to LTC state Transition from LTC state to CI state = Transition from Healthy to CI state
Scenario 2	CI state to LTC state = Healthy to LTC state × 2 LTC state to CI state = Healthy to CI state × 1
Scenario 3	CI state to LTC state = Healthy to LTC state × 1 LTC state to CI state = Healthy to CI state × 2
Scenario 4	CI state to LTC state = Healthy to LTC state × 2 LTC state to CI state = Healthy to CI state × 2
Scenario 5	CI state to LTC state = Healthy to LTC state × 5 LTC state to CI state = Healthy to CI state × 1
Scenario 6	CI state to LTC state = Healthy to LTC state × 1 LTC state to CI state = Healthy to CI state × 3
Scenario 7	CI state to LTC state = Healthy to LTC state × 5 LTC state to CI state = Healthy to CI state × 3

Note: From Scenario 2 to 6, we set the long-term care (the critical illness) incidence rates in the critically ill (long-term care) state to be a multiple of that in the healthy state. For example, in Scenario 2, the transition from the CI state to the LTC state is twice of that from the healthy state to the LTC state, while the transition from the LTC state to the CI state is the same as in the benchmark scenario.

the results of optimal portfolio in retirement.

5.1 Health transition matrix

In our benchmark case, because of data limitations, we assume that the age-specific transition probability between the critically ill state and the long-term care state is the same as if the transition is from the healthy state to each of the two poorer health states. In this section we consider six additional assumptions for the transitions between the poorer health states. We set the long-term care (the critical illness) incidence rates in the critically ill (long-term care) state to be a multiple of that in the healthy state. The six assumptions are summarised in Table 5. The remaining calibration procedure of the health transition matrix is kept the same as in the benchmark study. In Scenario 6 and 7, the multiplier for the transition from the long-term care state to the critically ill state is set to three rather than five. This is because that adopting the latter will result in a transition probability larger than one at late ages as the incidences of critical illness at that time are already high.

Table 6 compares the optimal insurance amounts under the six health transition scenarios described in Table 5 with the benchmark results for retirees with different economic backgrounds. The different transitions between poorer health states slightly affect the optimal insurance demand. If the multiplier for the incidence of needing long-term care in the critically ill state is higher than that for the incidence of critical illness in the long-term care state, and all else remains unchanged, the demand for the long-term care insurance increases. Conversely, if the multiplier for the incidence of critical illness in the long-term care state is higher, other unchanged, we find the demand for the long-term care insurance reduces. When both transitions in the poorer health states increase, the annuity demand decreases, but we find mixed results for the health-related

Table 6: Optimal portfolio with different underlying health transition processes

	CI to LTC	× 1	× 2	× 1	× 2	× 5	× 1	× 5
	LTC to CI	× 1	× 1	× 2	× 2	× 1	× 3	× 3
Wealth = 1 million, Pension = 3,000	Annuity	10	0	5	0	0	10	0
	CII	30	30	35	30	25	30	30
	LTCI	5	5	5	5	10	0	5
	Wealth gain	12%	14%	12%	13%	21%	12%	15%
Wealth = 1 million, Pension = 1,000	Annuity	40	35	40	40	35	40	35
	CII	25	25	25	25	25	25	25
	LTCI	5	10	5	5	10	5	10
	Wealth gain	32%	50%	25%	37%	98%	21%	59%
Wealth = 150,000, Pension = 3,000	Annuity	0	0	0	0	0	0	0
	CII	12	12	13	13	11	13	13
	LTCI	2	2	0	0	4	0	2
	Wealth gain	40%	46%	45%	47%	57%	49%	55%
Wealth = 150,000, Pension = 1,000	Annuity	14	12	15	13	9	15	11
	CII	0	0	0	0	0	0	0
	LTCI	1	3	0	2	6	0	4
	Wealth gain	66%	84%	59%	73%	>100%	57%	99%

Note: Results are for males at age 60 at specified wealth and pension profiles. The optimisation is derived with the health transition assumptions specified in each scenario and other benchmark parameters. Allocation values are in ¥10,000. The wealth gain is the extra amount of the initial retirement wealth required for those without insurance to achieve the same utility with optimal insurance.

insurance. Overall, the results of optimal portfolio for each economic profile generally hold compared with our benchmark results, and the results for retirees with higher wealth but a lower pension are least affected by these assumptions.

5.2 Product pricing

In our benchmark study, all the insurance products are priced separately. For example, the annuity is priced with the industry mortality curve for pension business without taking into account explicitly the possibility of being critically ill or needing long-term care. This can result in a diminishing insurance market due to adverse selection (see, e.g., Finkelstein and Poterba, 2002, 2004; Laitner et al., 2018). To examine the impact of pricing on optimal portfolio, we consider a "joint" pricing approach based on a unified health transition table covering all the health states involved for the three insurance products. The price is the expected present value of the sum of the discounted future insurance payments based on 2,000,000 simulated health projections, and we add the same 15% loading.

First, we use the joint health transition matrix to price all the insurance products and compare its results with that from the benchmark study, and we check how the results change with alternative underlying health transition matrices (pricing and evaluation unmatched). Next, we check how the results change when the matrix for pricing matches the underlying health transition matrix across the different health transitions (pricing and evaluation matched). We use the same additional health transition assumptions introduced in Section 5.1.

In the scenario with benchmark assumptions for the underlying health transition (Table 7, first column),

Table 7: Optimal portfolio with insurance priced by the joint health transition matrix, evaluated under different underlying health transition processes

	CI to LTC LTC to CI	× 1 × 1	× 2 × 1	× 1 × 2	× 2 × 2	× 5 × 1	× 1 × 3	× 5 × 3
Wealth = 1 million,	Annuity	35	35	40	35	30	40	35
Pension = 3,000	CII	30	30	35	30	25	30	30
	LTCI	5	5	5	5	5	0	5
	Wealth gain	15%	16%	15%	15%	19%	15%	14%
Wealth = 1 million,	Annuity	45	45	45	45	35	50	45
Pension = 1,000	CII	25	25	25	25	25	25	25
	LTCI	5	5	5	5	10	0	5
	Wealth gain	40%	59%	31%	44%	>100%	27%	67%
Wealth = 150,000,	Annuity	0	0	0	0	0	0	0
Pension = 3,000	CII	13	10	10	13	10	14	13
	LTCI	2	3	0	2	3	0	2
	Wealth gain	23%	30%	23%	27%	43%	25%	37%
Wealth = 150,000,	Annuity	15	13	15	14	10	15	13
Pension = 1,000	CII	0	0	0	0	0	0	0
	LTCI	0	2	0	1	5	0	2
	Wealth gain	>100%	>100%	93%	>100%	>100%	89%	>100%

Note: Results are for males at age 60 at specified wealth and pension profiles. The products are priced by the same joint health transition matrix. The optimisation is derived with the assumptions of health transitions specified in each scenario and other benchmark parameters. Allocation values are in ¥10,000. The wealth gain is the extra amount of the initial retirement wealth required for those without insurance to achieve the same utility with optimal insurance.

we find that, contrary to the results where insurance products are priced separately (Table 2), the annuity demand increases from 10% to 35% of the retirement savings for the wealthiest retirees. For the other three economic profiles, the results are similar compared with the benchmark study, respectively. In the other six scenarios where the pricing matrix and the evaluation matrix do not match (Table 7, Column 2 to 6), we find that the increase of annuity demand remains substantial for those with a higher pension and higher wealth. The other results of optimal insurance generally agree with our benchmark study while larger changes are more often to occur for those with a smaller wealth when one transition is boosted up and the other stays the same.

When the pricing matrix and the evaluation matrix match for each scenario (Table 8), compared with the unmatched situations (Table 7, Column 2 to 6), the results in the six additional health transition scenarios are generally robust. The largest difference occurs for the least wealthy group where full annuitisation is optimal across all health transition assumptions.

Regarding the changes of welfare with optimal insurance due to pricing, we find that the welfare increases slightly for those with higher wealth. For the lower wealth group, the welfare decreases slightly for those holding more critical illness but increases substantially for those hold more annuity. This is caused by a less costly annuity and a slightly more expensive critical illness insurance when the joint health transition matrix is used for pricing.

Table 8: Optimal portfolio with insurance priced by a joint health transition matrix, evaluated with matched underlying health transition processes

	CI to LTC LTC to CI	× 1 × 1	× 2 × 1	× 1 × 2	× 2 × 2	× 5 × 1	× 1 × 3	× 5 × 3
Wealth = 1 million, Pension = 3,000	Annuity	35	35	35	35	30	35	25
	CII	30	25	30	30	25	30	30
	LTCI	5	5	5	5	5	5	5
	Wealth gain	15%	12%	10%	10%	13%	9%	8%
Wealth = 1 million, Pension = 1,000	Annuity	45	40	40	40	45	40	45
	CII	25	30	25	25	30	30	30
	LTCI	5	10	5	5	10	5	5
	Wealth gain	40%	51%	26%	38%	93%	21%	55%
Wealth = 150,000, Pension = 3,000	Annuity	0	0	0	0	0	0	0
	CII	13	11	10	13	10	10	11
	LTCI	2	2	0	2	3	0	2
	Wealth gain	23%	27%	22%	25%	29%	22%	25%
Wealth = 150,000, Pension = 1,000	Annuity	15	15	15	15	15	15	15
	CII	0	0	0	0	0	0	0
	LTCI	0	0	0	0	0	0	0
	Wealth gain	>100%	>100%	77%	91%	>100%	74%	>100%

Note: Results are for males at age 60 at specified wealth and pension profiles. The products are priced by the matched joint health transition matrix. The optimisation is derived with the assumptions of health transitions specified in each scenario and other benchmark parameters. Allocation values are in ¥10,000. The wealth gain is the extra amount of the initial retirement wealth required for those without insurance to achieve the same utility with optimal insurance.

5.3 Risk aversion, time preference and bequest motives

We set the risk aversion parameter ρ to 2, 4, 5 and compare the results with our benchmark case where $\rho = 3$. Then we vary the subjective discount factor β to 0.96 and 0.985 (benchmark: 0.999). After that, we vary the strength of the bequest motives b to 0 (no bequest motives), 5, 10, and 100 (benchmark: 50). These values are within the range of the parameters from similar studies (e.g., Friedman and Warshawsky, 1990; Peijnenburg et al., 2017; İmrohoroğlu and Zhao, 2018).

The top panel in Table 9 lists the optimal allocated retirement savings to the available insurance products with respect to ρ for retirees with ¥1 million retirement savings and ¥3,000 monthly pension. This population segment is least affected by budget constraints. Overall, we find that our benchmark results for optimal portfolio is robust, except that in the less risk averse case where $\rho = 2$ the annuity demand reduces to zero. For other economic profiles we find the demand for the health-related insurance increases with risk aversion. Regarding the time preference, the results are similar (middle panel, Table 9). For retirees with more pension or wealth, we find a stable demand for the health-related insurance while the annuity demand grows as the subjective discount factor increases. Regarding to the bequest motives, we find that the annuity demand decreases with the strength of bequest motives for the wealthiest group (bottom panel, Table 9). For example, the demand for annuity reduced to zero when the strength of bequest is high from ¥300,000 when the bequest motive is low. The demand for health-related insurance increases if bequest motives are absent.

Table 9: Optimal portfolio with different risk aversion ρ, time preference β, and bequest b

		$\rho = 2$	$\rho = 3$	$\rho = 4$	$\rho = 5$
Wealth = 1 million, Pension = 3,000	Annuity	0	10	10	10
	CII	35	30	30	30
	LTCI	5	5	5	5
	Wealth gain	9%	12%	17%	24%

		$\beta = 0.96$	$\beta = 0.985$	$\beta = 0.999$
Wealth = 1 million, Pension = 3,000	Annuity	0	5	10
	CII	30	30	30
	LTCI	5	5	5
	Wealth gain	11%	11%	12%

		$b = 0$	$b = 5$	$b = 10$	$b = 50$	$b = 100$
Wealth = 1 million, Pension = 3,000	Annuity	30	30	25	10	0
	CII	40	30	30	30	30
	LTCI	10	5	5	5	5
	Wealth gain	21%	17%	15%	12%	11%

Note: Results are for males at age 60 with ¥1 million wealth and ¥3,000 pension. The optimisation is derived with the ρ, β, and b specified in each scenario and other benchmark parameters. Allocation values are in ¥10,000. The wealth gain is the extra amount of the initial retirement wealth required for those without insurance to achieve the same utility with optimal insurance.

Table 10: Optimal portfolio with different levels of government subsidy

		Subsidy = 0.59	**Subsidy = 0.83**	**Subsidy = 1.40**
Wealth = 150,000, Pension = 1,000	Annuity	13	13	15
	CII	0	0	0
	LTCI	2	2	0
	Wealth gain	>100%	>100%	>100%

Note: Results are for males at age 60 with ¥150,000 wealth and ¥1,000 pension. The optimisation is derived with the (annual) Subsidy specified in each scenario and other benchmark parameters. Allocation values are in ¥10,000. The wealth gain is the extra amount of the initial retirement wealth required for those without insurance to achieve the same utility with optimal insurance.

5.4 Government subsidy

We test three assumptions of government subsidy which serves as a consumption floor in our model. The values are based on the provincial data "Dibao", i.e., the monthly subsidy for those without enough income to live. We use the minimum (¥491) and the maximum (¥1,170) published by the government in 2020 September to compare with our benchmark (the average, ¥687). For the more financially vulnerable group, e.g., those with a low pension and low wealth (Table 10), we find that the retirees tend to trade the annuity for the long-term care insurance when the subsidy decreases. For those with a higher pension or wealth, the results generally agree with our benchmark study as the subsidy amount is less a concern for them.

6 Discussion

Currently, China's government is expanding its public insurance coverage, especially in the process of developing social long-term care insurance and enlarging the coverage of critical illness insurance. Our benchmark results provide the optimal insurance amounts for annuity, critical illness insurance, and long-term care insurance for retirees with different economic backgrounds. Governments can refer to these values to reflect on the coverage of the social insurance programs offered by cities with different economic developments. For example, our result predicts a small demand for long-term care insurance, which provides a theoretical support for China's government to provide basic long-term care services at the moment. Because our definition for long-term care requirement is in line with the insurance practice in China, our result makes it easier for the government, local communities to work with insurers to design innovative insurance solutions. The low demand for long-term care insurance may be hindered by insufficient medical services for critical illness. Our results show that a lower choice of health investment is better than an adequate one, and it will reduce the demand for critical illness substantially. This is likely caused by the high price of advanced medical services in China, and our results suggest that the government should set a higher reimbursement level and include more advanced medical services in the Reimbursement Drug List. Additionally, we find that the welfare gain with optimal insurance is much larger for less affluent retirees, compared with the better-off, and this inform the government that optimal insurance is one way to reduce the inequality in retirement.

For insurance companies, it is important to target different population segments with the right insurance products. For example, retirees with a low pension are predicted to use about more than 40% of their retirement savings to purchase life annuities, while retirees with an average pension and high wealth are expected to purchase critical illness insurance with 30% of their wealth. However, in a developing country context, a very practical and important question for retirement planning is how to optimise insurance purchase with a limited retirement wealth as there can be uninsurable risks or savings motives such as children's education and marriage. It is therefore natural to ask how to make insurance products more affordable. In developing countries like China, the financial market is immature, and the regulatory body tends to set conservative requirements to avoid solvency risks. One possibility is to explore how to reduce potential adverse selection in separate insurance markets. For example, to consider bundled retirement insurance products. Bundling longevity and health or care insurance products may serve as a natural hedge thus reduces the impact of adverse selection and brings down the insurance premium. Assuming that the bundled longevity and health insurance products are priced accordingly to a joint health transition matrix, in this analysis, the price for the annuity, the critical illness insurance, and the long-term care insurance is 78Another natural question to ask is, if a retiree cannot afford the whole optimal insurance upon retirement, or he only intends to spend a small proportion of his wealth for insurance, or he already has purchased part of the insurance, what is the next insurance to purchase with a small portion of his retirement wealth that will generate the largest welfare? Our results show that, for example, for a retiree without any private insurance who wants to use ¥10,000 (¥50,000) of the ¥150,000 (¥1 million) retirement savings for additional insurance, if he has a monthly pension of ¥3,000, purchasing the long-term care insurance will yield the largest welfare gain, while if he has a monthly pension of ¥1,000, purchasing the annuity is the best choice. These results can also assist the policy makers to decide which insurance is a priority for local governments and what amount is optimal under budget constraints.

7 Conclusion

This paper developed a life-cycle model of retirement where an individual retiree chooses a portfolio consisting of a life annuity, critical illness insurance, long-term care insurance, and a savings account to maximise his lifetime utility. The model allowed for stochastic transitions between different health states and stochastic critical illness and long-term care expenditures. We allowed for choices of health investment and we considered different weights on the utility of consumption in different health states. We predicted the optimal insurance for retirees in China with several typical combinations of retirement savings and pension. We found that the demand for critical illness insurance was high for retirees with an average pension, but it also largely depended on the choice of health investment, i.e., a substantial demand was predicted if sufficient medical treatments for critical illness were preferred, and such demand diminished if a lower health investment was preferred. We also found a high demand for annuity for retirees with a low pension, and a small demand for long-term care insurance for retirees with different economic backgrounds. Our results suggested it was optimal for a retiree to choose a lower level of medical services, compared with an adequate one. One reason could be the high price of advanced medical services in China relative to the improvements in post-illness mortality development. Allowing for health-state dependent utility and health investment simultaneously results in different trade-offs among the three insurance products, depending on a retiree's economic background. Our results confirm that optimal insurance choices differ for retirees with different financial backgrounds. Our study suggests that policy makers in developing countries to expand social insurance should consider regional economic development and financial budget to decide the direction for further insurance to maximise the welfare outcome. Insurance companies should target the population segments with the right insurance products. Designing bundled product pricing with a joint health transition matrix can release precautionary savings to purchase annuity for more affluent retirees.

In further work, it would be interesting to consider married households with correlated health transitions. Future research could also consider an extended framework to incorporate the role of children in retirement provision and the role of housing. Another interesting direction is to examine the stated demand for retirement insurance products. Furthermore, it would be interesting to consider the role of housing as an additional source of retirement funding (see, e.g., Hanewald et al., 2020).

References

Ai, J., Brockett, P. L., Golden, L. L., and Zhu, W. (2017). Health State Transitions and Longevity Effects on Retirees' Optimal Annuitization. *Journal of Risk and Insurance*, 84(S1):319–343.

Ameriks, J., Briggs, J., Caplin, A., Shapiro, M. D., and Tonetti, C. (2020). Long-Term-Care Utility and Late-in-Life Saving. *Journal of Political Economy*, 128(6):2375–2451.

Barr, N. and Diamond, P. (2010). Pension reform in china: issues, options and recommendations. *China Economic Research and Advisory Programme*. http://econ.lse.ac.uk/staff/nb/Barr_Diamond_China_Pensions_2010.pdf.

Benartzi, S., Previtero, A., and Thaler, R. H. (2011). Annuitization Puzzles. *Journal of Economic Perspectives*, 25(4):143–164.

Blake, D., Wright, D., and Zhang, Y. (2014). Age-dependent investing: Optimal funding and investment strategies in defined contribution pension plans when members are rational life cycle financial planners. *Journal of Economic Dynamics and Control*, 38:105–124.

Brown, J. (2009). Understanding the role of annuities in retirement planning. In Lusardi, A., editor, *Overcoming the Saving Slump*, pages 178–206. University of Chicago Press.

Carroll, C. D. (2006). The method of endogenous gridpoints for solving dynamic stochastic optimization problems. *Economics Letters*, 91(3):312–320.

Chen, X. and Ji, X. (2017). The Effect of House Price on Stock Market Participation in China: Evidence from the CHFS Microdata. *Emerging Markets Finance and Trade*, 53(5):1030–1044.

Cocco, J. F., Gomes, F. J., and Maenhout, P. J. (2005). Consumption and Portfolio Choice over the Life Cycle. *The Review of Financial Studies*, 18(2):491–533.

Davidoff, T., Brown, J. R., and Diamond, P. A. (2005). Annuities and Individual Welfare. *American Economic Review*, 95(5):1573–1590.

Edwards, R. D. (2008). Health Risk and Portfolio Choice. *Journal of Business & Economic Statistics*, 26(4):472–485.

Epstein, L. G. and Zin, S. E. (1989). Substitution, Risk Aversion, and the Temporal Behavior of Consumption and Asset Returns: A Theoretical Framework. *Econometrica*, 57(4):937–969.

Fang, H. and Feng, J. (2018). The Chinese Pension System. NBER Working Paper 25088, National Bureau of Economic Research.

Fang, P., Pan, Z., Zhang, X., Bai, X., Gong, Y., and Yin, X. (2018). The effect of critical illness insurance in China. *Medicine*, 97(27).

Feldstein, M. and Ranguelova, E. (2001). Individual Risk in an Investment-Based Social Security System. Working Paper 8074, National Bureau of Economic Research.

Finkelstein, A., Luttmer, E. F. P., and Notowidigdo, M. J. (2013). What Good is Wealth Without Health? The Effect of Health on the Marginal Utility of Consumption. *Journal of the European Economic Association*, 11(suppl_1):221–258.

Finkelstein, A. and Poterba, J. (2002). SELECTION EFFECTS IN THE UNITED KINGDOM INDIVIDUAL ANNUITIES MARKET. *The Economic Journal*, 112(476):28–50.

Finkelstein, A. and Poterba, J. (2004). Adverse Selection in Insurance Markets: Policyholder Evidence from the U.K. Annuity Market. *Journal of Political Economy*, 112(1):183–208.

Friedman, B. M. and Warshawsky, M. J. (1990). The Cost of Annuities: Implications for Saving Behavior and Bequests. *The Quarterly Journal of Economics*, 105(1):135–154.

Gerking, S., Adamowicz, W., Dickie, M., and Veronesi, M. (2017). Baseline risk and marginal willingness to pay for health risk reduction. *Journal of Risk and Uncertainty*, 55(2-3):177–202.

Gyrd-Hansen, D. (2017). A Stated Preference Approach to Assess whether Health Status Impacts on

Marginal Utility of Consumption. *Health Economics*, 26(10):1224–1233.

Hanewald, K., Bateman, H., Fang, H., and Wu, S. (2020). Is there a demand for reverse mortgages in China? Evidence from two online surveys. *Journal of Economic Behavior & Organization*, 169:19–37.

Hanewald, K., Jia, R., and Liu, Z. (2019). Why is inequality higher among the old? Evidence from China. CEPAR Working Paper 2019/10, ARC Centre of Excellence in Population Ageing Research, Sydney, Australia.

Hou, Z., Van de Poel, E., Van Doorslaer, E., Yu, B., and Meng, Q. (2014). Effects of NCMS on access to care and financial protection in China. *Health Economics*, 23(8):917–934.

Huang, H.-Y., Shi, J.-F., Guo, L.-W., Zhu, X.-Y., Wang, L., Liao, X.-Z., Liu, G.-X., Bai, Y.-N., Mao, A.-Y., Ren, J.-S., Sun, X.-J., Zhang, K., He, J., and Dai, M. (2016). Expenditure and financial burden for common cancers in China: a hospital-based multicentre cross-sectional study. *The Lancet*, 388:S10.

Koijen, R. S., Van Nieuwerburgh, S., and Yogo, M. (2016). Health and Mortality Delta: Assessing the Welfare Cost of Household Insurance Choice. *The Journal of Finance*, 71(2):957–1010.

Laitner, J., Silverman, D., and Stolyarov, D. (2018). The Role of Annuitized Wealth in Post-retirement Behavior. *American Economic Journal: Macroeconomics*, 10(3):71–117.

Li, H., Liu, G. G., and Glaetzer, C. (2013a). Financing Innovative Medicines in Mainland China: The Role of Commercial Health Insurance. *Chinese Studies*, 02(03):128.

Li, J. and Feng, X. L. (2017). Health care-seeking behaviours and health expenditures in adults aged 45 years and older in China, 2011–2013. *Tropical Medicine & International Health*, 22(5):638–654.

Li, M., Zhang, Y., Zhang, Z., Zhang, Y., Zhou, L., and Chen, K. (2013b). Rural-Urban Differences in the Long-Term Care of the Disabled Elderly in China. *PLOS ONE*, 8(11):e79955.

Li, Y., Ying, C., Sufang, G., Brant, P., Bin, L., and Hipgrave, D. (2013c). Evaluation, in three provinces, of the introduction and impact of China's National Essential Medicines Scheme. *Bulletin of the World Health Organization*, 91(3):184–194.

Liu, G. G., Vortherms, S. A., and Hong, X. (2017). China's Health Reform Update. *Annual Review of Public Health*, 38(1):431–448.

Lockwood, L. M. (2012). Bequest motives and the annuity puzzle. *Review of Economic Dynamics*, 15(2):226–243.

Lu, B., He, W., and Piggott, J. (2014). Should China introduce a social pension? *The Journal of the Economics of Ageing*, 4:76–87.

Lu, B., Liu, X., and Piggott, J. (2015). Informal Long Term Care in China and Population Ageing: Evidence and Policy Implications. *Population Review*, 54(2).

Lu, B., Mi, H., Zhu, Y., and Piggott, J. (2017). A Sustainable Long-Term Health Care System for Aging China: A Case Study of Regional Practice. *Health Systems & Reform*, 3(3):182–190.

Meng, Q., Xu, L., Zhang, Y., Qian, J., Cai, M., Xin, Y., Gao, J., Xu, K., Boerma, J. T., and Barber, S. L. (2012). Trends in access to health services and financial protection in China between 2003 and 2011: a cross-sectional study. *The Lancet*, 379(9818):805–814.

Mitchell, O. S., Poterba, J. M., Warshawsky, M. J., and Brown, J. R. (1999). New Evidence on the Money's Worth of Individual Annuities. *American Economic Review*, 89(5):1299–1318.

Pan, X.-F., Xu, J., and Meng, Q. (2016). Integrating social health insurance systems in china. *The Lancet*, 387(10025):1274 – 1275.

Pang, G. and Warshawsky, M. (2010). Optimizing the equity-bond-annuity portfolio in retirement: The impact of uncertain health expenses. *Insurance: Mathematics and Economics*, 46(1):198–209.

Peijnenburg, K., Nijman, T., and Werker, B. J. (2017). Health Cost Risk: A Potential Solution to the Annuity Puzzle. *The Economic Journal*, 127(603):1598–1625.

Reichling, F. and Smetters, K. (2015). Optimal Annuitization with Stochastic Mortality and Correlated Medical Costs. *American Economic Review*, 105(11):3273–3320.

Shen, Y. and Sherris, M. (2018). Lifetime asset allocation with idiosyncratic and systematic mortality risks. *Scandinavian Actuarial Journal*, 2018(4):294–327.

Sun, X.-J., Shi, J.-F., Guo, L.-W., Huang, H.-Y., Yao, N.-L., Gong, J.-Y., Sun, Y.-W., Liu, G.-X., Mao, A.-Y., Liao, X.-Z., Bai, Y.-N., Ren, J.-S., Zhu, X.-Y., Zhou, J.-Y., Mai, L., Song, B.-B., Liu, Y.-Q., Zhu, L., Du, L.-B., Zhou, Q., Xing, X.-J., Lou, P.-A., Sun, X.-H., Qi, X., Wang, Y., Cao, R., Ren, Y., Lan, L., Zhang, K., He, J., Wang, J.-L., Dai, M., and The listed authors are on behalf of the Health Economic Evaluation Working Group, C. S. P. i. U. C. C. (2018). Medical expenses of urban Chinese patients with stomach cancer during 2002–2011: a hospital-based multicenter retrospective study. *BMC Cancer*, 18(1):435.

Sun, Y., Gregersen, H., and Yuan, W. (2017). Chinese health care system and clinical epidemiology. *Clinical Epidemiology*, 9:167–178.

Tengstam, S. (2014). Disability and Marginal Utility of Income: Evidence from Hypothetical Choices. *Health Economics*, 23(3):268–282.

Viscusi, W. K. and Evans, W. N. (1990). Utility functions that depend on health status: estimates and economic implications. *The American Economic Review*, pages 353–374.

Wan, C., Bateman, H., Fang, H., and Hanewald, K. (2020). The demand for longevity, health and long-term care insurance after COVID-19. CEPAR Working Paper, ARC Centre of Excellence in Population Ageing Research, Sydney, Australia.

Wang, Q., Zhou, Y., Ding, X., Ying, X., Wang, Q., Zhou, Y., Ding, X., and Ying, X. (2017). Demand for Long-Term Care Insurance in China. *International Journal of Environmental Research and Public Health*, 15(1):6.

Wang, X. and Wang, C. (2020). How Does Health Status Affect Marginal Utility of Consumption? Evidence from China. *International Journal of Environmental Research and Public Health*, 17(7).

Wu, D., Yu, F., and Nie, W. (2018). Improvement of the reduction in catastrophic health expenditure in China's public health insurance. *PLOS ONE*, 13(4):e0194915. Publisher: Public Library of Science.

Wu, S., Bateman, H., Stevens, R., and Thorp, S. (2017). Reducing the under-insurance puzzle by product design: Experimental evidence of life care annuity demand. CEPAR Working Paper 2017/17, ARC Centre of Excellence in Population Ageing Research, Sydney, Australia.

Yaari, M. E. (1965). Uncertain Lifetime, Life Insurance, and the Theory of the Consumer. *The Review of Economic Studies*, 32(2):137–150.

Yang, W., Jingwei He, A., Fang, L., and Mossialos, E. (2016). Financing institutional long-term care for the elderly in China: a policy evaluation of new models. *Health Policy and Planning*, 31(10):1391–1401.

Yin, X., Xu, Y., Man, X., Liu, L., Jiang, Y., Zhao, L., and Cheng, W. (2019). Direct costs of both inpatient and outpatient care for all type cancers: The evidence from Beijing, China. *Cancer Medicine*, 8(6):3250–3260. Publisher: John Wiley & Sons, Ltd.

Yip, W. C.-M., Hsiao, W. C., Chen, W., Hu, S., Ma, J., and Maynard, A. (2012). Early appraisal of China's huge and complex health-care reforms. *The Lancet*, 379(9818):833–842.

Yogo, M. (2016). Portfolio choice in retirement: Health risk and the demand for annuities, housing, and risky assets. *Journal of Monetary Economics*, 80:17–34.

Zhang, A., Nikoloski, Z., and Mossialos, E. (2017). Does health insurance reduce out-of-pocket expenditure? Heterogeneity among China's middle-aged and elderly. *Social Science & Medicine*, 190:11–19.

Zhang, Y., Vanneste, J., Xu, J., and Liu, X. (2019). Critical Illness Insurance to alleviate catastrophic health expenditures: new evidence from China. *International Journal of Health Economics and Management*, 19(2):193–212.

Zhao, W. (2018). Does health insurance promote people's consumption? New evidence from China. *China Economic Review*.

Zhen, Z., Feng, Q., and Gu, D. (2015). The Impacts of Unmet Needs for Long-Term Care on Mortality Among Older Adults in China. *Journal of Disability Policy Studies*, 25(4):243–251.

Zheng, W., Liu, Z., and Jia, R. (2019). How private sector participation improves retirement preparation: A case from China. *The Geneva Papers on Risk and Insurance - Issues and Practice*, 44(1):123–147.

Zhu, H. and Walker, A. (2018). Pension system reform in China: Who gets what pensions? *Social Policy & Administration*, 52(7):1410–1424. Publisher: John Wiley & Sons, Ltd.

Zhu, L., Xu, H., and Cui, X. (2016). Suggestion on Critical Illness Insurance in China. *Value in Health*, 19(7):A816. Publisher: Elsevier.

İmrohoroğlu, A. and Zhao, K. (2018). The chinese saving rate: Long-term care risks, family insurance, and demographics. *Journal of Monetary Economics*, 96:33–52.

Appendix A Pricing of insurance products

In our model, the life annuity, critical illness insurance, and long-term care insurance can be purchased by male individuals at age 60 by making a one-off payment. We priced the three products in an actuarially fair way based on gender and age. We assumed a real discount rate of 1.5% for each year in the future.[30] In addition, we assumed a 15% loading for all products, which is consistent with Mitchell et al. (1999) for longevity insurance and Karaca-Mandic et al. (2011) for health insurance.

We used the industry standard mortality and incidence rates tables provided by the China Bank and Insurance and Regulatory Commission (CBIRC) to price standalone annuities and critical illness insurance products. For the life annuity, we used the mortality curves for pension business for males and females starting at age 60 and 55, respectively. For the critical illness insurance, we used the incidence rate curves and the mortality curves for health insurance business for 25 diseases for males and females starting at age 60 and 55, respectively.[31] The insured period was lifetime for all three products. However, for critical illness insurance the contract ended if one payment was made, and for long-term care insurance, the payments would only be made when three or more ADLs were triggered.[32] For simplicity and a cleaner interpretation, we assumed that these curves were unchanged in the future.

For long-term care insurance, the industry health transition table is not available, therefore we estimated the health transition rates based on data from the CHARLS survey. We used all the CHARLS data: 2011 (Wave 1), 2013 (Wave 2), and 2015 (Wave 4).[33] A 2-year transition, that is, from 2011 to 2013 or from 2013 to 2015, was observable at each age for both genders by the longitudinal survey design. As the sample size for certain transitions could be limited, we pooled the first (2011-2013) and the second (2013-2015) transition data together, based on which we estimated a 1-year transition at each age for each gender. We only used data for respondents in the starting years (2011 or 2013) that were at least 35 years old.[34] We excluded observations with missing information for ADL status or death information.

We defined four health states: Healthy, Fair (1-2 ADLs), Disabled (3 or more ADLs, LTCI payable), and Dead. Different from the health states used in the life-cycle model, the inclusion of a Fair state here was to control the result such that the estimation of the transition Healthy-LTC was close to the insurance population. The recovery from states Fair or Disabled were allowed and Dead was an absorbing state. We

[30] We assumed a constant 3.5% nominal discount rate (industry standard in China) and a constant 2% inflation rate (Approximately an average of the CPI during the period 2010-2019 in China).

[31] Chinese insurance companies also use the mortality curves for pension business for a more defensive price.

[32] Instead of a lifetime coverage, we also tested the price with a shorter coverage, that is, age 60-80 for males, and age 55-85 for females. The differences compared with a lifetime coverage are not substantial because the cumulative survival chances at later ages are very small.

[33] Wave 3 is a separate survey to collect supplementary information about life history.

[34] We conducted sensitivity tests using a subset with ages between 45-84 and using aggregated data with a 10-year age group starting from 35-45. The impact on product pricing was not immaterial.

modelled the health transitions in a Markov framework. To estimate the health transition probabilities, we used a multinomial logit model based on a series of tests.[35] The model was estimated separately by gender. The dependent variable was each respondent's health state observed in the follow-up wave (2013 or 2015), and the explanatory variables were the respondent's age and health state in the initial wave (2011 or 2013).[36]

Based on the fitted multinomial logit model we predicted the 2-year transition rates for females starting at age 55 and for males from age 60 to 104. Both females and males were assumed dead at age 105. We calculated the 1-year transition probability matrix at age x based on the Markov property with the following conversion formula:

$$\boldsymbol{P}_x^{\text{2-year}} = \boldsymbol{P}_x^{\text{1-year}} \times \boldsymbol{P}_x^{\text{1-year}}, \tag{8}$$

where $\boldsymbol{P}_x^{\text{1-year}}$ is the probability of 1-year transition at each state at age x. As in the case of the life annuity and the critical illness insurance, we assumed that the estimated transition rates for long-term care were stable in the future. The price of the long-term care insurance was determined recursively accordingly to the standard formula.

For bundled products, we used the calibrated health transition matrix 1 and set the post-illness mortality to be an average of the mortality from receiving adequate medical services and that from receiving a quarter of the adequate services. We assumed the same 1.5% real discount rate and the 15% loading. 2,000,000 simulations are used to calculate the price for each insurance component.

Appendix B Numerical solution

The original optimisation problem has four choice variables: consumption at each period c_t, allocations for annuity ω_{a}, critical illness insurance ω_{c}, and long-term care insurance ω_{l}. This makes the standard 1-dimension EGM not directly applicable. However, our problem has a nested structure, and we first solve the optimal consumption problem conditioning on exogenous insurance states, and then we conduct a grid search under budget constraints for the optimal insurance choice based on the already obtained policy functions in

[35] We considered a probit model, which had been used to estimate the transition probabilities in a similar context in the U.S. by Yogo (2016) and Koijen et al. (2016). We also have tested models like ordered logit, probit, or cloglog, and multinomial logit models have the best performance in terms of AIC and deviance residuals. We also tested Poisson GLM and non-parametric smoothing techniques for each of the possible transitions and we did not find substantial differences in price.

[36] We did not distinguish between urban and rural population for pricing, as the insurance price for annuity or critical illness insurance is the same for them in China.

each exogenous insurance state. To see that, the original Bellman equation in our optimisation problem is:

$$V_t(M_t, H_t) = \max_{c_t,\omega_{\mathrm{a}},\omega_{\mathrm{c}},\omega_{\mathrm{l}}} E_t\left\{u(c_t|H_t) + \beta\Big[\sum_{j=1}^{3}\pi_t(H_t, j)V_{t+1}(M_{t+1}, H_{t+1} = j) + \pi_t(H_t, 4)v(M_{t+1})\Big]\right\},$$

$$\begin{aligned}
&\text{s.t.}\\
A_t &= M_t + P_t + \text{Annuity}_t + \text{CII}_t + \text{LTCI}_t - \text{CostCI}_t - \text{CostLTC}_t - c_t,\\
M_{t+1} &= RA_t,\\
A_t &\geqslant 0,\\
c_t &\geqslant S,\\
\omega_{\mathrm{a}}, \omega_{\mathrm{a}}, \omega_{\mathrm{l}} &\geq 0,\\
\omega_{\mathrm{a}} + \omega_{\mathrm{c}} + \omega_{\mathrm{l}} &\leqslant 1.
\end{aligned}$$

The optimal solution of the above problem can be obtained by solving the sub-problems in each exogenous insurance state $(\omega_{\mathrm{a}}, \omega_{\mathrm{c}}, \omega_{\mathrm{l}})$ defined below and finding their maximum:

$$V_t^{\omega_{\mathrm{a}},\omega_{\mathrm{c}},\omega_{\mathrm{l}}}(M_t, H_t) = \max_{c_t} E_t\left\{u(c_t|H_t) + \beta\Big[\sum_{j=1}^{3}\pi_t(H_t, j)V_{t+1}(M_{t+1}, H_{t+1} = j) + \pi_t(H_t, 4)v(M_{t+1})\Big]\right\},$$

$$\begin{aligned}
&\text{s.t.}\\
A_t &= M_t + P_t + \text{Annuity}_t + \text{CII}_t + \text{LTCI}_t - \text{CostCI}_t - \text{CostLTC}_t - c_t,\\
M_{t+1} &= RA_t,\\
A_t &\geqslant 0,\\
c_t &\geqslant S.
\end{aligned}$$

We use a three-dimension grid to discretise the insurance amount from 0 to 100% of the initial wealth. We use a minimum amount of 50,000 for allocation in the case of 1 million retirement savings, and 10,000 in the case of 150,000 retirement savings. Next, we solve the sub-problem for each of the exogenous insurance state to derive the optimal consumption. After that, we use simulations to project future scenarios for an individual and calculate the realised lifetime utility of consumption for each simulation. The optimal insurance choice is the insurance state that yields the maximal average utility across all simulated scenarios and is within the budget constraint. This two-step approach essentially transforms the original problem with four control variables to #3-D-grid sub-problems where the standard 1-D EGM can be applied. We use an adaptive grid to focus on the most dedicated part that needs fine tuning and test the size of the grid. We use parallel computing with the computing clusters provide by UNSW Katana to further speed up.