Back to The Funding Ratio! Improving Incentives and Retirement Security in Defined Contribution Plans with a Hedgeable Liability Measure*

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Abstract

Defined Contribution (DC) plans have no formal liabilities or defined targets in terms of retirement income for their beneficiaries, which leads to distorted performance metrics and incentives. To align pension fund manager incentives with the replacement income objectives of their beneficiaries, we define a new form of pension liability designed for DC plans, in which accrued benefits are equal to the affordable retirement income derived from past and current contributions to each retirement account. The proposed liability is *hedgeable*, meaning that the promised pension benefit can be financed with certainty with the given contributions to the fund. The new liability metric can be used to design performance metrics and asset allocation strategies that foster retirement security in DC plans.

Keywords: target date funds, target income, pension funds, manager incentives, asset allocation.

1 Introduction

Is it possible to reconcile the financial sustainability advantage of defined-contribution (DC) pension schemes and the retirement security provided by defined-benefit (DB) retirement plans? This paper proposes a retirement benefits accrual rule designed for DC plans that enables an adequate performance measurement and incentives design for pension fund management companies. As a result, it fosters retirement security while maintaining the financial sustainability nature of the DC scheme.

This question has become increasingly relevant given the massive transition from DB to DC pension schemes that has been taking place globally in the last three decades, both in

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nationwide public pension schemes as well as corporate pension plans.[1] The trend from DB to DC pension plans transfers the responsibility for retirement security to individuals, and as a consequence a large portion of the population is unlikely to be able to sustain their standard of living in retirement (Poterba, 2014). That transition has been fueled by the widespread defunding events observed in public and corporate DB plans in the United States (Novy-Marx and Rauh, 2011; Merton, 2014) and elsewhere (OECD, 2016; Mitchell and Piggott, 2016). This situation has occurred in spite of the development of sound liability-driven techniques to manage portfolios (e.g. Sharpe and Tint, 1990; Rudolf and Ziemba, 2004; Detemple and Rindisbacher, 2008; Martellini and Milhau, 2012, as well as references therein) and despite the fact that the sponsoring institution in DB plans has a strong incentive to manage assets in a way that avoids shortfalls relative to the promised benefits, because the latter constitute a liability for the institution. One possible cause for the systematic failure to avoid underfunding in DB plans is a structural problem in the current definitions of pension liabilities. Indeed, the way retirement benefits are usually defined in DB plans creates unhedgeable liabilities because the level of accrued benefits has a loose relation with the contributed funds.[2]

On the other hand, there is accumulating evidence that pension and investment risks are not properly managed within DC funds, as pointed out by several works such as Impavido et al. (2010), Merton (2014), Nijman and van Soest (2019), and van Bilsen et al. (2020). For example, the strategies followed by the widespread *target date funds*, use as "safe" assets (relatively) short-term government bonds, which reflect an asset-only perspective in their design, instead of focusing on retirement benefits.[3] While investment in short-term bonds commands low volatility in terms of asset value, the level of pension benefits that such investment can yield is as volatile as an investment in an equity index (see Merton, 2014, and Section 4.1).[4] In other

[1]Recent data show that in the United States, more than 70% of pension schemes follow a DC design, including employer-sponsored and individual saving accounts (Investment Company Institute, 2021). Globally, assets in DC plans surpassed DB assets for the first time in the seven largest pension markets in 2018 (Thinking Ahead Institute, 2019).

[2]In a typical DB plan for U.S. state employees, "an active worker accrues the right to a periodic benefit upon retirement that equals a flat percentage of his final (or late-career) salary times his years of service with the employer" (Novy-Marx and Rauh, 2011). Hence, the pension benefit depends on contribution time, and the latter only has a loose relationship with the level of contribution dollars needed to finance the promised benefits.

[3]For instance, the bond benchmark index used by target date funds, such as the LifePath Index 2055 Fund of BlackRock, which is the Bloomberg Barclays US Aggregate Bond Index, presents a duration fluctuating around an average of 4.9 years between 1978 to 2019. Blackrock reports $200 billion in "LifePath Target Date fund's assets."

[4]To illustrate this point, Merton (2014) presents an example of a worker with $1 million in retirement assets invested in T-bills. If these assets are converted into an inflation-protected annuity with a return of 4% to 5% he will receive "*an annual income of $40,000 to $50,000. But now suppose he retires a few years later, when the return on the annuity has dropped to 0.5%. His annual income will now be only $5,000*" plus inflation adjustments during the rest of his life.

words, investment strategies in DC funds with an asset-only focus leave beneficiaries exposed to the main investment risk in the retirement problem, which is interest rate risk. Consequently, pension benefits remain very uncertain even towards the end of the accumulation period. This mismatch between the duration of DC funds' bond portfolios and the terms of the pension cashflows is referred to as the *"duration puzzle in life-cycle investment"* (van Bilsen et al., 2020).

It is highly plausible that the explanation for the persistence of the *duration puzzle* is the inadequacy of current incentives in DC systems. Indeed, regulatory incentives for pension fund managers of DC plans, such as performance metrics and payment functions, usually have an asset-only focus. For example, the management fees of managers in DC plans in most cases are either a fixed proportion of assets under management (AUM) or a fixed portion of mandatory contributions, usually proportional to labor income (see Han and Stańko, 2020). In some cases, management fees are complemented with a "performance fee" proportional to *asset* returns in excess of a benchmark portfolio, whenever the excess return is positive.[5]

The missing focus on retirement income in DC plans' objectives and incentives contrasts with the liability-driven strategies and performance metrics that are largely in use by fund managers in DB systems.[6] In the absence of a formal definition for pension liabilities in DC plans, which constitutes an explicit target level of income benefits for the affiliates, and given the current asset-based payment functions and performance measures of DC pension fund managers, it is perhaps not surprising that investment risks are not managed to the best interests of the beneficiaries. After all, the emergence of sound liability-driven investment and risk management strategies in the DB pension space are intimately linked to the presence of explicit pension liabilities, which leads to the use of the assets-to-liability ratio, or the surplus (i.e., assets minus liabilities), as performance metric.

In this context, we address central problems in DB and DC systems by introducing a new pension liability definition that is *hedgeable* (see Section 2), in the sense that the accrued benefits can be financed with the given contributions with certainty. In Section 3 we discuss how, according to contract theory principles, the corresponding liability-driven performance metrics for pension funds lead to the key incentive alignment between the agent (the pension

[5]The benchmark is usually a "balanced" or static mix of indices of several asset classes, or a peer-based benchmark portfolio.

[6]For instance, according to their investment policy statement, the U.S. Pension Benefit Guaranty Corporation *"Utilize Liability Driven Investment (LDI) techniques to minimize funded status volatility and the risk of future deficits"* (PBGC, 2019).

fund management company) and the principal (the beneficiaries). We then introduce a new class of simple liability-driven asset allocation rules called *target income strategies*, which during the accumulation phase secure a strictly increasing and known level of retirement income benefits in DC plans, while capturing part of the upside potential of risky assets. In Section 4 we present an empirical analysis comparing the target income strategies with the typical glide-path allocation of target date funds, and find that the benefits in terms of risk management (e.g., derisking as retirement approaches and securing retirement benefits during the accumulation phase) are very substantial. Section 5 concludes, and the appendix collects technical derivations.

2 A Hedgeable Liability Definition for DC Plans

The first step towards effective risk management in pension funds is to define a safe asset for the accumulation period with respect to the retirement benefits that it generates. In accumulation, the proper safe asset is well-known to be an inflation-linked deferred annuity, designed to deliver a lifetime replacement income stream in retirement that guarantees a fixed purchasing power in terms of consumption goods (see Yaari, 1965; Brown, 2001). While they can be regarded as safe assets for retirement, the observed demand for annuities remains low. To try and understand this so-called "annuity puzzle," Brown (2001) recognizes that a life-cycle portfolio choice model only partially explains the empirical choices of individuals concerning annuitization (Brown, 2001). In a related effort, Pashchenko (2013) surveys various reasons that explain the low level of annuitization, such as the existence of "pre-annuitized" wealth (Social Security and DB plan benefits), adverse selection ruling out groups with higher mortality, minimum investment requirements, high surrender charges, and the perceived cost-inefficiency of annuities, which are not sold at an actuarially fair price (Friedman and Warshawsky, 1988, 1990), the fact that they do not contribute to bequest objectives, and the fact that annuitized wealth cannot be recovered in the form of capital even if the beneficiary experiences a severe health problem that would generate large expenses (Peijnenburg et al., 2017). As a result of these limitations, if deferred inflation-linked annuities are, in principle, the safe retirement assets, they cannot be regarded as such in practice.

Against this backdrop, one may instead use so-called *retirement bonds* as a proxy for the safe retirement asset (Martellini and Milhau, 2020). A retirement bond is a fixed-income security that pays $1 of replacement income every period, e.g., month or year, subject to a cost of living adjustment (COLA), starting at the retirement date T, for a period given by the life expectancy of the cohort after retirement, e.g., $M = 20$ years[7], thus providing replacement income, say from age 65 to age 85 (see Merton and Muralidhar, 2017, for the case of a closely related type of bond called retirement SeLFIES, with cash-flows indexed with respect to aggregate per capita consumption to hedge standard-of-living risk).

One key difference between retirement bonds and deferred annuities is that the latter offer longevity risk protection while the former do not. This is not a severe limitation, however, since protection against extreme longevity risk beyond the maturity date of the retirement bond for a given retiree reaching age 85 can be achieved by purchasing a deferred late-life annuity at retirement date in individual DC plans (Horneff et al., 2020), or by risk-sharing within members in collective decumulation solutions (Milevsky and Salisbury, 2015; Fullmer, 2019). In principle, if enough people of a given cohort held retirement bonds, as a group, they could exchange such bonds for life (indexed) annuities with an insurance company with a rate close to one-to-one in terms of real dollars of retirement. This is because such bonds would represent a hedging asset for an insurance company issuing such annuities for individuals in that cohort because the insurer can use the cash-flows of the bonds it obtains from the individuals that die before the life expectancy of the cohort to pay the annuity cash-flows of the individuals that survive past the life expectancy. In this sense, the price of the retirement bond is a proxy for the cost of financing one dollar of consumption throughout the life of each individual after retirement, taking into account the potential risk-pooling benefits that can be tapped into the payout phase.

Retirement bonds are currently not traded in fixed-income markets. However, their fair price at any date t during the accumulation phase (B_t), can be obtained by standard no-arbitrage arguments, that is, by discounting future replacement income cash flows at the zero-coupon rates of appropriate maturities (See Appendix A for details about the pricing of retirement bonds). That retirement bonds can be priced by arbitrage arguments implies that their payoffs can also be replicated using standard sovereign bonds via a suitable dynamic hedging strategy (see Mantilla-Garcia et al., 2019). Hence, in practice, B_t would be the (rebased) share price of an investable proxy of the corresponding retirement bond. Note that B_t would be the price of the retirement bond for the cohort of people with target retirement date T, and there would be a retirement bond per cohort.

[7]The life expectancy at age 65 for a generic US individual was 19.5 years in 2018 (see Table B in National Vital Statistics Reports, 2021: Arias et al., 2021), hence 20 years after rounding.

When the retirement date arrives, and the affiliate needs to start receiving replacement income from their retirement account, the accumulated assets are exchanged for a stream of cashflows. Different types of decumulation products exist, but as discussed above, the price of a retirement bond provides a good proxy for the exchange rate of wealth to income. Furthermore, if the individual already has part of its retirement assets annuitized (hence having some protection against longevity), they could use the retirement bond as the decumulation product to secure a known level of income during the life expectancy of their cohort. Although the bond does not hedge extra longevity risk, the fact that it remains liquid even throughout the payout phase has at least two important advantages relative to annuity contracts, namely providing flexibility in case of liquidity needs (related to health issues for instance) and serving bequest motives.

Remark 1 *If the retirement bond is the decumulation product used, then the price of this bond at any date during the accumulation period, $t \leq T$, constitutes an exact exchange rate of accumulated assets for a known level of retirement income benefits. This is simply because with the current retirement assets A_t, beneficiaries could buy $\tilde{N}_t = \frac{A_t}{B_t}$ retirement bonds. Without any further transactions, they will receive $\tilde{N}_t$ indexed dollars per year with certainty from $T+1$ until $T+M$. For this reason, we use interchangeably the terms retirement bond and retirement (benefits) unit.*

Definition 1 (Securable retirement income or fund value in retirement units, $\tilde{N}_t$) *If the cost of acquiring one retirement unit at time "t" is B_t, then the value of a portfolio in terms of retirement units is denoted:*

$$\tilde{N}_t := \frac{A_t}{B_t} \tag{1}$$

The latter quantity also constitutes the number of real dollars of income throughout retirement that the individual could secure at any time $t \leq T$ during the accumulation phase, by selling all assets and using the proceeds to buy $\tilde{N}_t$ retirement bonds. The bonds to be used for this valuation, are the ones that start paying cashflows at the target retirement date T of the beneficiary.

In standard DC plans, the level of income benefits $\tilde{N}_T$ that a beneficiary will receive in the payout phase, remains fully unknown during the accumulation phase, regardless of how much contribution the beneficiary makes to its retirement account. While $\tilde{N}_t$ increases with the contributions to the retirement account, it also fluctuates proportionally to the relative performance of the assets in the fund with respect to the cost of a retirement unit B_t. Indeed, a drop in asset value, an increase in B_t, or a combination of the two, can outweigh the effect of the extra contributions.

Remark 2 *DC plans have been considered by convention or "by definition" to be "always fully funded" (Yermo, 2003). In that sense, $\tilde{N}_t$ could be interpreted as the retirement income benefits that the members of a standard DC plan have accrued until any time t during accumulation. Using the treasuries' yield curve to discount the cashflows from $\tilde{N}_t$ retirement bonds (which is equivalent to use the retirement bond as discount factor), the present value of those accrued benefits is $A_t = \tilde{N}_t B_t$. Under this interpretation, the funded status of the DC plan is always 100%, regardless of how the asset portfolio is managed. Moreover, the level of accrued benefits $\tilde{N}_t$ can decrease over time, which stands in stark contrast with the dynamics of accrued benefits in DB plans, where the latter always increase with every additional contribution year from the beneficiary, and remains constant otherwise. Hereafter we propose a way to define a hedgeable liability metric for DC plans, in which the accrued benefits of plan members increase with each extra contribution dollar and remains constant otherwise.*

Measuring Liabilities in DC Plans

Pension fund managers of standard DC plans do not have any commitment or objective to fulfil in terms of retirement benefits. In contrast, the beneficiaries bear the risk of ending up with insufficient replacement income, and only learn about this upon retirement, at which points they cannot do much about it. Furthermore, while performance metrics and management fee structures in DC plans only focus on asset returns, the retirement income level, $\tilde{N}_t$, also depends on the variations in the cost per retirement unit B_t, which is even more volatile than the value of an equity index (see Section 4.1). As a consequence, pension fund managers in DC plans have no explicit incentives to manage the risks related to the (large) variations in the "exchange rate" of assets to retirement income. This constitutes a major misalignment in risk exposures between the plan managers and the plan members, and a disconnection between the plan managers' incentives, and the beneficiaries' retirement income objectives. In order to address this central problem of DC plans, we propose to introduce a formal measure of "liability" or commitment, expressed in terms of a defined level of retirement income that is proportional to the financial value of the contributions received by the pension fund.

In general, the total pension liability is the aggregation of the present value of all the plan members' benefits. Each plan member accrues benefits as a counterpart to their periodic contributions to the plan and/or the contributions done in their behalf by plan sponsors. In the Accumulated Benefit Obligation (ABO) and the Projected Benefit Obligation (PBO) accounting methods, a standard benefits accrual rule in DB plans for an individual after s_t periods (e.g. years) of service can be expressed in number of retirement units as[8]:

$$N_t^{DB} = \alpha \times s_t \times \hat{N}_T \tag{2}$$

where α is a constant percentage called the *pension benefit factor*, and $\hat{N}_T$ denotes the reference labor income to which the benefits are pegged.[9]

There are several reasons for which the accrued benefits in DB plans following rule (2) cannot be guaranteed in general, without the need of supplementary contributions. First, variations in the reference labor income to which benefits are pegged (usually the final or late-career salary), are not hedgeable, because there is no investment vehicle in the market that correlates perfectly with changes in real labor income per individual. Second, benefit accruals are the same every year regardless of the dollar amount contributed each year, and whether the contribution is done at the beginning or towards the end of the accumulation period. Hence, there is a disconnection between the financial value of the contributions and the accrued benefits in DB systems (see Appendix C for a more formal and detailed development of the previous arguments). As a result, DB liability definitions eventually create shortfalls which require contributions from the plan sponsor.

Instead, we propose a new accrual rule for which the benefits accumulated are directly proportional to the affordable retirement income that would have been generated with the past contributions to each retirement account, and the resulting pension liability or commitment is *hedgeable*. We define a hedgeable pension liability as one that can be financed with certainty with the contributions to the fund using a suitable replicating strategy, here investing in the retirement bond. Assume that the series of contribution dollars are given, although unknown *ex-ante*, as it would be the case if contributions are equal to a fixed proportion of the labor income of each individual. Also assume every individual has the option to use each of their

[8]Recall that each retirement unit/bond already embeds COLAs after the retirement date.

[9]In the ABO method, such reference is equal to current salary, while in the PBO the reference income includes an estimation of the expected increase in salary.

periodic dollar contributions C_t to buy $n_t = \frac{C_t}{B_t}$ retirement bonds. In general, this strategy secures with certainty an incremental benefit (in retirement income dollars) equal to $\Delta N_t = n_t$, with a dollar contribution amount of C_t at any time t during the accumulation phase. This leads to the following hedgeable definition of accrued pension benefits.

Definition 2 (Accrued Benefits, DC Liability and Affordable Retirement Income N_t) *Let C_t be the value of the contribution to the pension fund at time "t," and $n_t = \frac{C_t}{B_t}$ the number of retirement bonds that can be purchased with that contribution. If all the periodic contributions until time "t" were used to buy retirement bonds, then the number of real dollars of replacement income that the affiliate could afford without taking any risk, by any date during the accumulation phase $t \leq T$ is:*

$$N_t = \sum_{i=1}^{t} n_i \tag{3}$$

In general, pension liabilities are the present value of accrued benefits. Hence, the liability of the pension fund at any time "t" during the accumulation period is defined as the present value of the cash-flows that pay with certainty N_t retirement bonds:

$$L_t := N_t B_t \tag{4}$$

Remark 3 *At the plan level, the global liability portfolio can simply be obtained via aggregation of individual liabilities.*

Remark 4 *Definition 2 is an observable and objective measure of liability, as it does not depend on subjective estimates of expected salary increments or future service/contributions.*

Remark 5 *Unlike $\tilde{N}_t$, the accrued benefits level N_t is a non decreasing quantity at all times.*

Remark 6 *From an economic standpoint, discount rates should reflect the risk of the discounted cash-flows (Novy-Marx, 2013). The liability Definition 2 in Eq.* (4) *uses the Treasury yield curve to calculate the present value of the accrued benefits (because B_t is the present value of a series of certain cashflows). This coincides with the logic behind the use of a retirement bond, which is to provide a stream of cash-flows to secure consumption in the payout phase.*

Remark 7 *The number L_t also represents the current dollar value of a fund in which all past and present contributions have been invested in retirement bonds.*

3 Retirement Income Incentives and Target Income Strategies

Using the hedgeable liability definition for DC plans introduced in Section 2, it is possible to design incentives for pension fund management companies in DC plans to manage assets during the accumulation phase in ways that downside risk in terms of retirement income, while pursuing the aspirational goal of an income level that would exceed the level affordable without risk taking (i.e., N_T).

According to contract theory, a well-designed incentive scheme should depend on a performance measure that is informative about the agent's actions, and aligns with the principal's objective (Baker, 1992). However, it is difficult to find a measure satisfying both conditions in several contexts. It is more common to find so-called *broad* measures that the agents cannot control/affect well, but that induce high alignment of preferences, or *narrow* measures that the agents control well but imply low alignment of preferences (Hall, 2015). If a measure is not informative about the agent's actions, that means the agent has low control over the observed outputs that the principal receives/observes, and also has high risk exposure to the volatility of the performance measure. That results in weak incentives for the agent. On the other hand, if the performance measure is not aligned with the principal's objectives, the agent's effort is not useful for the principal, and the incentives are distorted.

As previously mentioned, the pension liability of DB plans is unhedgeable, meaning that the manager is unable to prevent funding deficits with certainty. Hence, DB funding ratios tend to be *broad* measures of performance, because the agent has very limited control over the output, and a high risk exposure to its variability. On the other hand, management fees in DC plans usually have the same structure used by regular asset management companies, i.e., a fixed percentage of year-end asset levels:[10]

$$Q_t^{\$} = b\ A_t \quad \text{with} \quad 0 < b < 1 \tag{5}$$

We argue that the payment function (5) provides inadequate incentives in the context of pension plans for two main reasons. First, the performance measure, i.e., increase in asset value , is not in the same units as the principal's benefits, which results in distorted incentives for the

[10] Another standard payment structure for pension fund managers is to charge a fixed percentage of the contributions to retirement accounts (see Han and Stańko, 2020). The latter fee type has the problems of both, the broad and the narrow metrics because, on the one hand it does not incentivize efforts to improve retirement income benefits for affiliates, as it is the principal, not the manager, that controls contributions (when the latter are not mandatory), and it also has the problem of being delinked from the level of pension benefits.

manager. Indeed, if the asset value in a given year increases, but at the same time the "exchange rate" of assets to retirement income $1/B_t$ decreases by a larger percentage, the payment for the manager would increase, even though the affiliate has experienced a loss in terms of securable retirement income $\tilde{N}_t$. Hence, the payment function (5) induces an asset-only focus on pension fund strategies, instead of measuring performance in terms of retirement benefits. Second, unlike regular investment funds the investor (principal) in pension fund schemes receives benefits from the fund only after their retirement date, and these benefits are spread over about 20 years on average after that date. In other words, there is a large mismatch in the benefit schedules of the principal and the agent (the fund manager) in pension funds, which does not exist in the delegated portfolio management arrangement of regular investment funds.

Note that the agent and principal's benefit units mismatch would not be resolved by paying the agent a number of retirement bonds $b\tilde{N}_t$, because in principle the agent can sell them at the market price B_t, in which case his payment in current dollars would be effectively bA_t, which would take us back to the payment function 5. This constitutes another example of the irrelevance result in incentives design of delegated portfolio management (see Admati and Pfleiderer, 1997; Stoughton, 1993). One solution to irrelevance situations is to restrict the manager from undoing, with transactions in their personal portfolios, the incentives given by the payment function. In this case, one could try restricting the agent (e.g., by regulation) from selling the retirement bonds $b\tilde{N}_t$, so that the agent's payment schedules would match with the principal's one. However, notice that while the units and schedules align under the selling restriction, the risk exposures do not. To see this, note that the number of retirement bonds (hence the future income) that the agent accrues during the principal's accumulation period, is strictly increasing. On the other hand, the principal's accrued retirement income benefits in a DC system without liability, which are equal to $\tilde{N}_t$, fluctuates all the time during the accumulation phase. Hence, it would be possible that after paying many retirement bond units to the agent, the principal loses many more units if the value of $\tilde{N}_t$ decreases (either due to asset losses, or increases in B_t). Furthermore, given the "forced" position in retirement bonds that the manager holds, the latter would have a strong incentive to take on more risk, most likely beyond what would be optimal for the principal.

Using our the liability measure as a benchmark to compare the year-end asset levels of the pension fund would alleviate these concerns. Recall indeed that Definition 2 sets the accrued benefits of a DC plan as the pension cash-flows that would have been secured if all past and present contributions had precisely been invested in retirement bonds. Note that such a risk-free investment policy is not necessarily optimal since beneficiaries may wish/need to reach aspirational levels of replacement income that may not be affordable in the absence of risk-taking. However, these liabilities can be seen as a natural benchmark against which can be measured the value-added by the risk-taking decisions of the pension fund manager. Once a formal liability portfolio process L_t is obtained, the corresponding funding ratio, and the surplus/shortfall of a DC plan (following any kind of investment strategy) can be measured similarly to defined-benefit pension plans.

Definition 3 (Funding Ratio (FR_t) and Surplus (S_t) for DC plans) *Given the liability process L_t from Definition 2, the corresponding funding ratio is given by:*

$$FR_t := \frac{A_t}{L_t} = \frac{\tilde{N}_t}{N_t}$$

and the Surplus or Shortfall in current dollars is:

$$S_t := A_t - L_t$$

A funding ratio greater than 1 means that purchasing power in terms of replacement income has increased since the beginning of the period relative to the riskless alternative, and a ratio lower than 1 that it has decreased. In other words, an investment strategy is successful ex-post if it leads to an increase in replacement income, that is, if it outperforms the retirement bond liability portfolio over a given period.[11]

First, note that the DC funding ratio and surplus align the performance measures of the manager, with the benefits of the principal. To see this, note that if the level of securable retirement income $\tilde{N}_t$ increases (decreases) on a given year, either due to increases in the asset value A_t or to decreases (increases) in the retirement unit's cost B_t, then the payment for the manager also increases (decreases).[12]

Second, note that the securable level of retirement income, $\tilde{N}_t$, mixes the responsibility of

[11]Note that if the first date t_0 at which the pension liability is estimated is posterior to the creation of the plan, t_0 can be taken as the starting date of a fresh risk management perspective and treat A_{t_0} as if it was an initial contribution made at the start of a new pension plan. In that case, since $A_{t_0} = C_{t_0}$, then the initial funding ratio given the Definition 2, is $FR_0 = \frac{A_0}{L_0} = 1$. To see this, note $n_{t_0} = \frac{C_{t_0}}{B_{t_0}}$, and hence $L_{t_0} = \frac{C_{t_0}}{B_{t_0}} B_{t_0} = C_{t_0}$. An alternative approach would be to perform ex-post backfilling starting from the origin of the pension plan as long as a recording schedule of past contributions is available.

[12]To see this, recall that $L_t = N_t B_t$, and N_t only changes its value with new contributions.

the pension fund manager (i.e., the investment strategy), with the contributions to the fund. On the other hand, the funding ratio and surplus measures are less dependent on the contributions, as the latest appear in both the numerator and denominator (or cancel out in S_t),

$$\begin{aligned} S_{t+1} &= A_{t+1} - L_{t+1} \\ &= A_t(1 + r^A_{t+1}) + C_{t+1} - L_t(1 + r^B_{t+1}) - C_{t+1} \\ &= A_t(1 + r^A_{t+1}) - L_t(1 + r^B_{t+1}) \end{aligned} \tag{6}$$

In that sense, the funding ratio and the surplus are more accurate metrics of investment performance that $\tilde{N}_t$. Furthermore, it means that the fund manager has more control over the realized values of the performance metrics compared to the assets level in the standard payment function (5), thus making these more balanced performance metrics in the sense that they provide stronger non distorted incentives.

A related metric of performance in the context of pension fund management is the *excess coverage*, discussed in Detemple and Rindisbacher (2008). They derive the optimal strategy of a pension plan manager/sponsor maximizing expected terminal excess coverage, which is defined as

$$E_t = A_t - \kappa L_t \qquad \text{with} \qquad 0 < \kappa < 1 \tag{7}$$

which is the excess of liquid wealth over a minimum liability coverage κ. There is a fundamental difference when the excess coverage is calculated relative to our hedgeable definition of pension liability for DC plans, which is the fact that underfunding below the minimum tolerable coverage level κ can be avoided in all cases and at all times by the fund management company, using the simple asset allocation rules described hereafter. In that sense, the excess coverage for DC plans is a more balanced performance measure, i.e., less broad, than the excess coverage of DB plans.

A simple but powerful form of incentive would be to require pension fund managers of DC plans to report the funded status, or funded surplus relative to the DC liability definition 2. Another form of incentive would be to create regulations fostering strategies that control funding status in DC plans (as the target income strategies presented hereafter), similar to the existing legal incentives to use target-date funds in the U.S. consisting in including them as a default option in pension plan menus. Finally, an even stronger incentive, would be to have a contract design in which the payment is proportional to the excess coverage (7).[13]

Target income strategies

In DB arrangements, plan sponsors are commonly required to make supplementary contributions whenever the funded status falls below a predefined percentage level, κ. This implies a strong liability-driven (and target-income driven) incentive. Using the hedgeable DC liability Definition 2, this type of goal can be introduced in DC plans as the objective of maintaining a minimum level of funded status:

$$FR_t = \frac{\tilde{N}_t}{N_t} \geq \kappa \quad \forall t \tag{8}$$

Note that the objective in Eq. (8) of maintaining a minimum level of funded status fosters retirement security as it is equivalent to secure a level of retirement income that increases with every new contribution, equal to

$$\tilde{N}_t \geq \kappa N_t \quad \forall t \tag{9}$$

If the liability portfolio N_t is defined in terms of an investable alternative for B_t, i.e., a retirement bond or a reference liability-hedging portfolio that replicates its value, then it is straightforward for the pension fund management company to *secure* the increasing level of retirement income in Eq. (9) for its beneficiaries, by holding a number of retirement bonds (or shares of the corresponding replicating fund), at all times, equal to κN_t, and investing the remaining capital in any risky performance-seeking portfolio (PSP) of choice.

This simple buy-and-hold strategy has several advantages. First, it would increase retirement security in DC systems since it protects a never decreasing minimum level of retirement income for beneficiaries known at every step during the accumulation phase. Second, it provides an important signal to beneficiaries, because the secured level κN_t is expressed in income dollar terms, which is a metric that most people can relate to. Third, the secured income floor level also provides beneficiaries with a higher level of control over their future prospects. Indeed they do not only observe but also can take action to increase their minimum level of secured pension

[13]Note that for such payment function, $Q_t^{\$} = b\ (A_t - \kappa L_t)$, if the funded status falls below the minimum tolerance level κ, then the fund management company would share losses with the beneficiaries. The latter in the sense that $Q_t^{\$} < 0$, and that money paid as penalty would be reinvested in the fund to partially compensate the beneficiaries for the loss. This is similar in spirit to the additional contributions that plan sponsors have to make in a DB schemes in cases of severe defunding. However, the hedgeable property of the liability implies that the penalties can be avoided in all cases by the fund management company, by using the simple asset allocation strategies discussed below.

benefits κN_t by making potentially extra contributions. Thus the income level N_t increases with extra contributions and stays constant otherwise in real terms (in nominal dollars, the minimum income secured also varies with the CPI level). We refer to this strategy in short as FR, as it ensures that the funding ratio always stays above a constant predefined proportion κ. Investors with higher loss aversion would prefer strategies with a higher protection level κ.

Clearly, the protection level κ is a key parameter as it determines the riskiness of the strategy. Common values for the minimum funding ratio levels to be protected in DB plans are 80% and 70%.[14] Nonetheless, the objective of protecting at all times a fixed minimum funding status level often given to the sponsor/manager of DB plans, has been criticized in the sense that it can result in overly conservative asset allocation strategies, implying a high opportunity cost for beneficiaries.[15] This kind of criticising also applies to the FR strategy described above. Indeed, the resulting proportion of assets allocated to the PSP is[16]

$$\omega_t^{PSP} = \frac{\tilde{N}_t - \kappa N_t}{\tilde{N}_t} = \frac{A_t - \kappa L_t}{A_t} \tag{10}$$

Some of the variations in the PSP weight ω_t^{PSP} will depend of the movement of the relative value of the PSP with respect to B_t. However, ω_t^{PSP} will be largely determined by the protection parameter level κ, as the strategy simply buys an extra number $\kappa \times n_t$ of retirement bonds at every new contribution date t, and holds them until maturity.[17] In the historical simulations presented in Section 4.2, we find that the average equity allocation across scenarios of that strategy, stays roughly constant at $1 - \kappa$. As illustrated in Section 4.2, the FR strategy with constant $\kappa = 0.7$ tends to have very limited upside potential, as its exposure to the PSP remains relatively low throughout the accumulation period.

In general, there is no particular reason to include a restriction to protect at all times a constant minimum level of funding ratio. In fact, any possible path of the funding ratio during the accumulation phase that end up in the same level of the funding ratio level at the retirement

[14] In the U.S., corporate DB plans and private sector multiemployer plans had a statutory mandate to maintain their funding level above 80 percent at all times (NASRA, 2012), while credit rating agencies use funding ratio as a general indicator of a public plans financial health. For example, "Fitch generally considers a funded ratio of 70% or above to be adequate and less than 60% to be weak, while noting that the funded ratio is one of many factors considered in Fitch's analysis of pension obligations." (Fitch, 2011).

[15] For instance, Brainard (2011) argues that "The fact that a plan is underfunded is not necessarily a sign of fiscal or actuarial distress; many pension plans remain underfunded for decades without causing fiscal stress for the plan sponsor or requiring benefits to be reduced." See also Martellini and Milhau (2012) for a formal analysis of the opportunity cost of short-term funding constraints.

[16] The second equality follows from the definitions $\tilde{N}_t = \frac{A_t}{B_t}$ and $N_t = \frac{L_t}{B_t}$ (see Eqs. (1) and (2)).

[17] To see this, note that the strategy consist in holding at all times a number of retirement bonds equal to $\kappa N_t = \kappa \sum_{i=1}^{t} n_i$.

date, will generate the same level of retirement income. In that sense, we propose instead to define an increasing protection proportion process κ_t, that is expected to converge to a target protection level *at retirement* of κ. In other words, instead of keeping a constant risk budget parameter throughout the accumulation period, we propose securing an increasing proportion κ_t of the affordable retirement income N_t, so as to take more risk when the investors are younger. This follows a similar logic to the asset allocation dynamic of target date funds, but decreasing risk as measured in number of retirement income dollars, instead of focusing on the variations in asset value only. In Section 4.2 we show that the change of measurement units from current dollars to retirement units, and using retirement bonds instead of short-term bonds as the safe asset, constitutes a very large difference in income benefit terms.

Particularly, we propose defining the proportion κ_t as:

$$\kappa_t^{CFR} = \left(1 - (1-\kappa)\frac{\hat{N}_T}{N_t}\right)^+ \tag{11}$$

with $0 < \kappa < 1$. The process κ_t^{CFR} never decreases, and increases (only) with every new contribution n_t. Hence, if there are frequent contributions, the protected proportion κ_t^{CFR} increases as the retirement date approaches. The parameter $\hat{N}_T$ is a target level of affordable retirement income, and is also the target level of cumulative contributions in number of retirement bonds. This level should be set taking into account the characteristics of each individual, particularly age and expected income.[18]

In general, the actual level of total contributions that will be reached by the retirement date, i.e., N_T, is unknown. This is particularly true if the contributions are defined as a fixed percentage of labor income (which is often the case in DC plans).[19] Hence, from the definition in Eq. (11), it follows that:

$$\kappa_T^{CFR} \begin{cases} \kappa_T^{CFR} = \kappa, & \text{if } N_T = \hat{N}_T \\ \kappa < \kappa_T^{CFR} < 1, & \text{if } N_T > \hat{N}_T \\ 0 < \kappa_T^{CFR} < \kappa, & \text{if } N_T < \hat{N}_T \end{cases} \tag{12}$$

[18]In the empirical simulation in Section 4.2, we set it as the expected level of income before retirement, based on the average age-income profiles per education level, constructed from data for average labor income series for workers in the United States (U.S.) for age-income profiles per education level based on the Panel Study of Income Dynamics (PSID), as in Cocco et al. (2005).

[19]In principle, it is possible to vary the percentage of labor income contributed to the retirement account, such that a given path of predefined contributions that sum to $\hat{N}_T$ is met at each point in time. However, we focus here on the case of current contribution paths in DC plans.

Furthermore, the minimum retirement income that is ultimately insured by the strategy equals the target floor level plus the difference between the target and the total level of contributions:

$$\kappa_T^{CFR} N_T = \kappa \hat{N}_T + (N_T - \hat{N}_T)$$

The above implies that if an overshoot in contributions occurs, relative to the target level, i.e., $N_T > \hat{N}_T$, then the minimum level of retirement income that the strategy secures, would be higher than the target floor level, i.e., $\kappa_T^{CFR} N_T > \kappa N_T > \kappa \hat{N}_T$. Conversely, if an undershoot in contributions occurs, a proportion lower than κ of N_T is insured. The latter situation does not necessarily result in a lower level of retirement income. In fact, in those cases, the strategy takes more risk by allocating more to the performance-seeking portfolio, which may increase the changes of achieving an income level of $\hat{N}_T$ (or even higher). However, the resulting level of retirement income in the most unfavorable scenarios, would be lower.[20]

Consider the simple strategy of holding a number of retirement bonds (or shares of the corresponding replicating fund) at all times equal to $\kappa_t N_t$. This implies an allocation to the risky PSP of

$$\omega_t^{PSP} = \frac{\tilde{N}_t - \kappa_t N_t}{\tilde{N}_t} = \frac{A_t - \kappa_t L_t}{A_t} \tag{13}$$

and $\omega_t^{LHP} = 1 - \omega_t^{PSP}$, where LHP stands for liability-hedging portfolio which in this case is the retirement bond. Note that κ_t cannot be any arbitrary unrestricted process. For instance, if κ_t increases and at the same time the value of the portfolio in retirement units $\tilde{N}_t$ decreases enough, then the condition $\tilde{N}_t \geq \kappa_t N_t$ could be violated. In Appendix D, we show that a sufficient condition for the allocation rule (13) to ensure that $\tilde{N}_t \geq \kappa_t N_t$, at every time t for every possible value for the risky assets' returns and interest rates levels, is that the dynamics of κ_t satisfies:

$$\Delta\kappa_t \leq (1 - \kappa_t) \frac{n_{t+1}}{N_{t+1}} \tag{14}$$

where $\Delta\kappa_t = \kappa_{t+1} - \kappa_t$. Hence, even in the extreme event that the risky PSP has a return of -100%, following the simple allocation rule (13) with a κ_t process that satisfies condition

[20] In practice, a possible modification of the strategy, is to make revisions of the target income level $\hat{N}_T$, depending on whether the expected overshoot or undershoot is too large, as the uncertainty about the contributions resolves over time. While securing a lower proportion of N_t, which implies a higher risk exposure, can increase the chances of achieving the target level of retirement income $\hat{N}_T$ or even higher, it also increases the chances of a larger shortfall. Indeed, depending on the level of risk aversion of the investor, the expected surplus might not compensate the extra level of risk taken. Conversely, if the contributions grow past the initially planned target $\hat{N}_T$, the proportion of affordable income that is secured is higher than κ. In that case, the investor may prefer to take more risk, by revising upwards the target level $\hat{N}_T$.

(14), guarantees that the portfolio's value remains above its floor, $\kappa_t N_t$, and the funding ratio is always above the level κ_t.

Note that the allocation rule Eq. (13) together with condition (14) define a general class of *target income strategies*, as the process κ_t can take several forms as long as it respects condition (14). In Appendix E, we show that the process κ_t^{CFR}, defined in Eq. (11), satisfies the condition (14) with equality, hence the floor income process $\kappa_t^{CFR} N_t$ can also be secured following the allocation rule (13). Also note the simpler FR strategy that uses a constant risk budget, i.e., $\kappa_t = \kappa$ at all times also satisfies the condition (14) because $\Delta\kappa_t = 0$, for $\kappa < 1$.

The target income strategies provide retirement income security, as they protect a minimum level of secured income $\kappa_t N_t$ that never decreases, regardless of changes in interest rates and equity prices, and this level increases with every new contribution to the retirement account. This means that during the years prior to the retirement date, each beneficiary knows what is the minimum level of income benefits to expect, ensuring that no 'last minute' disappointments occur when the accumulated assets are converted into retirement income. It also means that the engagement level of plan members might increase, as they can also observe the direct impact of an extra contribution in the minimum income level that they can secure.

4 Empirical Analysis

4.1 Measuring asset risk in retirement units

Hereafter we present the result of historical analysis of the target income strategies introduced in the previous section and a comparison with the 'glidepath' allocation strategy followed by typical target date funds (TDF). The asset allocation strategy of TDFs is typically defined in terms of a decreasing equity allocation over time, with a common practice consisting of taking it equal to 100 minus the investor's age (as a percentage of assets). We simulate the performance of cohorts of investors that contribute to their retirement account during 35 years and assume a retirement age of 65 years. Hence, each investor starts to contribute to her retirement account at age 30, and the initial allocation to equities is 100-30 = 70 percent. The portfolio is rebalanced to have an allocation to equities that decreases linearly every month until it reaches 35% at the retirement age of 65. TDFs typically allocate the rest of the capital to a bond index fund with relatively low duration, e.g., 5 years approximately[21]. The reason is that these bond portfolios

[21] For instance, the Duration of the Bloomberg Barclays US Aggregate Bond Index during the period 1978-01-01 to 2019-09-30 presented a mean-reverting behavior around an average value of 4.9 years. This bond index is

have a relatively low short-term volatility, which is (mistakenly) considered as an indication that they represent low-risk components for plan members. As we illustrate hereafter, these assets are actually as risky as an equity index when risk is more appropriately measured in terms of retirement income dollars, due to the large duration mismatch they exhibit with the retirement cashflows that the portfolios aim to finance during the decumulation phase. In terms of expected retirement benefits, TDFs are not using an asset that allows the strategy to decrease risk effectively as the retirement date approaches, which results in a substantial welfare loss for individuals.

To illustrate this, we use historical data for US government bond yields[22] and the Nelson and Siegel (1987) model to simulate both the returns of a bond index with a constant duration of 5 years and the returns of the retirement bonds described in Appendix A. Because nominal bond yields are available since 1962 for several maturities while real bond yields have shorter history, we choose to simulate nominal retirement bonds thus allowing for a long sample period, including relatively long periods of increasing interest rates and decreasing interest rates. The retirement bonds we simulate pay a series of nominal cash-flows growing at a fixed COLA rate of 2% per year.

Several authors have reported 'numerical difficulties' estimating the parameters of the Nelson-Siegel model.[23] In order to avoid parameter estimation issues, we follow the 'data-based' definitions in Diebold and Li (2006), with the level factor proxied as the 10-year yield, the slope as the difference between the 10-year and the short-term yield, and the curvature as the twice the medium-term yield minus the sum of the short-term and 10-year yields. From 1962-01-0 to 1976-05-31, dates during which the 2-year yield ('TCMNOM_Y2') is not available in WRDS, we use the 5-year yield ('TCMNOM_Y5') as the medium term rate instead and from 1962-01-02 to 1997-01-01, dates for which the CRSP 3-month yield ('NFCP_M3') is not available, we use the 1-year yield ('TCMNOM_Y1') as the short-term rate. The switch in the yields that we use in the middle of the sample is done solely to have historical simulations that span the periods displaying the most significant movements in government yields, but the resulting time series has no abrupt changes around the switch dates.[24] Figure 1 presents the time series of the resulting

used as the bond benchmark by Target-Date funds, such as the LifePath Index 2055 Fund of BlackRock (they report $200 billion invested in "LifePath Target Date fund's assets").

[22]The source for the yields data is the database of the Federal Reserve of Saint Louis (https://fred.stlouisfed.org/series).

[23]For instance, Gilli et al. (2010) show that in certain ranges of the parameters, the model is badly conditioned, thus estimated parameters are unstable given small perturbations of the data. Indeed, the optimization problem for the parameter estimation is not convex and has multiple local optima.

[24]Indeed, the last 1-year yield used was 5.57% on 1997-01-01 followed by the first 3-month yield used of 5.30%

Nelson-Siegel model parameters used for the historical simulations.

The dataset includes monthly time series of the CRSP S&P 500 equity index total returns, the Nelson-Siegel parameters series described above from 1962-01-02 to 2019-12-30, and average labor income series for workers in the United States (U.S.) for age-income profiles based on the Panel Study of Income Dynamics (PSID), i.e., the largest longitudinal U.S. dataset containing information on labor income and individual control variables. The methodology for the age-income profiles follows Cocco et al. (2005). We simulate retirement accounts for an average individual in each of all the possible cohorts in the sample during their accumulation phase, assuming that each individual makes monthly contributions during 35 years until retirement. The monthly contributions are set to 12.4% of the estimated monthly income, which is the mandatory contribution in the U.S. (OECD, 2017). The real monthly income series is estimated as the annual income numbers from the age-income profiles divided by 12. The average income profile presents an inverted u-shape, reaching a maximum level by age 46 of 38.6k per year, and decrease to 26.38k by age 65 (see Figure 1 in Cocco et al., 2005).

We simulate the behavior of 277 retirement accounts, which is the number of months from 1996-12-31 (35 years after the sample's start) until 2019-12-30. For each cohort, we compare the hypothetical historical performance of the TDF with the two target income strategies introduced in Section 3.

For illustration purposes, Figure 2 displays the cumulative return of the equity index, the constant duration bond index and the retirement bond for two of the simulated periods. The left Panel corresponds to the simulation of the cohort retiring in 1996-12-31 (first cohort in the sample), which experiences a significant increase in interest rates levels, and the right Panel to the cohort retiring in 2019-12-31 (last cohort), which shows a large decrease in the level of interest rates. In the former period, the value of the equity and short-duration indices presents a larger increase than the retirement bond of the cohort, while for the latter period, the opposite occurs.

More generally, Table 1 presents summary statistics of the returns for the equity index, the constant 5-year duration bond index and each of the 277 retirement bonds. The median annualized return in the sample, across the 277 periods of 35 years each, is 11.32% for the equity index, 8.11% for the constant duration index and 8.68% for the retirement bond. The

on 1997-01-02. The last 5-year yield used was 7.73% on 1976-05-31 and the first 2-year yield on 1976-06-01 was 7.26%.

annualized return for the retirement bonds ranges from 3.1% to 21.2%, compared to 6.2% to 8.8% for the bond index, and 9.5% to 12.4% for the equity index (see Panel A). In particular, the quantiles 75% and above of the annualized return distribution are higher for the retirement bond than for the equity and bond indices, which indicate that in many scenarios, there is the potential of losses in terms of retirement income dollars, relative to the lowest risk alternative of investing in the corresponding retirement bond. Panel E in Table 1 presents the relative drawdown (RDD) of these two indices with respect to the corresponding retirement bond for each of the 277 sample periods. The RDD is defined as the maximum drawdown of the relative value process. The latter is $Z_t = \frac{A'_t}{B_t}$, where A'_t denotes the asset value, and B_t is the price of the retirement bond of the respective cohort. Thus, the RDD measures the maximum cumulative loss in number of retirement bonds that an investment in those indices could secure at each point in time during the accumulation phase. Despite the high correlation displayed between the bond index and the retirement bonds (ranging between 0.82 and 0.9, as shown in Panel D), its RDD ranges from 73.9% to 98%, which is very similar and even slightly higher than the RDD of the equity index for most quantiles (the RDD of the equity index ranges from 74.8% to 97.7%, and its correlation with the retirement bonds from 0.05 to 0.28). Hence, the constant-duration bond index is as risky as the equity index, when its variations are measured in units of relevance for pension fund affiliates.

In contrast, if one were to measure the risk of these assets in current dollars during the accumulation phase, instead of doing it in retirement income dollars, the assessment would have been opposite to the previous one, and misleading in the context of a pension fund. For instance, as Panel B of Table 1 shows, while an investment in retirement bonds has by definition zero variations in terms of the number of securable retirement income dollars, its value in current dollars varies significantly during the accumulation phase, and presents significantly higher short-term volatility than the equity and bond indices. The annualized standard deviation of its monthly returns ranges between 31.8% and 49.6% compared to average volatilities of 14.4% to 15.9% for the equity index and 6.1% to 9.3% for the constant-maturity bond index. Nonetheless, investors in a pension fund usually do not have any benefits during the accumulation phase from their retirement accounts, and hence, these variations do not correspond to the relevant units for them, as the asset level alone, is a misleading metric of the expected level of benefits.

4.2 Target income strategies vs. target date funds

A widespread rule of thumb to set a target level of replacement income that would approximately sustain living standards after retirement, corresponds to a replacement rate of 70%. Hence, to simulate the four target-income strategies described in the previous section, we set $\kappa = 0.7$ and $\hat{N}_T$ equal to the average labor income at age 65 in real dollars.[25] In the historical period analyzed, the final level of contributions N_T obtained from investing 12.4% of the average income figures, is more than the target level of contribution $\hat{N}_T$ for 275 out of the 277 cohorts, and the minimum contribution ratio, i.e., the lowest level observed for $\frac{N_T}{\hat{N}_T}$ is 99.3%. As discussed in Eq. (12) in Section 3, the effect of an 'overshoot' in contributions for the strategy with horizon derisking (denoted CFR), relative to the target contribution level $\hat{N}_T$, is securing a proportion of the affordable retirement income above the predefined level $\kappa = 0.7$ by the retirement date, i.e., $\kappa_T > \kappa$. The effect of an 'undershoot' on the expected contributions, is that the strategy ends up securing a lower proportion of the affordable retirement income N_T. Notice that a lower secured proportion does not necessarily result in a lower level of retirement income. In fact, in those cases, the strategy takes more risk, which may increase the changes of achieving a higher income level $\hat{N}_T$. However, the resulting level of retirement income in the most unfavourable scenarios, would be lower. Table 2 presents a distribution summary of the resulting proportion κ_T across the 277 simulated cohorts, and for the age-income profile disaggregated by the three education levels, and for the average income across the three, which is the one used for the strategies simulation. The proportions obtained range between 0.69 and 0.92.

Figure 3 presents the securable level of retirement income, $\tilde{N}_t$, for the TDF, the two target income strategies, and their Floor value, $\kappa_t N_t$, at each point in time for the first cohort (left panels) and last cohort (right panels). The left panels present the simulation of the scenario with increasing interest rates (from January 1962 to December 1996), which represents a favorable scenario for the equity index and the TDF relative to the performance of the retirement bond over the 35 year period, while the right panel represents a less favorable for the indices relative to the retirement bond (from January 1990 to December 2019). In the first scenario, the TDF yields a level of retirement income 5% higher than the target income strategy without derisking, denoted by FR, but the strategy with an increasing protection level κ_t yields a level of retirement income 44% higher than the TDF in that scenario. Furthermore, the TDF displays

[25] The average income at age 65 from the age-income profiles data, across the three education levels is approximately 26.38k per year, or 2198 per month. The base year from the real income data is 1992.

a much larger level of variations towards the end of the sample, particularly when compared with the strategy without derisking, as observed in the left panels of Figure 3.

In the less favorable scenario for the equity index (from January 1985 to December 2019), the more protective strategy without derisking, FR, yields an increase in the level of retirement income with respect to the TDF of 75%, while the strategy with derisking presents an increment of 56.4%. More notably, such increase in benefits occurs with a much lower level of variations in $\tilde{N}_t$ than the TDF, as observed in the right panels of Figure 3.

One important advantage of the target income strategies over the TDFs, is that they generate a relevant and clear signal to pension fund affiliates during the accumulation phase: the minimum level of retirement income that has been secured with all previous contributions to the retirement account, at every point in time, $\kappa_t N_t$. Indeed, for the target income strategies introduced, this minimum level of secured income never decreases, regardless of the movements in interest rates and equity returns, and increases with every new contribution to the retirement account.[26] Hence, each pension affiliate knows the level of retirement income that they have secured at every moment during the accumulation phase (a number that will also increase with future COLAs embedded in the retirement bond), which is an instrumental piece of information to take consumption/savings decisions, particularly during the latter part of the accumulation phase. More importantly, this also means that at the retirement date, there are no bad surprises in terms of the benefits received, when the conversion from assets to retirement income occurs. This is because the income level is in every case above the Floor value $\kappa_T N_T$, which will be close and above to $\kappa_{T-1} N_{T-1}$ and all the preceding values for it, which only increase proportionally to the contributions. Such information comes in a form that is perhaps the only financial figure that virtually any adult understands: current income dollars. Indeed, the units of the Floor value process $\kappa_t N_t$ are monthly income dollars in real terms, and the latter can be easily converted to current nominal dollars.[27]

In contrast, although the value of TDFs can also be expressed in the number of securable retirement dollars, $\tilde{N}_t$, the latter is very volatile for that kind of strategy, even during the period close to retirement (see the right panels in Figure 3), and it has no minimum level (besides zero).

As discussed in Section 2, the affordable level of retirement income, $\tilde{N}_t$, mixes elements

[26] For the two strategies with ratchet effect (i.e., $FR^\star$ and $CFR^\star$), the floor value $\kappa_t N_t$ increases with every contribution but also whenever a new maximum surplus level is attained.

[27] For instance, if the cashflow paid on the first month by each retirement bond, is a dollar plus a fixed COLA adjustment of 2%, then the floor value $\kappa_t N_t$ can be deflected with that rate to express current dollars.

that fall under the responsibility of the pension fund manager (i.e., the investment strategy) and elements that fall under the responsibility of beneficiaries, namely the contributions to the fund. In that sense, a more accurate metric of investment performance is the funding ratio introduced in Definition 3. Figure 4 presents the funding ratio series of the TDF and the two target income strategies over the aforementioned periods. In the less favorable scenario (right panels), the TDF presents a final funding ratio of 0.57, while the FR and CFR strategies present a final funding ratio of 1.01 and 0.897. In the more favorable period for equities, the TDF displays a final funding ratio of 1.4, while the FR and CFR strategies present final funding ratios of 1.33 and 2.0.

More generally, Table 3 presents the distribution summary of the funding ratio across the 277 scenarios for the TDF and the two target income strategies. Panel A displays a distribution summary of the resulting funding ratio at the retirement date. The median value for the final funding ratio of the TDF is 0.55, while for the FR and CFR strategies is 0.94 and 1.12, respectively. Moreover, all the reported quantiles of the TDF's funding ratio distribution are dominated by the corresponding quantiles of the two target income strategies. In fact, in 269 out of the 277 scenarios the FR strategy results in a higher final funding ratio than the TDF. The strategy with derisking (CFR) end up with a higher funding ratio than the TDF in all scenarios.

At the same time, Panels B-E in Table 3 show that the risk, measured as the volatility and the maximum drawdown of the funding ratio series, is higher for the TDF than for the two target income strategies[28]. Indeed, the median funding ratio volatility of the TDF over the first 10 years of the accumulation period, across the 277 scenarios, is 44.6%, while the same value for the target income strategies is 12.8%, and 35.5% (see Panel B). This result is not surprising, given that the TDF does not include the retirement bond, which is a save asset with zero variations in funding ratio. On the other hand, the target income strategies use the retirement bond to control their level of risk over time. Indeed, the contrasts in the level of risk between the TDF and the target income strategy with derisking, becomes even sharper as the retirement date approaches. The median funding ratio volatility of the TDF is 26% higher than the one of CFR during the first 10 years of accumulation, while over the last 10 years of accumulation, it is more than twice the level of CFR.[29]

[28]The funding ratio volatility is the annualized standard deviation of the monthly series of relative changes of the funding ratio.

[29]To see this compare the median value of the TDF with the respective values for strategies CFR in panel B

A significant improvement in terms of risk control, such as the one discussed above for the target income strategies, could also be achieved using the simpler strategy of using the same deterministic glide-path of TDFs to allocate between the risky PSP and the safe block of the portfolio, if the latter constitutes a retirement bond (or a LHP replicating its returns) instead of short-term bonds. However, there is hardly any reason to follow such arbitrary deterministic rule for the main asset allocation decision of the overall portfolio. To this point, Levy and Levy (2021) show that the glide-path allocation rule is suboptimal, and propose an alternative strategy that also integrates a horizon effect but with a wealth-maximizing criteria.

There are two fundamental differences between the target income strategy with derisking (CFR), and the strategy proposed in Levy and Levy (2021). First, the latter focuses on absolute wealth maximization, instead of retirement income, and as a result, the safe asset in that strategy is short-duration bonds (instead of retirement bonds in the target income strategies). The second difference is that the proposed target income strategies are heuristic rules designed to achieve the objective of securing a minimum known level of retirement income, without relying in any parameter estimates, given an investable alternative for the retirement bonds. Hence, while optimal allocation strategies maximizing utility of expected retirement income should improve welfare in expectation, they are likely to be highly dependent on parameters often hard to estimate, such as expected returns. For this reason, the target income strategies proposed favor simplicity of implementation, as the glide-paths of target-date funds, but have the advantage of being designed under the criteria of securing an increasing known level of retirement income, during the accumulation phase.

Interestingly, the equity allocation of the CFR strategy over the lifecycle displays a very similar pattern to the one of the optimal strategy proposed by Levy and Levy (2021). Figure 5 presents the dynamics of the average allocation to equities across scenarios, of the three strategies simulated. As observed in the figure, while the strategy without derisking (FR) starts with an allocation to the equity index of 30% (i.e., $1 - \kappa$) and remain somewhat close to that level across the accumulation period, the strategy with derisking (CFR) starts with an allocation to equities of 100% at the start of the accumulation period and decreases at a high rate arriving at a value of 26% at the retirement date. The allocation pattern of CFR, starting at 100% and decaying exponentially, is similar to the one observed in the bottom panel in Figure 4 of Levy and Levy (2021).

of Table 3.

Furthermore, the target-income strategies not only integrate the horizon effect, but also other key lessons from optimal portfolio theory.[30] First, the target-income strategies integrate the feature of an increasing bonds to stocks ratio on the level of risk aversion. That feature, has been common in the asset allocation by professional investment advisors (Canner et al., 1997), and rationalized in the theoretical works of Brennan and Xia (2000) and Munk et al. (2004) under interest rate uncertainty. Everything else equal, the target-income strategies defined with a higher protection level κ parameter have a higher bond-to-equity allocation ratio.

Second, note that the risk-management objective in target income strategies of protecting a floor value of assets defined as a fraction of pension liabilities, is also present in the preferences defined in works such as Sharpe and Tint (1990), Detemple and Rindisbacher (2008), and Martellini and Milhau (2012). Finally, it is worth stressing the fact that portfolio theory, since the classic works of Merton (1969, 1971, 1973), indicates the importance of hedging against changes in the opportunity set, such as variations in interest rates, particularly when the investment horizon is relatively long. A prescription confirmed by the works of Omberg (1999), Sørensen (1999), Brennan and Xia (2000) and Viceira and Campbell (2001), who find that optimal portfolios should hedge variations in interest rates by investing in the zero-coupon bond expiring at the investment horizon of the investor (or a portfolio of regular coupon-paying bonds that replicates the zero-coupon bond). The problem solved in those papers concerns an investor with a consumption objective at a single date in the future. When generalized to a problem when the objective is financing consumption at a series of future dates (the payout period), naturally the hedging asset is a bond that pays indexed cashflows on those dates (or a liability hedging portfolio that aims to replicate the value of the hedging asset, as in Detemple and Rindisbacher, 2008; Martellini and Milhau, 2012). This central piece of advice is widely disregarded by pension fund managers of DC plans, such as as standard target-date fund strategies.

Note that for simplicity, and to make the target income strategy more comparable to the target date strategy, we have used as the risky PSP an equity index. However, portfolio theory implies that such PSP should be a diversified portfolio maximizing expected utility, which in principle should hold all kinds of invest-able asset classes that command a risk premium.

[30]The horizon effect in optimal allocation strategies (consisting of a higher allocation to equities for younger investors), was first referred as a puzzle in Samuelson (1963) and later reconciled with rational portfolio theory in works such as Samuelson (1989), Samuelson (1994), Kim and Omberg (1996), Campbell and Viceira (1999), Barberis (2000), Wachter (2002) and Munk et al. (2004).

5 Conclusion

The price of retirement bonds (or the value of replicating portfolios for these bonds) is an valuable piece of information since it enables individual investors to know (*i*) how much replacement income the current value of their retirement assets can secure at any point in time, and (*ii*) how much retirement income they could have generated if all their previous contributions had been invested in retirement bonds, which are the true risk-free assets in decumulation. We leverage the latter piece of information to provide a formal measure of pension liabilities for DC plans, which we define as the present value of the retirement income benefits that each individual could have secured with their previous contributions, without taking any risk.

Using the proposed DC liability definition we can use the corresponding funding ratio as performance metric in DC plans. This performance metric would focus the efforts of the pension fund managers towards the objective of increasing retirement income benefits, beyond the affordable level that can be secured risk-free, with retirement bonds. Furthermore, it creates a strong incentive for pension fund managers to control the primary investment risk their affiliates face, namely the variations in the conversion rate of assets to retirement income. Indeed, using historical data from U.S. treasury yields to value retirement bonds, we show that when assets' value is measured in units of retirement benefits (which is what matters for pension fund beneficiaries) an equity index and a bond portfolio with relatively short and constant duration have the same level of risk on average. This finding has important implications for the design of life-cycle products for the accumulation phase. Taking into account the variations in the conversion rate of assets to retirement income, we find that the popular target date funds, which use short-duration bonds as the "safe" asset in their asset allocation strategy, do a poor job at reducing the variations in the expected level of retirement income benefits as the retirement date approaches.

As an alternative, we introduce simple asset allocation rules that we call target income strategies, which secure an essential level of retirement income regardless of the fluctuations in risky asset prices and interest rates, but at the same time invest in risky securities to generate the upside potential required to achieve the aspirational goal of a higher income, which exceeds the maximum level affordable without risk-taking.

Appendix A Pricing of retirement bonds

The no-arbitrage price of the retirement bond at a date t in the accumulation phase is:

$$B_t = \sum_{h=1}^{M} (1+\pi)^{T+h} e^{-(T+h-t)R_{t,T+h-t}} \tag{15}$$

where cashflows are paid on $T+h$ periods ahead for h in $\{1, 2, .., M\}$, where T is the initial time to retirement, M the expected duration of the payout phase (e.g., 20 years), $R_{t,T+h-t}$ denotes the yield of a zero bond paying \$1, $T+h-t$ years ahead, and π is a COLA rate (e.g., 2% annually). The discount rates can be inferred from coupon-paying bond prices through standard bootstrapping techniques or fitting them to yield curve models such as Nelson and Siegel (1987). Note that the fixed COLA adjustment only covers the impact of expected inflation. If an exact hedge of inflation risk is preferred, then the cashflows would be expressed in real terms, which simply means that the discount rates $R_{T,h}$ in equation (15) should be real yields instead of nominal bond yields.

If a hedge for salary increments in real terms was preferred as well, then real yields would be used as discount rates in Eq. (15), and π would be the expected salary growth in real terms. However, having extra hedging for unexpected inflation risk and matching an expected real salary growth imply an increment in the cost of financing future consumption, reflected in a higher price B_t. We leave the analysis of the cashflow indexing tradeoff in retirement bonds for future research. In our empirical section, we opt for using nominal yields (and set π as the inflation target of the Federal Reserve), in order to have the longest available time period for the analysis.

Merton and Muralidhar (2017) argue that sovereign states should issue another type of closed-related bonds, which they call retirement SeLFIES for Standard of Living indexed, Forward-starting, Income-only Securities, and make them available to investors in accumulation (see also Muralidhar, 2015; Muralidhar et al., 2016; Kobor and Muralidhar, 2018; Martellini et al., 2018). SeLFIES are similar to the retirement bonds above, except that their cash-flows are indexed to aggregate per capita consumption. One key advantage of this feature is that it provides investors with a hedge of standard-of-living risk, while inflation-linked retirement bonds only provide a hedge against cost-of-living risk. This feature, however, introduces a form of market incompleteness and makes SeLFIES non-redundant securities that existing fixed-income instruments cannot replicate.

Appendix B Dynamics of fund value in retirement units $\tilde{N}_t$

Denote the returns of the pension fund assets r^A, thus we have that the value of retirement assets at each point in time is:

$$A_{t+1} = A_t(1 + r^A_{t+1}) + C_{t+1} \tag{16}$$

where C_{t+1} is the contribution to the fund at time $t + 1$. The affordable replacement income evolves by definition as

$$\Delta N_{t+1} = N_{t+1} - N_t = n_{t+1} = \frac{C_{t+1}}{B_{t+1}} \tag{17}$$

Hence N_{t+1} never decreases, and only increases with new contributions measured in retirement units, i.e., n_{t+1}. On the other hand, the value change of a pension fund invested in any kind of assets, measured in terms of retirement units is:

$$\tilde{n}_{t+1} := \Delta\tilde{N}_{t+1} = \tilde{N}_{t+1} - \tilde{N}_t = \frac{A_{t+1}}{B_{t+1}} - \frac{A_t}{B_t} \tag{18}$$

Replacing (16) in (18) we have,

$$\begin{aligned} \tilde{n}_{t+1} &= \frac{A_t(1 + r^A_{t+1}) + C_{t+1}}{B_{t+1}} - \frac{A_t}{B_t} \\ \tilde{n}_{t+1} &= \frac{A_t(1 + r^A_{t+1})}{B_{t+1}} - \frac{A_t}{B_t} + \frac{C_{t+1}}{B_{t+1}} \end{aligned}$$

The latter can be expressed in the following way by replacing $B_{t+1} = B_t(1 + r^B_{t+1})$ in the denominator and recalling that $\tilde{N}_t = \frac{A_t}{B_t}$ and $n_{t+1} = \frac{C_{t+1}}{B_{t+1}}$:

$$\tilde{n}_{t+1} = \frac{A_t(1 + r^A_{t+1})}{B_t(1 + r^B_{t+1})} - \frac{A_t}{B_t} + n_{t+1} \tag{19}$$

$$\tilde{n}_{t+1} = \tilde{N}_t \left(\frac{1 + r^A_{t+1}}{1 + r^B_{t+1}} - 1 \right) + n_{t+1} \tag{20}$$

Notice that $\tilde{n}_{t+1} - n_{t+1}$ is proportional to the percentage change in the relative value of the assets with respect to the price of the retirement bond. To see this, define $A'_{t+1} := A'_t(1 + r^A_{t+1})$ with $A'_0 = A_0$, i.e., the cumulative returns of investments in the pension fund, and $Z_t := \frac{A'_t}{B_t} \quad \forall t$ and notice that $r^Z_{t+1} = \frac{Z_{t+1}}{Z_t} - 1 = \frac{1+r^A_{t+1}}{1+r^B_{t+1}} - 1$, which are the relative returns of the investments in the pension fund. Thus, using definition (18) in equation (20) we obtain:

$$\tilde{N}_{t+1} = \tilde{N}_t(1 + r^Z_{t+1}) + n_{t+1}$$

and

$$\tilde{n}_{t+1} = \tilde{N}_t r^Z_{t+1} + n_{t+1}. \tag{21}$$

Appendix C Discussion on DB liabilities

Let us point some caveats in the current approaches to define pension liabilities in DB plans. The broadest liability measure is the Present Value of Benefits (PVB), which forecasts all future possible contributions and expected salary growth for current workers. The Entry Age Normal (EAN) and the Projected Benefit Obligation (PBO), account different fractions of the PVB (Winklevoss, 1993; Lenze et al., 2009). The EAN recognizes future liabilities in proportion to the ratio of the present value of a worker's wages earned to date and the present value of lifetime wages. The PBO accounts for projection of future wage growth, but it does not include expected future service/contribution. The Accumulated Benefit Obligation (ABO) only accounts for the benefits that workers would be entitled to based on today's accumulated years of service and salary level.

The cash flows benefits accrued for an active worker under all aforementioned DB accounting methods (ABO, PBO, EAN, and PVB) converge at retirement. Such accrued benefit equals a flat percentage of the final (or late-career) salary times the number of service/contribution years.[31] For instance, if the pension benefit factor of the DB plan is 2% and an employee has worked for 30 years and has an average wage in the last several years of work equal to \$35,000, the employee will be entitled to a pension of \$24,000 ($= 2\% \times 30 \times \$35,000$) per annum when she retires, plus any COLAs her plan offers. This implies a replacement rate of $60\% = 2\% \times 30$.

The accrued pension benefits for an individual after s_t periods (e.g. years) of service, in the ABO and PBO accounting methods, can be expressed in number of retirement units as[32]:

$$N^{DB}_t = \alpha \times s_t \times \hat{N}_T \tag{22}$$

[31] As standard accounting methods we mean the Present Value of Benefits (PVB), the Entry Age Normal (EAN) and the Projected Benefit Obligation (PBO) and the Accumulated Benefit Obligation (ABO).

[32] Recall that each retirement unit/bond already embeds COLAs after the retirement date.

where α is a constant percentage called the *pension benefit factor*, the number of contribution years until time t is $s_t = \sum_{i=1}^{t} \mathbf{1}_{i=c}$, where $\mathbf{1}_{i=c} = 1$ if the individual contributed on period i and zero otherwise, and $\hat{N}_T$ denotes the reference labor income to which the benefits are pegged. In the ABO method, such reference is equal to current salary, while in the PBO the reference income includes an estimation of the expected increase in salary. Note this implies a defined benefit in terms of replacement rate of $RR^{DB} = \frac{N_T^{DB}}{\hat{N}_T} = s_T \times \alpha$, and for 100% contribution density,

$$RR^{DB} = T \times \alpha \tag{23}$$

To set a number for the benefit factor, one may define a target replacement rate for a given number of contribution years. For instance, if the target replacement rate is $RR^{DB} = 70\%$, and the expected contributions years are 35, then $\frac{RR^{DB}}{T} = \frac{70\%}{35} = \alpha = 2\%$.

There are several reasons for which the benefit defined in Eq. (22) cannot be guaranteed without the need of supplementary contributions. First, note that the benefit level to be guaranteed N_T^{DB} is only known until retirement date, because the reference income $\hat{N}_T$ is unknown during accumulation, as it depends on the final and unknown income of the individual upon retirement.[33] Furthermore, there is no investment vehicle in the market that correlates perfectly with changes in real labor income per individual, nor on the average labor income. Hence, variations in the reference labor income are not hedgeable.[34]

Second, in every contribution year, the accrued benefits in terms of replacement rate increase by the same proportion

$$\frac{\Delta N_t^{DB}}{\hat{N}_T} = \alpha \tag{24}$$

Note that the such benefit increment is the same regardless of the amount contributed each year. Furthermore, the increment in benefits Eq. (24) is also the same whether the contribution is done at the beginning or towards the end of the accumulation period. Of course, in general, a dollar contributed many years prior retirement can finance a much higher level of pension benefits than a dollar contributed on the last year of the accumulation period (if those contribution dollars are invested). Hence, there is a definite disconnection between the financial value of the contributions and the accrued benefits in DB systems, as the formula that determines the

[33] While some of the variations in that number might washout when aggregating the benefits across individuales, in general (and most likely), they wont cancel out completely (and they can even all grow if there is a systematic increase in labor income in the economy).

[34] The exception to this is Uruguay, where there are government bonds with coupons indexed to the average salary in the country. However, even if changes in the reference income where hedge-able (as in Uruguay), in general the benefit in Eq. (22) still implies an unhedgeable liability for the other two reasons explained hereafter.

periodic increment in accrued benefits (Eq. 22) treats all contributions equal, ignoring their timing, and even their amount. This type of liability definition will inevitably create a shortfall and eventually require contributions from the plan sponsor.

In contrast, the strategy of investing all contributions in retirement bonds, secures a variable pension benefit factor of

$$\alpha_t = \frac{\Delta N_t}{\hat{N}_T} = \frac{n_t}{\hat{N}_T} \tag{25}$$

which results in a replacement rate of $\sum_{i=1}^{T} \alpha_t$. By setting a constant reference income target $\hat{N}_T$, in principle, it is possible to secure a target replacement rate $\bar{RR}$ relative to that reference income. To achieve that, define any given path of contributions in number of retirement units, i.e., $\{\hat{n}_1, \hat{n}_2, \ldots \hat{n}_T\}$ that satisfies $\sum_{t=1}^{T} \hat{n}_t = \bar{RR} \times \hat{N}_T$, which implies that the periodic dollar contribution amounts should vary over time in order satisfy $C_t = \hat{n}_t B_t$, at every time t. In order to maintain a constant benefit factor, as in DB plans, the extra condition $\hat{n}_t = n = \alpha \hat{N}_T$ must also hold for all $t \leq T$. In general, there is no particular reason to add that extra constraint.[35] Moreover, DB plans usually have the same predefined benefit factor level α that applies to all beneficiaries, regardless of the age or income profile. This rigidity prevents the system from accommodating beneficiaries that would like to secure the same replacement rate as other beneficiaries that entered the plan earlier, even if those late comers were willing to compensate the time "lost" by doing larger (or large enough) contributions in dollar terms. This is because DB plans (arbitrarily) define the incremental benefit depends only in contribution time instead of contribution funds, as if all contributions were equal, regardless of their amount and timing.

As explained above, in order to secure a given target level of total retirement benefits $\bar{RR} \times \hat{N}_T$ (or a replacement rate relative to that reference income level), the dollar contribution amounts to the retirement account C_t need to be adjusted so as to meet at each point in time a predefined path of contributions n_t that sum to $\bar{RR} \times \hat{N}_T$. Such variable and unknown contribution path C_t implies potential supplementary contributions, which may be covered by some plan sponsor, the beneficiaries or a combination. Depending on the nature and source of these supplementary contributions, an important level of uncertainty may remain over the possibility to achieve such objective. On the other hand, assuming that the series of contributions dollars are given, as it would be the case if contributions are equal to a fixed proportion of realized

[35] In fact, the predefined path for n_t can be set under the criteria of minimizing variations in the percentages of labor income required, based on reasonable estimates for labor income paths and for B_t that take into account the current and long term levels of interest rates and age-income profiles (and any other extra information relevant to forecast the contributions).

labor income of each individual, and allowing for a variable benefit factor equal to Eq. (25), leads to the hedgeable definition of accrued pension benefits.

Appendix D Condition on the dynamics of κ_t

Following the allocation rule (13), the dynamics of $\tilde{N}_t$ are

$$\tilde{N}_{t+1} \geq \kappa_t N_t + (\tilde{N}_t - \kappa_t N_t)(1 + r_{t+1}^{Z_{PSP}}) + n_{t+1} \tag{26}$$

where $r_{t+1}^{Z_{PSP}} = \frac{1+r_{t+1}^{PSP}}{1+r_{t+1}^{B}} - 1$ is the relative return of the PSP with respect to the return of the retirement bond. Replacing Eq. (26) in the liability-driven constraint (9),

$$\tilde{N}_{t+1} \geq \kappa_{t+1} N_{t+1} \tag{27}$$

$$\kappa_t N_t + (\tilde{N}_t - \kappa_t N_t)(1 + r_{t+1}^{Z_{PSP}}) + n_{t+1} \geq (\kappa_t + \Delta\kappa_t) N_t + \kappa_{t+1} n_{t+1}$$

$$(\tilde{N}_t - \kappa_t N_t)(1 + r_{t+1}^{Z_{PSP}}) \geq \Delta\kappa_t N_t - (1 - \kappa_{t+1}) n_{t+1} \tag{28}$$

Note that since $r_{t+1}^{PSP} > -1$, then $r_{t+1}^{Z_{PSP}} > -1$. Hence, if at the previous time period the floor income was respected, i.e., $\tilde{N}_t - \kappa_t N_t \geq 0$, then the left hand side of Eq. (28) is positive, which means that a sufficient condition to meet condition (27) on $t+1$ would be:

$$\Delta\kappa_t N_t - (1 - \kappa_{t+1}) n_{t+1} \leq 0$$

$$\Delta\kappa_t N_t - n_{t+1} + (\kappa_t + \Delta\kappa_t) n_{t+1} \leq 0$$

$$\Delta\kappa_t N_{t+1} - (1 - \kappa_t) n_{t+1} \leq 0$$

$$\Delta\kappa_t \leq (1 - \kappa_t) \frac{n_{t+1}}{N_{t+1}} \tag{29}$$

if at the previous period $\tilde{N}_t - \kappa_t N_t \geq 0$ held. Recall that by convention $\tilde{N}_0 = N_0$, hence the condition holds at the initial time $t = 0$, i.e., $\tilde{N}_0 - \kappa_0 N_0 \geq 0$ for $\kappa_0 \leq 1$. Hence, iteratively, the floor condition Eq. (27) is met for every $t \geq 1$ if the increase in κ satisfies Eq. (29), which is Eq. (14) in the main text.

Appendix E Condition check on κ_t^{CFR}

Rewrite condition (14) as

$$\Delta\kappa_t N_{t+1} \leq (1-\kappa_t)n_{t+1} \tag{30}$$

On the one hand, given definition (14) of κ_t^{CFR}, we have:

$$\begin{aligned}\Delta\kappa_t^{CFR} &= (1-\kappa)\hat{N}_T\left(\frac{1}{N_t} - \frac{1}{N_{t+1}}\right)\\ \Delta\kappa_t^{CFR}N_{t+1} &= (1-\kappa)\hat{N}_T\left(\frac{N_{t+1}-N_t}{N_t}\right)\\ \Delta\kappa_t^{CFR}N_{t+1} &= (1-\kappa)\hat{N}_T\left(\frac{n_{t+1}}{N_t}\right)\end{aligned} \tag{31}$$

On the other hand, replacing the definition (14) of κ_t^{CFR} on the right hand side of condition (30), we obtain the same expression as Eq. (31). Hence, κ_t^{CFR} satisfies the condition (14) with equality.

Appendix F Condition check on $\kappa_t^{FR^\star}$

First consider the case in which a new maximum surplus $S_t^\star = \tilde{N}_{t^\star} - N_{t^\star}$ is attained, hence $t = t^\star$. The condition to be satisfied is

$$\begin{aligned}&\tilde{N}_{t^\star} \geq \kappa_{t^\star}^{FR^\star} N_{t^\star}\\ &\tilde{N}_{t^\star} \geq \kappa\left(1 + \frac{S_{t^\star}}{N_{t^\star}}\right)N_{t^\star}\\ &\tilde{N}_{t^\star} \geq \kappa(N_{t^\star} + S_{t^\star})\\ &\tilde{N}_{t^\star} \geq \kappa\tilde{N}_{t^\star}\end{aligned}$$

which is satisfied for $\kappa \leq 1$. Now consider the other case in which the surplus' running maximum is still the same as in the previous period, i.e., $S_{t+1}^\star = S_t^\star$. On the left hand side of the equivalent condition (30), given definition (??) of $\kappa_t^{FR\star}$, we have:

$$\Delta\kappa_t^{FR\star} = \kappa\left(\frac{S^\star_{t+1}}{N_{t+1}} - \frac{S_{t^\star}}{N_t}\right)$$
$$\Delta\kappa_t^{FR\star} = \kappa S^\star_t\left(\frac{1}{N_{t+1}} - \frac{1}{N_t}\right)$$
$$\Delta\kappa_t^{FR\star} N_{t+1} = \kappa S^\star_t\left(\frac{N_t - N_{t+1}}{N_t}\right)$$
$$\Delta\kappa_t^{FR\star} N_{t+1} = -\kappa S^\star_t\left(\frac{n_{t+1}}{N_t}\right) \tag{32}$$

On the other hand, replacing the definition of $\kappa_t^{FR\star}$ on the right hand side of condition (30), we have

$$(1-\kappa_t^{FR\star})n_{t+1} = \left(1-\kappa\left(1+\frac{S_{t^\star}}{N_t}\right)\right)n_{t+1}$$
$$(1-\kappa_t^{FR\star})n_{t+1} = \left(1-\kappa-\kappa\frac{S_{t^\star}}{N_t}\right)n_{t+1} \tag{33}$$

Note that the last term in Eq. (33) is the same as the rhs of Eq. (32). Hence, for $0 \le \kappa \le 1$ $\kappa_t^{FR\star}$ satisfies the condition (14).

Appendix G Condition check on $\kappa_t^{CFR\star}$

First consider the case in which a new maximum surplus $S^\star_t = \tilde{N}_{t^\star} - N_{t^\star}$ is attained, hence $t = t^\star$. The condition to be satisfied is

$$\tilde{N}_{t^\star} \ge \kappa_{t^\star}^{CFR\star} N_{t^\star}$$
$$\tilde{N}_{t^\star} \ge \left(\kappa_{t^\star} + \kappa\frac{S_{t^\star}}{N_{t^\star}}\right)N_{t^\star}$$
$$\tilde{N}_{t^\star} \ge \kappa_t N_{t^\star} + \kappa S_{t^\star}$$
$$\tilde{N}_{t^\star} \ge \left(1-(1-\kappa)\frac{\hat{N}_T}{N_{t^\star}}\right)N_{t^\star} + \kappa S_{t^\star}$$
$$\tilde{N}_{t^\star} \ge N_{t^\star} - (1-\kappa)\hat{N}_T + \kappa S_{t^\star}$$
$$\tilde{N}_{t^\star} \ge \kappa\tilde{N}_{t^\star} + (1-\kappa)(N_{t^\star} - \hat{N}_T)$$
$$\tilde{N}_{t^\star}(1-\kappa) \ge (1-\kappa)(N_{t^\star} - \hat{N}_T)$$
$$\tilde{N}_{t^\star} - N_{t^\star} \ge -\hat{N}_T \tag{34}$$

where above we assumed $0 \leq \kappa \leq 1$. Since $\tilde{N}_0 = N_0$, then for any $t > 0$, $\tilde{N}_{t^\star} \geq N_{t^\star}$ so condition (34) is satisfied.

Now consider the other case in which the surplus' running maximum is still the same as in the previous period, i.e., $S^\star_{t+1} = S^\star_t$. On the left hand side of the equivalent condition (30), given definition (**??**) of $\kappa_t^{CFR^\star}$, we have:

$$\Delta\kappa_t^{CFR^\star} N_{t+1} = \left(\Delta\kappa_t^{CFR} + \Delta\kappa_t^{FR^\star}\right) N_{t+1} \tag{35}$$

Replacing Eq. (31) and Eq. (32) on Eq. (35),

$$\Delta\kappa_t^{CFR^\star} N_{t+1} = (1-\kappa)\hat{N}_T \left(\frac{n_{t+1}}{N_t}\right) - \kappa S_t^\star \left(\frac{n_{t+1}}{N_t}\right) \tag{36}$$

On the other hand, replacing the definition of $\kappa_t^{CFR^\star}$ on the right hand side of condition (30), we have

$$(1-\kappa_t^{CFR^\star}) n_{t+1} = \left((1-\kappa)\frac{\hat{N}_T}{N_t} - \kappa\left(\frac{S_{t^\star}}{N_t}\right)\right) n_{t+1} \tag{37}$$

Note that the Eq. (37) is equal to Eq. (36). Hence, for $0 \leq \kappa \leq 1$, $\kappa_t^{CFR^\star}$ satisfies the condition (14) with equality.

Appendix H Bound on funding ratio drawdown of $FR^\star$ and $CFR^\star$

The target income strategy $CFR^\star$ ensures at all times that,

$$\tilde{N}_t \geq \kappa_t^{CFR^\star} N_t$$

Hence, the funding ratio of the strategy, FR_t, is bounded from below:

$$\begin{aligned}
FR_t &\geq \kappa_t^{CFR^\star} \\
FR_t &\geq \kappa_t^{CFR} + \kappa\left(\frac{S_{t^\star}}{N_t}\right) \\
FR_t &\geq \kappa_t^{CFR} + \kappa\left(\frac{\tilde{N}_{t^\star}}{N_t} - \frac{N_{t^\star}}{N_t}\right)
\end{aligned}$$

if there are no contributions from $t^\star$ until t, then $N_t = N_{t^\star}$, and

$$\begin{aligned}
FR_t &\geq \kappa_t^{CFR} + \kappa\left(\frac{\tilde{N}_{t^\star}}{N_{t^\star}} - 1\right) \\
FR_t &\geq \kappa_t^{CFR} + \kappa FR_{t^\star} - \kappa \qquad (38)\\
FR_t &\geq 1 - (1-\kappa)\frac{\hat{N}_T}{N_{t^\star}} + \kappa FR_{t^\star} - \kappa \\
FR_t &\geq (1-\kappa)\left(1 - \frac{\hat{N}_T}{N_{t^\star}}\right) + \kappa FR_{t^\star} \\
\frac{FR_t}{FR_{t^\star}} - 1 &\geq (\kappa - 1)\left(\frac{\hat{N}_T}{N_{t^\star}} - 1\right)\frac{1}{FR_{t^\star}} + \kappa - 1 \\
\frac{FR_t}{FR_{t^\star}} - 1 &\geq (\kappa - 1)\left(\frac{\tilde{N}_{t^\star} + \hat{N}_T - N_{t^\star}}{\tilde{N}_{t^\star}}\right) \qquad (39)
\end{aligned}$$

Hence, whenever there are no contributions from $t^\star$ on, but they had reached or passed the target level by that time, i.e., $N_{t^\star} \geq \hat{N}_T$, then the ratio to the right of Eq. (39) is lower than 1, which means that the max drawdown[36] in funding ratio of strategy $CFR^\star$ is bounded by $\kappa - 1$.

From the definition, of κ_t^{FR}, it follows that the lower bound of the funding ratio for the strategy $FR^\star$ is similar to Eq. (38) but with κ instead of κ_t^{CFR}, whenever there are no contributions from $t^\star$ on. Hence, we have:

$$\begin{aligned}
FR_t &\geq \kappa + \kappa FR_{t^\star} - \kappa \\
FR_t &\geq \kappa FR_{t^\star} \\
\frac{FR_t}{FR_{t^\star}} - 1 &\geq \kappa - 1 \qquad (40)
\end{aligned}$$

Hence, whenever there are no contributions from $t^\star$ on, the max drawdown in funding ratio of strategy $FR^\star$ is bounded by $\kappa - 1$.

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[36]The max drawdown is the maximum percentage drop observed in the funding ratio, until current time t, relative to its running maximum $FR_{t^\star}$ (i.e., lhs of Eq. 39).

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Appendix I Tables and Figures

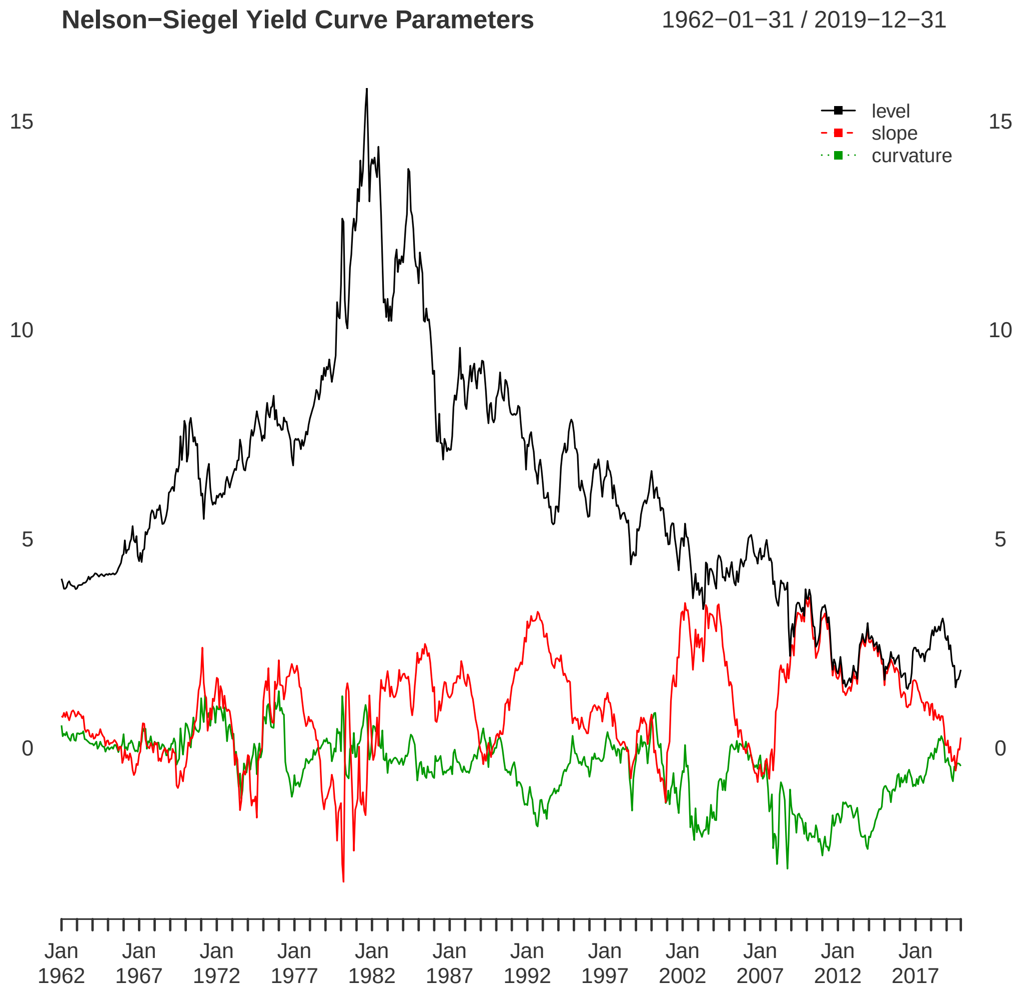

Figure 1: Yield Curve Parameters for Nelson Siegel Model. We use the 'data-based' definitions in Diebold and Li (2006), setting $\lambda = 0.0609 * 12$, the level factor is proxied by the 10-year yield, the slope as the difference between the 10-year and the short-term yield, and the curvature as the twice the medium-term yield minus the sum of the short-term and 10-year yields. For periods for which the 2-year yield ('TCMNOM_Y2') is not available (1962-01-02 to 1976-05-31), we use the 5-year yield ('TCMNOM_Y5') as the medium term rate instead, and for periods for which the CRSP 3-month yield ('NFCP_M3') is not available (1962-01-02 to 1997-01-01) we use the 1-year yield ('TCMNOM_Y1') as the short-term rate.

Table 1: Distribution summary across all accumulation periods of assets' annualized return and volatility, correlation with the corresponding retirement bond (R.bond), and relative drawdown (RDD) with respect to the R.bond. All figures are in percentage terms.

	Min	10%	25%	50%	75%	90%	Max
				Panel A: Return			
R.bond	3.14	3.79	5.96	8.68	11.58	16.4	21.2
BondIndex	6.23	7.04	7.84	8.11	8.45	8.61	8.82
EquityIndex	9.54	10.6	11.02	11.32	11.81	12.15	12.45
				Panel B: Volatility			
R.bond	31.76	33.82	36.25	40.4	43.65	47.41	49.6
BondIndex	6.13	6.51	8.89	9.12	9.24	9.28	9.35
EquityIndex	14.44	14.67	14.94	15.26	15.4	15.63	15.88
				Panel C: Correlation			
R.bond	100	100	100	100	100	100	100
BondIndex	81.98	83.4	89.04	89.59	89.87	90.06	90.45
EquityIndex	5.14	10	15.8	20	24.47	27.01	27.9
				Panel D: RMDD			
R.bond	0	0	0	0	0	0	0
BondIndex	73.9	79.82	86.4	93.27	96.61	97.21	98.01
EquityIndex	74.82	79.41	84.85	91.51	95.73	96.88	97.68

Table 2: Distribution summary across all simulated cohorts of the secured proportion of affordable retirement income, κ_T, obtained from investing 12.4% of the average labor income, for each of the age-income profiles.

	Min	10%	25%	50%	75%	90%	Max
No high school	0.69	0.75	0.80	0.86	0.90	0.92	0.92
High school	0.70	0.75	0.81	0.86	0.90	0.91	0.92
College	0.70	0.76	0.82	0.87	0.91	0.92	0.93
Average	0.70	0.75	0.81	0.86	0.90	0.92	0.92

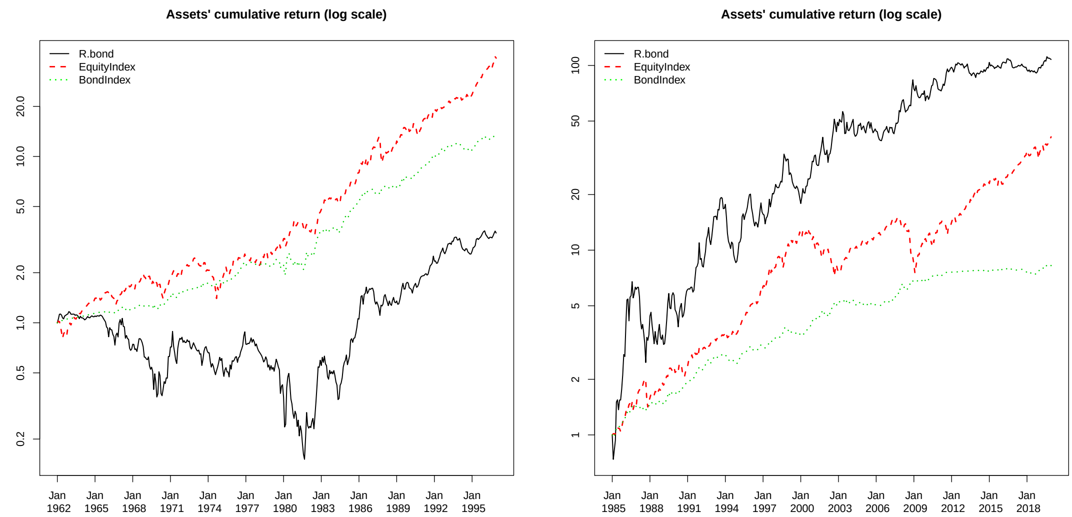

Figure 2: Cumulative Returns for the equity index, the 5-year constant duration bond index, and the retirement bond corresponding to the cohort retiring at the end of each sample period. The left Panel presents the data corresponding to the cohort retiring in 1996-12-31, and the right Panel for the cohort retiring in 2019-12-31.

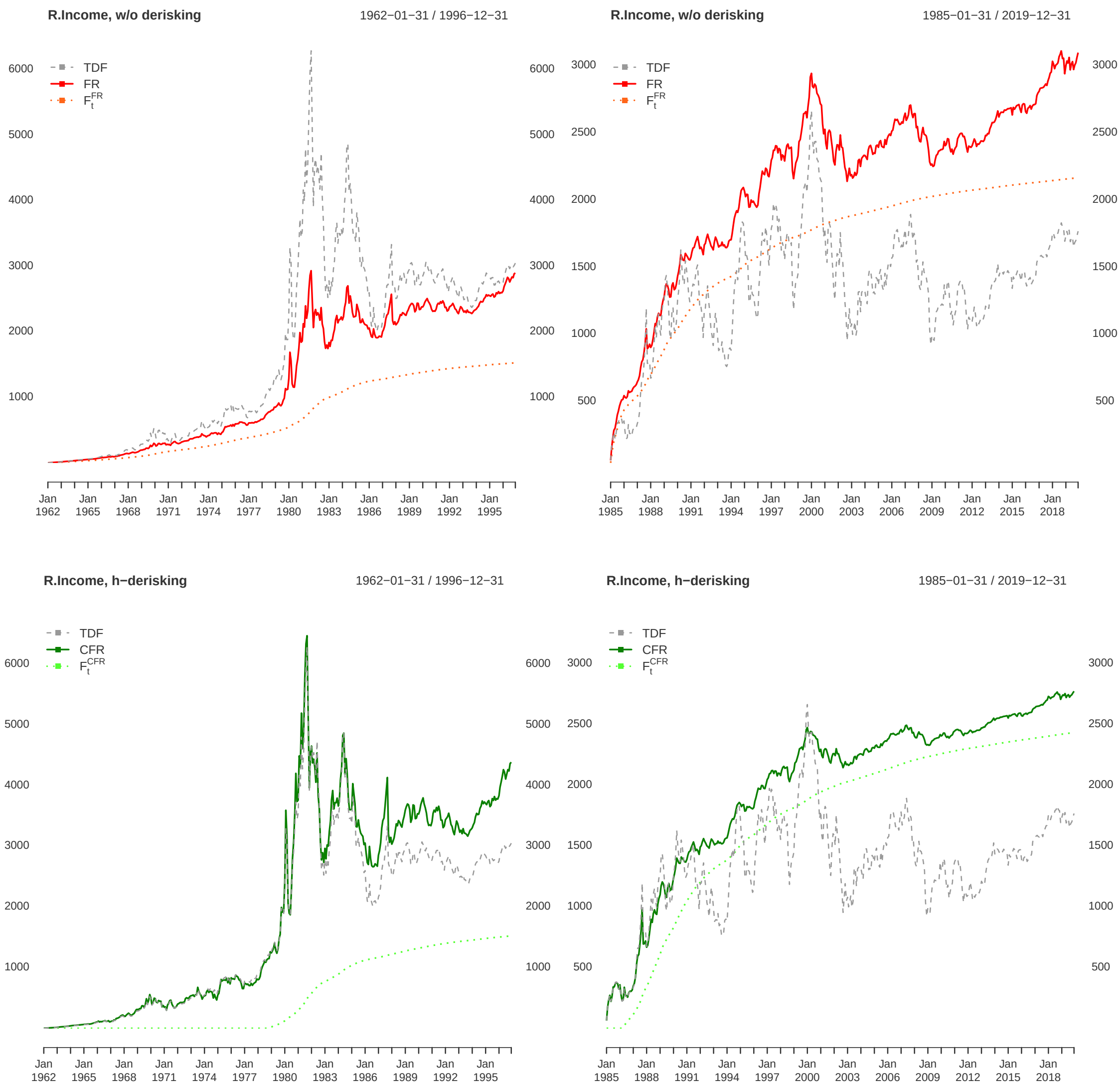

Figure 3: Securable monthly retirement income dollars from investing the monthly contributions in the target date fund or the target-income strategies with $\kappa = 0.7$. The figure also presents the lower bound protected by the target-income strategies. Units are constant dollars from December 1992. The left panels correspond to the historical simulation from January 1962 to December 1996. The right panels correspond to the historical simulation from January 1990 to December 2019.

Table 3: Distribution summary across all accumulation periods for the target date fund and target income strategies' final funding ratio (FR) across the 277 cohorts, FR volatility over the first 10 years of accumulation, FR volatility over the last 10 years of accumulation, and maximum drawdown (MDD) of FR over the first 10 years of accumulation and over the last 10 years of accumulation. All figures are in percentage terms.

	Min	10%	25%	50%	75%	90%	Max
Panel A: Final FR							
TDF	24.2	28.6	36.2	55.2	82.5	132.9	143.5
FR	79.7	80.8	82.1	94.4	101.1	141.2	155.4
CFR	85.9	90.9	96.5	112	128	209.4	228.7
Panel B: Vol(FR) First 10 years							
TDF	26.6	29.8	33.1	44.6	56	61.7	64.6
FR	8.6	10.8	12	12.8	28.1	30.5	31.1
CFR	9.6	18	29.1	35.5	47.7	49.7	50.5
Panel C: Vol(FR) Last 10 years							
TDF	9	9.6	11.4	13.3	14.5	15.2	16
FR	2.3	2.5	3	5.6	6.8	7.3	7.7
CFR	0.1	0.4	1	6.2	8.1	8.6	11.4
Panel D: MDD(FR) First 10 years							
TDF	42.1	59.3	61	78.3	89.6	94.1	94.3
FR	15.4	25.6	31	39.6	59.7	64.1	66.6
CFR	28.7	47.7	58.7	62.1	73.3	76.2	79.9
Panel E: MDD(FR) Last 10 years							
TDF	24.7	27.7	33.5	39	44.3	47.3	51.6
FR	4.6	10.4	12.1	14.8	28.1	30.7	32.3
CFR	0.5	1.3	4	21.2	29.3	33.7	37

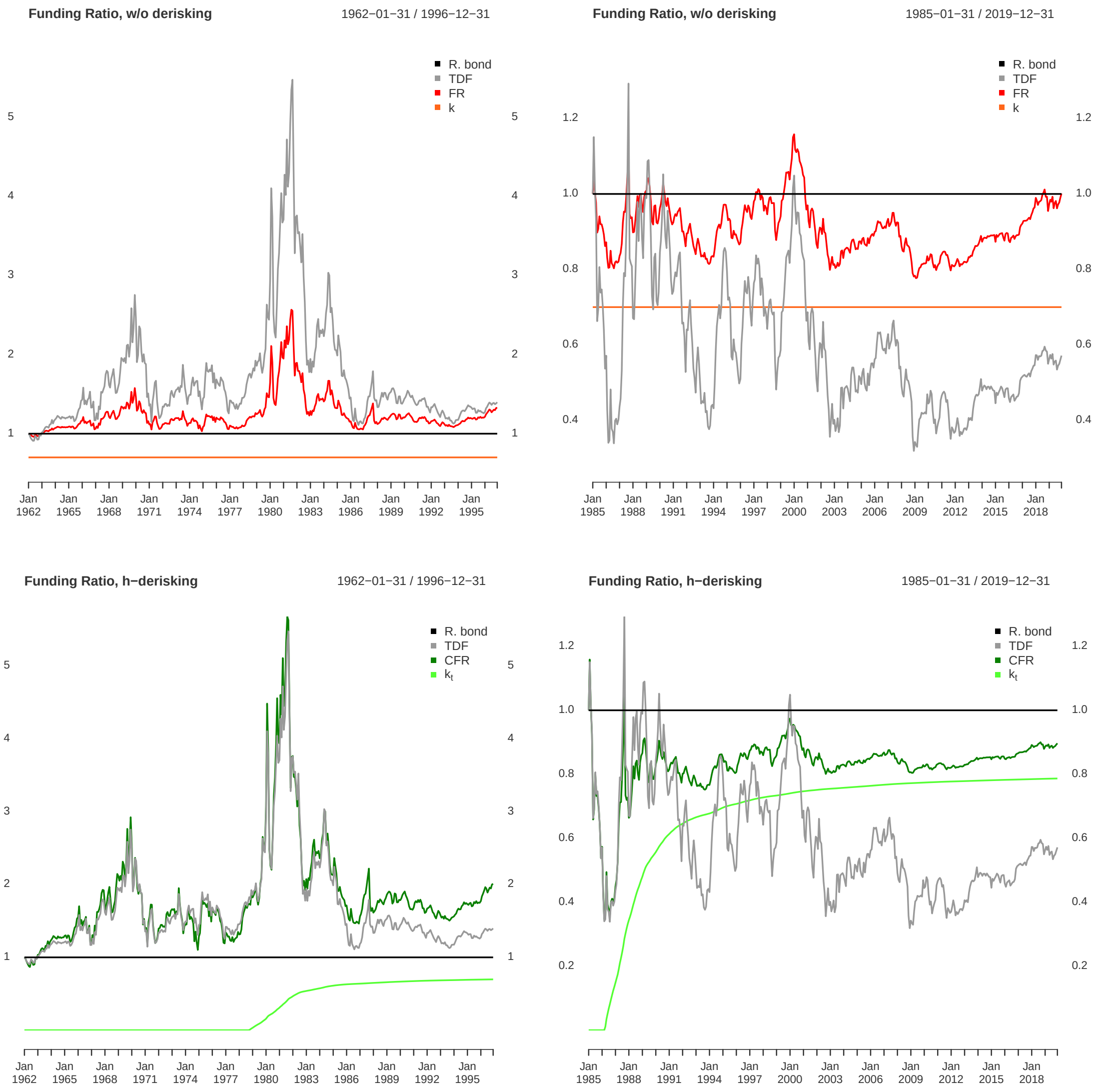

Figure 4: Funding ratio of target date fund (TDF) and of funding ratio of the target-income strategies with $\kappa = 0.7$. The figure also presents the lower bound protected by the target-income strategies. The left panels correspond to the historical simulation from January 1962 to December 1996. The right panels correspond to the historical simulation from January 1990 to December 2019.

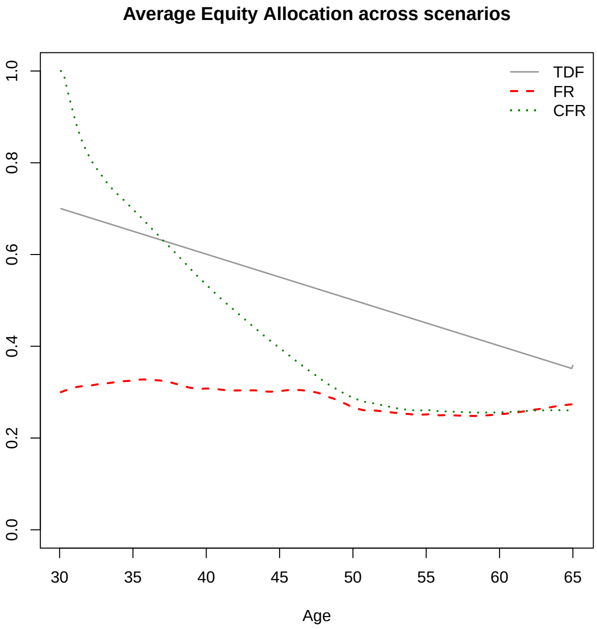

Figure 5: Average equity allocation across all 277 scenarios of the target date fund (TDF) and the target income strategies.