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A New Method to Estimate Risk and Return of Non-traded Assets from Cash Flows: The Case of Private Equity Funds*

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A New Method to Estimate Risk and Return of Non-traded Assets from Cash Flows: The Case of Private Equity Funds

We develop a new methodology to estimate abnormal performance and risk exposure of non-traded assets from cash flows. Our methodology extends the standard internal rate of return approach to a dynamic setting. The small-sample properties are validated using a simulation study. We apply the method to a sample of 958 private equity funds. For venture capital funds, we find a high market beta and underperformance before and after fees. For buyout funds, we find a relatively low market beta and no evidence for outperformance. We find that self-reported net asset values significantly overstate fund values for mature and inactive funds.

JEL classification: C51; G12; G23

Keywords: Risk exposure; Abnormal return; Private equity

I. Introduction

The estimation of risk exposure (beta) and abnormal performance (alpha) is at the heart of financial economics. Since Jensen's (1968) time-series regression approach to determine the alpha and beta of a mutual fund, a large literature has been dedicated to refining measures of risk and return (see Cochrane, 2005a, for an overview). However, the case of a non-traded asset for which we only observe cash flows has received little attention. For example, investors in private equity funds give away cash at different points in time and receive dividends at other points in time during the finite life of the fund. In this paper, we propose a methodology to measure risk and abnormal return in such a context and apply it to a sample of private equity funds.

Our method can be understood as follows. Consider the usual internal rate of return (IRR) calculation, where the net present value (NPV) of the cash flows of a fund is set zero using a constant discount rate. Our method extends this approach by using a dynamic discount rate. For example, in case of a market model, the discount rate in period t equals $1 + r_{f,t} + \alpha + \beta r_{m,t}$. Then, if one assumes that the alpha and beta are the same across a cross-section of funds, a natural approach is to find the alpha and beta that provide the best fit of this cross-section of cash flows. This boils down to finding the alpha and beta that bring the NPVs of (portfolios of) funds closest to zero.

We show that this can be written as a Generalized Method of Moments (GMM) estimation and that this method is asymptotically consistent. Importantly, our method does not require an assumption for the probability distribution of one-period returns. This is a key contribution because it is basically impossible to estimate this distribution when an asset is not traded.[1]

An important benefit of our method is that it only needs data on aggregate fund-level investments and dividends to identify alpha and beta. This is the typical nature of the data faced in practice for non-traded assets. We therefore do not need information on (i) the link between dividends and investments at the project level, (ii) the number of projects a fund invests in or (iii) the date at which a project investment is officially written off.

We validate our approach by simulations. We generate cash flows assuming a market model with a given pair of alpha and beta, and different non-standard one-period return distributions. In each case, and despite the fact that we do not use information on the true return distribution, we obtain alpha and beta estimates that are very close to the true values.

We apply our method to a trillion-dollar asset class: private equity funds. These funds are financial intermediaries that are typically classified as venture capital focused or buyout focused, and invest in a series of projects. They are not publicly traded and investors in these funds observe only a stream of cash flows for about 10 years until the fund is liquidated. Hence time-series estimation techniques cannot be

[1] One could compute the cross-sectional distribution of IRRs. However, this distribution of IRRs is not the same as the probability distribution of the underlying one-period returns. In fact, several probability distributions of one-period returns can be consistent with a given IRR distribution.

applied to measure risk and abnormal return.

In addition to our methodology, we contribute to the private equity literature in two ways. First, we provide an estimate of the cost of capital. This is an essential figure in practice. For example, venture capitalists need an estimate of the cost of capital to take their investment decisions. Second, we provide an estimate of abnormal return. An ongoing debate in this literature is whether these financial intermediaries (i.e. private equity funds) add value. A large literature points out that once a company is in the hand of private equity funds its free cash flow increases (see Cumming, Wright and Siegel, 2007, for a survey). However, private equity funds may pay too much for the companies they buy and face large transaction costs when buying and selling. As a result, the benefits they bring to the companies they buy may not translate into superior returns for their principals (i.e. the investors). In that case the value they create is captured by other agents. A related issue is that the fees private equity funds charge to their investors may be too large. So they may outperform before fees but underperform after fees. In this case, they would add value but investors would be paying too much for this financial intermediation. Our paper brings unique evidence on this important debate. We are the first paper to decompose private equity returns into what comes from systematic risk and alpha, and to show this decomposition both before fees and after fees.[2]

[2] The cash flows that are provided to us are net of fees. We simulate a typical fee structure on these cash flows to obtain cash flows gross of fees. No studies have had access to the exact fees charged by a fund. Existing papers also use simulations to quantify fees (e.g. Metrick and Yasuda, 2010).

Our dataset contains the cash flows (dividends and investments) of 958 mature private equity funds between 1980 and 2003. Mature funds are those that are more than 10 years old (the typical fund duration). For funds that are not reported as liquidated, we predict their final market value using a statistical model. This model relates the realized market value to fund characteristics at each age and is estimated using the subsample of liquidated funds.

We find that, after fees, venture capital funds have a market beta of 2.7. Given the high equity premium over our sample period, the cost of capital for venture capital (after fees) was about 27% per annum (using the CAPM). For buyout funds we estimate an after-fee market beta of 1.3, which implies that the cost of capital was 15% (using the CAPM). Note however that the cost of capital naturally varies over time and looking forward, we expect it to be due to a lower equity premium.

For venture capital, we find strong negative abnormal performance. The alpha is equal to -12% per annum with the CAPM. Before fees, the underperformance is reduced to -8.5%, but remains statistically significant. We also consider a three-factor Fama-French model and show that venture capital returns resemble those of small growth stocks. Relative to these small growth stocks, there is still underperformance of VC, but less so given that small growth stocks perform poorly too. For buyout funds, abnormal performance is (slightly) negative both before and after fees, and for both the CAPM and the three-factor model, but it is always statistically insignificant.

The results on VC may be surprising, but notice that learning about alpha and beta is not trivial. In fact, our paper is one of the first to estimate alphas and betas of private equity funds. Over our sample period, investors had less data than we have, especially so in the 1980-1993 period, when they had to take their investment decisions (funds have a duration of ten years). Notice also that investors may have observed absolute returns but these do not appear alarming (15% IRR on average). It is the risk correction that makes the alpha negative for venture capital funds and some casual evidence indicates that investors underestimated the risk of venture capital funds.[3] Also, for buyout funds, where we do not observe underperformance, the beta is similar to that of public equity, hence the potential underestimation of beta may not have been an issue.

Our statistical model for final market values predicts that the value of non-liquidated funds beyond the typical liquidation age (10 years) is around 30% of the self-reported Net Asset Value (NAV). In contrast, for funds that are eventually liquidated, market values are close to NAV. This substantial discrepancy comes from the fact that the non-liquidated funds are larger, have not distributed dividends for a long time, and have not updated their NAV for a long time; these characteristics are significantly associated with poorer subsequent cash flows according to our statistical

[3] Thomson Venture Economics (TVE) is the historical provider of benchmarks for the private equity industry. Individual investors did not have sufficient data to estimate the risk and return of venture capital themselves (until maybe recently). TVE has been providing benchmarks for a long time, but reported a beta for the first time only in 2001 (to our knowledge). In their "2001 Investment Benchmarks Report" they report a beta for venture capital of 0.84. Other firms providing data for asset allocation decisions (e.g. Ibbotson Associates) used estimates from Chen, Baierl and Kaplan (2002), who report a correlation between venture capital and public equity of only 4%. These low betas result from the use of quarterly Net Asset Values, which are known to be stale.

model.

Finally, we show that our estimates for alpha and beta are reasonably robust to various changes in the empirical settings. Also, in an appendix we show that our method can readily incorporate fund characteristics in the alpha and beta, such as size, experience, and geographic focus.

The rest of the paper proceeds as follows. Section II discusses related literature. Section III described our approach and presents a numerical example. Section IV discusses econometric properties of the method and presents a simulation study. Section V describes the private equity industry, our data, and the model for final market values. Section VI presents the empirical results and robustness checks. Section VII concludes.

II. Related Literature

Cochrane (2005b) and Korteweg and Sorensen (2010) assess the alpha and beta of US venture-backed companies. They observe valuations of projects at each financing round.[4] They can therefore compute a return. If one observes a return for each investment, a standard nonlinear least squares approach can be used. However, their data have two important features: (i) missing financing rounds and (ii) sample selection

[4] Venture Capital (VC) funds invest in distinct projects in so-called rounds. A return from round n to $n+1$ is observed only if i) project valuation post-money at round n is observed, ii) there is a subsequent valuation round $n+1$ which happens only if investments do well enough, and iii) project valuation pre-money at round $n+1$ is observed. Cochrane (2005b) reports that a return could not be computed in 58% of the cases and a subsequent round (item 2) is not observed in 23% of the cases. Korteweg and Sorensen (2010) use an improved sample and cannot compute a return in 36% of the cases.

bias (only companies that perform well get a new financing round). Consequently, they need to assume a parametric structure for both the return distribution (e.g. assume lognormally distributed returns) and the selection equation.

Our method is designed for cases where representative fund-level cash flows are observed, hence cases where (i) no return can be computed without restrictive assumptions[5] and (ii) there are no significant sample selection biases.[6] Point (i) means that we cannot use standard nonlinear least squares. Point (ii) means that we can avoid making distributional assumptions which is an important feature of our approach.

The advantage of the Cochrane-Korteweg-Sorensen (CKS) data compared to those we use in the empirical section is that they have more disaggregated information, which (i) may lead to more precise estimates of risk and (ii) allows for an analysis of risk and return as a function of project characteristics. Note also that in the empirical section, we do not measure exactly the same object as CKS. CKS would measure the return of Google from its valuation at the first round of venture financing until the IPO date. In contrast, we observe what investors paid and received from the investment in Google.[7]

[5] A return can be computed if and only if the cash flow stream consists only of one investment and one dividend. When there are intermediary cash flows, one can only compute a return at the cost of making a so-called re-investment assumption. An internal rate of return is such a return. Because our method does not require the observation of a return, we do not require assumptions on how paid-out dividends are reinvested by the investor.

[6] The data we use in the empirical section include the full cash flow history of a large number of funds. Hence, there are no (a priori) significant sample selection biases.

[7] An extreme example is the eBay IPO. Benchmark Partners return in eBay was 20 times the investment at the IPO. This is what CKS would observe. However, investors received the eBay stocks 6 months after the IPO, when the price had increased by more than 3000% making their stake worth 700 times the investment.

In terms of empirical results, the beta for venture capital reported by Korteweg and Sorensen (2010) is close to our estimate, while Cochrane's estimate for beta is somewhat lower (1.9). The alpha, however, is negative in our case, both before fees and after fees while CKS report large positive before-fee alphas.

Our paper is also related to Kaplan and Schoar (2005) and Phalippou and Gottschalg (2009) who benchmark private equity funds to the S&P 500 index, effectively assuming a CAPM with beta equal to one. Our results suggest that the benchmark for venture capital is much higher. In addition, we show via a statistical model that NAVs reported by mature funds (beyond their 10th anniversary) are exaggerated but not worthless; hence, we predict final fund market values that are in between the Kaplan-Schoar assumption (market value of non-liquidated mature funds is equal to the reported NAV) and the Phalippou-Gottschalg assumption (market value of non-liquidated mature funds is equal to zero). Also related is the work of Jones and Rhodes-Kropf (2004), who estimate the risk and return of private equity funds from the time series of returns constructed from NAVs.

Another study proposing a risk adjustment for buyout investments is that of Ljungqvist and Richardson (2003). They match buyout investments to similar publicly traded companies, assume a certain leverage and propose a beta close to unity. Finally, Moskowitz and Vissing-Jorgensen (2002) document returns obtained by entrepreneurs (mainly family businesses). The asset class they study is distinct from ours (although both are called "private equity") but with similar characteristics (illiquid, skewed return distribution). Like them, we find puzzlingly low returns.

III. Methodology and Illustrative Example

In this section we introduce our approach to estimate alpha and beta from a cross-section of cash flows. In Section III.A we describe our approach analytically and relate it to the commonly used internal rate of return (IRR). In Section III.B we provide a numerical example to illustrate our method.

A. Methodology

The input to our method is a panel of cash flow data for N (portfolios of) funds. For portfolio i $(i = 1, ..., N)$, we observe a series of cash flows between the inception date (denoted t_{0i}) and the liquidation date (denoted L_i). The cash flows consist of investments (sometimes called "takedowns") and dividends. We denote the amount invested by portfolio i at time t by T_{it}. Similarly, we denote the dividend by D_{it}. When facing such a cash flow stream investors typically calculate the internal rate of return of portfolio i as follows:

$$\sum_{t=t_{0i}}^{L_i} \left[\frac{D_{it} - T_{it}}{(1 + IRR_i)^{t-t_{0i}}} \right] = 0 \qquad (1)$$

Our approach extends this standard IRR calculation by incorporating exposure to realized market returns. Instead of using a constant discount rate, we specify a discount rate that is different each period, and equal to $1 + r_{f,t} + \alpha_i + \beta_i r_{m,t}$ in period t, where $r_{m,t}$ is the excess stock market return and $r_{f,t}$ the risk-free rate. This is simply the discount rate of a standard market model and it depends on two unknown parameters, α_i and β_i.[8]

If we replace IRR_i by $1 + r_{f,t} + \alpha_i + \beta_i r_{m,t}$ in equation (1), we obtain

$$\sum_{t=t_{0i}}^{L_i} \left[\frac{D_{it} - T_{it}}{\prod_{s=t_{0i}+1}^{t} (1 + r_{f,s} + \alpha + \beta r_{m,s})} \right] = 0 \qquad (2)$$

With this single equation and two unknowns we cannot solve for α_i and β_i. Thus, a necessary assumption is that there is a common parametric structure for α_i and β_i across portfolios. In the simplest case of $\alpha_i = \alpha$ and $\beta_i = \beta$, our method then looks for values of α and β that bring all the NPVs as close as possible to zero for the cross-section of N portfolios. The most straightforward way to do this is to solve the following least-squares optimization

$$\min_{\alpha,\beta} \sum_{i=1}^{N} [NPV_i(\alpha, \beta)]^2 \qquad (3)$$

where

$$NPV_i(\alpha, \beta) = \sum_{t=t_{0i}}^{L_i} \left[\frac{D_{it} - T_{it}}{\prod_{s=t_{0i}+1}^{t} (1 + r_{f,s} + \alpha + \beta r_{m,s})} \right] \qquad (4)$$

which is simply the present value of the net cash flows of portfolio i. Hence, our method discounts all dividends and investments back to the starting the date of

[8]Our approach allows for exposure to multiple factors. For simplicity, we explain the method using a one-factor market model. In the empirical analysis we incorporate multiple factors.

a portfolio using time-varying discount rates. In Section IV.A, we show that this estimator can be written as a Generalized Method of Moments (GMM) estimator that is asymptotically consistent. It is important to note that in equation (3) we use the realized market returns $r_{m,t}$ when discounting cash flows, and not expected returns. As discussed below, this is essential to identify β from a cross-section of portfolios.[9]

B. Numerical Example

We now discuss various aspects of our method using a simple numerical example. There are three funds. We assume a CAPM economy with no idiosyncratic shocks, a risk-free rate set to zero, $\alpha = -10\%$, and $\beta = 3$. These alpha and beta are close to those found for venture capital in the empirical section of this paper.

Funds invest in projects. Each project costs 100. Fund 1 invests in one project at the beginning of year 1 and in one project at the beginning of year 3. Fund 1 holds each project for one year. The market returns are chosen so that there is a multiple solution problem for fund 1's IRR.

The resulting cash flows are shown in the table below and are obtained as follows. The liquidating dividend of the first project of fund 1 is $100(1-10\%+3*40\%) = 210$. That of the second project is $100(1-10\%-3*26\%) = 12$. Fund 1 has three IRRs:

[9] Note that one needs to put an upper bound on α when performing the optimization in (3). This is to exclude the possibility that α tends to infinity. From equation (4), one sees that if α tends to infinity then the net present value may decrease in absolute terms, and hence the goal function in (3) may be decreasing for sufficiently high values of α. In our application, putting an upper bound of 100% per year suffices to avoid this numerical issue.

-81%, -56% and 48%. The chosen IRR would usually be the latter one (48%) because for a cost of capital in the range of 0% to 20%, the NPV of fund 1 is positive; so its IRR should be positive.

Fund 2 and fund 3 invest in one project each, and hold them for two years. Each project pays an intermediary dividend. Fund 2 pays an intermediary dividend equal to 70% of its value and fund 3 pays an intermediary dividend equal to 50% of its value.[10]

The cash flows for fund 2 are therefore as follows. The value of the project after one year is $100(1 - 10\% + 3 * 10\%) = 120$. The intermediate dividend is thus 84 and the liquidating dividend is $36(1 - 10\% - 3 * 26\%) = 4.32$. The same calculation is made for fund 3. The IRR of fund 2 is -30% and that of fund 3 is -72%.

Cash flows (year end)

Year	Market return	Fund 1	Fund 2	Fund 3
0		-100		
1	40%	210	-100	
2	10%	-100	84	-100
3	-26%	12	4.32	6
4	0%			5.4

To find the true underlying alpha and beta from the cash flows shown in the table

[10] These numbers are chosen to obtain multiple solutions when the system shown below is exactly identified.

above, the econometrician can set the NPV equal to zero as in equation (3). In this example, this means that she solves a system of three equations (one for each fund) in two unknowns

$$
\begin{aligned}
100+\frac{100}{(1+\alpha+0.4\beta)(1+\alpha+0.1\beta)} &= \frac{210}{1+\alpha+0.4\beta}+\frac{12}{(1+\alpha+0.4\beta)(1+\alpha+0.1\beta)(1+\alpha-0.26\beta)} \\
100 &= \frac{84}{(1+\alpha+0.1\beta)}+\frac{4.32}{(1+\alpha+0.1\beta)(1+\alpha-0.26\beta)} \qquad (5) \\
100 &= \frac{6}{(1+\alpha-0.26\beta)}+\frac{5.4}{(1+\alpha-0.26\beta)(1+\alpha)}
\end{aligned}
$$

We use Mathematica software to solve this system and obtain a unique solution $(-10\%, 3)$, which is the correct solution.

This example provides several insights. First, it shows that comparing IRRs across funds is not necessarily informative about fund performance. In this case, all three funds have the same alpha (skill) and beta (risk exposure), but have very different IRRs as they were exposed to different market returns.

Second, the example shows how the method is robust to multiple-solution issues. Although fund 1 has three IRRs, a unique solution for (α, β) is obtained. A related issue is that each equation is a polynomial and can therefore have multiple solutions. To illustrate this, suppose that we observed only cash flows of fund 1 and 2. In this case, there would be three (α, β) pairs satisfying the two equations: $(-19\%, 1.17)$, $(-10\%, 3)$ and $(1.5\%, 3.6)$. If we had only fund 1 and 3, we would again have three solutions: $(-215\%, -4.49)$. $(-72\%, 0.13)$, and $(-10\%, 3)$. If we had only fund 2 and

3, we would have three solutions: $(-163\%, -2.34)$, $(-102\%, 8.44)$ and $(-10\%, 3)$. Hence, with two funds and two unknowns we have a multiple-solution issue due to the polynomial nature of the equations.

However, when we use all three equations, the system is overidentified and we obtain a unique and "correct" solution: $(-10\%, 3)$. What is required for this uniqueness is that the funds are active in different time periods, so that they are subject to different market returns (so that there is no redundant equation). Having an overidentified system of equations thus seems to solve the multiple-solution problem. Although we do not have a formal proof for uniqueness of the solution, we always find a unique optimum i) when we changed the parameters in this example, ii) when we run many Monte Carlo simulations (Section IV.C) and iii) in the empirical analysis (Section VI).

A third insight from this example concerns the identification of alpha and beta. Beta is identified by having each fund going through different market returns. Essentially, if beta is positive, funds exposed to high market returns should have higher dividends compared to funds exposed to low market returns, and the extent to which this is the case identifies the magnitude of beta.

Alpha measures the abnormal performance. What we do is basically searching for the level of alpha that makes the NPVs across funds as close to zero as possible. If all funds have high dividends (compared to beta and market realizations), alpha will be high. To see this in our numerical example, if we would set alpha to 0 and beta to 3, the right hand side of the three equations in (5) is lower than the left hand side. This means that the present value of dividends is lower than the investments for all funds, indicating a negative alpha. As we decrease alpha from zero towards -10%, the present value of dividends increases; when alpha is -10% the three present values of dividends are equal to the present values of investments.

In sum, alpha affects the NPVs of different funds in the same way, thus changing the level of all NPVs, while the effect of beta on each funds' NPV is different and depends on the market returns that the fund was subject to.

IV. Validating and Implementing the Methodology

In this section we discuss the several steps to validate and implement our methodology. We first set our two assumptions in Section IV.A and provide the main proposition stating that our estimator is asymptotically consistent. Section IV.B discusses how to group funds into portfolios and how to calculate standard errors. Section IV.C uses simulations to validate our methodology and assess its small-sample properties.

A. Assumptions and Consistency of the Estimator

We focus on portfolios of funds and call these portfolios "fund-of-funds" (i.e. FoFs). Each FoF i invests in n_i projects, where each project requires an initial investment and pays out a dividend.[11] Hence, at the FoF level we observe a series of investments and dividends at different points in time.

[11] In the empirical section, we have fund-level data, with each fund making investments in multiple projects. We then form portfolios of funds, hence effectively we form portfolios of projects.

Assumption 1: The latent return $R_{ij,t}$ on a dollar invested in project j of FoF i in period t is generated by a linear factor model with idiosyncratic shocks. For example, in case of a one-factor market model we assume

$$R_{ij,t} = r_{f,t} + \alpha_i + \beta_i r_{m,t} + \varepsilon_{ij,t} \tag{6}$$

where $r_{f,t}$ is the risk-free rate, $r_{m,t}$ is the market return in excess of the risk-free rate, $\varepsilon_{ij,t}$ and $r_{m,s}$ are independent for all t and s, $\varepsilon_{ij,t}$ and $\varepsilon_{ij,s}$ are independent if $t \neq s$, and $E[\varepsilon_{ij,t}] = 0$. In Section IV.B we discuss the assumption we make on the correlation of $\varepsilon_{ij,t}$ across projects for the calculation of standard errors. But to obtain a consistent estimate of risk and abnormal performance, we can keep this correlation unspecified.

Assumption 1 states that, as long as the project is in the hand of the fund, its value grows by $1 + r_{f,t} + \alpha_i + \beta_i r_{m,t} + \varepsilon_{ij,t}$ each period. This is somewhat similar to the reinvestment assumption needed for IRR calculations: we assume that all projects of a fund are driven by the same return process until the project is terminated.[12]

Assumption 2: Some cross-sectional restrictions are placed on the α_i and β_i parameters.

As discussed in Section III.A, since we observe only a cross-section of funds, we

[12] Note that private equity funds directly pass through any dividends to their investors once a project is terminated. For example, Lerner and Schoar (2004, page 7) state: "The general partners (the private equity fund's managers) invest the capital raised from limited partners, typically large institutional and individual investors, in entrepreneurial or restructuring funds. After the firms go public or are sold, the proceeds (whether in the form of equity or cash) are divided between the limited and general partners, leading to a close alignment of the incentives of the two parties."

need to impose some cross-sectional restrictions to prevent having an underidentified system.

It is important to note that these two assumptions are shared with Cochrane (2005) and Korteweg and Sorensen (2010). In addition, note that we do not make assumptions about what happens to the dividends after they have been paid out to the investors. We do not need such assumptions and this is confirmed by our simulations below. We simulate investments and dividends without any assumptions about how dividends are reinvested by the fund investor, and when applying our approach we find the "correct" alpha and beta.

We can now formulate the main result on the estimator in equation (3).

THEOREM I: Given Assumptions 1 and 2, the estimator in (3) can be written as a first-step GMM estimator and is asymptotically consistent as the number of projects or funds in each portfolio of funds (FoF) tends to infinity.

Proof: Internet Appendix 1.

It is important to note that our method and asymptotics are cross-sectional: we fix the number of FoFs N, but let the number of projects (or number of funds) per portfolio, n_i, go to infinity to analyze consistency of our estimator. In Internet Appendix 1 we also argue that our method remains consistent when allowing for the possibility of exit timing (e.g. early exit and dividend payment in case of good performance).

B. Portfolio Formation and Inference

In order to lower the effect of idiosyncratic shocks as much as possible, we construct portfolio of funds (FoFs). As discussed in Section III, to identify β, it is essential that the different FoFs are exposed to different market returns. We thus form FoFs based on the date of the investment. With fund-level data, it means that we group funds based on their starting year (called vintage year).

To obtain standard errors, we use a cross-sectional bootstrap technique. We resample the funds with replacement within each FoF, and then re-estimate alpha and beta. Repeating the process 1,000 times yields the bootstrap distribution of alpha and beta. Intuitively, it is conceivable that projects within a fund are correlated, as funds may specialize in certain sectors or invest in related projects. By resampling at the fund level, we thus assume that the idiosyncratic shocks of projects are perfectly correlated within a given fund but that idiosyncratic shocks to projects are uncorrelated across funds.

The assumption on the independence of project idiosyncratic shocks between funds (within and across FoFs) is not needed for consistency but is required for our inference. To test whether it is a reasonable assumption, we propose two tests. First, we perform a block bootstrapping. We group funds into four blocks (2x2 sort on EU/US focus and fund size) within each vintage year. Within each block, the shocks are thus assumed to be perfectly correlated. This enables to gauge the validity of the independence assumption *within* vintage years. Second, we compute a pricing error for each of the 14 moment conditions (for the 14 vintage years we have in our data). This enables to gauge the validity of the independence assumption *between* vintage years. We conjecture that if there is significant cross-vintage-year dependence, pricing errors will be autocorrelated. Empirical results of these tests support our assumptions and are discussed in Section VI.A.

C. Small-sample Properties: A Monte Carlo Simulation

As discussed above, our GMM-style methodology generates asymptotically consistent estimates of α and β. To assess its performance in small samples, we run a Monte Carlo experiment. Results are shown in Table 1.

[INSERT TABLE 1 HERE]

We simulate project-level cash flows and aggregate these to obtain fund-level cash flow streams of investments and dividends. This matches the type of data we use in the empirical application below. Next, we apply our NPV-based estimation method to these simulated fund-level data. We aim to mimic the size and characteristics of our main dataset (venture capital funds). At the beginning of year = 1980,...,1993, 50 funds are started. They all invest $1 per project and start 3 projects from year 1 to 5. Hence, a fund has 15 projects in total.[13] The quarterly growth in the (latent) value of project j of fund i follows a simple market model

$$\frac{V_{ij,t+1}}{V_{ij,t}} = 1 + \alpha + r_f + \beta(R_{m,t+1} - r_f) + \varepsilon_{ij,t+1} \tag{7}$$

[13]This number matches the venture capital sample that we describe below.

where $R_{m,t+1}$ is i.i.d. shifted lognormal over time, i.e. $R_{m,t+1} = e^x - c$, where x is normally distributed with mean μ_m and variance σ_m^2, and c is a constant. Similarly, $\varepsilon_{ij,t}$ is i.i.d. shifted lognormal across projects and over time. We use a shifted lognormal distribution in order to make sure that the project return is bounded below by -100%. Given the large volatility of project returns, using a normal distribution would generate returns below -100% with nonnegligible probability. In Internet Appendix 2 we describe in detail how we calibrate the parameters of the shifted lognormal distributions. In short, for the market return we match S&P 500 data and for the idiosyncratic volatility we match Cochrane's (2005b) estimate (leading to an annual idiosyncratic volatility of 108% per year). Risk-free rate is set to 4% p.a. The "true" α is set to zero and "true" β is set to one.[14]

The timing of dividend payments is endogenous, using the process assumed by Cochrane (2005b). That is, the probability that a project exits at time t is given by the logistic function $\frac{1}{1+e^{-a(\ln(V_{ij,t})-b)}}$. Hence, a project is more likely to exit as it reaches higher values. In addition, a project is more likely to exit if it reaches a low value. The probability of exiting is given by $max(0, \frac{k-V_t}{k})$. The parameter values are taken from Cochrane (2005b): $a = 1, b = 3.8, k = 0.25$. Finally, if a project is still alive after 5 years, it is liquidated and a dividend equal to its value is paid.[15]

[14]We have also used different pairs of true parameters ($\alpha = -10\%, 0\%$, or 10% p.a., and $\beta = 1, 2$, or 3), and find that all the results presented in this section hold true with these combinations as well.

[15]Note that we do not make any assumptions on how dividends are reinvested after being paid out to the investor.

Following the argument in the previous section, we group all the funds with the same vintage year into a FoF. We thus have 14 moment conditions (one for each vintage year).

In the third column of Table 1 we see that our NPV-based estimation methodology generates across simulations an average beta of 0.98, very close to the true value of 1, and an average alpha of 0.05% per month, again close to the true value of 0%.

The performance of our method depends naturally on the size of idiosyncratic shocks. We therefore present results with different levels of idiosyncratic volatility. Column 4 shows the results with a relatively low idiosyncratic volatility (25% per annum)[16] and column 5 shows results with very high idiosyncratic volatility (150% per annum). Our estimator is very precise when idiosyncratic volatility is relatively low. When there are very high idiosyncratic shocks, the precision is lower. In addition, we find an alpha that is too high on average and a beta that is too low. This small-sample bias is due to (i) nonlinearities in the moment conditions and (ii) the fact that a high alpha helps to decrease the absolute size of the NPVs (see (4)). This means that in small samples with extremely volatile projects one would underestimate the beta and overestimate the alpha.

Next, we provide numerical evidence of the statistical consistency of our method by increasing the number of projects per fund from 15 to 50 (see Table 1 column 6)

[16] This corresponds to the Ang, Hodrick, Xing and Zhang (2006) estimate for the highest idiosyncratic volatility quintile of US stocks. Their Table 6 provides total volatility estimates across quintile portfolios. We correct these total volatilities for market volatility to obtain idiosyncratic volatility.

and then from 50 to 100 (Table 1, column 7). We see that the estimator converges towards the true value with increased precision, as expected.

Finally, we redo the simulation using an alternative exit rule (Table 1, column 8). The probability of exit is simply a function of the contemporaneous market return. All projects exit if the quarterly market return is higher than its 95th percentile. Hence, the expected duration is 20 quarters (i.e. 5 years). In this case, exit only depends on market-wide returns and not individual project returns. Project exit is thus highly clustered. This may reduce the information in the hand of the econometrician. Yet, results are satisfactory as they are similar to the benchmark results.

V. Data

A. Data Source

Data on both private equity fund cash flows (net of fees) and quarterly NAVs are from Thomson Venture Economics. We have separate observations for investments and dividends. This dataset is the most comprehensive source of financial performance of both US and European private equity funds and has been used in previous studies (e.g., Kaplan and Schoar, 2005). It covers an estimated 66% of both venture capital funds and buyout funds (Phalippou and Gottschalg, 2009).

We consider all funds (with size over $5 million) raised between 1980 and 1993 as they have reached their normal liquidation age (10 years) at the end of our sample period (2003). We construct venture capital fund-of-funds and buyout fund-of-funds based on vintage years. We exclude vintage years with less than 10 funds; this excludes buyout funds raised between 1980 and 1983 but does not affect venture capital funds.

[INSERT TABLE 2 HERE]

This leaves us with 958 funds, of which 686 have a Venture Capital (VC) objective and 272 have a buyout (BO) objective. In total, we have 25,800 cash flows. Descriptive statistics are reported in Table 2. We present these separately for funds that were fully liquidated by the end of 2003, and for funds that were not yet liquidated end of 2003 and thus report a positive NAV at the end of our sample period. Our descriptive statistics are similar to what has been reported in the literature. We see that liquidated funds and non-liquidated funds have similar characteristics, although liquidated buyout funds are smaller.

[INSERT TABLE 3 HERE]

In Table 3 we present the annual cash flows of each vintage-year fund portfolio (FoF). It shows how investments are typically done in the first years, and that from the fifth year onwards dividends are paid out. In Figure 1 we plot a histogram of the internal rate of returns (IRRs) of the individual funds. The graph shows large cross-sectional variation in these IRRs. This reflects the idiosyncratic risk faced by each fund, but also the fact that funds were subject to different market-wide shocks.

[INSERT FIGURE 1 HERE]

B. Estimating Final Market Values

Table 2 shows that two thirds of the funds report a positive NAV at the end of our sample period despite having passed their tenth anniversary. Existing work either treats these final NAVs as a final cash flow (Kaplan and Schoar, 2005) or writes them off (Phalippou and Gottschalg, 2009). One of the problems faced in the literature and which partly explains these simple choices is that the conversion of NAVs into a market value necessitates an estimate of systematic risk.

This section describes how we model the final market value of these non-liquidated funds. We take the fully liquidated funds at different ages, compute their realized market value (MV) as the net present value of subsequent cash flows where we discount with the pricing model estimated by our method. Then, for each age a=10,11,12, and 13, we separately estimate the following model

$$\ln(1 + MV_{a,i}(\alpha, \beta)) = b_{a0} + b'_{a1} X_{a,i} + \varepsilon_{a,i} \tag{8}$$

The vector of explanatory variables $X_{a,i}$ includes $\ln(1 + NAV)$, the log of fund size, the log of the time elapsed since the last dividend distribution, the log of the time elapsed since the last NAV update, and fund's performance multiple excluding NAV (sum of capital distributed divided by sum of capital invested); where all variables are computed at age a.

Results from regression (8) are in Table 4 - Panel A. We find that a 1% increase in NAV leads to slightly less than 1% increase in market value and this elasticity decreases with age. Given the log-specification this implies that large funds have relatively lower residual market values.[17] Funds that have not paid a dividend for a long time or not updated their NAVs for a long time also have lower market values.

[INSERT TABLE 4 HERE]

Next, we use equation (8) to predict final market values for the non-liquidated funds.[18] Results of the prediction are shown in Table 4 - Panel B.[19] The ratio of total predicted market values to total reported NAVs is between 21% (age 12) and 38% (age 10). These low ratios are mainly due to the fact that non-liquidated funds (i) are larger, (ii) have not paid any dividends for about 3.5 years (versus 1 year for liquidated funds) and (iii) have not updated their NAVs for 2.5 years (versus 6 months for liquidated funds). We thus provide evidence that NAVs of old and inactive funds largely overstate the true market value.

The results described above require a joint estimation setup. To run regression (8) for the fully liquidated funds, we need $MV_{a,i}(\alpha,\beta)$ which depends on the discount rate and hence on α and β. In turn, to estimate α and β with GMM, we need the

[17] This can be seen as follows. Neglecting the other variables, we have $\ln(1+MV) = b\ln(1+NAV)$, which implies $1 + MV = (1 + NAV)^b$. If $b < 1$, large funds have relatively lower market value.

[18] Beyond the 13th anniversary, we observe very few funds with positive NAV that are subsequently liquidated. For funds older than 13 years, we predict the market value at the end of their 13th anniversary using the coefficients from the age 13 regression. This is why most of the funds in the prediction sample are in the age 13 category (N=434).

[19] The results described above are those obtained when estimating a one-factor market model. We find similar coefficients for the three-factor model.

predicted values of $MV_{a,i}(\alpha, \beta)$ for the non-liquidated funds. We thus simultaneously estimate equation (8) and α and β from equation (3).

A valid concern in this analysis is that liquidated funds are inherently different from funds that were not yet liquidated end of 2003, so that regression (8), estimated using liquidated funds, cannot be used to predict market values of non-liquidated funds. We deal with this issue in several ways. First, we present characteristics of liquidated and non-liquidated funds in Table 2 (size, multiple, first-time fund or not, US versus Europe) and find that the two groups are similar in these dimensions (except that liquidated buyout funds are smaller). Second, we perform a wide range of robustness checks on (8). We start with a model using only the NAV as explanatory variable and then add variables step by step. We also add vintage-year fixed-effects to the model in (8). The predicted market values across all these models range from 29% to 46% of the NAV, hence we always find market values well below the NAVs. We also find that the estimated α and β are very similar across these different specifications (non-tabulated results). Finally, and most importantly, we show in the robustness section that if we make two extreme assumptions about the true value of the NAVs the estimated α and β do not change dramatically. This further shows that results are robust to changes in the model that converts NAVs into market values.

VI. Risk and Return Estimates

In this section, we report the estimates of risk and abnormal return of private equity funds and present several robustness checks.

A. Benchmark Results

Table 5 - Panel A shows results after fees. We find that venture capital funds have a particularly large market beta (2.73). This implies a large cost of capital. Taking a naive estimate for the equity premium of 8% (the average over our time period) and a risk free rate at 5%, the cost of capital is $5\% + 2.73 * 8\% \approx 27\%$ according to the CAPM. The abnormal return (alpha) is significantly negative at -1.09% per month, thus about -12% annual. The large negative abnormal performance of venture capital is a direct result of the large beta. Using a back-of-the-envelope calculation, venture capital funds have an average IRR of 15%, a cost of capital of 27%, hence the abnormal return is about -12%.

[INSERT TABLE 5 HERE]

We also consider the three-factor Fama-French model and find that VC returns resemble those of small growth stocks. Relative to these small growth stocks, there is still underperformance of VC, but much less so given that small growth stocks perform poorly too. The alpha decreases to -0.74% per month or -8.5% annual.[20]

Buyout funds have a very different risk profile. We find a lower market beta (1.31). Consequently, the cost of capital is relatively low at $5\% + 1.31 * 8\% \approx 15\%$,

[20] Note that with the three-factor Fama-French model, the precision of the estimates decrease. Given that we perform a cross-sectional estimation, the parameter estimates are correlated to some extent and the correlations between these parameter estimates are higher for the three-factor model. This makes it harder to precisely pin down the different risk exposures. Still, the exposures to SMB and HML make intuitive sense.

and there is a negative but insignificant alpha of -0.41% per month. According to the three-factor model of Fama-French, the alpha is even lower but still insignificant.

So far, all results reported are net of fees, since all cash flows in our sample are net of fees. To assess the impact of fees, we add simulated fees to the original cash flows. We assume a standard fee structure consisting of a 2% management fee (on committed capital), 20% carry with an 8% hurdle rate (see Metrick and Yasuda, 2010). The results in Table 5-Panel B show that fees affect both beta and alpha. Adding back fees increases the annualized alpha by 3% to 4%. Hence, a substantial part of the underperformance of VC funds is due to fees. Table 5-Panel B also shows an interesting effect of fees on the betas. Given the hurdle rate, fees are nonlinear and this increases the beta when we add fees. The increase in beta is 0.15 for VC funds and slightly larger (0.23) for BO funds. This means that the fee bill is actually lower than if one simply measures the difference between after-fee performance and before-fee performance. For example, Phalippou and Gottschalg (2009) assess fees to be 6% annual, but maintain the assumption that beta is one.

As mentioned in Section IV.B, standard errors are derived under the assumption that the idiosyncratic shocks are perfectly correlated within a fund, but independent between funds (both within and between FoFs). We have proposed two tests to gauge the validity of this assumption. First, we repeat all the inference in Table 5 with block bootstrapping instead of simple bootstrapping. We group funds within each FoF by geographical focus and size, thereby assuming that funds of similar size and geographical focus have perfectly correlated pricing errors. The standard errors obtained in this way are extremely similar (non-tabulated). Second, we compute the autocorrelation of FoF pricing errors, which we report underneath each specification in Table 5. We find that the autocorrelation is negative in most specifications and never significantly positive.[21] These results are a strong indication that the independence assumption we make for inference is reasonable on this dataset.

Finally, in Internet Appendix 3 we show how one can make the alpha and beta a function of fund characteristics (fund size, fund experience, and US versus Europe focus). Overall, we do not find significant effects of these variables on alpha and beta, although US funds seem to have slightly better performance than European funds.

B. Reality Check

To gain confidence in our beta estimates, we investigate the time series of "dividend yields" for venture capital funds and buyout funds. The dividend yield in year t for fund i is defined as the sum of all dividends paid over this year divided by fund size. To obtain an aggregate dividend yield, we take the average across all funds that are in their divestment phase, i.e. fund age is between 4 and 10 years. Figure 2 shows the resulting series. On the same graph, we plot the 5-year moving annual average of the S&P 500 returns. The idea is that if the stock-market does well during 5 years - which is the average duration of an investment - then a high (low) beta asset

[21] In unreported results, we find that the autocorrelation of the IRR per vintage year is positive. This suggests that, once we correct for the overlapping market return exposure of different vintage years, the residual returns are not correlated.

will distribute larger (smaller) dividends in the following year. On the figure, it is apparent that the first pick of the stock-market in 1995 and the rally of 1998-1999 go hand-in-hand with a huge spike in dividend yield for venture capital funds. When the stock market went down the following three years, so did the dividends. Our high estimate for the venture capital beta reflects these features of the data. Interestingly, the same figure shows that buyout fund dividends are smoother across years. Figure 2 is thus consistent with our empirical estimates of risk exposure.

[INSERT FIGURE 2 HERE]

Yet, one may still wonder how a leveraged buyout investment can have a relatively low beta. Kaplan and Stein (1990) find that in highly leveraged transactions, a sharp increase in debt coincides with a "surprisingly small" increase in equity beta. Hence, increased leverage may have a countervailing effect on the asset beta. In addition, a related puzzle appears to be present for stocks. E.g. Korteweg (2005) talks about a "leverage puzzle" as he, like other researchers, finds no positive relation between leverage and expected stock returns.

An important caveat here is that our estimates for buyout funds are less precise than for venture capital funds. This is probably due to the smaller sample for buyout funds.

C. Economic Interpretation

As mentioned in the introduction, an ongoing debate in this literature is whether private equity funds add value. To answer this question, one needs to know the alpha before fees. For venture capital, it is negative according to both the CAPM and the three-factor model of Fama-French. A classic interpretation of this negative before-fee alpha would be that the price paid by VC funds to acquire assets is too high. One reason could be that VC funds have underestimated the systematic risk of their investments and thus used a too low discount rate. Another reason could be that there is too much money chasing too few opportunities (Gompers and Lerner, 2000). This overcapacity could be created by a number of investors that are not performance maximizing. In VC, particularly in Europe (where performance is lowest), large amounts are invested by government-sponsored bodies to stimulate local economies.[22]

Obviously, if alphas are negative before fees, they get only worse after fees. This raises the question of why investors accept to pay 4% per year for negative alphas. First, as mentioned above, there may be some side benefits of investing in venture capital. Second, our paper shows that measuring risk is not trivial and is only possible when a substantial set of funds are liquidated or close to liquidation. Over our sample period, investors had less data than we have, especially so in the early years. Lack of data and high idiosyncratic volatility make learning difficult. Investors may observe absolute returns but widely used proxies do not seem alarming (e.g. 15% IRR on average). It is the risk correction that makes the alpha negative for venture capital

[22] For example, the European Investment Fund (EIF), sponsored by the European Union, "now accounts for about 10% of the early-stage venture capital market in the EU, with a portfolio of EUR 2.4bn invested in about 180 funds from across the EU and the Candidate Countries." See http://www.eif.org/about/news/quarterly-newsletter-issue-1.htm.

funds. One may expect that, in the future, alphas move towards zero as some investors may learn. Our estimate of beta, however, may be a reasonable estimate looking forward.

D. Comparison with IRRs

As mentioned above, our approach extends the standard static IRR approach to a dynamic setting with time-varying discount rates. This allows us to decompose the return into abnormal performance and risk exposure. One would then expect that the time-series average of the estimated time-varying discount rates is close to the static IRR. However, this average "dynamic IRR" will not be exactly equal to the static IRR due to the complex relation between the NPV and the discount rates. Only if there is a single investment and dividend is the average of the time-varying discount rates equal to the static IRR.[23] In addition, each fund-of-funds will be subject to idiosyncratic shocks, which generates further discrepancy between the static IRR of a FoF (which also reflects idiosyncratic shocks that the funds were subject to) and the dynamic IRR implied by the market model (which only depends on the market returns and the estimates for alpha and beta).

[INSERT TABLE 6 HERE]

In Table 6 we compare the average dynamic IRRs with the static IRRs across the different vintage years. To calculate the average dynamic IRR for each vintage

[23]This can be shown as follows. The IRR solves $-I + \frac{D}{(1+IRR)^T} = 0$, while (in case a of perfect fit) our approach implies $-I + \frac{D}{\Pi_{t=1}^{T}(1+r_{f,t}+\alpha+\beta r_{m,t})} = 0$. In this case the IRR equals the geometric average of the discount rates.

year, we need to take the average of the market return and risk-free rate over the period during which the funds were active. Intuitively, the years in the middle of a funds' life should be more important than the first year or the final years. We opt for a simple approach. We compute when the average investment is done. We find it to be when the fund is 1.5 year old. Similarly, we compute the average time at which dividends are paid out and find it to be 6.5 years later for venture capital funds and 5 years later for buyout funds.[24] The (monthly) average dynamic IRR is equal to alpha plus the average risk free rate plus beta times the average market return minus the average risk free rate. This number is then annualized and compared to the annual static IRR. The average dynamic IRR is 18% (14%) versus a static IRR of 15% (15%) for venture-capital funds (buyout funds). The correlation between the static and dynamic IRRs across vintage years equals 66% for venture-capital but only 2% for buyout. The lower correlation for buyout is driven by the extreme static IRR for vintage-year 1985 (which has only 12 funds). Without this year the correlation equals 29%.

Table 6 thus shows that the average discount rate (i.e. dynamic IRR) is rather close to the static IRR, which is reassuring. It also shows that the realized discount rates vary substantially over time. As mentioned earlier, this variation is not recognized by the standard IRR, but is incorporated in our approach.

[24] To find these average investment/dividend times, we proceed like when computing a duration for a bond. We weight the time of the relevant cash flows by their present value, using the IRR as discount rate.

E. Robustness

In this subsection, we investigate the robustness of our results. Table 7 shows estimates of alpha and beta (CAPM model) for different samples and different methodological choices. The default results (those shown in Table 5) are included in the first line for convenience.

[INSERT TABLE 7 HERE]

First, we study the impact of using different values for the final market value of non-liquidated funds. We re-estimate abnormal return and risk with i) final NAVs treated as fair market value (as in Kaplan and Schoar, 2005) and with ii) writing final NAVs off (as in Phalippou and Gottschalg, 2009). The VC beta equals 2.79 (when final NAV is treated as market value) versus 2.30 (when final NAV is written off). The estimate we find in the main analysis is between these two values. Similar results are observed for buyout funds. Interestingly, the effect on abnormal performance is minimal. This is because writing off NAVs decreases both beta and raw performance. Hence, although we cannot be certain that our treatment of final NAVs is correct, the relatively small sensitivity of our estimates to the treatment of NAVs is reassuring.

The second result is that varying the number of FoFs has little impact on the estimates. For each vintage year, we sort funds by size and create either 2, 3 or 4 portfolios. We always find similar alpha and beta estimates.

Our third result is that changing the time period does not significantly change estimates. This is especially true for VC funds. Given the nature of our data, we provide a sense of the impact of the sample time period (funds raised between 1980 and 1993) by adding and removing one vintage year.

Fourth, we estimate the alpha and beta for the subsample of liquidated funds. Liquidated funds have somewhat higher alphas, and liquidated VC funds have somewhat higher betas, while liquidated BO funds have somewhat lower betas. But, overall, we still find substantial underperformance for VC funds, a high beta for VC funds and a much lower beta for BO funds.

The fifth result is that using a different weighting of the moment conditions (FoFs) does not substantially change the estimated risk and abnormal performance. In the benchmark case we simply add cash flows of funds into FoFs, and equally weight these FoFs. This can be seen as value-weighting, since larger funds with larger cash flows have a larger impact on the pricing error of a FoF. We therefore also consider a weighting scheme that weights by the inverse of FoF size to undo this implicit value weighting. This does not affect the VC results, while for buyouts the alpha increases and the beta decreases. We also consider a weighting scheme where more weight is given to FoFs which consist of more funds. This is achieved by using a weight equal to (the square root of) the number of funds per FoF divided by the size of the FoF.[25] This weighting does not change the VC results, while for buyout funds the alpha is a bit higher and beta a bit lower.

[25] We use the square root of the number of funds per FoF, since precision grows with the square root of the number of observations.

Finally, we run our estimations using different indices for the market portfolio. We begin by using a different market portfolio for the non-US focused funds. In the above analysis we have used one market portfolio for all funds. Implicitly we have assumed that financial markets are integrated. We now assume that financial market are perfectly segmented and thus use non-US stock indices for non-US focused funds (with returns in US dollars to be consistent with the cash flow currency). The indices come from the website of Ken French. We use the Europe index and find that this hardly affects the alpha and beta estimates. Next, we use the NASDAQ for venture capital funds. We find a lower beta (1.47 instead of 2.73) and a higher alpha (-0.45% versus -1.09%). This is consistent with the idea that venture capital funds resemble more Nasdaq stocks. Similarly, when we use the small-growth portfolio of Fama-French, we obtain a beta of 1.62. Alpha, however, turns positive consistent with the fact that the small growth portfolio has had historically a very low performance. This result is similar to what Cochrane (2005b) finds with venture capital projects.

VII. Conclusion

We develop a new econometric methodology to estimate the risk and return of an asset using cash flow data. Our method extends the standard IRR calculations to a dynamic setting, and solves for the abnormal return and risk exposure that best fit the cross-section of private equity fund cash flows. A simulation study shows that the small-sample properties of our method are satisfactory.

We apply it to a large sample of private equity funds. We find that venture capital funds have a high CAPM-beta, while buyout funds have a much lower CAPM-beta. Venture capital funds have a significantly negative alpha, both before and after fees. Buyout funds also have negative alphas, but these are statistically insignificant. Our model indicates that the net asset values reported by funds that are inactive and mature are highly upward biased estimates of their market value.

Our method can be used for other limited life non-traded private partnerships (e.g. mezzanine debt funds and some real estate funds) and for corporate investments in case the CFO observes a stream of cash flows from a division/project but no market values.

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Table 1: Monte Carlo Simulations

Each year (from 1980 to 1993), 50 funds are started. Each fund invests $1 per project, starts 3 projects per year from year 1 to 5, leading to a total of 15 projects per fund. The project return follows a one-factor market model. Market returns and error terms are drawn from shifted-lognormal distributions. Market returns are matched to the empirical distribution of the S&P 500 (Appendix 2). In the benchmark case, idiosyncratic volatility is matched to that of Cochrane (2005) (108% per year, see Appendix 2 for details). Idiosyncratic volatility is set to 25% p.a. and 150% p.a. in the lower and higher volatility case respectively. Columns 6 and 7 show results when sample size is increased from 15 projects per fund to 50 and 100 projects per fund. For the first five columns, the project liquidation rule is that of Cochrane (2005). For the last column, the project liquidation rule is: liquidates if market return that quarter is above 17% (its 95^{th} percentile). This exercise is repeated 1000 times with true alpha set to zero and true beta set to one. The mean, standard deviation, and inter-quartile range of the 1,000 estimated pair of parameters (alpha, beta; monthly frequency) are displayed.

	True values	Changing idiosyncratic volatility			Increasing sample size		Changing the
		Benchmark estimates	Lower	Higher	50 projects	100 projects	liquidation rule
Mean Alpha	0.00%	*0.05%*	0.00%	0.22%	0.01%	0.01%	-0.01%
Std Alpha		*0.20%*	0.03%	0.46%	0.10%	0.07%	0.18%
Inter-Quartile		*[-0.06% 0.15%]*	[-0.01% 0.02%]	[-0.05% 0.41 %]	[-0.04% 0.07%]	[-0.03% 0.04%]	[-0.10% 0.08%]
Mean Beta	1.00	*0.98*	1.00	0.88	1.00	1.00	1.00
Std Beta		*0.32*	0.06	0.65	0.16	0.12	0.29
Inter-Quartile		*[0.80 1.15]*	[0.97 1.03]	[0.54 1.22]	[0.91 1.09]	[0.94 1.06]	[0.85 1.16]

Table 2: Descriptive Statistics

This table shows descriptive statistics for our sample. Statistics are shown separately for liquidated and non-liquidated funds and for venture capital funds (Panel A) and buyout funds (Panel B). The following statistics are shown: (i) the average and the median of the amount committed to funds in million of 2003 U.S. dollars (size); (ii) the total final Net Asset Value (NAV) reported (December 2003), total capital distributed and total capital invested; (iii) the total and realized multiple; (iv) the proportion of first time funds; (v) the proportion of funds that are US focused; and (vi) the number of cash flows and the number of funds.

Panel A: Venture Capital funds

		Fully liquidated funds	Not fully liquidated funds	All funds
Mean size	($ million)	78	98	91
Median size	($ million)	50	55	52
Sum NAV	($ billion)	0	8	8
Sum Distributed	($ billion)	24	57	81
Sum Invested	($ billion)	11	29	40
Total Multiple: (NAV+Distributed)/invested		2.24	2.27	2.26
Realized Multiple: Distributed/invested		2.24	1.99	2.06
Average (value-weighted) IRR		9.8%	12.1%	11.4%
First time funds		55%	41%	46%
US-focused funds		82%	76%	78%
Number of cash-flows		5,799	11,060	16,859
Number of funds		251	435	686

Panel B: Buyout funds

		Fully liquidated funds	Not fully liquidated funds	All funds
Mean size	($ million)	219	455	371
Median size	($ million)	87	146	133
Sum NAV	($ billion)	0	20	20
Sum Distributed	($ billion)	27	102	129
Sum Invested	($ billion)	13	67	80
Total Multiple: (NAV+Distributed)/invested		2.06	1.80	1.85
Realized Multiple: Distributed/invested		2.06	1.51	1.60
Average (value-weighted) IRR		16.6%	13.7%	14.3%
First time funds		55%	58%	57%
US-focused funds		49%	56%	53%
Number of cash-flows		3,060	5,878	8,941
Number of funds		97	175	272

Table 3: Cash-flows of each Fund-of-Funds

The table shows the net cash flows of each fund-of-funds (one per vintage year) used in the empirical analysis, aggregated per year.

Panel A: Cash flows of venture capital fund-of-funds ($ million)

	Vintage year													
	1980	1981	1982	1983	1984	1985	1986	1987	1988	1989	1990	1991	1992	1993
1980	-283	0	0	0	0	0	0	0	0	0	0	0	0	0
1981	-243	-227	0	0	0	0	0	0	0	0	0	0	0	0
1982	-76	-195	-449	0	0	0	0	0	0	0	0	0	0	0
1983	73	-166	-245	-734	0	0	0	0	0	0	0	0	0	0
1984	10	2	-161	-561	-849	0	0	0	0	0	0	0	0	0
1985	123	49	-27	-413	-530	-648	0	0	0	0	0	0	0	0
1986	336	118	55	-189	-481	-383	-1120	0	0	0	0	0	0	0
1987	229	207	129	278	-263	-261	-767	-1140	0	0	0	0	0	0
1988	50	82	121	91	-115	-112	-668	-837	-625	0	0	0	0	0
1989	80	63	153	264	235	59	-536	-563	-497	-1270	0	0	0	0
1990	90	70	176	206	362	119	117	-226	-598	-717	-346	0	0	0
1991	176	397	173	332	353	271	205	3	-240	-756	-276	-886	0	0
1992	205	71	98	585	502	210	361	108	-41	-1038	-305	-185	-503	0
1993	1138	42	60	273	470	274	494	796	430	-329	-263	-106	-614	-640
1994	5	79	60	318	364	367	849	638	681	-120	49	-146	-584	-599
1995	32	4	84	362	359	256	625	1171	1034	1131	668	230	-57	-419
1996	32	9	36	411	383	262	1424	1116	1384	2222	951	752	941	128
1997	11	4	28	301	274	287	2074	827	735	2302	786	283	1715	885
1998	0	0	9	31	62	201	1099	602	456	1260	398	104	954	273
1999	0	0	0	100	26	2	398	800	348	2032	547	1532	834	923
2000	0	0	0	79	7	7	258	617	278	1141	496	416	1154	4669
2001	0	0	0	0	11	14	241	14	241	687	70	28	403	332
2002	0	0	0	0	3	0	1	2	7	460	44	18	74	172
2003	0	0	0	0	1	0	0	0	0	2	152	437	304	705
IRR	18%	10%	4%	8%	6%	6%	12%	12%	16%	15%	24%	17%	30%	32%

Panel B: Cash flows of buyout fund-of-funds ($ million)

	Vintage year									
	1984	1985	1986	1987	1988	1989	1990	1991	1992	1993
1984	-734	0	0	0	0	0	0	0	0	0
1985	-766	-215	0	0	0	0	0	0	0	0
1986	244	-560	-707	0	0	0	0	0	0	0
1987	276	78	-205	-2075	0	0	0	0	0	0
1988	1487	987	262	-1932	-1728	0	0	0	0	0
1989	41	1439	101	-89	-1281	-4810	0	0	0	0
1990	552	28	155	-1421	-1643	-790	-2230	0	0	0
1991	131	88	69	-267	-701	-618	-559	-550	0	0
1992	456	95	424	-188	-853	-650	-891	-326	-1153	0
1993	452	645	171	806	2258	571	-74	-345	-859	-1905
1994	130	214	208	1632	1020	802	685	-157	-1439	-1176
1995	663	315	48	1848	2075	2292	1767	43	608	-1994
1996	1146	287	453	3265	2554	1365	1022	439	1815	-318
1997	340	142	742	884	973	2044	2455	546	1747	1223
1998	573	418	973	1783	1138	1173	1773	502	1868	1967
1999	77	24	524	911	768	571	1074	395	505	3537
2000	0	3	0	2942	900	532	351	127	1168	1702
2001	0	0	0	5	649	498	43	50	692	1038
2002	0	0	0	16	60	3094	51	5	261	711
2003	0	0	0	0	0	0	346	156	625	2224
IRR	26%	56%	21%	11%	12%	8%	16%	9%	23%	16%

Table 4: Final Market Value

Panel A shows the relation between fund market value (MV) and fund characteristics for the sample of liquidated funds. Market Value (MV) at a given age is computed as the present value of the subsequently realized cash flows using the market model to discount. Fund characteristics include Net Asset Value (NAV), fund size, time elapsed since last dividend distribution (LastDiv), time elapsed since last NAV change (LastNAV), and Profitability Index (present value of dividends over present value of takedowns). Standard errors are show between parentheses. The estimation is done separately for each age. This regression is estimated simultaneously with the GMM estimation of alpha and beta. Panel B shows summary statistics of the liquidated sample and non-liquidated sample; including the predicted Market Values of non-liquidated funds using the estimated statistical model (Panel A). *, **, and *** indicate significance at 10%, 5%, and 1% levels, respectively.

Panel A: Market values as a function of fund characteristics – liquidated sample

	Dependent variable: ln(Market Value)			
	Age 10	Age 11	Age 12	Age 13
Constant	-0.08	-0.19	*-0.47	-0.32
	(0.22)	(0.28)	(0.26)	(0.35)
ln(1+NAV)	***0.89	***0.84	***0.83	***0.72
	(0.04)	(0.05)	(0.05)	(0.06)
ln(Size)	*0.09	*0.11	**0.13	*0.13
	(0.06)	(0.06)	(0.06)	(0.07)
ln(LastDiv)	**-0.09	-0.08	-0.02	**-0.16
	(0.04)	(0.05)	(0.05)	(0.07)
ln(LastNAV)	***-0.24	***-0.22	***-0.28	**-0.17
	(0.04)	(0.05)	(0.05)	(0.07)
Profitability Index	-0.02	0.15	*0.22	***0.39
	(0.05)	(0.10)	(0.12)	(0.14)
Adj. R-square	0.70	0.68	0.71	0.66
N-observations	280	226	182	136

Panel B: Summary Statistics

	Fully liquidated funds				Not fully liquidated funds			
	Age 10	Age 11	Age 12	Age 13	Age 10	Age 11	Age 12	Age 13+
(mean) NAV	35.32	30.96	21.15	19.76	78.47	48.55	56.09	55.89
(mean) Size	121.21	122.77	122.08	123.42	234.81	202.30	140.89	203.93
(mean) LastDiv	14.00	12.75	11.97	11.40	37.09	36.58	57.46	41.03
(mean) LastNAV	7.39	5.82	5.60	5.26	22.22	23.54	37.62	29.77
(mean) PI	0.90	0.92	0.94	0.99	0.82	0.94	0.59	0.81
NAV/Size	0.23	0.16	0.09	0.07	0.33	0.24	0.40	0.27
MV/NAV	1.00	1.00	1.06	1.13				
Predicted_MV/NAV					0.38	0.34	0.21	0.28
N-observations	280	226	182	136	79	50	50	431

Table 5: Risk and Abnormal Performance of Private Equity Funds

This table shows monthly abnormal performance (Alpha) and risk loadings using either a one-factor market model (S&P 500; specs 1 and 3) or the three-factor Fama-French model (specs 2 and 4). Panel A shows the results with the original (net-of-fee) cash flows. Panel B shows the results with the simulated gross-of-fee cash flows. Gross-of-fee cash flows are obtained by adding fees to the net-of-fees cash flows. Fees are assumed to be made of 2% management fee and 20% carry with an 8% hurdle rate (see text for details). Estimation is executed by GMM with joint estimation of final market values (see text for details). Moment conditions are equally weighted. Standard errors are obtained by bootstrapping and are show between parentheses. Below each specification, the autocorrelation of the pricing errors across vintage years is reported with its corresponding standard error. *, **, and *** indicate significance at 10%, 5%, and 1% levels, respectively.

Panel A: Net-of-fee

	Venture Capital		Buyout	
	Spec 1	Spec 2	Spec 3	Spec 4
Alpha	***-1.09%	*-0.74%	-0.41%	-0.97%
	(0.21)	(0.41)	(0.58)	(0.70)
Beta_Market	***2.73	***2.37	**1.31	***1.71
	(0.55)	(0.55)	(0.66)	(0.65)
Beta_SMB		0.94		-0.92
		(0.65)		(0.65)
Beta_HML		-0.24		1.43
		(0.67)		(1.17)
Number obs.	686	686	272	272
Error autocorrelation	-0.09	-0.19	0.30	0.29
	(0.28)	(0.30)	(0.30)	(0.33)

Panel B: Gross-of-fee

	Venture Capital		Buyout	
	Spec 1	Spec 2	Spec 3	Spec 4
Alpha	***-0.76%	-0.37%	-0.12%	-0.79%
	(0.20)	(0.36)	(0.98)	(0.69)
Beta_Market	***2.88	***2.52	**1.54	***2.12
	(0.59)	(0.52)	(0.68)	(0.66)
Beta_SMB		*1.14		-1.03
		(0.61)		(0.67)
Beta_HML		-0.27		1.55
		(0.66)		(1.23)
Number obs.	686	686	272	272
Error autocorrelation	0.02	-0.11	0.31	0.30
	(0.28)	(0.30)	(0.31)	(0.34)

Table 6: Internal Rate of Returns and Average Discount Rates

This table compares IRR with the average discount rate (labeled dynamic IRR) we estimated for each fund-of-funds (FoFs); there is one FoF per vintage year. The first three columns show the vintage year, the number of funds, and the total size of each FoF. Dynamic IRR is alpha plus average risk free rate plus beta times the average market return minus average risk free rate. All averages are geometric and the monthly dynamic IRR is compounded to obtain the displayed (annualized) IRR. The life of a venture capital FoF starts 1.5 years after the first month of its vintage year and ends 6.5 years later. The life of a buyout FoF starts at the same time but ends 5 years later. The last column shows the IRR of the FoF. At the bottom of the table, the size-weighted means are shown.

Panel A: Venture Capital funds

Year	N_funds	Size	Dynamic IRR	IRR
1980	19	3795	0.08	0.18
1981	25	1465	0.26	0.10
1982	28	2156	0.18	0.04
1983	58	4220	0.18	0.08
1984	68	4878	0.15	0.06
1985	62	3916	0.09	0.06
1986	56	6158	0.06	0.12
1987	79	7353	0.10	0.12
1988	58	5541	0.14	0.16
1989	76	7640	0.20	0.15
1990	37	2862	0.28	0.24
1991	35	3992	0.34	0.17
1992	34	3490	0.39	0.30
1993	51	4862	0.33	0.32
Average			**0.18**	**0.15**

Panel B: Buyout funds

Year	N_funds	Size	Dynamic IRR	IRR
1984	12	3092	0.15	0.26
1985	12	2669	0.11	0.56
1986	18	3451	0.04	0.21
1987	37	20654	0.13	0.11
1988	36	16390	0.05	0.12
1989	35	18044	0.08	0.08
1990	36	10710	0.13	0.16
1991	23	3588	0.15	0.09
1992	26	8086	0.25	0.23
1993	37	14233	0.29	0.16
Average			**0.14**	**0.15**

Table 7: Robustness Tests

This table shows monthly abnormal performance (Alpha) and risk loading (Beta) using a one-factor market model. Parameter estimates are shown for different treatment of final NAVs, number of fund-of-funds, time periods, sub-samples, weighting of moment conditions, and benchmarks. Other benchmarks include Nasdaq composite index, and indices from Kenneth French's webpage (Small growth 5x5, and European market return in dollars).

	Venture capital		Buyout	
	Alpha	Beta	Alpha	Beta
Base estimation	*-1.09%*	*2.73*	*-0.41%*	*1.31*
Other treatment of final NAV (by default, it is inferred from a statistical model)				
. Full write-off	-0.95%	2.30	-0.23%	0.98
. Treated as final market value	-0.96%	2.79	-0.35%	1.50
Changing the number of fund-of-funds per vintage year (by default it is one per vintage year)				
. 2 fund-of-funds per vintage year	-1.07%	2.82	-0.41%	1.32
. 3 fund-of-funds per vintage year	-1.09%	3.10	-0.44%	1.43
. 4 fund-of-funds per vintage year	-1.04%	3.00	-0.52%	1.55
Changing the last vintage year to be included (by default it is 1993)				
. Add one year (i.e. include 1994)	-1.08%	2.72	-0.27%	1.06
. Remove one year (i.e. include 1994)	-1.07%	2.59	-0.50%	1.47
Using only liquidated funds	-0.88%	3.62	0.57%	0.65
Changing the weighting of moment conditions (by default it is equally weighted)				
. Weighted by inverse of fund-of-funds size	-1.11%	2.83	0.50%	0.46
. Weighted by inverse of average fund size	-1.16%	2.90	-0.14%	1.03
Changing the benchmark (by default it is the S&P 500 for all funds)				
. European stock index for European funds	-1.05%	2.99	-0.27%	1.20
. Nasdaq index for venture capital funds	-0.45%	1.47		
. Small-growth index for venture capital funds	1.46%	1.62		

Figure 1: Histograms of (annual) Internal Rates of Return of Individual Funds

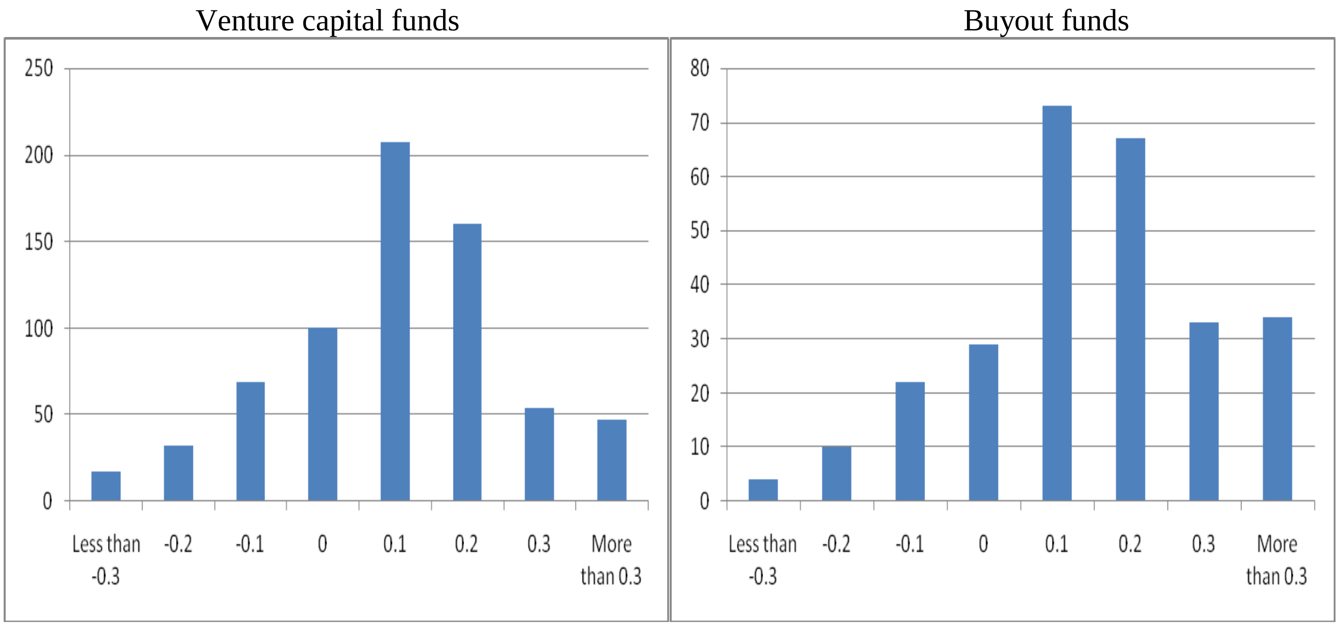

Figure 2: Fund Dividend Yields

Dividend yield is the sum of the dividends paid divided by fund size. We plot the average of the next 12 months dividend yields of funds in their 4th to 10th year. S&P 500 returns are the 5-year cumulative returns, divided by 5. Time spans 1990 to 2003.