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# Pricing of Commercial Real Estate Securities during the 2007-2009 Financial Crisis

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## Abstract

We study the relative and absolute pricing of CMBX contracts (commercial real estate derivatives) during the recent financial crisis. Using a structural CMBX pricing model we find little systematic mispricing relative to REIT equity and options. We do find short-term deviations from this relative pricing relationship that are statistically and economically significant. In particular, the CMBX market temporarily overreacts to news announcements. We provide evidence that this temporary mispricing is caused by price pressure due to hedging activities. Finally, an absolute pricing analysis provides no substantial evidence that CMBX contracts traded at fire sale levels during the crisis.

*JEL classification:* G1, G13, G14

*Keywords:* CMBX, REIT, financial crisis, commercial real estate, capital structure arbitrage

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# 1. Introduction

In this paper we provide a detailed analysis of the pricing of CMBX contracts, which are credit default swaps on a portfolio of Commercial Mortgage-Backed Securities (CMBS). In particular, we study to what extent this asset-backed securities market was affected by the recent financial crisis. While it is well documented how the market for residential mortgage-backed securities has collapsed, with these securities trading at fire-sale levels, there is less work on the performance of the market for commercial real estate derivatives during the crisis period. In addition, we study whether investors overpriced senior CMBX tranches before the crisis. Coval, Jurek and Stafford (2009a) argue that investors overpriced tranchised securities because they relied on credit ratings instead of systematic risk.

We address these issues in two ways. First, we perform a relative pricing exercise. In contrast to residential mortgage-backed derivatives (like ABX contracts), for CMBX contracts there is a closely related and liquid asset class, Real Estate Investment Trusts (REITs), with both stock and option data. Given that we have reliable daily CMBX price data, this makes the commercial real estate market an ideal place to study the pricing of mortgage-backed securities. We therefore develop a pricing model to evaluate CMBX prices relative to stock and option prices of REITs and the S&P 500 index, and use this model to empirically test for the existence of relative mispricing. Our relative pricing exercise allows us to test for both long-term and short-term mispricing in the CMBX market, most importantly by analyzing whether CMBX and REIT returns can be predicted by the degree of relative mispricing and by studying mispricing around news announcements.

Of course, it is possible that CMBX and REITs are priced “correctly” relative to each other, even when both assets are mispriced in terms of their absolute price levels. Therefore, our second analysis focuses on the absolute pricing of CMBX contracts. To this end, we adjust our CMBX pricing model for potential mispricing of REITs. We do this by using NAVs (net asset values) of REITs as input to the pricing model, instead of equity market

values of REITs. In an additional exercise, we study the commercial mortgage default rates that are implied by CMBX prices and compare these to historical default rates.

To illustrate the contracts and pricing, consider Fig. 1. The price of a CMBX contract is quoted as \$100 minus the price of protection on \$100 notional, and Fig.1 plots the price for the CMBX 1 AA index, which references a portfolio of 25 CMBS tranches which insure losses between 10.5% and 12.5% on the underlying portfolio. At the lowest price of \$19.60 on April 15, 2009 it cost \$80.40 per \$100 notional to insure losses on the CMBS bonds.

The first key result of this paper is that we find little evidence for persistent relative mispricing between CMBX contracts and REITs. Actual CMBX price levels are, overall, reasonably in line with prices implied by an option-based CMBX pricing model, calibrated to equity and option prices for REITs and the S&P 500 index. For the AA tranche this is illustrated by the dashed line in Fig. 1. In particular, we neither find evidence that investors overpriced these high-rated securities before the crisis nor that fire sales led to too low prices during the crisis.

The second key result of this paper is that our pricing model shows that over shorter horizons there are temporary deviations from this relative pricing relation: the CMBX mispricing predicts subsequent CMBX returns, and a trading strategy that exploits this temporary mispricing earns abnormal returns that are statistically and economically significant. We show that this evidence of short-term inefficiencies is robust to various parameter settings for the pricing model. We also provide evidence that suggests that this temporary CMBX mispricing is due to hedging pressure by banks hedging their commercial real estate exposure.

Our third key result is that we do not find evidence for substantial absolute mispricing. This conclusion is reached on the basis of two analyses. First, instead of inserting REIT equity prices into our CMBX pricing model, we use net asset values (NAVs) of REIT equity (as reported by Green Street Advisors). These NAVs are supposed to measure the “fundamental” value of REIT equity, and differ from market prices if REIT equity is mispriced in the market. We find that using the NAV instead of REIT equity prices as

input to the model does not lead to very different model prices. The NAV and REIT equity prices do differ substantially during the peak of the crisis, but in this period the model price of the CMBX tranches considered is not very sensitive to the REIT asset value; in a similar way that an out-of-the money option is not very sensitive to changes in the underlying price (low delta).

In addition to this NAV-based analysis, we study default rates implied by CMBX prices and find that these are not excessively high compared to historical default rates. In sum, although CMBX prices went down substantially during the crisis period, it does not seem to be the case that these contracts traded at firesale prices. Interestingly, Stanton and Wallace (2011) study derivatives based on subprime residential mortgage-backed securities (ABX indexes) and find strong evidence that actual prices were too low and inconsistent with any reasonable choice of mortgage default rates.

Our model is set up as follows. The value of a commercial property is driven by exposure to stock market returns, sector-level property returns and idiosyncratic shocks. The model includes the three main commercial property sectors: retail, apartment, and office. Defaults on commercial mortgage loans occur whenever the property value is below a default threshold at maturity (hence for simplicity we abstract from term defaults). We calibrate the model each day to data on stock index (S&P 500) returns and option prices, and REIT equity returns and option prices for 15 REITs in the different sectors. Our calibration approach extends the standard way of calibrating Merton's (1974) firm value model to equity values and volatilities (see for example Vassalou and Xing, 2004). We do not use data on the underlying CMBS contracts since these are not liquidly traded. We then price the CMBX contracts. Importantly, we do not calibrate any parameters to the CMBX prices, which allows for a clean and transparent analysis of relative pricing. CMBX contracts are priced by simulating property values for the pool of loans and assessing the loss distribution due to loan defaults. We recalibrate the latent property values, volatilities, correlations, and stock-market exposure each day.

We focus our empirical analysis on three liquid CMBX series 1 tranches: the AJ tranche, which insures portfolio losses between 12.5% and 20%, the AA tranche which insures losses between 10.5% and 12.5%, and the A tranche, which insures losses between about 7.7% and 10.5%. We find a close correspondence between market and model prices. The correlation between model and market prices is in the range of 86% – 96%. We also find that the model does a reasonable job pricing the AJ tranches of series 2 through 5 (which are also liquidly traded), but that there is a modest downward trend in the actual price relative to the model price as one moves to newer series, consistent with a downward trend in underwriting standards, which is (partially) recognized by the market but not accounted for in the model. As a robustness check we consider model extensions with (i) jumps in the asset value dynamics and (ii) a stochastic interest rate process, and find similar results.

We do, however, find evidence of short-term episodes of mispricing. We first establish that the model mispricing predicts subsequent CMBX returns with t-statistics between 2.3 and 3.3 across tranches. To assess the economic significance of the predictability in CMBX returns, we analyze a simple trading strategy in CMBX and REIT stocks that exploits the model mispricing. Assuming realistic transaction costs, we find an annualized Sharpe ratio of 2.25 for a strategy based on the AJ, AA and A tranches. To put this figure in perspective, Duarte, Longstaff, and Yu (2007) analyze five popular fixed-income arbitrage strategies and find a maximum Sharpe ratio of 1.20. We show that the trading strategy earns a significant alpha when we correct for the exposure to several equity and bond market risk factors.

Next, we use the model to study the response of the CMBX market to news announcements. We stick to an objective set of news days to prevent data snooping. We find that the two days following the news day, the CMBX market continues to move (on average) in the same direction as it did on the news day itself (relative to the REIT market). This could be either due to initial underreaction or subsequent overreaction. Our results point to overreaction, since the (average) price reverses within five days of the announcement to a price around

the closing level of the announcement day.<sup>1</sup> Despite the short sample period, these results are statistically significant.

The evidence above raises the question why this apparent market inefficiency is not arbitrated away instantaneously. We argue that the short-term mispricing can occur due to hedging pressure effects. The sample period includes the financial crisis and many banks were very concerned with hedging risk exposure to real estate assets, e.g. to satisfy regulatory value-at-risk requirements. These hedging needs could create a temporary price pressure on the instruments used for hedging, like CMBX. This would explain why we find convincing evidence of predictability for CMBX, and only limited evidence of predictability for REITs. Also our finding that following news announcements the CMBX market overreacts for two days, seems consistent with banks rebalancing their hedge portfolio in a rush following news. We provide further evidence supportive of this interpretation by regressing the change in CMBX mispricing on the return on a bank stock index (in excess of the market return). We find that CMBX mispricing has a positive exposure to bank stock returns, in line with the hedging pressure hypothesis: as banks get closer to distress, they short CMBX tranches for hedging, pushing down CMBX prices relative to REIT stock and option prices. In line with this hedging hypothesis, we find that most of the abnormal returns of the trading strategy are generated at the height of the crisis period in 2008 and 2009.

Our paper builds on a recent literature studying tranching derivative structures. Recent work of Stanton and Wallace (2010) analyzes CMBS contracts before and during the recent financial crisis, and finds that the collapse of this market was mainly caused by rating agencies who gradually lowered subordination levels to levels that provided insufficient protection given the assigned rating. They do not use a formal pricing model, but regress CMBX returns on variables such as default dynamics and REIT returns and find that these “fundamentals” explain most of the CMBX return variation, which is consistent with our findings. Using an

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<sup>1</sup>This contrasts studies on US stocks, like Chan (2003), that find that the price continuation (or decrease in price reversal) following news announcements is more consistent with underreaction to news.

equity- and option-based pricing model for corporate collateralized debt obligations (CDOs), Coval, Jurek, and Stafford (2009a) show that prices of senior tranches on a pool of credit default swaps were higher than model-based predictions before the crisis. Coval, Jurek, and Stafford (2009b) find that model and market prices became more in line during the crisis. Diamond and Rajan (2009) argue that the short-term incentives of traders led them to invest in products with substantial tail risk, which is a key feature of tranching securities. Keys, Mukherjee, Seru, and Vig (2010) provide evidence that securitization led to lax screening of residential mortgage loans. In contrast, for the CMBX market we find no evidence that investors overpriced senior tranches before the crisis.

Much of the attention in this recent literature has been devoted to tranching products based on corporate loans, credit default swaps and residential mortgage loans.<sup>2</sup> We complement these various papers by looking at tranching derivative products in the market for commercial real estate loans (CMBX). This is interesting for several reasons. First, because CMBX are derivative contracts on commercial property values, we can study their relative value to other liquid assets that are also directly dependent on commercial property values, like REIT stock and options.<sup>3</sup> Second, while by now it is widely accepted that large losses on subprime securities will materialize, the jury is still out on CMBX. For example the delinquency rate on the CMBS deals referenced by CMBX 1 was only 4.50% per March 2010; in contrast the delinquency rate for subprime deals referenced by the ABX 06-01, issued around the same time as the CMBX 1 series, was 41.61% per March 2010.<sup>4</sup> Third, we study the market

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<sup>2</sup>Longstaff (2008) finds contagion in financial markets, with lower-rated tranches of subprime-mortgage based tranches (ABX) leading the Treasury bond and stock market. Other papers have focused on understanding the pricing of subprime mortgages, including Demyanyk and Van Hemert (2011) and Mian and Sufi (2009).

<sup>3</sup>Cannon and Cole (2010) report that REITs have become increasingly liquid over the period 1988 to 2007, with average bid-ask spreads for REITs of less than 20 basis points for 2004 to 2007.

<sup>4</sup>The delinquency rate is defined as the fraction of loans that are 60 or more days late on payments. Reported figures are from the March 2010 CMBX and ABX monthly reports by JPMC.



development before, during and after the crisis period. This allows us to see whether the crisis led to fire sales or market breakdowns, and whether investors updated their beliefs concerning tranching securities, leading to more accurate pricing in this market. Fourth, we analyze the time series of mispricing of CMBX contracts in detail and report several short-term inefficiencies.

Our paper also builds on the literature applying the Merton (1974) contingent-claim approach for pricing bonds. Titman and Torous (1989) and Childs, Ott, and Riddiough (1996) apply such an approach to CMBS contracts, while Christopoulos, Jarrow, and Yildirim (2008) test the pricing of CMBS contracts using a reduced-form approach. Downing, Stanton, and Wallace (2007) use the Titman and Torous (1989) model to analyze CMBS over the 1996–2005 period and provided an early warning that subordination levels were decreasing over time, while implied volatility estimates remained roughly constant. Kau, Keenan, and Yildirim (2009) study CMBS default probabilities using REIT property-type indexes. Also, there is a large literature studying the empirical determinants of commercial mortgage defaults, including VanDell, Barnes, Hartzell, Kraft, and Wendt (1993), Follain and Ondrich (1997), Ciochetti, Deng, Gao, and Yao (2002), Ambrose and Sanders (2003), and Ciochetti, Deng, Lee, Shilling, and Yao (2003). To the best of our knowledge, none of these papers study the CMBX derivative contract, nor do they test the efficiency of the commercial real estate market during the 2007–2009 financial crisis, like this paper does.

The paper proceeds as follows. In Section 2 we discuss the CMBS and CMBX market. In Section 3 we present the option model, and in Section 4 we discuss the data and calibration results. Section 5 presents results on pricing CMBX contracts, several empirical tests for inefficiencies and a test of the hedging pressure hypothesis. In Section 6 we discuss the main news events that affected the CMBS market and we analyze the CMBX market response to those announcements. Section 7 explores the absolute pricing of CMBX and REITs. Section 8 concludes.

## 2. Overview of the commercial real estate debt market

In this section we provide a brief overview of the US commercial real estate debt market. We start by discussing commercial mortgage-backed securities. Next we explain the CMBX contracts on a basket of securitized commercial mortgages.

### *2.1. Commercial mortgage-backed securities*

Commercial real estate mortgage loans are collateralized by income-producing properties like offices, shopping malls, hotels, and apartment buildings. The dominant contract type is a fixed-rate mortgage that amortizes over 20-30 years, but matures in 10 years, resulting in a large payment at maturity. Most commercial mortgages have prepayment or call protection mitigating largely the risk of strategic refinancing when mortgage rates decline, in contrast to agency-backed residential mortgages that allow for prepayment at no or little cost.

During the first quarter of 2009, 21% of the \$3.47 trillion commercial mortgage market was securitized.<sup>5</sup> A commercial mortgage-backed security (CMBS) is backed by a pool of commercial mortgages, typically diversified across property type and location.

The mortgage originator issues a range of CMBS (also referred to as tranches) that are different along several dimensions, most importantly with respect to credit enhancement and maturity. In Fig. 2 we illustrate a typical CMBS deal. The super-senior tranches (A-1, A-2, A-3, A-4) have 30% credit enhancement (subordination level). Super-senior tranches differ with respect to the timing of principal repayment, the A-1 being the first and the A-4 being the last to receive principal. The mezzanine (A-M) tranche absorbs deal losses between 20% and 30% and the junior AAA (A-J) tranche absorbs losses between 12% and 20%. The A-1, A-2, A-3, A-4, A-M, and A-J tranches all had an AAA credit rating on the issuance date. Only the remaining tranches (Subs), that absorb losses between 0% and 12%, got an original

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<sup>5</sup>We use data from the Board of Governors of the Federal Reserve System reported at <http://www.federalreserve.gov/econresdata/releases/mortoutstand/current.htm>. In comparison, 60% of the \$11.0 trillion US residential mortgage market was securitized.

credit rating below AAA. Note that Fig. 2 represents a basic structure; in practice many variations to this basic structure exist. Appendix A gives a detailed example on a CMBS deal.

## *2.2. CMBX contracts*

A CMBX contract is an agreement between a buyer and a seller on protection on 25 reference CMBSs. The buyer of protection pays a fixed coupon each month. The seller of protection pays a floating payment equal to the amount of the shortfall in the scheduled interest and principal payments, a settlement type referred to as pay-as-you-go.

The basket of reference CMBS and contractual details are standardized and set by Markit, a financial information services company. When a new CMBX contract is introduced by Markit, the fixed coupon is also set. For buyers and sellers that enter into the CMBX contract at a later date, the same fixed coupon applies; the exchange of an up-front payment allows the contractors to take into account the current market value of protection.

The size of the up-front payment at a given point in time is often translated into either (i) a spread, which allows for a comparison with the CMBS yield spread to swaps, or (ii) a price per \$100 notional, which is approximately equal to \$100 minus the up-front payment. We used the latter in Fig. 1.

A CMBX series consists of several tranches, each referencing a different set of 25 CMBSs. However, the different tranches of a given series all reference the same 25 underlying CMBS deals. The CMBX AAA tranche references super-senior CMBSs with about 30% credit enhancement, where typically the last-pay (A-4 in Fig. 2) CMBS are used. The CMBX AJ, AA, A, BBB, BBB- reference CMBS increasingly lower in the capital structure of the 25 CMBS deals for a particular series.

The CMBX series 1 became effective on March 7th, 2006. Markit initially planned to introduce a new series each half year, but the introduction of CMBX series 6, planned for October 25th, 2008, was postponed due to a lack of new issuance.

### 3. An options-based CMBX valuation model

An investor selling protection using a CMBX contract has to compensate the protection buyer in case of defaults on the referenced CMBS, which is most likely to occur when property prices drop. Therefore, in essence the protection seller writes a derivative on property values, creating a fundamental link between the CMBX and the REIT market. REIT stock prices are informative on the current value of properties. REIT option prices contain additional information on the (risk-neutral) probability distribution of future property values. In the CMBX valuation model described below we make explicit this fundamental link between the CMBX and the REIT market. In addition, we incorporate exposure to market-wide systematic risk using the S&P 500 index and options on this index.

We first describe the model for property values and how REITs and CMBX contracts can be priced using this model. Then we discuss the calibration procedure. Our approach is to calibrate the model using data on REIT returns, REIT options, S&P 500 returns and options, and then price the CMBX contract out of sample. Hence, we avoid fitting the model to CMBX prices directly.

#### *3.1. Model for property values*

We distinguish properties in the three dominant sectors in commercial real estate: office, retail and apartments. Consider commercial property  $i$  in sector  $j$ ,  $j = 1, \dots, 3$ . Let the value of the property be denoted by  $V_{ij}$ . We assume that the return on a property is driven by (i) market-wide shocks, (ii) real estate sector-level shocks, and (iii) an idiosyncratic shock. Specifically, the process of  $V_{ij}$  under the risk-neutral measure is

$$\begin{aligned} \frac{dV_{ij}}{V_{ij}} &= (r - q)dt + \beta_j \sigma_S dW_0 + \gamma_j dW_j + \sigma_j dZ_{ij} \\ \frac{dS}{S} &= rdt + \sigma_S dW_0, \end{aligned} \tag{1}$$

where  $r$  denotes the risk-free rate,  $q$  the dividend rate,  $\frac{dS}{S}$  the return on the S&P 500 index

driven by Brownian motion  $dW_0$ ,  $dW_j$  a Brownian motion representing sector-level shocks for sector  $j$ , and  $dZ_{ij}$  a property-specific shock.<sup>6</sup> All factors are orthogonal to each other, except that the sector-level shocks  $dW_j$  are correlated with each other

$$\text{Corr}(dW_j, dW_k) = \rho_{jk}dt, \quad j, k = 1, \dots, 3. \quad (2)$$

### 3.2. Pricing REITs

The model in (1) can be used to price REITs. A REIT invests in commercial properties, typically financed partially by debt and partially by equity. We assume that each REIT invests in one sector  $j$ , and that each REIT is fully diversified so that the idiosyncratic shocks  $dZ_{ij}$  average out. Empirically, we make sure we only include large REITs that are well diversified. Let  $\bar{V}_j$  denote the total value of the properties of a REIT in sector  $j$ . The return on the portfolio of properties of a REIT is then equal to

$$\frac{d\bar{V}_j}{\bar{V}_j} = (r - q)dt + \beta_j \sigma_S dW_0 + \gamma_j dW_j. \quad (3)$$

We follow Merton (1974) and assume a simple capital structure where the debt of a REIT has face value  $D_j$  and maturity  $T_j$ . The payoff of a REIT at  $T_j$  is then given by  $\max(\bar{V}_j(T_j) - D_j, 0)$ , a call option on a portfolio of properties. Given the process in (1), the value of REIT equity, denoted  $E_j$ , is then simply given by the Black-Scholes call price formula.

### 3.3. Pricing CMBX contracts

We now discuss the payoffs and pricing of CMBX contracts.

#### 3.3.1. Payoff structure of CMBX

Put briefly, a CMBX is a portfolio of 25 CMBS tranches, where each tranche is based on a pool of commercial real estate loans. More precisely, a CMBX contract with \$100 notional

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<sup>6</sup>Yildirim (2008) finds evidence that defaults are clustered according to property type and geographical area. Our model picks up the property-type effect through the sector-level shocks, that are common to all properties in each sector (or “property-type”).

insures losses on 25 CMBS deals, each up to \$4. For a contract insuring the losses between a fraction  $CE^L$  and  $CE^H$  of notional, the value  $P(CE^L, CE^H)$  is defined as \$100 plus the present value of the fixed payments received by the protection seller,  $CF^{fixed}$  per dollar notional, minus the present value of the floating payments made by the protection seller,  $CF^{floating}$  per dollar notional:

$$P(CE^L, CE^H) = 100 + 4 * \sum_{k=1}^{25} \sum_{t=1}^T e^{-r_f t} E^Q \left( CF_{t,k}^{fixed} - CF_{t,k}^{floating} \right), \quad (4)$$

where the summation is over CMBS deals  $k$  and payment dates  $t$ , and where  $E^Q$  denotes the risk-neutral expectation.<sup>7</sup> The fixed payments are a fixed annual coupon rate,  $c$ , times the fraction of notional outstanding. For simplicity we assume that the underlying loans are interest-only, which means that the fraction of notional outstanding equals one minus the cumulative tranche loss (as a fraction of notional) per the previous period,  $L_{t-1,k}^{tranche}$ . The fixed cash flow is then given by

$$CF_{t,k}^{fixed} = (1 - L_{t-1,k}^{tranche}) * c. \quad (5)$$

The floating payment equals the change in the cumulative tranche loss

$$CF_{t,k}^{floating} = L_{t,k}^{tranche} - L_{t-1,k}^{tranche}. \quad (6)$$

Finally, the cumulative tranche loss is the proportion of the  $CE^L$ - $CE^H$  tranche that is hit by the cumulative loss on the entire portfolio,  $L_{t,k}^{ptf}$ ,

$$L_{t,k}^{tranche} = \frac{\max \left\{ L_{t,k}^{ptf} - CE^L, 0 \right\} - \max \left\{ L_{t,k}^{ptf} - CE^H, 0 \right\}}{CE^H - CE^L}. \quad (7)$$

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<sup>7</sup>To keep the notation simple, we set the length of one time period equal to the time between payment dates.

Deal losses occur in case of a default. We define  $D_{t,k}^i$  as a default indicator function taking the value one if at time  $t$  loan  $i$  of CMBS deal  $k$  defaults. The size of the loss is the (original) weight of loan  $i$  in CMBS deal  $k$ , denoted  $w_k^i$ , times one minus the property value per dollar loan amount,  $\tilde{V}_{t,k}^i$ , at the time of default. The portfolio loss is then obtained by summing losses across  $I$  loans

$$L_{t,k}^{ptf} = L_{t-1,k}^{ptf} + \sum_{i=1}^I w_k^i * D_{t,k}^i * \max \left\{ 1.0 - \tilde{V}_{t,k}^i, 0 \right\}. \quad (8)$$

Default occurs at maturity ( $t = T$ ) if the value per dollar loan amount drops below a default trigger value, which is set equal to the loan amount to be paid at maturity.

### 3.3.2. Dynamics collateral value

The value of properties per dollar loan amount  $\tilde{V}_{t,k}^i$  (across loans  $i$  and CMBS portfolios  $k$ ) is the key driver of defaults and losses on the CMBX contracts. The Loan-to-Value (LTV) is the most common ratio to examine the probability of default and expected losses on a CMBS. We can thus use these LTVs at issuance, and set the initial property value per dollar loan amount for a particular loan,  $\tilde{V}_{0,k}^i$ , equal to the inverse of the LTV ratio.

We link the evolution of these property values to the value of properties underlying REIT contracts. We assign each loan in the CMBX contract to one of the three REIT sectors  $j, j = 1.., 3$ . To price a contract at time  $t$ , Eq. (1) implies that the property value change for a loan between the contract introduction date 0 and the pricing date  $t$  is equal to the return on REIT property values for the given sector,  $\frac{\bar{V}_{jt}}{\bar{V}_{j0}}$ , times a factor that captures that property values within a sector are subject to idiosyncratic shocks,

$$\tilde{V}_{t,k}^i = \frac{1}{LTV_{0,k}^i} * \frac{\bar{V}_{jt}}{\bar{V}_{j0}} * e^{-0.5\sigma_j^2 t + \sigma_j \sqrt{t}\varepsilon_i}, \quad (9)$$

where  $\varepsilon_i$  has an i.i.d. standard normal distribution. This gives the risk-neutral dynamics for the stochastic variables  $\tilde{V}_{t,k}^i$ , and we can compute the present value in (4) by discounting the

risk-neutral expected payoffs at the risk-free rate.

### 3.3.3. Simplifying assumptions

While we believe the model above captures some of the main features that drive CMBX prices, it is important to notice that the model is simplified along several dimensions, including the following. First, we do not model amortization of the loan balance and instead assume all loans are interest only. Second, we neglect the presence of defeasance options in commercial mortgage loans (Dierker, Quan, and Torous, 2005). Third, we assume defaults only occur at maturity and thus abstract from term defaults for both REITs and CMBX. Fourth, we do not allow for extensions upon maturity, neither do we explicitly model bankruptcy costs. Fifth, REITs are actively managed and hence we neglect any embedded growth options that are reflected in the value of REITs. Finally, we assume risk-free rates are constant and thus independent from property value movements. In reality though, one could expect a relation between property prices and interest rates, although there is little empirical evidence on this relation. In Section 4.1 we empirically analyze the relation between CMBX returns and interest rate changes, and find low correlations generally.

### 3.4. Calibration of the pricing model to REIT and S&P 500 data

We calibrate the pricing model in (1) to daily data on returns of REITs and the S&P 500 index, and options on these assets. We have the following parameters: sector-level property volatilities  $(\gamma_1, \dots, \gamma_3)$ , betas  $(\beta_1, \dots, \beta_3)$ , sector correlations  $(\rho_{12}, \rho_{13}, \rho_{23})$ , idiosyncratic property volatilities  $(\sigma_1, \sigma_2, \sigma_3)$ , stock market volatility  $\sigma_S$ , and the latent values of the sector-level REIT properties on each day,  $\bar{V}_{1t}$ ,  $\bar{V}_{2t}$ , and  $\bar{V}_{3t}$ . Our procedure is an extension of the standard approach to extract asset values and asset volatilities from equity prices and equity volatilities in a Merton (1974) firm value model (see for example Vassalou and Xing, 2004).

Most of the parameters are calibrated using the following set of restrictions. First, the total equity value of a REIT for each sector  $j$

$$E_j = BS(\beta_j^2 \sigma_S^2 + \gamma_j^2, \bar{V}_j, D_j, T_j), \quad j = 1, \dots, 3, \quad (10)$$



where  $BS(.,.,.,.)$  denotes the Black-Scholes call price formula as a function of volatility, price, strike and maturity, respectively. Second, the variance of REIT equity returns,

$$\begin{aligned} Var\left(\frac{dE_j}{E_j}\right) &= \left(\frac{\bar{V}_j}{E_j} \frac{\partial E_j}{\partial \bar{V}_j}\right)^2 Var\left(\frac{d\bar{V}_j}{\bar{V}_j}\right) \\ &= \left(\frac{\bar{V}_j}{E_j} \frac{\partial E_j}{\partial \bar{V}_j}\right)^2 (\beta_j^2 \sigma_S^2 + \gamma_j^2) dt, \quad j = 1, \dots, 3. \end{aligned} \quad (11)$$

Third, the covariance of REIT equity with the stock market index,

$$Cov\left(\frac{dE_j}{E_j}, \frac{dM}{M}\right) = \left(\frac{\bar{V}_j}{E_j} \frac{\partial E_j}{\partial \bar{V}_j}\right) \beta_j \sigma_S^2 dt, \quad j = 1, \dots, 3, \quad (12)$$

Fourth, the covariance of REIT returns across different sectors,

$$\begin{aligned} Cov\left(\frac{dE_j}{E_j}, \frac{dE_k}{E_k}\right) &= \left(\frac{\bar{V}_j}{E_j} \frac{\partial E_j}{\partial \bar{V}_j}\right) \left(\frac{\bar{V}_k}{E_k} \frac{\partial E_k}{\partial \bar{V}_k}\right) \beta_j \beta_k \sigma_S^2 dt \\ &+ \left(\frac{\bar{V}_j}{E_j} \frac{\partial E_j}{\partial \bar{V}_j}\right) \left(\frac{\bar{V}_k}{E_k} \frac{\partial E_k}{\partial \bar{V}_k}\right) \rho_{jk} \gamma_j \gamma_k dt, \quad j, k = 1, \dots, 3, \end{aligned} \quad (13)$$

and, finally, the variance of the stock index return  $V\left(\frac{dS}{S}\right)$  which is equal to  $\sigma_S^2 dt$ .

These restrictions are matched empirically on each calibration day. Specifically, Eq.(10) is matched to the observed REIT equity market capitalization. Eq. (11) is matched to the implied volatility of three-month at-the-money (ATM) options on REIT equity.<sup>8</sup> Eq. (12) is matched to the historical covariance between REIT equity returns and stock index returns. Eq. (13) is matched to the historical covariance between REIT equity returns in different sectors. For the historical covariances we use daily returns and an exponential weighting scheme to give more weight to recent observations. We choose the decay of the

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<sup>8</sup>Formally, an option on REIT equity is a compound option in our model, so that the implied volatility does not exactly measure the volatility of REIT equity. We have also implemented a calibration method where we use the compound option model of Geske (1979) to price the equity option. This gives very similar results.

exponential weights such that an observation six months ago has half the weight of the most recent observation. Finally,  $\sigma_S$  is matched to the price of three-month ATM S&P 500 index options.

As discussed below, we identify the level of REIT debt  $D_j$  using balance sheet information. We set the maturity of (zero-coupon) REIT debt equal to five years, close to the duration of a typical commercial real estate loan.

Finally, the volatility of idiosyncratic property value shocks,  $\sigma_j$  in Eq. (1), cannot be identified from these highly diversified REITs. To calculate these we base ourselves on existing work on estimating property value volatilities implied by loan-level prices. Most recently, Downing, Stanton, and Wallace (2007) estimate the return volatility of an individual property implied by loan-level prices. They report (liquidity-adjusted) total volatility levels of 23.8%, 19.7% and 21.5% for office, apartments and retail sectors, respectively. They also estimate unlevered return volatilities at the REIT level (i.e., for portfolios of loans), at 15.5%, 10.7% and 15.5%. From these estimates one can obtain the idiosyncratic variance of property values by subtracting the REIT-level variance from the total variance levels. This gives idiosyncratic volatility levels of 18.1% (office), 16.5% (apartments) and 14.9% (retail). Older work by Titman and Torous (1989), Ciochetti and Vandell (1999), and Holland (2000) reports estimates of total implied volatility between 15% and 19%, a bit lower than the estimates of Downing, Stanton, and Wallace (2007). In our benchmark analysis we therefore set the idiosyncratic volatility levels at 75% of the levels implied by Downing, Stanton, and Wallace (2007) (i.e., at 0.75 times 18.1%, 16.5% and 14.9%). We later check the robustness of our results to this choice. In the absence of time-series data on these volatilities, we assume that these volatilities are constant over time.

In Appendix B we discuss how we deal with new issuance of bonds and stocks in the calibration approach, and we also discuss how we correct for a potential survivorship bias that results from our selection of large REITs with option data available (as discussed below).

## 4. Data and calibration results

This section discusses the data used to calibrate the CMBX pricing model, presents the calibration results and provides details on the CMBX contracts used for the analysis.

### 4.1. Data

We select all REITs that (i) are present in the Dow Jones US Real Estate Index, (ii) have options data available, and (iii) fall into one of three largest sectors: apartments, office and retail. We end up with 15 different REITs, four in the apartments sector, six in the office sector, and five in retail.<sup>9</sup> For these REITs we obtain daily equity returns and total market capitalization from Center for Research in Security Prices (CRSP), three-month ATM implied volatilities from Optionmetrics, and debt in current liabilities and long-term debt from Compustat. We set the total debt ( $D_j$ ) equal to debt in current liabilities plus long-term debt, and update this value annually.

The model specifies the behavior of a REIT per sector. To obtain a representative REIT per sector, we average all relevant measures (the covariances, implied volatilities and equity market capitalizations entering Eq. (10)-(12)) across REITs in a given sector on each day. For the market capitalization we average the ratio of the equity value  $E$  and debt  $D$  across REITs per sector, rewriting restriction (10) as  $E_j/D_j = BS(\beta_j^2 \sigma_S^2 + \gamma_j^2, \bar{V}_j/D_j, 1, T_j)$ .

Daily S&P 500 index returns and S&P 500 option prices come from CRSP and Optionmetrics, respectively. The risk-free interest rate,  $r_f$  is determined using the 5- and 10-year swap rate and interpolating linearly to obtain a swap rate for a  $\tau = T - t$  time-to-maturity.

We set the dividend rate  $q$  equal to 0% in the baseline case. We choose this zero value for two reasons. First of all, even though a property generates income in the form of rents, this income rate could be very low in the most relevant states for our pricing exercise: the severely distressed states in which many buildings are vacant and property prices are low. Second, to keep matters tractable, in our model we assume loans underlying CMBX are interest-only

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<sup>9</sup>These are the REIT tickers: AVB, BRE, BXP, CLI, DDR, DRE, EQR, HIW, KIM, LRY, REG, SPG, TCO, UDR, VNO.

loans while in reality some of the underlying loans will be amortizing. Also, some loans are of a 5-year maturity rather than the 10-year maturity assumed in the pricing exercise. Setting the income rate to zero can be thought of as a short-cut modeling assumption to account for the fact that income is used to amortize and pay off shorter-term loans, which we think is a reasonable assumption for the relevant distressed states. As a robustness check we will also show results for a -2% and 2% dividend rate  $q$  and show that model prices are higher and lower respectively, but that the dynamics of the model price are hardly affected.

An alternative to the short-cut modeling assumption above would be to model the exact timing of cash flows, as well as all the cash flow allocation rules to the different tranches. Modeling the exact timing of cash flows involves many additional assumptions, which seems less relevant for our study of the efficiency of the commercial real estate market. Modeling the cash flow allocation rules at the most granular level is a daunting task. In Appendix A we illustrate this and discuss the details on one of the 25 CMBS deals referenced by the CMBX 1 indexes, which we consider representative for the universe of deals underlying the CMBX indexes.

#### *4.2. Calibration results*

As discussed above, we recalibrate the model each day to the prevailing levels of implied volatilities, covariances, and equity and debt values. This gives us a daily time series for the parameters and latent sector-level property values  $\bar{V}_{1t}/D_1$ ,  $\bar{V}_{2t}/D_2$ , and  $\bar{V}_{3t}/D_3$ . In Fig. 3a-3d we plot the daily calibrated values for these latent property values, sector-level property volatilities  $(\gamma_1, \dots, \gamma_3)$ , betas  $(\beta_1, \dots, \beta_3)$ , and sector correlations  $(\rho_{12}, \rho_{13}, \rho_{23})$ . Fig. 3a shows that the latent sector-level property values have fallen substantially during the crisis. To put this dramatic value decrease in perspective, we compare this to the decrease in NAVs reported by Green Street Advisors.<sup>10</sup> The lowest NAV value is about 56% lower compared to the June 2006 level, while for the latent property values the lowest value is 70% lower

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<sup>10</sup>Green Street Advisors report price/NAV ratios, we convert these to NAVs using the REIT stock index value for the price level.

than in June 2006 (averaged across sectors). Hence, our latent property values are not very different from industry estimates for the NAV of REITs.

We then turn to the betas, capturing the exposure of latent property values to the stock market index, which have increased substantially during the crisis, doubling from levels of around 0.7 to levels around 1.4 during the crisis (Fig. 3b).<sup>11</sup> Obviously, such an increase in systematic risk has a substantial impact on the prices of CMBX contracts. Fig. 3c shows even more dramatic behavior for property-value volatilities; these increase by a factor three to four in the crisis period. Finally, Fig. 3d shows that in general, property-value correlations across REIT sectors are quite high, that they further increase during the crisis, and come down after the crisis. In sum, given that the value of tranching CMBX structures depend on underlying property values, betas, volatilities and correlations, it is clear that all these four drivers of CMBX prices will strongly affect CMBX prices during the crisis period.

Our model is simplified as we assume constant correlations and volatilities. A simple check on the validity of this assumption is to calculate the correlations between the time series of calibrated property values, property value correlations and volatilities. These numbers (unreported) show that, indeed, the daily changes in the calibrated parameters are correlated, but these correlations are not extremely high. For example, the usual “leverage-effect” correlation between property value changes and volatility changes is only -27%, whereas for the stock market index this correlation is typically much more negative. Still, implementing a model with stochastic volatilities and correlations is definitely interesting, yet challenging because the additional parameters to estimate raise issues like parameter stability and overfitting.

### *4.3. CMBX contract settings*

By now, five CMBX series have been issued. For most of this paper we will focus on series 1, which has the longest price history and also is the most liquidly traded series. To

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<sup>11</sup>Fei, Ding and Deng (2010) also show that REIT returns became more strongly correlated with stock market returns during 2007 and 2008.

illustrate the latter, in Table 1 we report the daily traded volume (dollar notional amounts in millions), based on the median value reported in a survey of six major brokers we conducted in the third quarter of 2009. The AM tranche did not trade yet in 2009 and is thus omitted from the table. Of the different tranches considered, the AJ is traded most. This makes sense as the AJ tranche is much thicker than more subordinated tranches and thus the underlying CMBS have a larger notional value. In particular for the tranches below AJ, series 1 is traded much more than the other four series. As a robustness check, we discuss in Section 5.5 of the paper the results for the AJ tranches of series 2 through 5.

We have daily midquotes for these CMBX contracts, as provided by Markit. For the CMBX 1 series, we focus on the AM, AJ, AA and A tranches, see Table 2 for the overview. We do not analyze the AAA tranche because of the complex structure of this tranche and because our model does not discriminate between first-pay (A1) and last-pay (A4) super-senior AAA tranches, while CMBX AAA tranches only reference last-pay tranches. The AJ tranche was introduced later and the AM tranche has only been traded recently. Time 0 refers to March 7, 2006, the day CMBX 1 was initiated. The maturity date, time  $T$ , is assumed to be exactly ten years later, March 7, 2016. The AM, AJ, AA, and A tranches have a fixed coupon between  $25bp$  and  $84bp$  and insure deal losses in the  $(19.92\%, 29.76\%)$ ,  $(12.50\%, 19.92\%)$ ,  $(10.45\%, 12.50\%)$  and  $(7.71\%, 10.45\%)$  intervals respectively.

To keep the computation manageable, we evaluate the CMBX value based on one representative CMBS deal. This representative deal is assumed to consist of 30 loans of equal size, for which the initial inverse LTVs are calibrated to the inverse LTVs of the 30 largest loans in CMBX 1, reported in the November 2008 Citi CMBX report, scaled to match the 1.45 mean inverse LTV across the 25 CMBS deals.<sup>12</sup>

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<sup>12</sup>As a technical note, to determine the model mispricing we correct for accrued dividends for REITs and accrued coupons for CMBX. This is to prevent REIT prices to have a predictable quarterly jump at the dividend date, and, similarly, to prevent the CMBX price to have a monthly jump when it goes ex-coupon. The accrual correction has a negligible effect on the results, but we consider it theoretically correct. Returns

#### 4.4. Numerical procedure

For each date, for each of the tranches, we determine the present value of the cash flows under the CMBX contracts in Eq. (4) by means of a Monte Carlo simulation method. We simulate 50 random paths for the risk-neutral distribution using Eq. (1) with one hundred time steps to maturity  $T - t$ . We double the number of random paths to one hundred by using the negated simulated standard normal shocks as well, a method referred to as antithetic variates. This avoids the need of doing a larger number of simulations. We use the same paths for the different dates and tranches to facilitate comparison. To check the accuracy of this method, we have also quadrupled the number of paths and time steps for the benchmark parameter setting, and find that this leads to negligible changes in the resulting CMBX prices.<sup>13</sup>

#### 4.5. CMBX interest rate exposure

As mentioned earlier, we assume constant interest rates in our pricing model. In Appendix C we formally assess the impact of a pricing model with stochastic interest rates, by extending our model with a one-factor Vasicek model for the short rate. The analysis shows that the effect of stochastic interest rates on CMBX prices is quite small, and mainly depends on the correlation between property values and interest rates. To provide some guidance on this correlation, we regress CMBX returns on changes in one-year, two-year, five-year and ten-year swap rates. We do this both for daily returns and weekly overlapping returns, for the AJ, AA and A CMBX 1 contracts that we focus on. We do not find significant interest rate exposure in any of these regressions, and the  $R^2$  statistics are between 6% and 10%. We also interact the interest rate variables with the level of the REIT index implied volatility, to see if

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for REIT stocks and CMBX take into account dividend and coupons respectively.

<sup>13</sup>An additional check on the numerical procedure is obtained by looking at a simplified setting with identical and perfectly correlated loans. It is straightforward to show that we can apply the Black and Scholes (1973) model for CMBX prices in this special case. Comparing the numerical and closed-form prices provides a powerful check on the accuracy of the procedure.

there is any dynamic interest rate exposure, but find no evidence for such an effect. Moreover, we also study the whether the change in CMBX mispricing can be explained by interest rate changes using similar daily and weekly regressions, and find essentially no evidence for a significant interest-rate effect. This suggests that risk-free rates are not strongly related to commercial property values, and supports our simplifying assumption of constant interest rates.

## 5. Relative pricing: empirical results

In this section we present the resulting model CMBX prices and compare these with actual prices. We first present the model fit, then we explore to what extent model mispricing leads to statistically significant and economically meaningful predictability in CMBX returns.

### *5.1. Global patterns of model and actual prices*

We start with the CMBX 1 AA contract which we discussed briefly in the introduction. In Fig. 1 we plot the actual CMBX 1 AA price versus the model price. We see that, globally, the option-based model generates prices that are in line with actual prices. First of all, before the crisis the model predicts small losses for the AA tranche in line with market prices around \$100. This result is in contrast to Coval, Jurek, and Stafford (2009a), who show that investors overpriced senior corporate CDO tranches before the crisis. Hence, we find no evidence that investors relied “too much” on the credit rating when pricing these products. As the crisis starts to unfold from the Fall of 2007 onwards, both actual and model prices drop severely to levels just below \$20 at the end of 2008, later recovering to levels around \$60. The actual-model price differential is on average \$8.22 and the correlation between the model price and actual price is 96%. Put differently, the model explains 91% of the actual price variation, measured by the  $R^2$  of a regression of actual prices on model prices. Keeping in mind that the option-based model is not calibrated to CMBX prices but only to REITs, these results provide some support for the absence of substantial relative mispricing between the REIT market and the CMBX market.



In Fig. 4, we present results for the other tranches: AM, AJ and A, and we repeat the AA graph for convenience. The AM tranche has only been traded since February 2010, and for this tranche we see a reasonable correspondence between model and market prices for the short sample period (until April 29, 2010). For the AJ and A tranches we also observe that, globally, model and market prices are in line, with model-market price correlations of 88.6% and 95.8%, respectively. We have also calculated these correlations using rolling windows of one year and three months, respectively. These are shown in Fig. 5. Again, we find correlations that are typically high (between 50% and 95%). We do see a few short periods where the model and market prices have somewhat lower correlations (and more so when we use a shorter window for the rolling correlations). This is in line with the short-term mispricing results that we discuss in Section 5.2 below. The average actual-model differential is \$16.73 for the AJ tranche and \$6.34 for the A tranche, hence the model seems to underprice these tranches slightly, but, as discussed above, given the out-of-sample nature of our approach, model and market prices are remarkably close.

In Table 3 we report the mean and standard deviation of pricing errors and model-market price correlations for various alternative parameter settings. We see that lowering the dividend rate or idiosyncratic property volatility lowers the average pricing error, but typically increases the standard deviation of the pricing error. The opposite happens when we increase the dividend rate or idiosyncratic property volatility. Hence, obtaining a better fit than the baseline setting is not trivial. The most important result in Table 3 is that for all parameter settings the correlations between market and model prices remain very high. This implies that assumptions on the dividend rate and idiosyncratic volatility affect the overall level of model prices, but have little effect on the time variation.

In sum, our first main empirical result is that, overall, there is little evidence of large and permanent relative mispricing between REIT stock and option markets and CMBX markets. This is quite remarkable given the complexity of the products and associated pricing models, and given that we study a crisis period. We thus find no evidence of a breakdown of the

market of securitized products leading to “distressed” pricing of such products.

### *5.2. Short-term mispricing: statistical significance*

Although model and market prices are reasonably in line with each other overall, there seem to be some short-term mispricing patterns. Specifically, in this subsection we study the predictive power of the actual-model price differential for the subsequent returns on CMBX tranches and a REIT index, and for the change in the implied volatility for this REIT index. For the index we use the Dow Jones REIT index. The independent variable is the one-day lagged actual-model price differential. We lag by one day to avoid any issues with nonsynchronous timing of the daily prices. We consider a specification without contemporaneous control variables (Panel A), as well as a specification where we include the contemporaneous return (or volatility change) of the other two assets as independent variables (Panel B). For example, for the regression with the CMBX AJ return as dependent variable, the contemporaneous REIT index return and the change in the REIT option-implied volatility are added as independent variables. This is a simple way to study whether current mispricing predicts price changes relative to other assets.

Table 4 shows that CMBX returns are predicted by the actual-model price differential with a negative sign, indicating that when the actual CMBX price is higher than the model price, the actual CMBX price is predicted to go down. This result obtains without (Panel A) and with (Panel B) contemporaneous control variables. We also see that mispricing across tranches contains similar information, suggesting that the mispricing is not tranche-specific but similar across tranches. Inspecting the results with the REIT index return as dependent variable, the AJ mispricing has some predictive power without controls (Panel A), but not when we include controls (Panel B). This implies that, relative to movements in the CMBX and REIT option market, REIT returns are not predictable. In other words, the current mispricing does not predict that REIT prices move away from CMBX and REIT option prices. For the results with the change in REIT option implied volatility as dependent variable we find no evidence of predictability.

Looking at the  $R^2$  of the regressions without controls (not reported) confirms that predictability is mainly present in the CMBX market. The  $R^2$  lies between 1.7% and 3.5% for predicting CMBX returns, which is remarkably high for daily returns, while the  $R^2$  for predicting REIT returns and REIT implied volatilities is between 0.1% and 0.6%.

In sum, these regression results provide evidence for temporary mispricing of CMBX contracts relative to REIT equity and option markets, which leads to reversals in the CMBX prices in subsequent days. There is much less evidence that REIT equity and option markets are subject to such temporary mispricing. In Section 6 we show that such CMBX price reversals occur in particular around important news announcements.

### *5.3. Short-term mispricing: economic significance*

In this subsection we analyze the economic significance of the short-term predictability in CMBX prices as uncovered in Section 5.2. We do this by means of a simple trading strategy. In theory, if Eq. (1) holds and if one can trade all assets continuously without frictions, it is possible to construct the portfolio of equities and options (of REITs and S&P 500 index) that dynamically replicates the CMBX contract (except for the idiosyncratic property shocks), but in practice this is complicated by the considerable transaction costs on options. We therefore focus on a simpler strategy trading only CMBX and REIT stocks.

The model view is summarized by the standardized raw signal,  $S_t^{raw}$ :

$$S_t^{raw} = -\frac{(P_t^{actual} - P_t^{model}) - \mu(P_s^{actual} - P_s^{model}|_s = 0, \dots, t)}{\sigma(P_s^{actual} - P_s^{model}|_s = 0, \dots, t)}, \quad (14)$$

with the mean  $\mu(\cdot)$  and standard deviation  $\sigma(\cdot)$  for the standardization determined using data up to time  $t$  (hence an expanding window). The standardized value is often referred to as the z-score. It is important that we only use data known at time  $t$  for the standardization; the use of future data would result in a severe look-ahead bias. The use of a standardization is a common trick with two main benefits: (i) the back test results are more robust to alternative model calibrations, because the results are invariant to linear transformations

of the mispricing, and (ii) it leads to more stable signal values in the face of time-varying volatility. To avoid excessive trading, we define a trade signal which has low turnover. At time  $t$  we invest in a CMBX tranche an amount proportional to the trade signal:

$$\begin{aligned} S_t^{trade} &= S_{t-1}^{trade} + 0.2, \text{ if } S_t^{raw} - S_{t-1}^{trade} > 0.5 \\ S_t^{trade} &= S_{t-1}^{trade} - 0.2, \text{ if } S_t^{raw} - S_{t-1}^{trade} < -0.5. \end{aligned} \tag{15}$$

In words, we increase (decrease) the trade signal by 0.2 if the raw-trade signal differential exceeds 0.5 (-0.5). The trade signal value at the start of the sample period is set to zero.

The CMBX position is hedged with a Dow Jones REIT index position, with the hedge ratio determined as the slope coefficient (beta) in a regression of overlapping five-day CMBX returns on a constant and the contemporaneous REIT stock returns, using data from time 0 to  $t$  (i.e, again an expanding window).<sup>14</sup>

To provide conservative results, we (i) use a trade lag of one day, meaning that the trade signal at time  $t$  is used for a trade at time  $t+1$  leading to a strategy return between time  $t+1$  and  $t+2$ , (ii) assume a realistic round-trip transaction cost (bid-ask spread) of \$1 per \$100 notional CMBX.<sup>15</sup> Transaction cost on the REIT index hedge are an order of magnitude smaller and ignored in the analysis. We start the back test July 3th, 2008, which provides sufficient data for determining the first standardized signal value and the first hedge ratio, and is well before the turbulent fourth quarter of 2008. The test runs through the end of our sample period, April 29, 2010. We present results for the AJ, AA and A tranche, and an equally-weighted combination of these three tranches, denoted combo.

Table 5 reports the annualized Sharpe ratios of these strategies, net of transaction costs.

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<sup>14</sup>Using five-day, rather than daily, returns is to reduce any effects of nonsynchronous timing of CMBX versus REIT prices.

<sup>15</sup>We have talked to CMBX traders and found that \$1 transaction costs are representative across the main brokers and over time. As a robustness check, we also consider results at double the transaction cost, \$2 round-trip.

Using the baseline model, The annualized net Sharpe ratio is 1.43, 2.37, 2.51, and 2.25 for the AJ, AA, A and combo, respectively.<sup>16</sup> To put this in perspective, Duarte, Longstaff, and Yu (2007) back test five popular fixed-income arbitrage strategies and find a maximum Sharpe ratio of 1.20. Remarkably, this highest Sharpe ratio is for the capital structure arbitrage strategy implemented with credit default swaps; of the five strategies considered the most similar in spirit to the strategy explored in this paper.

Of course, the Sharpe ratio is perhaps not the best performance measure, for example because it does not punish for negatively skewed return distributions. However, for all strategies examined the skewness of the return series is positive. Another concern is the small and, arguably, special sample period used for the analysis. For the AA and A series we have a longer time series available and can start the trading strategy in January 2007. This leads to Sharpe ratios of 1.97 and 2.05, respectively, slightly lower than the baseline values but still economically high.

We provide further checks on robustness of this result by varying the options-based pricing model in several ways. First, we vary the lag period, to see whether it matters how quickly one needs to trade on the signal. Second, we use daily returns to calculate hedge ratios. Third, we change the trade signal and trade size parameters in Eq. (15). Fourth, we double the transaction costs. Fifth, we use a lower or higher dividend rate  $q$ . Finally, we vary the level of the idiosyncratic volatility of property values in the pricing model ( $\sigma_j$ ). In all these robustness checks, we always find Sharpe ratios well above one and typically above two. In fact, results do not vary strongly across all these variations, which shows that the z-score adjustment is a good way to generate robust trading signals.

Another concern with the results from the trading strategy could be that the strategy returns reflect systematic risk. To address these concerns, we regress the strategy returns

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<sup>16</sup>The Sharpe ratio is defined as the mean of the daily return divided by the standard deviation of the daily return, times the square root of 260 to annualize (260 being a typical number of business days in a year).

on a series of commonly used risk factors. We follow Duarte, Longstaff, and Yu (2007) and use factors for the stock market (the three Fama-French factors, momentum, and a sector index for banks), the government bond market (three-month T-bill return and return on Merrill Lynch long-term treasury bond index), and the corporate bond market (return on Merrill Lynch A/BBB industrial and bank corporate bond indexes). We perform the regressions using both daily and weekly overlapping returns, correcting for serial correlation using Newey-West (1987). We study the baseline sample period (July 2008 - April 2010) and the extended period for the AA and A tranches (January 2007 - April 2010).

Table 6 reports the t-statistics of the alpha of the trading strategies (the constant term in the regression) and the  $R^2$  of the regressions. We do not report the exposures to the risk factors since these are mostly insignificant. Across all assets and specifications, we find that only nine out of 130 of the factor exposure coefficients are significant at the 5% level. The results for the alphas show that, despite the short sample period, the strategy generates significantly positive alphas in all cases except for the AJ tranche. In addition, the  $R^2$  of the factor regressions are quite low, which suggests that the strategy returns do not reflect systematic risk but rather short-term pricing inefficiencies specific to the CMBX market.

It is important to put these results in perspective. As we argue below, the short-term CMBX mispricing could be caused by acute hedging pressure by banks, which is important during this crisis period, but likely less relevant in normal times. Hence, we would expect much less mispricing and smaller alphas in normal times. Indeed, when we calculate Sharpe ratios of the trading strategy returns per calendar year, we find that the largest gains are made in 2008 and 2009, at the height of the crisis.

#### *5.4. What causes the short-term mispricing?*

One potential driver of the temporary mispricing effects documented above could be hedging activities of banks. By shorting CMBX contracts, banks can hedge the exposure to commercial real estate in their loan portfolio. Such hedging activities could become more important when banks are close to or in distress. This could lead to selling pressure for

CMBX contracts, and, if there are limits to arbitrage, to temporary mispricing between CMBX contracts and REIT stock and option prices.

To examine this hypothesis, we perform the following analysis. We regress the change in mispricing for a given CMBX tranche on the return of a US bank stock index. The hedging pressure hypothesis discussed above would imply a positive coefficient on the bank stock index: as bank stock prices decrease, banks short more CMBX contracts, leading to lower CMBX prices (relative to REIT stock and option prices), and thus a negative change in the CMBX mispricing.

We perform the regression using either daily or weekly overlapping returns, and include lagged bank stock returns since hedging activities could take several days. We control for market-wide stock price movements by including (lags of) the S&P 500 return. We also subtract the S&P 500 return from the bank stock return to isolate bank-sector movements.

Table 7 presents the t-statistics of the sum of the contemporaneous and lagged effects of the bank stock return and of the S&P 500 return. These results provide support for the hedging pressure hypothesis: we find significantly positive coefficients on the bank stock return for all CMBX tranches, and small and insignificant exposure to S&P 500 returns.

To further illustrate the hedging hypothesis, we focus on April 2010. In April 2010 market prices increased substantially while the REIT-based model price remains fairly stable. Consistent with our model, broker reports make note of the market move and cannot find a clear fundamental driver. E.g. in the April 16 weekly CMBS report by Bank of America (Lehman, 2010) it is said that “The CMBS market has seen a violent rally...” and “... very little has changed over the past week on the fundamental front...”.

To understand these price movements, we look at returns on the index for bank stocks and the S&P 500 returns in this period. We find that, in April 2010, the daily CMBX returns (equally-weighted average across tranches) had a correlation of 27% with bank stock returns, while the correlation between CMBX returns and S&P 500 returns was negative (-35%). Even the correlation between model CMBX and market CMBX returns was slightly

negative (-2%). This suggests that there were some non-fundamental movements in CMBX prices. The fact that these were correlated with bank stock returns is in line with our hedging hypothesis. In this month with positive returns for the CMBX tranches and bank stocks, it would suggest that banks were reducing their hedge positions (“short covering”). Lehman (2010) also mentions short covering as a potential reason for the CMBX price increases in April 2010.

### *5.5. Other mispricing patterns*

In this subsection we discuss two additional forms of mispricing that seem important. First, the pricing of the AJ tranche relative to the AA tranche deserves attention. Second, we find and discuss some mispricing patterns for other CMBX series. In Appendix D we discuss a third form of mispricing during the Quantitative Easing 2 (QE2) period.

#### *5.5.1. AJ versus AA tranche*

The market prices for the AJ tranche are typically well above the AA tranche price (\$14.2 price difference on average), while the model predicts a smaller difference (\$8.8 on average). This “relative-relative” mispricing mainly occurs during the crisis period. Because of the lack of a substantial gap between the AJ and AA model prices, we have analyzed an extension of our pricing model with systematic jumps. Potentially, by allowing for jumps a larger gap can be generated, if these jumps are such that they typically wipe out the AA tranche but not the AJ tranche. We have therefore extended the model to allow for common jumps to all property values. We do this by allowing the stock market index to jump. Given the exposure (beta) of property values to the stock market index, property values also jump when the stock index jumps. Each day, we calibrate the jump parameters to S&P 500 index option prices (across different strikes), as explained in detail in Appendix E of the paper.

The results for the jump model show that jumps do not really help to explain the AJ-AA gap. On average, including jumps leads to prices that are lower by \$1.3 (AJ), \$1.2 (AA) and 1.0\$ (A), and the correlation between diffusion model prices and jump-diffusion model prices is above 99.7% for all tranches.



There are two explanations for this result. First, the CMBX tranches have a long maturity so that part of the jump effect is averaged out. Second, even though we find that S&P 500 index options exhibit substantial jump risk, most of the total equity volatility generated by the model is still due to the diffusion component. Given the extremely high volatility implied by option prices, it is essentially impossible to explain these high implied volatilities with only downward jumps. Then, given that the model has diffusion shocks with substantial volatility (and jumps), it cannot generate a sufficient number of “events” that wipe out the AA tranche but not the AJ tranche. In other words, the diffusion model and jump-diffusion model are quite similar in terms of the distributions they generate.

It thus seems that the AJ and AA market prices reflected beliefs that losses up to 12.5% were very likely, while losses beyond 12.5% were assessed to be much less likely. Our jump-diffusion results show that such a pattern is hard to generate when matching the model to observed prices of S&P 500 options and REIT equity options.

### *5.5.2. Pricing results for the AJ tranche of series 2 to 5*

In Fig.6 we plot the actual and model price for the AJ tranche of series 2 through 5. To construct these plots, for each series we replace the series-specific values reported in Table 2 with the values corresponding to the series at hand. The main insight that is that moving to newer series, the actual-model price differential decreases somewhat. This seems in line with a general trend of deteriorating underwriting standards;<sup>17</sup> a consideration recognized by the market to some extent, but not explicitly modeled by us.

The model does a worse job matching the prices of some of the lower tranches, in particular the AA and A tranches of series 4 and 5 (not reported). This can partially be explained by the fact that these tranches are less liquidly traded; see our discussion in Section 4.3 and Table 1. An additional effect is that our model abstracts from term defaults, which is a less suitable assumption for these tranches as they have a high coupon in combination

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<sup>17</sup>For example by using pro forma underwriting, i.e. providing loans based on optimistic expectations of future property income growth.

with (presumably) a high perceived probability of default in the near term, leading to a substantial upward bias in the model price.

## 6. Response to news announcements

In this section we provide further evidence of short-term price reversals of CMBX prices by studying price patterns after important news announcements. First, we first provide a general discussion of the main news events relevant for the CMBS market. Next we analyze how the CMBX market responded to those news announcements.

### *6.1. General discussion of the main CMBS news events*

In Table 8 we report the main CMBS news events, defined by events that satisfy two criteria (i) Bank of America sent out a special report on the event, (ii) Bloomberg reported the event. We add the second criterion because two of the special broker reports were not on a new topic and no mentioning on Bloomberg could be found around the report issue date. We take this formal approach to the definition of news to prevent data snooping that could bias the statistical tests on the response to news later in this section.

We now briefly discuss a few examples of these news events. On August 15th, 2008, news broke that a mortgage loan collateralized by the Riverton apartments in Harlem, New York City was transferred to the special servicer due to imminent default. The loan, referenced by the CMBX 3 series, was particularly worrying because it shone light on the practice of pro forma underwriting, using projected rather than realized cash flows for reported multiples as the debt-service coverage ratio (DSCR). On November 18th 2008, two large loans on the Promenade Shops at Dos Lagos and the Westin portfolio, referenced by the CMBX 5 series, were reported to be 30-days delinquent and were transferred to the special servicer. In the spring of 2009 good and bad news announcements took turns. Good news came from the Government that expanded the Term Asset-Backed Loan Securities Facility (TALF) to include new and later also legacy CMBSs. Also the Public-Private Investment Program (PPIP) was introduced. The bad news came predominantly from S&P, which became more

strict on credit ratings for CMBS.

## 6.2. Short-term market response to news

Next we analyze the market response to the news announcements included in Table 8. Similar to the predictive regressions, we look at how current mispricing affects future CMBX returns. More precisely, on an announcement day, event time zero, we calculate the sign of the change in mispricing, thus splitting news events into events where the actual CMBX price increased more than the model price and events where the opposite happened. We define the change in the mispricing,  $\tilde{R}_t$ , as the actual return minus the model return (Eq. (16)). To measure whether on subsequent days the market moves in the same or opposite direction, we introduce the signed return,  $R_t^{sign}$  (Eq. (17)). Now the cumulative response at event time  $t$ ,  $R_t^{cum}$  is the sum of signed returns from time zero to event time  $t$ :

$$\tilde{R}_t = R_t^{actual} - R_t^{model} \quad (16)$$

$$R_t^{sign} = R_t * \text{sign}(\tilde{R}_0) \quad (17)$$

$$R_t^{cum} = \sum_{s=0}^t R_s^{sign}. \quad (18)$$

The signed cumulative returns thus capture whether CMBX prices move in the same direction (positive signed return) or opposite direction (negative signed return) of the initial change in mispricing. We apply Eq. (18) to compute the average and median cumulative response across all events, and we plot these in Fig. 7.

On the day of the announcement, the average signed  $\tilde{R}_0$  is equal to about 2.4%. The average cumulative response reaches a peak two days later at 4.6%. This could be consistent with either initial underreaction or subsequent overreaction to news. The sharp decrease in the cumulative response two to five days after the news announcement points to overreaction. Also the median cumulative response peaks two days after the announcement and sharply reverts over the subsequent three days, suggesting the pattern is not explained by extreme

outliers. These results are consistent with the predictive regressions in Table 4.

To support these graphical results with statistical evidence, we proceed by investigating the statistical significance of the overreaction to news on the days following a news announcement. In Table 9 we show the t-statistics for the beta coefficients of the following two regression specifications

$$R_t = \alpha + \sum_{lag=1}^6 \beta_{lag} R_{t-lag} D_{t-lag} + \sum_{lag=1}^6 \gamma_{lag} R_{t-lag} + \varepsilon_t \quad (19)$$

$$R_t = \alpha + \sum_{lag=1}^6 \beta_{lag} \text{sign}(R_{t-lag}) D_{t-lag} + \sum_{lag=1}^6 \gamma_{lag} \text{sign}(R_{t-lag}) + \varepsilon_t, \quad (20)$$

where  $D_t$  is a dummy variable taking a value of one on a news day. Notice that the beta coefficients measure the effect of news on price continuation and reversal *in addition* to general short-term price continuation and reversal captured by the  $\gamma$ -coefficients (not reported). The results in Table 9 show evidence consistent with Fig. 7, with mostly price continuation for the first two lags, and reversals for lags three to six. Statistical significance is strongest for the reversal after a few days.

## 7. Absolute mispricing of CMBX and REITs

So far, we have focused on the relative pricing of CMBX and REITs. Even when there is no persistent relative mispricing, it could be that the absolute price levels of CMBX and REITs deviated from their fundamental values during the crisis. Stanton and Wallace (2011) study the absolute price levels of ABX contracts which reference subprime residential MBS. They find strong evidence that ABX prices were too low during the crisis period. In this section we study whether CMBX contracts were also underpriced at the height of the crisis.

### 7.1. Using REIT NAVs

Our first analysis uses net asset values (NAVs) for REIT equity. Several financial institutions report such NAVs, which are based on valuation models for commercial properties.

These NAVs are supposed to capture the fundamental value of REITs, and whenever REIT equity prices deviate from NAVs, this is interpreted as mispricing of REIT equity by the market. Fig. 8 shows the ratio of market price to NAV reported by three institutions: JP Morgan, Green Street Advisors (GSA), and Bank of America. It illustrates that there are some periods where NAVs deviate from market prices, but on average they are reasonably in line with each other. During the crisis, REIT equity market prices were a bit lower than their NAVs, with the largest discount for the NAVs from GSA. In the analysis below we use the NAVs of GSA, since these deviate most from market prices during the crisis.<sup>18</sup>

To study the absolute price levels of CMBX contracts, we proceed by calibrating our CMBX pricing model to REIT NAV levels (using (Eq. (10))), instead of market prices of REIT equity. REIT NAVs are obtained by using the price-to-NAV ratio to correct the market prices for REIT equity. Under the assumption that NAVs correctly assess the fundamental REIT equity value, this approach generates NAV-based “fundamental” CMBX model prices. We compare these with actual CMBX prices to see whether there was any absolute mispricing.

We show results for the AA tranche in Fig. 9, where we plot the “fundamental” CMBX model prices, as well as the difference between these prices and model prices from our benchmark pricing model. We see that using NAVs instead of equity market prices has a small effect on the CMBX prices. This is because, according to our model, property values go down during the crisis to a level where the tranches are “out-of-the-money”, in the sense that their intrinsic value is zero. Hence, the entire value of the tranche is due to the option value. Then, the sensitivity of these tranches to property values (the delta) of such out-of-the-money options is low.<sup>19</sup> This implies that the tranche values are not very sensitive to the value of REIT equity. Hence, whether REITs traded at a discount or premium is less relevant during the crisis.

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<sup>18</sup>We use the NAVs reported by GSA in the figure at <https://www.greenstreetadvisors.com/i/premNAV.png>, and manually convert this graph into numbers.

<sup>19</sup>Other “greeks”, such as the vega, are also small for such out-of-the-money options.

The situation changes in the aftermath of the crisis: as the tranches become in-the-money again, there is an effect of the REIT discount/premium. This can be seen in Fig. 9, where the NAV-based model price deviates from the equity-based model price in the later part of the sample. Since NAVs are below market prices in this period, the NAV-based model prices are a bit lower than the equity-based prices and also lower than observed CMBX prices. Hence, we do not find evidence that the CMBX market prices were too low during the crisis period.<sup>20</sup>

## 7.2. Market-implied default rates

Our second approach is to calculate default rates on commercial properties as implied by CMBX prices. We do this in the following way. First note that we have prices for the A, AA and AJ tranches of the CMBX 1 series, which cover losses between 7.7% to 10.5%, 10.5% to 12.5%, and 12.5% to 19.9%, respectively. Now assume that loan defaults cluster into events that knock out an entire tranche, and that such events happen after  $\tau$  years (if at all) with risk-neutral loss probability  $Q_{CE^L, CE^H}$  for a tranche with attachment points  $CE^L$  and  $CE^H$ . Then the pricing Eq. (4) simplifies to

$$P(CE^L, CE^H) - 100 - 4 * \sum_{k=1}^{25} \sum_{t=1}^T e^{-r_f t} CF_{t,k}^{fixed} = -100e^{-r_f \tau} Q_{CE^L, CE^H}, \quad (21)$$

from which we can solve for  $Q_{CE^L, CE^H}$  given the observed tranche price and coupon. Using these tranche-loss probabilities one can calculate the probability that losses will be in one of the four ranges implied by the AJ, AA and A tranches of the CMBX 1 series, 0 to 7.7%, 7.7% to 10.5%, 10.5% to 12.5%, and 12.5% and higher. These probabilities are equal to  $1 - Q_{7.7, 10.5\%}$ ,  $(Q_{7.7, 10.5\%} - Q_{10.5, 12.5\%})$ ,  $(Q_{10.5, 12.5\%} - Q_{12.5, 19.9\%})$  and  $Q_{12.5, 19.9\%}$ , respectively. Next, we use this distribution to obtain an expected default rate on the entire commercial

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<sup>20</sup>We have also regressed CMBX prices on the Price-to-NAV ratios to see if the REIT discount/premium is related to CMBX prices, both in levels and first differences. In line with the analysis in this section, we find little evidence that the REIT discount/premium explains CMBX prices.

mortgage pool, using the following additional simplifying assumptions: (i) a fixed recovery rate at the historical average of 60% (Downing, Stanton, and Wallace, 2007), (ii) within the 0% - 7.7% range, losses are uniformly distributed with expected loss equal to  $1/2 \cdot 7.7\%$ , (iii) for all subsequent ranges, the loss in case of an event is equal to the upper attachment point  $CE^H$ , in line with the assumption that the tranche is fully wiped out in case of a tranche default event, and (iv) for the range of 12.5% and beyond, we assume that the tranche event causes losses equal to 19.9% (the upper attachment point of the AJ tranche). This final assumption is obviously very important and we assess the impact of it below. Using these assumptions we can calculate the risk-neutral expected default rate on the entire pool of commercial mortgage loans as

$$\frac{1}{1-R} \left( \begin{array}{l} (1 - Q_{7.7\%,10.5\%}) \cdot 1/2 \cdot 7.7\% + (Q_{7.7\%,10.5\%} - Q_{10.5\%,12.5\%}) \cdot 10.5\% + \\ (Q_{10.5\%,12.5\%} - Q_{12.5\%,19.9\%}) \cdot 12.5\% + Q_{12.5\%,19.9\%} \cdot 19.9\% \end{array} \right), \quad (22)$$

where  $R$  is the percentage recovery rate. For example, with a recovery rate of 60% a fraction equal to  $\frac{1}{1-0.6} \cdot 7.7\% = 19.3\%$  of the mortgage pool needs to default to generate losses equal to 7.7%.

In Fig. 10 we plot this risk-neutral expected default rate for each day in our sample for  $\tau$  equal to one year. We see that it peaks in April 2009 at almost 40%, then dropping to about 20% in April 2010. In Table 10 we assess the sensitivity of these results to various input parameters. Table 10 reports the maximum default rate over the sample for different parameter settings. First, we lower the recovery rate to 40%, as it is conceivable that recovery rates are lower than the long-term average in crisis periods. This lowers the maximum default rate from 40% to almost 27%. Second, we change the timing of the default events,  $\tau$ , to three years from the pricing date. This has a relatively small effect on the implied default rates. Finally, we change the assumption on the maximum loss. In the benchmark setting, we assumed the worst tranche event led to a loss of 19.9%. We then increase the loss associated

with this event to 40%. Note that with a 60% recovery rate, an event with a loss of 40% implies that *all* loans in the underlying portfolio default. Under these extreme assumptions, the maximum expected default rate increases substantially to 73%. If we use a 40% recovery rate instead, the highest default rate during the crisis period equals 49%.

Are these implied default rates reasonable or too high? This is obviously not an easy question to answer, but we argue that these default rates are not excessively high. One can compare these numbers to long-term commercial mortgage default rates for 1972-2002 (Esaki and Goldman, 2005). They report an average lifetime default rate of 19.6%, with the highest lifetime default rate equal to 32% for the 1986 cohort. These are cumulative lifetime default rates, and most defaults happen between three and seven years of origination. Obviously, the maximum implied default rates in Table 10 are higher and refer to shorter horizons, but this seems not unreasonable given the severity of the crisis. In addition, note that the market-implied default rates in Table 10 are risk-neutral default rates. In the corporate credit risk literature, several articles have found that risk-neutral default probabilities are at least two to three times larger than actual default rates (Driessen, 2005; Berndt, Douglas, Duffie, Ferguson, and Schranz, 2008). Finally, note that these default rates are less extreme than what Stanton and Wallace (2011) report for the ABX market for residential mortgage loans. They find that ABX prices imply default rates of essentially 100% at recovery rates of 34%, which is extremely far from historical averages.

In addition to the overall expected default rate, we also study the default rates at the tranche level. Eq. (21) gives the risk-neutral tranche default rates  $Q_{CEL,CEH}$  implied by CMBX prices using our simple model. At the height of the crisis, these tranche default rates (that give the probability that a tranche is fully wiped out) are around 80% for the AA and A tranches, and around 60% for the AJ tranche. We compare these high risk-neutral tranche-level probabilities with the historical distribution of losses. Using the default rates reported by Esaki and Goldman (2005) and using various levels for the recovery rate, we plot the fraction of the tranche that would have been wiped out for cohorts starting 1972



to 1997 in Fig. 11, for recovery rates of 60% (the historical average) and 40%, respectively. There are two reasons why using a recovery rate below the historical average is appropriate. First, it is conceivable that recovery rates are lower in periods with high default rates (see Altman, Brady, Resti, and Sironi, 2003 for such evidence for the corporate bond market). Second, the risk-neutral recovery rate that is relevant for pricing CMBX contracts is lower than the actual recovery rate if there a risk premium associated with recovery risk.

Fig. 11 shows that, with a recovery rate of 60%, the A tranche is fully wiped out in two out of 26 cohort-years, the AA tranche in one cohort-year, and the AJ tranche has essentially never any losses. However, with a recovery rate of 40%, tranche losses are much more prevalent. The AA and A tranches would have been fully wiped out for 11 and five out of the 26 cohort-years, respectively. The effect of a change in the recovery rate is so strong because especially the AA and A tranches are quite thin. Hence, a relatively small increase in the default rate or loss rate can have a substantial effect on these CMBX tranches.<sup>21</sup> In sum, we see that it is not uncommon for AA and A tranches to be wiped out fully. In this light, the market-implied tranche-default probabilities do not seem excessively high.

Overall, these results are reasonably consistent with the analysis based on NAVs of REITs: we do not find strong evidence that absolute price levels in the CMBX market were too low at the height of the crisis, in contrast to the ABX market.

## 8. Conclusion

We analyze the efficiency of the CMBX market during the recent financial crisis. We do this by comparing actual CMBX prices with prices from a Merton-style structural option pricing model. This model is calibrated to stock and option prices for the S&P 500 index and

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<sup>21</sup>Note that the sensitivity of thin tranches to default or loss rates is only large conditional on the tranches being close to at-the-money. If thin tranches are away from at-the-money, their sensitivity to changes in default or loss rates is actually very low. We thank the anonymous referee for pointing this out. The results in Fig. 11 thus show that these tranches are quite close to at-the-money in several years.

several REITs, and not to CMBX prices, which allows for a clean relative pricing analysis. We find that, on the one hand, the general price swings experienced by the CMBX indexes are largely explained by our structural option model. On the other hand, at shorter horizons, we find predictability in CMBX prices relative to REIT prices that is both statistically significant and economically meaningful, as evidenced by significant abnormal returns from a model-based trading strategy. We provide evidence that this temporary mispricing of CMBX contracts is caused by hedging pressure of banks.

In addition to this relative pricing analysis, we also study the absolute pricing of CMBX contracts, and do not find strong evidence that these assets were substantially mispriced before or during the crisis period.

## Appendix A. Example of a CMBX 1 constituent

In this appendix we discuss the JPMCC 2005-LDP4 deal, which is one of the 25 CMBS deals referenced by the CMBX 1 indexes and is illustrative of the type of deals underlying the CMBX indexes. Presented information is based on (i) the JPMC February 2010 CMBX Monthly update, (ii) the JPMC February 2010 CMBX Reference Obligation Performance Appendix and (iii) the official prospectus supplement from JPMC, dated September 22, 2005. The prospectus supplement is 471 pages; a first indication that the deal structure is quite complex, and that the model setup used in this paper is necessarily simplified.

The three most-used property types are the ones we explicitly model in our paper: office, retail, and multifamily, accounting for 28.77%, 35.47%, 19.52% of the CMBS deal balance respectively. It is reported that 73.25% of loans are interest only, for either the full term or part of the term. More than 99% of the pool has prepay protection.

In Table 11 we provide information on the 24 different tranches of the deal. The CMBX AAA, AJ, AA, A, BBB tranches reference the A-4, A-J, B, D, G tranches of the JPMCC 2005-LDP4 deal respectively. From the A-M tranche down in the capital structure, the losses are allocated simply in order of seniority. This is why in this paper we mainly focus on the CMBX AM, AJ, AA, and A indexes, which reference these relatively more straightforward tranches below the AAA, rather than the CMBX AAA index, which references the last-cash flow tranche of the most senior layer of tranches, the A-4 in case of this deal.

The allocation for the most-senior tranches, those with 30% original subordination, is more complex and one has to work through the 471-page prospectus (and this is for just one of the 25 constituents of CMBX 1). For this particular deal the underlying loans are split up in two loan groups. Principal payments from loans in group two first go to the A-1A class. In the prospectus it can be seen that the A-1A class is not offered to the public. In a 2006 Nomura Synthetic CMBS Primer it is explained that the A-1A class usually is a carved-out class bought by Freddie Mac and Fannie Mae consisting of multifamily loans that conform

the investment rules specified in their charters. Principal payments from group one, and principal payments from group two after the A-1A is paid off, first go to the A-SB class in the amount of the “planned principle balance schedule,” then to the A-1, A-2 and A-2FL, A-3A1, A-3A2, A-4. As one can see in Table 11, the A-2 and A-2FL have the same factor and weighted average life (WAL), but differ because the A-2FL is packaged with a swap contract that converts the fixed-rate coupon payments into floating payment at a 0.36% spread over one-month LIBOR. Hence only for the A-2FL class is the coupon reported in Table 11 not a fixed rate, but the spread over one-month LIBOR. Interest payments to the A-1, A-1A, A-2, A-2FL, A-3A1, A-3A2, A-4, A-SB are pro rata, which explains why they have the same reported original subordination of 30% in Table 11.

Notice that the structure displayed in Fig. 2 is indeed a simplified situation, as it just illustrates that the A-1, A-2, A-3, A-4 tranche are paid off sequentially, but ignores the existence of special tranches that may exist, like the A-1A, A-2FL, and A-SB tranche for the JPMCC 2005-LDP4 deal.

The balance per February 2010 is \$156*m* lower than the \$2677*m* original balance, implying a factor 0.94 of the balance is still outstanding. Looking at the deal structure in the figure below, the \$156*m* total balance reduction can be broken up into \$85*m* for fully paying off of the A-1 tranche, \$8*m*, \$32*m*, \$28*m*, for partially paying off the A1-A, A-2, and A-2FL tranches respectively, and \$4*m* balance reduction for the non-rate (NR) tranche.

## **Appendix B. Adjusting the calibration for asset issuance and survivorship bias**

If companies issue debt or equity, their asset value grows non-organically. We want to purge the asset value from this effect to be able to interpret the change in asset value as a

measure for property value change. That is, we want to measure organic growth. We use

$$\left(\frac{\bar{V}_{jt}}{D_{j0}}\right)_{adj} = \frac{\mu\left(\frac{V_{jt}}{D_{jt}}\right) - \mu\left(\frac{D_{jt}-D_{j0}}{D_{jt}}\right) - \mu\left(\frac{(Sh_{jt}-Sh_{j0})P_{jt}}{D_{jt}}\right)}{\mu\left(\frac{D_{j0}}{D_{jt}}\right)}, \quad (23)$$

where  $\mu$  is the mean over all REITs in a given sector,  $P_{jt}$  denotes the stock price of a REIT in sector  $j$  at time  $t$ ,  $Sh_{jt}$  denotes number of shares outstanding, and  $\mu\left(\bar{V}_{jt}/D_{jt}\right)$  is obtained from the calibration procedure discussed in Section 3. The organic growth needed for Eq. (9) is then given by  $\left(\frac{\bar{V}_{jt}}{D_{j0}}\right)_{adj} / \left(\frac{\bar{V}_{j0}}{D_{j0}}\right)_{adj}$ .

A second calibration adjustment is because of a potential bias towards selecting larger REITs by requiring REITs to have option data. Also we could have a survivorship bias by selecting REITs that were constituents of the index in 2009. We control for this bias by calculating the ratio of the cumulative equity return on the broad IYR index over the cumulative return on our selected set of REITs. We use this ratio to adjust the organic growth discussed above. In practice, this adjustment is small and always below 10%.

## Appendix C. CMBX pricing model with stochastic interest rates

This appendix describes an extension of the CMBX pricing model, where interest rates are driven by a one-factor Vasicek model. Specifically, we have under the risk-neutral measure

$$dr = k(m - r)dt + \sigma_r dW_r, \quad (24)$$

with correlation between  $dW_0$  and  $dW_r$  equal to  $\bar{\rho}$ . It is easy to show that this leads to an effective correlation between  $\frac{d\bar{V}_j}{\bar{V}_j}$  and  $dW_r$  equal to

$$\rho_{r,j} = \frac{\beta_j \sigma_S \bar{\rho}}{\sqrt{\beta_j^2 \sigma_S^2 + \gamma_j^2}}. \quad (25)$$

Bonds are priced according to the standard Vasicek pricing formulas,  $P(T) = A(T)e^{-B(T)r}$ . In this case the option pricing formula for a call option on a REIT is (Rabinovitch, 1989)

$$\begin{aligned}
E_j &= \bar{V}_j N(d_{1j}) - D_j P(T) N(d_{2j}) \\
d_{1j} &= (\ln(\bar{V}_j/D_j) - \ln(P(T)) + \phi_j/2)/\sqrt{\phi_j} \\
d_{2j} &= d_{1j} - \sqrt{\phi_j} \\
\phi_j &= \beta_j^2 \sigma_S^2 T + \gamma_j^2 T + (T - 2B(T) + \frac{1 - e^{-2kT}}{2k}) \frac{\sigma_r^2}{k^2} - 2\rho_{r,j} \sqrt{\beta_j^2 \sigma_S^2 + \gamma_j^2 (T - B(T))} \frac{\sigma_r}{k}.
\end{aligned} \tag{26}$$

For pricing CMBX tranches, this Vasicek model leads to the following pricing expression

$$P(CE^L, CE^H) = 100 + 4 * \sum_{k=1}^{25} \sum_{t=1}^T E^Q \left( \exp\left(-\int_0^t r_s ds\right) CF_{t,k}^{fixed} - \exp\left(-\int_0^t r_s ds\right) CF_{t,k}^{floating} \right). \tag{27}$$

This has two implications. First, the fixed coupon  $CF_{t,k}^{fixed}$  should be discounted by the appropriate discount factor  $P(t)$ . Second, deal losses should be discounted by the realized short-rate path.

For the mean-reversion and volatility parameters we use parameters estimated by de Jong (2000):  $k = 0.0222$ , and  $\sigma_r = \sqrt{1.9980/10000} = 0.0141$ . For the long-term mean  $m$  we use 4%, and for the correlation  $\bar{\rho}$  we use three values, 50%, 0%, and -50%. Using Eq. (25), this corresponds to correlations between property values and interest rates of -33%, 0%, and 33% (averaged over time and across sectors).

We then price CMBX contracts with this model. The effect of stochastic interest rates on the model CMBX prices is quite small. First, with  $\bar{\rho} = 0\%$ , we find that CMBX prices are slightly lower compared to the benchmark model, about \$1 on average across tranches. This is due to the different shape of the term-structure of interest rates in the Vasicek model (compared to the flat term structure in the benchmark model).

When we move from  $\bar{\rho} = -50\%$  to  $\bar{\rho} = 50\%$ , the model CMBX prices go down by about

2.5\$ on average (across tranches). This shows that, if property value shocks are correlated with interest-rate shocks, the price of a CMBX will be affected. For example, if interest rates decrease when property values decrease (positive correlation), CMBX prices will be lower since losses are higher in present value terms (since they are discounted at lower interest rates). Still, the effects are not very large and do not affect our results in a major way.

## Appendix D. Impact of QE2

In this appendix we study CMBX prices through July 15th, 2011, and argue that the second round of quantitative easing (QE2) by the US Federal Reserve had some impact on the market in the second half of 2010 and first half of 2011. We lack the data to update all our results through July 15th, 2011, but we can update the baseline-case model prices, provided we augment implied volatility data from OptionMetrics, available through October 2010, with implied volatility data from Bloomberg.

In Fig. 12 we plot the actual-model price differential for the AM, AJ, AA and A tranches of CMBX series 1 through July 15th, 2011. We argue that the results for 2010 and 2011 in Fig. 12 are supportive of a “search for yield” phenomenon where the Fed pushes down Treasury yields under QE2 and CMBS (and by no arbitrage CMBX) is considered a substitute investment by bond investors (but REIT stock and options are not to the same extent).<sup>22</sup>

First, the timing of changes in the actual-model price differential seems in line with the search for yield story. In the second half of 2010, when QE2 was first anticipated and officially announced on November 3, 2010, actual prices rise relative to model prices substantially. In 2011, when investors anticipated the end of QE2 and later QE2 was officially done, actual prices drop substantially again for all tranches and get much closer to the model price.

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<sup>22</sup>Gambacorta (2009) discusses how monetary policy can generate a “search for yield”. Several newspaper articles mention the potential effects of QE2, see for example the Wall Street Journal of March 17, 2011, where it is mentioned that “The Fed’s program to buy \$600 billion in U.S. Treasury bonds, known colloquially as QE2, has kept interest rates low, spurring bond investors to buy riskier assets in search of yield.” Krishnamurthy and Vissing-Jorgensen (2011) discuss the effects of QE2 on long-term interest rates.

Second, as can be seen in Fig. 12 for the 2010H2-2011H1 period: first the AM actual-model price differential peaks, then the AJ differential peaks, then the AA differential peaks, and finally the A differentials peak. Also this seems consistent with a search for yield story, where rising prices mean that investors have to invest increasingly lower in the capital structure in order to obtain a high yield, and the associated demand pressure pushes up actual prices relative to model prices in order of seniority.

## Appendix E. CMBX pricing model with jumps

In this appendix we describe an extension of the CMBX pricing model, allowing for jumps to stock prices and property values. We start with the following model with jumps to the stock market,  $dJ$ , following Bates (1991), under the risk-neutral measure:

$$\begin{aligned}\frac{dV_{ij}}{V_{ij}} &= (r - q - \lambda\beta_j k)dt + \beta_j\sigma_S dW_0 + \beta_j k dJ + \gamma_j dW_j + \sigma_j dZ_{ij} \\ \frac{dS}{S} &= (r - \lambda k)dt + \sigma_S dW_0 + k dJ,\end{aligned}\tag{28}$$

where the jump size  $k$  and the risk-neutral jump intensity  $\lambda$  are constant over time. This is a special case of Bates since we only use fixed-size jumps, equal to  $k$ . We impose that  $k > -1$  and that  $\beta_j k > -1$ .

The return on the portfolio of properties of a REIT is then equal to

$$\frac{d\bar{V}_j}{\bar{V}_j} = (r - q - \lambda\beta_j k)dt + \beta_j\sigma_S dW_0 + \beta_j k dJ + \gamma_j dW_j.\tag{29}$$

Given the process in (29), the value of REIT equity, denoted  $E_j$ , is then valued as follows (Merton (1976), Bates (1991))

$$E_j = e^{-rT} \sum_{n=0}^{\infty} [e^{-\lambda T} (\lambda T)^n / n!] [\bar{V}_j e^{b^{(n)T}} N(d_{1n}) - D_j N(d_{2n})],\tag{30}$$



with

$$\begin{aligned}
d_{1n} &= (\ln(\bar{V}_j/D_j) + b(n)T + (\beta_j^2\sigma_S^2T + \gamma_j^2T)/2) / \sqrt{\beta_j^2\sigma_S^2T + \gamma_j^2T} \\
d_{2n} &= d_{1n} - \sqrt{\beta_j^2\sigma_S^2T + \gamma_j^2T} \\
b(n) &= r - q - \lambda\beta_jk + n \ln(1 + \beta_jk)/T.
\end{aligned} \tag{31}$$

Using Ito's lemma for jumps we can derive the dynamics of  $E_j$

$$\begin{aligned}
dE_j &= (\dots)dt + \frac{\partial E_j}{\partial \bar{V}_j} \bar{V}_j (\beta_j\sigma_S dW_0 + \gamma_j dW_j) + \Delta E_j dJ \\
\Delta E_j &= E((1 + \beta_jk)\bar{V}_j) - E(\bar{V}_j),
\end{aligned} \tag{32}$$

where  $E(\cdot)$  denotes the REIT equity value as a function of the underlying value.

The first step in the model calibration is that the parameters of the jump process are calibrated to S&P 500 option prices. Specifically, to calibrate the jump parameters (intensity  $\lambda$ , jump size  $k$ ) and the stock price diffusion volatility  $\sigma_S$ , we use daily data on three-month index put options with various strike levels (both in-the-money and out-of-the-money options) from OptionMetrics.<sup>23</sup> These options can be priced using a similar option pricing expression as (30). Each day, we fit the parameters by minimizing the sum of the squared percentage pricing errors of all index options.

The calibration of the remaining model parameters is similar to the calibration of the model without jumps (Section 3.4), but the expressions for the calibration restrictions change due to the jump process. We do this by first-order Taylor approximation of  $\Delta E_j$ . For the

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<sup>23</sup>We use the so-called "standardized" option price data, which has option prices for three-month options with deltas ranging from -0.80 to -0.20 (in steps of 0.05).

variance of REIT returns this gives

$$\begin{aligned} Var\left(\frac{dE_j}{E_j}\right) &= \left(\frac{\bar{V}_j}{E_j} \frac{\partial E_j}{\partial \bar{V}_j}\right)^2 (\beta_j^2 \sigma_S^2 + \gamma_j^2) dt + \left(\frac{\Delta E_j}{E_j}\right)^2 \lambda dt \\ &\simeq \left(\frac{\bar{V}_j}{E_j} \frac{\partial E_j}{\partial \bar{V}_j}\right)^2 (\beta_j^2 \sigma_S^2 + \gamma_j^2) dt + \left(\frac{\bar{V}_j}{E_j} \frac{\partial E_j}{\partial \bar{V}_j}\right)^2 \beta_j^2 k^2 \lambda dt. \end{aligned} \quad (33)$$

Similarly, the covariance of REIT equity with the stock market index is given by

$$Cov\left(\frac{dE_j}{E_j}, \frac{dS}{S}\right) \simeq \left(\frac{\bar{V}_j}{E_j} \frac{\partial E_j}{\partial \bar{V}_j}\right) \beta_j \sigma_S^2 dt + \left(\frac{\bar{V}_j}{E_j} \frac{\partial E_j}{\partial \bar{V}_j}\right) \beta_j k^2 \lambda dt, \quad j = 1, \dots, 3, \quad (34)$$

and the covariance of REIT returns across different sectors is

$$\begin{aligned} Cov\left(\frac{dE_j}{E_j}, \frac{dE_k}{E_k}\right) &\simeq \left(\frac{\bar{V}_j}{E_j} \frac{\partial E_j}{\partial \bar{V}_j}\right) \left(\frac{\bar{V}_k}{E_k} \frac{\partial E_k}{\partial \bar{V}_k}\right) \beta_j \beta_k \sigma_S^2 dt \\ &+ \left(\frac{\bar{V}_j}{E_j} \frac{\partial E_j}{\partial \bar{V}_j}\right) \left(\frac{\bar{V}_k}{E_k} \frac{\partial E_k}{\partial \bar{V}_k}\right) \rho_{jk} \gamma_j \gamma_k dt + \left(\frac{\bar{V}_j}{E_j} \frac{\partial E_j}{\partial \bar{V}_j}\right) \left(\frac{\bar{V}_k}{E_k} \frac{\partial E_k}{\partial \bar{V}_k}\right) \beta_j \beta_k k^2 \lambda dt, \quad j, k = 1, \dots, 3. \end{aligned} \quad (35)$$

Next we discuss the calibration results for this jump-diffusion model. Fig. 13 plots the parameters related to the stock price process ( $\sigma_S$ ,  $\lambda$ , and  $k$ ). We see that, as the crisis unfolds, the jump intensity increases and the jump size becomes much more negative, as expected. In fact, in unreported results we see that the product  $\lambda k$ , the expected jump effect, is very closely related to the implied volatility skew of S&P 500 options (out-of-the-money put implied volatility minus at-the-money put implied volatility). This skew steepened substantially during the crisis, leading to larger jump effects. Still, this jump process does not explain the total volatility level: the diffusion volatility  $\sigma_S$  also increases during the crisis to levels above 40%.

CMBX tranches are priced by simulating this jump-diffusion model, in a similar way as the diffusion model. In addition to the benchmark jump model where we calibrate both  $\lambda$  and  $k$ , we have also fixed  $\lambda$  at 0.5 and 2, respectively, and calibrated  $k$  daily given this value for  $\lambda$ . This gives similar pricing results for the CMBX tranches.

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Table 1: Daily traded notional, median estimate across six major brokers

This table reports the daily traded volume (notional dollar amounts in millions) for different series-tranche combinations. Reported are the median values across six major brokers from a survey we conducted in the third quarter of 2009.

	Series 1	Series 2	Series 3	Series 4	Series 5
AJ	150	100	88	100	50
AA	51	25	38	38	10
A	76	30	28	38	10

Table 2: CMBX contract settings

This table reports the CMBX contract parameters for the CMBX 1 series used for the baseline pricing analysis.

Name	Symbol	Value
Start date	$t = 0$	March 7th, 2006, the introduction date of CMBX 1
Maturity date	$t = T$	March 7th, 2016
Fixed coupon	$c$	50bp, 84bp, 25bp, 35bp per annum for the AM, AJ, AA, A tranche
Lower attachment point	$CE^L$	19.92%, 12.50%, 10.45%, 7.71% for the AM, AJ, AA, A tranche
Higher attachment point	$CE^H$	29.76%, 19.92%, 12.50%, 10.45% for the AM, AJ, AA, A tranche
Number of loans per CMBS	$I$	30
Original weight loan	$w_k^i$	3.33%, we assume 30 loans per CMBS, all of equal size
Initial collateral value	$1/LTV_{0,k}^i$	Calibrated to the inverse LTV of the top 30 loans in CMBX, reported in the November 2008 Citi CMBX report.



Table 3: Robustness of model fit

The table shows the average pricing error, standard deviation of the pricing error, and the correlation between the model and market price for various choices for the dividend rate  $q$  and the idiosyncratic volatility  $\sigma_j$ . The pricing error is defined as the actual price minus the model price in dollars.

	Average error			Std. error			Correl. with actual		
	AJ	AA	A	AJ	AA	A	AJ	AA	A
Baseline	16.73	8.22	6.34	9.76	8.43	7.93	86.99	95.45	95.81
-2% dividend rate	12.19	4.19	1.85	11.39	8.26	8.04	85.60	95.71	95.81
+2% dividend rate	21.42	13.29	11.71	8.38	8.23	7.88	88.48	95.04	95.72
Idiosyncratic volatility at 75%	15.20	6.35	4.16	10.33	8.44	8.02	86.73	95.59	95.82
Idiosyncratic volatility at 125%	18.73	10.73	9.50	9.08	8.32	7.75	87.33	95.27	95.86

Table 4: Predictive regressions

The table reports Newey-West corrected t-stats (25 lags); each for a different regression. The dependent variables considered are the returns on the CMBX 1 AJ, AA, and A tranches, as well as the REIT index return, and the change in the implied volatility of the REIT index. The independent variable for which the t-stat is reported is the model-implied mispricing for one of the three tranches considered, lagged by one day. In Panel A we do not include control variables. In Panel B, for the regressions with CMBX as dependent variable, we include as controls the contemporaneous REIT return and REIT implied volatility change. For the regressions with REIT return (option implied volatility) as dependent variables we include as controls the contemporaneous CMBX return (of the tranche for which we use the mispricing) as well as the contemporaneous change in REIT implied volatility (REIT return). t-statistics on these control variables are not reported.

Panel A: No controls					
Dependent variable	AJ	AA	A	REIT stock	REIT option
AJ mispricing	-2.33	-2.77	-2.63	-1.91	0.58
AA mispricing	-2.25	-2.87	-2.64	-2.40	1.17
A mispricing	-2.66	-3.27	-3.04	-2.56	1.43

Panel B: Contemporaneous return other two assets as control					
Dependent variable	AJ	AA	A	REIT stock	REIT option
AJ mispricing	-2.33	-2.76	-2.60	-0.58	-0.75
AA mispricing	-2.01	-2.67	-2.43	-0.35	-0.33
A mispricing	-2.36	-3.06	-2.83	-0.45	-0.35

Table 5: Net Sharpe ratios of trading strategies: Robustness analysis

The table shows the net Sharpe ratio of the baseline-case model and several alternative specifications. The sample period for the back test is June 30, 2008 to April 29, 2009. For the baseline case model we lag the trade signal by one day, the hedge ratio is determined using a beta based on five-day overlapping returns, the trade signal and beta are determined using an expanding window, the trade trigger is a z-score of 0.50, the maximum trade size is the amount corresponding to a z-score of 0.2, and the round-trip transaction cost are \$1 per \$100 notional CMBX.

	AJ	AA	A	Combo
Baseline	1.43	2.37	2.51	2.25
Lag = 0	1.40	2.27	2.33	2.14
Lag = 2	1.39	2.42	2.60	2.28
Hedge ratio based on non-overlapping daily returns	1.88	2.64	2.99	2.68
Trade trigger at $z = 0.25$ instead of $z = 0.50$	1.37	2.05	2.26	2.03
Trade trigger at $z = 1.00$ instead of $z = 0.50$	1.27	2.23	2.76	2.22
Trade size cap at $z = 0.1$ instead of $z = 0.20$	1.42	2.34	2.63	2.26
Trade size cap at $z = 0.4$ instead of $z = 0.20$	1.54	2.31	2.43	2.24
Position size cap at $z = +/ - 2.0$	1.53	2.32	2.44	2.27
Double transaction cost	1.31	2.20	2.29	2.07
-2% dividend rate	1.33	2.28	2.58	2.16
+2% dividend rate	1.62	2.21	2.05	2.16
Idiosyncratic volatility at 75%	1.45	2.27	2.60	2.21
Idiosyncratic volatility at 125%	1.53	2.28	2.20	2.19

Table 6: Alpha of trading strategies

The table reports the t-statistic of the constant term (alpha) and the  $R^2$  for regressions of trading strategy returns on a constant and eight risk factor returns: the return on the Datastream US bank stock index, the three Fama-French factors and the Carhart momentum factor, 3-month US T-bill return, the return on the Merrill Lynch Treasury bond index for maturities beyond ten years, and A/BBB Merrill Lynch corporate bond index returns for the industrial and bank sectors. Results are presented using either daily or weekly overlapping returns (correcting for autocorrelation using Newey-West), and for two sample periods. Four trading strategies are analyzed, based on the AJ, AA, and A tranches of the CMBX 1 series, and an equally weighted portfolio of these three strategies, denoted Combo.

Daily returns						
	July 2008 - April 2010			January 2007 - April 2010		
	t-stat alpha	$R^2$	Risk factors	t-stat alpha	$R^2$	Risk factors
AJ	1.39	12.4%	Yes	-	-	-
AA	2.29	7.0%	Yes	1.84	6.4%	Yes
A	2.38	3.6%	Yes	2.03	3.3%	Yes
Combo	2.14	7.4%	Yes	-	-	-
Weekly returns						
	July 2008 - April 2010			January 2007 - April 2010		
	t-stat alpha	$R^2$	Risk factors	t-stat alpha	$R^2$	Risk factors
AJ	1.18	10.5%	Yes	-	-	-
AA	2.61	12.1%	Yes	2.35	8.6%	Yes
A	2.32	8.3%	Yes	2.16	5.1%	Yes
Combo	2.17	10.1%	Yes	-	-	-

Table 7: Regression of CMBX mispricing on bank stock returns

The change in CMBX mispricing is regressed on returns on the Datastream US bank stock index (in excess of the S&P 500 return) and the S&P 500 return. This is done using either daily or weekly overlapping returns, and using both contemporaneous and effects and lags (nine lags in case of daily returns and one lag in case of weekly returns). The table reports the t-statistics of the sum of coefficients for the contemporaneous and lagged effects, correcting for autocorrelation using Newey-West. Sample period is May 31, 2006 through April 29, 2010, except for the AJ tranche where data start January 4, 2008.

	Daily returns		Weekly overlapping returns	
	Bank excess return	S&P-500 return	Bank excess return	S&P-500 return
	t-statistic	t-statistic	t-statistic	t-statistic
AJ	2.61	-0.18	3.12	-0.50
AA	1.94	0.09	2.50	0.11
A	2.29	-0.02	3.13	0.04

Table 8: Main news events

This Table reports the news announcements for which both (i) Bank of America issued a special report, and (ii) Bloomberg reported the event. The first column presents the title of the report. The second column presents the date and time (EST) Bloomberg reported on the announcement.

<b>Title report</b>	<b>Date, Time Bloomberg</b>
Riverton Apartments - Big CMBX.3 Loan - In Trouble	20080815, 01:50:29
Peter Cooper & Stuyvesant Two Loans Cause Downgrade	20080922, 17:26:31
Peter Cooper & Stuyvesant Two Loans Cause More Downgrades	20081016, 09:46:04
GGP CMBS Loans: Balloon Loan Pays Down, ARD Loan Extends	20081020, 03:22:42
Fitch Follows and lowers ratings on PCV & Stuy Town	20081029, 16:36:55
Circuit City to close 155 stores	20081103, 16:59:15
JPMCC 2008-C2 (CMBX.5): 2 Big Loans 30-days Delinquent	20081118, 13:45:05
MLMT 2008-C1: DBSI Loans To Special Servicing	20081121, 11:18:24
Maguire Says GG10 Hope Street Loan is Current	20090115, 06:44:46
BACM 2006-3: Big Interest Shortfall on Boscov's Loans	20090108, 07:58:08
Goody's Files for Bankruptcy Protection Again	20090114, 08:33:29
Peter Cooper / Stuy Town Update	20090122, 13:53:59
Treasury launches program for Distressed Legacy Assets	20090323, 08:01:52
S&P follows suit - says recent vintage downgrade risk is high	20090406, 15:06:01
TALF: Clarification by the Treasury Caps Max Leverage	20090406, 13:05:47
Fitch's turn and S&P follow-up	20090408, 11:53:56
CMBX 5: Appraisal reductions lead to interest shorfalls	20090414, 17:15:25
GGP files for bankruptcy includes many CMBS loans	20090416, 01:58:14
TALF for new issue CMBS	20090501, 14:01:53
Legacy CMBS TALF Guidelines Better than expected	20090519, 14:00:07
S&P proposal puts CMBS Legacy TALF in jeopardy	20090526, 14:42:14
S&P finalizes methodology change	20090626, 12:23:16
CMBS TALF Update - July 16th first legacy subscription	20090702, 16:12:41
Peter Cooper / Stuy Town deals on watch by Moody's (again)	20090714, 16:40:10
NY Fed announces accepted and rejected TALF collateral	20090722, 18:46:29
TALF Extended	20090817, 09:02:12
TALF Round 2: 83 Accepted, 3 Rejected	20090826, 18:39:29
New guidelines on modification rules for REMICs	20090915, 11:44:53
Legacy TALF round 3: All accepted	20090923, 17:22:16
Legacy TALF round 4: 5 rejected, 81 accepted	20091027, 22:07:45
More on GGP's Restructuring Specific loan extension info	20091202, 07:29:37

Table 9: Response to news

The table shows the t-statistic (Newey West, ten lags) for the coefficients of two multiple regressions. The dependent variable in both cases is the CMBX 1, AA tranche return at time  $t$ . For the first specification the independent variables are a constant and the sign of the CMBX return at a lag of one to six days, with and without interaction with a dummy taking the value of one if at time  $t - lag$  there was a news announcement. For the second specification the independent variables are a constant and the return on the AA tranche at a lag of one to six days, with and without interaction with a dummy taking the value of one if at time  $t - lag$  there was a news announcement. Only the t-stats on the dependent variables with interaction with the news dummy are reported.

Variable	Using $sign(R_{t-lag})D_{t-lag}$	Using $R_{t-lag}D_{t-lag}$
	T-stat	T-stat
$\alpha$	-0.01	-0.66
$\beta_1$	0.67	-0.33
$\beta_2$	1.41	1.42
$\beta_3$	-2.16	-1.31
$\beta_4$	-2.92	-1.97
$\beta_5$	-1.72	-1.35
$\beta_6$	-0.43	-1.66
$R^2(\%)$	6.31	9.15

Table 10: Maximum expected risk-neutral default rates

This table reports the maximum of the time series of expected risk-neutral default rates. These default rates are implied by CMBX prices as described in Section 7.2. The table reports the maximum of these rates over the time series from Jan 4, 2008 to April 29, 2010, for various choices for the timing of defaults ( $\tau$ ), the recovery rate, and the maximum loss.

		Recovery rate			
		60%	60%	40%	40%
Maximum loss	Default time ( $\tau$ )	1	3	1	3
19.9%		40%	41.5%	26.6%	27.6%
30.0%		65.09%	58.6%	37.4%	39%
40.0%		72.87%	76.4%	48.6%	50.9%



Table 11: CMBX 1 constituent: JPMCC 2005-LDP4

This table contains details on the JPMCC 2005-LDP4 CMBS deal, obtained from the JPMC February 2010 CMBX Monthly update, the JPMC February 2010 CMBX Reference Obligation Performance Appendix and the official prospectus supplement from JPMC, dated September 22, 2005.

Class	Balance (\$m)					Current Rating			Subord. (%)		Cum. Int.
	Orig.	Curr.	Factor	WAL	Coupon	Moody's	S&P	Fitch	Orig.	Curr.	Shortfall (\$)
A-1	84.71	0.00	0.00	0.00	4.61	WR	na	PIF	30.00	31.70	0
A-1A	396.35	388.03	0.98	4.04	4.89	Aaa	na	AAA	30.00	31.70	0
A-2	227.32	195.77	0.86	0.33	4.79	Aaa	na	AAA	30.00	31.70	0
A-2FL	200.00	172.24	0.86	0.33	0.36	Aaa	na	AAA	30.00	31.70	0
A-3A1	179.93	179.93	1.00	1.17	4.87	Aaa	na	AAA	30.00	31.70	0
A-3A2	75.00	75.00	1.00	2.37	4.90	Aaa	na	AAA	30.00	31.70	0
A-4	580.27	580.27	1.00	5.23	4.92	Aaa	na	AAA	30.00	31.70	0
A-SB	130.38	130.38	1.00	2.53	4.82	Aaa	na	AAA	30.00	31.70	0
A-M	267.71	267.71	1.00	5.43	5.00	Aaa	na	AAA	20.00	21.08	0
A-J	204.13	204.13	1.00	5.50	5.04	Aaa	na	BBB	12.38	12.98	0
B	50.20	50.20	1.00	5.50	5.13	Aa2	na	BB	10.50	10.99	0
C	23.42	23.42	1.00	5.50	5.05	Aa3	na	BB	9.63	10.06	0
D	46.85	46.85	1.00	5.50	5.12	A2	na	B	7.88	8.21	0
E	23.42	23.42	1.00	5.50	5.15	Baa1	na	B-	7.00	7.28	0
F	40.16	40.16	1.00	5.56	5.15	Ba1	na	B-	5.50	5.68	0
G	26.77	26.77	1.00	5.58	5.15	Ba2	na	B-	4.50	4.62	0
H	30.12	30.12	1.00	5.58	5.15	B1	na	B-	3.38	3.43	0
J	10.04	10.04	1.00	5.58	4.67	B2	na	CCC	3.00	3.03	0
K	13.39	13.39	1.00	5.58	4.67	B3	na	CCC	2.50	2.50	0
L	13.39	13.39	1.00	6.72	4.67	Caa1	na	CCC	2.00	1.97	0
M	6.69	6.69	1.00	8.50	4.67	Caa1	na	CCC	1.75	1.70	0
N	3.35	3.35	1.00	8.50	4.67	Caa3	na	CCC	1.63	1.57	0
NR	33.46	29.46	0.88	99.00	4.67	NR	na	na	—	—	637,966
P	10.04	10.04	1.00	8.50	4.67	Caa3	na	CCC	1.25	1.17	57,244

Fig. 1: Actual and model price CMBX 1, AA tranche

The figure shows the actual prices for the CMBX 1, AA tranche from May 31, 2006 to April 29, 2010. It also shows prices based on our CMBX pricing model, using REIT equity market prices. The CMBX 1 series was introduced March 7th, 2006. We lack the data for the first two-and-a-half months.

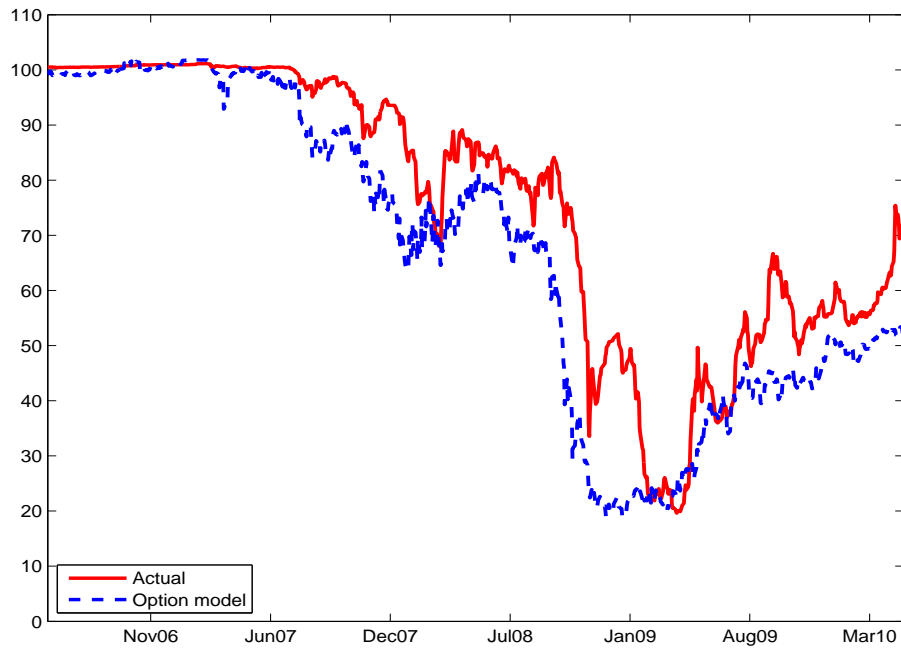


Fig. 2: Basic structure for the cash flow allocation to different CMBS tranches

The figure illustrates the basic structure for the cash flow allocation to different CMBS tranches.

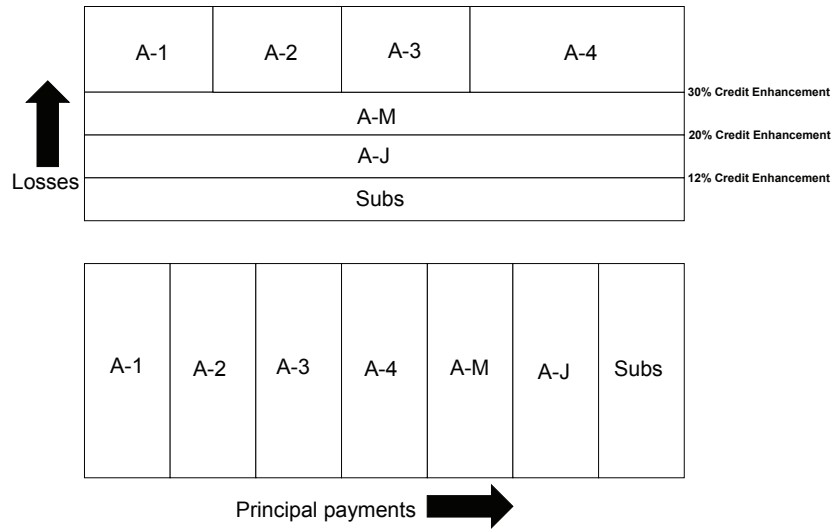


Fig. 3: Calibrated property values, betas, volatilities, and correlations

The different figures show daily calibrated values for the latent sector-level property values relative to debt,  $V_{jt}/D_j$ , betas of property values with respect to the S&P 500 index  $\beta_j$ , volatilities of property values at the sector level  $\gamma_j$ , and correlations across property sectors  $\rho_{jk}$ ,  $j, k = 1, \dots, 3$ .

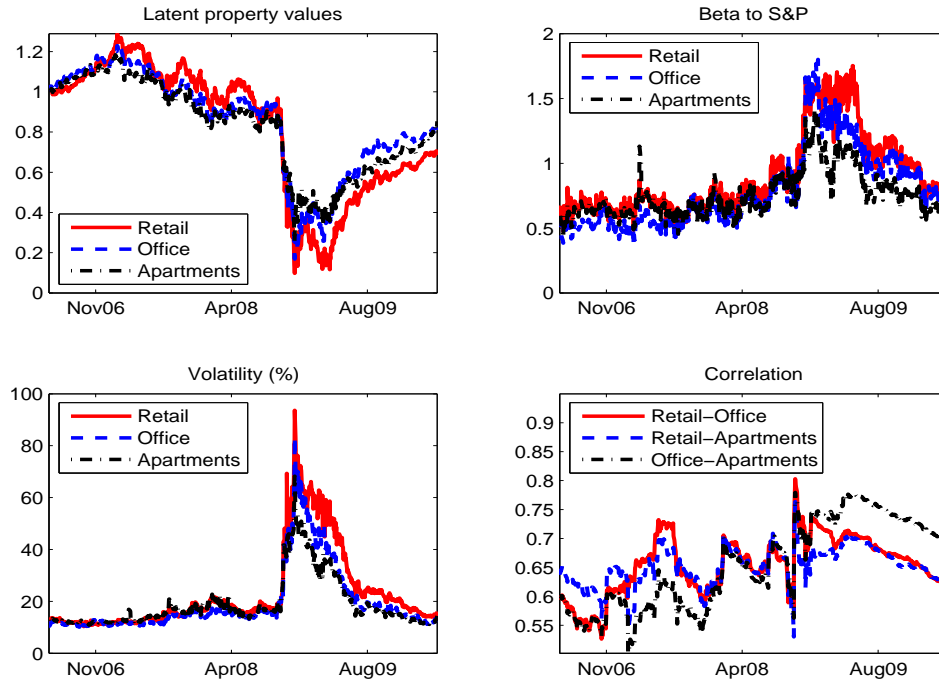


Fig. 4: Actual and model prices for the CMBX 1 series for different tranches

The figure shows the actual and option model prices for the CMBX 1, AM, AJ, AA, A tranches from May 31, 2006 to April 29, 2010. The CMBX 1 series was introduced March 7th, 2006. For the AJ tranche prices are available as of January 4, 2008, for the AM tranche as of February 9, 2010.

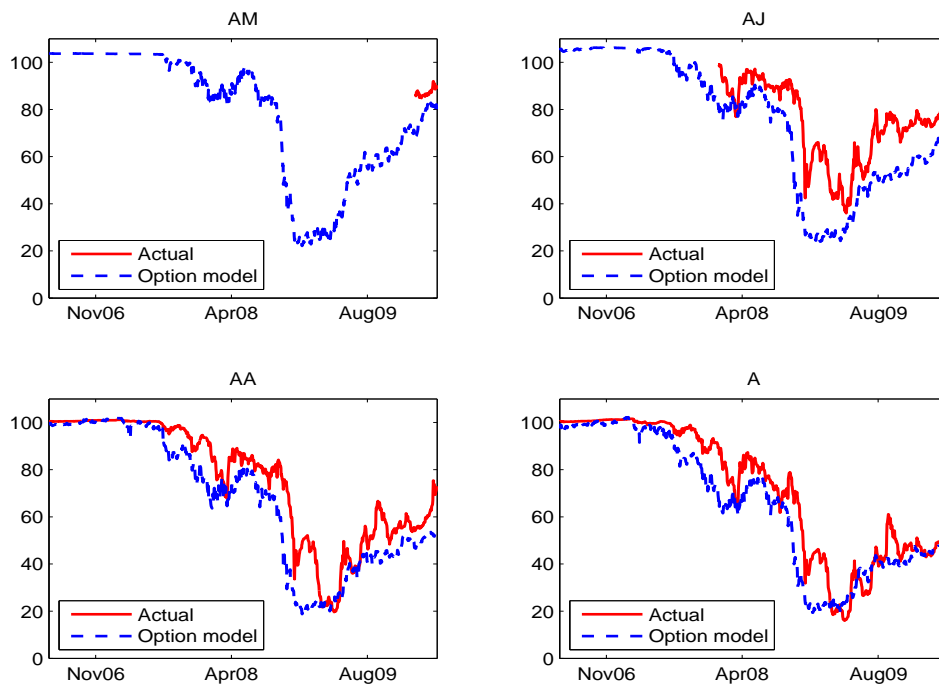


Fig. 5: Correlation between market and model CMBX series 1 prices: one-year and three-month rolling windows

The figure shows the correlation between market CMBX series 1 prices and CMBX model prices calculated using our options-based pricing model, for the AJ, AA and A tranches. The correlations are calculated using a rolling window of one year (upper panel) and three months (lower panel).

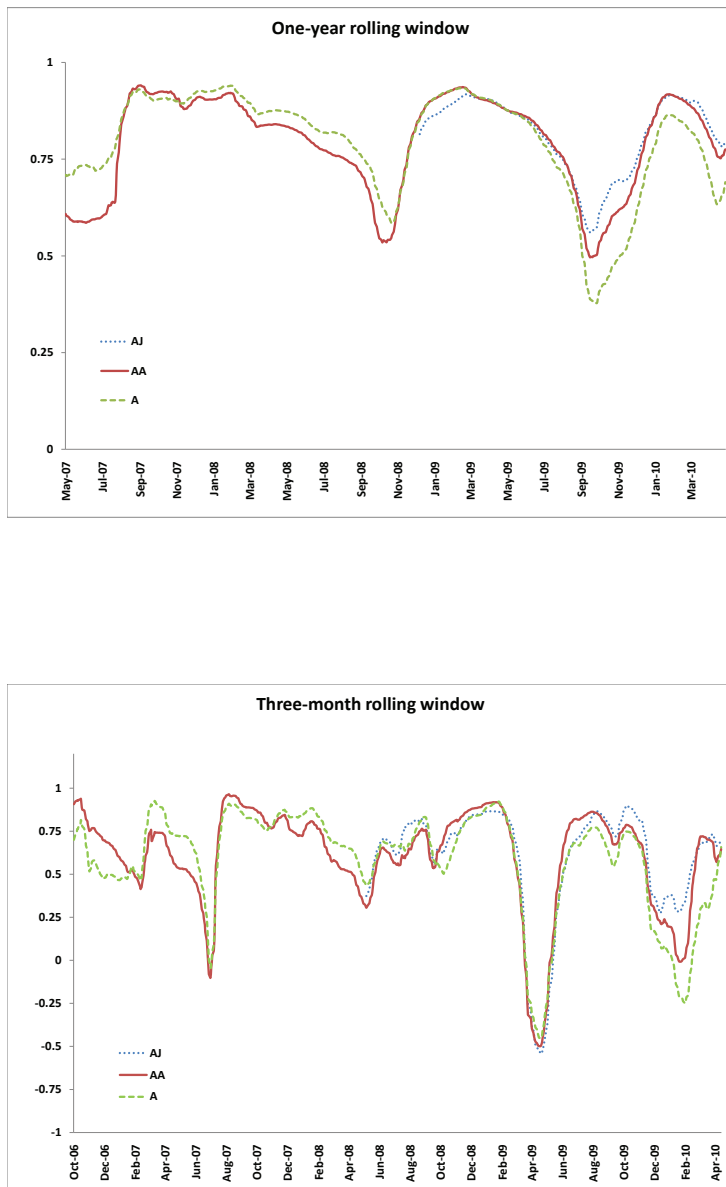


Fig. 6: Actual and model price CMBX 2-5, AJ tranche

The figure shows the actual and option model price for the AJ tranche of series 2 through 5.

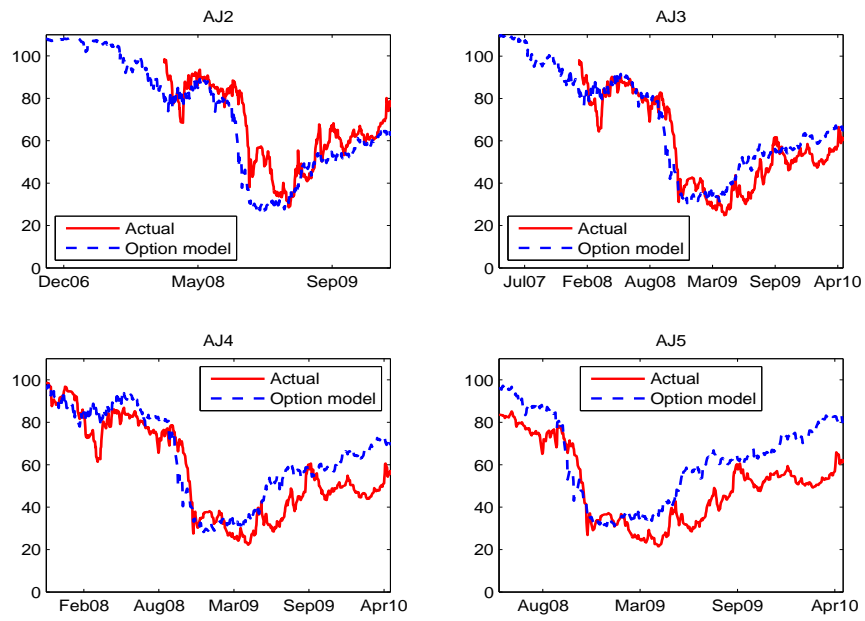


Fig. 7: Cumulative response CMBX 1, AA tranche to a news announcement at time zero

The Figure shows the average and median cumulative response of the CMBX 1, AA tranche to a news announcement at time zero. The cumulative response is the sum of the signed returns, where the signed return is computed as the CMBX 1, AA return after the announcement, interacted with the sign of the change in the mispricing on the announcement day.

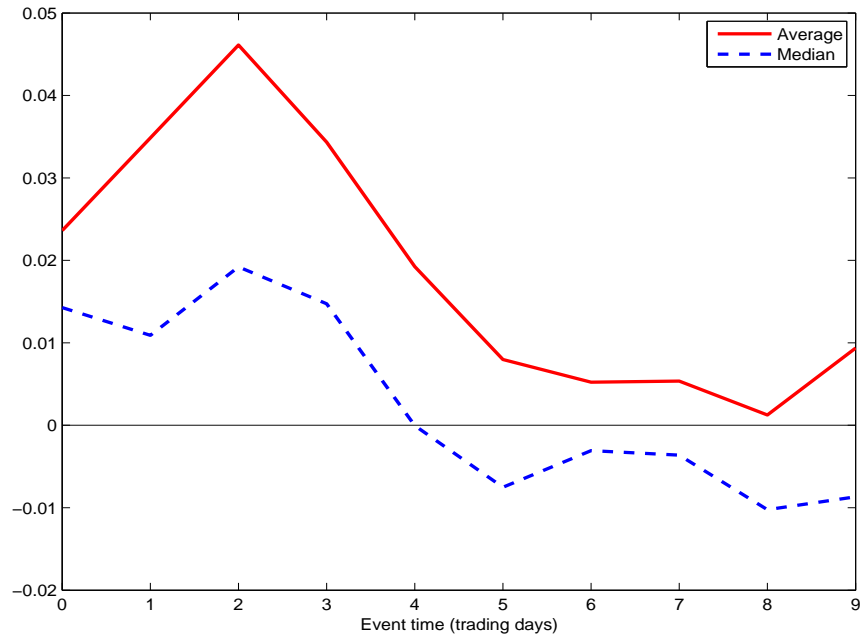




Fig. 8: Price-to-NAV ratio for REITs

The figure shows the average price-to-Net Asset Value ratio for a large cross-section of US REITs as reported by Bank of America, JP Morgan, and Green Street Advisors.

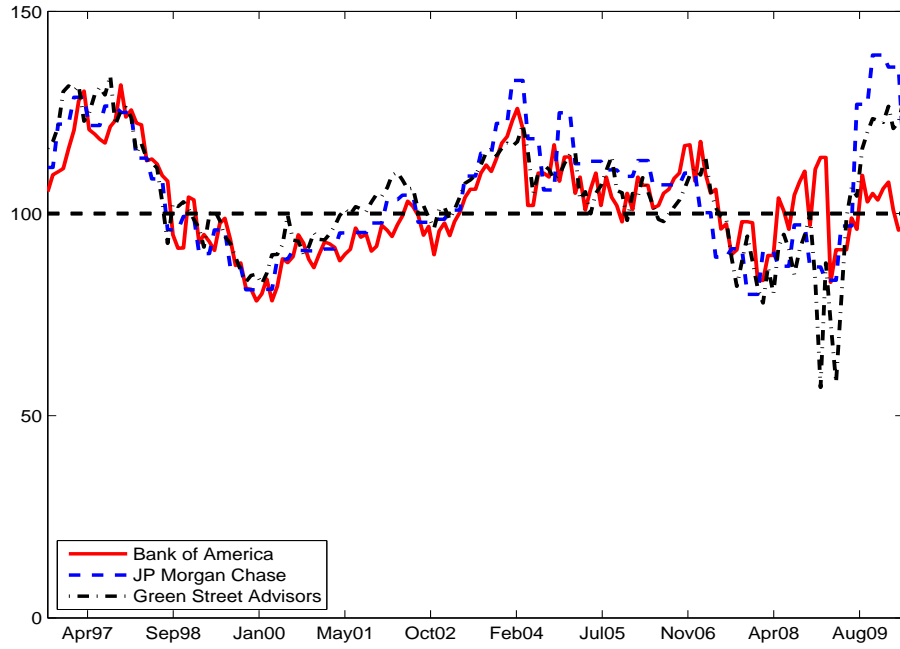


Fig. 9: Actual and NAV-adjusted model price CMBX 1, AA tranche

The figure shows the actual prices for the CMBX 1, AA tranche from May 31, 2006 to April 29, 2010. It also shows prices based on our CMBX pricing model, using REIT equity market prices with an adjustment for the price discount/premium to NAV reported by Green Street Advisors. The difference between these model prices and the unadjusted benchmark model prices is also shown. The CMBX 1 series was introduced March 7th, 2006. We lack the data for the first two-and-a-half months.

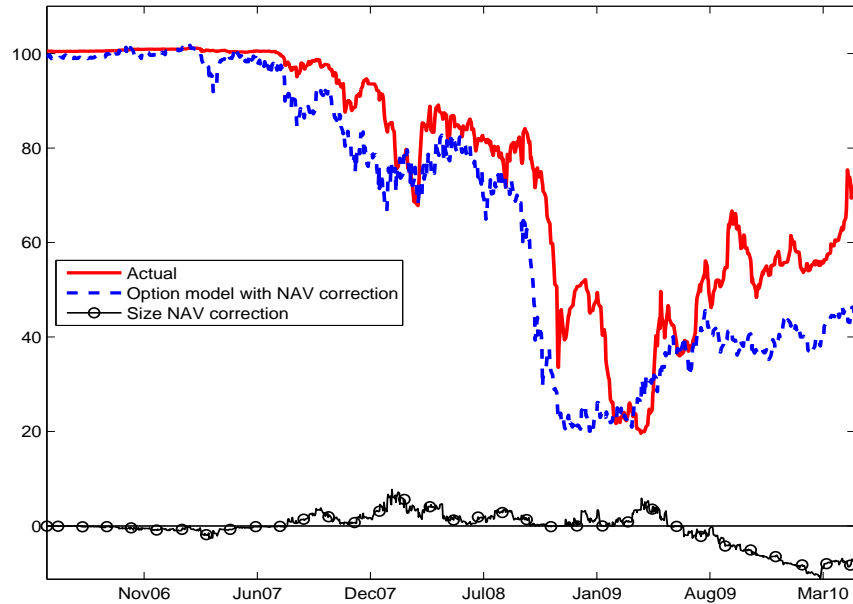


Fig. 10: Implied risk-neutral default rate for deals referenced by CMBX 1 (%)

The figure shows the implied risk-neutral default rate for deals reference by CMBX 1 (%). The sample period runs from January 4th, 2008 (when the CMBX 1 AJ tranche was introduced) until April 29th, 2010.

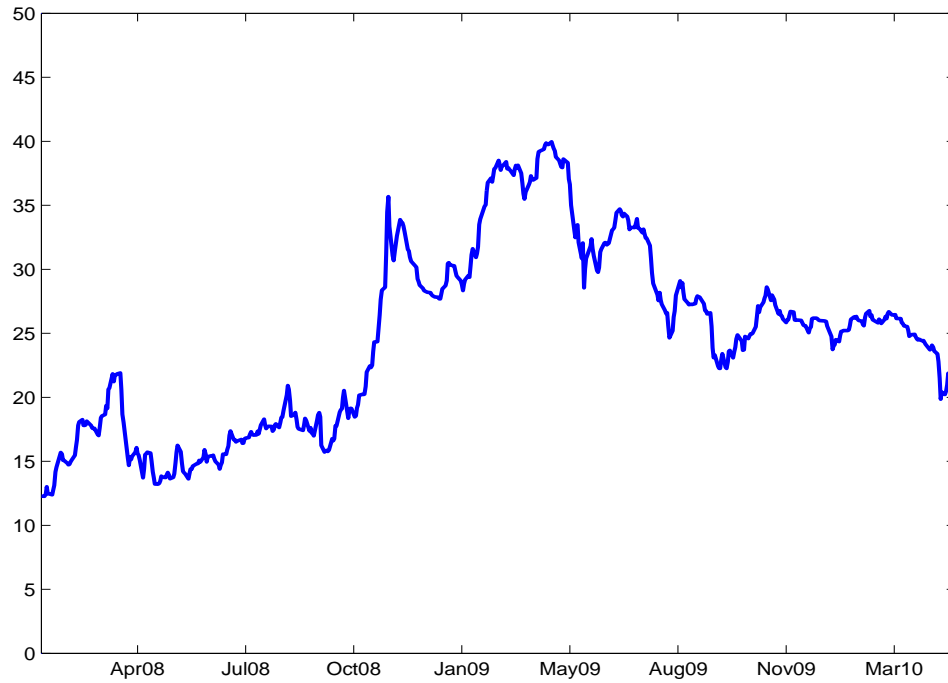


Fig. 11: CMBX tranche losses based on historical default rates: 60% versus 40% recovery

The figures show percentage losses on CMBX AJ, AA and A tranches, given historical default rates of cohort-years from 1972 to 1997 (Esaki and Goldman (2005)), assuming 60% and 40% recovery, respectively.

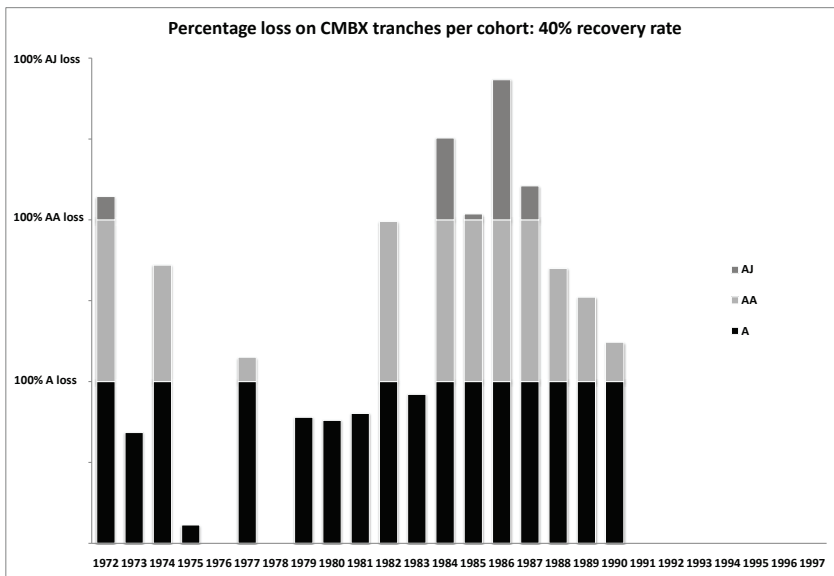
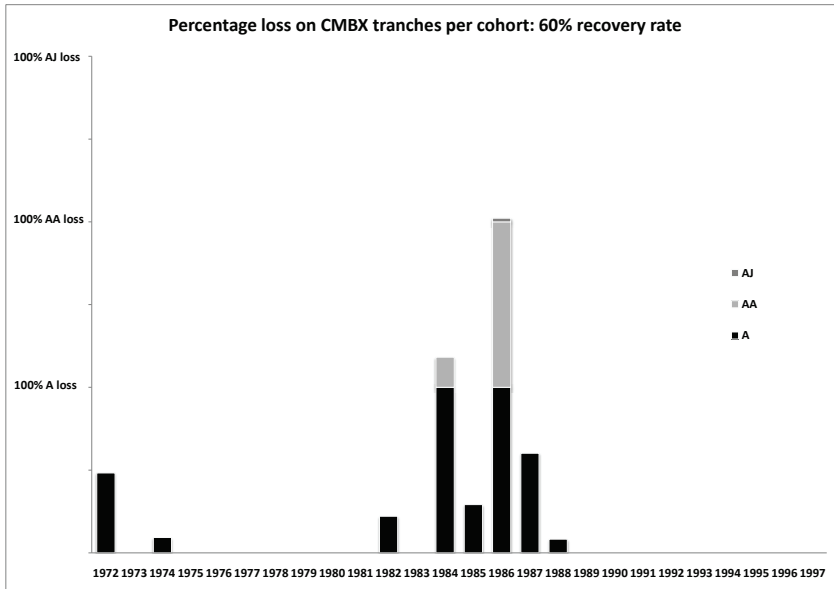


Fig. 12: CMBX series 1 pricing errors: May 2006 to July 2011

The figure shows the difference between market CMBX series 1 prices and model CMBX prices, calculated using the options-based CMBX pricing model, for the sample period May 31, 2006 to July 15, 2011. The pricing model uses OptionMetrics option price data until October 2010, and option prices from Bloomberg for the rest of the sample. These pricing errors are shown for four tranches: AM, AJ, AA and A.

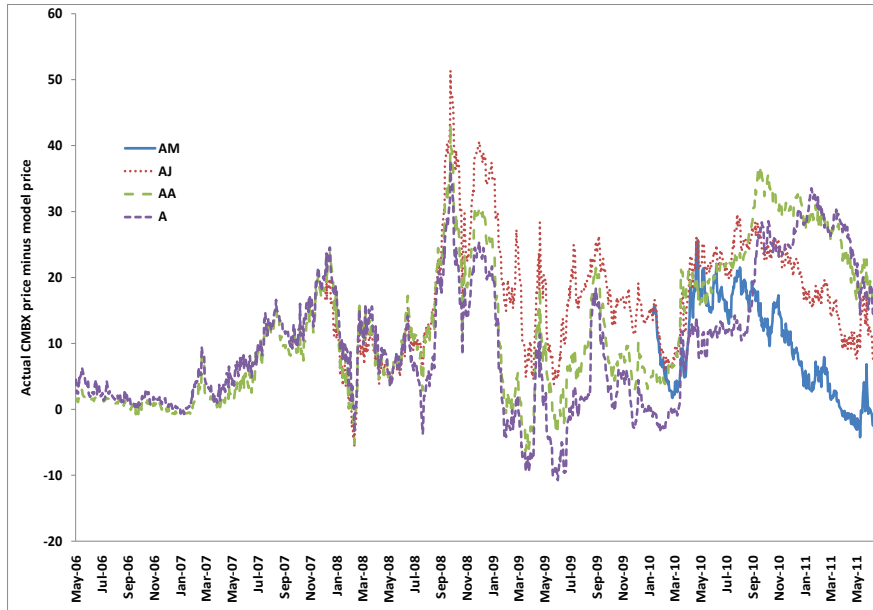


Fig. 13: Daily calibrated parameter values: jump-diffusion model

The figure shows daily calibrated parameter values for three parameters in the jump-diffusion model: the diffusion volatility, the annual jump intensity (divided by ten), and the jump size. Parameters are calibrated to prices of S&P 500 index options.

